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A Two-sector Model of Endogenous Growth
with Leisure Externalities

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Abstract

This paper considers the impact of leisure preference and leisure externalities on growth and labor supply in a Lucas [12] type model, as in Gómez [7], with a separable non-homothetic utility and the assumption that physical and human capital are both necessary inputs in both the goods and the education sectors. In spite of the non-concavities due to the leisure externality, the balanced growth path is always unique, which guarantees global stability for comparative-static exercises. We find that small differences in preferences toward leisure or in leisure externalities can generate substantial differences in hours worked and growth, which may play a significant role in explaining differences in growth paths between the US and Europe, in addition to the mechanisms uncovered in Prescott [16] relying on differing marginal tax rates on labor income. Our model indicates, however, that a higher preference for leisure or leisure externality implies less growth but also less education attainment, which seems counterfactual.

Keywords: leisure externalities, two-sector model, labor supply, economic growth.

JEL Classification: O41, E24.

1. Introduction

Leisure and leisure externalities are clearly part of the cultures of various communities, such as the US, Europe, Latin America or Asia. The possible influence of preference for leisure and leisure externalities has been already recognized in the literature and studied in particular by Gómez [7]. Gómez [7] adapted the Lucas [12] two sectors endogenous growth model to include both leisure and leisure externalities in preferences. However, Gómez assumed a homothetic utility function, which implied that leisure externalities have no impact on growth along a balanced growth path (BGP). The present paper employs instead a non-homothetic utility, separable in consumption and leisure, as assumed in Benhabib and Perli [3] and Ladrón-de-Guevara et al. [10, 11], who already introduced preferences for leisure in a Lucas [12] type model with human capital. Such non-homotheticity permits leisure externalities to influence growth along a BGP. However, the above works all assumed that physical capital was not an input in the education technology, with the awkward consequence that there may exist multiple BGPs and indeterminacy.

The present work introduces in Gómez [7] model the assumption that physical capital and human capital are both necessary inputs in the two sectors. This feature was present in earlier works by Bond et al. [4], Mino [14], Mulligan and Sala-i-Martin [15], and Stokey and Rebelo [18], and implied the uniqueness of BGP. The main theoretical contribution of the present work is to show that this feature preserves the uniqueness of BGP when one introduces preference toward leisure as in Benhabib and Perli [3] and Ladrón-de-Guevara et al. [10, 11], and even when one introduces leisure externalities as in Gómez [7], in spite of the resulting non-concavities of the utility function (Proposition 1). The second theoretical contribution is that this framework allows us to evaluate the impact on working hours of different structural parameters such as total factor productivity and the intensity of preference for leisure and leisure externalities (Proposition 2).

We analyze and quantify the effects of household differences in the size of leisure and leisure externalities in order to shed light on working hours and economic growth in America and Europe. For Europe, we view these differences in part as a positive leisure externality and the implied *keeping up with the Joneses effect*, and partly as a slightly larger weight on the preference for leisure in order to highlight the culture of leisure in European societies. For the U.S., we impose a negative leisure externality and the implied *running away from the Joneses effect*, and also a slightly smaller weight on the preference for leisure in order to characterize the workaholic American labor market. We find that small differences in the degree of leisure externalities and the weight of preference for individual leisure can account for a large fraction of differences in hours worked between Americans and Europeans. Thus, different preferences for leisure and/or leisure externalities might be a part of the reasons why the US and Europe display different growth paths and numbers of hours worked. We should make clear that this is not the core contribution of this work. Moreover, our model has a counterfactual implication concerning differing education attainments in the US and Europe. The

education attainment data indicate narrowing differences between the US and European countries, especially Germany, in the 2000s which goes against our model's prediction.

A roadmap for this paper is as follows. A model of two-sector endogenous growth with leisure and leisure externalities in the household utility function is studied in Section 2. In Section 3 we characterize the BGP and examine comparative-static exercises. We offer quantitative assessment regarding the effects of leisure and leisure externalities on the labor supply and economic growth. Finally, concluding remarks are offered in Section 4.

2. The Model

This section builds the basic analytical framework. This framework draws on the Lucas [12] two-sector model, extended to include leisure by Benhabib and Perli [3], and Ladrón-de-Guevara, et al. [10, 11], and augmented to a general technology by Bond et al. [4] and Mino [14].

The representative agent is endowed with L units of time; l units are allocated to leisure and the remaining $L-l$ units to working. She obtains utility from consumption and leisure. In addition, her utility is affected by the average leisure level in society, \bar{l} . An agent's lifetime utility is as follows.

$$U = \int_0^{\infty} u(c, l, \bar{l}) e^{-\rho t} dt,$$

where

$$u(c, l, \bar{l}) = \ln c + \psi \frac{(l\bar{l}^\gamma)^{1-\sigma}-1}{1-\sigma}, \quad \gamma(1-\sigma) < \sigma, \sigma > 0, \sigma \neq 1, \quad (\text{P})$$

where c is consumption and $\rho > 0$ is the time preference rate.

Following Benhabib and Perli [3] and Ladrón-de-Guevara [10, Section 4.2], we use a form of felicity that is separable in consumption and leisure with a unit *intertemporal elasticity of substitution* (hereafter, IES) for consumption that is different from the IES of leisure.¹ In this type of utility, the marginal rate of substitution (MRS) between consumption and leisure is not homothetic along a BGP. Leisure externalities may then influence competitive equilibrium in the long run. The parameter ψ is the intensity of leisure preferences relative to consumption with $\psi > 0$ because leisure is in general a normal good. The larger ψ , the higher the utility is from an additional unit of leisure. Also $\sigma > 0$ is the

¹ As in Ladrón-de-Guevara [11, p. 613], only two forms of felicity are consistent with a BGP in our two-sector endogenous growth model. The general form of our separable felicity is $u(c, l, \bar{l}) = \ln c + f(l, \bar{l})$ as noted in Ladrón-de-Guevara [10, Section 4.2]. As a constant IES is necessary in order to be consistent with a BGP, we use the parametric form of the felicity from leisure assumed in Benhabib and Perli [3]. An alternative felicity is the non-separable form of $\frac{[c^\theta f(l, \bar{l})]^{1-\sigma}-1}{1-\sigma}$, with the special case of $\frac{[c^\theta (l\bar{l}^\gamma)^\psi]^{1-\sigma}-1}{1-\sigma}$ used in Lucas [12] that imposes the same IES for consumption and leisure. In that formulation, the marginal rate of substitution is homothetic in consumption and leisure, and hence leisure externalities do not affect the allocation in the long run.

reciprocal of the IES of leisure. The parameter γ denotes the degree of leisure externalities. To assure that felicity is concave in pure leisure time, we impose $(1+\gamma)(1-\sigma) < 1$ and thus, $\gamma(1-\sigma) < \sigma$. Therefore, it is required that the degree of leisure externalities be not too large. We also require $\sigma \neq 1$. When $\sigma = 1$, our utility function is reduced to the Cobb-Douglas utility function which is homothetic in consumption, leisure and leisure externalities. In this case, the MRS between consumption and leisure is independent of leisure externalities. Without the intra-temporal substitution effect between consumption and leisure externalities, leisure externalities do not influence long-run growth.

An agent's utility is positively influenced by the average leisure level in an economy if $\gamma > 0$, and negatively affected if $\gamma < 0$. Following existing studies on the consumption externality (Dupor and Liu [5]), we may call the individual leisure *admiring* if $u_3 > 0$, or equivalently $\gamma > 0$, and leisure *jealous* if $\gamma < 0$. Alesina et al. [1] referred to the case $u_3 > 0$ as a *social multiplier*. An agent's marginal utility of leisure may be affected by the leisure externality. The leisure externality can be described as *keeping up with the Joneses* if $u_{23} > 0$, or equivalently $\gamma(1-\sigma) > 0$ (e.g., Gali [6]) or as *running away from the Joneses* if $\gamma(1-\sigma) < 0$ (e.g., Dupor and Liu [5]).

The economy is composed of two production sectors: the goods sector x and the education sector y . The two sectors have general technologies which use both physical capital and human capital as inputs, as in Bond, et al. [4], Mino [14], Mulligan and Sala-i-Martin [14] and Stokey and Rebelo [18]. For simplicity, the Cobb-Douglas form is employed for each one:

$$x = A(sk)^\alpha [(L-l)uh]^{1-\alpha}$$

$$y = B[(1-s)k]^\beta [(L-l)(1-u)h]^{1-\beta},$$

in which k is physical capital and h is human capital with given initial values $k(0)$ and $h(0)$. The variable s denotes the share of physical capital and u is the share of human capital allocated to the goods sector. Both technologies are assumed to exhibit constant returns to scale in order to be consistent with perpetual growth. The parameter $\alpha \in (0,1)$ is the income share of physical capital in the goods sector, and $\beta \in (0,1)$ is the income share of physical capital in the educational sector; A and B are the technology coefficients, or factor productivities, in the goods and the educational sector, respectively.

The Lucas [12] model with leisure in utility was studied by Benhabib and Perli [3] and Ladrón-de-Guevara et al. [10, 11] neither of whom allowed the education technology to use physical capital as an input, with the implication that there may exist multiple BGPs and indeterminacy. We employ a model in which both the goods sector and education technology need physical capital as an input. This feature was introduced already by Bond et al. [4], Mino [14], Mulligan and Sala-i-Martin [15], and Stokey and Rebelo [18] in models of this type with neither a utility for leisure nor any leisure externality. These works showed that the choice of two inputs in the Lucas two-sector model is

important in two aspects. First, this choice guarantees a unique BGP, which is useful when conducting comparative-static exercises. Second, technical progress in the goods sector is beneficial to economic growth instead of neutral as asserted in the earlier literature. Our contribution below is to show that this choice leads to the same conclusions in the present model, even after having introduced leisure in the utility function, or even with leisure externalities, in spite of the resulting non-concavity in preferences.

While the goods output is used either for consumption or for the formation of physical capital, the education output can only serve the accumulation of human capital. For simplicity, we assume there is no depreciation for physical and human capital. Their laws of motion are as follows.

$$\dot{k} = A(sk)^\alpha [(L-l)uh]^{1-\alpha} - c, \quad k(0) \text{ given}, \quad (1a)$$

$$\dot{h} = B[(1-s)k]^\beta [(L-l)(1-u)h]^{1-\beta}, \quad h(0) \text{ given}. \quad (1b)$$

Eq. (P), (1a) and (1b) are the basic framework in this model. Our model reduces to Benhabib and Perli [3] and Ladrón-de-Guevara, et al. [11] if $\beta = \gamma = 0$ and to Bond, et al. [4] and Mino [14] if $\psi = \gamma = 0$. Moreover, our model technology reduces to Gómez [7] if $\beta = 0$ and $\sigma = 1$.

2.1 Optimization

The representative agent's optimization problem is to maximize the lifetime utility (P) by choosing between consumption, leisure, and investment in the goods and the education sectors, all of which are subject to the constraints in (1a)-(1b) and the given initial stocks of physical and human capital, $k(0)$ and $h(0)$. Let μ and λ be the co-state variable associated with k and h , respectively. Thus, μ is the shadow price of capital in terms of consumption, while λ is the shadow price of human capital in terms of consumption. The necessary conditions are

$$\frac{1}{c} = \mu, \quad (2a)$$

$$\psi l^{-\sigma} \bar{l}^{\gamma(1-\sigma)} = (1-\alpha)\mu \frac{x}{L-l} + (1-\beta)\lambda \frac{y}{L-l}, \quad (2b)$$

$$\mu\alpha \frac{x}{s} = \lambda\beta \frac{y}{1-s}, \quad (2c)$$

$$\mu(1-\alpha) \frac{x}{u} = \lambda(1-\beta) \frac{y}{1-u}, \quad (2d)$$

$$\dot{\mu} = \mu\rho - \mu\alpha \frac{x}{k} - \lambda\beta \frac{y}{k}, \quad (2e)$$

$$\dot{\lambda} = \lambda\rho - \mu(1-\alpha) \frac{x}{h} - \lambda(1-\beta) \frac{y}{h}, \quad (2f)$$

along with the transversality conditions,

$$\lim_{t \rightarrow \infty} e^{-\rho t} \mu(t) k(t) = 0, \quad (2g)$$

$$\lim_{t \rightarrow \infty} e^{-\rho t} \lambda(t) h(t) = 0. \quad (2h)$$

While (2a) equates the marginal utility of consumption to the shadow price of capital, (2b) equates the marginal utility of leisure to the marginal cost of leisure when labor is reduced by one unit, that is, the marginal utilities derived from forgone goods and from forgone educational output. It is worth noting that because of $\sigma \neq 1$, leisure externalities affect the labor-leisure tradeoff in (2b) and thus the MRS between consumption and leisure ((2a) divided by (2b)). Thus, leisure externalities can have an effect on the allocation. Equations (2c) and (2d) allocate factors optimally between the two sectors: (2c) equates the marginal products of capital, and (2d) equates the marginal products of human capital. Finally, (2e) and (2f) are Euler equations, and (2g) and (2h) are the two usual transversality or no ‘‘Ponzi scheme’’ conditions on physical and human capital.

Dividing (2c) by (2d) yields s as an increasing function of u :

$$s := s(u) = \alpha(1 - \beta)u / [\beta(1 - \alpha) + u(\alpha - \beta)]. \quad (3)$$

Intuitively, physical and human capitals are complements in production. As human capital inputs increase in the goods sector, physical capital must increase in that sector, too.

2.2 Equilibrium

The equilibrium, with $l(t) = \bar{l}(t)$, defines time paths of $\{k(t), b(t), c(t), l(t), s(t), u(t), \lambda(t), \mu(t)\}$ which satisfy equations (1a), (1b) and (2a)-(2f). We will simplify the equilibrium conditions by transforming them into a three-dimensional dynamical system with state vector $(m, q, p) \equiv (c/k, b/k, \lambda/\mu)$. We briefly sketch the transformation.

First, (2a) implies $\frac{\dot{c}}{c} = -\frac{\dot{\mu}}{\mu}$, while (2c) and (2d) imply, respectively,

$$y = \frac{\alpha}{\beta} \frac{\mu}{\lambda} \frac{1-s}{s} x, \quad (4a)$$

$$x = \frac{1-\beta}{1-\alpha} \frac{\lambda}{\mu} \frac{u}{(1-u)} y. \quad (4b)$$

If we substitute these three relationships into (2e), (1b), (2f) and (1a), we obtain:

$$\frac{\dot{c}}{c} = -\frac{\dot{\mu}}{\mu} = \alpha A \left[\frac{sk}{(L-l)uh} \right]^{\alpha-1} - \rho, \quad (5a)$$

$$\frac{\dot{h}}{h} = \frac{\alpha}{\beta} A \frac{\mu}{\lambda} (1-s) \left(\frac{u}{s} \right)^{1-\alpha} \left(\frac{k}{h} \right)^{\alpha} (L-l)^{1-\alpha}. \quad (5b)$$

$$\frac{\dot{\lambda}}{\lambda} = \rho - (1-\alpha) A \frac{\mu}{\lambda} \left(\frac{sk}{uh} \right)^{\alpha} (L-l)^{1-\alpha}, \quad (5c)$$

$$\frac{\dot{k}}{k} = As^\alpha \left(\frac{k}{h}\right)^{\alpha-1} [(L-l)u]^{1-\alpha} - \frac{c}{k}. \quad (5d)$$

Following Bond, et al [4], then (4a)-(5d) yield:

$$\frac{\dot{m}}{m} = -\rho + m + As^\alpha u^{1-\alpha} q^{1-\alpha} (L-l)^{1-\alpha} \left(\frac{\alpha}{s} - 1\right), \quad (6a)$$

$$\frac{\dot{q}}{q} = As^\alpha u^{1-\alpha} q^{-\alpha} (L-l)^{1-\alpha} \left(\frac{\alpha}{\beta} \frac{1}{p} \frac{1-s}{s} - q\right) + m, \quad (6b)$$

$$\frac{\dot{p}}{p} = As^\alpha u^{-\alpha} q^{-\alpha} (L-l)^{1-\alpha} [\alpha q \frac{u}{s} - (1-\alpha) \frac{1}{p}]. \quad (6c)$$

Thus, (6a)-(6c) is a three-dimensional dynamical system if l , u and s are all functions of m , q and p . To derive these functions, use (2c) and (3) to obtain

$$L-l = \left[\left(\frac{B}{A}\right)^{\frac{1}{\alpha-\beta}} \left(\frac{\alpha}{\beta}\right)^{\frac{-\beta}{\alpha-\beta}} \left(\frac{1-\beta}{1-\alpha}\right)^{\frac{(1-\beta)}{\alpha-\beta}} \left(\frac{s}{u}\right)^{-1} p^{\frac{1}{\alpha-\beta}} q \right]^{-1},$$

This, together (2b) and (2d), leads to

$$l = \left[\frac{1-\alpha}{\psi} \left(\frac{\alpha}{\beta}\right)^{\frac{-\alpha\beta}{\alpha-\beta}} \left(\frac{1-\alpha}{1-\beta}\right)^{\frac{-\alpha(1-\beta)}{\alpha-\beta}} A^{\frac{-\beta}{\alpha-\beta}} B^{\frac{-\alpha}{\alpha-\beta}} \frac{1}{m} qp^{\frac{\alpha}{\alpha-\beta}} \right]^{\frac{-1}{\sigma-\gamma(1-\sigma)}} \equiv l(m, p, q). \quad (7a)$$

Moreover, (2d) and (3) and (7a) yield

$$u = \frac{\alpha(1-\beta)}{\alpha-\beta} \left(\frac{1-\alpha}{1-\beta}\right)^{\frac{1-\beta}{\alpha-\beta}} \left(\frac{\alpha}{\beta}\right)^{\frac{\beta}{\alpha-\beta}} \left(\frac{B}{A}\right)^{\frac{-1}{\alpha-\beta}} [L-l(m, q, p)]^{-1} q^{-1} p^{\frac{-1}{\alpha-\beta}} + \frac{\beta(1-\alpha)}{\beta-\alpha} \equiv u(m, p, q). \quad (7b)$$

Finally, s depends on u alone by eq. (3); eq. (7) then says that $s=s(m, q, p)$.

2.3 The balanced growth path

On any *Balanced Growth Path* (BGP) we have $\dot{m}=\dot{q}=\dot{p}=0$ and thus the state variables m , q and p are constant. Along that path the fractions l , u and s are also constant, while c , k and h grow at the same rate, as do μ and λ .

To determine the BGP, first, we use (6b), along with (7a), (6c) and (3), to obtain

$$p = p(u) = \left[\frac{\alpha}{\rho(1-\beta)} \right]^{\frac{(\alpha-\beta)}{1-\alpha}} \left(\frac{\alpha}{\beta}\right)^{\beta} \left(\frac{1-\alpha}{1-\beta}\right)^{1-\beta} A^{\frac{1-\beta}{1-\alpha}} B^{-1} (u - \beta)^{\frac{(\alpha-\beta)}{1-\alpha}}. \quad (8a)$$

Next we substitute (8a) and (3) into (6c) and obtain:

$$q = q(u) = (1-\alpha) \left(\frac{1-\beta}{A}\right)^{\frac{1-\beta}{1-\alpha}} \left(\frac{\alpha}{\rho}\right)^{\frac{\beta-\alpha}{1-\alpha}} \left(\frac{\beta}{\alpha}\right)^{\beta} \left(\frac{1-\beta}{1-\alpha}\right)^{1-\beta} \frac{B}{\beta(1-\alpha)+u(\alpha-\beta)} \left(\frac{1}{u-\beta}\right)^{\frac{\alpha-\beta}{1-\alpha}}. \quad (8b)$$

Finally, if we substitute (8b) and (3) into (6a), we obtain

$$m = m(u) = \frac{\rho[(1-\alpha)+u(\alpha-\beta)]}{\beta(1-\alpha)+u(\alpha-\beta)}. \quad (8c)$$

It is obvious that once we determine a unique value for the share u of human capital in the goods sector along a BGP, we can obtain a unique state vector (m, q, p) from (8a), (8b) and (8c), respectively. Note that consumption, physical capital or human capital should be nonnegative, and so should the shadow prices of physical and human capital. Thus, m , q and p must be nonnegative in a BGP.

To determine u , we substitute (8a)-(8c) into (7b) and obtain a single equation (A1a) in Appendix 1, in which u is the only unknown. In particular, if we denote

$$M \equiv \left\{ L - \left[\left(\frac{1}{1-\alpha} \right)^{1+\beta} \rho^{\frac{1-\alpha+\beta}{1-\alpha}} \left(\frac{1}{1-\beta} \right)^{1-\beta} \left(\frac{\alpha}{\beta} \right)^\beta (A\alpha)^{\frac{-\beta}{1-\alpha}} \frac{u}{B} \right]^{\frac{1}{\sigma-\gamma(1-\sigma)}} \right\},$$

Then we obtain the following result.²

Proposition 1. *Assume that*

(i) $(\alpha A)^{\frac{\beta}{1-\alpha}} B(1-\alpha) \left(\frac{1-\beta}{1-\alpha} \right)^{1-\beta} \left(\frac{\beta}{\alpha} \right)^\beta \rho^{\frac{-(1-\alpha+\beta)}{1-\alpha}} M > 1,$

(ii) $a > \beta$ and $\beta / (1-\beta) > (a-\beta),$

(iii) *the unique solution u to equation (A1a) satisfies $u \geq \beta$.*

Then there exists a unique BGP.

In proposition 1, while condition (iii) guarantees $p \geq 0$ in a BGP, and condition (ii) assures the existence of a BGP. Finally, condition (i) ensures uniqueness. As there exists a unique u^* in a BGP, we can use that value of u^* to solve for the unique values of m^* , q^* and p^* in a BGP.

Notice that our BGP is unique even though the felicity is non-concave due to leisure and leisure externalities. This result is in a sharp contrast to that obtained by Benhabib and Perli [3] and Ladrón-de-Guevara et al. [10, 11]. These authors found multiple BGPs in a Lucas-type [12] model with consumption and pure leisure time. In spite of using a utility function of a similar type, our model possesses a unique BGP. Bond et al. [4] and Mino [14] employed a two-sector human capital-based model with only consumption in utility and found a unique BGP. By extending their model to include leisure and leisure externalities, in spite of the resulting non-concave utility function, the result of a unique BGP remains unchanged in our model.

The reason for this difference in results is that earlier studies assumed that human capital is the only input in the education sector. This corresponds to $\beta = 0$ in our model. In that situation, the education technology is linear in the level of human capital. Indeed, if $\beta = 0$ in our model, the LHS(u)

² See the proof in Appendix 1.

and $RHS(u)$ in the appendix equation (A1a) are so nonlinear that $LHS(u)$ and $RHS(u)$ are no longer monotone in u . As a result, there are possibly multiple BGPs in our model. However, when $\beta > 0$, the education technology is strictly concave in the level of human capital. The strict concavity of the education technology in the level of human capital offsets the non-concavity of the utility function and delivers a unique BGP. Uniqueness is important for our analysis as it assures global stability in the comparative-static analysis that follows.

3 Effects on Aggregate Labor Supply

In this section we characterize the effects on aggregate labor supply along a BGP.³ While we are more interested in the effects of changes in the degree of leisure externalities (γ) and the intensity of leisure preference relative to consumption (ψ), we will also analyze the impact of technical change in both sectors (A, B). We start with the comparative-static exercises, followed by calibration exercises.

3.1 Comparative-static analysis

Eq. (8a)-(8c) allow us to calculate the effects of changes in structural parameters on aggregate labor supply as follows (See Appendix 2 for derivation).

$$\begin{aligned}
 \frac{dl}{dA} &= \frac{\partial l}{\partial A} + \frac{\partial l}{\partial u} \frac{\partial u}{\partial A} \stackrel{0}{=} \quad \text{if } \beta \geq 0, \\
 &\quad (-) \quad (+) \quad (-) \\
 \frac{dl}{dB} &= \frac{\partial l}{\partial B} + \frac{\partial l}{\partial u} \frac{\partial u}{\partial B} < 0, \\
 &\quad (-) \quad (-) \quad (-) \\
 \frac{dl}{d\psi} &= \frac{\partial l}{\partial \psi} + \frac{\partial l}{\partial u} \frac{\partial u}{\partial \psi} > 0, \\
 &\quad (+) \quad (-) \quad (+) \\
 \frac{dl}{d\gamma} &= \frac{\partial l}{\partial \gamma} + \frac{\partial l}{\partial u} \frac{\partial u}{\partial \gamma} \stackrel{\geq}{\leq} 0 \quad \text{if } \sigma \stackrel{\leq}{\geq} 1. \tag{9} \\
 &\quad (+) \quad (-) \quad (+) \leq \\
 &\quad (-) \quad (-) \quad (-)
 \end{aligned}$$

For the effects of higher productivity, we note that a higher productivity in the goods sector (A) has no effect on labor supply if the education sector does not require physical capital ($\beta = 0$). Now, as the education sector requires physical capital ($\beta > 0$), the labor supply and the fraction of the human capital allocated to the education sector are increased. This is because higher productivity in the goods sector represents a human capital-saving improvement and thus labor is relocated to the education sector. A higher intensity of leisure preference relative to consumption (ψ) increases the marginal utility of leisure; leisure goes up and the labor supply goes down. Since leisure and consumption are

³ To ensure there is only one equilibrium path toward the unique BGP when doing comparative-static analysis, we need the BGP to be a saddle. In earlier versions we discussed the conditions that guarantee the saddle property.

complements, it is necessary to allocate a larger fraction of the labor supply to the goods sector in order to produce a sufficient amount of consumption goods.

For changes in the intensity of the leisure externality, γ could be positive or negative. When $\gamma > 0$, the agent admires leisure enjoyed by others, and the average level of leisure is a complement to each agent's leisure. In this case there is a social multiplier, as in Alesina et al. [1]. In contrast, when $\gamma < 0$, the agent feels jealous about other's leisure and the average level of leisure in society becomes a substitute for individual leisure. It follows that an increase in γ will either increase our admiration toward other people's leisure plans or reduce the jealousy we feel about them. Depending on whether the initial value of γ is positive or negative, the impact of changes in γ can be stated in Table 1.

[Insert Table 1 about here]

The effect of a larger intensity of leisure externalities ($d\gamma > 0$) depends on how the agent's marginal utility of leisure, $l^{-\sigma} \bar{l}^{\gamma(1-\sigma)}$, is affected by the leisure externality. In particular, the sign of $\gamma(1-\sigma)$ controls whether the agent will increase or decrease her leisure level. Thus, the crucial determinant of the effect of a larger intensity of leisure externalities is whether $\gamma(1-\sigma) > 0$ which signals a *keeping up with the Joneses effect*, or $\gamma(1-\sigma) < 0$ which signals a *running away from the Joneses effect*.

Suppose $\gamma(1-\sigma) > 0$. Then, $(1-\sigma)d\gamma > 0$ means a stronger "keeping up with the Joneses" effect which leads to higher leisure. Thus, for a given level of leisure, the marginal utility of leisure increases in the leisure externality which leads the agent to choose a higher level of leisure. As labor supply drops, it is necessary to allocate a larger fraction of labor to the goods sector in order to produce a sufficient amount of goods. Alternatively, suppose $\gamma(1-\sigma) < 0$. Then, $(1-\sigma)d\gamma < 0$ means a higher "running away from the Joneses" effect, which leads to lower leisure by reversing the earlier causal chain. As the labor supply goes up, the agent will allocate a smaller fraction of labor to the goods sector in order to produce a sufficient amount of goods. These results are summarized as follows.

Proposition 2. *In the long run, the labor supply is:*

- (i) *an increasing function of total factor productivity in both the goods and education sectors;*
- (ii) *a decreasing function of the intensity of leisure in preferences;*
- (iii) *a decreasing function of the strength of the "keeping up with the Joneses" effect and an increasing function of the strength of the "running away from the Joneses" effect.*

3.2 Calibration

This subsection quantifies the effects on the labor supply of changes in the two parameters in relation

to preferences toward leisure: changes in the intensity of leisure relative to consumption and changes in the degree of leisure externalities. We calibrate the model in a BGP to reproduce key features representative of the U.S. economy in annual frequencies.

The time endowment is assumed to be $L=100$ units. The leisure time is chosen at $l^*=75$ in consistence with the fraction of time allocated to market at around 25 percent, as pointed out by Prescott [17]. Human capital is as large as physical capital according to Kendrick [9]; accordingly, we choose the ratio of human capital to physical capital at $q^*=h/k=1$. Imai and Keane [8] found that the IES for labor is 3.82 in the US. Prescott [17] pointed out this labor supply elasticity is equivalent to an IES for leisure of 1.2 if the fraction of productive time allocated to market is 0.25. Following this line of wisdom, we set $\sigma = 0.83$. Moreover, we choose the physical capital share in the goods sector at $\alpha = 0.36$. Under Condition (ii) in proposition 1, the physical capital share in the educational sector is less than that in the goods sector; thus, we choose $\beta = 0.3$. The rate of time preference (ρ) is set at 4%. We use $\gamma = 0$ in our benchmark parameterization.

The remaining parameters of technology and preference are chosen so that our model is consistent with various facts characterizing the US macroeconomy. The fraction of labor allocated to the goods sector (u) is chosen to match the $\phi = 2\%$ long-run economic growth rate in per capita income, and we obtain $u^*=0.7667$. From Eq. (6c) and (3) we also compute the ratio of the shadow price of human capital to the shadow price of physical capital at $p^*=1.8824$. Given this, we use (3) and (8c) to compute the fraction of capital allocated to the goods sector and the ratio of consumption to capital at $s^*=0.8118$ and $m^*=0.1153$, respectively. Finally, using (7a), (7b) and (8a), we obtain $A=0.02203$, $B=0.0096$, and $\psi=1.41045$. Under benchmark parameter values, the BGP is unique.⁴ The equilibrium value are $L-l^*=25$, $u^*=0.7667$, $s^*=0.8118$, $(L-l^*)u^*=19.1675$, $(L-l^*)(1-u^*)=5.8325$, and $\phi = 2\%$.

Now, we examine the effects of differences in preferences by deviating the degree of leisure externalities from the benchmark value $\gamma=0$ by 5% and the intensity of the leisure parameter from its benchmark value $\psi=1.41045$ by 5%. The quantitative results are in Table 2.

The results in Table 2 may contrast the labor supply in two similar economies, Europe and the US, which seem to differ in their preference for vacations and leisure. First, suppose that Europe has a culture of leisure with a “keeping up with the Joneses” effect at $\gamma = 0.05$ and Americans are workaholics with a “running away from the Joneses” effect at $\gamma = -0.05$. Then, our quantitative results suggest that Americans supply labor hours that are about $\Delta = 13.80\%$ higher than Europeans (c.f. percentage difference between Columns 2 and 3, Table 2). Second, suppose that the culture of leisure

⁴ We have shown that the unique BGP is a saddle point under the benchmark parameter values. When the parameter values are changed below, we have also shown that, for these chosen values, the unique BGP satisfies the saddle-path stability.

in Europe is captured by a higher intensity of leisure at $\psi = 1.48$, whereas the workaholism in the US is represented by a lower intensity of leisure at $\psi = 1.34$. Then, the labor supply in the US is about $\Delta = 19.09\%$ higher than that in Europe (c.f. the second row in Columns 4 and 5, Table 2).

[Insert Table 2 about here]

Finally, if we make somewhat extreme assumptions that the culture of leisure in Europe is partly captured by keeping up with the Joneses effect at $\gamma = 0.05$ and partly by a higher intensity of leisure at $\psi = 1.48$, whereas the workaholism in the US is partly signified by a running away from the Joneses effect at $\gamma = -0.05$ and a lower intensity of leisure at $\psi = 1.34$. Then, the labor supply in the US is about $\Delta = 35.53\%$ higher than that in Europe (c.f. the second row in Columns 6 and 7, Table 2). Thus, a small difference in preferences toward leisure seems to one of the reasons for differences in hours worked per person between the US and Europe after 1993-1996.⁵

4. Concluding Remarks

This paper studies the impact of leisure preference and leisure externalities on growth and the labor supply in a Lucas [12] type model, as in Gómez [7], with a separable non-homothetic utility and under the assumption that physical and human capital are both necessary inputs in both sectors. A separable non-homothetic utility in consumption and leisure permits leisure externalities to impact the allocation along the BGP. In spite of the non-concavities due to the leisure externality, the BGP is always unique, which guarantees global stability for comparative-static exercises. We find that small differences in preferences toward leisure or leisure externalities can generate substantial differences in working hours and growth, which may play a significant role in explaining differences in growth paths between the US and Europe, in addition to the mechanisms uncovered in Prescott [16] relying on differing marginal tax rates on labor income. Our model indicates however that a higher preference for leisure or leisure externality also implies less education attainment, which seems counterfactual.

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⁵ Our model predicts the observation that during the 1990s a growing discrepancy in years of schooling and educational attainment would accompany the observed increasing difference in work hours. Recent data show this prediction to be controversial because differences in years of schooling years have shrunk, in particular between Germany and the U.S. According to Barro and Lee [2], years of schooling in Germany, Italy, France and US were, respectively, 8.0, 7.6, 7.5 and 12.1 in 1990, 10, 8.6, 9.6 and 12.7 in 2000, and 11.8, 9.6, 10.5 and 12.2 in 2010. We thank a referee for pointing this limitation of our paper.

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Appendix 1: Proof of Proposition 1

Proof: First, Condition (iii) in proposition 1 assures $p \geq 0$ along any BGP.

Next, to determine u , if we substitute in (8a)-(8c), we may rewrite (7b) as

$$LHS(u) = RHS(u), \quad (A1a)$$

where $LHS(u) \equiv \frac{1}{L-l(u)}$,

$$RHS(u) \equiv (\alpha A)^{\frac{\beta}{1-\alpha}} (1-\alpha)^{\frac{1-\beta}{1-\alpha}} \left(\frac{\beta}{\alpha}\right)^{\beta} [\rho(1-\beta)]^{\frac{1-\alpha+\beta}{1-\alpha}} B(u-\beta)^{\frac{1-\alpha+\beta}{1-\alpha}}.$$

The function $l(u) = l(m(u), p(u), q(u))$ is defined in (7a), and with the use of (8a)-(8c), can be rewritten as

$$l(u) = \{(1-\alpha)^{-2} \rho^{\frac{1-\alpha+\beta}{1-\alpha}} \left(\frac{1}{1-\beta}\right)^{\frac{1-\alpha+\beta}{1-\alpha}} \left(\frac{\alpha}{\beta}\right)^{\beta} \left(\frac{1-\alpha}{1-\beta}\right)^{1-\beta} [A\alpha(u-\beta)]^{\frac{-\beta}{1-\alpha}} \frac{\psi}{B} [1-\alpha+u(\alpha-\beta)]\}^{\frac{1}{\sigma-\gamma(1-\sigma)}}. \quad (A1b)$$

It is obvious that $RHS(u)$ is increasing in u even if $\gamma = 0$.

Moreover, $LHS(u)$ is decreasing in u if l is decreasing in u . To see this, we use (A1b) to obtain

$$\frac{\partial l}{\partial u} = -\frac{(1-\alpha)[\beta-(\alpha-\beta)(u-\beta)]+\beta u(\alpha-\beta)}{(1-\alpha)(u-\beta)} \frac{1}{(1-\alpha)+u(\alpha-\beta)} \frac{l(u)}{[\sigma-\gamma(1-\sigma)]}, \quad (A1c)$$

where the concavity of the utility function in l requires $[\sigma-\gamma(1-\sigma)] > 0$. Eq. (A1c) is negative if condition (ii) in proposition 1 is met. Then, under condition (ii) in proposition 1, $LHS(u)$ is decreasing in u . Moreover, at $u = 0$,

$$l(0) = \{(1-\alpha)^{-1} \rho^{\frac{1-\alpha+\beta}{1-\alpha}} \left(\frac{1}{1-\beta}\right)^{\frac{1-\alpha+\beta}{1-\alpha}} \left(\frac{\alpha}{\beta}\right)^{\beta} \left(\frac{1-\alpha}{1-\beta}\right)^{1-\beta} [A\alpha(-\beta)]^{\frac{-\beta}{1-\alpha}} \frac{\psi}{B}\}^{\frac{1}{\sigma-\gamma(1-\sigma)}} < 0$$

and thus $LHS(0) > 0 > RHS(0)$. Therefore, a negatively sloping $LHS(u)$ and a positively sloping $RHS(u)$ must intersect.

Finally, the intersection is unique if $LHS(1) < RHS(1)$. This is true under condition (i) in proposition 1. ■

Appendix 2: Comparative-static effects on aggregate labor supply/leisure in eq. (9)

Total differentiation to (A1a)-(A1b) with respect to A , B , ψ and γ yields

$$\frac{du}{dA} = \frac{-\beta}{A} \frac{l+(L-l)[\sigma-\gamma(1-\sigma)]}{(L-l)[\sigma-\gamma(1-\sigma)]} \frac{(u-\beta)}{T(u)}, \quad (A2a)$$

$$\frac{du}{dB} = -\frac{l+(L-l)[\sigma-\gamma(1-\sigma)]}{B(L-l)[\sigma-\gamma(1-\sigma)]} \frac{(1-\alpha)(u-\beta)}{T(u)}, \quad (A2b)$$

$$\frac{du}{d\psi} = \frac{l}{\psi} \frac{1}{(L-l)[\sigma-\gamma(1-\sigma)]} \frac{(+\alpha \quad \partial \beta)}{T(u)} \quad (A2c)$$

$$\frac{du}{d\gamma} = \frac{l}{L-l} \frac{(1-\sigma) \ln l}{[\sigma-\gamma(1-\sigma)]} \frac{(1-\alpha)(u-\beta)}{T(u)}, \quad (A2d)$$

where $T(u) = 1-\alpha + \beta + \frac{\{(1-\alpha)[\beta-(u-\beta)(\alpha-\beta)]+\beta[u(\alpha-\beta)]\}}{(L-l)[1-\alpha+u(\alpha-\beta)]} \frac{l(u)}{\sigma-\gamma(1-\sigma)} > 0$ under condition (ii) in proposition 1.

Thus, we obtain

$$\begin{aligned}
\frac{du}{dA} &\leq 0 \quad \text{if } \beta \geq 0, \\
\frac{du}{dB} &< 0, \\
\frac{du}{d\psi} &> 0, \\
\frac{du}{d\gamma} &\begin{cases} \geq 0 \\ \leq 0 \end{cases} \quad \text{if } \begin{cases} \sigma \leq 1 \\ \sigma \geq 1 \end{cases}.
\end{aligned} \tag{A2e}$$

Moreover, equation (A1b) suggests that $\frac{\partial l}{\partial A} = -\frac{\beta}{1-\alpha} \frac{l(u)}{A[\sigma-\gamma(1-\sigma)]} \leq 0$ if $\beta \geq 0$, $\frac{\partial l}{\partial B} = -\frac{l(u)}{B[\sigma-\gamma(1-\sigma)]} < 0$, $\frac{\partial l}{\partial \psi} = \frac{l(u)}{\psi[\sigma-\gamma(1-\sigma)]} > 0$ and $\frac{\partial l}{\partial \gamma} = \frac{(1-\sigma)l(u)\ln l(u)}{\sigma-\gamma(1-\sigma)} \leq (\geq) 0$ if $\sigma \geq (\leq) 1$. Using these relationships and those in (A2e), we obtain the following results.

$$\begin{aligned}
\frac{\partial l}{\partial A} &= \frac{\partial l}{\partial A} + \frac{\partial l}{\partial u} \frac{\partial u}{\partial A} \\
&\quad (-) \quad (-) \quad (-) \\
&= \frac{-\beta l}{(1-\alpha)A[\sigma-\gamma(1-\sigma)]} \left\{ 1 + \frac{(\alpha-\beta)(1-\alpha)(u-\beta)-\beta[1-\alpha+u(\alpha-\beta)]}{[1-\alpha+u(\alpha-\beta)]} \frac{l+(L-l)[\sigma-\gamma(1-\sigma)]}{(L-l)[\sigma-\gamma(1-\sigma)]T(u)} \right\} \\
&= \frac{-\beta l}{(1-\alpha)A[\sigma-\gamma(1-\sigma)]} \frac{(1-\alpha)[1-\alpha+u(\alpha-\beta)]+(\alpha-\beta)(1-\alpha)(u-\beta)}{[1-\alpha+u(\alpha-\beta)]T(u)} < 0.
\end{aligned} \tag{A2f}$$

$$\begin{aligned}
\frac{\partial l}{\partial B} &= \frac{\partial l}{\partial B} + \frac{\partial l}{\partial u} \frac{\partial u}{\partial B} \\
&\quad (-) \quad (-) \quad (-) \\
&= \frac{-\beta l}{B[\sigma-\gamma(1-\sigma)]} \left\{ 1 + \frac{(\alpha-\beta)(1-\alpha)(u-\beta)-\beta[1-\alpha+u(\alpha-\beta)]}{[1-\alpha+u(\alpha-\beta)]} \frac{l+(L-l)[\sigma-\gamma(1-\sigma)]}{(L-l)[\sigma-\gamma(1-\sigma)]T(u)} \right\} \\
&= \frac{-\beta l}{B[\sigma-\gamma(1-\sigma)]} \frac{(1-\alpha)[1-\alpha+u(\alpha-\beta)]+(\alpha-\beta)(1-\alpha)(u-\beta)}{[1-\alpha+u(\alpha-\beta)]T(u)} < 0.
\end{aligned} \tag{A2g}$$

$$\begin{aligned}
\frac{\partial l}{\partial \psi} &= \frac{\partial l}{\partial \psi} + \frac{\partial l}{\partial u} \frac{\partial u}{\partial \psi} \\
&\quad (+) \quad (-) \quad (+) \\
&= \frac{1}{\psi} \frac{l}{[\sigma-\gamma(1-\sigma)]} \left\{ 1 + \frac{(\alpha-\beta)(1-\alpha)(u-\beta)-\beta[1-\alpha+u(\alpha-\beta)]}{[1-\alpha+u(\alpha-\beta)]} \frac{l}{(L-l)[\sigma-\gamma(1-\sigma)]T(u)} \right\} \\
&= \frac{1}{\psi} \frac{l}{[\sigma-\gamma(1-\sigma)]} \frac{(1-\alpha+\beta)}{T(u)} > 0.
\end{aligned} \tag{A2h}$$

$$\begin{aligned}
\frac{\partial l}{\partial \gamma} &= \frac{\partial l}{\partial \gamma} + \frac{\partial l}{\partial u} \frac{\partial u}{\partial \gamma} \\
&\quad (\pm) \quad (-) \quad (\pm) \\
&= \frac{(1-\sigma)l\ln(l)}{[\sigma-\gamma(1-\sigma)]} \left\{ 1 + \frac{(\alpha-\beta)(1-\alpha)(u-\beta)-\beta[1-\alpha+u(\alpha-\beta)]}{[1-\alpha+u(\alpha-\beta)]} \frac{l}{(L-l)[\sigma-\gamma(1-\sigma)]T(u)} \right\} \\
&= \frac{(1-\sigma)l\ln(l)}{[\sigma-\gamma(1-\sigma)]} \frac{(1-\alpha+\beta)}{T(u)} \begin{cases} \geq \\ \leq \end{cases} 0 \quad \text{if } \begin{cases} \sigma \leq \\ \sigma \geq \end{cases} 1.
\end{aligned} \tag{A2i}$$

Table 1 Meanings for changes in the degree of leisure externalities

	$\gamma > 0$	$\gamma = 0$	$\gamma < 0$
$d\gamma > 0$	Admiration goes up.	Admiration effect created.	Jealousy reduced.
$d\gamma < 0$	Admiration reduced.	Jealousy created.	Jealousy goes up.

Table 2 Comparative Static Results

	Benchmark	$\psi=1.4105$	$\psi=1.48$	$\psi=1.34$	$\psi=1.48$	$\psi=1.34$
γ	0 [#]	0.05	-0.05	0	0	-0.05
$L-l^*$	25.0000	23.3949	26.6231	22.9125	27.2860	21.3487
u^*	0.7667	0.7883	0.7471	0.7953	0.7397	0.8197
Δ (%)			13.8		19.09	35.53

[#]: $\gamma=0$, $\psi=1.41045$. Other parameters: $\rho=0.04$, $\alpha=0.36$, $\beta=0.3$, $\sigma=0.83$, $\gamma=0$, $A=0.02203$, $B=0.0096$, $L=100$. Benchmark equilibrium: $L-l^*=25$, $u^*=0.7667$, $s^*=0.8118$, $(L-l^*)u^*=19.1675$ and $\phi = 2\%$.
 Δ : Percentage difference in $(L-l^*)$ from the previous column.