A Note on the Displacement Effects of Marginal Wage Subsidies

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ABSTRACT

The efficiency of marginal wage subsidies, which are paid only for a firm’s additional employment exceeding some reference level, suffers from intra-industry competition effects. Some firms might expand employment, but mainly at the cost of crowding out employment in other firms. Hence, critics expect that marginal subsidies are equivalent to general subsidies and create large fiscal costs without much gain in employment. We develop a model that specifically focuses on the displacement effects of marginal wage subsidies. Our results show that marginal subsidies generally create larger output and employment effects and are fiscally more efficient than general subsidies despite their between-firm displacement effects.
1. INTRODUCTION

One of the standard textbook answers to the possible reasons for unemployment is wage rigidity. Wages are set at a level exceeding that at which the labor market clears. Consequently, labor demand falls short of labor supply and unemployment occurs. One approach to reduce this type of unemployment is to subsidize wages to price people back into employment. The wage paid by the employer is moved closer to its full employment level, while the wage received by the employee remains unchanged.

Even though, for a given level of real wages, a general wage subsidy can succeed in reducing unemployment, it will create large fiscal costs because in order to stimulate employment at the margin, the subsidy has to be paid for all employees. Hence, a general wage subsidy creates large windfall gains for employers that receive the subsidy for all already employed workers without even creating a single new job. The large costs of a general wage subsidy constitute an almost insurmountable political obstacle to its implementation.

Apparently, the problem of subsidizing all workers can be avoided by restricting the subsidy to additional employment. Only if a firm hires a new employee in excess of its incumbent workforce at some reference date will it receive a subsidy for this additional worker. Such a scheme is called a marginal wage subsidy. Marginal wage subsidization has been proposed by Layard and Nickell (1980) and was subsequently discussed by, inter alia, Chiarella and Steinherr (1982), Rehn (1982), Hart (1989), Haveman (1996), and Lin (1999). The arguments in favor of a marginal subsidy compared to a general subsidy seem obvious: if only additional employment is subsidized, the same employment effect could be created at much lower costs.

Critics challenge this view because there seem to be various ways to get around the “additionality” requirement. For example, an incumbent firm could establish a second, new firm to which it outsources all its employees. Since the new firm’s em-

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1 Empirical investigations into the reasons for downward wage rigidity are provided by Bewley (1999), Agell and Lundborg (2003), Franz and Pfeiffer (2006), and Agell and Bennmarker (2006).

2 Such a wage subsidy was first proposed by Kaldor (1936). Various types of wage subsidies have recently been suggested by Haveman (1996), Orszag and Snower (2000, 2003), Phelps (1997), Snower (1994), and Schöb and Weimann (2003, 2005).

3 The specific marginal subsidy scheme discussed in this paper was recently proposed for Germany by Schöb and Weimann (2003, 2005) and initiated some debate among economists as well as politicians (see e.g. Bothfeld et al., 2006; German Council of Economic Experts, 2006; Sinn, 2006).
ployment level on the reference date would be zero, all workers hired in the new firm would have to be subsidized, but net employment would not change. To prevent such outsourcing, one has to set economic incentives that make it more profitable to expand employment in the already existing firm instead of outsourcing. Double marginal subsidization provides such an incentive (see Schöb and Weimann, 2003, 2005; Knabe et al., 2006). If a firm hires a new worker in excess of its reference employment level, it receives the subsidy not only for the new employee, but also for one incumbent employee. This effectively doubles the marginal labor cost reduction if employment is expanded in the incumbent firm and provides an effective means to prevent outsourcing compared to regular marginal subsidies.

Even though double marginal subsidization can prevent displacement of unsubsidized by subsidized workers at the level of an individual firm, one must be careful when transferring the firm-level effects of a marginal subsidy to more aggregate levels of the economy. As has been pointed out by Layard and Nickell (1980), even though some firms expand their production, hire new workers and obtain the subsidy, they do so mainly at the expense of other firms which have to reduce employment or are driven out of the market. In an extreme case, only fully subsidized firms could survive in the market, and double marginal and general subsidies would be equivalent.

We refer to this process, in which subsidy-induced employment expansions in some firms cause employment reductions in other firms, as between-firm displacement. Even though a thorough analysis of these displacement effects is crucial for definite economic policy advice, the literature on this topic is rather sparse. Oswald (1984) shows that a marginal wage subsidy, while increasing employment at the firm-level, affects the number of firms in equilibrium in an ambiguous way. The net effect on employment at the industry level is generally ambiguous. Luskin (1986) shows that marginal subsidies will be fiscally more efficient than general subsidies only if industry product demand is relatively inelastic. If this condition is not met, however, the opposite holds and general subsidies are more favorable than marginal subsidies.

The results of Oswald (1984) and Luskin (1986) have to be taken with caution. Both models rely heavily on assumptions that do not correspond to effectively implemented or proposed subsidy schemes. For example, the subsidy scheme analyzed by Oswald (1984) combines a marginal wage subsidy for employment above some reference employment level with a marginal tax on layoffs below this level. Real-life marginal wage subsidy scheme, however, have always been asymmetric in the sense
that they subsidize new jobs, but do not punish firms that lay off workers. Since this asymmetry affects the extent to which firms are able to lay off workers if their competitors expand, this assumption is critical for the analysis of between-firm displacement. Luskin’s results hinge on the assumption that, with marginal subsidies, new entrants will receive the subsidy only if they expand their employment above the same reference level as incumbent firms. This assumption, however, is at odds with actual marginal wage subsidy programs, all of which have individualized reference levels for incumbent firms, while firms founded after the reference date typically receive the subsidy for all their workers (cf. Schmidt, 1979, p. 342). Moreover, the policy implications of both studies suffer from the ambiguity of their results.

In this paper, we resume the discussion of the industry-level effects of marginal wage subsidies. In line with the existing literature (Oswald, 1984; Luskin, 1986), we restrict our analysis to the partial equilibrium effects of a marginal subsidy targeted at a single competitive industry. While these studies appear to have sacrificed practical relevance for formal elegance, this paper’s contribution consists in developing a model of marginal wage subsidies that pays close attention to the institutional features of real-life marginal subsidy programs. For example, our model has firm-specific reference employment levels (contrary to Luskin, 1986) and restricts the subsidy to employment expansions without penalizing employment reductions below the reference employment level (contrary to Oswald, 1984). With this novel way of modeling marginal subsidies, we can overcome the ambiguities of previous studies and unambiguously identify the differential impact of general and marginal wage subsidies in the presence of between-firm displacement. Under relatively weak assumptions, we are able to show that marginal wage subsidies create more employment at less fiscal costs than general subsidies despite between-firm displacement at the industry level.

We will proceed as follows. In Section 2, we present a graphical argument to visualize the effects of double marginal subsidization on a firm’s decisions and on the industry equilibrium. In Section 3, the model is analyzed formally. Section 4 concludes.

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4 In the 1970s, many countries experimented with marginal employment subsidies. Examples are the New Jobs Tax Credit in the United States (see Perloff and Wachter, 1979; Bishop and Haveman, 1979), the French Prime d’incitation à la création d’emploi (see Kopits, 1978), the Small Firms Employment Subsidy in Great Britain (see Layard, 1979), and the Lohnkostenzuschüsse in Germany (see Schmidt, 1979). All of these programs were asymmetric, i.e. they subsidized employment expansions but did not tax layoffs.
2. A GRAPHICAL ARGUMENT

To grasp the different effects of double marginal wage subsidies (DMS) compared to general wage subsidies (GS), it is helpful to look at a diagram showing marginal and average cost curves in order to predict the optimizing behavior of firms.

The upper part of Figure 1 shows the marginal and average cost curves for a firm without subsidies (MC, AC), for a newly founded firm receiving a single subsidy (MC\textsubscript{S}, AC\textsubscript{new}), and the marginal cost curve for an incumbent firm receiving the double marginal subsidy as long as employment has not doubled (MC\textsubscript{DMS}). Prices are denoted by \( p \), a firm’s output level by \( y \), and the initial price and output levels are \( p_0 \) and \( y_0 \), respectively. To keep the analysis as simple as possible, we assume that labor is the only factor of production, that the production function exhibits a diminishing marginal product of labor, and that the wage rate is constant. Hence, the marginal cost schedule is increasing with the level of output. When entering the market, firms have to pay start-up costs which are sunk afterwards.\(^5\)

With DMS, an incumbent firm will not receive the subsidy for output levels below \( y_0 \) because it has not created any new jobs. Its marginal cost schedule is given by MC. If the firm raises its output above \( y_0 \) and consequently hires more workers, the additional employment qualifies for the marginal subsidy. With DMS, the firm then receives twice the subsidy rate \( s \) per new employee because for each new employee, one already employed worker is subsidized as well. The effect on marginal costs is equivalent to a marginal subsidy of \( 2s \); the corresponding marginal cost curve is given by MC\textsubscript{DMS}. If the firm doubles its employment, all of its incumbent workers will be subsidized too. This point corresponds to output level \( y_2 \) in Figure 1.\(^6\) At \( y_2 \), the marginal cost curve jumps up to MC\textsubscript{S}. None of the incumbent workers is left unsubsidized, so further employment expansions will only receive the single subsidy.

A new firm entering the market has no employment on the reference date, and so all its employees receive the single subsidy. Its marginal cost curve is thus given by

\(^5\) Sunk costs are ubiquitous for firms. Baumol et al. (1983, p. 494) define sunk costs as the share of capital investment costs which cannot be recouped by resale of the asset. Examples for such investments are highly specific physical assets, the gathering of information before a firm enters a specific industry, costs of organizing the new operation, product-differentiating sales efforts etc. (Martin, 1993, p. 306). Hence, it is plausible to assume that sunk start-up costs play a role in intra-industry competition.

\(^6\) Note that with diminishing marginal product of labor, \( y_2 < 2y_0 \).
the entire $MC_S$-curve. However, a newly entering firm has to bear start-up costs, and so its average cost curve is given by $AC_{new}$. The lowest price at which new firms start entering the market is denoted by $\tilde{p} \equiv \min_y AC_{new}(y)$.

With the knowledge of its marginal and average cost curves, we can derive an incumbent firm’s supply function. For very low output prices, the firm’s supply function is given by the marginal cost curve $MC$, i.e. it will shrink compared to its initial
production level $y_0$. For higher prices, there will be a critical price at which the firm makes the same profit either by shrinking to some production level less than $y_0$ or by expanding production to a level above $y_0$ and receiving the subsidy. In Figure 1, this critical price is denoted by $\bar{p}$, and the two production levels yielding equal profits are $y''$ and $y'$, respectively.\(^7\) For prices above $\bar{p}$, the firm’s supply curve is given by the double subsidy marginal cost curve $MC_{DMS}$ as long as employment has not doubled. Since employment exceeding twice the initial level receives only the single subsidy, the firm will always supply $y_2$ for all prices between $MC_{DMS}(y_2)$ and $MC_S(y_2)$. For all prices above this interval, the firm’s supply curve is given by $MC_S$.

With the knowledge of marginal and average cost curves, we can derive the industry’s aggregate supply function shown in the lower part of Figure 1 (solid line), where $n_0$ denotes the number of firms in initial equilibrium and $Y$ stands for the industry’s output level. For visual clarity, we have assigned numbers to the different sections of the supply curve. In Section 1, the output price is less than the critical price $\bar{p}$. All incumbent firms reduce their production level. No subsidy is paid out. In Section 2, the output price is exactly $\bar{p}$. Some incumbent firms expand to $y'$, while others contract to $y''$. The number of firms contracting and expanding depends on the quantity of industry output demanded. Hence, this is the section in which between-firm displacement occurs because some of the incumbent firms expand at the cost of other incumbent firms. The additional employment in expanding firms has to be subsidized. In Section 3, all incumbent firms expand their employment. Most, but not all, workers are subsidized. In Section 4, all incumbent firms double their employment. All workers are subsidized. In Section 5, the output price is $\bar{p}$. All incumbent firms double their employment and produce $y_2$. The industry product demand not supplied by the incumbent firms is then served by new entrants, each of which produces an output level of $MC_S^{-1}(\bar{p})$. All workers are subsidized.

The lower part of Figure 1 can be used to compare the equilibria obtained with DMS when displacement is taken into account and when it is neglected. If there were no displacement between firms, all firms would supply along their marginal cost curve with double subsidies, $MC_{DMS}$. The industry supply curve would then be given by the dashed-dotted line in the lower part of Figure 1, and the resulting market equilibrium would occur in point $A$. As we have argued above, however, point $A$ cannot be an

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\(^7\) By expanding from $y''$ to $y'$, a firm makes inframarginal losses on all units between $y''$ and $y_0$, but inframarginal profits on all units between $y_0$ and $y'$ (shown by the shaded areas in Figure 1). At $\bar{p}$, the gains and losses exactly balance, so that the same level of profit arises at both output levels.
equilibrium because the output price would be so low that all firms would want to reduce their output levels below $y_0$. Hence, the equilibrium price cannot be lower than the level at which firms decide to shrink and between-firm displacement sets in. Since this price is given by $\tilde{p}$, taking displacement effects into account shifts the equilibrium with DMS from point $A$ to point $B$, which impairs the predicted equilibrium output and employment effects of DMS.

With GS, the derivation of the industry’s supply curve is straightforward (dashed line in the lower part of Figure 1). An incumbent firm is completely subsidized at the single rate, and so its supply curve is given by $MC_S$. For new firms, there is no difference between DMS and GS – in either case, all their employees are subsidized at the single rate. Hence, new firms will enter the market if the output price is at least $\bar{p}$. The resulting equilibrium is indicated by point $C$.

Figure 1 compares the industry supply curves both under DMS and GS. For output levels below $\tilde{Y}$, the industry supply curve with GS is below the industry supply curve with DMS. The DMS-curve is above the GS-curve for output levels between $\tilde{Y}$ and $n_0y_2$. Both curves coincide for output levels above $n_0y_2$ where new firms enter the market. A sufficient condition for a favorable output effect of DMS compared to GS is that the industry supply curve with GS is above the DMS-curve at the initial output level, i.e. $\tilde{Y} < n_0y_0$. In this case, any non-increasing demand function will result in at least the same, but generally larger equilibrium output and employment effects with DMS compared to GS. Furthermore, it is clear that a favorable employment effect of DMS is also sufficient to ensure higher fiscal efficiency (measured by fiscal costs per newly created job) because DMS create more new jobs without subsidizing more already existing jobs than GS.

Although the graphical analysis might be helpful to illustrate the industry-level effects of DMS and GS, it does not give a definite answer under which conditions favorable output and employment effects of DMS are secured. To derive these conditions, we will turn a formal analysis in the next section.

### 3. A FORMAL ANALYSIS

Suppose that each firm has a production technology $y = f(l)$, with $f' > 0$, $f'' < 0$,

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8 In Figure 1, $\tilde{Y} < n_0y_0$ means that point $C$ has to be to the left of – or exactly at the same point as – point $B$. 

where \( y \geq 0 \) is a firm’s output and \( l \geq 0 \) its labor input. This production technology results in a variable cost function \( C(y) \) with \( C' > 0 \) and \( C'' > 0 \). Let marginal costs be denoted by \( \text{MC}(y) \equiv C'(y) \). The wage rate is denoted by \( w \), the single subsidy rate by \( s \), and start-up costs by \( F \). Total market demand is a non-increasing function of the price level denoted by \( D(p) \) with \( D' \leq 0 \).

Each firm maximizes its profits by producing the level of output at which the output price \( p \) equals its marginal production costs. With general subsidies, the profit-maximizing output level is then implicitly determined by

\[
p = \begin{cases} 
\text{MC}(y), & \text{if } y \leq y'', \\
(1 - 2s)\text{MC}(y), & \text{if } y' \leq y < y_2, \\
(1 - s)\text{MC}(y), & \text{if } y_2 < y,
\end{cases}
\]

where the firm is restricted to \( y = y_2 \equiv 2f^{-1}(y_0) \) (i.e. twice the initial employment level) if \( p \in [(1 - 2s)\text{MC}(y_2), (1 - s)\text{MC}(y_2)] \). Intuitively, this restriction refers to the case where the output price is at a level at which a firm would like to expand to more than twice its initial employment level if it could receive the double subsidy also beyond \( y_2 \). Since expanding beyond \( y_2 \) is only subsidized by the single rate, however, the firm will not find this expansion profitable if the output price is less than the firm’s marginal cost at the single subsidy. Hence, in this price interval, the firm’s profit-maximizing output level is exactly \( y_2 \). The output levels \( y'' \) and \( y' \) are implicitly determined by \( \tilde{p} = \text{MC}(y') \) and \( \tilde{p} = (1 - 2s)\text{MC}(y'') \), where \( \tilde{p} \), the price level at which the firm is indifferent between expanding or contracting under double marginal subsidies, is implicitly determined by the condition

\[
\tilde{p}\text{MC}^{-1}(\tilde{p}) - wf^{-1}(\text{MC}^{-1}(\tilde{p})) = \begin{cases} 
\tilde{p}\text{MC}^{-1}\left(\frac{\tilde{p}}{1 - 2s}\right) -wl_0, & \text{if } \tilde{p} \leq (1 - 2s)\text{MC}(y_2), \\
-(1 - 2s)w f^{-1}\left(\text{MC}^{-1}\left(\frac{\tilde{p}}{1 - 2s}\right)\right) -l_0, & \text{if } \tilde{p} > (1 - 2s)\text{MC}(y_2).
\end{cases}
\]

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The LHS of (2) is the profit if a firm decides to shrink, while the RHS is the profit if the firm expands. The piecewise definition is necessary because the indifference might occur either at a price level where the firm wants to expand to less than twice its initial employment level (upper part) or where it hits the restriction that it can receive the double subsidy for at most twice its initial employment level (lower part).

By inverting the firm-level supply functions and adding up over all $n_0$ incumbent firms, one obtains the industry supply function with DMS:

$$Y_{DMS}(p) = \begin{cases} 
0, & \text{if } 0 \leq p \leq \text{MC}(0), \\
\left[n_0 \text{MC}^{-1}(\tilde{p}), n_0 \text{MC}^{-1} \left(\frac{\tilde{p}}{1-2s}\right)\right], & \text{if } \text{MC}(0) < p < \tilde{p}, \\
n_0 \text{MC}^{-1} \left(\frac{p}{1-2s}\right), & \text{if } \tilde{p} < p \leq (1-2s)\text{MC}(y_2), \\
n_0 y_2, & \text{if } (1-2s)\text{MC}(y_2) < p < \bar{p}, \\
\left[n_0 y_2, \infty\right], & \text{if } p = \bar{p}, 
\end{cases}$$

(3)

and the industry supply function with GS:

$$Y_{GS}(p) = \begin{cases} 
0, & \text{if } 0 \leq p \leq (1-s)\text{MC}(0), \\
\left[n_0 \text{MC}^{-1} \left(\frac{p}{1-s}\right)\right], & \text{if } (1-s)\text{MC}(0) < p < \tilde{p}, \\
n_0 \text{MC}^{-1} \left(\frac{\tilde{p}}{1-s}\right), & \text{if } \tilde{p} < p \leq \bar{p}, \\
\left[n_0 \text{MC}^{-1} \left(\frac{\bar{p}}{1-s}\right), \infty\right], & \text{if } p = \bar{p}, 
\end{cases}$$

(4)

In (3) and (4), the price level $\tilde{p}$ at which new firms enter the market is given by

$$\tilde{p} = \min_y \left[\frac{F + (1-s)C(y)}{y}\right].$$

(5)

We can now turn to the analysis of the output, employment, and fiscal effects of the two types of subsidies. For this, we will make use of the following two lemmas.

**Lemma 1** Comparing two industry equilibria, in each of which the same level of output is produced, but which differ in the number and/or size of individual firms, the
level of employment is always smaller in the equilibrium in which, with the same or larger number of firms, all firms have the same output level.

**Proof:** See Appendix.

Lemma 1 shows that for a given level of output, employment is larger under DMS due to a technical inefficiency in the production sector. If all firms have the same strictly concave production function, different firm sizes mean that the marginal productivity of labor is not equalized between firms. Moreover, reducing the number of firms means that each firm has to increase its level of production. This lowers the marginal productivity of labor to inefficiently low levels. Hence, a small number of large, differently sized firms require more labor inputs to produce a given output than a large number of small, equally sized firms.\textsuperscript{9}

As in the previous section, we define a critical level of industry output $\tilde{Y}$ by the point at which the industry supply curves with DMS and GS intersect: $\tilde{Y} = n_0MC^{-1}[\hat{\beta}/(1 - s)]$. One can now show that if the production function $f(l)$ is homogenous of some degree $h \in [0, 1]$, then $\tilde{Y}$ will always be to the left of the initial output level.

**Lemma 2** For all homogenous one-factor production functions with a degree of homogeneity between 0 and 1, it holds that

$$n_0y_0 > \tilde{Y}. \quad (6)$$

**Proof:** See Appendix.

With the help of Lemmas 1 and 2, we can now easily show that DMS have favorable output and employment effects compared to GS.

**Proposition 1** For any demand function $D(p)$ with $D' \leq 0$ and any homogenous production function with a degree of homogeneity between 0 and 1, a double marginal subsidy will have at least the same output effect and a larger employment effect than an equal-rate general subsidy.

**Proof:** See Appendix.

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\textsuperscript{9} In this paper’s partial-equilibrium context, this property is desirable because it increases employment. A general-equilibrium analysis of its welfare effects, however, is beyond the scope of this paper.
Proposition 1, combined with Lemma 2, states that the demand curve intersects the industry supply curves in a range where the DMS-curve lies on or below the GS-curve, which causes the equilibrium output with DMS to be larger than (Sections 2, 3 and 4) or as large as (Section 5) with GS. The reason that DMS may result in higher equilibrium output lies in their rent reduction effect. Under both subsidy schemes, incumbent firms are to some degree protected from the competition of new entrants by the sunk character of start-up costs, and hence obtain rents. Under GS, for \( p < \bar{p} \) demand is large enough to support all incumbent firms, each of which expands its production and receives the subsidy for all its employees. Under DMS, the double marginal subsidy provides a larger incentive for each firm to expand production than the single subsidy. If total market demand is not large enough to support all incumbent firms’ desired output expansion, incumbent firms will compete for the expansion, so that some firms increase their output at the expense of other firms. The subsidy thus triggers competition between incumbent firms that, in industry equilibrium, results in lower prices, lower rents, and larger output under DMS than under GS.

Proposition 1 also postulates that DMS always have a larger employment effect than GS. While this is obvious when DMS yield a larger output effect than GS, it holds even if both subsidy schemes have the same output effect. The reason lies in the technical inefficiency described by Lemma 1. With GS, any given output level is produced by equally sized firms, while with DMS either some firms expand while others contract or all incumbent firms become larger than they would be under GS. Thus, the favorable output effect of DMS always causes an even stronger favorable employment effect.

The last point to analyze is the fiscal efficiency of both subsidy schemes. Our measure of fiscal efficiency is the subsidy expenditure per net job created (as in Luskin, 1986). Under DMS, this measure corresponds to

\[
\frac{\phi swL_{DMS}}{L_{DMS} - L_0},
\]

where \( L_{DMS} \) is total equilibrium employment and \( L_0 = n_0l_0 \) is total initial employment. \( \phi \) denotes the share of subsidized workers in total employment. If \( p \in [\bar{p}, (1 - 2s)MC(y_2)] \) (Sections 2 and 3 in Figure 1), incumbent firms have not doubled their employment and some, but not all, workers are subsidized: \( \phi \in ]0, 1[ \). If \( p \geq (1 - 2s)MC(y_2) \) (Sections 4 and 5 in Figure 1), incumbent firms have doubled
their employment so that all their workers are subsidized. Since new entrants are always fully subsidized, we have $\phi = 1$.

Under GS, subsidy expenditures per net job created are given by

$$\frac{swL_{GS}}{L_{GS} - L_0},$$

where $L_{GS}$ is total equilibrium employment.

By comparing (7) and (8), one can determine the fiscally more efficient subsidy scheme.

**Proposition 2** Double marginal subsidies are fiscally more efficient than general subsidies.

The proof of Proposition 2 is straightforward. Even if all workers had to be subsidized under DMS ($\phi = 1$), its favorable employment effect (Proposition 1) is sufficient for favorable fiscal efficiency because the costs of subsidizing already existing jobs are spread out among more new jobs, thereby reducing the fiscal burden per new job created.

4. **CONCLUSION**

In this paper, we have analyzed whether a marginal wage subsidy can serve as a better policy measure to reduce unemployment than a general wage subsidy despite its intra-industry displacement effects. Our aim was to determine the conditions under which a double marginal wage subsidy yields better output and employment effects than a general subsidy, and to clarify which of the subsidy schemes is fiscally more efficient.

Our results show that a double marginal subsidy generally leads to lower equilibrium prices, and hence to a higher equilibrium output level, than an equal-rate general subsidy. The reason for this result is that double marginal subsidies give a larger marginal stimulus for output expansion than equal-rate general subsidies. The resulting increase in industry-level supply drives down the market price more strongly than with general subsidies, which raises output and employment at the industry level. If double marginal subsidies depress prices below a critical level, however, between-firm displacement occurs. Only some incumbent firms can expand their production,
while all other incumbents are forced to reduce their size substantially. This displace-
ment raises the fiscal costs of marginal subsidies and reduces their employment effects. 
Nevertheless, we were able to show that between-firm displacement is not sufficient 
to counteract the positive effects of double marginal subsidies completely. Our results 
show that double marginal subsidies generally result in lower prices as well as larger 
output and employment than general subsidies despite their displacement effects.

Furthermore, the employment effect of a double marginal wage subsidy generally 
exceeds that of the general subsidy even if both yield the same output effect. This 
results from a technical inefficiency induced by double marginal subsidization. With 
decreasing returns to scale at the firm level, an efficient production structure requires 
all firms to have the same size. Double marginal subsidies, however, cause some firms 
to expand and others to contract, which distorts the production structure and increases 
the amount of labor necessary to produce the same level of output. In our partial-
equilibrium context, this technical inefficiency yields favorable employment effects, 
such that even in those cases where the output effect is equivalent under both subsidy 
schemes, employment is nevertheless larger with marginal wage subsidies. Finally, 
the fiscal effect, measured by the subsidy expenditures per new job, always favors the 
double marginal subsidy.

By using a partial-equilibrium model that focuses on the displacement compe-
tition between firms in the same industry, we have shown that if policymakers want 
to implement a wage subsidy program, but are concerned about its fiscal cost, they 
should prefer a double marginal subsidy to a general subsidy despite its between-firm 
displacement effects.
APPENDIX

Proof of Lemma 1 Suppose that in one sector, output is produced by $n_1$ firms, each of which has a production level of $y$. In the other sector, there are $n_2$ firms, of which $\gamma n_2$ firms each produce $y'$ and $(1 - \gamma) n_2$ firms each produce $y''$. If total output is the same in both sectors, we have

$$n_1 y = n_2 [\gamma y' + (1 - \gamma) y''].$$  \hspace{1cm} (A1)

Employment in the first sector is given by $n_1 f^{-1}(y)$, which by inserting (A1) becomes

$$n_1 f^{-1} \left( \frac{n_2}{n_1} (\gamma y' + (1 - \gamma) y'') \right),$$  \hspace{1cm} (A2)

while employment in the second sector is given by

$$n_2 [\gamma f^{-1}(y') + (1 - \gamma) f^{-1}(y'')].$$  \hspace{1cm} (A3)

From the definition of the production function ($f' > 0$, $f'' < 0$), we know that the inverse production function $f^{-1}(y)$ is strictly convex. Thus, from Jensen’s inequality we have

$$f^{-1}(\gamma y' + (1 - \gamma) y'') < \gamma f^{-1}(y') + (1 - \gamma) f^{-1}(y''), \quad \text{if } y' \neq y'', \hspace{1cm} (A4)$$

and from the definition of a convex function we have

$$f^{-1} \left( \frac{n_2}{n_1} (\gamma y' + (1 - \gamma) y'') \right) \leq \frac{n_2}{n_1} f^{-1}(\gamma y' + (1 - \gamma) y''), \quad \text{if } n_2 \leq n_1. \hspace{1cm} (A5)$$

Combining (A4) and (A5) yields
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\[
n_1 f^{-1} \left( \frac{n_2}{n_1} (\gamma y' + (1 - \gamma) y'') \right) < n_2 [\gamma f^{-1}(y') + (1 - \gamma) f^{-1}(y'')] \tag{A6}
\]

if \( y' \neq y'' \) and \( n_2 \leq n_1 \), which proves the lemma.

**Proof of Lemma 2**  The condition \( n_0 y_0 > \bar{Y} \) can be written as

\[
(1 - s)MC(y_0) > \bar{p}, \tag{A7}
\]

where \( \bar{p} \) is implicitly defined in (2) by

\[
\bar{p}MC^{-1}(\bar{p}) - wf^{-1}(MC^{-1}(\bar{p})) - \bar{p}MC^{-1} \left( \frac{\bar{p}}{1 - 2s} \right) + wl_0 + (1 - 2s)w \left[ f^{-1} \left( MC^{-1} \left( \frac{\bar{p}}{1 - 2s} \right) \right) - l_0 \right] = 0. \tag{A8}
\]

With \( y = f(l) \), the marginal and inverse marginal cost functions are given by

\[
MC(y) = \frac{w}{f'(f^{-1}(y))}, \tag{A9}
\]

\[
MC^{-1}(p) = f \left( f'^{-1} \left( \frac{w}{p} \right) \right). \tag{A10}
\]

Inserting into (A8) gives

\[
\bar{p}f \left( f'^{-1} \left( \frac{w}{\bar{p}} \right) \right) - wf'^{-1} \left( \frac{w}{\bar{p}} \right) - \bar{p}f \left( f'^{-1} \left( (1 - 2s)\frac{w}{\bar{p}} \right) \right) + wl_0 + (1 - 2s)w \left[ f'^{-1} \left( (1 - 2s)\frac{w}{\bar{p}} \right) - l_0 \right] = 0. \tag{A11}
\]

If \( f(l) \) is homogenous of degree \( h \), this is rewritten as

\[
\left[ 1 - (1 - 2s)^{\frac{h-1}{h}} \right] \left[ \bar{p}f \left( f'^{-1} \left( \frac{w}{\bar{p}} \right) \right) - wf'^{-1} \left( \frac{w}{\bar{p}} \right) \right] + 2swl_0 = 0. \tag{A12}
\]

If Condition (A7) is true, we must have

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\[ [1 - (1 - 2s)^{\frac{h}{s-1}}] \left[ (1 - s)MC(y_0) f \left( f'^{-1} \left( \frac{w}{(1 - s)MC(y_0)} \right) \right) \right] - w f'^{-1} \left( \frac{w}{(1 - s)MC(y_0)} \right) + 2swl_0 < 0, \]  

(A13)

because a price higher than \( \bar{p} \) must yield a larger profit for an expanding firm than for a contracting firm. Since \( (1 - s)MC(y_0) = (1 - s)w/f'(l_0) \), this condition becomes

\[ \frac{1 - (1 - 2s)^{\frac{h}{s-1}}}{(1 - s)^{\frac{h}{s-1}}} \left[ \frac{f(l_0)}{l_0 f'(l_0)} - 1 \right] + 2s < 0, \tag{A14} \]

which, by applying Euler’s theorem, can be rewritten as

\[ \frac{1 - (1 - 2s)^{\frac{h}{s-1}}}{(1 - s)^{\frac{h}{s-1}}} \left[ \frac{1}{h} - 1 \right] + 2s < 0. \tag{A15} \]

This condition holds for all values \( h \in ]0, 1[ \) and \( s \in ]0, 1/2[ \). This proves the claim.

**Proof of Proposition 1** Let \( p_{DMS} \) and \( p_{GS} \) be defined by \( D(p_i) = Y_i(p_i) \) with \( i \in \{\text{DMS, GS}\} \). From the supply functions (3) and (4), it follows that

\[ p > \bar{p} \Rightarrow Y_{GS}(p) \leq Y_{DMS}(p). \tag{A16} \]

The initial equilibrium is defined by \( D(p_0) = n_0y_0 \). With \( \bar{p} < p_0 \) and \( D'(p) \leq 0 \), we always have \( D(\bar{p}) = Y_{GS}(\bar{p}) \) (Lemma 2), and hence \( p_{GS} > \bar{p} \). Under this condition, (A16) implies that \( D(p_{GS}) = Y_{GS}(p_{GS}) \leq Y_{DMS}(p_{GS}) \) and thus \( p_{DMS} \leq p_{GS} \). Therefore, we have \( D(p_{DMS}) \geq D(p_{GS}) \), i.e. the output effect is always at least as large with DMS as with GS.

We now have to show that employment is larger under DMS than under GS. If \( p_{GS} \in [\bar{p}, \bar{p}] \), and hence \( p_{DMS} \in [\bar{p}, \bar{p}] \), the number of firms is equal to \( n_0 \) under both subsidy schemes. Under GS, each firm produces \( MC^{-1}[p_{GS}/(1 - s)] \), while under DMS some firms produce \( MC^{-1}[p_{DMS}/(1 - 2s)] \) and others produce \( MC^{-1}(p_{DMS}) \). Lemma 1 shows that for \( D(p_{DMS}) = D(p_{GS}) \) and this production structure, employment is larger under DMS. This holds, a fortiori, for \( D(p_{DMS}) > D(p_{GS}) \).
If $p_{GS} = \tilde{p}$, all firms produce the same quantity $MC^{-1}[\tilde{p}/(1 - s)]$ under GS, and the number of firms is given by $D(\tilde{p})/MC^{-1}[\tilde{p}/(1 - s)] \geq n_0$. Under DMS, either $p_{DMS} < \tilde{p}$, in which case the number of firms is equal to $n_0$, or $p_{DMS} = \tilde{p}$, in which case the number of firms is equal to $n_0 + [D(\tilde{p}) - n_0y_2]/MC^{-1}[\tilde{p}/(1 - s)] \leq D(\tilde{p})/MC^{-1}[\tilde{p}/(1 - s)]$ since, by assumption, $\arg \min_y AC_{new}(y) \leq y_2$. Hence, under GS the number of firms is never smaller than under DMS and firms are always of equal size. In this case, Lemma 1 shows that the employment effect always favors DMS.
REFERENCES


邊際工資補貼的取代效果

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摘 要

邊際工資補貼只對超過某標準的廠商僱用量進行補貼，然而此種補貼方式有產業內的競爭問題，某些廠商可能增加僱用量，但卻排擠其他廠商的僱用量。因此，批評者一般認為，邊際補貼無異於全體補貼，有巨大財政成本而增加就業效果卻有限。本文針對邊際工資補貼的取代效果建立一個模型，分析結果顯示，一般而言，即使存在廠商間的取代效果，邊際補貼比全體補貼創造更大的產出和就業量，且在財政上較具效率。