Review of Economic Dynamics 52 (2024) 21–45

Contents lists available at ScienceDirect

Review of Economic Dynamics

journal homepage: www.elsevier.com/locate/red

Full Length Article

Optimal taxation in the life cycle with human capital investment[☆]

Been-Lon Chen [*], Fei-Chi Liang

Academia Sinica, Taiwan

ARTICLE INFO

JEL classification:
E62
H21
J24

Keywords:
Optimal capital and labor taxes
Human capital accumulation

ABSTRACT

This paper studies optimal taxes in a lifecycle model with unverifiable human capital investment inseparable from regular consumption. The planner faces asymmetric information regarding agents' exogenous abilities and endogenous human capital. Agents deviate in two ways: misreporting ability and mis-investing in human capital. We characterize the distortions in a model with i.i.d. shocks and full human capital depreciation. Distortions are characterized by capital wedges that are positive over the life cycle, labor wedges that are negative early and positive later in the life cycle, and net human capital wedges that are positive in the life cycle. These wedges serve as mechanisms to eliminate the distortion to consumption due to inseparability from education expenditure. Calibrate to U.S. data, we show numerically that these results apply in a richer model with persistent shocks and non-full human capital depreciation. Simulation suggests that average capital wedges are positive in all working periods, with progressive capital wedges in contemporary skills, average labor wedges are negative in early and positive in later periods, with hump-shape in skills and nonzero at the top and the bottom of the skill distribution, and net human capital wedges are positive and regressive in skills, indicating that human capital subsidies are in favor of the high skilled.

1. Introduction

People receive an education when they are young and go on to accumulate human capital thru training and learning by doing over the life cycle. A wide range of goods and services have components of consumption and human capital investment, and it is difficult to distinguish consumption activities from education purposes.[1] The difficulty in separating spending for normal consumption from education has been known in policy debates on how to design the tax system to foster human capital accumulation. As Grochulski and Piskorski (2010) put it, this measurement problem lies in the fact that, in reality, there is a human capital investment component in

☆ We would like to thank Loukas Karabarbounis (the Co-ordinating Editor), the anonymous associate editor and two referees for useful suggestions. We also thank Jang-Ting Guo, Dirk Krueger, Pei-Cheng Yu, and the participants of seminars at several universities and the conferences at American Economic Association, Asian Meeting of the Econometric Society, Association for Public Economic Theory, and Society of Advanced Economic Theory for comments.

* Correspondence address at: The Institute of Economics, Academia Sinica, 128 Academia Road Section 2, Taipei 11529, TAIWAN.
E-mail address: bchen@econ.sinica.edu.tw (B.-L. Chen).

[1] As Lazear (1977) pointed out, education is normal consumption and, like other normal goods, an increase in income produces increases in schooling purchase. According to Bovenberg and Jacobs (2005), books, computers and travelling costs are difficult to verify, since individuals may misrepresent expenditure for consumption as investment in education. Indeed, as early as in 1961, Schultz (1961) was aware of the difficulty in distinguishing education from consumption.

https://doi.org/10.1016/j.red.2023.11.004
Received 3 June 2022; Received in revised form 12 November 2023;
Available online 23 November 2023

normal consumption expenditure and a consumption value in the human capital investment activity such as education and training. Income tax policies affect the choice of investment between physical and human capital investment, and human capital investment in turn affects income distribution. The interaction between human capital and the tax system thus calls for a study of optimal income tax policies in a model with endogenous human capital over the life cycle. The vast majority of the literature on optimal income taxes assumes that productivity is exogenous in place of being the result of human capital decisions made throughout life.

This paper studies a dynamic Mirrlees model, in which agents accumulate privately observed human capital through education expenditure over the life cycle, but private regular consumption may be disguised as expenses for education purposes.[2] Our model extends the Stantcheva (2017) model with observable human capital investment to one with unobservable human capital investment. Agents in our model can deviate in two ways: by misreporting their ability and by mis-investing in their human capital. Mis-investing in human capital has persistent effects: an agent who has invested the wrong amount of human capital in the past will face different trade-offs today, and different incentives to report truthfully. It is precisely this complexity in possible deviation strategies that makes the problem complex. Even so, when the focus is on the case of fully depreciated human capital, we successfully characterize the distortions of the constrained efficient allocations in the problem. Earlier, in a dynamic Mirrlees model with unobservable investment in human capital only in the initial period, Grochulski and Piskorski (2010) studied the constrained efficient allocation by assuming that human capital may be fully depreciated due to risky bad shocks. Our model changes parts of their setting and adds value to Grochulski and Piskorski (2010).

First, under the case of fully depreciated human capital, our theoretical findings are that the distortions in the constrained efficient allocations are described by a capital wedge that is positive in the life cycle, a labor wedge that is negative early and positive later in the life cycle, and a human capital wedge (implicit human capital subsidy) that is positive in the life cycle.

Stantcheva (2017) and Grochulski and Piskorski (2010) obtained positive capital wedges due to skill shocks, which is an insurance effect against future risks.[3] We find a new mechanism, as consumption and education expenses are inseparable. As a result, agents may increase consumption by reducing unobservable human capital investment (hereafter, HCI), dubbed the HCI effect for simplicity. A higher-skill shock today exerts two HCI effects: one is reducing education expenses for consumption today and the other is reducing education expenses for consumption tomorrow. While today's HCI effect enhances the positive capital wedge from the insurance effect, tomorrow's HCI effect counteracts the positive capital wedge from the insurance effect. The net capital wedge is generally positive, unless tomorrow's HCI effect is so strong that it completely offsets the sum of today's HCI effect and the insurance effect. In particular, in the terminal period, without tomorrow's HCI effect, there is only today's HCI effect, so the capital wedge is larger than the otherwise positive capital wedge arising from the insurance effect.

Our negative labor wedges in the early life cycle are different from those of positive labor wedges in Stantcheva (2017) and positive labor wedges for low-skilled workers as well as negative labor wedges for high-skilled workers in Grochulski and Piskorski (2010). Positive labor wedges in these papers are to prevent high-skilled workers from pretending low-skilled and reducing labor effort, dubbed the shirk-preventing effect. However, with inseparable consumption and education expenses in our model, there is the skill-fostering effect that calls for negative labor wedges to induce agents to work more, so more consumption from under-investing in human capital is less attractive. The skill-fostering effect dominates the shirk-preventing effect in early periods, so the labor wedge is negative. In later periods, the HCI decreases and the skill-fostering effect phases out, so the labor wedge is positive. Intuitively, the deviation strategies involve shirking and underinvesting in human capital. Since it is more efficient if agents invest more in human capital, a negative tax on labor income in early periods reduces agents' incentives to underinvest in human capital in face of positive labor income taxes in the future needed for redistribution purpose. Thus, the negative labor income tax reduces the deadweight loss of redistributive taxation in the future.

Next, in the quantitative analysis, we allow for partially depreciated human capital. We find that the average capital wedge is positive over the life cycle due to positive today's HCI effects and insurance effects dominating negative tomorrow's HCI effects. In addition, the average labor wedge is negative in the early life cycle since the negative skill-fostering effect dominates, but it is positive in later working periods as positive shirk-preventing effect dominates. These findings are different from those in Farhi and Werning (2013) and Stantcheva (2017), who uncovered positive average capital wedge over the life cycle caused by insurance effects, and positive average labor wedge over the life cycle owing to shirk-preventing effects.

Moreover, we quantify the wedges against skill types and break down the wedges to different sources. First, the capital wedge is positive and progressive in all except very high skills, in which the source from today's HCI effects is positive and progressive in all except very high skills, which itself dominates positive and regressive insurance effects and negative and progressive tomorrow's HCI effects. This is a new result. Second, the labor wedge is positive and hump-shaped in skills, wherein the source of the shirk-preventing effect is positive and hump-shaped, while that of the skill-fostering effect is negative and regressive. Third, removing the effects of capital and labor wedges from the positive and hump-shaped human capital wedge, the net human capital wedge is positive but regressive in skills, for the high skilled benefit more from HCIs. Finally, individual overall tax, as a share of aggregate overall tax, is progressive in skills and positive in all except the bottom of the skill distribution, wherein individual overall tax is zero or even negative. This indicates that our constrained efficient allocation indeed redistributes the resource from the high skilled to the low skilled.

[2] One way to think about unobservable investments in human capital – in terms of consumption goods – is that if the government tries to subsidize education expenses, as prescribed by Stantcheva (2017)'s model, agents have an incentive to call their regular consumption human capital expenditure and avoid paying taxes all together.

[3] Models of exogenous skills (Diamond and Mirrlees, 1978) also found positive capital wedges due to the insurance effect.

Different from the positive capital wedge in skills due to the insurance effect in existing dynamic Mirrlees models (e.g., Farhi and Werning, 2013), our positive and progressive capital wedge in all except very high skills is due to positive and progressive today's HCI effects in skills dominating the insurance and tomorrow's HCI effects. Moreover, unlike the positive average labor wedge due to the standard positive shirk-preventing effect in models without private HCIs (e.g., Stantcheva, 2017) and without HCIs (e.g., Farhi and Werning, 2013), our negative average labor wedge in the early life cycle is caused by the negative skill-fostering effects dominating the positive shirk-preventing effect. Moreover, our labor wedge is hump-shaped in skills, like that in the new dynamic public finance literature (e.g., Golosov et al., 2006); yet, as the negative skill-fostering effect dominates the positive shirk-preventing effect, our labor wedge is negative at the top and the bottom of the skill distribution, different from the standard zero-tax result at the top and the bottom of the skill distribution. Note that our nonzero-tax result comes from private HCIs, different from the nonzero-tax result in Farhi and Werning (2013), which arises from a moving support with the top and bottom bounds of the productivity being functions of the previous period's productivity.

Finally, in evaluating the benefit of our tax system, we find a quantitatively large welfare gain within our second-best economy over the status quo tax system. We also assess a simple linear age-dependent tax system and find that it performs very well in that it can yield a large welfare gain within our second-best economy. Lastly, we numerically verify that the allocation solved by the envelope condition is ex post incentive compatible, so our solution to the relaxed program is indeed the solution to the full program.

1.1. Related literature

There is a longstanding literature that emphasizes endogenous skill acquisition (Becker, 1964; Ben-Porath, 1967). In particular, the literature posits that human capital is accumulated over the life cycle, underscoring the need for a life-cycle model (Cunha and Heckman, 2007). A body of empirical work documents risky returns to human capital investment and thus earnings (e.g. Meghir and Pistaferri, 2004). Our model incorporates these facts by assuming that agents invest in human capital over the life cycle and are subject to skill shocks.

There is a growing body of the new dynamic public finance literature, which extends the optimal taxation pioneered by Mirrlees (1971) to a dynamic setting.[4] The literature typically considers exogenously evolving abilities, thus abstracting from endogenous skill acquisition. We contribute to this literature by considering endogenous skills, which change over time based on human capital investment. Our paper is closely related to the dynamic Mirrlees literature that studied optimal income taxation with monetary investment in human capital. Most related papers are Stantcheva (2017) and Grochulski and Piskorski (2010).

In Stantcheva (2017), agents accumulate human capital over the life cycle through observable education expenses. Besides positive labor wedge for redistribution purposes and positive capital wedge for insurance purposes, Stantcheva uncovered optimal education subsidies to observable education expenses, which not only prevent agents from shirking but also offset capital and labor income tax-induced distortions to learning.[5] In our model, the positive capital wedges arise from not only insurance purposes but also the HCI effect that emerges from distortions due to inseparable consumption and education expenses. Moreover, our negative labor wedge early in the life cycle arises, as the positive shirk-preventing effect is dominated by the negative skill-fostering effect. Thus, our tax policy is to offset distortions due to inseparable consumption from education expenses.

In an earlier version, Stantcheva (2014, Section 7) extended Stantcheva (2017) to one with unobservable human capital and derived the condition for labor wedges and the modified inverse Euler equation for capital wedges, but their signs were not determined. There are three differences. First, skill shocks arrive before agents' HCIs are model in our model,[6] but after HCIs are made in Stantcheva (2014). Second, the education expense affects the human capital without a time lag in Stantcheva (2014, 2017), while the human capital is accumulated with a one-period lag in our model. The different timings of skill shocks and HCIs enable us to separate the skill-fostering from the shirk-preventing effect, which work with a one-period difference. Thirdly, our model determines the signs of labor and capital wedges. We analytically obtain negative labor wedges in early periods of the life cycle. Moreover, we obtain a modified inverse Euler equation, which analytically separates today's positive HCI effect from tomorrow's negative HCI effect on capital wedges.

Next, in Grochulski and Piskorski (2010), human capital is increased by unobservable education expenses in the initial period and is subject to stochastic depreciation shocks later in the life cycle. They found positive labor wedges for low-skilled types and negative labor wedges for high-skilled types, and positive capital wedges due to insurance effects against skill depreciation shocks that come after HCIs are made. Our model is different. Firstly, skill shocks arrive before HCIs are made. Next, unobservable HCIs are made over the life cycle. As a result, for all skill types, labor wedges are negative early and positive later in the life cycle; and capital wedges are positive due to not only insurance but also HCI effects.

[4] The Mirrlees approach considers an unrestricted direct revelation mechanism (e.g., Kocherlakota, 2005; Golosov et al. 2006; and Farhi and Werning, 2013), as opposed to the Ramsey approach that specifies ex ante instruments available to the government (e.g., Judd, 1985; Chamley, 1986; and Chen and Lu, 2013).

[5] An earlier static model by Bovenberg and Jacobs (2005) also found positive optimal labor taxes for redistribution and education subsidies to eliminate the adverse impact of redistributive taxes on observable education expenses.

[6] Numerous studies find that cognitive ability (such as learning ability) and noncognitive traits (such as personality traits and health) affect agents' performance and human capital formation and are a strong predictor of schooling attainments and wages (e.g., Heckman et al., 2006). Borghans et al. (2008) also present the evidence that both cognitive and personal traits evolve over the life cycle. Along this line, we assume that skill shocks like cognitive and noncognitive abilities arrive before agents invest in human capital. The setup is in line with that in a study by Huggett et al. (2011), who built a human capital technology wherein investment in human capital is made after an agent's learning ability shock is realized.

Moreover, Kapička (2015) and Kapička and Neira (2019) studied Mirrlees income taxes in models with observable education expenses and unobservable learning efforts over the life cycle. Due to risky HCIs, optimal income taxes decrease with age in Kapička (2015), and optimal tax policies balance redistribution, insurance, and incentives in Kapička and Neira (2019). With incentives to learn and work, increases of labor supply in one period induce changes in HCIs and affect the disutility of working in other periods in these two papers. As unobservable HCIs make preferences over labor supply non-separable across age, optimal labor income taxes depend on whether labor is complementary or substitutable across age and how incentives between learning and work are balanced. Different from their unobservable learning time for HCIs, we consider observable and unobservable expenditure for HCIs, and our labor wedge is negative in early and positive in later age, thus implying optimal labor income taxes to increase with age. Kapička and Neira (2019) also found positive capital wedges but come from the moral hazard problem to elicit higher learning effort today. By contrast, our positive capital wedges come from not only the insurance but also from the tradeoff between unobservable consumption and education expenses.[7]

Note that Allen (1985) explored whether long-term contracts can improve on a series of short-term contracts, and Cole and Kocherlakota (2001) studied whether efficient consumption allocation can be decentralized via a competitive asset market. Their consumption is unobservable, as agents can borrow or save secretly. In contrast, consumption is unobservable in our model, as it is indiscernible from education expenses. Moreover, we study whether unobservable consumption affects the optimal capital and labor income taxes.

Finally, in models with an extensive margin of the labor supply, Diamond (1980) and Saez (2002) found obtained negative labor wedges for the working poor at the bottom of the income distribution, because the labor force participation effect dominates the incentive effect of higher income earners. Contrarily, we obtain negative labor wedges for all workers, because consumption is indistinguishable from education expenditure.

We organize this paper as follows. Section 2 presents a two-period model and characterize the sign of the capital, labor and net human capital wedges of the constrained efficient allocation. Section 3 extends the model to T periods. In Section 4, we offer numerical analysis. Finally, concluding remarks are offered in Section 5.

2. An illustrative example: a two-period model

To introduce the main ideas in the simplest way, we start with a two-period model, extended to a T-period model in the next section, wherein agents' choice and shock history affects human capital investment.

2.1. The environment

Preference. The economy consists of a continuum of agents who live for two periods. An agent obtains utility from consumption and disutility from working, with a utility function represented by:

$$u(c_1) - \phi(l_1) + \beta[u(c_2) - \phi(l_2)],$$

where $0 < \beta < 1$ is the discount factor, c_t is consumption, and l_t is work effort in period t. An agent provides at most $\bar{l} > 0$ work effort in a period. We assume that $u(\cdot)$ satisfies the Inada condition and $u' > 0 > u''$, and $\phi(\cdot)$ satisfies $\phi', \phi'' > 0$, $\phi(0) = 0$, $lim_{l\to 0}\phi'(l) = 0$, and $lim_{l\to\bar{l}}\phi'(l) = \infty$. An agent with human capital h_t and effort l_t supplies $z_t = l_t h_t$ units of effective labor. At the initial period $t = 1$, an agent consumes, spends on education to accumulate human capital h_2, or saves to form physical capital k_2 in the next period.

Final goods production. In each period, the representative firm uses aggregate physical capital K_t and aggregate effective labor Z_t to produce final goods using the technology $F(K_t, Z_t)$. The technology is neoclassical, which satisfies constant returns to scale and is strictly increasing and concave in K_t and Z_t. The physical capital depreciates at the rate of δ_k.

Human capital technology. There are two kinds of education expenses, verifiable x_t and non-verifiable y_t. The human capital technology is

$$h_{t+1} = H(x_t, y_t, \theta, h_t) \tag{1}$$

where θ is a skill shock over a fixed support $\Theta = \left[\underline{\theta}, \overline{\theta}\right]$. The technology is increasing in x_t, y_t, θ, h_t and concave in x_t and y_t. That is, H_x, $H_y, H_\theta, H_h \geq 0$, and $H_{xx}, H_{yy} \leq 0$. Agents are assumed to be endowed with the same human capital level h_1 when born, while agents with larger skill shocks have advantages to acquire human capital than those with smaller shocks. Ability shocks arrive before education investment is made, which is different from Grochulski and Piskorski (2010) and Stantcheva (2017), wherein skill shocks come after the investment is made. This timing is the key to separate the shirk-preventing effect from the skill-fostering effect, which give a positive and a negative labor wedge, respectively.

Agents' types and reporting strategy. A skill shock θ is an innate ability, which is private information, referred to as an agent's type.[8] The probability of a skill shock θ is $\pi(\theta)$, with $0 \leq \pi(\theta) \leq 1$ for each $\theta \in \Theta$. Suppose that the law of large numbers applies; then,

7 Bovenberg and Jacobs (2010) introduced non-verifiable HCIs via time, so consumption is separable from education.

8 Although skill shocks are time varying, agent's types in period 2 do not affect the economy in a simple two-period model, for HCIs are not needed in the terminal period. Thus, agents only need to report their types in period 1. In a T-period model latter, the history of types affects the allocation in later periods.

$\pi(\theta)$ indicates the probability density function (p.d.f.) of the type θ agents. Suppose that the social planner designs a direct revelation mechanism, in which agents report ability types in each period and the social planner directly specifies the allocation to agents according to agents' reported types. Let $\sigma(\theta)$ denote an agent's reporting strategy, specifying a reported type σ conditional on the true type θ. Some notations are in order. If a is observable, $a(\theta)$ denotes the allocation to true type θ. If a is unobservable, $a^{\sigma}(\theta)$ denotes the allocation to true type θ whose reported type is σ. When $\sigma = \theta$, the agent truthfully reports the type and thus, $a(\theta)$ denotes the allocation to the truth-telling agent.

Resource constraints. Given the initial capital K_1, the initial human capital h_1, and the government expenditure G_1 and G_2, an allocation A, consisting of consumption $c_t(\theta)$, non-verifiable and verifiable education expense $y_t(\theta)$ and $x_t(\theta)$, human capital $h_t(\theta)$, labor effort $l_t(\theta)$, effective labor $z_t(\theta)$ and capital $k_t(\theta)$, is ***feasible*** if it satisfies the following resource constraints.

$$\int_{\Theta}[c_1(\theta)+y_1(\theta)+x_1(\theta)]\pi(\theta)d\theta+K_2\leq F(K_1,Z_1)+(1-\delta_k)K_1-G_1, \tag{2a}$$

$$\int_{\Theta}c_2(\theta)\pi(\theta)d\theta\leq F(K_2,Z_2)+(1-\delta_k)K_2-G_2, \tag{2b}$$

where $K_2 = \int k_2(\theta)\pi(\theta)d\theta$, $Z_t = \int z_t(\theta)\pi(\theta)d\theta$.

Hicksian coefficient of complementarity. In determining the direction of distortion caused by the non-verifiable education expenses, the Hicksian coefficient of complementarity between ability θ and non-verifiable education expenses y_t, denoted by $\rho_{y\theta}$, is a crucial parameter, which is defined by

$$\rho_{y\theta}\equiv\frac{H_{y\theta}H}{H_yH_\theta},$$

where H_j is the partial derivative of the human capital technology H with respect to $j\in\{y,\theta\}$ and $H_{y\theta}$ is the second-order partial derivative of H with respect to y and θ. When $\rho_{y\theta}>0$, higher type agents have higher marginal returns to non-verifiable education expenses y. A multiplicative form $H=\theta\psi_3(x_t,y_t,h_t)$ gives $\rho_{y\theta}=1$, while a separable form $H=\psi_1(x_t,y_t)+\psi_2(\theta,h_t)$ yields $\rho_{y\theta}=0$. A constant elasticity of substitution (CES) form $H=(\alpha_1y_t^{1-\rho}+\alpha_2\theta^{1-\rho})^{1/1-\rho}\psi_4(x_t,h_t)$ means that $\rho_{y\theta}$ is a constant and equals ρ.

Information structure. In our environment, agents' skill shocks θ, non-verifiable education expenses y_1, consumption in the first period c_1, and human capital h_2 and work effort l_2 in the second period are private information. Although y_1 and c_1 are not observable, their sum $c_1+y_1\equiv d_1$ is, since it is inferable from the budget constraint. Initial human capital h_1, capital k_t and effective labor z_t, $t=1$, 2, verifiable education expenses x_1, and consumption in the second period c_2 are publicly observable. Note that work effort l_1 is inferable from initial human capital and individual effective labor in the first period z_1, and is thus observable.

2.2. Agent's problem - individual optimal non-verifiable education expenses

As consumption is indistinguishable from education expenses, the agent reporting the strategy $\sigma(\theta)$ trades off consumption and education expenses, given that their sum $c_1^{\sigma}(\theta)+y_1^{\sigma}(\theta)$ is consistent with the reported type $c_1(\sigma)+y_1(\sigma)\equiv d_1(\sigma)$. Given the allocation $\{d_1(\sigma),\ x_1(\sigma),\ c_2(\sigma),\ z_1(\sigma),\ z_2(\sigma)\}$ that the social planner assigns for truth-telling agents, an agent θ with a reported type σ chooses the allocation $\{c_1,\ y_1,\ h_2\}$ to solve:

$$\max_{c_1,y_1,h_2}u(c_1)-\phi\left(\frac{z_1(\sigma)}{h_1}\right)+\beta\left[u(c_2(\sigma))-\phi\left(\frac{z_2(\sigma)}{h_2}\right)\right], \tag{3}$$

s.t. $c_1+y_1=d_1(\sigma)$ and $h_2=H(x_1(\sigma),y_1,\theta,h_1)$, with h_1 given.

Although the allocation $\{c_1,\ y_1,\ h_2\}$ is not observable, the social planner knows that agents with a reporting strategy $\sigma(\theta)$ would solve the above problem by choosing private allocation $\{c_1^{\sigma}(\theta),\ y_1^{\sigma}(\theta),\ h_2^{\sigma}(\theta)\}$. Denote $\phi_h\left(\frac{z_t}{h_t}\right)\equiv-\phi'\left(\frac{z_t}{h_t}\right)\frac{z_t}{(h_t)^2}<0$. The above problem yields the following proposition. See Appendix A.1.2.

Proposition 1. *When there are non-verifiable education expenses, the optimal non-verifiable education expenses are determined by the following condition*

$$u'\left(c_1^{\sigma}(\theta)\right)=-\beta\phi_h\left(\frac{z_2(\sigma)}{h_2^{\sigma}(\theta)}\right)H_y\left(x_1(\sigma),y_1^{\sigma}(\theta),\theta,h_1\right). \tag{4}$$

As the second-order condition is negative, the optimal condition (4) *is the necessary and sufficient condition.*

Proposition 1 says that agents with a reporting strategy $\sigma(\theta)$ spend on non-verifiable y_1 until the decrease in the marginal utility of consumption today equals the increase in the discounted marginal utility of leisure tomorrow (or the decrease in the discounted marginal disutility of labor) resulting from higher human capital. The following lemma investigates the signs of $\frac{\partial y_1^{\sigma}(\theta)}{\partial\theta}$ and $\frac{\partial h_2^{\sigma}(\theta)}{\partial\theta}$, and the proof is relegated to Appendix A.1.3.

Lemma 1. (i) if $\rho_{y\theta} \leq 2$, then $\frac{\partial y_1^\sigma(\theta)}{\partial\theta} < 0$,

(ii) if $\rho_{y\theta} \geq \frac{H_{yy}H}{H_y^2}$, then $\frac{\partial h_2^\sigma(\theta)}{\partial\theta} > 0$.

(iii) if there are only verifiable education expenses, then $\frac{\partial y_1^\sigma(\theta)}{\partial\theta} = 0$ and $\frac{\partial h_2^\sigma(\theta)}{\partial\theta} > 0$.

The results $\frac{\partial y_1^\sigma(\theta)}{\partial\theta} < 0$ and $\frac{\partial h_2^\sigma(\theta)}{\partial\theta} > 0$ hold when $\rho_{y\theta}$ is neither too large nor too small; i.e., $\frac{H_{yy}H}{H_y^2} \leq \rho_{y\theta} \leq 2$. A separable form, a multiplicative form, and the Cobb-Douglas form all meet $\frac{H_{yy}H}{H_y^2} \leq \rho_{y\theta} \leq 2$. This result $\frac{\partial y_1^\sigma(\theta)}{\partial\theta} < 0$ holds when $\rho_{y\theta}$ is sufficiently small, as it gives rooms for high types to benefit from cutting off HCIs when pretending to be low types, while the result $\frac{\partial h_2^\sigma(\theta)}{\partial\theta} > 0$ holds when $\rho_{y\theta}$ is sufficiently large, since it gives rooms for high types to benefit from more leisure by providing less labor effort if pretending to be low types.

When there are no non-verifiable education expenses, agents cannot cut off HCIs without being caught. Moreover, high types can generate high productivity regardless of the range of $\rho_{y\theta}$, as shown in Lemma 1(iii).

2.3. *Incentive compatibility*

Now, we establish the incentive compatible constraint. Let the lifetime utility of an agent with reporting strategy $\sigma(\theta)$ be denoted by

$$W^\sigma(\theta) \equiv u\left(c_1^\sigma(\theta)\right) - \phi(l_1(\sigma)) + \beta\left[u(c_2(\sigma)) - \phi\left(l_2^\sigma(\theta)\right)\right]. \tag{5a}$$

where $c_1^\sigma(\theta)$ must satisfy Proposition 1, when there are non-verifiable education expenses. Then, an allocation A is ***incentive-compatible*** if

$$W(\theta) \geq W^\sigma(\theta), \forall\sigma, \theta \in \Theta. \tag{5b}$$

The envelope condition is derived as follows. Incentive compatibility (IC) constraints in (5a)-(5b) imply that, for all θ, the incentive compatibility constraint is as follows:

$$\begin{aligned} W(\theta) &= \max_{\sigma\in\Theta} W^\sigma(\theta) \\ &= \max_{\sigma\in\Theta} u\left(c_1(\sigma) + y_1(\sigma) - y_1^\sigma(\theta)\right) - \phi\left(\frac{z_1(\sigma)}{h_1}\right) + \beta\left[u(c_2(\sigma)) - \phi\left(\frac{z_2(\sigma)}{h_2^\sigma(\theta)}\right)\right]. \end{aligned} \tag{5c}$$

If we take the derivative with respect to (true) skill shocks, there are two direct effects on unobserved variables, namely, unobserved education expenses y_1 and the human capital h_2, and indirect effects on the allocation through the report. By the first-order conditions of the agent, all indirect effects are jointly zero and only the two direct effects remain. This leads to the agent's envelope condition as follows:

$$\dot{W}(\theta) = -u'(c_1(\theta))\frac{\partial y_1(\theta)}{\partial\theta} + \beta\phi'\left(\frac{z_2(\theta)}{h_2(\theta)}\right)\frac{z_2(\theta)}{(h_2(\theta))^2}\frac{\partial h_2(\theta)}{\partial\theta}, \tag{5d}$$

where $\dot{W}(\theta) \equiv \frac{\partial W(\theta)}{\partial\theta}$ and $\frac{\partial h_2(\theta)}{\partial\theta} = H_y\frac{\partial y_1(\theta)}{\partial\theta} + H_\theta$.

The envelope condition unveils how a promised utility changes with types. To inspire the truth-telling, an incentive compatible allocation must stop agents from getting the benefit of misreporting. The first term in (5d) is the static utility gain from reducing non-verifiable education for consumption today, while the second term is the dynamic utility gain of leisure tomorrow from the benefit of higher shirking abilities.[9] It has been shown that the envelope condition is a necessary condition for incentive compatibility (Milgrom and Segal, 2002).

2.4. *The planning problem in the two-period model*

We proceed to envisage the social planner's dynamic programming problem. By comparing the second-best allocation in the social planner's problem to the allocation in the decentralization problem, we can understand the distortion in the second-best allocation relative to the laissez-faire allocation.

The social planner chooses allocations that maximize the following utilitarian social welfare:[10]

[9] As seen from Proposition 1, with skill shocks, higher-skill types need not spend non-verifiable education too much in order to yield higher skills in the future, which is beneficial for a shirking ability. Thus, their best misreporting strategy is to reduce non-verifiable education a little bit today for more consumption today and more leisure tomorrow.

[10] See Diamond (1998) and Tuomala (1990) concerning how the choice of the welfare function affects optimal taxes in a static framework. For more general social welfares, readers are referred to Saez and Stantcheva (2016) as to how tax policy is reformed under generalized social marginal weights, which is beyond the scope of this paper.

$$Max \int_{\Theta} W(\theta)\pi(\theta)d\theta,$$

subject to resource constraints (2a)-(2b) and the incentive compatibility constraint (5b). The solution of the planning problem above is a ***constrained efficient*** *allocation A*.

First-order approach. The relaxed planning problem replaces the IC constraint (5b) by the envelope condition (5d). Let λ_t be the shadow price of the resource constraint in t and $\mu(\theta)$ be the co-state variable associated with the envelope condition (5d). The Hamiltonian is relegated to Appendix A.1.1. The envelope condition is necessary but not sufficient to be incentive compatible. Proposition 2 below shows that, under the monotonicity condition (5e), the incentive-compatible constraint (5b) can be recovered from the envelope condition (5d), which implies that the relaxed planning problem is valid under the monotonicity condition. The proof of Proposition 2 is relegated to Appendix A.1.4.

Proposition 2. *Suppose that an allocation satisfies the envelope condition (5d), and*

$$-u'\left(c_1^\sigma(\theta)\right)\frac{\partial y_1^\sigma(\theta)}{\partial\theta}+\beta\phi'\left(\frac{z_2(\sigma)}{h_2^\sigma(\theta)}\right)\frac{z_2(\sigma))}{\left(h_2^\sigma(\theta)\right)^2}\frac{\partial h_2^\sigma(\theta)}{\partial\theta} \tag{5e}$$

is non-decreasing in σ for all $\theta\in\Theta$. Then, the allocation is incentive compatible, satisfying (5b).

The monotonicity condition (5e) is similar to those in Kapička (2013, Theorem 5) and Pavan et al. (2014, Corollary 1(ii)), and guarantees that the envelope condition is a sufficient condition. As Kapicka and Neira (2019, p280) put it, one can directly verify incentive compatibility numerically if this condition fails. Section 4 will numerically verify that the allocation solved by our relaxed problem implies ex post incentive compatibility.

2.5. Properties of the optimum and wedges

Wedges measure distortions in the second-best allocation relative to the laissez-faire allocation. Agents' work effort, consumption and non-verifiable education expenses are private information, which generate distortions. There are three marginal distortions in the second-best allocation, defined as the labor wedge τ_{z_t}, the capital wedge τ_{k_t} and the human capital wedge τ_{x_t} as follows:

$$(1-\tau_{z_t})u'(c_t)\equiv\phi'\left(\frac{z_t}{h_t}\right)\frac{1}{w_t h_t} \tag{6a}$$

$$u'(c_{t-1})\equiv(1-\tau_{k_t})\beta R_t E_{t-1}[u'(c_t)], \tag{6b}$$

$$(1-\tau_{x_t})u'(c_t)\equiv\beta H_x(x_t,y_t,\theta_t,h_t)E_t\left[\phi'\left(\frac{z_{t+1}}{h_{t+1}}\right)\frac{z_{t+1}}{(h_{t+1})^2}\right], \tag{6c}$$

where $w_t=F_Z(K_t,Z_t)$ and $R_t=F_K(K_t,Z_t)+(1-\delta_k)$.

The labor wedge is an *intratemporal* wedge, which measures the difference of household's MRS between labor and consumption today from a firm's MPL today (i.e., the wage rate). Similarly, the capital wedge is an *intertemporal* wedge, which measures the difference of household's MRS in consumption today and tomorrow from a firm's MPK tomorrow (i.e., the rental rate). The human capital wedge measures the gap between the marginal cost and the marginal benefit from observable education investment in human capital. The labor and capital wedges are defined as implicit labor and capital tax rates, which is standard in the dynamic taxation literature. Following Stantcheva (2017), the human capital wedge is defined as an implicit subsidy to observable HCIs x_t. The signs of those wedges for heterogenous types are studied in Propositions 3 to 5 below.

First, to derive the modified inverse Euler equation, we denote the following notation $\Omega_t(c_t,c_{t+1})$

$$\Omega_t(c_t,c_{t+1})\equiv\frac{1}{u'(c_t)}-\frac{1}{\beta R_{t+1}u'(c_{t+1})}. \tag{7a}$$

Proposition 3 below determines the capital wedge, followed by the corollary for the sign of the capital wedge. The proofs are relegated to Appendixes A.1.5 and A.1.6.

Proposition 3. *In the case of a separable utility, the modified inverse Euler equation is of the following form*

$$\frac{1}{u'(c_1(\theta))}=\frac{1}{\beta R_2\,u'(c_2(\theta))}+\frac{-\mu(\theta)u''(c_1(\theta))}{\lambda_1\pi(\theta)u'(c_1(\theta))}\frac{\partial y_1(\theta)}{\partial\theta}\ for\ \theta\in\left(\underline{\theta},\overline{\theta}\right), \tag{7b}$$

where $\frac{-\mu(\theta)u''(c_1(\theta))}{\lambda_1\pi(\theta)u'(c_1(\theta))}\frac{\partial y_1(\theta)}{\partial\theta}=\Omega_1(c_1(\theta),c_2(\theta))$ is the HCI effect.

Corollary 1.

(i) If $\rho_{y\theta} \leq 2$, then $\Omega_1 > 0$ and $\tau_{k_2}(\theta) > 0$, and the HCI effect and the capital wedge are positive for $\theta \in \left(\underline{\theta}, \overline{\theta}\right)$.

(ii) If there are only verifiable education expenses, the (inverse) Euler equation holds; that is,

$$\frac{1}{u'(c_1(\theta))} = \frac{1}{\beta R_2 u'(c_2(\theta))} \text{ for } \theta \in \left[\underline{\theta}, \overline{\theta}\right]. \tag{7c}$$

Then, capital wedge is zero: $\tau_{k_2}(\theta) = 0$ *for* $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$.

The modified inverse Euler Eq. (7b) indicates positive capital wedges $\tau_{k_2}(\theta)$ for $\theta \in \left(\underline{\theta}, \overline{\theta}\right)$ The positive capital tax comes from the *HCI effect*. The reason goes as follows. Due to indistinct consumption and education expenditure, agents may underreport their types by substituting away from education expenses toward consumption today, which distorts the MRS of consumption today and tomorrow. This generates an indirect tax on future consumption and thus a positive capital wedge even without time-varying skill shocks. Specifically, by reducing education for consumption today, agents increase consumption today, which gives the preference for savings to smooth consumption. However, as savings are observable, the deviators, who under-report skills, have to save as much as their reported types to avoid being caught. A positive capital wedge hurts the deviators more than the truth-tellers, and offsets the benefit from under-reporting. So, it is an efficient way to provide the incentives for agents to reveal their true type. The same result holds in the T-period model later.

By contrast, if there are only verifiable education expenses, the standard consumption Euler Eq. (7c) holds and thus, the capital wedge is zero. This is an application of the Atkinson and Stiglitz (1976) result on the non-optimality of indirect taxes if the preference is separable in consumption and labor. Yet, the Atkinson and Stiglitz (1976) result cannot apply when there are non-verifiable education expenses.

Next, we obtain the proposition and the corollary concerning the sign of the labor wedge. The proofs are relegated to Appendixes A.1.7 and A.1.8.

Proposition 4. *The labor wedge in the first period is induced by skill-fostering effect, and the labor wedge in the terminal period is induced by shirk-preventing effect. To be more specific, for* $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$,

$$\tau_{z_1}(\theta) = \underbrace{\frac{\mu(\theta)\phi'\left(\frac{z_1(\theta)}{h_1}\right)u''(c_1(\theta))}{\lambda_1 \pi(\theta) u'(c_1(\theta)) w_1 h_1} \frac{\partial y_1(\theta)}{\partial \theta}}_{\text{skill-fostering effect}}$$

$$\tau_{z_2}(\theta) = \underbrace{\frac{-\beta\mu(\theta)}{\lambda_2 \pi(\theta)[h_2(\theta)]^2 \mathrm{w}_2}\left[\phi''\left(\frac{z_2(\theta)}{h_2(\theta)}\right)\frac{z_2(\theta)}{h_2(\theta)} + \phi'\left(\frac{z_2(\theta)}{h_2(\theta)}\right)\right]\frac{\partial h_2(\theta)}{\partial \theta}}_{\text{shirk-preventing effect}}$$

Corollary 2.

(i) If $\rho_{y\theta} \leq 2$, the skill-fostering effect is negative, implying a negative labor wedge in the first period,

$$\tau_{z_1}(\theta) < 0, \text{ for } \theta \in \left(\underline{\theta}, \overline{\theta}\right)$$

(ii) If $\rho_{y\theta} \geq \frac{H_{yy}H}{(H_y)^2}$, the shirk-preventing is positive, implying a positive labor wedge in the terminal period,

$$\tau_{z_2}(\theta) > 0, \text{ for } \theta \in \left(\underline{\theta}, \overline{\theta}\right)$$

(iii) If there are only verifiable education expenses, $\tau_{z_1}(\theta) = 0$ and $\tau_{z_2}(\theta) > 0$ for $\theta \in \left(\underline{\theta}, \overline{\theta}\right)$

Intuitively, as non-verifiable education and consumption expenses are indistinguishable, high-skilled agents may underreport skill types and reduce education expenses for consumption. This calls for a negative labor wedge in the first period, which is the skill-fostering effect. The effect induces agents to work and consume more, and reduce HCIs. Despite positive labor income taxes for redistribution purpose in the second period, the negative labor income tax reduces the deadweight loss of redistributive taxation. Yet, if there are only verifiable education expenses, agents cannot reduce education expenses without being caught. Then, given the same initial human capital, consumption and labor are not distorted, so the labor wedge is zero in the first period. In the final (here, second) period, agents have no incentives to invest in human capital. This goes back the standard Mirrlees literature with only the shirk-preventing effect. Thus, the labor tax is positive to prevent agents from shirking. In the extension to T periods, we show a transition from the skill-fostering effect to the shirk-preventing effect in early periods, and labor wedge is negative in the early life cycle.

As Stantcheva (2017) pointed out, the human capital wedge may include several simultaneous distortions, such as labor and capital distortions, so (6c) may not reflect the distortion caused purely by observable education expenses. If there is a positive (or negative) labor wedge, labor income is taxed (or subsidized), so human capital is indirectly distorted downward (or upward). Similarly, if there is a positive capital wedge, individuals will transfer resources to the future via HCIs to avoid the capital tax. Hence, part of the subsidy on human capital simply nullifies some of the labor and capital distortions on human capital. Following Stantcheva (2017), we define the net human capital wedge purely caused by observable education expenses as follows:

Definition 1. The net wedge on observable human capital expenses, $\tau^n_{x_t}$ is defined as

$$\tau^n_{x_t} \equiv E_t[\tau_{x_t} - \mathscr{N}_{t+1} + \mathscr{K}_{t+1}],$$

where $\mathscr{N}_{t+1} = \frac{w_{t+1}z_{t+1}}{R_{t+1}h_{t+1}}H_x(x_t, y_t, h_t, \theta_t)\tau_{z_{t+1}}$ is the distortion caused by the (appropriately scaled) labor wedge $\tau_{z_{t+1}}$, and $\mathscr{K}_{t+1} = \frac{\beta z_{t+1}}{(h_{t+1})^2}\phi'\left(\frac{z_{t+1}}{h_{t+1}}\right)H_x(x_t, y_t, h_t, \theta_t)\Omega_t$ is the distortion caused by the capital wedge, and Ω_t is related to the modified inverse Euler equation $\Omega_t = \frac{1}{u'(c_t)} - \frac{1}{\beta R_{t+1}u'(c_{t+1})}$ defined in (7a).

We obtain the following proposition and corollary regarding the net human capital wedge. The proofs are relegated to Appendixes A.1.9 and A.1.10.

Proposition 5. *The net human capital wedge $\tau^n_{x_1}(\theta)$ is given as follows:*

$$\tau^n_{x_1}(\theta) = \frac{-\beta\mu(\theta)z_2(\theta)\phi'\left(\frac{z_2(\theta)}{h_2(\theta)}\right)H_x}{\lambda_1\pi(\theta)(h_2(\theta))^3}\frac{\partial h_2(\theta)}{\partial\theta}.$$

Corollary 3.

(i) If $\rho_{y\theta} \geq \frac{H_{yy}H}{(H_y)^2}$, then $\tau^n_{x_1}(\theta) > 0$ for $\theta \in \left(\underline{\theta}, \overline{\theta}\right)$.

(ii) If there are only verifiable education expenses, then $\tau^n_{x_1}(\theta) > 0$ for $\theta \in \left(\underline{\theta}, \overline{\theta}\right)$.

Denote by $\rho_{h\theta}$ the Hicksian complementarity between human capital h_t and ability θ. Stantcheva (2017) established a positive net human capital wedge under the condition $0 < \rho_{h\theta} < 1$. In her model, the ability shock arrives after HCIs, so an agent's productivity includes both human capital and ability. In our model, the ability shock comes before HCIs, so agent's productivity includes only human capital, and thus, $\rho_{h\theta} = 0$. Yet, our model obtains a positive net human capital wedge without requiring condition $\rho_{h\theta} > 0$.

3. A T-period model

In a T-period model, the skill shock θ_t is time-varying, and an agent's skill history is denoted by $\theta^t \equiv (\theta_1, \theta_2, ..., \theta_t)$. We denote the reporting strategy by $\sigma \equiv (\sigma_1(\theta^T), ..., \sigma_T(\theta^T))$ when an agent of type $\theta^T = (\theta_1, ..., \theta_T)$ specifies a reported type σ_t in period t, and the set of all possible reporting strategies is denoted by $\mathscr{R}$. The lifetime utility (5a) of an agent with reporting strategy σ can be extended to a T-period model as follows:

$$W^\sigma(\theta^t) = u(c^\sigma(\theta^t)) - \phi\left(\frac{z(\sigma^t)}{h^\sigma(\theta^{t-1})}\right) + \sum_{s=t+1}^{T}\beta^{s-t}\int\left[u(c^\sigma(\theta^s)) - \phi\left(\frac{z(\sigma^s)}{h^\sigma(\theta^{s-1})}\right)\right]\pi(\theta^s_{t+1})d\theta^s_{t+1}, \tag{8a}$$

where $h^\sigma(\theta^s) = H(x(\sigma^s), y^\sigma(\theta^s), \theta_s, h^\sigma(\theta^{s-1}))$ for all $s = t-1, t, ..., T$, and by abuse of notation, we denote $\pi(\theta^s_{t+1}) = \pi(\theta_{t+1})\pi(\theta_{t+2})...\pi(\theta_s)$ and $d\theta^s_{t+1} = d\theta_{t+1}d\theta_{t+2}...d\theta_s$.

3.1. Recursive relaxed planning problem with full depreciation of human capital

A T-period model is more complicated than a 2-period model, as an agent's past skill shocks affect HCIs over the rest of the period. To make the T-period model tractable, this subsection constructs the relaxed planning problem as a recursive form and derives the Bellman equation. To this end, we make the assumption that human capital is fully depreciated after one period, and we may rewrite the human capital (1) as follows:

$$h(\theta^t) = \mathbb{H}^t(x(\theta^t), y(\theta^t), \theta_t). \tag{8b}$$

Although the fully depreciated human capital is strict, it eliminates the persistent effects of HCIs so that we can construct the recursive relaxed planning problem in terms of Bellman equations and analyze the wedges. To be in line with Section 2, this subsection analyzes the planning problem under i.i.d. skill shocks θ_t with density function $\pi(\theta_t)$. In SubSection 3.2 later, we study the relaxed planning problem with partially depreciated human capital and persistent skill shocks θ_t with conditional density function $\pi(\theta_t|\theta_{t-1})$.

Agent's problem. In the T-period model, the Proposition 1 in two-period model also applies. The non-verifiable education expense condition for an agent with a reporting strategy $\sigma \in \mathscr{R}$ becomes

$$u^{'}(c^{\sigma}(\theta^{t})) = \beta \int \phi^{'}\left(\frac{z(\sigma^{t+1})}{h^{\sigma}(\theta^{t})}\right) \frac{z(\sigma^{t+1})}{h^{\sigma}(\theta^{t})^{2}}\mathbb{H}_{y}^{t}(x(\sigma^{t}), y^{\sigma}(\theta^{t}),\ \theta_{t})\pi(\theta_{t+1})d\theta_{t+1}. \tag{8c}$$

Besides, the Hicksian coefficient $\rho_{y\theta}$ in Section 2 is defined $\rho_{y\theta}^{t} \equiv \frac{\mathbb{H}_{y\theta}^{t}\mathbb{H}^{t}}{\mathbb{H}_{y}^{t}\mathbb{H}_{\theta}^{t}}$ in this T-period model. One can easily prove that Lemma 1 also applies in this T-period model, which is formally stated as follows.

Lemma 2. *With human capital technology (8b), we have*

(i) if $\rho_{y\theta}^{t} \le 2$, then $\frac{\partial y^{\sigma}(\theta^{t})}{\partial \theta_{t}} < 0$,

(ii) if $\rho_{y\theta}^{t} \ge \frac{\mathbb{H}_{yy}^{t}\mathbb{H}^{t}}{(\mathbb{H}_{y}^{t})^{2}}$, then $\frac{\partial h^{\sigma}(\theta^{t})}{\partial \theta_{t}} > 0$.

(iii) if there are only verifiable education expenses, then $\frac{\partial y^{\sigma}(\theta^{t})}{\partial \theta_{t}} = 0$ and $\frac{\partial h^{\sigma}(\theta^{t})}{\partial \theta_{t}} > 0$.

Later, in SubSection 3.2, we will consider a general case with non-full human capital depreciation and derive the general forms of optimal condition (8c) and Lemma 2.

Incentive compatibility. Follow Stantcheva (2017), we consider a deviation strategy $\widehat{\sigma} \equiv (\theta_1,..,\widehat{\theta}_t,..,\theta_T)$, wherein the agent reports strategies truthfully in all except period t (i.e., $\widehat{\sigma}_u(\theta^T) = \theta_u \ \forall \ u \neq t$) when the agent may deviate by specifying a reported type $\widehat{\sigma}_t(\theta^T) = \widehat{\theta}_t$. Denote the set of this particular deviation strategy $\widehat{\sigma}$ by $\widehat{R}_t(\theta^T) \equiv \{\widehat{\sigma} = (\widehat{\theta}_1, ..., \widehat{\theta}_T)|\widehat{\theta}_u = \theta_u \ \forall u \neq t, and \ \widehat{\theta}_t \in \Theta\}$[11] Like (5c), the incentive compatibility constraint is.

$$W(\theta^{t}) = \max_{\hat{\theta}_t \in \Theta} W^{\hat{\sigma}}(\theta^{t}),\ \forall \theta^{t} \in \Theta^{t}. \tag{9a}$$

Based on (9a), taking the derivative of (8a) with respect to θ_t gives the following envelope condition.

$$\dot{W}(\theta^{t}) = -u^{'}(c(\theta^{t}))\frac{\partial y(\theta^{t})}{\partial \theta_{t}} + \beta\Delta(\theta^{t}), \tag{9b}$$

where $\Delta(\theta^t) \equiv -\int\left[\phi_h\left(\frac{z(\theta^{t+1})}{h(\theta^{t})}\right) \frac{\partial h(\theta^{t})}{\partial \theta_t}\right]\pi(\theta_{t+1})d\theta_{t+1}$ denotes the dynamic utility gain. Condition (9b) is like (5d) in the two-period model, which includes a static utility gain and a dynamic utility gain.

Moreover, based on (8a), the expected lifetime utility of a type θ^t agent with a truth-telling strategy is

$$W(\theta^{t}) = u(c(\theta^{t})) - \phi\left(\frac{z(\theta^{t})}{h(\theta^{t-1})}\right) + \beta v(\theta^{t}), \tag{9c}$$

where $v(\theta^t) = \int W(\theta^{t+1})\pi(\theta_{t+1})d\theta_{t+1}$.

Recursive relaxed planning problem with Bellman equations. To make our problem tractable, we reformulate the problem from a utility maximization to a cost minimization problem to avoid tedious period-by-period resource constraints.[12] Following Stantcheva (2017) we focus on partial equilibrium, wherein the interest rate R_t and the wage rate w_t are treated as predetermined. Using the first-order approach, we replaces the IC constraint (9a) by the envelope condition (9b) and then through a suitable definition of state variables, we construct the following recursive relaxed planning problem.

Given past history θ^{t-1}, we take as given previous values for state variables $v(\theta^{t-1})$, $\Delta(\theta^{t-1})$, and $h(\theta^{t-1})$.[13] For any period t, conditional on the history of shocks in one period earlier θ_{t-1}, the entire history of shocks θ^{t-2} is redundant. Thus, the recursive relaxed planning problem is:

[11] In the text, we only focus on a one-deviation strategy, which is the standard way of the first-order approach. In Appendix A.3, we show that, under a proper condition, the incentive compatibility of the one-deviation strategy can be extended to a multi-deviation strategy. Fernandes and Phelan (2000) also provide a proof in their Theorem 2.1, showing that incentive compatibility of one-deviation strategy can be equivalent to general incentive compatibility.

[12] The same approach was used by Atkeson and Lucas (1992) and Farhi and Werning (2013). They studied a dual continuation problem, wherein the partial equilibrium of a dynamic incentive problem was analyzed without period-by-period resource constraints imposed upon the principal.

[13] In the first period, the state variables θ_{-} and Δ are absent, and $v = \underline{v}_1$ and $h = h_1$ are exogenously given.

$$\mathscr{K}(v,\Delta,h,\theta_{-},t)=\min\int\left[c(\theta)+y(\theta)+x(\theta)-w_t z(\theta)+\frac{\mathscr{K}(v(\theta),\Delta(\theta),h(\theta),\theta,t+1)}{R_{t+1}}\right]\pi(\theta)d\theta, \tag{9d}$$

subject to $W(\theta)=u(c(\theta))-\phi\left(\frac{z(\theta)}{h(\theta_{-})}\right)+\beta v(\theta)$,

$$\dot{W}(\theta)=-u'(c(\theta))\frac{\partial y(\theta)}{\partial\theta}+\beta\Delta(\theta),$$

where $v=\int W(\theta)\pi(\theta)d\theta$ and $\Delta=\int\phi'\left(\frac{z(\theta)}{h(\theta_{-})}\right)\frac{z(\theta)\frac{\partial h(\theta_{-})}{\partial\theta_{-}}}{[h(\theta_{-})]^2}\pi(\theta)d\theta$, with θ_{-} denoting past shocks and the minimization being taken over $c(\theta)$, $x(\theta),z(\theta),W(\theta),v(\theta)$ and $\Delta(\theta)$. Denote by P^{*FOA} the set of the recursive planning problem in (9d) and by X^{*FOA} the set of the resulting constrained efficient allocation. The recursive relaxed social planning problem is constructed and solved in Appendix A.2.1.

Properties of the optimum and wedges. We now analyze the properties of the optimum in terms of the sign of the capital wedge, the labor wedge and the human capital wedge. As the wedges in the T-period model depend on the history of types, the definitions in (6a)-(6c) are revised as follows:

$$(1-\tau_z(\theta^t))u'(c(\theta^t))\equiv\phi'\left(\frac{z(\theta^t)}{h(\theta^{t-1})}\right)\frac{1}{w_t h(\theta^{t-1})}, \tag{10a}$$

$$u'(c(\theta^t))\equiv(1-\tau_k(\theta^t))\beta R_{t+1}E_t\left[u'\left(c\left(\theta^{t+1}\right)\right)\right], \tag{10b}$$

$$(1-\tau_x(\theta^t))u'(c(\theta^t))\equiv\beta E_t\left[\phi'\left(\frac{z(\theta^{t+1})}{h(\theta^t)}\right)\frac{z(\theta^{t+1})}{[h(\theta^t)]^2}\mathbb{H}_x^t\right]. \tag{10c}$$

Let $\widehat{\mu}(\theta)$ be the co-state variable associated with the envelope condition $\dot{W}(\theta)$ and $\gamma(\theta_{-})$ be the shadow price associated of the state variable Δ in the relaxed planning problem. In Appendix A.2.1, we show that $\widehat{\mu}(\theta)>0$ and $\gamma(\theta_{-})<0$.[14] Based on the above definitions, the wedges in Proposition 3-5 of the two-period model now can be extended to the T-period model, which is shown in the following Proposition 6. The proof is relegated to Appendix A.2.2.

Proposition 6. *With technology (8b), the constrained efficient allocation in X^{*FOA} satisfies the following.*

(i) **Capital wedge** is determined by the following modified inverse Euler equation:

$$\frac{1}{u'\left(c\left(\theta^{t-1}\right)\right)}=E_{t-1}\left[\frac{1}{\beta R_t u'(c(\theta^t))}+\Omega_{t-1}\left(c\left(\theta^{t-1}\right),c(\theta^t)\right)\right],\text{ for }\theta^t\in\theta^t, t=2,3,\ldots T \tag{11a}$$

$$\textit{where } E_{t-1}[\Omega_{t-1}(c(\theta^{t-1}),c(\theta^t))]\equiv\underbrace{\frac{\widehat{\mu}(\theta^{t-1})u''(c(\theta^{t-1}))}{\pi(\theta^{t-1})u''(c(\theta^{t-1}))}\frac{\partial y(\theta^{t-1})}{\partial\theta^{t-1}}}_{\text{Current period's HCI effect}}+\underbrace{\frac{-1}{\beta R_t}E_{t-1}\left[\frac{\widehat{\mu}(\theta^t)u''(c(\theta^t))}{\pi(\theta^t)u'(c(\theta^t))}\frac{\partial y(\theta^t)}{\partial\theta^t}\right]}_{\text{Next period's HCI effect}}$$

(ii) **Labor wedge** is of the following form

$$\tau_z(\theta^t)=\underbrace{\frac{-\gamma\left(\theta^{t-1}\right)\frac{\partial h\left(\theta^{t-1}\right)}{\partial\theta_{t-1}}\left[\phi'\left(\frac{z(\theta^t)}{h\left(\theta^{t-1}\right)}\right)+\frac{z(\theta^t)}{h\left(\theta^{t-1}\right)}\phi''\left(\frac{z(\theta^t)}{h\left(\theta^{t-1}\right)}\right)\right]}{w_t\left[h\left(\theta^{t-1}\right)\right]^2}}_{\text{shirk-preventing effect}\begin{cases}=0 \text{ when } t=1\\ \neq 0 \text{ otherwise}\end{cases}}+\underbrace{\frac{-\widehat{\mu}(\theta^t)u''(c(\theta^t))\phi'\left(\frac{z(\theta^t)}{h\left(\theta^{t-1}\right)}\right)}{\pi(\theta^t)u'(c(\theta^t))w_t h\left(\theta^{t-1}\right)}\frac{\partial y(\theta^t)}{\partial\theta^t}}_{\text{skill-fostering effect}\begin{cases}=0 \textit{ when } t=T\\ \neq 0 \textit{ otherwise}\end{cases}} \tag{11b}$$

Note that only the skill-fostering effect is present in the first period, while in the terminal period, only the shirk-preventing effect is present.

(iii) **Net human capital wedge** is of the following form

[14] The sign $\widehat{\mu}(\theta)>0$ in Section 3 is different from the sign $\mu(\theta)<0$ in Section 2. The reason is that the planning problem in Section 3 is a social cost minimization, wherein the cost is raised to be incentive compatible, while the problem in Section 2 is a social welfare maximization, wherein the welfare is decreased to be incentive compatible.

$$\tau_x^n(\theta^t) = E_t\left[\frac{-\gamma(\theta^t)z(\theta^{t+1})}{R_{t+1}(h(\theta^t))^3}\phi'\left(\frac{z(\theta^{t+1})}{h(\theta^t)}\right)\mathbb{H}_{x,t}^t\frac{\partial h(\theta^t)}{\partial\theta_t}\right]. \tag{11c}$$

Remark. Proposition 6 establishes the capital wedge, the labor wedge, and the net human capital wedge in the general T-period model with random types. Their forms reduce to those in Propositions 3-5 in the two-period model, when the interest rate is set consistent with the marginal return to capital ex post, the product of the wage rate and human capital is set in line with the marginal return to labor ex post, and multipliers are adjusted such that $\widehat{\mu} = \frac{-\mu}{\lambda_1}$ and $\gamma = \frac{\beta\mu}{\lambda_2\pi}$.[15] Note that the planner's problem is formulated in the cost minimization in the T-period model, while it is a welfare maximization problem in the two period model. As a result, multipliers in the T-period model are different from those in the two-period model and thus, an adjustment is needed.

Capital wedge. Based on Proposition 6 (i), Corollary 1 for two periods is extended to T periods, as shown by the following Corollary 4, the proof of which is relegated to Appendix A.2.3.

Corollary 4.

(i) If $\rho_{y\theta}^{T-1} \leq 2$, then in the terminal period, as $\frac{\partial y(\theta^{T-1})}{\partial\theta^{T-1}} < 0$ and $\frac{\partial y(\theta^T)}{\partial\theta^T} = 0,$ then $\Omega_{t-1}(c(\theta^{t-1}), c(\theta^t)) > 0$ and

$$\frac{1}{u'(c(\theta^{T-1}))} > \frac{1}{\beta R_T}E\left[\frac{1}{u'(c(\theta^T))}\right]$$

which induces a larger capital wedge than that in the case when the standard inverse Euler equation holds.

(ii) If there are only verifiable education expenses, $\frac{\partial y(\theta^{t-1})}{\partial\theta^{t-1}} = \frac{\partial y(\theta^t)}{\partial\theta^t} = 0$ and the standard inverse Euler equation holds; that is, $E_{t-1}[\Omega_{t-1}(c(\theta^{t-1}), c(\theta^t))] = 0$.

If there are only verifiable education expenses, Corollary 4 shows that the modified inverse Euler Eq. (11a) reduces to the standard one. Then, the capital wedge is positive, which is the outcome in the standard dynamic Mirrlees literature due to an insurance effect.

In contrast, when there are non-verifiable education expenses, due to inseparable consumption and education expenses, $E_{t-1}[\Omega_{t-1}(c(\theta^{t-1}), c(\theta^t))] \neq 0$, the standard inverse Euler equation does not hold. Then, agents have incentives to reduce non-verifiable education expenses for consumption, thus an HCI effect. A higher skill shock today exerts two HCI effects that offset each other. One effect is to reduce non-verifiable education expenses toward period $t-1$'s consumption, which enhances the otherwise positive capital wedge from the insurance effect. By contrast, the other effect is to reduce non-verifiable education expenses toward next period's (period t) consumption, which offsets the otherwise positive capital wedge from the insurance effect. The net effect on the capital wedge is ambiguous, as it is not sure whether the next period's HCI effect is strong enough to dominate the sum of the current period's HCI effect and the insurance effect. Yet, in the terminal period, there is only the current period's HCI effect, so the capital wedge is unambiguously larger than the otherwise positive capital wedge arising from the insurance effect.

Labor wedge. Based on Proposition 6 (ii), Corollary 2 for two periods is extended to T periods, as shown by the following Corollary 5, the proof of which is relegated to Appendix A.2.4.

Corollary 5.

(i) If $\rho_{y\theta}^t \leq 2$, the skill-fostering effect is negative, implying $\tau_z(\theta) < 0$, for $\theta \in \left(\underline{\theta}, \overline{\theta}\right)$ in the first period.

(ii) If $\rho_{y\theta}^t \geq \frac{\mathbb{H}_{yy}^t\mathbb{H}^t}{(\mathbb{H}_y^t)^2}$, the shirk-preventing is positive, implying $\tau_z(\theta^T) > 0$, for $\theta_T \in \left(\underline{\theta}, \overline{\theta}\right)$ in the terminal period.

(iii) If there are only verifiable education expenses, the labor wedge is zero in the first period, and thus, $\tau_z(\theta_1) = 0$ for $\theta_1 \in \left[\underline{\theta}, \overline{\theta}\right]$ and positive in the rest of the periods and thus, $\tau_z(\theta^t) > 0$ for $\forall\theta_t \in \left(\underline{\theta}, \overline{\theta}\right)$ and $t \geq 2$.

If there are only verifiable education expenses, $\frac{\partial y(\theta)}{\partial\theta} = 0$. The labor wedge is zero in the first period, and, like the existing literature, there is only a shirk-preventing effect after period 1, so the labor wedge is positive in order to prevent agents from shirking. With non-verifiable education expenses, there is a skill-fostering effect. As there is only a skill-fostering effect in period 1, a negative labor wedge is optimal. After period 1, the skill-fostering effect dominates the shirk-preventing effect in the early life cycle, so a negative labor wedge is optimal, which is a mechanism to induce agents to work according to their true types and invest sufficiently on education. In

[15] When $T = 2$, the history of type is $\theta^t = (\theta_1, \theta_2)$, but θ_2 is redundant as there is no need to invest human capital in period 2. By setting $\theta_1 = \theta$ and adjusting multipliers, Propositions 6 reduce to Propositions 3-5, respectively.

the later life cycle, HCIs decrease and the skill-fostering effect phases out, so the labor wedge is positive.

Human capital wedge. As in Section 2, the human capital wedge (10c) is affected by capital or labor distortions, so we define a net human capital wedge that reflects only the distortion caused by observable education expenses. The net human capital wedge in Definition 1 still applies in the T-period model. Corollary 3 is extended to T periods, as shown by the following Corollary 6, with the proof relegated to Appendix A.2.5.

Corollary 6.

(i) If $\rho^t_{y\theta,t} \geq \frac{\mathbb{H}^t_{yy,t}\mathbb{H}^t}{(\mathbb{H}^t_{y,t})^2}$, then $\tau^n_x(\theta^t) > 0$ for $\theta_t \in \left(\underline{\theta}, \overline{\theta}\right)$.

(ii) If there are only verifiable education expenses, then $\tau^n_x(\theta^t) > 0$ for $\theta_t \in \left(\underline{\theta}, \overline{\theta}\right)$.

To summarize the wedges, our positive capital wedge comes from both the insurance effect and the HCI effect. This is a new mechanism, which is different from that in the existing Mirrlees models with exogenous skills in Diamond and Mirrlees (1978) and Farhi and Werning (2013), observable HCIs in Stantcheva (2017), and unobservable HCIs in Grochulski and Piskorski (2010). Moreover, the result adds value to Stantcheva (2014). We analytically separate today's positive HCI effect from tomorrow's negative HCI effect on the capital wedge, so as to assure when the capital wedge is larger or smaller than the capital wedge in the case with only observable HCIs.

Our negative labor wedge in the early life cycle is a new result in the dynamic Mirrlees literature. This is different from the positive labor wedge in models with exogenous skills and those with observable HCIs in Stantcheva (2017). Our result is also different from the model with unobservable HCIs at the beginning of the life cycle by Grochulski and Piskorski (2010), whose labor wedge is positive for low skills and negative for high skills. The labor wedge in these existing studies serves to induce agents to work according to their types, thus a shirk-preventing effect. However, in our model with inseparable consumption and education expenses, the deviation involves shirking and under-investing in human capital. As the effect from under-investing in human capital dominates the effect from shirking, the deviators have a stronger preference for leisure and a weaker preference for consumption than the truth-tellers. A labor subsidy makes it optimal to provide the effective labor supply and invest in human capital according to their true types, and is thus a skill-fostering effect. The result adds value to Stantcheva (2014) as well. We obtain a negative labor wedge in early periods of the life cycle, at least unambiguously in the first period, when the negative skill-fostering effect on the labor wedge dominates the positive shirk-preventing effect.

3.2. Non-full human capital depreciation and persistent shock

Now, we relax the full human capital depreciation and i.i.d. skill shocks θ_t assumptions. We consider partial human capital depreciation and persistent shocks θ_t with conditional density function $\pi(\theta_t|\theta_{t-1})$.

Human capital technology. With non-full human capital depreciation, an agent's human capital in period t is affected not only by skill shocks θ_t and HCIs faced in period t but also by the history of skill shocks θ^t and HCIs realized before period t. Thus, we rewrite the human capital technology (1) as follows:

$$h(\theta^t) = \mathbb{H}^t(\mathrm{X}(\theta^t), \mathrm{Y}(\theta^t),\ \theta^t), \tag{12}$$

where $\mathrm{X}(\theta^t) = (x(\theta^1), \ldots, x(\theta^t))$, and $\mathrm{Y}(\theta^t) = (y(\theta^1), \ldots, y(\theta^t))$. Note that, in the case of full human capital depreciation, $h(\theta^t)$ is independent of $h(\theta^{t-1})$; then, (12) reduces to the form $h(\theta^t) = \mathbb{H}^t(x(\theta^t), y(\theta^t),\ \theta_t)$ in (8b).

Agent's problem. We start with the non-verifiable education choice for the agent with reporting strategy $\sigma \in \mathscr{R}$. The optimal condition (8c) is extended to the model with non-full human capital depreciation as follows. The proof is relegated to Appendix A.2.6.

Proposition 7. *For a reporting strategy $\sigma \in \mathscr{R}$, the optimal non-verifiable human capital investment is determined by the following condition*

$$u'(c^\sigma(\theta^t)) = \sum_{s=t+1}^{T} \beta^{s-t} \int \left[\phi'\left(\frac{z(\sigma^s)}{h^\sigma(\theta^{s-1})}\right) \frac{z(\sigma^s)}{h^\sigma(\theta^{s-1})^2} \mathbb{H}^{s-1}_{y,t}\right] \pi(\theta^s_{t+1}) d\theta^s_{t+1},$$

where $\mathbb{H}^{s-1}_{y,t} \equiv \frac{\partial}{\partial y^\sigma(\theta^{s-1})} \mathbb{H}^{s-1}(\mathrm{X}(\sigma^{s-1}), \mathrm{Y}^\sigma(\theta^{s-1}),\ \theta^{s-1})$ *and* $\pi(\theta^s_{t+1}) \equiv \pi(\theta_{t+1}|\theta_t)\pi(\theta_{t+2}|\theta_{t+1})\ldots\pi(\theta_s|\theta_{s-1})$.

Hicksian coefficient of complementarity. With non-full human capital depreciation for the human capital technology $h^\sigma(\theta^s) = \mathbb{H}^s(\mathrm{X}(\sigma^s), \mathrm{Y}^\sigma(\theta^s),\ \theta^s)$ for $s \geq t$, the Hicksian coefficient of complementarity between y_t and θ_t in period t, denoted by $\rho^s_{y\theta,t}$, is defined as follows:

$$\rho^s_{y\theta,t} \equiv \frac{\mathbb{H}^s_{y\theta,t}\mathbb{H}^s}{\mathbb{H}^s_{y,t}\mathbb{H}^s_{\theta,t}}, \forall s \geq t$$

where $\mathbb{H}^s = \mathbb{H}^s(\mathrm{X}(\sigma^s), \mathrm{Y}^\sigma(\theta^s),\ \theta^s)$, $\mathbb{H}^s_{y,t} = \mathbb{H}^s_{y,t}(\mathrm{X}(\sigma^s), \mathrm{Y}^\sigma(\theta^s),\ \theta^s)$, and $\mathbb{H}^s_{\theta,t}$ and $\mathbb{H}^s_{y\theta,t}$ satisfy the following two equations, respectively:

$$\frac{d}{d\theta_t}\mathbb{H}^s(\mathrm{X}(\sigma^s), \mathrm{Y}^\sigma(\theta^s),\ \theta^s) = \mathbb{H}^s_{y,t} \cdot \frac{\partial y^\sigma(\theta^t)}{\partial \theta_t} + \mathbb{H}^s_{\theta,t},$$

$$\frac{d}{d\theta_t}\mathbb{H}^s_{y,t}(\mathrm{X}(\sigma^s), \mathrm{Y}^\sigma(\theta^s), \theta^s) = \mathbb{H}^s_{yy,t} \cdot \frac{\partial y^\sigma(\theta^t)}{\partial \theta_t} + \mathbb{H}^s_{y\theta,t}.$$

In the case of full human capital depreciation, $h^\sigma(\theta^s)$ is independent of θ_t and y_t, so the Hicksian coefficient of complementarity is zero: $\rho^s_{y\theta,t} = 0, \ \forall s > t$. Based on the new definition of $\rho^s_{y\theta,t}$, we obtain the following lemma. The proof is relegated to Appendix A.2.7.

Lemma 3. *Suppose* $\mathbb{H}^s_{\theta,t} \geq 0$ *for all* $s \geq t$.

(i) if $\rho^s_{y\theta,t} \leq 2$, then $\frac{\partial y^\sigma(\theta^t)}{\partial \theta_t} < 0$ for all $s \geq t$;
(ii) if $\frac{\mathbb{H}^t_{\theta,t}}{\mathbb{H}^t_{y,t}} \geq \frac{\mathbb{H}^s_{\theta,t}}{\mathbb{H}^s_{y,t}}$ and $\rho^s_{y\theta,t} \geq \frac{\mathbb{H}^s_{yy,t}\mathbb{H}^s}{(\mathbb{H}^s_{y,t})^2}$ for all $s \geq t$, then $\frac{\partial h^\sigma(\theta^t)}{\partial \theta_t} > 0$:
(iii) if there are only verifiable education expenses, then $\frac{\partial y^\sigma(\theta^t)}{\partial \theta_t} = 0$ and $\frac{\partial h^\sigma(\theta^t)}{\partial \theta_t} > 0$.

Thus, Lemma 2 under full human capital depreciation is extended to otherwise the same Lemma 3 except non-full human capital depreciation. The result $\frac{\partial h^\sigma(\theta^t)}{\partial \theta_t} > 0$ requires condition $\frac{\mathbb{H}^t_{\theta,t}}{\mathbb{H}^t_{y,t}} \geq \frac{\mathbb{H}^s_{\theta,t}}{\mathbb{H}^s_{y,t}}$ for all $s \geq t$. This condition says that the ratio of the marginal return of θ_t to the marginal return of y_t in the current period t is not lower than any future period $s > t$. The condition is explained as follows. When human capital is not fully depreciated, the ability type θ_t affects not only the next period's human capital $h^\sigma(\theta^t)$ but also all future period's human capital $h^\sigma(\theta^s)$ for $s > t$. The condition $\frac{\mathbb{H}^t_{\theta,t}}{\mathbb{H}^t_{y,t}} \geq \frac{\mathbb{H}^s_{\theta,t}}{\mathbb{H}^s_{y,t}}$ for $s \geq t$ requires that the effect of θ_t upon human capital $h^\sigma(\theta^t)$ in period t be relatively larger than those in future periods, so the effect of θ_t dominates the path of the period-t human capital $h^\sigma(\theta^t)$. If otherwise, the effect of future-periods' ability types dominates the effect of the period-t ability type, leading to an ambiguous sign of $\frac{\partial h^\sigma(\theta^t)}{\partial \theta_t}$. Moreover, it is natural to assume that the effect of θ_t on human capital fades over time.

Incentive compatibility and envelope condition. Recall the incentive compatibility constraint

$$W(\theta^t) = \max_{\hat{\theta}_t \in \Theta} W^{\hat{\sigma}}(\theta^t), \forall \theta^t \in \Theta^t, \tag{13a}$$

and the lifetime utility of an agent with reporting strategy $\sigma \in \mathscr{R}$

$$W^\sigma(\theta^t) = u(c^\sigma(\theta^t)) - \phi\left(\frac{z(\sigma^t)}{h^\sigma(\theta^{t-1})}\right) + \sum_{s=t+1}^{T}\beta^{s-t}\int\left[u(c^\sigma(\theta^s)) - \phi\left(\frac{z(\sigma^s)}{h^\sigma(\theta^{s-1})}\right)\right]\pi(\theta^s_{t+1})d\theta^s_{t+1}. \tag{13b}$$

The following envelope condition can be obtained by differentiating (13b) with respect to θ_t

$$\frac{\partial W^{\hat{\sigma}}(\theta^t)}{\partial \theta_t} = -u'(c^{\hat{\sigma}}(\theta^t))\frac{\partial y^{\hat{\sigma}}(\theta^t)}{\partial \theta_t} + \beta\int W^{\hat{\sigma}}(\theta^{t+1})\frac{d\pi(\theta_{t+1}|\theta_t)}{d\theta_t}d\theta_{t+1} + \sum_{s=t+1}^{T}\beta^{s-t}\int\Bigg[- u(c^{\hat{\sigma}}(\theta^s))\frac{\partial y^{\hat{\sigma}}(\theta^s)}{\partial \theta_t} - \phi_h\left(\frac{z(\widehat{\theta}^s)}{h^{\hat{\sigma}}(\theta^{s-1})}\right)\frac{\partial h^{\hat{\sigma}}(\theta^{s-1})}{\partial \theta_t}\Bigg]\pi(\theta^s_{t+1})d\theta^s_{t+1}. \tag{13c}$$

Conditioning on the truth-telling strategy, the envelope condition (13c) becomes

$$\dot{W}(\theta^t) = -u'(c(\theta^t))\frac{\partial y(\theta^t)}{\partial \theta_t} + \beta\Delta(\theta^t), \tag{13d}$$

where $\Delta(\theta^t) \equiv \int W(\theta^{t+1})\frac{d\pi(\theta_{t+1}|\theta_t)}{d\theta_t}d\theta_{t+1} - \sum_{s=t+1}^{T}\beta^{s-t-1}\int\left[u(c(\theta^s))\frac{\partial y(\theta^s)}{\partial \theta_t} - \phi_h\left(\frac{z(\theta^s)}{h(\theta^{s-1})}\right)\frac{\partial h(\theta^{s-1})}{\partial \theta_t}\right]\pi(\theta^s_{t+1})d\theta^s_{t+1}$.

As can be seen, with non-full human capital depreciation and persistent shocks, the envelope condition in (13d) is more complicated than (9b), making the analysis of the signs of wedges much more difficult.

Through replacing the envelope condition (9b) by (13d), the recursive relaxed planning problem (9d) in the model with full human capital depreciation and i.i.d. skill shocks can be extended to the model with non-full human capital depreciation and persistent skill shocks. The model with non-full human capital depreciation and persistent shocks and the envelope condition (13d) is then used for quantitative analysis in Section 4 below. We will find that the results derived in the model with fully depreciated human capital in subSection 3.1, such as negative labor wedges in the early periods, carry over in the general model with non-full human capital depreciation. Finally, we numerically verify that the envelope condition used to solve for the allocation in our model implies ex-post incentive compatibility.

4. Numerical analysis

Based on the model in subSection 3.2, this section offers numerical analysis to highlight the quantitative importance of our results. Our numerical analysis takes a middle position between a simple demonstration of the optimal mechanism and a careful calibration of quantitative implications for the wedge. The numerical analysis has four goals: firstly, to demonstrate the average capital, labor and human capital wedges over time; secondly, to illustrate the capital, the labor, and the human capital wedges for different skill types in some working periods; thirdly, to exhibit whether the capital, the labor, the human capital wedges, and even overall taxes are progressive or regressive in agents' types; fourthly, to highlight the redistribution effect in terms of the welfare gain and compare our history-dependent tax system with a simple age-dependent tax.

4.1. Calibration

A period in the model is considered to be a year. We calibrate our model to the US data, and then quantitatively illustrate the optimal tax policy in our model. The calibration proceeds as follows. We construct a baseline decentralized economy with the status quo income tax system in the US. The structure of our baseline economy is the same as the model in SubSection 3.2 except for no social planner. Agents are set to live 60 years, working for 40 years and then retiring for 20 years. In the baseline, some parameter values are set exogenously, based on the existing literature, and five parameter values are calibrated to match the moments from data. Table 1 lists all parameter values. Following Dyrda and Pedroni (2023), these five parameter values are calibrated to match the six macroeconomic aggregate data. Table 2 lists these macroeconomic aggregates, targeted statistics, and their model counterparts. Table 2 also lists cross-sectional income distribution used as targets for calibration and their model counterparts. The model provides a good fit of these macroeconomic aggregates and cross-sectional income distribution. We also list cross-sectional wealth distribution to Table 2, and our model fits well to the wealth distribution.

Status quo tax system. For the tax system, we follow Kapička and Neira (2019) and set a linear capital tax rate at $\tau_k^b = 37\%$, which is the value estimated by McDaniel (2007), a linear education subsidy rate at $\tau_x^b = 35\%$, which is also used by Stantcheva (2017), and a progressive labor income tax $T_l^b(w_t z_t)$ of the form, proposed by Gouveia and Strauss (1994), as follows:

$$T_l^b(w_t z_t) = \nu_0 \left[1 - (\nu_1 (w_t z_t)^{\nu_2} + 1)^{-\frac{1}{\nu_2}}\right],$$

where $\nu_0 = 0.182$, $\nu_1 = 0.008$ and $\nu_2 = 1.496$, estimated by Guner et al. (2014). As for the government expenditure G_t, following Peterman (2016), we assume that the government expenditure equals 17 percent of output. After subtracting the government expenditure, the remaining tax revenue is equally redistributed to agents as a lump-sum transfer LS_t. An agent's budget constraint is as follows:

$$c_t + \left(1 - \tau_x^b\right) x_t + y_t + k_{t+1} \le w_t z_t - T_l^b(w_t z_t) + \left(1 - \tau_k^b\right) R_t k_t + LS_t. \tag{14}$$

Preference. The utility function during working years takes the form $u(c_t) - \phi\left(\frac{z_t}{h_t}\right) = \log(c_t) - \frac{1}{\kappa}\left(\frac{z_t}{h_t}\right)^{\kappa}$. Following Farhi and Werning (2013), we set $\kappa = 3$, which implies the Frisch elasticity for labor of 0.5. The discount factor is set at 5 per annum, which gives $\beta = 0.95$.

Final goods production. The aggregate production technology takes the Cobb-Douglas form $F(K_t, Z_t) = K_t^{\alpha} Z_t^{1-\alpha}$ with the capital depreciation rate δ_k. Following Conesa et al. (2009), the capital share is set at $\alpha = 0.36$. The capital depreciation rate δ_k is calibrated to match the targets. The above production function implies that the intertest rate is $R_t = 1 + \alpha (K_t/Z_t)^{\alpha-1} - \delta_k$, and the wage rate is $w_t = (1-\alpha)(K_t/Z_t)^{\alpha}$.

Human capital technology. The human capital technology (1) takes the non-separable form between initial human capital h_t and investments in human capital (x_t, y_t) as follows.[16]

$$h_{t+1} = H(x_t, y_t, \theta_t, h_t) = \left((1-\omega)x_t^{\varepsilon} + \omega y_t^{\varepsilon}\right)^{\frac{\eta}{\varepsilon}} h_t^{1-\eta} \theta_t + (1-\delta_h) h_t, \tag{15}$$

where η is the degree of homogeneity of the technology, the ability θ_t and verifiable and non-verifiable education expenses x_t and y_t are in a multiplicative form, and x_t and y_t are in a CES form with the share parameter ω. The elasticity of substitution is $\rho_{xy} \equiv \frac{1}{1-\varepsilon}$. Following Huggett et al. (2011), the depreciation rate is set to $\delta_h = 0.02$. Following Tobing (2011), the share parameter value is set to $\omega = 0.6$.[17]

As for the value of ε, to the best of our knowledge, no paper investigates how verifiable and non-verifiable education expenses form human capital except van Ewijk and Tang (2007).[18] They formulated x and y in the Cobb-Douglas function. We take the form as a baseline case and set $\varepsilon = 0$, and thus, the elasticity of substitution between x and y is $\frac{1}{1-\varepsilon} = 1$. In SubSection 4.3 later, we deviate from

[16] The form (15) indicates $h_{t+2} = ((1-\omega)x_{t+1}^{\varepsilon} + \omega y_{t+1}^{\varepsilon})^{\frac{\eta}{\varepsilon}} H(x_t, y_t, \theta_t, h_t)^{1-\eta} \theta_{t+1} + (1-\delta_h) H(x_t, y_t, \theta_t, h_t)$, implying that the discounted sum of the future marginal productivity of x_t and y_t is affected by the shock θ_{t+1}. Thus, human capital investments x_t and y_t are risky.

[17] The parameter ω is non-verifiable education expenses as a share of total education expenses, and the value $\omega = 0.6$ means that the share of non-verifiable expenses is larger than that of verifiable expenses. To see whether our results are sensitive to the share of non-verifiable education expenses, we have simulated our model with a larger and a smaller parameter value of $\omega = 0.2$ and $\omega = 0.8$. We found that the values of ω do not change our quantitative results.

[18] In studying efficient progressive taxes and education subsidies in a model with trade unions, van Ewijk and Tang (2007) analyzed both unobservable and observable education investments.

Table 1
Parameter values in the baseline.

Definition	Symbol	Value	Source/Note
Parameter value set			
Disutility elasticity	κ	3	Farhi and Werning (2013)
Discount factor	β	0.95	Farhi and Werning (2013)
Elasticity of substitution	ε	0	Cobb-Douglas form
Share of non-verifiable education expenses	ω	0.667	van Ewijk and Tang (2007)
Depreciation rate of HC	δ_h	0.02	Huggett et al. (2011)
Capital share	α	0.36	Conesa et al. (2009)
Government expenditure, as a share of output	G	0.17	Peterman (2016)
Capital income tax rate	τ_k^b	0.37	McDaniel (2007)
Labor income tax function	(ν_0, ν_1, ν_2)	(0.182,0.008,1.496)	Guner et al. (2014)
Education subsidy rate	τ_x^b	0.35	Stantcheva (2017)
Parameter values calibrated			
Education degree	η	0.25	
Initial human capital	h_1	0.35	
SD log-ability	σ_ε	0.75	
Mean log-ability	μ_ε	0.35	
Depreciation rate of capital	δ_k	0.1	

Table 2
Target statistics and model counterparts.

Macroeconomic aggregates	Model	Target	Source
Education expense ratio	0.2	0.19	Stantcheva (2017)
Wage premium	1.9	1.8	Heathcote et al. (2010)
Income Gini	0.53	0.56	Kuhn and Ríos-Rull (2016, Table 5)
Variance of log income	0.96	0.94	Kuhn and Ríos-Rull (2016, Table 5)
Investment-output ratio	0.25	0.26	Dyrda and Pedroni (2023)
Capital-output ratio	2.5	2.5	Dyrda and Pedroni (2023)

Cross-sectional income distribution (Data Source: Kuhn and Ríos-Rull (2016, Table 7))

Quintiles	**1st**	**2nd**	**3rd**	**4th**	**5th**
Data	3	6.5	10.9	18.1	61.4
Model	4.46	6.88	9.91	16.64	62.11

Cross-sectional wealth distribution (Data Source: Kuhn and Ríos-Rull (2016, Table 7))

Quintiles	**1st**	**2nd**	**3rd**	**4th**	**5th**
Data	2.8	4.1	6.5	12.9	73.8
Model	2.65	4.09	6.53	12.6	74.13

the baseline by raising and lowering the parameter to $\varepsilon = -0.2$ and $\varepsilon = 0.2$, respectively, and investigate how the optimal taxes and subsidies change when x and y are more complementary or more substitutable than the baseline.

Initial conditions and targeted moments. We assume that the persistent skill shock θ_t is a geometric random walk $\theta_t = \epsilon_t \theta_{t-1}$ with ϵ_t from a lognormal distribution, $\log\epsilon_t \overset{i.i.d.}{\sim} N(\mu_\epsilon, \sigma_\epsilon^2)$. Initial human capital h_1 is identical for all agents. The degree of homogeneity η, initial human capital h_1, the depreciation rate of capital δ_k, and the mean μ_ϵ and the standard deviation (SD) σ_ϵ of the lognormal distribution are calibrated to match six targeted macroeconomic aggregates: the education ratio 0.19, the wage premium 1.8, the investment-output ratio 0.26, the capital-output ratio 2.5, and the Gini coefficient of income 0.56 and the variance of log income 0.94. These five parameters are determined by minimizing the sum of (absolute) differences between these six targeted values and the corresponding values of our model.

First, Stantcheva (2017) found that the education ratio of the net present value of lifetime education expenses to the net present value of lifetime income is 19 percent, which we accept. Next, the estimated value for the wage premium in the literature lies within 1.2 and 2.4.[19] Our calibration targets a medium value of 1.8, a value estimated by Heathcote et al. (2010) and also targeted by Kapička and Neira (2019). As our model does not have college choices, we go along with Stantcheva (2017), Kapička and Neira (2019) and Autor et al. (1998) to redefine the wage premium as the labor income of the top 42.7 percent relative to the bottom 42.7 percent in the

[19] The estimated range is 1.26-1.74 in Murphy and Welch (1992), 1.37-1.75 in Autor et al. (1998), 1.7-2.4 in Heathcote et al. (2005), and 1.2-2.2 in James (2012).

population. According to Dyrda and Pedroni (2023), the annual investment-output ratio is 0.26 and the annual capital-output ratio is 2.5. Finally, as for the Gini coefficient of income and the variance of log income, we target the values estimated by Kuhn and Ríos-Rull (2016), in which their income includes labor income, capital income, and government transfers. These values $\eta = 0.25$, $h_1 = 0.35$, $\delta_k = 0.1$, $\mu_\epsilon = 0.35$, and $\sigma_\epsilon = 0.75$ are calibrated to best approximately match the six targeted statistics. Finally, the calibrated values fit well to the cross-sectional income and wealth distribution.

4.2. Simulation results

We apply these parameter values from calibration to the second-best economy and calculate the policy functions with respect to the constrained efficient allocation of each type. Using the computed policy functions, we carry out Monte Carlo simulations with 100,000 agents evolving through periods $t = 1, 2, ..., T$. Note that agents do not work but consume the same after retirement, so all the wedges are zero after retirement. Therefore, we only focus on working periods $t = 1, 2, ..., 40$ in this subsection.

In the first period, we normalize $\underline{v}_1 = 0$ and solve the cost minimization problem (9d). In later periods $t = 2, ..., T$, the state variables $v(\theta)$, $\Delta(\theta)$, $h(\theta)$ are solved by the problem in the previous period, and then the policy functions are solved by using the cost minimization problem (9d). We also simulate otherwise the same model except that both $x(\theta)$ and $y(\theta)$ are observable. This serves to highlight the role of privately observed HCI by comparing the results of our model with and without private HCI. See the comparison in Fig. 1.

Fig. 1 shows higher average capital wedges, lower average labor wedges, and lower average human capital wedges over time (solid lines) in our model than the model without private HCI (dash lines, labelled no private HCI). The results indicate that non-verifiable education expenses increase average capital taxes and decreases average labor taxes and average human capital subsidies.

Capital wedge. As Fig. 1(a) shows, the average capital wedge is positive and decreasing over time in both our model and the model without private HCI, and the average capital wedge is higher in our model. The result comes from unobservable HCI. Proposition 6 indicates three sources of our capital wedge:

$$\tau_{k_t}(\theta^t) = \{\text{insurance effect}\} + \{\text{current HCI effect}\} + \{\text{next HCI effect}\}.$$

Fig. 2 decomposes the average capital wedge over time into effects of these three sources. As is standard, the insurance effect is positive and decreasing over time, which is the average capital wedge in the model with no private HCI in Fig. 1(a). The current HCI effect is positive and decreasing over time, while the next HCI effect is negative and decreasing over time. As the positive current HCI effect dominates the negative next HCI effect, the capital wedge in our model is higher than the model with no private HCI in Fig. 1(a).

To understand the capital wedge across contemporary skills, we simulate the capital wedge against skills over the life cycle. To save space, Fig. 3 presents the scatter plot in the mid-working period in $t = 20$. In Fig. 3(a), the capital wedge is positive, and progressive in all except very high skills. In Figs. 3(b)−3(d), we decompose the capital wedge into the following three effects. Fig. 3(b) is the insurance effect, which is positive and regressive; Fig. 3(c) is the current HCI effect, which is positive and progressive in all except very high skills; and Fig. 3(d) is the next HCI effect, which is negative and progressive. As the current HCI effect dominates other effects, the capital wedge is positive and progressive.

The Mirrlees literature obtained a positive capital wedge due to the insurance effect. As the insurance effect weakens over time, the average capital wedge tapers to zero when nearing retirement (e.g., Farhi and Werning, 2013; Stantcheva, 2017). By contrast, the positive current HCI effect and the negative next HCI effect are at work in our model. Our result shows that the next HCI effect is weaker than the current HCI effect and the insurance effect. As a result, the capital wedge is positive in all working periods. Moreover, due to the large current HCI effect, the capital wedge is positive and progressive except in very high skills.

Labor wedge. In Fig. 1(b), the average labor wedge is negative early and positive later in our model (solid lines), as compared to being positive in the model without private HCIs (dash lines). Proposition 6 indicates two sources of labor wedge:

$$\tau_{z_t}(\theta^t) = \{\text{shirk-preventing effect}\} + \{\text{skill-fostering effect}\}.$$

Fig. 4 decomposes the average labor wedge over the life cycle into the two sources. As can be seen, the shirk-preventing effect is positive, while the skill-fostering effect is negative. Moreover, the shirk-preventing effect is increasing over time, while the skill-fostering effect is decreasing and tapering to zero over time. The negative skill-fostering effect quantitatively dominates in earlier periods, so the average labor wedge is negative. As the shirk-preventing effect dominates later, the average labor wedge is then positive.

To understand the skill distribution of the labor wedge, we simulate the labor wedge against skills over the life cycle. Fig. 5 reports the scatter plot using the mid-working period in $t = 20$. In Fig. 5(a), the labor wedge is hump-shaped against skills, just like that in the Mirrlees model in Golosov et al. (2006) and Ales et al. (2015). The shirk-preventing effect is positive and hump-shaped in skills in Fig. 5 (b), while the skill-fostering effect is negative and regressive in skill types in Fig. 5(c).

In the Mirrlees literature, because of the shirk-preventing effect, a positive labor wedge is designed for the redistribution purpose to prevent skilled agents from shirking. Farhi and Werning (2013) studied a model with exogenous skills evolving according to a stochastic AR(1) process. Their labor wedge is positive and regressive against skills. Stantcheva (2017) analyzed a model with endogenous skills via verifiable education expenses. Her quantitative results suggest that the labor wedge may be regressive or progressive, depending on whether the Hicksian complementarity between skills and human capital is larger or smaller than 1. Different from these two papers, as our model considers unobservable HCIs, the labor wedge is determined by the interaction of the positive shirk-preventing effect and the negative skill-fostering effect. Thus, the labor wedge can be positive or negative, depending on these

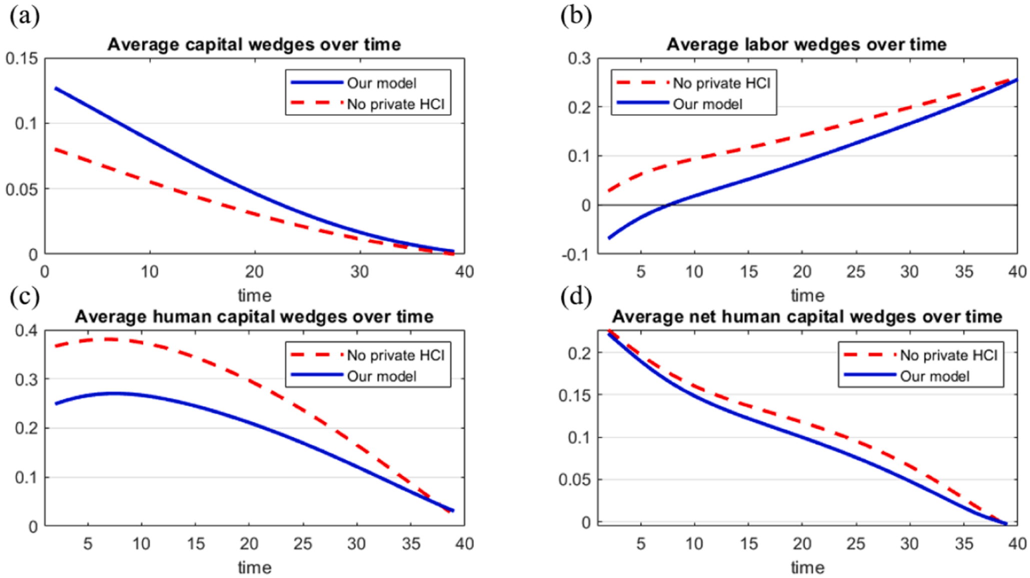

Fig. 1. Average capital, labor and (net) human capital wedges over time.

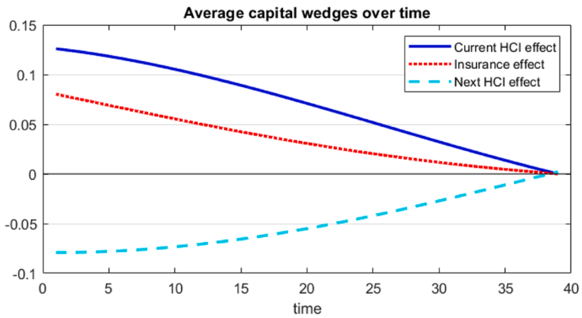

Fig. 2. Decomposition of the average capital wedge into effects of different sources.

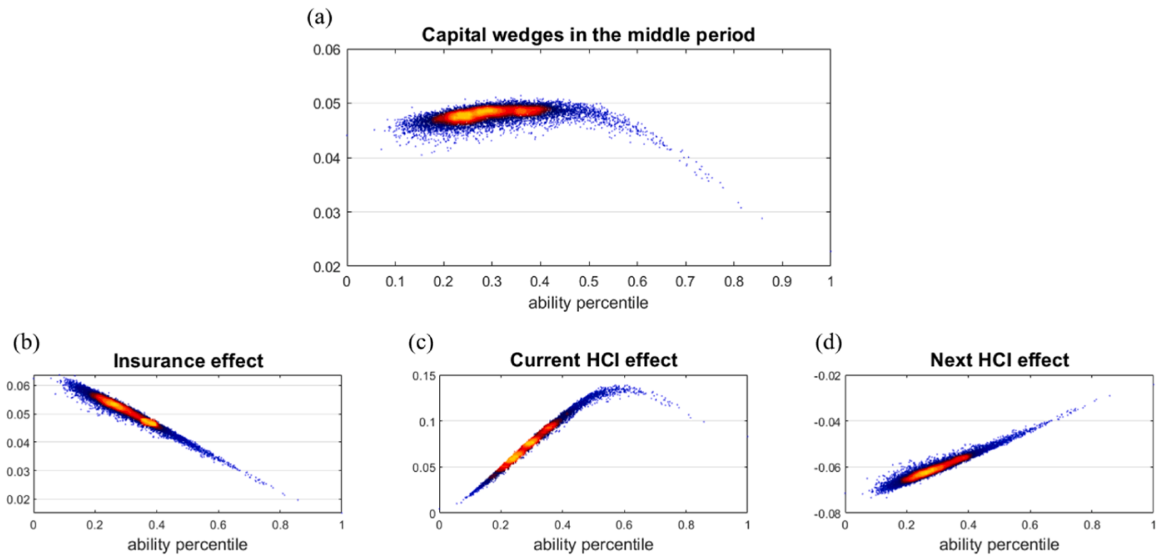

Fig. 3. Scatter plot of the capital wedge against skill types in $t = 20$ and the decomposition into effects of three sources.

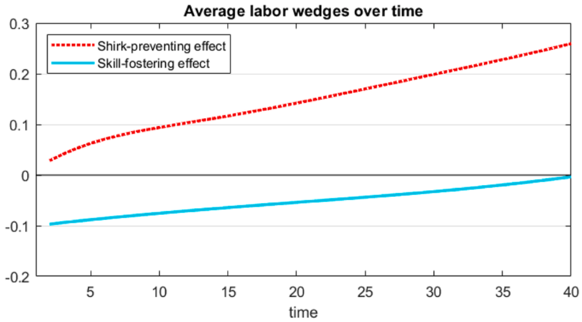

Fig. 4. Decomposition of the average labor wedge into effects of two sources.

two effects. The negative skill-fostering effect dominates the positive shirk-preventing effect in early periods, so the labor wedge is negative. In particular, even in a middle working period, as the shirk-preventing effect is zero and dominated by the skill-fostering effect at the top and the bottom of the skill distribution, the labor wedge is negative at the bottom and the top of the skill distribution.

Human capital wedge and net human capital wedge. In Figs. 1(c) and (1d), the average (net) human capital wedges are positive and decreasing over time in our model (solid lines) and the model with no private HCI (dash lines). The human capital wedge in our model is much lower than the model with no private HCI, but the net human capital wedge in our model is close to the model with no private HCI. To see why, according to Definition 1, the net human capital wedge $\tau^n_{x_t}$ is constructed from the human capital wedge τ_{x_t} by removing labor wedge distortions $\mathscr{N}_{t+1}$ and capital wedge distortions $-\mathscr{K}_{t+1}$. As the net effects from labor and capital distortions are eliminated, the net human capital wedge in our model is close to the model with no private HCI in Fig. 1(d). Figs. 1(a) and 1(b) indicate that our model has a lower labor wedge and a higher capital wedge, so the net effects of labor and capital distortions $(\mathscr{N}_{t+1} - \mathscr{K}_{t+1})$ in our model are lower than those in the model with no private HCI. As a result, the human capital wedge in our model is lower in Fig. 1 (c).

We also simulate the scatter plot of the human capital wedge and the net human capital wedge against skills over the life cycle. Fig. 6 reports the scatter plot using the mid-working period in $t = 20$. The human capital wedge is positive and hump-shaped in skills in Fig. 6(a), due to the effect from the hump-shaped labor wedge. Eliminating effects from labor and capital distortions, the net human capital wedge is positive and regressive in skills in Fig. 6(b), which means that the human capital subsidy is in favor of the high skilled, because the high skilled benefit more from HCI. Stantcheva (2017, p1965) also finds a regressive human capital subsidy as long as the Hicksian complementarity between human capital and skills is positive ($\rho_{h\theta} > 0$). By contrast, our model obtains a regressive human capital subsidy even if $\rho_{h\theta} = 0$.

Persistence of labor earnings process. Huggett et al. (2011, Fig. 2) showed that the mean earnings by age for ages 23–60 are hump-shaped in both their human capital model and the data. In Fig. 7(a) we compute the mean labor earnings by age for ages 21–60 in 40 periods in our model and compare them with the data in Huggett et al. (2011, Fig. 2), who calculated the mean earnings by age for ages 23 to 60 in the data from the Panel Study of Income Dynamics (PSID) 1969–2004. Huggett et al. (2011) normalized the mean earnings in the data to 100 at age 60, and our model follows the same normalization for the terminal period ($t = 40$). We find that the labor earnings are low early in life because the initial human capital is low, while the labor earnings fall later due to less investment in human capital. Fig. 7(a) shows that our simulation has a hump-shaped profile for mean labor earnings, which is close to the data in Huggett et al. (2011, Fig. 2).

Overall tax progressivity. The wedges do not directly implement the constrained efficient allocation in a market economy, and a tax system that implements the constrained efficient allocation is usually not unique.[20] To better understand the redistribution effect in the constrained efficient allocation, we simulate the individual tax, $Tax^I_t(\theta^t)$, which satisfies the following budget constraint for each type $\theta^t \in \Theta^t$.

$$c(\theta^t) + x(\theta^t) + y(\theta^t) + k(\theta^t) = w_t z(\theta^t) + R_t k(\theta^{t-1}) - Tax^I_t(\theta^t).$$

This individual tax is defined as individual labor and capital income after deducting individual spending on consumption, HCIs and savings. According to the aggregate resource constraints, the aggregate tax revenue, $Tax^A_t = \int Tax^I_t(\theta^t)\pi(\theta^t)d\theta^t$, is equal to the government expenditure G_t. Fig. 7(b) reports the scatter plot of the individual tax against skills as a share in the aggregate tax, $\frac{Tax^I_t(\theta^t)}{Tax^A_t}$, in the mid-working period $t = 20$. Overall, the tax system is progressive in skills. The individual tax is in general positive. The exception is in the bottom of the skill distribution, wherein the individual tax is zero and even negative, indicating that the tax system indeed redistributes the resource from the high skilled to the low skilled.

[20] In Appendix A.4, we construct a tax system with an income condition and a lump-sum tax that can directly use the capital and labor wedges as linear tax rates to implement the constrained efficient allocation in a market economy.

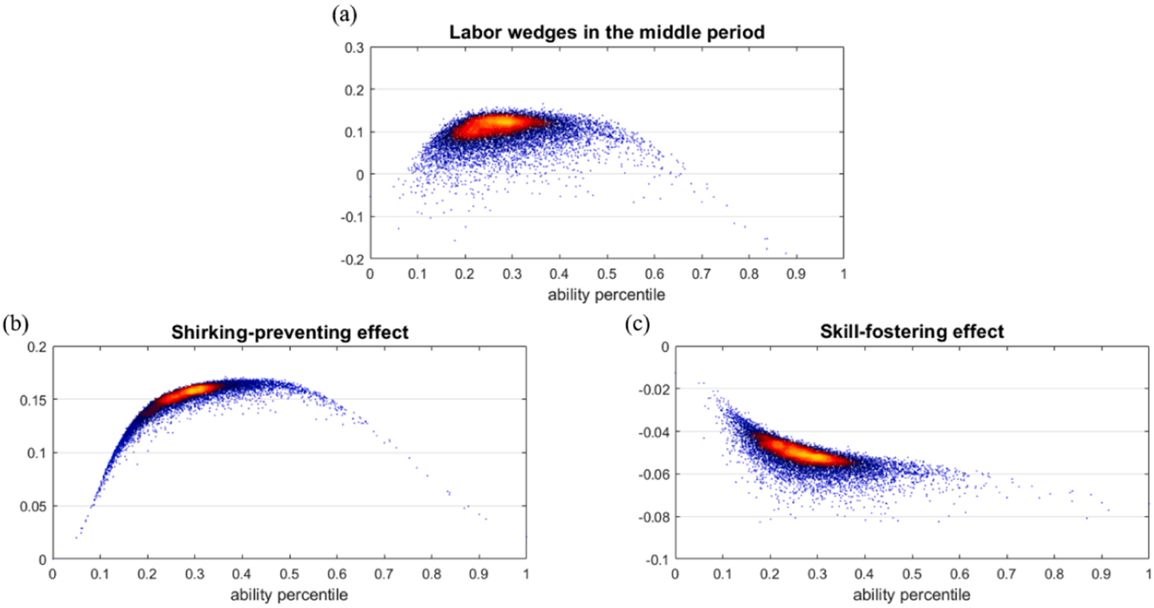

Fig. 5. Scatter plot of the labor wedge against skill types in $t = 20$ and the decomposition into effects of two sources.

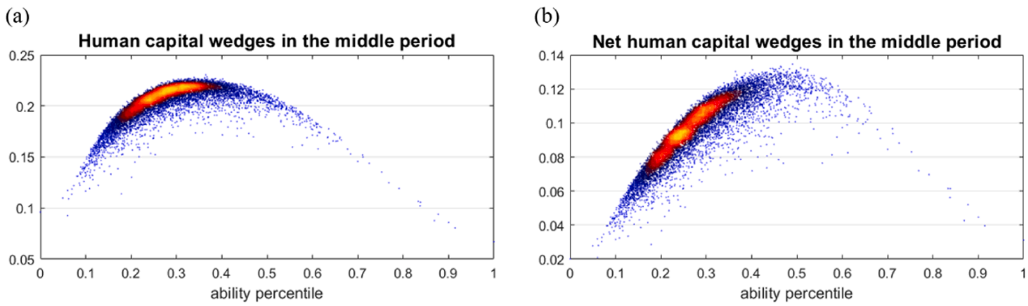

Fig. 6. Scatter plot of the human capital wedge and the net human capital wedge against skill types in $t = 20$.

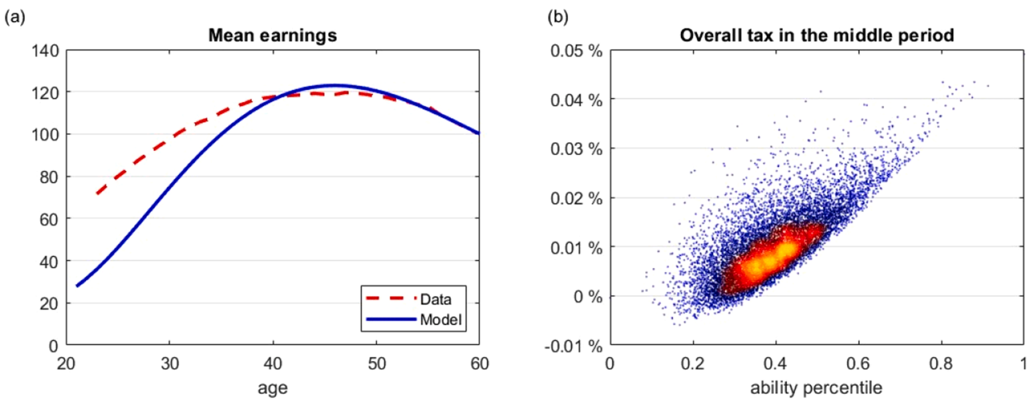

Fig. 7. (a) Mean of earnings by age (b) Scatter plot of individual taxes, as a share of aggregate taxes, against skill types in $t = 20$.
Note. In panel (a) we compute the mean labor earnings by age for ages 21–60 in 40 periods in our model and compare with the data in Huggett et al. (2011, Fig. 2), who computed the mean earnings by age for ages 23 to 60 in the data from the PSID 1969–2004. The data in Huggett et al. (2011, Fig. 2) is normalized to 100 at age 60, and our model follows the same normalization for the terminal period ($t = 40$).

4.3. Different elasticities of substitution between x and y

In the baseline we set $\varepsilon = 0$, so the human capital technology of verifiable and non-verifiable HCIs x and y in (15) is a Cobb-Douglas form. How will optimal taxes and subsidies respond to different elasticities of substitution between x and y? This subsection explores these issues by studying the cases when x and y are more complementary or more substitutable than the baseline.

First, Fig. 8 is average capital, average labor, average human capital, and average net human capital wedges over the life cycle under the baseline case and two other cases. Solid lines are under the baseline case $\varepsilon = 0$, dash lines are under the case $\varepsilon = -0.2$ when x and y are more complementary than the baseline case, and dotted lines are under the case $\varepsilon = 0.2$ when x and y are more substitutable than the baseline case. As can be seen, the capital and the labor wedges change little with the value ε, but the human capital and the net human capital wedges increase when the value ε decreases, as x and y are more complementary.

Next, we report the scatter plot of the human capital wedge and the net human capital wedge against skills under these three cases of ε in the middle working period $t = 20$ in Fig. 9. Clearly, both the human capital and the net human capital wedges increase when the value ε decreases. Intuitively, if x and y are more complementary, an increase in verifiable education expenses raises the marginal product of the non-verifiable education expense. Thus, a subsidy to verifiable education expenses not only directly increases verifiable education expenses, but also indirectly solicits non-verifiable education expenses. As a result, when x and y are more complementary, the human capital subsidy increases.

Though average capital and average labor wedges change little in different ε in Figs. 8(a)-8(b), we still can find that in the middle period, the average capital wedge is a little bit lower and the labor wedge is a little bit higher when ε decreases. Fig. 10 duplicates the capital wedge against skills and the decomposition into the insurance effect, the current HCI effect and the next HCI effect. Figs. 10(b)−10(d) indicate a decrease in the insurance and the current HCI effects against skills but an increase in the next HCI effect against skills when ε decreases. As the insurance and the current HCI effects dominate the next period's HCI effect, the capital wedge decreases against skills when ε decreases, as seen in Fig. 10(a).

Finally, we analyze how the labor wedge and the decomposition into the shirk-preventing effect and the skill-fostering effect change in different ε. Figs. 11(b)−11(c) suggest that shirk-preventing effect against skills decrease when ε decreases, while the skill-fostering effect increases when ε decreases. As a decrease in ε leads to the increase in the skill-fostering effect more than the decrease in the shirk-preventing effect, the labor wedge increases when ε decreases, as seen in Fig. 11(a).

4.4. Welfare gains and simple age-dependent policy

We now compute the welfare gains to answer the following questions. First, what are the welfare gains of the constrained efficient allocation in our model? Second, how much would the welfare gains increase if the frictions about non-verifiable HCIs were shut down? Third, if our history-dependent tax system is too complicated to be feasible, how well can a simple age-dependent tax policy do in our model?

To answer the first and the second questions, we compare the welfare gains of the constrained efficient allocation in our second-best economy relative to the status quo allocation in the economy with the status quo tax system illustrated in subSection 4.1. Let $W^{SQ}(c_t^{SQ}(\theta^t), l_t^{SQ}(\theta^t))$ be the welfare of the status quo economy (*SQ*), where $c_t^{SQ}(\theta^t)$ and $l_t^{SQ}(\theta^t)$ are, respectively, consumption and the labor supply of type θ^t. Let W^{SB} be the welfare of our second-best economy (*SB*). The welfare gain of the second-best economy from the status quo economy is defined in terms of consumption equivalence: the percentage increase in consumption in the second-best economy relative to the status quo economy. Let ω_{SB} denote the percentage increase in consumption. The welfare gain ω_{SB} solves $W^{SQ}((1+\omega_{SB})c_t^{SQ}(\theta^t), l_t^{SQ}(\theta^t)) = W^{SB}$. Our results indicate that the welfare gain of our second-best economy is 1.29 percent over the status quo economy. See the second row of Table 3. Moreover, when the information frictions of the non-verifiable HCIs are shut down, the welfare gain of the economy with no private HCI is 1.31 percent over the status quo economy, as shown in the fourth row of Table 3.

Finally, to answer the third question, we consider a simple age-dependent tax. In computing the welfare gain under a simple age-dependent tax policy, we follow the method in Farhi and Werning (2013) and Stantcheva (2017). We take a hint from the second-best and set the linear capital tax rates, the linear labor tax rates, and the linear human capital subsidy rates at each age to their cross-sectional averages at that age in the second-best. The number of tax variables is greatly reduced, making the problem numerically tractable. We find that, relative to the status quo economy, the welfare gain of the simple linear age-dependent policy is 1.12 percent, which is 86.8 percent of the welfare gains of our second-best economy. See the third row, Table 3. It is worth recapping that the average wedges change over time (ages) in our second-best economy (cf. Fig. 1). Thus, in terms of the overall welfare gain, it is reasonable that the simple age-dependent tax is close to our second-best economy.

4.5. Ex-post ICC verification

Our model studies the relaxed planning problem based on the first-order approach, which replaces the incentive compatible conditions with the envelope condition. Since the envelope condition is only a necessary condition, the solution to the relaxed program might not be a solution to the full program. This subsection numerically verifies whether the solution to the relaxed program is the solution to the full program. In other words, we verify whether the envelope condition used in our model implies ex-post incentive compatibility.

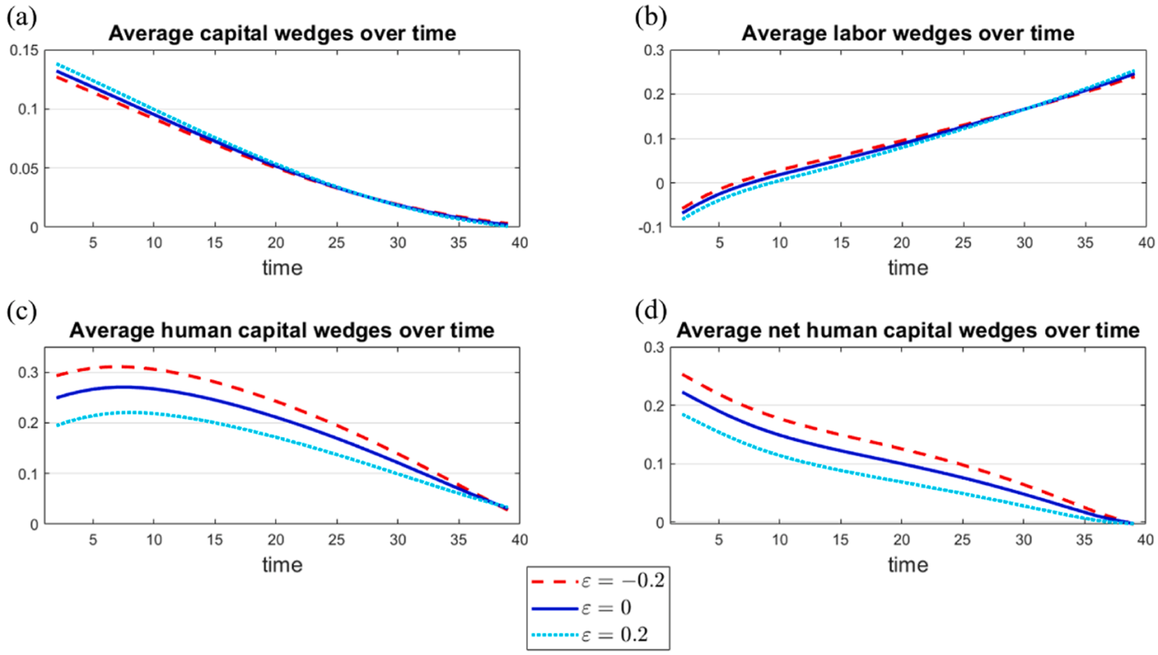

Fig. 8. Average wedges over time under different elasticities of substitution between verifiable HCI and non-verifiable HCI. (a) capital wedge (b) labor wedge (c) HC wedge (d) net HC wedge.

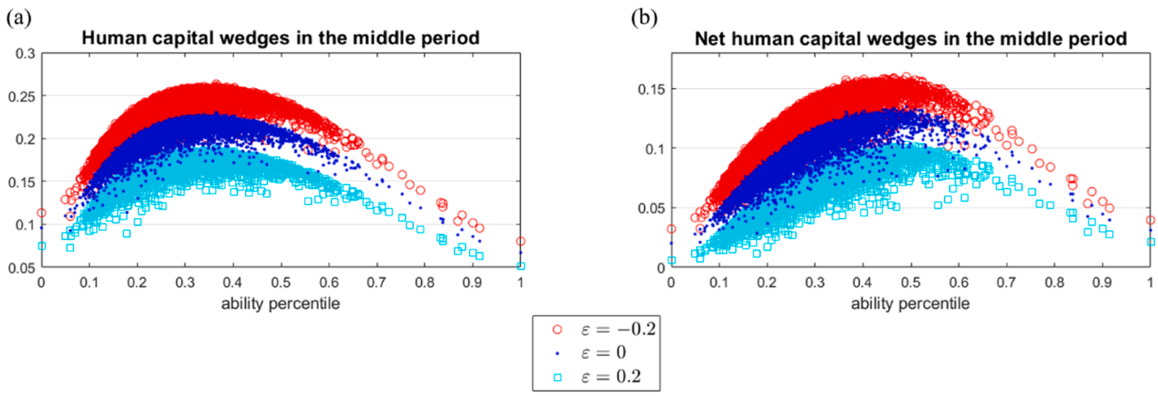

Fig. 9. Scatter plot of the human capital wedge and the net human capital wedge under different elasticities of substitution between verifiable and non-verifiable HCI in $t = 20$.

We simulate the utility gains of different reporting strategies in our model. Specifically, we simulate the utility of our model in all periods $t = 1, \ldots, 40$, and then calculate $W^{\sigma}(\theta^t) - W(\theta^t)$, as defined in (13b); that is, the welfare gain of reporting strategies σ from the truth-telling strategy in the model. We do the simulation based on the first-best allocation (without the envelope condition) and the constrained efficient allocation (with the envelope condition), respectively. Fig. 12 is the welfare difference, where the x-axis is the true type θ, the y-axis is the reported type σ, and the z-axis is the welfare difference of reported types σ from the truth-telling strategy θ. To save space, Fig. 12 only demonstrates the welfare difference for period $t = 20$ using the set of the values of the state variables $\mathscr{K}(v = 0.11, \Delta = 0.03, h = 0.68, \theta_{-} = 0.9, t = 20)$ as an example of ex post verification. As can be seen from Fig. 12(a), which is based on the first-best allocation without the envelope condition, agents can obtain a higher welfare by under-reporting their types. By contrast, from Fig. 12(b), which is based on the constrained efficient allocation with the envelope condition, agents cannot obtain a higher welfare from any misreporting strategy. The truth-telling strategy always gives the highest utility in our model (cf. the solid diagonal bold line). The comparison between Fig. 12(a) and Fig. 12(b) indicates that the envelope condition indeed generates the incentive-compatible allocation in our model and thus, the validity of the relaxed planning problem is guaranteed.

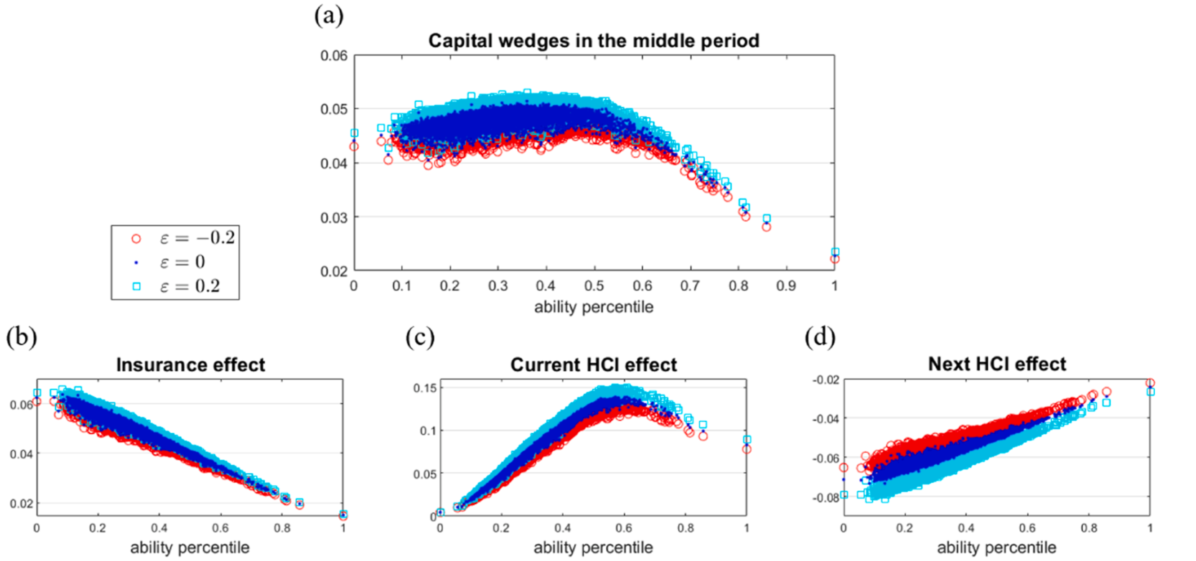

Fig. 10. Scatter plot of the capital wedge and the decomposition into effects of three sources under different elasticities of substitution between verifiable and non-verifiable HCI in $t = 20$.

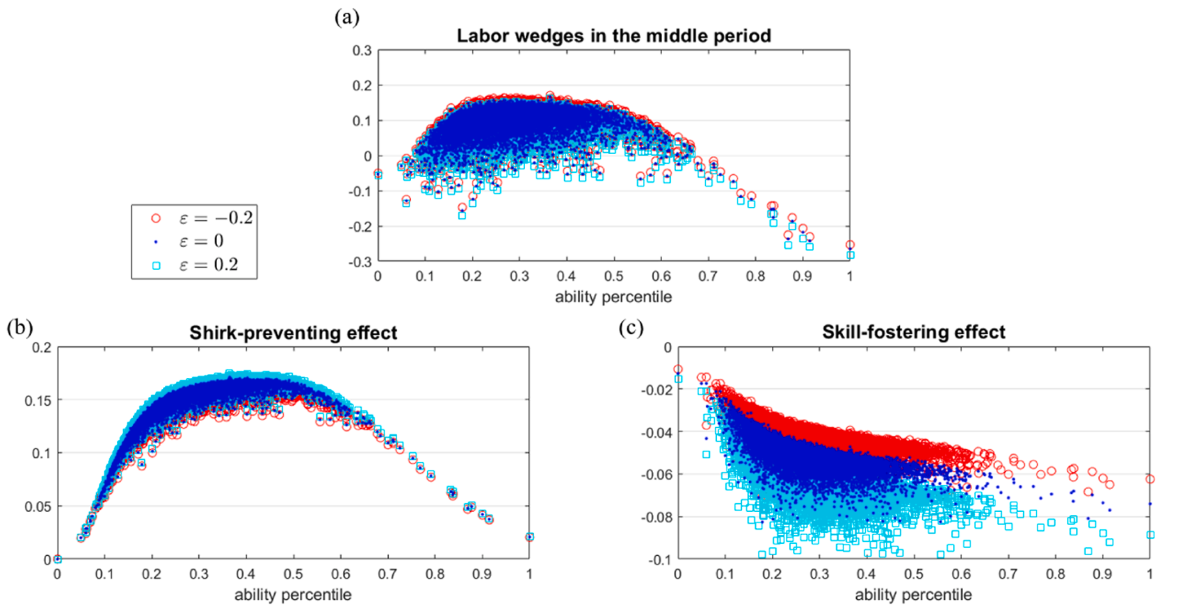

Fig. 11. Scatter plot of the labor wedge and the decomposition into effects of two sources under different elasticities of substitution between verifiable and non-verifiable HCI in $t = 20$.

Table 3
Welfare gains over status quo economy.

Economies	Welfare gains	As% of second-best
Status quo	0%	
Our second-best model	1.29%	
Simple age-dependent tax	1.12%	86.8%
No private HCI	1.31%	

Note: Welfare gains are in terms of consumption equivalence. Simple age-dependent tax is the welfare gain of an otherwise our model except with the simple age-dependent tax policy.

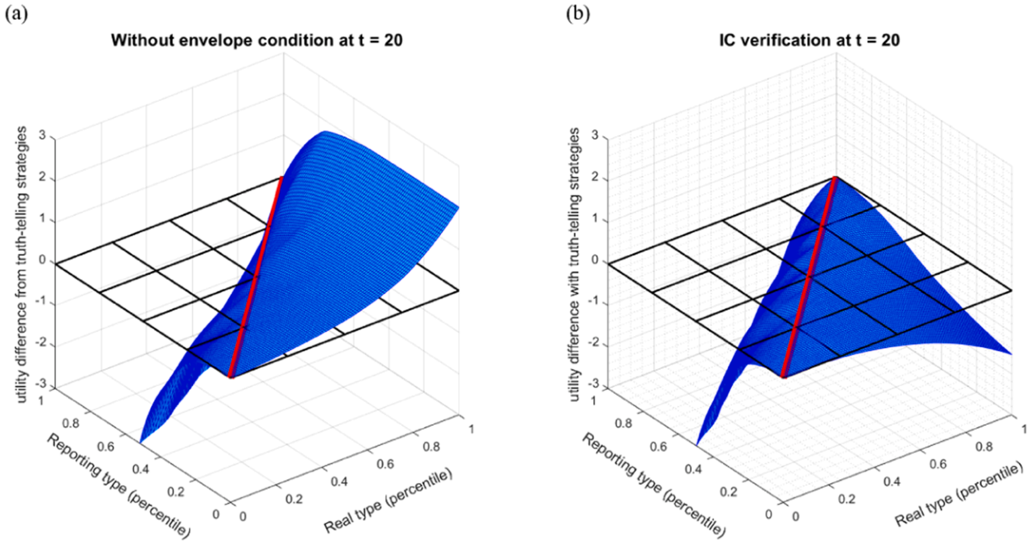

Fig. 12. Utility gains from different reporting strategies in our model ($t = 20$).
Note. The x-axis is the true type θ, the y-axis is the reported type σ, and the z-axis is $W^{\sigma}(\theta) - W(\theta)$ in (13b). We calculate the difference in lifetime utilities of the reporting strategies σ from the truth-telling strategy in periods 1, 2, …, 40 over the life cycle. The figure reports period $t = 20$ as an example. (a) Our model except without the envelope condition (first-best economy). (b) Our model (second-best economy).

5. Concluding remarks

This paper studies the wedges in a dynamic Mirrlees economy that adds unobservable HCIs to the model with observable HCIs over the life cycle. In the model, in addition to working and savings, agents choose consumption and education over the life cycle. The key feature is indistinguishable consumption from education expenses. The social planner chooses constrained efficient allocations that maximize the utilitarian social welfare subject to resource constraints and incentive-compatibility constraints. We characterize the capital wedge, the labor wedge, and the (net) human capital wedge in the resulting constrained efficient allocations.

When allowing for partially depreciated human capital, we quantitatively find that the average capital wedge is positive and decreasing over the life cycle, due to the current period's HCI effects and insurance effects dominating the next period's HCI effects. The average labor wedge is negative in the early life cycle since the skill-fostering effect dominates, but it is positive and increasing in the later life cycle as the shirk-preventing effect dominates. The average (net) human capital wedges are positive and decreasing over the life cycle. Moreover, the capital wedge is positive and progressive in all except very high skills due to the current period's HCI effects dominating insurance and the next period's HCI effects. In the middle period, the labor wedge is positive and hump-shaped in skills due to positive and hump-shaped shirk-preventing effects dominating negative and regressive skill-fostering effects. In particular, the labor wedge is negative at the bottom and the top of the skill distribution, departing from the standard zero-tax result in the existing Mirrlees literature. In the middle period, the human capital wedge is positive and hump-shaped in skills, due to the effect from the hump-shaped labor wedge. Eliminating effects from labor and capital distortions, the net human capital wedge is positive and regressive in skills, which means that the human capital subsidy is in favor of the high skilled, because the high skilled benefit more from HCI.

Overall, individual taxes are progressive in skills and positive in all except the bottom of the skill distribution, wherein individual taxes are zero or even negative, indicating that constrained efficient allocation indeed redistributes the resource from the high skilled to the low skilled. There is a large welfare gain of our second-best economy over the status quo tax system. We find that a simple linear age-dependent tax can yield a large welfare gain over the status quo tax system. Finally, if there is misrepresentation of physical capital investment for other investments, like misrepresentation of consumption for education, it is an interesting issue, but the problem is not trivial. Though it is out of the scope of this paper, it posits an avenue for future research.

Data availability

Email to Zimmermann

Supplementary materials

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.red.2023.11.004.

References

Ales, L., Kurnaz, M., Sleet, C., 2015. Technical change, wage inequality and taxes. Am. Econ. Rev. 105 (10), 3061–3101.
Allen, F., 1985. Repeated principal-agent relationships with lending and borrowing. Econ. Lett 17, 27–31.
Atkeson, A., Lucus Jr, R.E., 1992. On efficient distribution with private information. Rev. Econ. Studies 59, 427–453.
Atkinson, A.B., Stiglitz, J.E., 1976. The design of tax structure: direct versus indirect taxation. J. Public Econ. 6 (1), 55–75.
Autor, D., Katz, L.F., Krueger, A.B., 1998. Computing inequality: have computers changed the labor market? Q. J. Econ. 113 (4), 1169–1213.
Becker, G.S., 1964. Human Capital: A Theoretical and Empirical Analysis, With Special Reference to Education. University of Chicago Press.
Ben-Porath, Y., 1967. The production of human capital and the life cycle of earnings. J. Polit. Econ. 75 (4), 352–365.
Borghans, L., Duckworth, A.L., Heckman, J.J., ter Weel, B., 2008. The economics and psychology of personality traits. J. Human Resource 43 (4), 972–1059.
Bovenberg, A.L., Jacobs, B., 2005. Redistribution and education subsidies are Siamese twins. J. Public Econ. 89, 2005–2035.
Bovenberg, A.L., Jacobs, B., 2010. Human capital and optimal positive taxation of capital income. Int. Tax Public Financ. 17, 451–478.
Chamley, C., 1986. Optimal taxation of capital income in general equilibrium with infinite lives. Econometrica 54 (3), 607–622.
Chen, B.-L., Lu, C.-H., 2013. Optimal factor tax incidence in two-sector human capital-based models. J Public Econ 97, 75–94.
Cole, H.L., Kocherlakota, N.R., 2001. Efficient allocations with hidden income and hidden storage. Review of Economic Studies 68, 523–542.
Conesa, J.C., Kitao, S., Krueger, D., 2009. Taxing capital? Not a bad idea after all! Am. Econ. Rev. 99 (1), 25–38.
Cunha, F., Heckman, J.J., 2007. The technology of skill formation. Am. Econ. Rev. 97 (2), 31–47.
Diamond, P.A., 1980. Income taxation with fixed hours of work. J Public Econ 13, 101–110.
Diamond, P.A., 1998. Optimal income taxation: an example with a U-shaped pattern of optimal marginal tax rates. Am. Econ. Rev. 88 (1), 83–95.
Diamond, P.A., Mirrlees, J.A., 1978. A model of social insurance with variable retirement. J. Public Econ 10, 295–336.
Dyrda, S., Pedroni, M., 2023. Optimal fiscal policy in a model with uninsurable idiosyncratic income risk. Rev. Econ. Studies 90, 744–780.
Farhi, E., Werning, I., 2013. Insurance and taxation over the life cycle. Rev. Econ. Studies 80, 596–635.
Fernandes, A., Phelan, C., 2000. A recursive formulation for repeated agency with history dependence. J. Econ. Theory 91, 223–247.
Golosov, M., Tsyvinski, A., Werning, I., 2006. New Dynamic Public finance: A user's guide. *NBER Macroeconomic Annual*. MIT Press.
Gouvieia, M., Strauss, R., 1994. Effective federal individual income tax functions: an exploratory analysis. Natl. Tax J. 47 (2), 317–339.
Grochulski, B., Piskorski, T., 2010. Risky human capital and deferred capital income taxation. J. Econ. Theory 145, 908–943.
Guner, N., Kaygusuz, R., Ventura, G., 2014. Income taxation of U.S. households: facts and parametric estimates. Rev. Econ. Dyn 17 (4), 559–581.
Heathcote, J., Perri, F., Violante, G.L., 2010. Unequal we stand: an empirical analysis of economic inequality in the United States, 1967–2006. Rev. Econ. Dyn 13 (1), 15–51.
Heathcote, J., Storesletten, K., Violante, G.L., 2005. Two views of inequality over the life cycle. J. Eur. Econ. Assoc 3, 765–775.
Heckman, J.J., Stixrud, J., Urzua, S., 2006. The effects of cognitive and noncognitive ability on labor market. J. Labor Econ 24 (3), 411–482.
Huggett, M., Ventura, G., Yaron, A., 2011. Sources of lifetime inequality. Am. Econ. Rev. 101 (7), 2923–2954.
James, J., 2012. The college wage premium. *Economic Commentary*. Federal Res. Bank Cleveland (2012–10).
Judd, K.L., 1985. Redistributive taxation in a simple perfect foresight model. J Public Econ 28, 59–83.
Kapička, M., 2013. Efficient allocation in dynamic private information economics with persistent shocks: A first-order approach. Review of Economic Studies 80 (3), 1027–1054.
Kapička, M., 2015. Optimal Mirrleesean taxation in a Ben-Porath economy. Macroeconomics 7 (2), 219–248.
Kapička, M., Neira, J., 2019. Optimal taxation with risky human capital. Macroeconomics 11 (4), 271–309.
Kocherlakota, N.R., 2005. Zero expected wealth taxes: a Mirrlees approach to dynamic optimal taxation. Econometrica 73, 1587–1621.
Kuhn, M., Ríos-Rull, J.-V., 2016. 2013 Update on the U.S. earnings, income, and wealth distribution facts: a view from macroeconomics. Q. Rev. DC Nurses Assoc. 1–75.
Lazear, E., 1977. Education: consumption or production? J. Polit. Econ. 85 (3), 569–598.
McDaniel, C., 2007. Average Tax Rates On consumption, investment, Labor and Capital in the OECD 1950-2003. Arizona State University. Working Paper.
Meghir, C., Pistaferri, L., 2004. Income variance dynamics and heterogeneity. Econometrica 72, 1–32.
Milgrom, P., Segal, I., 2002. Envelope theorems for arbitrary choice sets. Econometrica 70, 583–601.
Mirrlees, J., 1971. An exploration in the theory of optimum income taxation. Rev. Econ. Studies 38, 175–208.
Murphy, K.M., Welch, F., 1992. The structure of wages. Q. J. Econ. 107 (1), 285–326.
Pavan, A., Segal, I., Toikka, J., 2014. Dynamic mechanism design: a Myersonian approach. Econometrica 82 (2), 601–653.
Peterman, W.B., 2016. The effect of endogenous human capital accumulation on optimal taxation. Rev. Econ. Dyn 21, 46–71.
Saez, E., 2002. Optimal income transfer programs: intensive versus extensive labor supply responses. Q. J. Econ. 117, 1039–1073.
Saez, E., Stantcheva, S., 2016. Generalized social marginal welfare weights for optimal tax theory. Am. Econ. Rev. 106 (1), 24–45.
Schultz, T.W., 1961. Investment in human capital. Am. Econ. Rev. 51, 1–17.
Stantcheva, S., 2014. Optimal taxation and human capital policies over the lifecycle. MIT Department of Economics Graduate Student Research Paper No. 12-02 available at. http://ssrn.com/abstract=2191089.
Stantcheva, S., 2017. Optimal taxation and human capital policies over the life cycle. J. Polit. Econ. 125 (6), 1931–1990.
Tobing, E., 2011. Taxation, human capital formation, and long-run growth with private investment in education. J. Asian Econ. 22, 48–60.
Tuomala, M., 1990. Optimal Income Taxation and Redistribution. Clarendon Press, Oxford.
van Ewijk, C., Tang, P.J.G., 2007. Union, progressive taxes, and education subsidies. Eur. J. Polit. Econ. 23, 1119–1139.