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ARTICLE

Land-price dynamics and macroeconomic fluctuations with general household preferences

Been-Lon Chen[1], Zheng-Ze Lai[2], and Shian-Yu Liao[3]

[1]Institute of Economics, Academia Sinica, Taipei, Taiwan
[2]Department of Economics, National Chengchi University, Taipei, Taiwan
[3]Department of Economics, Fu Jen Catholic University, New Taipei City, Taiwan
Corresponding author: Been-Lon Chen; Email: bchen@econ.sinica.edu.tw

Abstract

Through the collateral channel for entrepreneurs, a positive housing demand shock in Liu et al. [(2013) *Econometrica* 81, 1147–1184.] increases land prices and business investment, but consumption decreases on impact and there is thus a comovement problem. This paper improves Liu et al. [(2013) *Econometrica* 81, 1147–1184.] by adding general household preferences with broader intratemporal and intertemporal substitutions Bayesian estimation of our structural model based on aggregate US data suggests that the intratemporal substitution is larger than unity and the intertemporal substitution is smaller than unity. Our impulse responses show that a positive housing demand shock increases land prices, business investment, and consumption, which resolves the comovement problem. Moreover, the strength of the collateral channel linking land prices and business investment in our Bayesian DSGE model is larger than that in Liu et al. [(2013) *Econometrica* 81, 1147–1184.]. Housing demand shocks explain 39−43% of the variance of output and 41−47% of the variance of investment in our model, but the same shocks explain only 17−31% of the variance of output and 30−41% of the variance of investment in Liu et al. [(2013) *Econometrica* 81, 1147–1184.]. Variance decomposition reveals that housing demand shocks account for a larger share of the fluctuations in land prices, investment, employment, and output than other shocks. Using the marginal data density as the measure of fit for models, we find that our model can better explain the same US aggregate data.

Keywords: land prices; housing demand shocks; CES preferences; collateral constraints

1. Introduction

The financial crisis of 2007–2008 propelled the USA and world economies into the most severe global recession since the Great Depression. Triggered by the sudden and severe slump of the US housing market, the financial crisis shrank asset values tied to US real estate. This came with a sharp decline in housing and land prices and a harsh collapse in business investment, leaving a stark decrease in employment, output, and consumption. The crisis has sparked substantial interest in what drives house or land prices and how they affect macroeconomies. To understand the salient features on house prices, a strand of recent dynamic stochastic general equilibrium (DSGE) literature assumes that a subset of households is impatient and credit-constrained, and these households use houses or land as collateral to finance their consumption spending. See Iacoviello (2005), Iacoviello and Neri (2010), Justiniano et al. (2015), and Favilukis et al. (2017). Through the collateral channel for impatient households, these models are capable of explaining positive comovement between land prices and consumption, but in general they have difficulty accounting for positive comovement between land prices and business investment [Iacoviello and Neri (2010)].

To conquer the difficulty, another strand of literature assumes that firms, instead of households, are credit-constrained. By way of the collateral channel for entrepreneurs, a recent Bayesian DSGE model by Liu et al. (2013) has obtained positive comovement between land prices and business investment. In the Liu et al. (2013) model, the household preference is logarithmic in consumption and housing services. Land is used for housing services and production, and land owned by entrepreneurs serves as collateral. A positive residential housing demand shock increases land prices and raises entrepreneurs' collateralized loans, which increases business investment, employment, and output. However, the land price surge leads to a decrease of consumption on impact in Liu et al. (2013).[1] Empirical evidence using micro-data indicates that a rise in house prices increases consumer spending.[2] Moreover, the estimated Bayesian vector autoregression (BVAR) model in Liu et al. (2013) shows positive comovement between land prices and consumption. Thus, the Liu et al. (2013) model has difficulty delivering positive comovement between land prices and consumption.

The responses of business investment, consumption, and residential housing services to a land price increase depend on both the intratemporal and the intertemporal elasticity of substitution (henceforth, ES).[3] In this paper, we overcome the difficulty in the Liu et al. (2013) model by considering a general household preference with broader intratemporal and intertemporal ESs than the logarithmic preference in Liu et al. (2013). Specifically, our household preference allows for both intertemporal ES between consumption and residential housing services within a given period of time and the intertemporal ES for consumption over different time periods to deviate from unity. We will show that, as land prices rise in response to a positive housing demand shock, business investment increases owing to the relaxation of the collateral constraint for entrepreneurs, and consumption also increases if the intertemporal ES is smaller than unity and less than the intratemporal ES. Our reason goes as follows. When the intertemporal ES for consumption over time is less than unity and smaller than the intratemporal ES between consumption and housing services within a period, a positive housing demand shock will increase land prices largely, so housing services are substituted away toward consumption. As a result, land prices comove with both business investment and consumption, consistent with empirical evidence of micro-data and impulse responses of the estimated BVAR model in Liu et al. (2013).

Using US aggregate data, we estimate the intratemporal ES and the intertemporal ES within the structural model. Our estimates show that the intertemporal ES is less than unity and the intratemporal ES is larger than unity, which are consistent with the estimates in the existing literature.[4] In our Bayesian DSGE model, the impulse responses to positive residential housing demand shocks indicate that the land price increases largely and comoves with consumption and business investment. Moreover, the strength of the collateral channel linking land prices and business investment in our model is larger than that in Liu et al. (2013). In particular, shocks to residential housing demands explain 39−43% of the variance of output and 41−47% of the variance of investment in our model, but the same shocks explain only 17−31% of the variance of output and 30−41% of the variance of investment in Liu et al. (2013). As for the relative importance of different shocks, variance decomposition reveals that housing demand shocks account for a larger share of the fluctuations in the land price, investment, hours, and output than other shocks. Using the marginal data density as the measure of fit for models, we find that our model can better explain the same US aggregate data.

Other extensions of the collateral channel model of Liu et al. (2013) include Liu et al. (2016), which added the labor search and matching framework to the model of Liu et al. (2013) and explained the observed negative relation between land prices and unemployment and the fact that housing demand shocks have a large effect on the volatility of unemployment. Gong et al. (2017) extended the Liu et al. (2013) model to allow for a household utility that is separable in residential housing services and non-separable in consumption and leisure. Bahaj et al. (2019) allowed small and medium-sized enterprises in the Liu et al. (2013) model to use the directors' homes as collateral for business investment, so there are both the residential and corporate collateral channels. Davis et al. (2022) added a land development sector into the Liu et al. (2013) model to allow residential and commercial land to be imperfectly substitutable. Among these, Gong et al. (2017) are more related to our model, as they extend the household utility in Liu et al. (2013) to one that is separable between residential housing services and consumption. However, in response to the land price increase triggered by a positive housing demand shock, their consumption decreases. As a result, Gong et al. (2017) have a comovement problem.

Our paper is related to Barsky et al. (2003, 2007), Monacelli (2009), and Chen and Liao (2014), which also consider models with a general household preference for nondurables and durable housing services, and thus, there are both intratemporal and intertemporal substitutions. The difference is that we explore the effects of the residential housing demand shock, as opposed to the effects of the monetary policy shock analyzed in these existing papers.

Finally, our paper is also broadly related to the papers that envisage the amplification effect through the borrowing constraint. On this, the seminal work is Kiyotaki and Moore (1997). See also papers by Kocherlakota (2000), Cordoba and Ripoll (2004), and Cao and Nie (2017), which analyzed the quantitative significance of the amplification through the borrowing constraint.

The remaining of the paper is organized as follows. Section 2 sets up the model, while Section 3 is the estimation strategy. Section 4 studies the impulse responses of positive housing demand shocks and the role of different intertemporal and intratemporal elasticities of substitutions. Section 5 analyzes the relative importance of different structural shocks in terms of variance decompositions. Finally, Section 6 is the concluding remark.

2. The model

Our model is otherwise identical to that of Liu et al. (2013) except for a general household preference. The economy has a representative household and a representative entrepreneur. The household consumes goods and land services (housing) and supplies labor, while the entrepreneur consumes consumption goods only. The entrepreneur produces final goods using labor, capital, and land and needs external financing for investment. Due to imperfect contract enforcement, the borrowing capacity is constrained by the value of collateral assets, consisting of land and capital. As in Liu et al. (2013), we assume that the household is more patient than the entrepreneur, so that the entrepreneur's collateral constraint is binding in and near the steady-state equilibrium. The supply of land is fixed.

2.1 The representative entrepreneur

As in Liu et al. (2013), the entrepreneur's utility is given by

$$E\sum_{t=0}^{\infty}\beta^t[\log(C_{e,t}-\gamma_e C_{e,t-1})], \tag{1a}$$

where $C_{e,t}$ is entrepreneur's consumption, γ_e is the degree of entrepreneur's habit persistence, and $\beta\in(0,1)$ is the discount factor.

The entrepreneur produces final goods with the production technology given by

$$Y_t = Z_t[L_{e,t-1}^{\phi}K_{t-1}^{1-\phi}]^{\alpha}N_{e,t}^{1-\alpha}, \tag{1b}$$

where Y_t denotes output, $L_{e,t\text{-}1}$ is land, $K_{t\text{-}1}$ is capital, and $N_{e,t}$ is labor input. Parameters $\alpha\in(0,1)$ and $\phi\in(0,1)$ measure the output elasticities of these production factors.

As in Liu et al. (2013), the total factor productivity Z_t consists of a permanent component Z_t^p and a transitory component $v_{z,t}$ such that $Z_t = Z_t^p v_{z,t}$. The permanent component Z_t^p follows $Z_t^p = Z_{t-1}^p\lambda_{zt}$, where the growth rate λ_{zt} and the transitory component $v_{z,t}$ follow the stochastic

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process given, respectively, by

$$ln\lambda_{z,t} = (1-\rho_z)ln\overline{\lambda}_z + \rho_z ln\lambda_{z,t-1} + \sigma_z\varepsilon_{z,t}, \tag{2a}$$

$$lnv_{z,t} = \rho_{vz} lnv_{z,t-1} + \sigma_{vz}\varepsilon_{vz,t}. \tag{2b}$$

In (2a) and (2b), $\overline{\lambda}_z$ is the steady-state value of λ_{zt}. Parameters ρ_z and $\rho_{vz} \in (-1, 1)$ are the degree of persistence, σ_z and σ_{vz} are the standard deviations, and the innovations $\epsilon_{z,t}$ and $\varepsilon_{vz,t}$ are independent and identically distributed (*i.i.d.*) standard normal processes.

The entrepreneur faces the flow of funds constraint given by

$$C_{e,t} + q_{l,t}(L_{e,t} - L_{e,t-1}) - \frac{B_t}{R_t} = Z_t[L_{e,t-1}^{\phi} K_{t-1}^{1-\phi}]^{\alpha} N_{e,t}^{1-\alpha} - \frac{I_t}{Q_t} - w_t N_{e,t} - B_{t-1}, \tag{3}$$

where I_t is investment, $q_{l,t}$ is the land price, $B_{t\text{-}1}$ is the amount of matured debts, R_t is the gross real interest rate, w_t is the real wage rate, and B_t/R_t is the value of new debts.

As in Greenwood et al. (1997), there is the investment-specific technology change Q_t. Following Liu et al. (2013), we assume that $Q_t = Q_t^p v_{q,t}$, in which the permanent component Q_t^p follows $Q_t^p = Q_{t-1}^p \lambda_{qt}$, where the growth rate λ_{qt} and the transitory component $v_{q,t}$ follow the stochastic process given, respectively, by

$$ln\lambda_{q,t} = (1-\rho_q)ln\overline{\lambda}_q + \rho_q ln\lambda_{q,t-1} + \sigma_q\varepsilon_{q,t}, \tag{4a}$$

$$lnv_{q,t} = \rho_{vq} lnv_{q,t-1} + \sigma_{vq}\varepsilon_{vq,t}, \tag{4b}$$

where $\overline{\lambda}_q$ is the steady-state value of λ_{qt}, and the parameters ρ_q and $\rho_{vq} \in (-1, 1)$ are the degree of persistence, while σ_q and σ_{vq} are the standard deviations, and the innovations $\varepsilon_{q,t}$ and $\varepsilon_{vq,t}$ are *i.i.d.* standard normal processes.

The entrepreneur is endowed with $K_{t\text{-}1}$ units of capital and $L_{e,t\text{-}1}$ units of land initially. Capital is accumulated from investment that follows the law of motion given by

$$K_t = (1-\delta)K_{t-1} + \left[1 - \frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}} - \overline{\lambda}_I\right)^2\right] I_t, \tag{5}$$

where δ is the depreciation rate, $\overline{\lambda}_I$ is the steady-state growth rate of investment, and $\Omega > 0$ is the adjustment cost parameter.

The loan market is imperfect, and collateral is required in order to take out loans. The entrepreneur faces the following credit constraint

$$B_t \leq \varsigma_t E_t[q_{l,t+1}L_{e,t} + q_{k,t+1}K_t], \tag{6}$$

where $q_{k,t+1}$ is the shadow price of capital in unit of final goods, and ς_t is interpreted as a collateral shock.

Following Kiyotaki and Moore (1997), we interpret this type of credit constraint as reflecting the problem of costly contract enforcement. The credit constraint implies that, if the entrepreneur fails to repay the debt in the next period, the creditor can seize the collateral assets, which is the value of land and accumulated capital in the next period. As it is costly to liquidate the seized land and capital stock, the creditor can recover up to a fraction ς_t of the total value of collateral assets.

Following Liu et al. (2013), ς_t follows the stochastic process given by

$$ln\varsigma_t = (1-\rho_\varsigma)ln\overline{\varsigma} + \rho_\varsigma ln\varsigma_{t-1} + \sigma_\varsigma\varepsilon_{\varsigma,t}, \tag{7}$$

where $\overline{\varsigma}$ is the steady-state value of ς_t, and $\rho_\varsigma \in (-1, 1)$ is the persistent parameter, while σ_ς is the standard deviation, and the innovation $\varepsilon_{\varsigma,t}$ is an *i.i.d.* standard normal process.

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2.2 The representative household

The household's discounted utility function is given by

$$E\sum_{t=0}^{\infty}\beta^t A_t[U(C_{h,t},L_{h,t})-\psi_t N_{h,t}], \tag{8a}$$

where $C_{h,t}$ is consumption, $L_{h,t}$ is (durable) land services,[5] and $N_{h,t}$ is labor hours, wherein consumption and land services in period t are aggregated to the consumption bundle $U(C_{h,t},L_{h,t})$. Different from Liu et al. (2013), the consumption bundle is a constant-elasticity-of-substitution (henceforth, CES) form given by[6]

$$U\big(C_{h,t},L_{h,t}\big)=\frac{1}{1-1/\eta}[u(C_{h,t},L_{h,t})]^{\frac{1-1/\eta}{1-1/\zeta}} \text{ and}$$

$$u\big(C_{h,t},L_{h,t}\big)=(1-\chi)\left(\frac{C_{h,t}-\gamma_h C_{h,t-1}}{\Gamma_t}\right)^{1-\frac{1}{\zeta}}+\chi\left(L_{h,t}^{\varphi_t}\right)^{1-\frac{1}{\zeta}}, \tag{8b}$$

where χ denotes the relative weight between consumption and land services, and γ_h is the degree of the habit persistence. The household also obtains a negative utility from supplying labor hours, which gives an infinite Frisch labor supply elasticity, as in Liu et al. (2013).

Following Liu et al. (2016), consumption is scaled by the growth factor $\Gamma_t\equiv[Z_tQ_t^{(1-\phi)\alpha}]^{\frac{1}{1-(1-\phi)\alpha}}$ in order to be consistent with the balanced growth, where Z_t is the total factor productivity and Q_t is the investment-specific technology. In (8a), A_t is the household's patience factor and evolves as $A_t=A_{t-1}(1+\lambda_{a,t})$, where $\lambda_{a,t}$ represents a shock to the household's patience factor. Moreover, φ_t is a shock to the household's demand for land services, which is also labeled as the housing demand shock, and ψ_t is a shock to the labor supply. The stochastic processes of those shocks are the same as those in Liu et al. (2013), given as follows.

$$ln\lambda_{a,t}=(1-\rho_a)ln\overline{\lambda}_a+\rho_a ln\lambda_{a,t-1}+\sigma_a\varepsilon_{a,t}, \tag{9a}$$

$$ln\varphi_t=(1-\rho_\varphi)ln\overline{\varphi}+\rho_\varphi ln\varphi_{t-1}+\sigma_\varphi\varepsilon_{\varphi,t}, \tag{9b}$$

$$ln\psi_t=(1-\rho_\psi)ln\overline{\psi}+\rho_\psi ln\psi_{t-1}+\sigma_\psi\varepsilon_{\psi,t}, \tag{9c}$$

where $\overline{\lambda}_a,\overline{\varphi},\overline{\psi}>0$ are steady-state values of $\lambda_{a,t},\varphi_t,\psi_t$, while $\rho_a,\rho_\varphi,\rho_\psi\in(-1,1)$ are persistence parameters, and $\sigma_a,\sigma_\varphi,\sigma_\psi>0$ are the standard deviations of the innovation. The innovations $\varepsilon_{a,t}$, $\varepsilon_{\varphi,t}$, $\varepsilon_{\psi,t}$ are *i.i.d.* standard normal processes.

The function of the consumption bundle U_t has two parameters η and ζ. The parameter η is the intertemporal ES between consumption bundles across periods. For a higher value of η, the representative household is more willing to substitute consumption bundles over time. The consumption bundles across periods are perfect substitutes if $\eta\to\infty$ and perfect complements if $\eta\to 0$, and the bundle has the Cobb–Douglas function if $\eta\to 1$.[7] The parameter ζ represents the intratemporal ES between consumption and land services within a given period of time. For a higher value of ζ, the household is more willing to substitute one for the other. Consumption and land services are perfect substitutes within a given period as $\zeta\to\infty$ and perfect complements as $\zeta\to 0$. Taking the limit as $\zeta\to 1$ yields the Cobb–Douglas function. In the case if $\zeta=\eta$, the consumption bundle is separable in consumption and land services, and the intertemporal substitution effect is equal to the intratemporal substitution effect. The special case $\eta=\zeta=1$ yields the household's utility of the consumption bundle in Liu et al. (2013), wherein the intertemporal substitution effect equals the income effect, and equals the intratemporal substitution effect.

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The cross partial derivative of the bundle U_t with respect to the two goods is affected by η and ζ given by[8]

$$U_{C_hL_h} \equiv \frac{\partial}{\partial L_{h,t}}\left(\frac{\partial U_t}{\partial C_{h,t}}\right)$$
$$= \left(\frac{1}{\zeta} - \frac{1}{\eta}\right)(1-\chi)\,\chi\varphi_t\,(u_t)^{\frac{1-1/\eta}{1-1/\zeta}-2}\left(L_{h,t}^{\varphi_t\left(1-\frac{1}{\zeta}\right)-1}\right)\left(\frac{C_{h,t}-\gamma_h C_{h,t-1}}{\Gamma_t}\right)^{-\frac{1}{\zeta}}, \qquad (10)$$

which indicates the relative strength of the intratemporal and intertemporal tradeoffs. The sign of the cross partial derivative is determined by $(\eta - \zeta)$, which is positive if $(\eta - \zeta) > 0$ and negative if $(\eta - \zeta) < 0$. In particular, when $(\eta - \zeta) < 0$, the intertemporal tradeoff is smaller than the intratemporal tradeoff, and then, a higher housing price tends to substitute away from land services toward consumption. In the special case of the Liu et al. (2013) model, $(\eta - \zeta) = 0$, and a higher housing price will not substitute away from land services toward consumption.

The household faces the flow budget constraint given by

$$C_{h,t} + q_{l,t}(L_{h,t} - L_{h,t-1}) + \frac{S_t}{R_t} \le w_t N_{h,t} + S_{t-1}, \qquad (11)$$

where S_t is the risk-free bond. In the initial period, the household is endowed with $L_{h,t\text{-}1}$>0 units of land and with $S_{\text{-}1}$>0 units of the risk-free bond.

The household chooses $C_{h,t}$, $L_{h,t}$, $N_{h,t}$, and S_t to maximize the expected lifetime utility in (8a) and (8b) subject to (11) and the borrowing constraint $S_t \ge -\bar{S}$ for some large number $\bar{S}$.[9]

2.3 Equilibrium

There are four markets: the markets for final goods, land, labor, and loans. All markets are clear in the competitive equilibrium. First, the final goods market clearing condition is

$$C_t + \frac{I_t}{Q_t} = Y_t, \qquad (12a)$$

where $C_t = C_{h,t} + C_{e,t}$ is aggregate consumption. Next, the land market clearing condition is

$$L_{h,t} + L_{e,t} = \bar{L}. \qquad (12b)$$

Moreover, the clearing condition for the labor market is

$$N_{e,t} = N_{h,t} \equiv N_t. \qquad (12c)$$

Finally, the market clearing condition for loans is

$$S_t = B_t. \qquad (12d)$$

A competitive equilibrium is sequences of prices $\{w_t, q_{lt}, R_t\}_{t=0}^{\infty}$ and allocations$\{C_{h,t}, C_{e,t}, I_t, N_{h,t}, N_{e,t}, L_{h,t}, L_{e,t}, S_t, B_t, K_t, Y_t\}_{t=0}^{\infty}$ such that given the sequence of prices, (i) the allocations maximize the household's problem; (ii) the allocations solve the entrepreneur's problem; and (iii) all the markets clear.

2.4 The role of η and ζ in the land price in response to the housing demand shock

The fluctuations in the land price are vital in the propagation of housing demand shocks via the collateral channel into impacts on consumption, investment, and other variables. In response to a positive housing demand shock, when the land price is increased sufficiently, the land service can be substituted away toward consumption. Then, consumption comoves with the land price.

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This subsection analyzes the role that η and ζ play in the effect on the land price in response to a positive housing demand shock.

In the Appendix, we have derived the relationship between the land price $q_{l,t}$ and the housing demand shock φ_t, given as follows.

$$q_{l,t} = \beta E_t q_{l,t+1} \frac{\mu_{h,t+1}}{\mu_{h,t}} + \frac{\varphi_t \Lambda_t(\eta, \zeta)}{\mu_{h,t}}, \tag{13}$$

where $\Lambda_t(\eta, \zeta) \equiv A_t \chi L_{h,t}^{\varphi_t(1-1/\zeta)-1} \left[(1-\chi) \left(\frac{C_{h,t} - \gamma_h C_{h,t-1}}{\Gamma_t} \right)^{1-1/\zeta} + \chi \left(L_{h,t}^{\varphi_t} \right)^{1-1/\zeta} \right]^{\frac{1-1/\eta}{1-1/\zeta}-1}$, and $\mu_{h,t}$ is the Lagrange multiplier of the household's budget constraint in (11), which is the marginal utility of a household's consumption in t, and $\varphi_t \Lambda_t(\eta, \zeta)$ is the marginal utility of a household's land services in t.

In (13), the first term in the right-hand side, $\frac{\mu_{h,t+1}}{\mu_{h,t}}$, is the marginal rate of substitution (hereafter, MRS) of a household's consumption bundle between periods t and $t+1$. The second term in the right-hand side, $\frac{\varphi_t \Lambda_t(\eta, \zeta)}{\mu_{h,t}}$, is the MRS between a household's consumption and land services within a given period of time. When there is a positive housing demand shock (i.e. when φ_t increases), there are direct and indirect effects to increase the land price. The direct effect is through φ_t in the second term in (13), which directly increases the land price. The indirect effects act through affecting the MRS of a household's consumption bundles between periods t and $t+1$ and the MRS between a household's consumption and land services within a given period t of time. The indirect effects are where the intertemporal ES η and the intratemporal ES ζ play a role.

To illustrate the role of η and ζ, let us simplify the household's utility of the consumption bundle by assuming no consumption habits and no growth factors, so $\gamma_h = 0$ and $A_t = \Gamma_t = 1$. Thus, the household's consumption bundle in (8b) reduces to

$$U(C_{h,t}, L_{h,t}) = \frac{1}{1-1/\eta} \left[(1-\chi) (C_{h,t})^{1-\frac{1}{\zeta}} + \chi \left(L_{h,t}^{\varphi_t} \right)^{1-\frac{1}{\zeta}} \right]^{\frac{1-1/\eta}{1-1/\zeta}}. \tag{14}$$

It serves to denote $\Delta_t(\varphi_t) \equiv (1-\chi) (C_{h,t})^{1-\frac{1}{\zeta}} + \chi \left(L_{h,t}^{\varphi_t} \right)^{1-\frac{1}{\zeta}}$. Then, the marginal utility of a household's consumption and the marginal utility of a household's land services are, respectively, given by

$$\mu_{h,t} = (1-\chi) (C_{h,t})^{-\frac{1}{\zeta}} (\Delta_t)^{\frac{1-1/\eta}{1-1/\zeta}-1},$$

$$\varphi_t \Lambda_t(\eta, \zeta) = \chi \varphi_t L_{h,t}^{\varphi_t(1-1/\zeta)-1} (\Delta_t)^{\frac{1-1/\eta}{1-1/\zeta}-1}.$$

In the special case of the Liu et al. (2013) model, $\zeta = \eta = 1$, and (14) reduces to the log separable form given by $U_t = (1-\chi) log(C_{h,t}) + \chi \varphi_t log L_{h,t}$. As a result, the marginal utility of a household's consumption is $\mu_{h,t} = (1-\chi)(C_{h,t})^{-1}$, and the marginal utility of a household's land services is $\varphi_t \Lambda_t(1, 1) = \varphi_t \chi (L_{h,t})^{-1}$, which is independent of the marginal utility of a household's consumption. In this case, the relationship between the land price $q_{l,t}$ and the housing demand shock φ_t in (13) reduces to

$$q_{l,t} = \beta E_t q_{l,t+1} \frac{C_{h,t}}{C_{h,t+1}} + \frac{\chi \varphi_t}{1-\chi} \frac{C_{h,t}}{L_{h,t}}. \tag{15}$$

Then, a positive housing demand shock increases the land price only through the direct effect summarized by the term $\frac{\chi \varphi_t}{1-\chi}$.

By contrast, when the value of η and ζ deviates from unity, the indirect effects emerge. The relationship between the land price $q_{l,t}$ and the housing demand shock φ_t in (13) is now

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$$q_{l,t}=\beta E_t q_{l,t+1}\left(\frac{C_{h,t}}{C_{h,t+1}}\right)^{1/\zeta}\left[\frac{(1-\chi)C_{h,t}^{(1-1/\zeta)}+\chi L_{h,t}^{\varphi_t(1-1/\zeta)}}{(1-\chi)\,C_{h,t+1}^{(1-1/\zeta)}+\chi L_{h,t+1}^{\varphi_{t+1}(1-1/\zeta)}}\right]^{1-\frac{1-1/\eta}{1-1/\zeta}}+\frac{\chi\varphi_t}{1-\chi}\frac{C_{h,t}^{1/\zeta}L_{h,t}^{\varphi_t(1-1/\zeta)}}{L_{h,t}}. \tag{16}$$

Thus, a positive housing demand shock φ_t increases the land price not only through the direct effect $\frac{\chi\varphi_t}{1-\chi}$ as in Liu et al. (2013), but also via indirect effects. Two cases of the indirect effects are in order.

First, when $\eta=1$ as in Liu et al. (2013) but $\zeta\neq 1$ different from Liu et al. (2013), (16) is

$$q_{l,t}=\beta E_t q_{l,t+1}\left(\frac{C_{h,t}}{C_{h,t+1}}\right)^{1/\zeta}\left[\frac{(1-\chi)\,C_{h,t}^{(1-1/\zeta)}+\chi L_{h,t}^{\varphi_t(1-1/\zeta)}}{(1-\chi)\,C_{h,t+1}^{(1-1/\zeta)}+\chi L_{h,t+1}^{\varphi_{t+1}(1-1/\zeta)}}\right]+\frac{\chi\varphi_t}{1-\chi}\frac{C_{h,t}^{1/\zeta}L_{h,t}^{\varphi_t(1-1/\zeta)}}{L_{h,t}}. \tag{17}$$

Then, other than the direct effect via $\frac{\chi\varphi_t}{1-\chi}$, the positive housing demand shock φ_t also generates the indirect effects through $L_{h,t}^{\varphi_t(1-1/\zeta)}$ in the first term and the second term in the right-hand side, which affect the MRS of a household's consumption bundle between periods t and $t+1$ and the MRS between a household's consumption and land services within a given period t, respectively. Thus, different intertemporal and intratemporal consumption smoothing effects are both at work. In particular, via these two additional terms in (17), if $\zeta>1$, a positive housing demand shock φ_t increases the land price more than that in the case of $\eta=1$ and $\zeta=1$ in Liu et al. (2013).

Next, in addition to $\zeta\neq 1$, when it is also $\eta<1$, then other than the direct effect via $\frac{\chi\varphi_t}{1-\chi}$ and the indirect effect through $L_{h,t}^{\varphi_t(1-1/\zeta)}$ in the second term in (17), the positive housing demand shock φ_t also has an indirect effect working via $\left(L_{h,t}^{\varphi_t(1-1/\zeta)}\right)^{1-\frac{1-1/\eta}{1-1/\zeta}}$ $(=L_{h,t}^{\varphi_t(\frac{1}{\eta}-\frac{1}{\zeta})})$ in the first term in (16), which changes the MRS of a household's consumption bundle between periods t and $t+1$. Thus, comparing with the case $\eta=1$ and $\zeta\neq 1$ above, a different intertemporal consumption smoothing effect is at work. In particular, via the first term, when $\eta<1$ and $\eta<\zeta$, the household is less willing to substitute away from current consumption toward future consumption, and also more willing to substitute away from the land service toward current consumption. In this situation, a positive housing demand shock φ_t exerts a further additional indirect effect to increase the land price. As the land price is increased sufficiently, the land service is substituted away toward current consumption. As a result, consumption increases and comoves with land prices. Note that, if we combine the condition $\eta<1$ and $\eta<\zeta$ with the condition $\zeta>1$, we obtain the condition $\eta<1<\zeta$. Under $\eta<1<\zeta$, these indirect effects are even stronger. Then, a positive housing demand shock φ_t increases the land price even more than that in the case of $\eta<1$ and $\eta<\zeta$.

3. Estimation

We take a log-linearization of the equilibrium system around the steady state. Following Liu et al. (2013), we use the Bayesian approach to fit the log-linearized equilibrium system to the same six quarterly US time series as used by Liu et al. (2013): the relative price of land ($q_{l,t}$), the inverse of the quality-adjusted relative price of investment (Q_t), real consumption per capita (C_t), real investment per capita in consumption units (I_t), real nonfinancial business debts per capita (B_t), and per capita hours worked (N_t). All these series are constructed in line with the corresponding series in Greenwood, et al. (1997), Cummins and Violante (2002), and Davis and Heathcote (2007). The sample covers the period from 1975: Q1 to 2010: Q4.

In the Bayesian estimation, a system of measurement equations links observable variables to state variables. By setting prior distributions and updating the joint distribution through the information contained in the observed data, the posterior distribution of the parameter set θ can be

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well approximated by a Markov chain Monte Carlo (MCMC) algorithm, and eventually, the value of the parameter set is obtained by maximizing the likelihood function. Yet, with binding credit constraints, the posterior kernel is filled with narrow but twisty ridges and local peaks. Thus, it is not only difficult to find the mode of the posterior distribution but also difficult to uncover the posterior mode of the built-in optimizing methods in the popular Dynare software.

In estimating structural parameters and shock parameters, our optimization routine follows Liu et al. (2013), which is based on Sims et al. (2008).[10] With an initial guess of the values of structural parameters and shock parameters, we use a combination of a constrained optimization algorithm and an unconstrained Broyden–Fletcher–Goldfarb–Shanno optimization algorithm to find a local peak. Then, the local peak is used to simulate a long sequence of MCMC posterior draws. These simulated draws are treated as different starting points in order for the optimization routine to find a potentially higher peak. We iterate this process, until the highest peak is found.

Our parameters are partitioned into three subsets: the structural parameters on which we have agnostic priors; the structural parameters for which we have the steady-state relations to construct informative priors; and the parameters which describe the shock processes. The prior distributions of these parameters follow from those in Liu et al. (2013). First, we employ the steady-state values to calibrate the values of $\{\alpha, \overline{\theta}, \ \overline{\psi}\}$.[11] Next, we apply the Bayesian method to estimate the structural parameters on which we have agnostic priors $\{\gamma_h, \gamma_e, \Omega, g_\gamma, \overline{\lambda}_q\}$. Then, we identify the structural parameters for which we have the steady-state relations to construct informative priors $\{\beta, \overline{\lambda}_a, \overline{\varphi}, \phi, \delta\}$. Finally, we adopt agnostic priors for the persistence and standard deviations of the shock processes $\{\rho_i, \sigma_i\}$ for the eight shock parameters $i \in \{a, z, \nu_z, q, \nu_q, \varphi, \psi, \theta\}$.

Although the household's utility in Liu et al. (2013), wherein these two ESs are $\eta = \zeta = 1$, is treated as the baseline model, we let the data and the model decide what the estimates of these two ESs should be. The existing literature has estimated the intertemporal ES η to be smaller than unity and the intratemporal ES ζ to be on average larger than unity.[12] Therefore, we set the prior distribution of η to be a standard uniform distribution $U(a, b)$ with (a, b)=(0, 1), which are the minimum and maximum values. Moreover, we set the prior distribution of ζ to be Gamma (a, b) with (a, b)=(1.00, 0.50), which has the shape parameter value $a = 1.00$ and the inverse scale parameter value $b = 1/2$. The prior distributions of other parameters are the same as those in Liu et al. (2013).

We report the prior distributions and the posterior modes of structural parameters and shock parameters under different values of η and ζ. Tables 1 and 2 report our preferred model when both η and ζ are estimated. The estimated value of η is 0.47 and the estimated value of ζ is 2.77. We will compare the impulse response functions in the baseline case ($\eta = 1$, $\zeta = 1$) with those in our preferred model ($\eta = 0.47$, $\zeta = 2.77$).[13]

As a robust check, we also consider two counterfactual economies ($\eta = 1$, $\zeta = 2.77$) and ($\eta = 0.47$, $\zeta = 1$), wherein one of the two values η and ζ is taken from our preferred model and the other is from the baseline case in Liu et al. (2013). The prior distributions and the posterior modes of structural parameters and shock parameters for the two counterfactual economies are reported in Tables 3 and 4 for the case ($\eta = 1$, $\zeta = 2.77$), and Tables 5 and 6 for the case ($\eta = 0.47$, $\zeta = 1$), respectively.

4. Impulse responses of a positive housing demand shock

This section analyzes the impulse responses on the business cycles of land prices and key macroeconomic variables. We investigate how different values of these two ESs affect the fluctuations in land prices and other variables in response to a positive housing demand shock. We start with the comparison of our preferred model ($\eta = 0.47$, $\zeta = 2.77$) with the baseline case ($\eta = 1$, $\zeta = 1$). As a robustness check, we also consider the comparison of our preferred model ($\eta = 0.47$, $\zeta = 2.77$) with the two counterfactual cases ($\eta = 1$, $\zeta = 2.77$) and ($\eta = 0.47$, $\zeta = 1$).

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Table 1. Prior distributions and posterior modes of the structural parameters

Parameter		Prior			Posterior		
Estimated	Description	Distribution	a	b	Mode	Low	High
γ_h	Household's habit persistence	Beta (a,b)	1.00	2.00	0.3943	0.3609	0.4940
γ_e	Entrepreneur's habit persistence	Beta (a,b)	1.00	2.00	0.7852	0.7225	0.8015
Ω	Capital adjustment cost parameter	Gamma (a,b)	1.00	0.50	0.3354	0.3108	0.4068
$100(g_\gamma$ - 1)	Steady-state growth rate of TFP	Gamma (a,b)	1.86	3.01	0.4718	0.4320	0.5646
$100(\lambda_q$ - 1)	Steady-state growth rate of IST	Gamma (a,b)	1.86	3.01	1.1101	0.9868	1.1859
Calibrated							
β	Discount factor		0.9842				
$\overline{\lambda}_a$	Steady-state growth rate of preference shock		0.0108				
ϕ	Share on land input		0.7496				
δ	Depreciation rate		0.0373				
$\overline{\varphi}$	Steady state of housing demand shock		0.4514				

Note: 1. Our estimated model: ($\eta = 0.47$, $\zeta = 2.77$).
2. "Low" and "High" denote the bounds of the 90% probability interval for the prior distribution.

Table 2. Prior distributions and posterior modes of the shock parameters

	Prior				Posterior		
Parameter	Description	Distribution	a	b	Mode	Low	High
ρ_a	Intertemporal preference shock	Beta (a,b)	1.00	2.00	0.8889	0.8851	0.9363
ρ_z	Permanent neutral technology shock	Beta (a,b)	1.00	2.00	0.6482	0.6084	0.6967
ρ_{vz}	Transitory neutral technology shock	Beta (a,b)	1.00	2.00	0.0010	0.0008	0.0043
ρ_q	Permanent shock to IST change	Beta (a,b)	1.00	2.00	0.6738	0.6037	0.6937
ρ_{vq}	Transitory shock to IST change	Beta (a,b)	1.00	2.00	0.4665	0.3647	0.5243
ρ_φ	Housing demand shock	Beta (a,b)	1.00	2.00	0.9998	0.9993	0.9998
ρ_ψ	Labor supply shock	Beta (a,b)	1.00	2.00	0.9884	0.9831	0.9929
ρ_ς	Collateral shock	Beta (a,b)	1.00	2.00	0.9752	0.9729	0.9838
σ_a	SD on Intertemporal preference shock	Inv-Gamma (a,b)	0.3261	1.45E-04	0.1648	0.1109	0.2230
σ_z	SD on Permanent neutral tech shock	Inv-Gamma (a,b)	0.3261	1.45E-04	0.0032	0.0024	0.0044
σ_{vz}	SD on Transitory neutral tech shock	Inv-Gamma (a,b)	0.3261	1.45E-04	0.0070	0.0064	0.0082
σ_q	SD on Permanent shock to IST change	Inv-Gamma (a,b)	0.3261	1.45E-04	0.0029	0.0026	0.0056
σ_{vq}	SD on Transitory shock to IST change	Inv-Gamma (a,b)	0.3261	1.45E-04	0.0038	0.0031	0.0039
σ_ϕ	SD on Housing demand shock	Inv-Gamma (a,b)	0.3261	1.45E-04	0.0525	0.0484	0.0550
σ_ψ	SD on Labor supply shock	Inv-Gamma (a,b)	0.3261	1.45E-04	0.0082	0.0068	0.0083
σ_ς	SD on Collateral shock	Inv-Gamma (a,b)	0.3261	1.45E-04	0.0122	0.0113	0.0132

4.1 Impulse responses of a positive housing demand shock in the baseline and our preferred models

First, in the baseline case ($\eta = 1$, $\zeta = 1$), the household's utility of the consumption bundle is logarithmic, as in Liu et al. (2013). Under the baseline case, when there is a change in the interest rate, the intertemporal substitution effect is equal to the income effect. We perform the impulse

Table 3. Prior distributions and posterior modes of the structural parameters

Parameter		Prior			Posterior		
Estimated	Description	Distribution	a	b	Mode	Low	High
γ_h	Household's habit persistence	Beta (a,b)	1.00	2.00	0.5624	0.5314	0.6010
γ_e	Entrepreneur's habit persistence	Beta (a,b)	1.00	2.00	0.6174	0.5778	0.6357
Ω	Capital adjustment cost parameter	Gamma (a,b)	1.00	0.50	0.1863	0.1711	0.2317
$100(g_\gamma$ - 1)	Steady-state growth rate of TFP	Gamma (a,b)	1.86	3.01	0.4890	0.4124	0.5252
$100(\lambda_q$ - 1)	Steady-state growth rate of IST	Gamma (a,b)	1.86	3.01	1.2299	1.2233	1.3338
Calibrated							
β	Discount factor		0.9856				
$\bar{\lambda}_a$	Steady-state growth rate of preference shock		0.0088				
ϕ	Share on land input		0.0695				
δ	Depreciation rate		0.0370				
$\bar{\varphi}$	Steady state of housing demand shock		0.0458				

Note: 1. Counterfactual economy: ($\eta = 1, \zeta = 2.77$).
2. "Low" and "High" denote the bounds of the 90% probability interval for the prior distribution.

Table 4. Prior distributions and posterior modes of the shock parameters

	Prior				Posterior		
Parameter	Description	Distribution	a	b	Mode	Low	High
ρ_a	Intertemporal preference shock	Beta (a,b)	1.00	2.00	0.9148	0.8900	0.9332
ρ_z	Permanent neutral technology shock	Beta (a,b)	1.00	2.00	0.2186	0.2118	0.2767
ρ_{vz}	Transitory neutral technology shock	Beta (a,b)	1.00	2.00	0.2870	0.2740	0.3841
ρ_q	Permanent shock to IST change	Beta (a,b)	1.00	2.00	0.5374	0.5354	0.6518
ρ_{vq}	Transitory shock to IST change	Beta (a,b)	1.00	2.00	0.2965	0.1706	0.3104
ρ_φ	Housing demand shock	Beta (a,b)	1.00	2.00	0.9997	0.9990	0.9998
ρ_ψ	Labor supply shock	Beta (a,b)	1.00	2.00	0.9728	0.9668	0.9766
ρ_ς	Collateral shock	Beta (a,b)	1.00	2.00	0.9778	0.9743	0.9807
σ_a	SD on Intertemporal preference shock	Inv-Gamma (a,b)	0.3261	1.45E-04	0.0649	0.0520	0.0793
σ_z	SD on Permanent neutral tech shock	Inv-Gamma (a,b)	0.3261	1.45E-04	0.0054	0.0049	0.0059
σ_{vz}	SD on Transitory neutral tech shock	Inv-Gamma (a,b)	0.3261	1.45E-04	0.0030	0.0028	0.0037
σ_q	SD on Permanent shock to IST change	Inv-Gamma (a,b)	0.3261	1.45E-04	0.0042	0.0033	0.0044
σ_{vq}	SD on Transitory shock to IST change	Inv-Gamma (a,b)	0.3261	1.45E-04	0.0029	0.0026	0.0034
σ_ϕ	SD on Housing demand shock	Inv-Gamma (a,b)	0.3261	1.45E-04	0.0465	0.0404	0.0551
σ_ψ	SD on Labor supply shock	Inv-Gamma (a,b)	0.3261	1.45E-04	0.0051	0.0047	0.0055
σ_ς	SD on Collateral shock	Inv-Gamma (a,b)	0.3261	1.45E-04	0.0114	0.0100	0.0125

responses of an increase in the housing demand shock by one standard deviation. The impulse is in Figure 1.

The impulse responses of the baseline case ($\eta = 1$, $\zeta = 1$) replicate exactly those in Liu et al. (2013, Figure 4). In response to a positive housing demand shock, the household increases the land service and decreases consumption on impact. The increase in the land price propagates

Table 5. Prior distributions and posterior modes of the structural parameters

Parameter		Prior			Posterior		
Estimated	Description	Distribution	a	b	Mode	Low	High
γ_h	Household's habit persistence	Beta (a,b)	1.00	2.00	0.5029	0.4909	0.5054
γ_e	Entrepreneur's habit persistence	Beta (a,b)	1.00	2.00	0.7122	0.7021	0.7412
Ω	Capital adjustment cost parameter	Gamma (a,b)	1.00	0.50	0.1457	0.1136	0.1598
$100(g_\gamma - 1)$	Steady-state growth rate of TFP	Gamma (a,b)	1.86	3.01	0.3047	0.2978	0.3285
$100(\lambda_q - 1)$	Steady-state growth rate of IST	Gamma (a,b)	1.86	3.01	1.1660	1.1459	1.1869
Calibrated							
β	Discount factor	0.9855					
$\bar{\lambda}_a$	Steady-state growth rate of preference shock	0.0089					
ϕ	Share on land input	0.0695					
δ	Depreciation rate	0.0367					
$\bar{\varphi}$	Steady state of housing demand shock	0.0457					

Note: 1. Counterfactual economy: ($\eta = 0.47$, $\zeta = 1$).
2. "Low" and "High" denote the bounds of the 90% probability interval for the prior distribution.

Table 6. Prior distributions and posterior modes of the shock parameters

	Prior				Posterior		
Parameter	Description	Distribution	a	b	Mode	Low	High
ρ_a	Intertemporal preference shock	Beta (a,b)	1.00	2.00	0.9182	0.9128	0.9188
ρ_z	Permanent neutral technology shock	Beta (a,b)	1.00	2.00	0.4801	0.4606	0.4900
ρ_{vz}	Transitory neutral technology shock	Beta (a,b)	1.00	2.00	0.0260	0.0226	0.0335
ρ_q	Permanent shock to IST change	Beta (a,b)	1.00	2.00	0.5666	0.5457	0.5731
ρ_{vq}	Transitory shock to IST change	Beta (a,b)	1.00	2.00	0.2309	0.2051	0.2310
ρ_φ	Housing demand shock	Beta (a,b)	1.00	2.00	0.9998	0.9994	0.9998
ρ_ψ	Labor supply shock	Beta (a,b)	1.00	2.00	0.9946	0.9869	0.9955
ρ_ς	Collateral shock	Beta (a,b)	1.00	2.00	0.9856	0.9799	0.9859
σ_a	SD on Intertemporal preference shock	Inv-Gamma (a,b)	0.3261	1.45E-04	0.1860	0.1720	0.1934
σ_z	SD on Permanent neutral tech shock	Inv-Gamma (a,b)	0.3261	1.45E-04	0.0040	0.0034	0.0042
σ_{vz}	SD on Transitory neutral tech shock	Inv-Gamma (a,b)	0.3261	1.45E-04	0.0038	0.0032	0.0042
σ_q	SD on Permanent shock to IST change	Inv-Gamma (a,b)	0.3261	1.45E-04	0.0043	0.0037	0.0046
σ_{vq}	SD on Transitory shock to IST change	Inv-Gamma (a,b)	0.3261	1.45E-04	0.0029	0.0023	0.0030
σ_ϕ	SD on Housing demand shock	Inv-Gamma (a,b)	0.3261	1.45E-04	0.0465	0.0448	0.0519
σ_ψ	SD on Labor supply shock	Inv-Gamma (a,b)	0.3261	1.45E-04	0.0120	0.0106	0.0120
σ_ς	SD on Collateral shock	Inv-Gamma (a,b)	0.3261	1.45E-04	0.0109	0.0095	0.0126

the shock and, by way of the expansions of net worth and the entrepreneur's borrowing constraint, triggers the dynamic financial multiplier through interactions between the land price and investment. With a logarithmic utility, the entrepreneur would smooth consumption over time by investing part of the loans, and this intertemporal smoothing incentive is reinforced by habit persistence. The entrepreneur's habit persistence dampens consumption and increases investment in responses to a shock that raises the land price. As a result, investment, labor hours, and output

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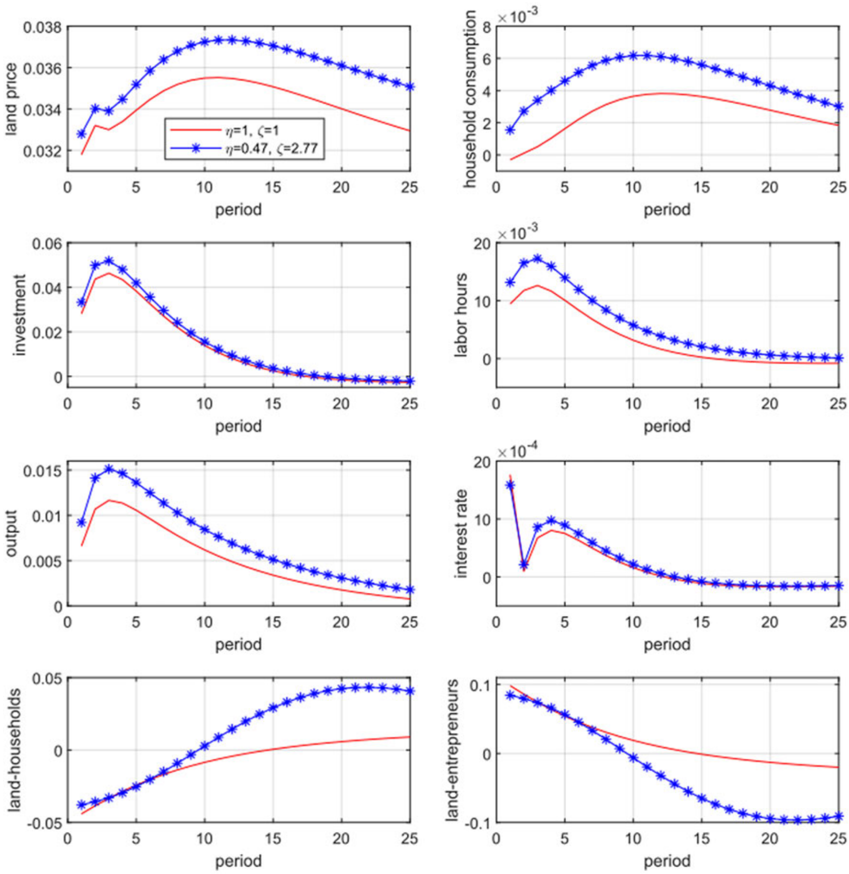

Figure 1. Impulse responses to a positive housing demand shock in our preferred model ($\eta = 0.47$, $\zeta = 2.77$) and Liu et al. (2013) ($\eta = 1$, $\zeta = 1$).

increase, but aggregate consumption decreases on impact and thus, does not comove with other aggregate variables.

Figure 1 also reports the impulse responses in our preferred model ($\eta = 0.47$, $\zeta = 2.77$). It is clear that aggregate consumption increases and comoves with land prices, business investment, labor hours, and output. Comparing our preferred model ($\eta = 0.47$, $\zeta = 2.77$) and the baseline model ($\eta = 1$, $\zeta = 1$), the willingness to substitute land services for consumption in a given period in our preferred model is larger than the willingness in the baseline model, but the willingness to substitute consumption across periods in our preferred model is smaller than that in the baseline model. Thus, in Figure 1, consumption increases more but the land price increases less in our preferred model than the baseline model. As the entrepreneurs increase investment and labor more in our preferred model than the baseline model, output also increases more in our preferred model.

4.2 Impulse responses of a positive housing demand shock in counterfactual economies

We turn to the counterfactual economies, starting with the counterfactual economy ($\eta = 1$, $\zeta = 2.77$). Figure 2 reports the impulse responses of a one-standard deviation increase in the housing demand shock in the counterfactual economy ($\eta = 1$, $\zeta = 2.77$), along with our preferred

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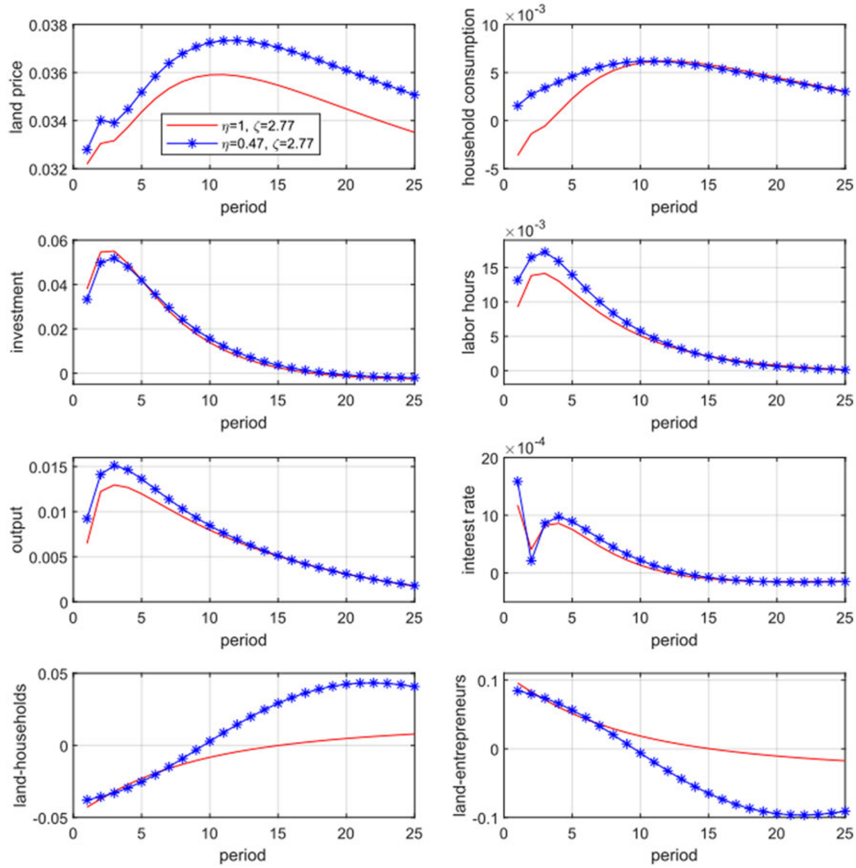

Figure 2. Impulse responses to a positive housing demand shock in our preferred model ($\eta = 0.47$, $\zeta = 2.77$) and the counterfactual economy ($\eta = 1$, $\zeta = 2.77$).

model ($\eta = 0.47$, $\zeta = 2.77$). In the economy ($\eta = 1$, $\zeta = 2.77$), the household's willingness to smooth consumption bundles over time ($\eta = 1$) is larger than that in our preferred model ($\eta = 0.47$) but smaller than the willingness to substitute land services for consumption in a given period in both models ($\zeta = 2.77$).

Figure 2 indicates that, in response to a positive housing demand shock, land price in this counterfactual economy increases more than our preferred model for all periods. However, aggregate consumption in this counterfactual economy decreases on impact in the first three periods. Thus, consumption does not comove with other aggregate variables on impact, as in Liu et al. (2013).

Next, we turn to the other counterfactual economy ($\eta = 0.47$, $\zeta = 1$). Figure 3 reports the impulse responses of a one-standard deviation increase in the housing demand shock in this counterfactual economy ($\eta = 0.47$, $\zeta = 1$), along with our preferred model ($\eta = 0.47$, $\zeta = 2.77$). In the counterfactual economy, the household's willingness to substitute land services for consumption in a given period ($\zeta = 1$) is smaller than that in our preferred model ($\zeta = 2.77$) but larger than the willingness to smooth consumption bundles over time in both models ($\eta = 0.47$).

Figure 3 implies that, in response to a positive housing demand shock, consumption increases in both models. Moreover, the land price and aggregate consumption in the counterfactual economy increases more than those in our preferred model for all periods. As the increase in the land price affects the entrepreneur's credit constraints and propagates the housing demand shock, the entrepreneur's investment increases more in the counterfactual economy than our preferred

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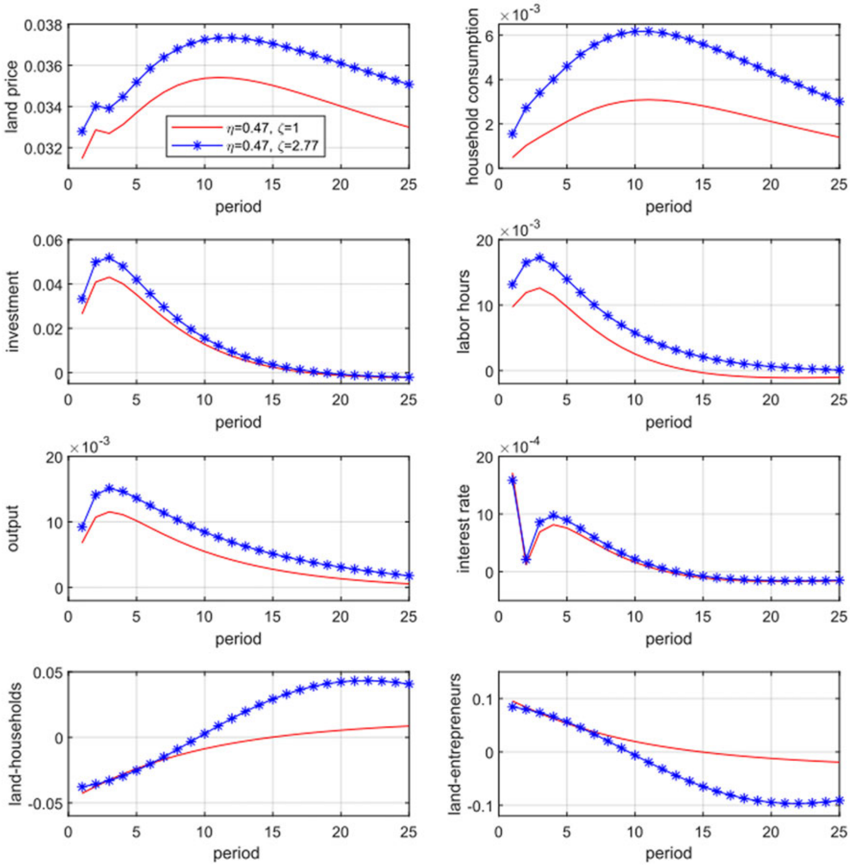

Figure 3. Impulse responses to a positive housing demand shock in our preferred model ($\eta = 0.47$, $\zeta = 2.77$) and the counterfactual economy ($\eta = 0.47$, $\zeta = 1$).

model. As investment increases, labor hours also increase, since these inputs are complements in production. As a result, output in this counterfactual economy also increases more than our preferred model.

Note that aggregate consumption comoves with investment and output in the counterfactual economy ($\eta = 0.47$, $\zeta = 1$) in Figure 3, different from the counterfactual economies ($\eta = 1$, $\zeta = 2.77$) in Figure 2. The result suggests that, in response to a positive housing demand shock, an intertemporal ES smaller than unity is needed for aggregate consumption to comove with other aggregate variables over time. Thus, the inconsistency of the impulse responses between the BVAR model and the DSGE model in Liu et al. (2013) comes from the intertemporal ES η being not smaller than unity.

5. Relative importance of different structural shocks

We have compared the impulse responses of a positive housing demand shock in models when the intertemporal and the intratemporal ESs are different from unity. In addition to the housing demand shock, other structural shocks also affect the fluctuations of the land price and other macroeconomic variables: patience shocks, labor supply shocks, collateral shocks, permanent shocks to the total factor productivity (henceforth TFP), transitory shocks to TFP, permanent

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Table 7. Variance decomposition of aggregate quantities

	Land price		Investment		Output		Hours	
	$\eta=1$ $\zeta=1$	$\eta=0.47$ $\zeta=2.77$	$\eta=1$ $\zeta=1$	$\eta=0.47$ $\zeta=2.77$	$\eta=1$ $\zeta=1$	$\eta=0.47$ $\zeta=2.77$	$\eta=1$ $\zeta=1$	$\eta=0.47$ $\zeta=2.77$
Patience shocks								
1Q	4.09	3.58	19.37	15.79	12.28	9.84	12.46	10.70
4Q	3.30	2.88	18.80	15.68	11.22	9.24	11.88	10.15
8Q	2.91	2.55	17.23	15.56	9.68	9.00	10.72	9.14
16Q	2.29	2.03	14.91	15.15	7.43	8.44	9.29	8.01
24Q	1.77	1.59	13.56	14.93	5.97	8.01	8.68	7.53
Housing demand shocks								
1Q	89.99	94.03	35.46	41.40	27.82	40.45	44.87	44.00
4Q	90.74	94.21	41.19	46.79	31.80	43.72	44.94	45.03
8Q	90.28	93.99	38.71	47.21	28.32	43.94	42.50	43.90
16Q	89.58	94.57	33.70	46.26	21.82	41.84	37.54	40.43
24Q	89.27	95.34	30.67	45.60	17.37	39.59	34.75	38.02
Labor supply shocks								
1Q	2.55	1.62	12.06	10.69	21.85	18.86	20.20	20.51
4Q	2.25	1.36	12.02	10.69	21.13	18.25	24.08	23.26
8Q	2.41	1.50	12.56	12.03	22.22	21.26	29.75	27.65
16Q	2.68	1.75	13.00	13.99	23.85	27.02	37.68	34.39
24Q	2.72	1.80	12.63	14.76	23.87	31.31	41.45	37.92
Collateral shocks								
1Q	0.00	0.08	12.33	13.59	9.17	13.84	13.82	15.05
4Q	0.11	0.04	16.08	16.81	12.12	16.26	13.09	14.48
8Q	0.25	0.13	14.65	16.15	10.32	15.34	11.56	12.97
16Q	0.35	0.25	12.42	15.25	7.38	13.24	9.99	11.21
24Q	0.29	0.21	11.51	15.36	5.74	12.05	9.83	10.88

Note: Baseline case ($\eta=1$, $\zeta=1$) is Liu et al. (2013).

shocks to investment-specific technology (henceforth IST), and transitory shocks to IST. To facilitate comparisons of the relative importance of these eight structural shocks in driving the impulse responses of the land price and other macroeconomic variables, we perform variance decompositions to each of these eight shocks in models when these two ESs are different from unity.

5.1 Variance decomposition of different shocks in different models

First, we start with comparisons of the baseline model and our preferred model. For these two models, Tables 7 and 8 report the variance decomposition of main aggregate variables across eight types of structural shocks at forecast horizons between the impact period (1Q) and six years after the shocks (24Q). Variance decompositions in Table 7 show that the housing demand shocks account for the lion's share of the fluctuations in the land price. For the fluctuations in the land price, the shares are high in our preferred model, accounting for 94% in all forecast horizons from 1Q to 24Q. Propagated by the collateral constraint via increases in land prices, the housing demand shocks drive large fluctuations in business investment, output, and labor hours. The

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Table 8. Variance decomposition of aggregate quantities

	Land price		Investment		Output		Hours	
	$\eta=1$ $\zeta=1$	$\eta=0.47$ $\zeta=2.77$	$\eta=1$ $\zeta=1$	$\eta=0.47$ $\zeta=2.77$	$\eta=1$ $\zeta=1$	$\eta=0.47$ $\zeta=2.77$	$\eta=1$ $\zeta=1$	$\eta=0.47$ $\zeta=2.77$
Permanent shocks to TFP								
1Q	1.97	0.13	1.13	0.00	6.93	2.79	0.43	0.23
4Q	3.19	0.47	5.64	1.44	17.14	0.49	0.61	0.88
8Q	3.84	0.43	9.19	2.44	25.20	0.30	1.27	1.59
16Q	4.88	0.24	12.71	2.83	35.70	0.24	1.49	1.69
24Q	5.68	0.17	14.41	2.82	42.82	0.21	1.42	1.61
Transitory shocks to TFP								
1Q	1.25	0.20	14.30	8.14	16.06	0.06	1.48	1.86
4Q	0.34	0.06	4.95	4.47	4.73	3.31	2.69	3.21
8Q	0.22	0.08	3.70	3.74	3.19	3.06	2.25	2.73
16Q	0.17	0.10	3.11	3.55	2.29	2.88	1.95	2.43
24Q	0.13	0.09	2.83	3.49	1.84	2.76	1.81	2.28
Permanent shocks to IST								
1Q	0.01	0.08	3.01	9.86	5.34	14.12	6.40	7.19
4Q	0.06	0.89	0.88	3.99	1.75	8.72	2.61	8.72
8Q	0.08	1.28	3.63	2.72	0.99	7.05	1.84	1.88
16Q	0.05	1.04	9.86	2.80	1.47	6.29	1.95	1.69
24Q	0.13	0.79	14.13	2.85	2.35	6.00	1.96	1.61
Transitory shocks to IST								
1Q	0.03	0.29	2.34	0.53	0.57	0.04	0.35	0.47
4Q	0.01	0.08	0.44	0.15	0.11	0.02	0.11	0.02
8Q	0.01	0.04	0.32	0.16	0.07	0.04	0.12	0.15
16Q	0.00	0.02	0.29	0.18	0.06	0.06	0.11	0.16
24Q	0.00	0.01	0.26	0.18	0.05	0.05	0.11	0.15

Note: Baseline case ($\eta=1$, $\zeta=1$) is Liu et al. (2013).

shares of fluctuations in output and investment are high in our preferred model, accounting for 39–43% and 41–47% in 1Q–24Q, respectively. For the shares of fluctuations in labor hours, except 1Q, our preferred model is higher than the baseline case.

The collateral shocks do not change land prices directly, but they impact the entrepreneur's borrowing capacity in a way similar to the housing demand shocks. The effects of collateral shocks are persistent. Table 7 indicates that collateral shocks account for about 5–16% of fluctuations in investment, output, and hours for all forecast horizons for the baseline model in Liu et al. (2013). For our preferred model, the shares of fluctuations in investment, output, and hours are slightly larger than those in Liu et al. (2013).

Moreover, the labor supply shocks also drive large fluctuations in output and labor hours. Yet, except the 16Q and 24Q in output and investment, the shares in our preferred model are less than those of Liu et al. (2013). Furthermore, the patience shocks can drive large fluctuations in investment, output, and labor hours. Yet, except output and investment in the 16Q and 24Q, the shares of patience shock in our preferred are smaller than the baseline model. Furthermore, as in Liu et al. (2013), permanent and transitory shocks to the total factor productivity (henceforth TFP) in Table 8 contribute little to fluctuations in the land price, investment, and labor hours in

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Table 9. Variance decomposition of aggregate quantities

	Land price		Investment		Output		Hours	
	$\eta=1$ $\zeta=2.77$	$\eta=0.47$ $\zeta=1$	$\eta=1$ $\zeta=2.77$	$\eta=0.47$ $\zeta=1$	$\eta=1$ $\zeta=2.77$	$\eta=0.47$ $\zeta=1$	$\eta=1$ $\zeta=2.77$	$\eta=0.47$ $\zeta=1$
Patience shocks								
1Q	3.95	4.08	15.62	16.20	9.00	10.61	10.53	11.14
4Q	3.24	3.40	15.56	16.21	8.95	9.76	10.04	10.33
8Q	2.86	3.09	15.75	16.13	9.01	9.23	9.24	8.98
16Q	2.27	2.52	15.54	15.61	8.86	8.22	8.12	7.61
24Q	1.78	2.00	15.35	15.25	8.62	7.45	7.49	7.07
Housing demand shocks								
1Q	94.40	90.58	38.17	41.01	31.96	41.10	37.39	43.18
4Q	94.73	90.63	42.90	45.74	37.39	43.20	38.41	43.21
8Q	94.69	90.13	43.60	45.21	37.48	42.18	37.23	40.37
16Q	95.25	90.49	42.87	43.48	35.33	38.29	33.57	35.27
24Q	95.98	91.11	42.37	42.50	33.11	34.81	31.01	32.10
Labor supply shocks								
1Q	1.01	4.52	11.07	14.59	21.43	22.87	25.07	24.03
4Q	0.77	4.16	10.52	14.84	20.61	23.78	28.13	28.24
8Q	0.89	4.51	11.75	16.55	23.35	28.33	32.97	33.70
16Q	1.05	5.18	13.70	19.18	28.90	36.50	40.89	41.42
24Q	1.09	5.54	14.47	20.46	33.32	42.52	45.45	45.73
Collateral shocks								
1Q	0.03	0.07	15.44	12.21	12.56	12.79	14.69	13.44
4Q	0.07	0.03	18.95	15.14	16.86	14.50	14.81	12.66
8Q	0.20	0.12	18.63	14.34	16.38	13.34	13.64	10.88
16Q	0.34	0.21	17.77	13.39	14.45	11.12	11.45	9.25
24Q	0.30	0.17	17.76	13.43	13.05	9.90	10.47	8.98

our preferred model, but permanent and transitory shocks to TFP account for a large fluctuation in output in the baseline case, but not in our preferred model. As in Liu et al. (2013), permanent and transitory shocks to investment-specific technology (henceforth IST) contribute little to land price fluctuations, and the fluctuations in investment, output, and labor hours.

Next, we look into the variance decomposition of the two counterfactual economies, ($\eta=1$, $\zeta=2.77$) and ($\eta=0.47$, $\zeta=1$) in Tables 9 and 10. Variance decompositions in Table 9 show that the housing demand shocks still account for the lion's share of the fluctuations in the land price for both counterfactual economies. Moreover, shocks to the TFP and shocks to IST in Table 10, both permanent and transitory, still contribute little to fluctuations in the land price, investment, and labor hours, as in Table 8.

Overall, as seen from all Tables 7, 8, 9, and 10, the housing demand shock has the strongest strength of the collateral channel linking residential house prices and firm investment in all these models. Shocks to residential housing demand explain 39–43% of the variance of output and 41–47% of the variance of business investment in our preferred model ($\eta=0.47$, $\zeta=2.77$), as compared to 31–37% and 34–43% of the variance of output and 38–43% and 41–45% of the variance of business investment in the two counterfactual economies ($\eta=1$, $\zeta=2.77$) and ($\eta=0.47$, $\zeta=1$), respectively, but the same shocks explain only 17–31% of the variance of output

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Table 10. Variance decomposition of aggregate quantities

	Land price		Investment		Output		Hours	
	$\eta=1$ $\zeta=2.77$	$\eta=0.47$ $\zeta=1$	$\eta=1$ $\zeta=2.77$	$\eta=0.47$ $\zeta=1$	$\eta=1$ $\zeta=2.77$	$\eta=0.47$ $\zeta=1$	$\eta=1$ $\zeta=2.77$	$\eta=0.47$ $\zeta=1$
Permanent shocks to TFP								
1Q	0.21	0.09	0.49	0.04	5.80	2.07	0.55	0.22
4Q	0.38	0.52	2.85	1.21	0.87	0.39	0.75	1.05
8Q	0.28	0.53	3.63	2.31	0.46	0.34	1.02	2.17
16Q	0.15	0.31	3.85	2.77	0.32	0.31	0.94	2.42
24Q	0.10	0.22	3.83	2.76	0.28	0.28	0.87	2.25
Transitory shocks to TFP								
1Q	0.09	0.26	6.85	7.90	0.08	0.48	0.53	2.82
4Q	0.03	0.08	4.07	3.77	2.20	2.87	2.13	2.91
8Q	0.06	0.09	3.19	3.19	2.13	2.52	1.83	2.30
16Q	0.07	0.10	2.99	2.98	2.04	2.23	1.61	1.95
24Q	0.07	0.09	2.95	2.91	1.98	2.03	1.51	1.78
Permanent shocks to IST								
1Q	0.08	0.12	11.77	7.63	19.04	10.07	10.86	4.70
4Q	0.72	1.11	4.99	2.77	13.10	5.47	5.62	1.46
8Q	1.02	1.49	3.31	2.12	11.15	4.01	3.98	1.44
16Q	0.86	1.16	3.11	2.44	10.06	3.28	3.33	1.93
24Q	0.66	0.87	3.11	2.53	9.61	2.96	3.12	1.95
Transitory shocks to IST								
1Q	0.24	0.28	0.59	0.41	0.13	0.00	0.38	0.46
4Q	0.07	0.08	0.17	0.11	0.02	0.02	0.11	0.14
8Q	0.03	0.04	0.15	0.14	0.02	0.06	0.10	0.16
16Q	0.02	0.02	0.17	0.15	0.03	0.07	0.09	0.15
24Q	0.01	0.01	0.17	0.15	0.03	0.06	0.09	0.13

and 30 − 41% of the variance of business investment in the baseline model ($\eta = 1, \zeta = 1$) in Liu et al. (2013).

5.3 Comparing different baseline values of η and different cases of ζ

In order to understand which model is favored by the data, we report the marginal data density (henceforth MDD). Given data set, the MDD measures how likely the model is supported by the data.[14] The MDD is the most comprehensive measure of fit. As in Liu et al. (2016), we estimate the MDD using three different methods based on different theoretical foundations. SWZ is the method developed by Sims et al. (2008), Mueller is the Mueller method described in Liu et al. (2011), and Bridge is the bridge-sampling method proposed by Meng and Wong (1996).

Table 11 reports the MDD values for different values of the intertemporal ES η and the intratemporal ES ζ in the baseline model ($\eta = \zeta = 1$), our preferred model ($\eta = 0.47, \zeta = 2.77$), and the two counterfactual economies ($\eta = 1, \zeta = 2.77$) and ($\eta = 0.47, \zeta = 1$). The results suggest

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Table 11. Measures of model fit for different values of η and ζ

Fit measure (log value)	$\eta=1$ $\zeta=1$	$\eta=0.47$ $\zeta=2.77$	$\eta=1$ $\zeta=2.77$	$\eta=0.47$ $\zeta=1$
MDD (SWZ)	2454.57	2632.10	2520.69	2529.14
MDD (Mueller)	2452.51	2625.06	2517.44	2526.95
MDD (Bridge)	2452.28	2624.60	2517.28	2526.87

that our preferred model has the largest MDD value among other models. Thus, our preferred model with the intertemporal ES less than unity and the intratemporal ES larger than unity is favored by the data. That is, the model is in favor of a household's utility with a complementary relationship for consumption bundles across periods but a substitutable relationship between consumption and land services within a given period.

6. Concluding remarks

In a recent Bayesian (DSGE) model with the collateral channel for entrepreneurs, Liu et al. (2013) have found that a positive shock to residential housing demand generates a mechanism that amplifies and propagates the shock through the joint dynamics of land prices and business investment. Their model showed that business investment increases in land prices and the housing demand shock accounts for about 90% of the fluctuations in land prices, as well as for large fluctuations in investment, output, and labor hours. Yet, their impulse responses have a difficulty, as their consumption decreases on impact in land prices, which is different from empirical evidence using micro-data and impulse responses from the estimated BVAR model in Liu et al. (2013). In this paper, we introduce a general household preference with broader intratemporal and intertemporal ESs than the logarithmic household preference in Liu et al. (2013. We find three different results.

First, we structurally estimate the values of the intertemporal ES and the intratemporal ES within the current model and estimate alternative models to fit the time series data. We find that the intertemporal ES is less than unity and the intratemporal ES is larger than unity. Moreover, we find that, in response to a positive housing demand shock, both consumption and business investment increase with land prices under our estimated values of the intertemporal ES and the intratemporal ES. Under our estimated values of the intertemporal ES and the intratemporal ES, there is a strong indirect effect, which increases the land price largely that causes residential land services to be substituted away toward consumption. As a result, consumption increases with land prices.

Second, we find that the strength of the collateral channel linking land prices and business investment in our Bayesian model is larger than that in Liu et al. (2013). In particular, shocks to residential housing demands explain 39—43% of the variance of output and 41—47% of the variance of business investment in our model, but the same shocks explain only 17—31% of the variance of output and 30—41% of the variance of business investment in Liu et al. (2013).

Finally, variance decomposition indicates that shocks to housing demand account for a larger share of the fluctuations in the land price, business investment, hours, and output than other structural shocks. Using the marginal data density as the measure of fit for models, we find that our model can better explain the US aggregate data.

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Notes

1 We should remark that, when Liu et al. (2011) used the model-generated data and treated it like the BVAR model, their land price surge led to a consumption increase. See Figure 5 in Liu et al. (2011).
2 See Muellbauer and Murphy (1997) and Campbell and Cocco (2007), which found evidence that a rise in house prices increases consumer spending in the UK and the US, respectively. See also Mian et al. (2013), which illustrated how an increase in housing net worth leads to a rise in consumption. When the initial housing endowment is revaluated, Berger et al. (2018) showed that a large positive effect of house prices on consumption in empirical work is consistent with the prominent income hypothesis, and this effect works just like a transitory shock to income. Kaplan et al. (2020) found that the boom-bust in house prices around the Great Recession explains half of the corresponding swings in nondurable expenditures through a wealth effect.
3 As Hall (1988) pointed out, the quantitative effect of shocks on consumption and investment depends on intertemporal substitution in consumption. Moreover, Ogaki and Reinhart (1998) emphasized that the intratemporal substitution between nondurables and durable goods is important for understanding the effects of the housing demand shock.
4 Ogaki and Reinhart (1998) and Piazzesi et al. (2007) estimated that the intertemporal ES is statistically and significantly less than unity and the intratemporal ES is statistically and significantly larger than unity. Moreover, Yogo (2006), and more recently Li et al. (2016), have found that the intertemporal ES is statistically and significantly smaller than unity and less than the intratemporal ES. Further, Flavin and Nakagawa (2008) estimated the intertemporal ES to be about a half, while Bajari et al. (2013) estimated the intratemporal ES to be statistically and significantly larger than unity.
5 As in Liu et al. (2013), we use the terms housing services and land services interchangeably.
6 The consumption bundle of a CES form is in line with the existing literature that considers consumption durables. See, for example, Barsky et al. (2003, 2007), Monacelli (2009), and Chen and Liao (2014).
7 We use standard Hicksian language here. Over time, two bundles are substitutes if $\eta > 1$ and complements if $\eta < 1$. In a period of time, two goods are substitutes if $\zeta > 1$ and complements if $\zeta < 1$.
8 Some papers refer to $U_{CL} < 0$ as the case in which C and L are substitutes, while $U_{CL} > 0$ is the case in which these two goods are complements. We refrain from such a language here, since the cross partial derivative of the felicity function captures both intertemporal and intratemporal tradeoffs.
9 See the Appendix for the first-order conditions of $C_{h,t}$ and $L_{h,t}$ in the household's problem.
10 The optimization routine in Sims et al. (2008) is coded in C/C++, downloadable at http://www.tzha.net/code. Compared with other optimization routines in Dynare 4.2, the optimization routine used in Liu et al. (2013) is efficient and can find the posterior mode. See the Appendix in Liu et al. (2013) for description of the data and the prior distributions.
11 As in Liu et al. (2013), the values of α and $\overline{\theta}$ are fixed at 0.3 and 0.75 in accordance with the data, respectively, and the values of $\overline{\psi}$ is adjusted so that the steady-state market hours are about 25% of time endowment.
12 Using a homothetic preference to estimate the intertemporal ES and the intratemporal ES, Ogaki and Reinhart (1998) found $\eta \in [0.32, 0.45]$ and $\zeta = 1.17$, while Piazzesi et al. (2007) attained $\eta \in [0.06, 0.20]$ and $\zeta \in [1.05, 1.25]$ and Flavin and Nakagawa (2008) discovered $\eta \in [0.54, 0.55]$. Recently, Bajari et al. (2013) estimated ζ =4.55.
13 Based on our preferred model ($\eta = 0.47$, $\zeta = 2.77$), when we use the model-generated data and treat it like the BVAR model, the land price surge also leads to a consumption increase, as in Liu et al. (2011). See Appendix Figure 1. However, as we will see, in response to a positive residential housing demand shock, consumption increases on impact in our preferred DSGE model ($\eta = 0.47$, $\zeta = 2.77$), as opposed to a consumption decrease on impact in Liu et al. (2013).
14 The DSGE-VAR approach, as proposed by Del Negro and Schorfheide (2004) and Del Negro et al. (2007), requires the number of shocks to equal the number of observed variables. Yet, under the framework of Liu et al. (2013), there are eight shocks and six observed variables, which makes the DSGE-VAR method infeasible. Thus, we report the value of MDD in the same way as in Gong et al. (2017).

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Mathematical Appendix

Our model generalizes the household utility function in Liu et al. (2013). The household's first-order conditions for nondurable consumption and land services and the effects of the elasticity of substitution between consumption and land services are the main differences between our model and Liu et al. (2013). The Appendix is for these two types of differences.

A1. Derivation of the first-order conditions of $C_{h,t}$ and $L_{h,t}$ in the household's problem

$$\max \ E\sum_{t=0}^{\infty}\beta^t A_t\left[\frac{(u_t)^{\frac{1-1/\eta}{1-1/\zeta}}}{1-1/\eta}-\psi_t N_{h,t}\right],$$

$$\text{s.t. } C_{h,t}+q_{l,t}\left(L_{h,t}-L_{h,t-1}\right)+\frac{S_t}{R_t}\leq W_t N_{h,t}+S_{t-1},$$

where $u_t\equiv u\left(C_{h,t},L_{h,t}\right)=(1-\chi)\left(\frac{C_{h,t}-\gamma_h C_{h,t-1}}{\Gamma_t}\right)^{1-1/\zeta}+\chi\left(L_{h,t}^{\varphi_t}\right)^{1-1/\zeta}$.

Let $\mu_{h,t}$ be the Lagrange multiplier of the budget constraint in period t. First, the first-order condition of $C_{h,t}$ is

$$\mu_{h,t}=\frac{A_t}{\Gamma_t}u_t^{\frac{1-1/\eta}{1-1/\zeta}-1}(1-\chi)\left(\frac{C_{h,t}-\gamma_h C_{h,t-1}}{\Gamma_t}\right)^{\frac{-1}{\zeta}}$$
$$-E_t\beta\gamma_h\frac{A_{t+1}}{\Gamma_{t+1}}u_{t+1}^{\frac{1-1/\eta}{1-1/\zeta}-1}(1-\chi)\left(\frac{C_{h,t+1}-\gamma_h C_{h,t}}{\Gamma_{t+1}}\right)^{\frac{-1}{\zeta}}. \tag{A1}$$

To be consistent with the balanced growth path (BGP), we denote the transformation of a variable consistent with the BGP by the variable with a tilde. Specifically, we denote

$$\tilde{C}_{h,t}\equiv\frac{C_{h,t}}{\Gamma_t},\ \tilde{\mu}_{h,t}\equiv\frac{\mu_{h,t}\Gamma_t}{A_t},\ \tilde{q}_{l,t}\equiv\frac{q_{l,t}}{\Gamma_t},$$

where $\Gamma_t\equiv[Z_t Q_t^{(1-\phi)\alpha}]^{1/[1-(1-\phi)\alpha]}$.

Then, condition (A1) can be written in terms of transformed variables as follows.

$$\tilde{\mu}_{h,t}=(1-\chi)\,\tilde{u}_t^{\frac{\zeta-\eta}{\eta(1-\zeta)}}\left(\tilde{C}_{h,t}-\frac{\gamma_h}{g_{\gamma,t}}\tilde{C}_{h,t-1}\right)^{\frac{-1}{\zeta}}$$
$$-E_t\beta\left(1+\lambda_{a,t+1}\right)\frac{\gamma_h}{g_{\gamma,t+1}}(1-\chi)\,\tilde{u}_{t+1}^{\frac{\zeta-\eta}{\eta(1-\zeta)}}\left(\tilde{C}_{h,t+1}-\frac{\gamma_h}{g_{\gamma,t}}\tilde{C}_{h,t}\right)^{\frac{-1}{\zeta}}, \tag{A2}$$

where $\tilde{u}_t=\left[(1-\chi)\left(\tilde{C}_{h,t}-\frac{\gamma_h}{g_{\gamma,t}}\tilde{C}_{h,t-1}\right)^{1-\frac{1}{\zeta}}+\chi\left(L_{h,t}^{\varphi_t}\right)^{1-\frac{1}{\zeta}}\right]$ and $g_{\gamma,t}\equiv\frac{\Gamma_t}{\Gamma_{t-1}}$.

Let a variable with an upper bar denote the steady state of the variable and a variable with a hat denote the variable in a percentage deviation from its steady state. If we take a log-linearization of (A2) around the steady state, we obtain

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$$\hat{\mu}_{h,t} = \frac{(1-\chi)\,\overline{u}^{\frac{1-1/\eta}{1-1/\zeta}-1}\left(\overline{C}_h - \frac{\gamma_h}{g_\gamma}\overline{C}_h\right)^{\frac{-1}{\zeta}}}{\overline{\mu}_h}$$

$$\times\left\{\left[\left(\frac{1-1/\eta}{1-1/\zeta}-1\right)\hat{u}_t - \frac{1}{\zeta}\left[\frac{g_\gamma \hat{C}_{h,t} + \gamma_h\left(\hat{g}_{\gamma,t} - \hat{C}_{h,t-1}\right)}{g_\gamma - \gamma_h}\right] - \beta\overline{\lambda}_a\frac{\gamma_h}{g_\gamma}\hat{\lambda}_{a,t+1}\right]\right.$$

$$\left.+\beta\left(1+\overline{\lambda}_a\right)\frac{\gamma_h}{g_\gamma}E_t\left(\hat{g}_{\gamma,t+1} - \frac{\zeta-\eta}{\eta(1-\zeta)}\hat{u}_{t+1} + \frac{1}{\zeta}\left[\frac{g_\gamma\hat{C}_{h,t+1} + \gamma_h\left(\hat{g}_{\gamma,t+1} - \hat{C}_{h,t}\right)}{g_\gamma - \gamma_h}\right]\right)\right\}, \tag{A3}$$

where $\hat{u}_t = \frac{1}{\overline{u}}\left(1-\frac{1}{\zeta}\right)\left\{(1-\chi)\,\overline{C}_h^{1-1/\zeta}\left(1-\frac{\gamma_h}{g_\gamma}\right)^{-1/\zeta}\left[\hat{C}_{h,t} + (\gamma_h/g_\gamma)\left(\hat{g}_{\gamma,t} - \hat{C}_{h,t-1}\right)\right] + \chi\overline{\varphi}\left(\overline{L}_h^{\overline{\varphi}}\right)^{1-1/\zeta}\left[\hat{L}_{h,t} + \left(\ln\overline{L}_h\right)\hat{\varphi}_t\right]\right\}$,

$$\hat{\lambda}_{a,t+1} = \frac{\lambda_{a,t+1} - \overline{\lambda}_a}{\overline{\lambda}_a},$$

$$\hat{g}_{\gamma,t} = \frac{g_{\gamma,t} - \overline{g}_\gamma}{\overline{g}_\gamma},$$

$$\overline{\mu}_h = (1-\chi)\left[1-\beta\left(1+\overline{\lambda}_a\right)\frac{\gamma_h}{g_\gamma}\right]\overline{u}^{\frac{1-1/\eta}{1-1/\zeta}-1}\left[\overline{C}_h\left(1-\frac{\gamma_h}{g_\gamma}\right)\right]^{\frac{-1}{\zeta}},$$

$$\overline{u} = \left[(1-\chi)\left(\overline{C}_h - \overline{C}_h\frac{\gamma_h}{g_\gamma}\right)^{1-\frac{1}{\zeta}} + \chi\left(\overline{L}_h^{\overline{\varphi}}\right)^{1-\frac{1}{\zeta}}\right].$$

Next, the first-order condition of $L_{h,t}$ is

$$\mu_{h,t}q_{l,t} = \beta E_t q_{l,t+1}\mu_{h,t+1} + \varphi_t\frac{A_t\chi L_{h,t}^{\varphi_t(1-1/\zeta)-1}}{\left[(1-\chi)\left(\frac{C_{h,t}-\gamma_h C_{h,t-1}}{\Gamma_t}\right)^{1-1/\zeta} + \chi\left(L_{h,t}^{\varphi_t}\right)^{1-1/\zeta}\right]^{1-\frac{1-1/\eta}{1-1/\zeta}}}. \tag{A4}$$

Condition (A4) is rewritten in terms of transformed BGP variables as follows.

$$\tilde{\mu}_{h,t}\tilde{q}_{l,t} = \beta E_t\left[\left(1+\lambda_{a,t+1}\right)\tilde{q}_{l,t+1}\tilde{\mu}_{h,t+1}\right] + \chi\varphi_t L_{h,t}^{\varphi_t(1-1/\zeta)-1}\tilde{u}_t^{\frac{1-1/\eta}{1-1/\zeta}-1}. \tag{A5}$$

Taking a log-linearization of (A5) around the steady state gives

$$\hat{\mu}_{h,t} + \hat{q}_{l,t} = \beta\overline{\lambda}_a E_t\hat{\lambda}_{a,t+1} + \beta\left(1+\overline{\lambda}_a\right)E_t\left(\hat{\mu}_{h,t+1} + \hat{q}_{l,t+1}\right) +$$
$$\left[1-\beta\left(1+\overline{\lambda}_a\right)\right]\left\{\left(\frac{1-1/\eta}{1-1/\zeta}-1\right)\hat{u}_t + \hat{\varphi}_t + \left[\overline{\varphi}(1-1/\zeta)-1\right]\hat{L}_{h,t} + \overline{\varphi}(1-1/\zeta)\left(\ln\overline{L}_h\right)\hat{\varphi}_t\right\}, \tag{A6}$$

where $\hat{q}_{l,t} = \frac{\tilde{q}_{l,t}-\overline{q}_l}{\overline{q}_l}$ and $\hat{\varphi}_t = \frac{\varphi_t - \overline{\varphi}}{\overline{\varphi}}$.

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A2. Effects of the elasticity of substitution between consumption and land services on land prices

The first-order condition of $L_{h,t}$ in (A4) can be rewritten as the following land Euler equation.

$$q_{l,t} = \beta E_t q_{l,t+1} \frac{\mu_{h,t+1}}{\mu_{h,t}} + \frac{\varphi_t \Lambda_t (\eta, \zeta)}{\mu_{h,t}}, \tag{A7}$$

where $\Lambda_t (\eta, \zeta) \equiv A_t \chi L_{h,t}^{\varphi_t(1-1/\zeta)-1} \left[(1-\chi) \left(\frac{C_{h,t} - \gamma_h C_{h,t-1}}{\Gamma_t} \right)^{1-1/\zeta} + \chi \left(L_{h,t}^{\varphi_t} \right)^{1-1/\zeta} \right]^{\frac{1-1/\eta}{1-1/\zeta} - 1}$, and the Lagrange multiplier of the budget constraint $\mu_{h,t}$ is in (A1).

The case of Liu et al. (2013) is under η=1 and $\zeta = 1$, which gives $\Lambda_t (1, 1) = \frac{A_t \chi}{L_{h,t}}$, and (A7) reduces to

$$q_{l,t} = \beta E_t q_{l,t+1} \frac{\mu_{h,t+1}}{\mu_{h,t}} + \frac{\varphi_t}{\mu_{h,t}} \frac{A_t \chi}{L_{h,t}}. \tag{A8}$$

So, the difference in terms of the propagation mechanism between our model and the model of Liu et al. (2013) lies in the term $\Lambda_t(\eta, \zeta)$ in (A7). When $\eta = 1$ but ζ deviates from unity, or when both values of η and ζ deviate from unity, then (A7) is different from (A8). As a result, in response to an increase in the housing demands, the fluctuations in the land price are different, and through the credit constraint, the fluctuations in other macroeconomic variables are different.

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