Credit Market Imperfections, Financial Activity and Economic Growth

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ABSTRACT

This paper develops a dynamic general-equilibrium model with production to examine the inter-relationships between the real and the financial sectors with and without credit market imperfections. Due to the moral hazard problem, credit rationing may be present, which is associated with a widened financial spread and low effective bank loans, compared to the unconstrained equilibrium. Credit rationing causes both the loan and the deposit rates to rise. In a generalized framework with intergenerational human capital accumulation, credit rationing discourages education investment and reduces output growth. In either unconstrained or constrained equilibrium, the long-run effects of a productivity improvement on real and financial activities depends crucially on where it is originated.

Keywords: Credit Constraints, Financial Activity, Endogenous Growth

JEL Classification: E44, D90

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I. Introduction

It is well-documented empirically and theoretically that the financial and real activities are inter-related.[1] Earlier empirical evidence suggests that relaxation of credit rationing raises the deposit rate, encourages financial savings and promotes financial deepening [for example, see Tsiang (1980) for the case of Taiwan and Diaz-Alejandro (1985) for the Latin American economies during the 1950s and 1960s].[2] However, recent cross-country econometric studies by Jappelli and Pagano (1994) and Liu and Woo (1994) indicate that there may exist a positive relationship between credit market imperfections and savings.[3] This latter result may be combined with the new growth theory to generate a positive growth effect of credit rationing, in contrasting with the conventional view. One may therefore wonder whether credit constraints are beneficial or harmful for financial development and economic growth. Underlying this empirical puzzle, there are also divergent theoretical predictions on how credit rationing influences the

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[1] For example, see Goldsmith (1969), McKinnon (1973), Greenwood and Jovanovic (1990), Bencivinga and Smith (1991), and King and Levine (1993). For an overview of the literature, see Pagano (1993) and Becsi and Wang (1997).

[2] More specifically, in the post-WWII period, Taiwan suffered a severe stagnation and its financial intermediation ratio was extremely low. Accompanied by a government policy toward reducing the financial market imperfection, a "Preferential Interest Rate" (at 125% per annum) was established for time deposits and credit ration was loosened. In the next decade, the Taiwan economy stabilized and grew rapidly by the 1960s. In the 1950s, Latin American countries had public development banks granting essentially negative real interest rates to favored borrowers (such as profitable non-traditional industries), leaving non-favored borrowers financing in expensive and unstable informal credit markets. This credit control was associated with negative real interest rates for depositors. Since 1960, several countries have undertaken financial reform, leading to higher savings and economic growth.

[3] Jappelli and Pagano consider that liquidity-constrained consumers may result in higher savings. They employ the Modigliani data set to show a strong negative effect of the maximum loan-to-value ratio (inversely related to the severity of credit constraints) on the national saving rate. Moreover, empirical results from the De Long-Summers and Barro-Wolf data sets suggest that this maximum loan-to-value ratio also has a negative effect on the average GDP growth rate (though statistically less significant in some cases). Liu and Woo argue that financial market imperfections induce agents to save more for lumpy investment in the future. Using 1975-85 data from 17 OECD countries plus Korea and Taiwan, they find that three measures of the financial intermediation ratio all have statistically significant negative effect on the private saving rate.

equilibrium rates of interest: while Azariadis and Smith (1993) find a lower equilibrium real interest rate in a constrained economy, Tsiddon (1992) and Bencivenga and Smith (1993) show the reverse. The present paper attempts to address these important but puzzling issues focusing on some plausible general-equilibrium channels through which credit constraints on firm borrowing influences long-run macroeconomic performance.[4]

In order to address the puzzling issues regarding the long-run effects of credit market imperfections on the rates of interest and economic growth, we design a stylized dynamic general-equilibrium model with three essential features. First, the consumer behavior, the producer behavior and the financial sector will all be fully specified so as to understand the determinants of the deposit and loan rates separately.[5] Second, while there are two possible methods of savings, in goods and in kind, the latter has received much less attention despite its empirical relevance.[6] We will incorporate intergenerational savings in kind and show that the effects of credit constraints may contrast with those via savings in goods. Finally, for the purposes of studying the long-run interactions between the real and financial sectors, we will differentiate technical progress originated in goods production from that in financial activity. In so doing, we allow for independent sources of financial and economic development through which the credit market consequences can be examined.

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[4] The presence of credit constraints on U.S. entrepreneurs and firms is empirically documented by Evans and Jovanovic (1989) and Gertler and Gilchrist (1994), respectively. For LCDs, Tybout (1983) uses micro data from Columbia to conclude that most of the small firms with total employees between 10 and 199 are credit rationed.

[5] Indeed, the conventional dynamic models developed by Tsiddon (1992), Azariadis and Smith (1993), and Bencivenga and Smith (1993) fail to determine both interest rates endogenously. In a static, pure-exchange framework, Williamson (1986) and Holmström and Tirole (1997) separate the loan from the deposit rates and study the effects of credit rationing. In Section IV, we compare carefully our results with theirs.

[6] For example, Lazear (1980) and Jacoby and Skoufias (1997) find strong relationships between credit market imperfections and human capital accumulation using data from U.S. college and rural Indian child education, respectively.

More specifically, we delineate the environment of the economy with three types of optimizing agents: households, banks and firms. In the basic model, the rate of economic growth is exogenous and individual human capital is a fixed proportion of the society's stock of knowledge. The young households work to receive wages and save for consumption during their retirement period. Banks employ labor and undertake financial productivity improvement to convert deposits into loan services. Firms hire workers and obtain loan services from the banking sector to invest in goods-production projects. In the presence of the moral hazard problem in that firms may borrow and abscond without repaying bank loans, it is optimal for banks to ration the credit.[7] We fully characterize the unconstrained and credit-constrained equilibrium, respectively, in Sections III and IV. In Section V, we then generalize the basic setup to allow for endogenous decision in home education through which human capital is accumulated perpetually and the rate of economic growth is endogenously determined.

The main findings of this paper can be summarized as follows. First, in the basic exogenous growth model, credit rationing causes both the *loan* and the *deposit* rate to rise, results in a widened financial spread and lowers effective bank loans. Second, in a human-capital based endogenous growth framework, credit rationing discourages *savings in kind* and *reduces* economic growth. Third, while technological advances in goods production is growth-enhancing, an increase in banking production efficiency generates no long-run growth effect under credit rationing. Their effects on effective bank loans, interest rates and the financial spread are generally different as well.

For the purposes of analytic convenience, we impose some seemingly arguable assumptions that greatly simplify the structure of the model. It may be informative at this moment to brief the reader whether these assumptions are innocuous to the main findings. First, to highlight the role of savings in kind, we assume forced savings in goods. By allowing for endogenous decision for savings in goods, there

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[7] Keynes (1964, pp. 144-145) illustrates the importance of this type of moral hazard problem.

may be a potentially positive savings effect of credit rationing. However, in the absence of direct lending from households to firms, there would be no first-order growth effect whereas allowing for a direct link, the growth consequence of credit rationing is generally ambiguous. Second, in modeling financial services, we consider only labor input without the use of capital. With capital entering financial production, our main results still hold whenever the financial sector is labor intensive compared to the goods sector. Finally, the current framework ignores the conventionally emphasized roles of financial intermediation in liquidity management (cf. Bencivenga and Smith 1991) and risk-pooling (cf. Greenwood and Jovanovic 1990). This is because the primary goal of this paper is to study the consequences of credit market imperfections rather than the emergence of financial intermediation. Nevertheless, the introduction of liquidity management may indeed reinforce the negative growth effect of credit rationing if long-term high-return investment projects are associated with higher degree of riskiness. The incorporation of risk-pooling, on the other hand, has no direct long-run consequence on the moral hazard induced detrimental growth effect of credit rationing. In the concluding section (Section VI), we provide a further discussion on endogenous savings in goods as well as risk-pooling in the presence of the adverse selection problem. Notably, none of these assumptions are critical to the results concerning the rates of interest.

There are a few closely related papers to the present work. Tsiddon (1992) establishes a moral-hazard low-growth trap with credit rationing on education loans in which economic growth is low and the interest rate is high. Azariadis and Smith (1993) consider a pure exchange model with adverse selection in that credit rationing raises individual savings and thus reduces the interest rate. Bencivenga and Smith (1993) also motivate credit rationing using adverse selection in physical capital investment and reconfirm the growth-retarding effect of credit rationing obtained in Tsiddon (1992). To motivate their empirical study, Jappelli and Pagano (1994) construct a simple overlapping-generations model with borrowing constraints on the consumption of the young, and argue that credit constraints encourage consumers to save and thus spur economic growth. In a Grossman-Helpman quality-ladders model with asymetric information, Shi (1996) establishes that credit rationing is growth-enhancing if it induces low-productivity firm to choose high-productivity technology. In contrast to these studies, our paper specifies completely an active financial intermediation sector, determining endogenously the deposit as well as the loan rates and differentiating technical progress originated in goods production from that in financial services. Also complement to previous work, our paper establishes endogenous credit rationing in a dynamic general-equilibrium model of investment loans in the presence of the moral hazard problem.[8]

II. The Basic Model

Time (indexed by *t*) is discrete. There are three separate theaters of economic activities: (i) each 2-period lived overlapping household (consumer/worker) is endowed with a unit of labor when young, who deposits wage incomes for future consumption, (ii) each infinitely lived producer is endowed with a production technology to manufacture the single final good using physical capital and credit facilitated by financial intermediation, and (iii) the financial sector simply converts banking deposits into loans.[9] There is a continuum of each type of economic agents (households, firms and banks) with unit mass.

Chart 1 displays the sequence of events. When young, a household works, receives pre-paid wages (apple tree) and deposits it to the financial sector. A bank then provides a loan (apple tree) to a goods producer, which subsequently manufactures the final good (apple) and pays back the loan with interests (in apples). Finally, the banks pay back the deposits with interests (in apples) to households at the end of the first period and the latter consume at the beginning of the second period (time is negligible between the end of the first and the beginning of the second periods).

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[8] This contrasts with the static, pure-exchange structure of Williamson (1986) and Holmström and Tirole (1997), the dynamic pure-exchange analysis of Azariadis and Smith (1993), the education-loan model of Tsiddon (1992) and the adverse-selection setup of Bencivenga and Smith (1993).

[9] We ignore, for the sake of simplicity, firm deposits and consumer loans. Indeed, one may reinterpret our bank loans as net loans (investment loans net of deposits) and bank deposits as net deposits (consumer deposits net of loans).

II.A. Households

Each household of generation t possesses a fraction of the society's knowledge stock and a unit time endowment when young, while consuming only during the second period. The latter assumption creates forced savings, which simplifies the analysis greatly. In Section V we relax this assumption by allowing savings in kind via human capital accumulation.

Each household allocates one unit of time endowment to goods production (ℓ) and bank operation (1-ℓ). Assuming perfect substitution, workers are always indifferent between the two activities. In the basic model, we consider exogenous human capital (or average knowledge stock, denoted h) and assume that final goods production employs unconsumed output and effective labor ($\ell_t h_t$) as inputs, taking the Cobb-Douglas functional form. In Section V below, we relax the exogeneity assumption allowing for human capital to be endogenously accumulated via home education. Across generations, the knowledge endowment is allowed to grow at a constant rate $g > 0$, that is, $h_{t+1} = (1+g)\, h_t$.

Since the focus of the paper is not on the formation of financial intermediation, we simply assume that all savings are channeled through the banking sector. Each household has an identical preference that is monotone increasing in consumption (c_{t+1}). In the absence of bequests, the representative household born in period t will consume all saved from the first period plus the interests (at a real deposit rate r_{t+1}).

II.B. Producers

Each producer utilizes current capital stock (k_t) and effective labor input ($\ell_t h_t$) to produce a single final good (y_t) which can be allocated to investment demand (i_t) and consumption goods supply (z_t). The production technology takes the Cobb-Douglas form: $y_t = A k_t^{\alpha} (\ell_t h_t)^{1-\alpha}$, where $A > 0$ and $\alpha \in (0, ½)$.[10] Next, we assume that producers are capable of converting bank loans (x_t) into fixed capital formation in

[10] The constant-returns assumption is made so that the model accepts balanced growth in the endogenous growth case, whereas the restriction that the capital share is less than half is consistent with empirical observations.

such an efficient fashion that $i_t = (1+\theta)x_t$, where $\theta > 0$. Implicitly, this more-than-proportional conversion captures the potential effect of external financing on real investment decisions as a result of bank's effective monitoring.[11] This setup is also consistent with a fractional loan-in-advance model as one unit investment requires less-than-one unit bank loan. This highly stylized structure may also be viewed to capture the potential liquidity management role of financial intermediation in the sense that financial loans enable capital deepening, leading to a higher rate of returns.

We also assume that the production transformation schedule is linear so the technology applies to both capital formation and consumption good production. Moreover, we follow Diamond and Yellin (1990) assuming that the goods producer is a residual claimer, i.e., it ingests the unsold consumption goods in a fashion consistent with lifetime value maximization. This ownership assumption avoids the unnecessary Arrow-Debreu redistribution from firms to consumers while maintaining the general equilibrium nature. Moreover, as we can see below this setup is consistent with conventional profit maximization where firms rent capital from external sources.

Denote δ as the (real) loan rate of interest and *w* as the (real effective) rate of wage. Then, the representative producer, at any given time *t*, will choose consumption goods supply, loan demand and labor demand to maximize its value (sum of present-discounted gross profit flows) subject to the capital evolution equation:

$$V(k_t) = \max_{\{i_\tau, x_\tau, \ell_\tau\}} \sum_{\tau=t}^{\infty} \left(\frac{1}{1+\sigma}\right)^{\tau} [y_\tau - (1+\delta_\tau)x_\tau - w_\tau \ell_\tau h_\tau]$$

$$s.t. \quad k_{\tau+1} = i_\tau + (1-d)k_\tau \tag{1}$$

[11] The use of financial instruments for a firm's capital formation has been illustrated by Robinson (1969, ch. 4) and emphasized in the finance literature [for example, see Vinala and Berges (1988)].

$$i_\tau = (1 + \theta) x_\tau \tag{2}$$

where σ > 0 denotes firm owner's (constant) rate of subjective discount and $d > 0$ the (constant) rate of capital depreciation, and recall that $y_\tau = A k_\tau^\alpha (\ell_\tau h_\tau)^{1-\alpha}$. The gross value $V(k_t)$ is crucial in determining the incentive constraint in the presence of moral hazard behavior. Under the specification of the technology and the assumption of the ownership structure, the factor demand functions are greatly simplified and the gross value of firm can be shown *linear* in k (and hence in x).

II.C. Banks

The reader may be reminded that this banking sector is designed mainly to specify the financial flows and to differentiate deposit from loan rates. Thus, we consider an extremely simple structure where each bank provides loan-deposit services to maximize periodic profits.[12] The bank's operation cost (including, for example, the monitoring cost of the firm's investment project and the operation of depositing and lending activity) is assumed to take a fixed coefficient form using labor as the only input. Specifically, provided that $x_t \leq a_t$, we have $(1-\ell_t) h_t = (1/\phi) x_t$, where $\phi > 0$ can be regarded as the cost-saving banking innovation and its inverse $(1/\phi)$ measures the unit labor requirement of banking operation. Notably, physical capital is excluded here for analytical convenience; by considering physical capital as an input in banking operation, the main results remain qualitatively unchanged as long as the banking sector is labor intensive compared to goods manufacturing. Moreover, for simplicity we assume that all consumer savings are financially intermediated and thus the bank deposit (a_t) can be expressed as: $a_t = w_t h_t$.[13]

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[12] The considerations of a infinitely lived bank would just complicate the analysis without altering the main findings.

[13] Recall that it is not the purpose of this paper to study the "formation" of financial intermediation. We take as given the existence of financial intermediation and assume that banks are "passive" in the deposit market, taking whatever households' savings and making decisions only for loans to producers. This assumption is innocuous as it is consistent with perfectly competitive banking sector.

Under this setup, deposits are transformed to loans via a costly financial intermediation process. In contrast to most of the existing literature, we allow for inter-sectoral reallocation of labor (between goods production and bank operation), which plays a crucial role in determining the effects of credit rationing and changes in the productivity and cost parameters on the real and financial activities, the loan and deposit rates and the rate of economic growth.

At any given period *t*, each bank earns profit flow from loan interest receipts ($\delta_t x_t$), net of the interest payments to its depositors ($r_t a_t$) and the labor cost ($w_t(1-\ell_t)h_t$):

$$\max_{\{x_t \le a_t\}} \; \delta_t x_t - r_t a_t - w_t(1-\ell_t)h_t = (\delta_t - \frac{w_t}{\phi})x_t - r_t a_t \tag{3}$$

where in its optimization that determines loan supply, each bank takes the amount of deposits as parametrically given. Obviously, by examining the flow of funds, the total amount of loans must not be greater than the total amount of deposits available from household savings.

Chart 2 provides a graphical summary of the structure of the basic model. The remainder of this Section presents the optimizing conditions for the representative household, firm and bank.

II.D. Optimization

We begin with the optimizing behavior of the representative household, which is trivial:

$$c_t = \frac{(1+r_t)w_{t-1}h_t}{1+g} \tag{4}$$

where in deriving this we have used the evolution equation of exogenous knowledge stock h_t = (1+g)h_{t-1} . That is, a household when young saves its entire working income for consumption when old, in which the total amount of consumption equals the sum of the principal and the interest.

To solve a firm's optimization problem, we apply the production function, (1) and (2) and utilize the now-standard dynamic programming technique transforming the infinite-horizon problem into the recursive Bellman equation:

$$V(k_t) \;=\; \max_{\{i_t,\ell_t\}} \left[A k_t^{\alpha} (\ell_t h_t)^{1-\alpha} - \left(\frac{1+\delta_t}{1+\theta}\right) i_t - w_t \ell_t h_t \right] + \frac{1}{1+\sigma} V(i_t + (1-d)k_t) \tag{5}$$

The first-order conditions for the firm's optimization imply (see Appendix A):

$$\frac{k_t}{\ell_t h_t} \;=\; \left[\frac{\sigma + d}{\alpha A} \left(\frac{1+\delta_t}{1+\theta} \right) \right]^{-\frac{1}{1-\alpha}} \tag{6}$$

$$(1-\alpha) A^{\frac{1}{1-\alpha}} \left[\frac{\sigma + d}{\alpha} \left(\frac{1+\delta_t}{1+\theta} \right) \right]^{-\frac{\alpha}{1-\alpha}} \;=\; w_t \tag{7}$$

In effect, (6) equates the marginal benefit of capital with its marginal cost, while (7) equates the marginal benefit of labor employment with its wage cost. Moreover, as we show in the Appendix A the gross value of firm can be expressed as

$$V(k_t) \;=\; (1+\sigma) \left(\frac{1+\delta_t}{1+\theta} \right) k_t \tag{8}$$

Thus, the value of the representative firm is linear in k (and hence x). The firm value per effective unit (V/h) is therefore bounded along a balanced growth path (where k and h grow at a common rate) if the loan rate is bounded.

From the bank's competitive profit condition, one gets:

$$r_t \;=\; \left(\delta_t - \frac{w_t}{\phi}\right) \frac{x_t}{w_t h_t} \tag{9}$$

which can be rewritten in terms of the net financial mark-up (or financial spread), defined as the ratio of the loan rate net of unit bank operation cost to the deposit rate:

$$\frac{\delta_t - w_t/\phi}{r_t} = \frac{w_t h_t}{x_t} = \frac{1}{x_t/a_t} \tag{10}$$

This expression suggests that the financial mark-up and the loan-deposit ratio are inversely related. We now illustrate the bank's optimizing conditions. Since the bank's objective function is linear in *x*, loan supply must reach the upper bound as long as $\delta > w/\phi$ (which is true under the bank's competitive profit condition):

$$x_t = a_t = w_t h_t \tag{11}$$

Thus, this implies the financial spread be unity. Finally, substituting (11) into the unit labor requirement equation of banking operation yields:

$$\ell_t = 1 - \frac{w_t}{\phi} = 1 - \frac{1}{\phi}\frac{x_t}{h_t} \tag{12}$$

III. Balanced Growth Equilibrium

We are now prepared to solve for the balanced growth equilibrium. Before proving the existence and deriving the comparative statics, we outline a number of equilibrium conditions. There is one more equilibrium condition in addition to those already imposed implicitly in Section II, including labor, deposit and loan market equilibrium and bank's zero profit condition. Goods market equilibrium requires that the total output be divided into investment and consumption. Since firm owners are residual claimers, goods demand must be equal to goods supply. Feasibility of goods allocation thus requires the following inequality to hold: $y_t - i_t = z_t \geq c_t$. With regard to feasible labor allocation, it requires that $\ell \in (0, 1)$. From (7) and (12), one can see that $\ell < 1$ is always satisfied. In order to guarantee $\ell > 0$, from (7) and (12), we impose:

Condition L: (Feasible Labor Allocation) $\delta \geq \delta^{\min} \equiv (1+\theta)\left[A^{\frac{1}{\alpha}}\left(\frac{\alpha}{\sigma+d}\right)\left(\frac{1-\alpha}{\phi}\right)^{\frac{1-\alpha}{\alpha}}\right]-1\,.$

This condition sets a lower bound for the equilibrium value of the loan rate.

III.A. Existence

We begin by defining the concepts of equilibrium and balanced growth equilibrium.

Definition 1: *A **perfect foresight equilibrium** (PFE) is a tuple* $\{c_t, i_t, k_t, a_t, x_t, V_t, \ell_t, \delta_t, r_t, w_t\}_{t=1}^{\infty}$ *such that (i) each of the representative agents (household, firm and bank) optimizes, (ii) each bank reaches zero profits, and (iii) labor, deposit, loan and goods markets all clear; that is, conditions (1), (2), (4), (6)-(9), (11) and (12) are met.*

Definition 2: *A **perfect foresight balanced growth equilibrium** (PFBGE) is a PFE such that all quantity variables, c, k, x and V grow at the same rate g as the public knowledge stock h, i.e.,* $m_{t+1} = g m_t,\ \ \forall\, t \geq 1$ *and* $m = c, k, a, x, V,$ *and all price variables and time allocation* ℓ *are constant.*

Using (1) and (2) with the definition of balanced growth, we obtain:

$$\frac{k}{h} = \left(\frac{1+\theta}{g+d}\right)\frac{x}{h} \tag{13}$$

*w*hich can be used together with the production function, (1), (4), (6) and (11) to express the goods feasibility condition as: $1+\delta_t \geq \alpha[(g+d)/(\sigma+d)][1+\theta+(1+r_t)/(1+g)]$. Using (7), (9) and (11), we show in the Appendix B that the above inequality is guaranteed by,[14]

[14] This condition is sufficient but not necessary.

Condition F: (Feasibility) $A\left[(1+g)\dfrac{\sigma+d}{g+d}-\alpha\right]^{\alpha} \;\geq\; (1+g)\,[\alpha(\sigma+d)]^{\alpha}\,[\phi(1+\theta)]^{1-\alpha}.$

To obtain a PFBGE, we proceed in a recursive manner. First, substituting (7) into (11), we obtain a "labor efficiency" (LE) schedule *given* deposit and loan market equilibrium:

$$\frac{x}{h} \;=\; (1-\alpha)A^{\frac{1}{1-\alpha}}\left[\frac{\sigma+d}{\alpha}\left(\frac{1+\delta}{1+\theta}\right)\right]^{\frac{-\alpha}{1-\alpha}} \tag{14}$$

which is obviously downward-sloping in (x/h, δ) space. Intuitively, when the loan rate is higher, capital accumulation slows down and by Pareto complementarity, the marginal product of labor is lower. As a consequence, the wage rate decreases and, given the fixed time endowment, labor income also reduces. Thus deposits and loan in effective unit are both lower, justifying the negative slope of the LE locus.

Then, utilizing (12) and (13), one can rewrite (6) to derive a "capital efficiency" (KE) schedule *given* loan demand as well as labor and goods market equilibrium:

$$\frac{x}{h} \;=\; \left\{\frac{1}{\phi}+\frac{1+\theta}{g+d}\left[\frac{\sigma+d}{\alpha A}\left(\frac{1+\delta}{1+\theta}\right)\right]^{\frac{1}{1-\alpha}}\right\}^{-1} \tag{15}$$

which is also downward-sloping in (x/h, δ) space. When the loan rate increases, capital accumulation becomes more costly and hence producers undertake factor substitution leading to a lower capital-labor ratio. Along the balanced growth path, the physical capital stock is increasing in loans. Under the fixed coefficient technology, labor in the banking sector is increasing in loans, so labor devoted to goods production is decreasing in loans. Therefore, the capital-labor ratio in the goods sector is unambiguously a monotone increasing function of loans. A reduction in the capital-labor ratio must be accompanied by a decrease in loans (per effective unit), which implies a downward-sloping KE locus.

We now plot the LE and KE loci in Figure 1 for the case where KE is steeper than LE, i.e.,

Condition S: (Slope Condition) $\delta \leq \delta^{max} \equiv (1+\theta)\left[\left(\frac{g+d}{1+\theta}\frac{1}{\phi}\frac{\alpha}{1-\alpha}\right)^{1-\alpha}\frac{\alpha A}{\sigma+d}\right]-1.$

Condition S essentially implies that when financial loan-driven capital formation becomes more efficient, the (gross) rate of returns on capital increases.[15] This is in the spirit of the Samuelson Correspondence Principle, ensuring that the direct effect dominates. Such a condition is imposed particularly for obtaining sensible comparative statics.

These two loci jointly determine the balanced growth equilibrium values of δ (denoted δ^E) and *x/h* (see the equilibrium point E in Figure 1 and detailed proof in the Appendix B), provided that:

Condition E: (Existence) $A^{\frac{1}{\alpha}} < \phi^{\frac{1-\alpha}{\alpha}}\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{g+d}{1+\theta}\right)$.

Then the equilibrium values of δ and *x/h* can be substituted into (2), (4), (6)-(9) and (11)-(13) to solve for other equilibrium quantities and prices. In particular, the deposit rate of interest can be derived as:

$$r = \delta - \frac{1-\alpha}{\phi}A^{\frac{1}{1-\alpha}}\left[\frac{\sigma+d}{\alpha}\left(\frac{1+\delta}{1+\theta}\right)\right]^{-\frac{\alpha}{1-\alpha}} \qquad (16)$$

which is strictly increasing in δ. In summary, we have:[16]

Proposition 1: (Existence) *Under Conditions E, F, L and S, there is a perfect foresight balanced growth equilibrium.*

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[15] To see this, define the gross rate of returns on capital as: $\Delta \equiv (1+\delta)/(1+\theta)$. Both the KE and LE loci are downward-sloping in (*x/h*, Δ) space and it is easily seen that $d\Delta/d\theta > 0$ iff Condition S is met. For detailed derivation, see the Appendix B.

[16] One may wonder if plausible sets of parameters would satisfy all the conditions required for existence. The answer is certainly positive. For example, we provide below a list of parameter values which are either commonly used in calibrated models or chosen for normalization purposes): $\alpha = 1/3$, $g = \sigma = 0.025$, $d = \theta = 0.05$, $A = 1$, $\phi = 3$. Under this parametrization, all conditions are met.

III.B. Comparative Statics

Straightforward comparative-static analysis enables us to examine how changes in A and ϕ affect the endogenous variables of our particular interest, including x/h, δ and r. Figures 2a and 2b display diagrammatically the effects of an increase in ϕ and A, respectively. The focus of the paper is to establish an unconstrained equilibrium and to illustrate the different effects of sectoral productivity changes. Thus, for brevity we omit other comparative static exercises.

When banking production becomes more efficient (i.e., ϕ increases), labor saving induces a reallocation from banking to goods sector, leaving the labor efficiency locus unchanged. Since labor and capital are Pareto-complements and capital formation is based on a fixed coefficient technology in terms of bank loans, the demand for loans increases for a given level of the loan rate. As a consequence, the KE locus shift rightwards (see the new KE′ locus and the new equilibrium point E′ in Figure 2a). These imply higher loans per effective unit (x/h) and lower loan and deposit rates (δ and r). Intuitively, by Pareto complementarity, the wage rate responds positively to bank loans; the resultant increase in the wage rate leads to a decrease in the loan rate due to the standard downward-sloping factor price frontier (i.e., factor substitution). The decrease in the deposit rate is a result of bank's zero profit condition. Thus, a cost-reducing bank innovation enables more bank loans and higher capital formation. These findings are consistent with the Schumpeterian view of financial development.

An increase in the goods production scaling factor A encourages sectoral reallocation toward goods production, leading to a higher demand for labor and demand for loans. The former causes a rightward shift of the LE locus (to LE′) whereas the induced demand for loans enhances capital accumulation and causes an rightward shift of the KE locus (to KE′). The new equilibrium point is thus at E′ and the effects on the effective loans and the loan rate are generally ambiguous. Intuitively, a higher goods productivity induces capital and hence loan demand. By Pareto complementarity and factor substitution, the loan rate is lower (similar to what described above). Through factor reallocation between sectors, more labor is devoted to goods production and by diminishing returns the marginal product of labor decreases. By Pareto complementarity, this lowers the demand for capital and bank loans, offsetting the direct productivity effect. Moreover, the wage rate is lower in response to a lower marginal product of labor. Using the relationship of factor price frontier (factor substitution), the loan rate must be higher, also offsetting previous effect from induced demand.

In summary, we conclude:

Proposition 2: (Characterization of the Unconstrained Equilibrium) *Under Conditions E,F, L and S, the uniquely determined PFBGE possesses the following properties:*

(i) *a cost-reducing banking innovation increases bank loans per effective unit, enhances capital formation, and lowers the loan and the deposit rates;*

(ii) *a more efficient goods production has ambiguous effects on bank loans, capital formation and loan and deposit rates.*

IV. Moral Hazard and Credit-Constrained Equilibrium

We turn next to examine what happens if moral hazard causes banks to ration investment loans. Conventionally, moral hazard behavior is modeled as for borrowers to take excessively risky projects after obtaining the loan from banks in that the lender can ensure that the money is invested but cannot appropriate the return (see Hart and Moore 1994 and papers cited therein). In this paper, we adopt a parsimonious form of moral hazard that is sufficient to capture Keynes' (1964) consideration of the lender's risk.[17] Specifically, the lender cannot ensure that the money lent is indeed invested and thus fails to ensure the repayment. An individual firm, in anticipating a low rate of returns on productive investment,

[17] Banerjee and Newman (1993) illustrate this type of moral hazard problem: "[an agent may] attempt to avoid his obligations by fleeting from his village, albeit at the cost of lost collateral" (p. 280).

may have an incentive to "take the money and run" (i.e., to abscond), without repaying the loan.[18] While both approaches generate the possibility of credit rationing, the latter is analytically much simpler. Moreover, we may reinterpret the absconding story as one similar to Hart and Moore in terms of ex post effort. That is, one can consider a borrower to exert an effort on an investment project upon obtaining the loan. Then absconding is equivalent to assuming such an effort is a step function, taking values of 0 and 1 only (the value of 0 implies taking the money and running while the value of 1 means undertaking the investment).

Assume that failing to repay the loan, an individual firm owner would have part of the productive capital stock seized and this fraction is denoted by η (later referred to as the unit absconding cost). Thus, the cost of absconding is measured by ηk and the value of taking the money (the amount of loan, x) and run is $x + (1-\eta)k$. Moreover, by absconding an individual producer would lose the value accrued from goods production and hence the value of production measures the opportunity cost of absconding.[19] The incentive compatibility constraint (IC) that eliminates this moral hazard behavior is therefore given by:

$$V(k) \ \geq \ x + (1-\eta)k \tag{17}$$

which implies that the value of undertaking production exceeds the net value of absconding.[20] Using (8) and (13), we can rewrite (17) as:

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[18] In modeling the absconding behavior, it is not necessary to fully specify the structure of uncertainty. Of course, the necessity for the moral hazard behavior to occur is that banks cannot detect the possibility of absconding *ex ante*. The incentive compatibility constraint can be thus written without probabilistic measurement.

[19] Our setup follows Kehoe and Levine (1993) in which "creditors can seize the assets of debtors who default on their debts" (p. 869).

[20] The reader may find our incentive compatibility constraint is analogous to the limited liability constraint (PAii) in Sappington (1983, p. 6).

$$\delta \geq D \equiv \frac{1}{1+\sigma}[(g+d)+(1-\eta)(1+\theta)]-1 \tag{18}$$

Notably, if D exceeds the unconstrained loan rate δ^E, credit rationing is present. In this case, equation (11) is no longer applicable, as is the LE schedule (14). Thus, the "effective" labor efficiency locus is now represented by the kinked dash line in Figure 3 where the shaded area represents the effective region in which the incentive compatibility constraint is met. Our framework is obviously different from the static, partial-equilibrium loanable funds model of Stiglitz and Weiss (1981) in which an increase in the loan rate encourages firms' to undertake riskier projects. In our equilibrium credit rationing model in the absence of an explicit specification of riskiness, banks' profit maximization implies a desire to have the maximum amount of loans subject to the incentive compatibility condition and other equilibrium conditions. Since an equilibrium must be along the downward-sloping capital efficiency locus, the maximum amount of incentive-compatible loans is attained at point R (see Figure 3) where $\delta = D$. Thus, in equilibrium, the incentive compatibility constraint (17) is binding, while equilibrium credit ration occurs in the sense that the amount of loans is below the unconstrained level (indicated by point E). The moral hazard behavior is not observed in equilibrium and the credit-constrained equilibrium is associated with an loan rate higher than that in the unconstrained equilibrium.[21]

The reader may ask if the loan contract specified above is optimal under our competitive setting. On the one hand, no individual bank would under-cut in the loan rate, since it would obviously result in a moral hazard problem, leading to a failure of loan repayments. On the other hand, if an individual bank would offer a higher loan rate, it would end up with no customers. Thus, the credit-constrained loan rate must be equal to the exogenous value D with no individual banks deviating in equilibrium.

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[21] Similar results are obtained in the moral hazard trap model of Tsiddon (1992) under a very different setting.

From the above discussion, we learn that the necessary and sufficient condition for the presence of credit rationing is to have the unconstrained equilibrium loan rate δ^E below D. From Proposition 2 and the definition of D in (18), it is easily seen that in order for credit rationing to occur, we need the unit absconding cost (η) and the unit labor requirement of bank operation ($1/\phi$) to be sufficiently low. More precisely, the necessary and sufficient condition to guarantee the presence of credit rationing is (see the derivation in the Appendix C),

Condition R: (Credit Rationing) $Q < \left[\frac{\sigma + d}{\alpha A} \left(\frac{1+D}{1+\theta} \right) \right]^{\frac{\alpha}{1-\alpha}}$ where Q solves $Q = (1-\alpha)A\left(\frac{1}{\phi} + \frac{1+\theta}{g+d} Q^{\frac{1}{\alpha}} \right)$.

Notice that the fixed point mapping of Q corresponds to the intersection of the KE and LE loci. Furthermore, to ensure the feasibility of loanable funds (i.e., $x < a$), we impose:

Condition N: (Loanable Funds Feasibility) $D > (1+\theta)\left[\frac{A^{1/\alpha}}{\phi^{(1-\alpha)/\alpha}} \left(\frac{\alpha}{\sigma + d} \right) \right] - 1$.

The wage rate under the constrained equilibrium is:

$$w^R = (1-\alpha)A\left[\frac{\sigma + d}{\alpha A} \left(\frac{1+D}{1+\theta} \right) \right]^{\frac{-\alpha}{1-\alpha}} \tag{19}$$

Since the presence of credit rationing is associated with a higher loan rate, it is clear that the wage rate is lower than that in unconstrained equilibrium. From (6), (13), (19) and the first equality of (12) (recalling that the second inequality of (12) involves the use of (11) which is invalid in the case of credit rationing), the deposit rate under the constrained equilibrium can be written as:

$$r^R = [(1-\alpha)A]^{\frac{1}{\alpha}} \left(\frac{g+d}{1+\theta} \right) (D - \frac{w^R}{\phi})(1 - \frac{w^R}{\phi})(w^R)^{\frac{1-\alpha}{\alpha}} \tag{20}$$

which yields the financial spread as: $\dfrac{D - w^R/\phi}{r^R} = \left\{ [(1-\alpha)A]^{\frac{1}{\alpha}} \left(\dfrac{g+d}{1+\theta} \right) (1 - \dfrac{w^R}{\phi})(w^R)^{\frac{1-\alpha}{\alpha}} \right\}^{-1}$.

Comparing the credit-constrained equilibrium with the unconstrained equilibrium (point E) and utilizing (9), (10) and (13), we can establish:

Proposition 3: (Credit-Constrained Equilibrium) *Under Conditions E, F, L, S, R and N, there is a perfect foresight balanced growth equilibrium with credit rationing. The presence of credit constraints causes the loan and the deposit rates and the financial spread to increase, and effective bank loans and the effective capital formation to decrease.*

While most of the results are straightforward, the finding concerning the deposit rate and financial spread deserves further comments. We show in the Appendix C that the deposit rate is unambiguously higher under credit rationing than in unconstrained equilibrium. This is because the positive banking marginal revenue effect of a higher loan rate dominates the negative loan reduction effect, leading to a higher bank profit and hence requiring a higher deposit rate to restore the zero profit condition. This contrasts with the pure exchange model of Azariadis and Smith (1993) in which credit rationing reduces intergenerational borrowing and the enlarged forced savings cause interest rates to fall. Moreover, our model suggests that a higher loan rate and a lower wage rate tends to increase the financial spread, whereas a higher deposit rate lowers the financial spread. Under Condition N which ensures that the loan-deposit ratio is lower under credit rationing, equation (10) then implies that the former effect must dominate the latter such that the financial spread is widened under credit rationing.

Our results regarding the size of loans and the loan rate can be compared with those in the existing literature, particularly the overlapping generations models with production by Tsiddon (1992), Bencivenga and Smith (1993) and Fender and Wang (1997). The moral-hazard trap in Tsiddon and the adverse-selection driven constrained equilibrium in Bencivenga and Smith are both associated with a higher (loan) rate of interest. In Fender and Wang, credit rationing on education loans discourages human capital accumulation, resulting in a lower loan rate under capital-skill complementarity. In our investment-loan production economy, credit rationing leads to lower bank loans and thus less capital accumulation. By diminishing returns, the marginal product of capital is higher, as is the constrained equilibrium rate of loan.

The results of the comparative-static analysis for changes in ϕ and A in the presence of credit rationing is summarized in Table 2 and the diagrammatic analysis is displayed in Figure 4. A cost-reducing banking innovation (a higher ϕ) shifts the KE locus rightward to KE $'$ which causes the effective bank loans to increase, but it has no effect on the loan rate (in the sense of a local analysis). Thus, the wage rate is not affected because banking innovation has no effect on the goods production technology (and the factor price frontier in the goods sector). However, the deposit rate increases due to banks' zero profit condition in response to an increase in the marginal profit. As a result, the financial spread is narrowed in response to a cost-reducing banking innovation. Since an increase in bank loans leads to higher capital accumulation and goods production, banking innovation will cause a negative correlation between real output and the financial spread, consistent with the empirical evidence in Scotese (1995) and Lehr and Wang (2000).[22]

An improvement in goods production efficiency (a higher A) shifts both the KE and LE loci rightward to KE $'$ and LE$'$ respectively. As long as the horizontal rightward shift of LE is less than that of KE, the new constrained equilibrium is still determined by the intersection of KE $'$ and the horizontal line D. In this case, the loan rate is not affected, whereas the bank loans in effective units increase. Moreover, an improvement in production efficiency raises the marginal productivity of labor which increases the

[22] Scotese (1995) estimates a structural vector autoregression model using post-World War II U.S. data, finding that financial innovations, measured by the structural disturbances to the inverse of the loan-deposit interest rate differential, are positively correlated with long-run movements in real output. Similar findings are also obtained in the multi-country study of Lehr and Wang (2000).

equilibrium wage rate. Although the direct effect of an improvement in production efficiency is to increase the deposit rate, the indirect effect through the wage rate is ambiguous. As a consequence, the change in financial spread is also ambiguous.

We can now summarize,

Proposition 4: (Characterization of the Constrained Equilibrium) *The comparative statics of changes in any productivity parameters in the presence of credit rationing are:*

(i) *an advancement in banking or goods productivity raises the effective bank loans;*

(ii) *while an advancement in banking productivity results in a higher deposit rate and a lower financial spread, an improvement in goods productivity has ambiguous effects on them.*

It may be of interest to compare our findings with those in the static, pure-exchange framework of Williamson (1986) in which credit rationing emerges as a result of costly bank operation. A common outcome is that the presence of credit rationing leads to a higher loan rate. When credit rationing is present, Williamson (1986) finds that an increase in the bank operation (monitoring) cost reduces the deposit rate (from maintaining zero profit), while a higher unit labor requirement (lower ϕ) in our model also results in a downward change in the deposit rate. In another static, pure-exchange model by Holmström and Tirole (1997), the presence of the moral hazard problem causes credit rationing. They, in a partial equilibrium setting, find that credit rationing implies a higher loan rate and a lower deposit rate. While a higher loan rate in their study is similar to our result, a lower deposit rate is different from ours. Different from both Williamson (1986) and Holmström and Tirole (1997), our model incorporates potential effects via intertemporal substitution and production factor reallocation.

V. Credit Rationing and Endogenous Growth

In this Section, we extend the basic framework to allow for endogenous growth.[23] The simplest way is to consider endogenous intergenerational human capital accumulation via home education. This approach to sustained human capital growth captures the idea of Lucas (1988), Stokey (1988) and Laing, Palivos and Wang (1995), and resembles the framework developed by Becker, Murphy and Tamura (1990) and Glomm and Ravikumar (1992). It enables us to determine the rate of economic growth within our dynamical system in a simple manner. As a result, we can examine the growth effects of credit rationing as well as the consequences of technological advances in credit-constrained equilibrium. Moreover, we are able to compare the adverse effect of credit rationing on the loan size between exogenous and endogenous growth. At the end of this Section, we demonstrate that the growth consequences of credit rationing depend crucially on the driving force of endogenous growth. In particular, we contrast the results obtained in the benchmark model with those incorporating either education loans or uncompensated positive spillovers via firm learning.

We assume that the human capital stock of the immediate offsprings grows at a rate $g = g(v)$ where the growth rate is an increasing function of the time effort the current generation devoted to home education (denoted v). In addition to the time input, home education also requires real resources of λvh and thus generation t's household consumption in this case should be modified to:

$$c_{t+1} = (1+r_{t+1})[w_t(1-v_t) - \lambda v_t]h_t \qquad (21)$$

where 1-v measures total labor devoted to market activities (goods production and banking services). Allowing for parental altruism, the lifetime utility of the representative household of generation t is then

[23] The endogenous growth theory is developed by Romer (1986) and Lucas (1988) in which knowledge or human capital is the main engine driving the perpetual growth of the economy.

given by: $\mathrm{U}_t = \ln\{(1+r_{t+1})[w_t(1-v_t) - \lambda v_t]h_t\} + (1+\rho)^{-1} \ln\{(1+r_{t+2})[w_{t+1}(1-v_{t+1}) - \lambda v_{t+1}][1+g(v_t)]h_t\}$,

where in maximizing its utility, the household born at t takes prices (w, r), human capital endowment (h) and the time allocation of its immediate offsprings (v_{t+1}) as given. Of course, we follow the conventional wisdom to assume a positive intergenerational discounting at rate $\rho > 0$ in order to capture less-than-100-percent altruism.

Manipulation of the first-order condition for home education effort gives (see Appendix D):

$$\frac{\left(1 + \dfrac{\epsilon_g}{1+\rho}\right)\lambda}{\dfrac{\epsilon_g}{1+\rho}\left(\dfrac{1-v_t}{v_t}\right) - 1} = (1-\alpha)A^{\frac{1}{1-\alpha}}\left[\frac{\sigma+d}{\alpha}\left(\frac{\delta}{1+\theta}\right)\right]^{-\frac{\alpha}{1-\alpha}}. \tag{22}$$

where $\epsilon_g = vg^{/}(v)/[1+g(v)]$ denotes the home education elasticity of human capital accumulation. From (21), generation t's consumption per effective unit becomes:[24]

$$\frac{c_{t+1}}{h_t} = \frac{1+\rho}{\epsilon_g}(1+r_{t+1})v_t(w_t+\lambda) \tag{23}$$

which is increasing in the deposit rate, the wage rate and the fraction of time devoted to home education.

It is clear from (22) that in order to ensure the existence of an interior solution for a steady-state value of v, we need:

Condition G: (Positive Endogenous Growth) $0 < v < \dfrac{\epsilon_g}{1+\rho}\left(1 + \dfrac{\epsilon_g}{1+\rho}\right)^{-1}$.

This condition implies that the growth effect dominates the wealth effect, so the fraction of time devoted to home education is nondegenerate and the endogenous rate of growth is positive. This condition also

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[24] Recall that h_t is the knowledge stock generation t faces whereas a generation-t household's consumption (when old) is c_{t+1}.

guarantees $v < 1$. Condition G and Condition L together thereby insure feasible labor allocation in the production sector, the banking sector and the home education sector. To simplify our analysis, we further assume that the home education elasticity of human capital accumulation is nearly constant. In this case, Condition G also implies that the net effect of the balanced growth wage rate on human capital accumulation is positive. Then, by the implicit function theorem, (22) can be used to write $v = v(\delta; A)$, where $\partial v/\partial\delta < 0$ and $\partial v/\partial A > 0$ (noting that ϕ has no direct effect on v). Intuitively, an increase in the loan rate reduces loan demand and physical capital accumulation, leading to a higher wage rate due to Pareto complementarity. Given Condition G, this discourages cross-generation human capital investment (i.e., a lower value of v). By similar arguments, an improvement in the productivity in the goods sector raises the marginal product of labor and the wage rate, thereby encouraging human capital investment.

In the endogenous growth framework, the qualitative results concerning the feasibility of economic activities, the existence of equilibrium, the emergence of credit rationing, and the feasibility of loanable funds remain unchanged. While the LE locus (equation (14)) is identical in both the endogenous and exogenous growth models, the KE locus (equation (15)) in now different due to the fact that the endogenous growth rate $g(v)$ is an increasing function of v and hence a decreasing function of δ. As a result, the conditions to guarantee the proper slopes of the LE and KE loci (Condition S), goods feasibility (Condition F) and the existence of an equilibrium (Condition E) need to be modified accordingly.[25] When Conditions G and N as well as modified Conditions S, F and E are met, we obtain a balanced growth equilibrium with a *positive* and *bounded* rate of endogenous growth which may be unconstrained or constrained (the latter

[25] We can show (in Appendix E) that only Conditions S, F and E require modifications, whereas Conditions N and R remain the same. Modified Conditions S, F, and E are combined to one restricting an appropriate range for δ (Condition D): $\delta^{\min} \leq \delta \leq \min\{\delta_1^{\max}, \delta_2^{\max}\}$, where $\delta^{\min}$, $\delta_1^{\max}$ and $\delta_2^{\max}$ solve $\frac{[1+g(v(\delta^{\min}))]^{1-\alpha}}{g(v(\delta^{\min}))+d} = \frac{\alpha}{A^{1/\alpha}}[\phi(1+\theta)]^{(1-\alpha)/\alpha} + \frac{\alpha}{\sigma+d}$, $1+\delta_1^{\max} = [g(v(\delta_1^{\max}))+d]^{1-\alpha}(1+\theta)^{\alpha}\left(\frac{1}{\phi}\frac{\alpha}{1-\alpha}\right)^{1-\alpha}\frac{\alpha A}{\sigma+d}$ and $g(v(\delta_2^{\max}))+d = (1+\theta)\frac{1-\alpha}{\alpha}\frac{A^{1/\alpha}}{\phi^{(1-\alpha)/\alpha}}$, respectively. This holds for sufficiently large A and ϕ.

emerges if Condition R is met).

In the unconstrained equilibrium, an increase in banking productivity (ϕ) lowers the loan rate which enhances the investment in human capital. As a result, the economic growth rate is higher. A higher productivity in goods production (A) has a positive direct effect on human capital investment, as well as an indirect ambiguous effect on the loan rate (due to the shifts of both KE and LE loci). This leads to ambiguous effect on the investment in human capital. If the direct effect dominates the indirect effects, however, a productivity enhancement in the goods sector spurs economic growth.

In the presence of credit rationing, the characterization of the constrained equilibrium remain qualitatively unchanged, because the properties of the LE and the KE loci are the same as before and because the constrained loan rate is still pegged at a higher level than the otherwise unconstrained value.[26] However, the positive effect of credit rationing on the loan rate discourages human capital investment and hence reduces savings in kind. As a result, economic growth is lower in the constrained equilibrium. The growth-retarding effect of credit rationing differs from Shi (1996) where credit constraints may induce low-productivity firm to choose high-productivity technology and thus enhance economic growth. Moreover, the reduction in savings in kind resulting from credit market imperfections under our framework obviously contrasts with Azariadis and Smith (1993) and Jappelli and Pagano (1994), in which credit constraints raise savings in goods and spur output growth. The negative growth effect of credit rationing is, however, consistent with the theoretical finding in the investment loan model of Bencivenga and Smith (1993) where the adverse selection problem is present, as well as the education loan models of Tsiddon (1992) and Fender and Wang (1997) where the moral hazard problem is considered. Due to the discrepancy in the long-run growth consequence of credit market imperfections, we will discuss at the end of this section how

[26] More precisely, while the lefthand side of (18) (with equality) is still linear and increasing in δ, the righthand is now decreasing in δ (since g depends positively on v and v negatively on δ). The equality produces implicitly a pegged loan rate (quantitatively different than but qualitatively similar to D).

robust the detrimental growth effect of credit rationing is with regard to the model specification.

Finally, when the economy is credit rationed, the effects of a productivity improvement in the real and financial sector are different than those in unconstrained equilibrium. This is mainly due to the absence of an effect via the loan rate. On the one hand, lacking a direct effect, a cost-reducing banking innovation generates no effect on human capital investment nor economic growth. On the other, a technological advancement in goods production creates unambiguously a positive effect on growth, which is different from the finding in Bencivenga and Smith (1993). Thus, the growth implications of productivity enhancement under credit rationing in our paper depend crucially on whether it is originated in the real or financial sector.

In summary, we have:

Proposition 5: *With endogenous human capital accumulation, the presence of credit rationing raises loan rate and discourages human capital accumulation and thus output growth. In credit-constrained equilibrium, a cost-reducing banking innovation has no long-run growth effect, whereas an increase in goods productivity spurs economic growth.*

To the end, we examine the robustness of the detrimental growth effect of credit market imperfections. One may wonder if such a conclusion is a result of our intergenerational human capital accumulation setup. We may extend our framework in a straightforward manner to incorporate (i) pecuniary education spending following Fender and Wang (1997) and (ii) firm learning in the Romer (1986) convention that drives endogenous growth. More specifically, pecuniary education spending may require bank financing and hence be subject to credit constraints. As a consequence, credit market imperfections may restrict on education loans, lowering human capital investment and economic growth. When we allow the economy's aggregate effective capital to create an uncompensated positive spillover effect via firm learning, it influences the rate of economic growth positively, i.e., $g = g(v, k/h)$, with $\partial g/\partial v$

> 0 and $\partial g/\partial(k/h) > 0$. In this case, credit rationing has an additional effect to the detrimental force through human capital accumulation: its reduction in effective bank loans leads to lower effective physical capital, creating another channel to suppress economic growth. In summary, the detrimental growth effect of credit rationing is robust to the above extensions of the present model. However, in the second extension, the effective capital can influence the rate of economic growth. As a consequence, a cost-reducing banking innovation can increase effective bank loans and effective capital, thereby fostering output growth.

VI. Concluding Remarks

This paper develops a dynamic general-equilibrium model with production to study the long-run consequences of credit market imperfections. Our results suggest that credit rationing unambiguously causes the loan rate rise and the financial spread to widen, corroborating with empirical evidence in Tsiang (1980) and Diaz-Alejandro (1985). By allowing for endogenous human capital accumulation, we find that as a result of reduced savings in kind, credit rationing creates a detrimental growth effect, which is in sharp contrast to findings in Jappelli and Pagano (1994) and Liu and Woo (1994), lending theoretical support to the conventional view.

In our stylized model, we assume that households only consume when old and thus face forced savings when young. Although savings in kind is permitted under the endogenous growth setup, households' decision on savings in goods remains trivial. A natural extension of our model is to allow for intertemporal consumption-saving choice through which the effect of credit rationing may alter. In particular, credit rationing results in two opposing effects. On the one hand, it reduces the wage rate which decreases savings in goods. On the other, it increases the deposit rate, and thus increases savings in goods provided that the substitution effect of a higher deposit rate on savings dominates the associated wealth effect. In the absence of direct lending from households to firms, there would be no first-order growth effect. By allowing a direct link between household savings and firm capital formation, the former retards capital accumulation whereas the later enhances it. Should the former effect dominates the later, the net effect of credit rationing are to reduce savings in goods and thus the detrimental effect of credit rationing remains. Only when the latter effect is the dominant force, one may obtain results consistent with Jappelli and Pagano (1994) and Liu and Woo (1994). This therefore provides an empirically testable hypothesis concerning the long-run effects of credit rationing on household savings and economic growth.

Another extension is to allow for two types of investment firms, one undertaking high-risk, high-return projects and another initiating low-risk, low-return projects, in a fashion analogous to Azariadis and Smith (1993), Bencivenga and Smith (1993) and Shi (1996). In this case, the adverse selection problem is present and credit rationing emerges for an entirely different reason. It may be interesting to compare the results with ours, especially for the credit rationing effects on the loan size and the loan and deposit interest rates in a pooled equilibrium.

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Appendix

(Not Intended for Publication)

A. Derivation of the Firm's Optimization Conditions:

From (5) we can derive the first-order conditions for i and ℓ, respectively, as:

$$\left(\frac{1}{1+\sigma}\right) V'_{t+1} = \frac{1+\delta_t}{1+\theta} \tag{A1}$$

$$A(1-\alpha)\left(\frac{k_t}{\ell_t h_t}\right)^{\alpha} = w_t \tag{A2}$$

The Benveniste-Scheinkman condition is:

$$V'_t = A\alpha k_t^{\alpha-1}(\ell_t h_t)^{1-\alpha} + \frac{1-d}{1+\sigma} V'_{t+1} \tag{A3}$$

Guess $V(k)$ is linear in k: $V(k_t) = V_0 + V_1\, k_t$. Then (A1) and (A3) imply

$$V_1 = \alpha A \frac{1+\sigma}{\sigma+d}\left(\frac{k_t}{\ell_t h_t}\right)^{-(1-\alpha)} = (1+\sigma)\left(\frac{1+\delta_t}{1+\theta}\right) \tag{A4}$$

From (A2) and (A4) we get (6) and (7). Substituting (A4) into the above V(k) expression and using (5) and (A2), we can prove $V_0 = 0$ and derive the linear value function (8).

B. Conditions for the Balanced Growth Equilibrium:

Condition F: (Feasibility) $A\left[(1+g)\frac{\sigma+d}{g+d} - \alpha\right]^{\alpha} \geq (1+g)\,[\alpha(\sigma+d)]^{\alpha}\,[\phi(1+\theta)]^{1-\alpha}$.

Feasibility requires $y - i \geq c$. From the production function, (2), (11) and (13), the feasibility condition can be rewritten as $A\left(\frac{k}{h\ell}\right)^{-(1-\alpha)} \geq (g+d) + \frac{c}{k}$. Substituting (4), (6), (9) and (13) into this inequality, one can see that the feasibility holds if and only if $\frac{1+\delta}{1+\theta} \geq \alpha(\frac{g+d}{\sigma+d})(1+\frac{1}{1+g}\frac{1+r}{1+\theta})$, and by substituting into (9) and (7), this inequality can be re-written as

$$\left[\frac{1+g}{\alpha}\frac{\sigma+d}{g+d}-1\right]\Delta \geq (1+g)-\frac{(1-\alpha)A^{1/(1-\alpha)}}{\phi(1+\theta)}\left(\frac{\alpha}{\sigma+d}\right)^{\alpha/(1-\alpha)}\Delta^{-\alpha/(1-\alpha)} \tag{A5}$$

where $\Delta \equiv \frac{1+\delta}{1+\theta}$. As one can see, the LHS of (A5) is linear in Δ whereas the RHS is increasing and concave in Δ with a positive horizontal intercept and an asymptote 1+g as Δ approaches infinity. Define a critical value $\Delta_c \equiv \alpha A\left\{(\sigma+d)^{\alpha}\left[\phi(1+\theta)\left(\frac{1+g}{\alpha}\frac{\sigma+d}{g+d}-1\right)\right]^{(1-\alpha)}\right\}^{-1}$ such that the slope of LHS is equal to the slope of RHS at which (LHS - RHS) is minimized. Thus, if (LHS - RHS) is nonnegative at Δ_c, the above inequality (A5) must always hold for any value of Δ. This yields Condition F.

Condition S: (Slope condition) $\delta \leq \delta^{\max} \equiv (1+\theta)\left[\left(\frac{g+d}{1+\theta}\frac{1}{\phi}\frac{\alpha}{1-\alpha}\right)^{1-\alpha}\frac{\alpha A}{\sigma+d}\right]-1$

Differentiating (14) and (15) gives:

$$\frac{-d(x/h)|_{LE}}{d\delta} = \left.\frac{x}{h}\right|_{LE}\frac{1}{2+\theta+\delta}\frac{\alpha}{1-\alpha},$$

$$\frac{-d(x/h)|_{KE}}{d\delta} = \left.\frac{x}{h}\right|_{KE}\frac{\frac{1+\theta}{g+d}\left[\frac{\sigma+d}{\alpha A}\left(\frac{1+\delta}{1+\theta}\right)\right]^{\frac{1}{1-\alpha}}\left(\frac{1}{2+\theta+\delta}\right)\left(\frac{1}{1-\alpha}\right)}{\frac{1}{\phi}+\left(\frac{1+\theta}{g+d}\right)\left[\frac{\sigma+d}{\alpha A}\left(\frac{1+\delta}{1+\theta}\right)\right]^{\frac{1}{1-\alpha}}}$$

respectively. At the intersection of both LE and KE loci, i.e., $\left(\frac{x}{h}\right)_{LE} = \left(\frac{x}{h}\right)_{KE}$, the KE locus is steeper than the LE locus in (x/h, δ) space if $\frac{1+\theta}{g+d}\left[\left(\frac{\sigma+d}{\alpha A}\right)\left(\frac{1+\delta}{1+\theta}\right)\right]^{\frac{1}{1-\alpha}} < \frac{1}{\phi}\frac{\alpha}{1-\alpha}$, which can be rewritten as that specified in the Condition S.

Condition E: (Existence) $A^{\frac{1}{\alpha}} < \phi^{\frac{1-\alpha}{\alpha}}\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{g+d}{1+\theta}\right)$.

By equating the RHS of (14) and that of (15) and solving the resulting equation for δ, one can solve for the equilibrium δ^E. Thus, we can prove the existence of balanced growth equilibrium by showing such an equation has a solution for Q. Equating the RHS of both (14) and (15) yields

$$\frac{1}{\phi}+\frac{1+\theta}{g+d}\left[\frac{\sigma+d}{\alpha A}\left(\frac{1+\delta}{1+\theta}\right)\right]^{\frac{1}{1-\alpha}} = \frac{1}{(1-\alpha)A}\left[\frac{\sigma+d}{\alpha A}\left(\frac{1+\delta}{1+\theta}\right)\right]^{\frac{\alpha}{1-\alpha}}$$

Defining $Q \equiv \left[\frac{\sigma+d}{\alpha A}\left(\frac{1+\delta}{1+\theta}\right)\right]^{\frac{\alpha}{1-\alpha}}$, the above equation can then be rewritten as:

$$Q = H(Q) \equiv (1-\alpha)A\left[\frac{1}{\phi}+\frac{1+\theta}{g+d}Q^{1/\alpha}\right] \tag{A6}$$

Notice that $H^{/}(Q)=\frac{1-\alpha}{\alpha}\frac{1+\theta}{g+d}AQ^{(1-\alpha)/\alpha}$ is continuous and increasing in Q; moreover, $H'(Q = 0) = 0$. Let $\bar{Q}$ be such that $H^{/}(\bar{Q}) = 1$. To show the existence of an equilibrium, it is sufficient to show that $H(\bar{Q}) < \bar{Q}$. One can easily compute $\bar{Q} = \left[\frac{g+d}{1+\theta}\frac{1}{A}\frac{\alpha}{1-\alpha}\right]^{\alpha/(1-\alpha)}$. By substituting this result into $H(\bar{Q}) < \bar{Q}$, one can derive Condition E.

C. Moral Hazard, Credit Rationing and Proposition 3

Condition R: (Credit Rationing) $Q < \left[\frac{\sigma+d}{\alpha A}\left(\frac{1+D}{1+\theta}\right)\right]^{\frac{\alpha}{1-\alpha}}$ where Q solves $Q=(1-\alpha)A\left(\frac{1}{\phi}+\frac{1+\theta}{g+d}Q^{\frac{1}{\alpha}}\right)$.

Recall that $Q = \{[(\sigma+d)/\alpha A][(1+\delta)/(1+\theta)]\}^{[\alpha/(1-\alpha)]}$ in unconstrained equilibrium. When $D > \delta^E$ (the unconstrained equilibrium level of the loan rate), credit rationing occurs. Thus Condition R follows from setting δ = D.

Condition N: (Loanable Funds Feasibility) $D > (1+\theta)\left[\dfrac{A^{1/\alpha}}{\phi^{(1-\alpha)/\alpha}}\left(\dfrac{\alpha}{\sigma+d}\right)\right]-1$.

When the credit rationing constraint (18) is binding, the wage rate (w^R) obtained from (7) is

$$w^R = (1-\alpha)A\left[\frac{\sigma+d}{\alpha A}\left(\frac{1+D}{1+\theta}\right)\right]^{\frac{-\alpha}{1-\alpha}} \tag{A7}$$

Notice that w^R is decreasing in *D*. Moreover, from the bank's zero profit condition, one can derive the deposit rate in constrained equilibrium (r^R) as

$$r^R = [(1-\alpha)A]^{\frac{1}{\alpha}}\left(\frac{g+d}{1+\theta}\right)(D-\frac{w^R}{\phi})(1-\frac{w^R}{\phi})(w^R)^{\frac{1-\alpha}{\alpha}} \tag{A8}$$

It can be easily shown that $\left.\dfrac{d\ln r^R}{d\delta}\right|_{\delta=D} \propto \dfrac{1-\alpha}{\alpha}\dfrac{1}{w^R}\left(2+\theta+w^R/\phi\right)+\dfrac{1}{\phi}\dfrac{D-w^R/\phi}{1-w^R/\phi} > 0$. Furthermore, from the bank's zero profit condition, the constrained equilibrium financial markup is derived as:

$$q \equiv \frac{\delta-\frac{w}{\phi}}{r}\left(=\frac{a/h}{x/h}\right) = \left\{[(1-\alpha)A]^{\frac{1}{\alpha}}\left(\frac{g+d}{1+\theta}\right)(1-\frac{w^R}{\phi})(w^R)^{\frac{1-\alpha}{\alpha}}\right\}^{-1}. \tag{A9}$$

One can verify that $\dfrac{dq}{dD} > 0$ if and only if $D > (1+\theta)\left[\dfrac{A^{1/\alpha}}{\phi^{(1-\alpha)/\alpha}}\left(\dfrac{\alpha}{\sigma+d}\right)\right]-1$. As one might notice that this is also the condition for x < a since q = 1 when the loan rate is δ^E (see equation (10)) and $D > \delta^E$ (which is required for credit rationing to occur). As a result we have Condition N.

D. Derivation of the Optimizing Condition for Home Education:

The first-order condition for the home education effort is:

$$w_t + \lambda \;=\; \frac{\epsilon_g}{1+\rho}\left[w_t \left(\frac{1-v_t}{v_t} \right) - \lambda \right] \tag{A10}$$

Substituting the wage rate equation (7) into (A10), we obtain (22).

E. The Endogenous Growth Model

In the following we modify the conditions for the slope of LE and KE loci (Condition S), the feasibility (Condition F), the existence (Condition E), the existence of credit rationing (Condition R), and the loanable funds feasibility (Condition N).

Condition S′: $\delta \leq \delta_1^{max}$ where δ_1^{max} solves $1+\delta_1^{max} = [g(v(\delta_1^{max})) + d]^{1-\alpha}(1+\theta)^{\alpha}\left(\frac{1}{\phi}\frac{\alpha}{1-\alpha}\right)^{1-\alpha}\frac{\alpha A}{\sigma+d}$.

When the rate of economic growth is endogenous, the loan rate would affect education decision ($\partial v/\partial\delta < 0$), which in turn affects the economic growth rate ($g'(v)$>0). Taking this feedback effect into account, the right hand side of Condition S is also affected by δ. Rearranging the inequality in the Condition S, one obtains

$$(1+\delta) \leq [g(v(\delta)) + d)^{1-\alpha}(1+\theta)^{\alpha}\left(\frac{1}{\theta}\frac{\alpha}{1-\alpha}\right)^{1-\alpha}\frac{\alpha A}{\sigma+d} \tag{A11}$$

Notice that the left hand side of (A11) is bounded below by unity and increasing linearly in δ, whereas the right side is decreasing in δ. For sufficiently large *A*, the right hand is strictly greater than one as δ approaches zero. This ensures the existence of a value of $\delta = \delta_1^{max} > 0$ at which (A11) holds for equality. It is clear that for all $\delta \leq \delta_1^{max}$, the inequality in (A11) must hold, which yields Condition S′ for the endogenous growth case.

Condition F′: $\delta^{\min} \leq \delta$ where $\delta^{\min}$ solves $\dfrac{[1+g(v(\delta^{\min}))]^{1-\alpha}}{g(v(\delta^{\min}))+d} = \dfrac{\alpha}{A^{1/\alpha}}[\phi(1+\theta)]^{(1-\alpha)/\alpha} + \dfrac{\alpha}{\sigma+d}$.

Rearranging the terms in Condition F, one obtains

$$\frac{(1+g)^{1-\alpha}}{g+d} \;\geq\; \frac{\alpha}{A^{1/\alpha}}[\phi(1+\theta)]^{(1-\alpha)/\alpha} + \frac{\alpha}{\sigma+d}. \tag{A12}$$

Thus, goods feasibility requires g to satisfy the inequality in (A12). Notice that the left hand side of (A12) decreases in g and hence increases in δ, while the right hand side is independent of g or δ. Thus, goods feasibility requires a lower bound on the loan rate, as specified in Condition F′.

Condition E′: $\delta \;\leq\; \delta_2^{\max}$ where $\delta_2^{\max}$ solves $g(v(\delta_2^{\max})) + d = (1+\theta)\dfrac{1-\alpha}{\alpha}\dfrac{A^{1/\alpha}}{\phi^{(1-\alpha)/\alpha}}$.

By similar arguments in the derivation of Condition S′ above, the right side of Condition E is affected by δ in the endogenous growth case. By rearranging terms, we have:

$$g(v(\delta)) + d \geq (1+\theta)\frac{1-\alpha}{\alpha}\frac{A^{1/\alpha}}{\phi^{(1-\alpha)/\alpha}} \tag{A13}$$

While the right hand side of (A13) decreases in δ, the left hand side is a positive constant. A sufficiently large value of *A* ensures that (A13) may holds for equality at $\delta = \delta_2^{\max} > 0$. Then, for all $\delta \leq \delta_2^{\max}$, the inequality in (A13) must hold which leads to Condition E′.

We now combine Conditions S′, F′ and E′ together to obtain:

Condition D: $\delta^{\min} \leq \delta \leq \min\{\delta_1^{\max}, \delta_2^{\max}\}$,

which will be met with sufficiently large values of *A* and ϕ.

Condition R′: $Q'(\delta^{\mathrm{E}}) \le \left(\dfrac{\sigma+d}{\alpha A}\left(\dfrac{1+D}{1+\theta}\right)\right)^{\frac{\alpha}{1-\alpha}}$ where $Q'(\delta^{\mathrm{E}}) \equiv \left(\dfrac{\sigma+d}{\alpha A}\left(\dfrac{1+\delta^{E}}{1+\theta}\right)\right)^{\frac{\alpha}{1-\alpha}}$.

This condition follows directly from Condition R and the fact that the unconstrained equilibrium loan rate δ^{E} must be less than D in order for credit rationing to occur.

Condition N′: $D > (1+\theta)\left[\dfrac{A^{1/\alpha}}{\phi^{(1-\alpha)/\alpha}}\left(\dfrac{\alpha}{\sigma+d}\right)\right]-1$.

From the proof of Condition N, we know it is sufficient to show $\dfrac{dq}{dD}>0$. Notice that

$$\frac{dq}{dD} = \frac{\partial q}{\partial w}\frac{\partial w}{\partial D}+\frac{\partial q}{\partial g}\frac{\partial g}{\partial D} \tag{A14}$$

The major difference between the endogenous and exogenous growth cases lies in the fact that $\dfrac{\partial g}{\partial D} < 0$ in the former case whereas $\dfrac{\partial g}{\partial D} = 0$ in the latter. In the proof of Condition N, we have already shown that $D > (1+\theta)\left[\dfrac{A^{1/\alpha}}{\phi^{(1-\alpha)/\alpha}}\left(\dfrac{\alpha}{\sigma+d}\right)\right]-1$ implies $\dfrac{\partial q}{\partial w} < 0$. The remaining three partial derivatives on the right hand side of (A14) are all negative. Thus, Condition N remains valid in the endogenous growth case.

Table 1: Comparative Statics without Credit Rationing

Effect of	*x/h*	δ	*r*	*w*
ϕ	+	–	–	+
A	?	?	?	?

Table 2: Comparative Statics with Credit Rationing

Effect of	*x/h*	δ	*r*	*w*	$(\delta - w/\phi)/r$
ϕ	+	*0*	+	*0*	–
A	+	*0*	?	+	?

Figure 1: Perfect Foresight Steady-State Equilibrium

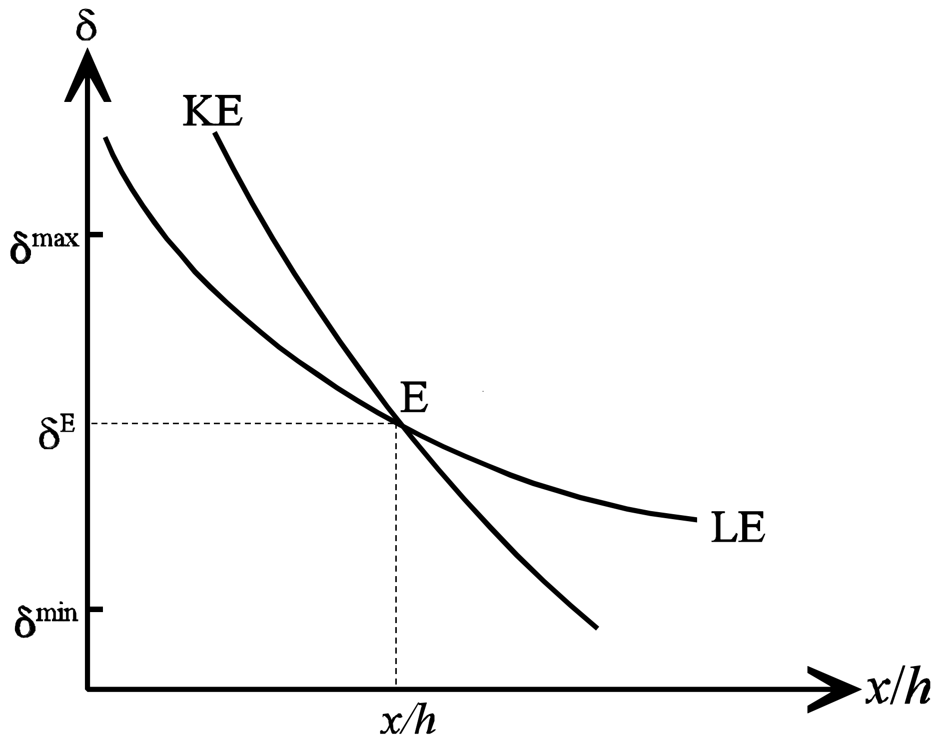


Figure 2a: Effects of an Increase in ϕ

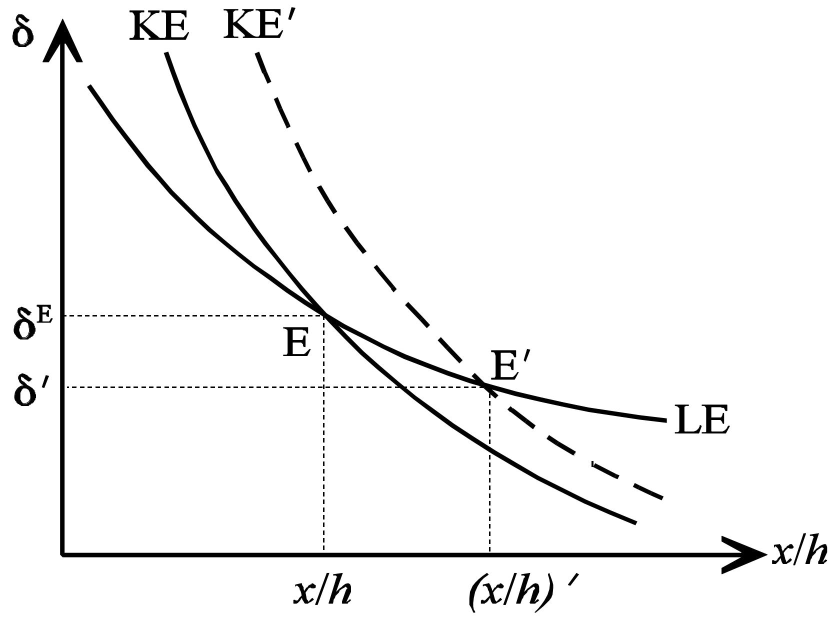


Figure 2b: Effects of an Increase in A

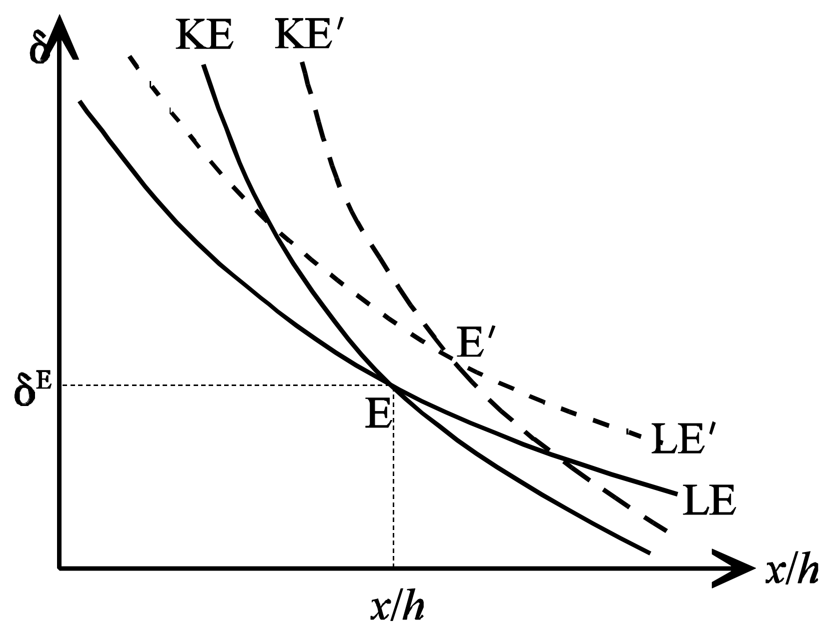


Figure 3: Credit-Constrained Steady-State Equilibrium

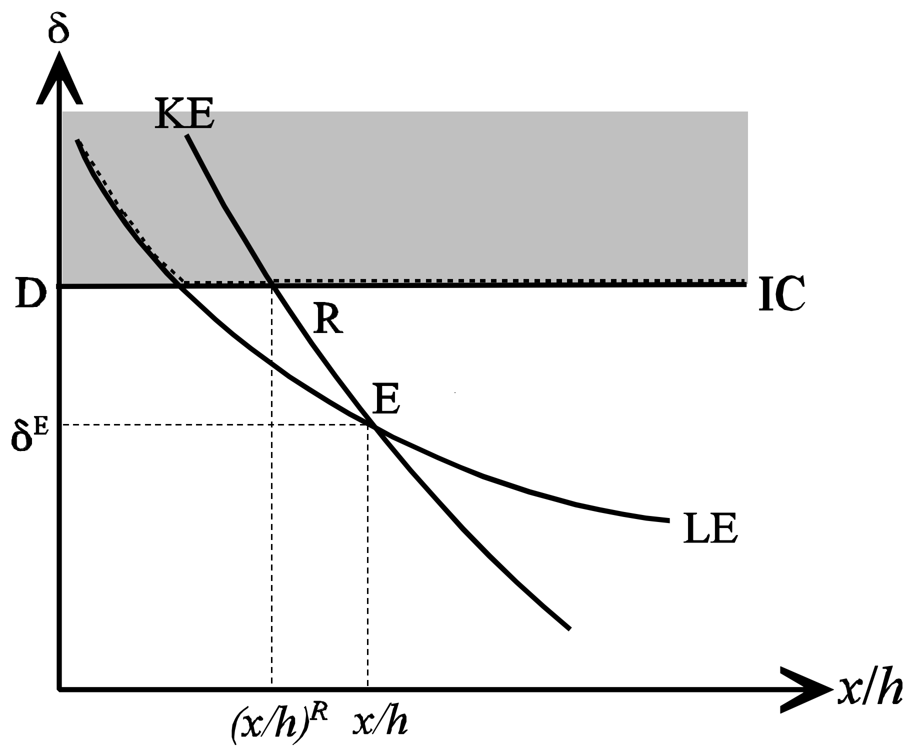


Figure 4a: Effects of an Increase in with Credit Rationing

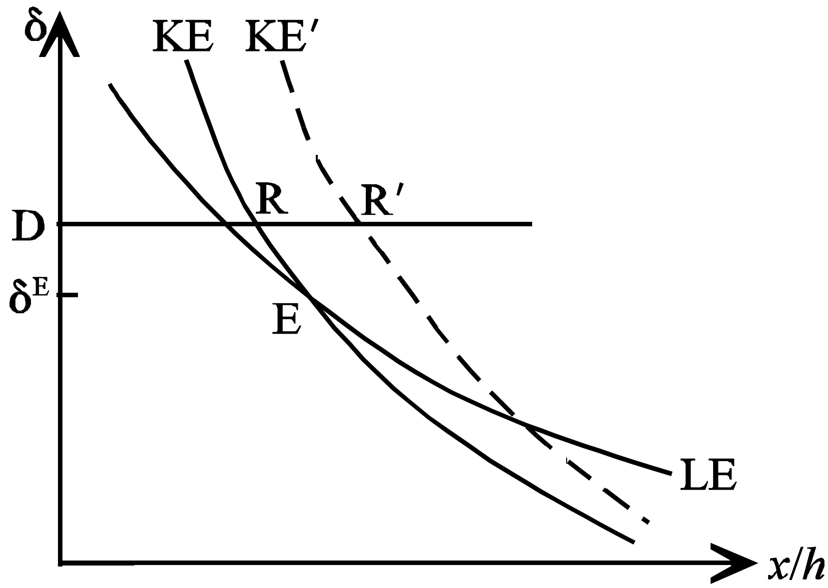


Figure 4b: Effects of an Increase in A with Credit Rationing

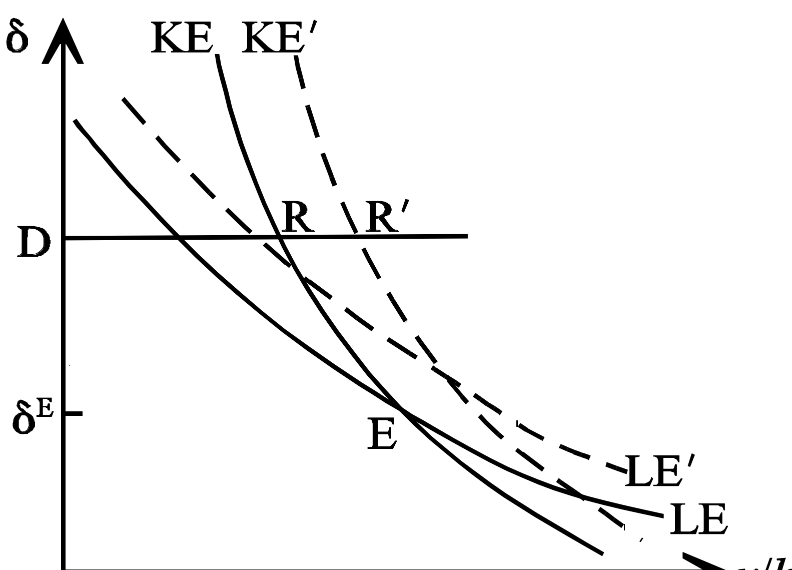


Chart 1: The Sequence of Events

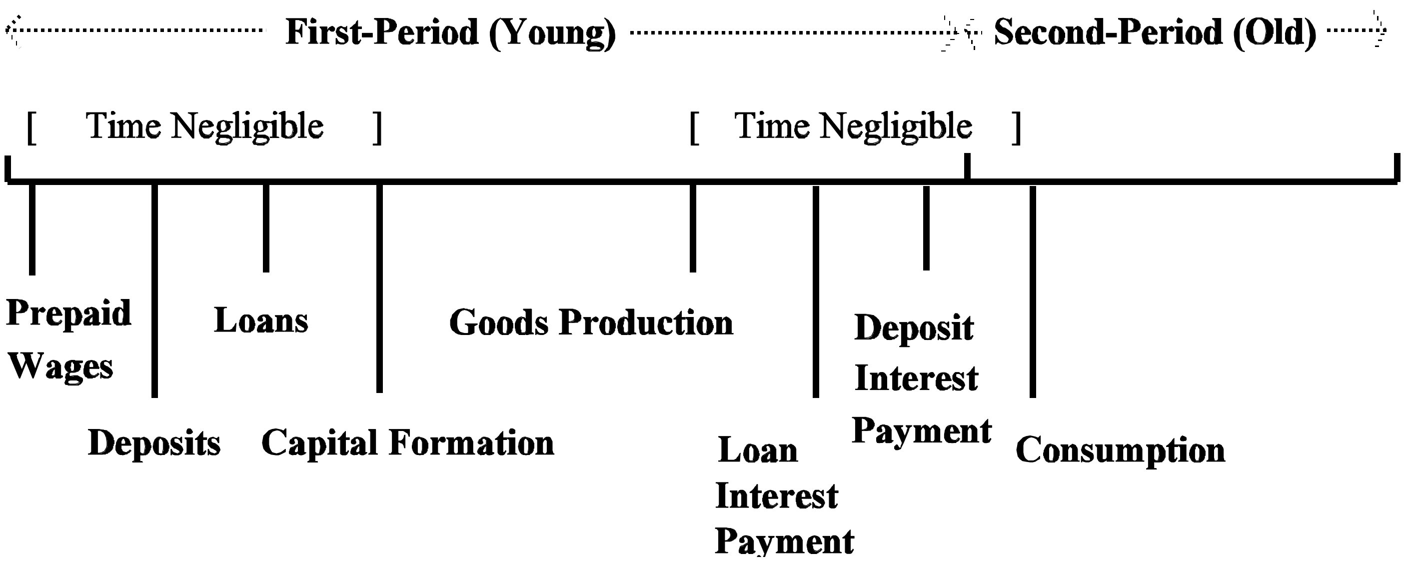
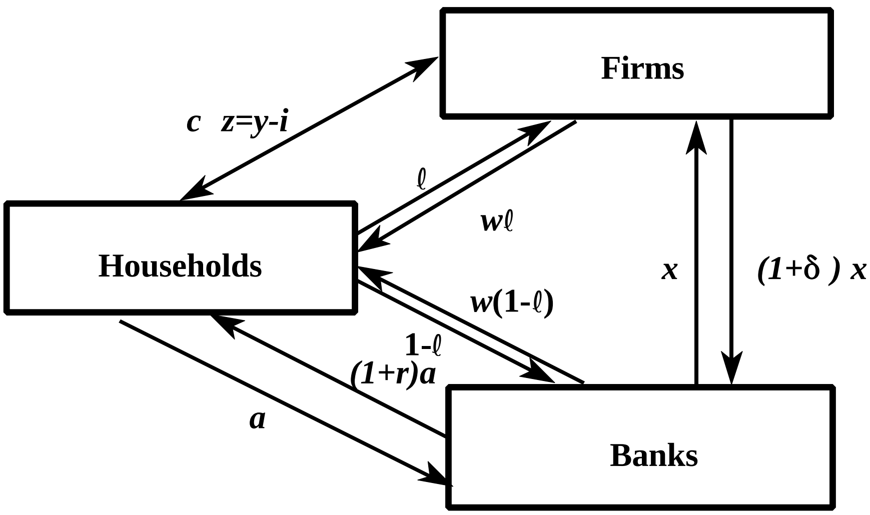


Chart 2: The Structure of the Basic Model

Firm Production: $y = A\, k^{\alpha}\, (\ell h)^{1-\alpha}$

Firm Profit: $_{f} = \; _{t}\; 1/(1+\sigma)^{t}\, [y - (1+\delta)x - w\ell h]$



Bank Production: $x = \phi\, [(1-\ell)h]$

Bank Profit: $_{b} = (\delta-\mu)x - ra - w(1-\ell)h$