

Education Attainment and Structural Transformation

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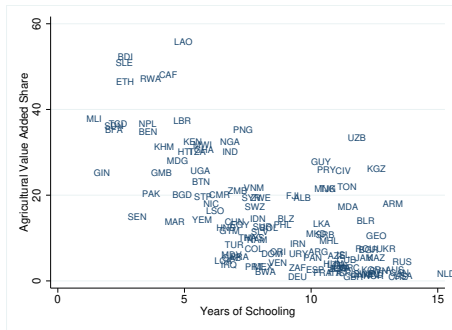
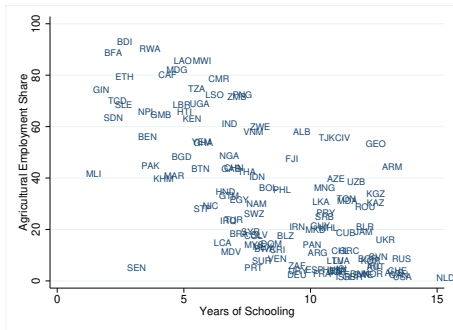
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Motivation

- Structural Transformation due to:
 - 1 Income Effect: Increased income reduces the demand for food relative to other goods
 - 2 Productivity Effect: Increased relative productivity of agricultural sector

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 - 1 Income Effect: Increased income reduces the demand for food relative to other goods
 - 2 Productivity Effect: Increased relative productivity of agricultural sector
- We emphasize the importance of education in structural transformation

- Correlation between sectoral allocation and education



- Nonagricultural sector has higher human capital intensity
- Individuals make endogenous education decision
- Places that are wealthier and with better education policies results in larger stock of human capital
- More people self-selected into nonagricultural sector
- Rybczynski Theorem

- We develop a lifecycle framework to study the role of human capital accumulation in determining the process of structural transformation and cross-country productivity difference
 - Life-cycle schooling and work decision
 - Quality and stock of human capital differ by talent, personal wealth, government policies, and schooling
 - Intensity of human capital differs by sectors
- Calibrate the model using U.S. data and carry out cross-country analysis

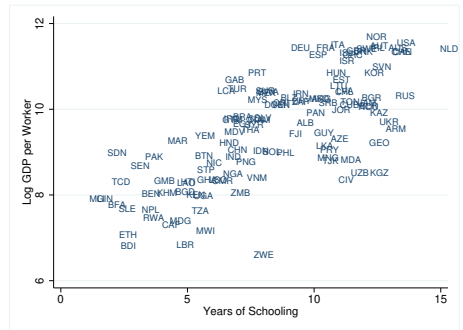
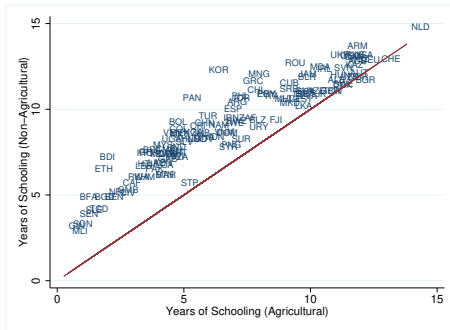
- There are three main groups of literature
 - ① Sectoral choice based on exogenous human capital
 - Lagakos & Waugh (2013), Pozio, Rossi & Santangelo (2021)
 - Human capital exogenous (policy, macroeconomic response)
 - Cheung (2022): International comparison and productivity gap

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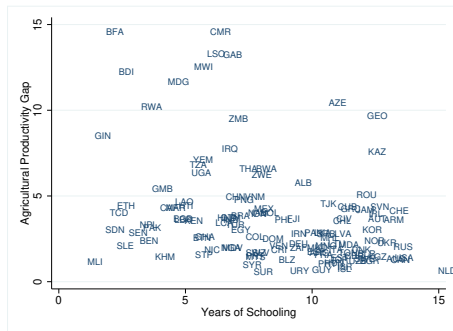
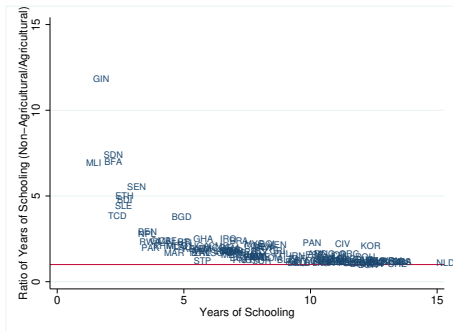
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 - 3 Structural transformation
 - Acemoglu & Guerrieri (2008) on Rybczynski Theorem, Herrendorf, Rogerson & Valentinyi (2014)
 - No emphasis on education
 - Buera, Kaboski, Rogerson & Vizcaino (2021): exogenous sectoral productivity progress

Empirical Facts

- Sectoral distribution of human capital, effect of human capital on productivity



- Education reduces sectoral schooling gap and agricultural productivity gap



Model

- Lifecycle model, similar to Córdoba & Ripoll (2013), with heterogeneity and sectoral allocation
- Individuals differ by their endowment of 1) initial wealth b , 2) ability ψ , and 3) farming skill l . The distribution assume to follow:

$$G(b, \psi, l) = G^b(b)G^{\psi,l}(\psi, l)$$

- Individuals make schooling decision $\{s, e_p(\tau)\}$, knowing $\{b, \psi, l\}$, to determine human capital $h(s)$
- Given $h(s)$, individual make sectoral choice among other decisions

- Individual in sector i solves

$$V_i(b; \psi, l) = \max_{c(\tau), e_p(\tau), s, \kappa(s)} \int_6^T e^{-\rho(\tau-6)} u(c(\tau)) d\tau$$

s.t.

$$\int_6^s e^{-r(\tau-6)} [c(\tau) + e_p(\tau)] d\tau + e^{-r(s-6)} \kappa(s) \leq b$$

$$\int_s^T e^{-r(\tau-6)} c(\tau) d\tau \leq \int_s^R e^{-r(\tau-6)} w_i(h(s), \tau - s; \psi, l) d\tau + e^{-r(s-6)} \kappa(s)$$

$$h(s) \leq z_h \left[\int_6^s \psi (e_p(\tau) + e_g(\tau))^\alpha d\tau \right]^{\frac{\gamma}{\alpha}}$$

$$e_p(\tau) \geq 0 \text{ for all } \tau$$

$$\kappa(s) \geq 0$$

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$$\kappa(s) \geq 0$$

- $w_i(h(s), \tau - s; \psi, l) = \tilde{w}_i(h(s); \psi, l) e^{v_{1i}(\tau-s) + v_{2i}(\tau-s)^2}$
- The sectoral choice S is such that

$$S = \arg \max_{S \in \{0,1\}} \{S V_a(b|\psi, l) + (1 - S) V_m(b|\psi, l)\}$$

- The utility function $u(c(\tau)) = \frac{c(\tau)^{1-\sigma}}{1-\sigma}$, where

$$c(\tau) = \frac{\tilde{c}(\tau) - p_a(\tau)\bar{c}}{[\zeta p_a(\tau)^{1-\eta} + (1-\zeta)p_m(\tau)^{1-\eta}]^{\frac{1}{1-\eta}}}$$

- The above is a indirect utility function comes from maximizes

$$\left[\zeta^{\frac{1}{\eta}} (c_a - \bar{c})^{\frac{\eta-1}{\eta}} + (1-\zeta)^{\frac{1}{\eta}} c_m^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

subject to

$$p_a(\tau)c_a(\tau) + p_m(\tau)c_m(\tau) \leq \tilde{c}(\tau)$$

- There are two sectors in the economy, both use efficiency unit of labor $\xi_i(h(s); \psi, l)$ as only input, where

$$\xi_a(h(s); l) = \left[\theta_a h(s)^{\frac{1}{\phi_a}} + (1 - \theta_a) l^{\frac{1}{\phi_a}} \right]^{\phi_a}$$

$$\xi_m(h(s); \psi) = \left[\theta_m h(s)^{\frac{1}{\phi_m}} + (1 - \theta_m) \psi^{\frac{1}{\phi_m}} \right]^{\phi_m}$$

we assume $\theta_m > \theta_a$

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- Denote $x = \{b, \psi, l\}$, the sectoral production function:

$$Y_i = A_i \iint_{\Omega_i} \xi_i(h(s); \psi, l) e^{\nu_{1i}(\tau-s) + \nu_{2i}(\tau-s)^2} N(\tau; x) dG(x) d\Pi(\tau)$$

or, in steady state, age distribution is stationary and all cohorts are identical

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- The firm maximization problem implies that

$$w_i(h(s), \tau - s; \psi, l) = \hat{w}_i \zeta_i(h(s); \psi, l) e^{\nu_{1i}(\tau-s) + \nu_{2i}(\tau-s)^2}$$

- FOC with respect to consumption $c(\tau)$ for $\tau \in [6, s]$ (λ_1) and $\tau \in [s, T]$ (λ_2)

$$J \equiv \frac{\lambda_1}{\lambda_2} = \frac{u_c(c^S(s))}{u_c(c^W(s))} = e^{(\rho-r)(F-6)} \frac{u_c(c(6))}{u_c(c(F))} \geq 1$$

- If $J > 1$, then $c^S(s) < c^W(s)$, constrained and $\kappa(s) = 0$

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- If $J = 1$, then $c^S(s) = c^W(s)$, not constrained and

$$\kappa(s) = \frac{\frac{b-E^*}{D_6^s} - \frac{l_i(s)}{D_s^T}}{e^{-r(s-6)} \left[\frac{1}{D_6^s} + \frac{1}{D_s^T} \right]} > 0$$

where $E^* = \int_6^s e^{-r(\tau-6)} e_p(\tau) d\tau$, $l_i(s) = \int_s^R e^{-r(\tau-6)} w_i(h(s), \tau - s; \psi, l) d\tau$ and $D_x^y = \int_x^y e^{-r(\tau-6)} \left(e^{(\rho-r)(\tau-6)} \right)^{-\frac{1}{\sigma}} d\tau$

- The importance of b , which can be affected by family size, family income, among other things.

- Consider the following optimal schooling:

$$\begin{aligned} & \frac{\partial}{\partial s} \int_s^R e^{-r(\tau-6)} w_i(h(s), \tau - s; \psi, l) d\tau \\ &= J e^{-r(s-6)} \left[\frac{u(c^W(s)) - u(c^S(s)) + u_c(c^S(s))c^S(s) - u_c(c^W(s))c^W(s)}{u_c(c^S(s))} + e_p(s) \right] \\ &= RHS_{J>1} \end{aligned}$$

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 &= RHS_{J>1}
 \end{aligned}$$

- Notice that with $J = 1$, $RHS_{J=1} = e^{-r(s-6)} e_p(s)$
- So, $RHS_{J>1} > RHS_{J=1}$ for our assumed utility function (applies to power and log utilities).
- Budget constrained individuals face higher marginal cost on schooling.**

- Assume no public education (i.e. $e_g = 0$), call optimal private education expenditure in this hypothetical regime $\hat{e}^*(\tau)$:

$$\begin{aligned} & \hat{e}^*(\tau) \\ &= \left[\frac{\psi \gamma z_h^{\frac{\alpha}{\gamma}} h(s)^{1-\frac{\alpha}{\gamma}} \int_s^R e^{-r(\tau-s)} \tilde{w}'_i(h(s), \tau-s; \psi, l) e^{v_{1i}(\tau-s)+v_{2i}(\tau-s)^2} d\tau}{J} \right]^{\frac{1}{1-\alpha}} e^{\frac{r(\tau-6)}{1-\alpha}} \\ &= \hat{e}(0) e^{\frac{r(\tau-6)}{1-\alpha}} \end{aligned}$$

- Now, introduce public education $e_g(\tau) \equiv e_g$ for $\tau \in [\underline{s}, \bar{s}]$ define total education investment $e(\tau) = e_p(\tau) + e_g(\tau)$

$$e(\tau) = \begin{cases} \hat{e}^*(\tau) & \text{for } \tau \leq \min\{s, \underline{s}\} \\ e_g & \text{for } \min\{s, \underline{s}\} \leq \tau \leq s_g \\ \hat{e}^*(\tau) & \text{for } s_g \leq \tau \leq s \end{cases}$$

where $s_g \equiv \min\{s, \bar{s}, \max[\underline{s}, s_{UG}]\}$ and s_{UG} is defined as $\hat{e}^*(s_{UG}) = e_g$

Government Policy $\{\underline{s}, \bar{s}, e_g\}$ and Education

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- As $\hat{e}^*(\tau)$ is increasing in τ , for all $\tau > s_{UG}$, $\hat{e}^*(\tau) > e_g$ (private investment)
- s_g is the (potential, contingent on $\min\{s, \underline{s}\} \leq s_g$) upper bound age that an individual solely depends on public education investment Example
- Finally, one's human capital will be:

$h(s)$

$$= z_h \psi^{\frac{\gamma}{\alpha}} \hat{e}(0)^\gamma \left[\int_0^{\min\{s, \underline{s}\}} e^{\frac{r\alpha(\tau-6)}{1-\alpha}} d\tau + \int_{\min\{s, \underline{s}\}}^{s_g} \left(\frac{e_g}{\hat{e}(0)}\right)^\alpha d\tau + \int_{s_g}^s e^{\frac{r\alpha(\tau-6)}{1-\alpha}} d\tau \right]^{\frac{\gamma}{\alpha}}$$

Numerical Analysis

- Distribution of (ψ, l, b)

- Assume ψ, l, b follows log-normal, i.e. $\ln(y) \sim \mathcal{N}(\mu_y, \sigma_y)$ with cdf $G^y(y)$ and

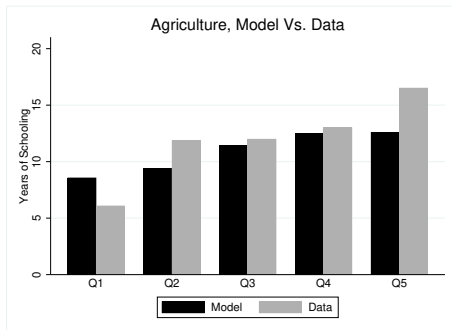
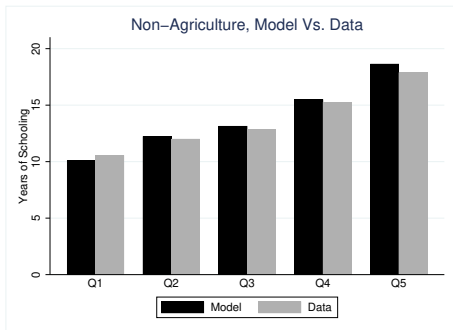
$$G(\psi, l, b) = G^b(b)G^{\psi, l}(\psi, l)$$

- $G^{\psi, l}(\psi, l)$ with $\rho_{\psi, l}$ determining the extent of dependence in Frank copula
- Set $\mu_\psi = \mu_l = 1$ and calibrate $\mu_b, \sigma_x, \rho_{\psi, l}$

Parameters	Value	Target
Panel A: Predetermined		
Talent	$\mu_\psi = \mu_l = 1$	Normalize
Preference	$\rho = 0.03, \zeta = 0.005, \sigma = 1.5, \eta = 0.85$	Preset or Literature
Human capital	$z_h = 1, \underline{s} = 6, \bar{s} = 18$	Normalize or Data
Experience	$v_{1,a} = 0.0254, v_{2,a} = -0.0004,$ $v_{1,m} = 0.0382, v_{2,m} = -0.0006$	IPUMS USA
Production	$A_a = 1$	Normalize
Life exp. & retirement	$T = 76.6, R = 65$	Data
Panel B: Calibrated		
Production	$A_m = 0.33, \theta_m = 0.82, \phi_m = 4.00,$ $\theta_a = 0.74, \phi_a = -3.16$	1. Agri. Wage Gap, 2. Var. Agr. Wage, 3. Var Non-agr. Wage, 4. Agr. Emp. Share, 5. Agr. V.A. Share, 6. Agr. School Years, 7. Non-Agr. School Year, 8. Private Exp. on School, 9. Public Exp. on School, 10. Agr. Return to School, 11. Non-agr. Return to School, 12. Wealth-Wage Ratio, 13. S.D. log Wealth, 14. Non-agr. Price Gap
Talent/ Wealth	$\sigma_\psi = 0.45, \sigma_l = 0.47, \rho_{\psi l} = 10.03,$ $\mu_b = 5.65, \sigma_b = 0.63$	
Human Capital	$\alpha = 0.25, \gamma = 0.26, e_g = 6.76$	
Preference	$\bar{c} = 0.15$	

Target	Numerically	Data	Model
Agri. Wage Gap	$\frac{w_m}{w_a}$	1.427	1.469
Var. Agr. Wage	$Var(w_a)$	0.144	0.153
Var. Non-agr. Wage	$Var(w_m)$	0.224	0.220
Agri. Emp. Share (%)	$\frac{L_a}{Y}$	1.50	1.51
Agri. V.A. Share (%)	$\frac{V_a}{Y}$	1.10	1.03
Agri. School Years	s_a	11.55	11.01
Non-Agr. School Years	s_m	13.18	13.92
Private Exp. on School (%)	$\frac{E_p}{Y}$	2.10	2.74
Public Exp. on School (%)	$\frac{E_g}{Y}$	4.95	5.56
Agri. Return to School	$\frac{\partial w_a}{\partial s}$	0.050	0.056
Non-Agr. Return to School	$\frac{\partial w_m}{\partial s}$	0.075	0.074
Wealth-Income Ratio	$\frac{W_i}{w_i}$	2.45	1.92
S.D. log Wealth	$SD(\ln(W_i))$	11.41	10.52
Non-agr. Price Gap	$\frac{p_m}{p_a}$	1.60	1.60

- Distribution of Sectoral Years of Schooling



Counterfactual Experiment

	Baseline	$\{A_a, A_m\} +10\%$	$\bar{s} +10\%$	$e_g +10\%$
Agr. YOS	11.01	10.63	11.34	10.96
Non-Agr. YOS	13.92	13.51	14.23	13.92
Agr. HC	1.00	0.98	1.65	1.16
Non-Agr. HC	1.00	0.97	1.16	1.12
Agr. Prod. Gap	1.47	1.45	1.41	1.46
Agr. Emp. Share	1.51%	1.38%	1.46%	1.50%
Agr. V.A. Share	1.03%	0.95%	1.04%	1.03%

- Choose Bangladesh (BGD) (10th percentile in world income distribution in 2005)
- 10 parameters difference:
 - Preset:
 - Parameters: $\underline{s}, \bar{s}, T, \mu_b, \sigma_b$
 - Calibrated:
 - Parameters: $A_a, A_m, \theta_a, \theta_m, e_g$
 - Targets: GDP ratio, agricultural employment share, sectoral years of schooling, public schooling expenditure

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 - Targets: GDP ratio, agricultural employment share, sectoral years of schooling, public schooling expenditure
- How to set μ_b, σ_b ?
 - Assume that wealth concentration, GDP per capita and fertility rate will affect mean of inter vivo transfer in Bangladesh $E(b_{BGD})$

$$E(b_{BGD}) = e^{\mu_b, USA + \sigma_b, USA / 2} \times \frac{\text{wealth/income}_{BGD}}{\text{wealth/income}_{USA}} \times \frac{\text{gdppc}_{BGD}}{\text{gdppc}_{USA}} \times \frac{\text{tfr}_{USA}}{\text{tfr}_{BGD}}$$

- Back out σ_b, BGD using the top 10% wealth concentration relative to USA and σ_b, USA

	Model	Data
Agricultural Output per Worker (Relative to U.S.)	114	79.2
Non-Agricultural Output per Worker (Relative to U.S.)	31.9	3.39
Agricultural Productivity Gap ($\frac{p_m Y_m / N_m}{p_a Y_a / N_a}$)	3.28	3.68

Conclusion

- We develop a lifecycle framework to study the role of human capital quality in determining the process of structural transformation and cross-country productivity difference
- Education decision and quality are important for structural transformation and sectoral productivity differences
- Future Work:
 - Schooling decreasing with TFP, endogenize b and e_g to incorporate government spending
 - Calibrate the model to fit international data
 - See how education affects structural transformation and productivity (c.f. Bils and Klenow (2000))

Thank You

- Economy with education policy $[\underline{s}, \bar{s}] = [6, 18]$
- s depends positively on one's general endowment
- $s_{ug} = \frac{1-\alpha}{r} \ln\left(\frac{e_g}{\hat{e}(0)}\right) + \underline{s}$, depends negatively on $\hat{e}(0)$ which is related to one's general endowment
- $s_g \equiv \min\{s, \bar{s}, \max[\underline{s}, s_{ug}]\}$, the (potential, contingent on $\min\{s, \underline{s}\} \leq s_g$) upper bound age that an individual solely depends on public education investment

Individual	Schooling until age	s_{ug}	s_g	Limited by
A (PhD)	30	5	6 (no s_g)	\underline{s}
B (College Grad)	22	14	14	s_{ug}
C (College Drop Out)	19	20	18	\bar{s}
D (No Schooling)	4	24	4 (no s_g)	s

Examples of Government Education Policies

Individual	Schooling until age	s_{ug}	s_g	Limited by
A (PhD)	30	5	6 (no s_g)	\underline{s}
B (College Grad)	22	14	14	s_{ug}
C (College Drop Out)	19	20	18	\bar{s}
D (No Schooling)	4	24	4 (no s_g)	s

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