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Capital flows and wealth distribution in a global economy with asymmetric countries[☆]

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ABSTRACT

Focusing on international differences in firms' production efficiency, financial constraints, and savers' time discount rates, this study explores how these factors affect a country's long-run asset position and the global distribution of wealth. Using a dynamic two-country model with financial frictions and firm heterogeneity, we clarify the roles of these asymmetric factors in the long-run equilibrium of the world economy. Due to the tractability of our model. we can obtain key results analytically using simple graphical expositions. Numerical examples are also provided to evaluate the comparative statics in the steady state and transition dynamics of the global economy.

1. Introduction

Neoclassical growth theory predicts that if two countries are identical apart from their initial capital stocks, capital flows from the capital-rich country to the capital-poor one. However, empirical evidence frequently contradicts this prediction. To account for observed international capital flow patterns, researchers have introduced country-specific differences. First, although technological knowledge may diffuse globally, production technologies often remain country-specific. For example, if production relies on internationally immobile human capital and a rich country possesses more human capital, the rate of return on physical capital may be higher in the richer country, attracting capital inflows from poorer countries (Lucas, 1990). Second, despite financial globalization, international capital markets remain imperfect. Country-specific financial frictions can generate capital flow patterns that deviate from standard neoclassical predictions (Gertler and Rogoff, 1990; Antras and Caballero, 2009; Matsuyama, 2014; Buera and Shin, 2017; Wang et al., 2017). Third, cross-country differences in the impatience of savers can influence capital flows (Buiter, 1981). Finally, country-specific fiscal policies, in particular, differences in tax regimes, can significantly shape global wealth distribution (Frenkel et al., 1991; Gordon, 1986; Turnovsky, 1997).

In this study, we integrate these country-specific differences into a unified, dynamic two-country model with financial frictions and firm heterogeneity. We examine the long-run behavior of a global economy composed of asymmetric countries in a tractable framework. Specifically, we explore how differences in production technology, financial frictions, time discount rates, and tax policies jointly determine long-run asset positions and wealth distribution across countries. Our approach contrasts with the traditional two-country model without financial frictions, in which the steady-state wealth distribution depends primarily on each country's initial wealth holdings.[1]

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[1] See, for example, Ono and Shibata (1992) and Chapter 6 in Turnovsky (1997).

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In our two-country model, both countries produce homogeneous final goods. This enables international lending and borrowing to operate as intertemporal trade in these goods. Each country has two types of agents, namely, homogeneous workers and heterogeneous entrepreneurs. Every entrepreneur owns a firm. These firms are subject to continuous idiosyncratic technological shocks, resulting in heterogeneous productivity. The amount of physical capital a firm can utilize is subject to financial constraints and is proportional to the entrepreneur's net worth. The interplay between financial constraints and heterogeneous capital productivity generates an endogenously determined cutoff for capital efficiency. Entrepreneurs engage in production only if their capital efficiency exceeds this cutoff; otherwise, they become rentiers. The efficiency cutoff affects the productivity of the aggregate production technology in each country and is determined by the country's aggregate wealth-to-capital ratio. This structure enables analysis of how financial constraints and productivity heterogeneity influence capital allocation and aggregate productivity across countries.

We assume that entrepreneurs in both countries can borrow from or lend to each other via internationally traded bonds. Accordingly, the no-arbitrage condition between real and financial assets implies that the return on capital in both countries equals the real interest rate on these bonds, ensuring a common return on capital. As this return is influenced by the capital efficiency cutoff, the wealth-to-capital ratios of both countries are interdependent. Consequently, a country's aggregate productivity depends on the wealth of domestic and foreign entrepreneurs. Thus, the total factor productivity (TFP) of each country's aggregate production function is shaped by the global distribution of entrepreneurial wealth. This feature differentiates our model from traditional two-country frameworks without financial frictions, where aggregate TFP is determined solely by domestic production technologies.

Using this framework, we first verify the existence and stability of the world economy's steady-state equilibrium. Subsequently, we analyze the determinants of long-run asset positions and wealth distribution between the two countries. Our findings indicate that firm productivity, credit constraints, time discount rates, and tax policies jointly affect long-run wealth distribution. However, long-run asset positions, reflected in international lending and borrowing patterns, depend exclusively on differences in entrepreneurs' time discount rates. Moreover, we perform a comparative statics analysis to explore how unilateral changes in financial or real sector regimes in one country influence long-run outcomes in the other. A financial regime change is modeled as a permanent shift in firms' financial constraints, while a real regime change involves a permanent alteration in the technology parameter of the aggregate production function. Owing to the tractability of our model, these comparative statics can be depicted using simple graphical representations. To supplement our theoretical insights, we provide numerical examples that quantify the long-run effects of these regime changes and reveal the transitional dynamics of key economic variables.

1.1. Related literature

1.1.1. Empirical studies

The empirical literature on the macroeconomic effects of financial frictions is extensive, so we focus on the research most closely related to our study.

First, we assume that each firm's investment is subject to a stock-based collateral constraint, as in Kiyotaki and Moore (1997). Numerous empirical studies have confirmed that capital market imperfections influence firms' investment behavior. For example, Fazzari et al. (1988) and Hubbard et al. (1995) used data on U.S. manufacturing firms to show that financial constraints negatively affect investment. Generally, these studies emphasize that investment is positively related to cash flow, which is often associated with larger net assets. This supports our model's assumption that greater net worth enables higher levels of investment. Recently, Lian and Ma (2021) conducted a detailed empirical analysis of borrowing constraints, finding that 20% of non-financial U.S. firms faced stock-based borrowing constraints, while the remainder were subject to earnings-based constraints.[2] However, most Japanese firms were subject to stock-based borrowing constraints. Although not all empirical findings align precisely with our model's formulation, they affirm that financial frictions play a significant role in investment decisions.

Second, our model highlights the crucial role that capital market imperfections play in shaping international capital flows. The 2007–2008 financial crisis prompted a surge in empirical investigations that support the relevance of our framework. These studies demonstrated that financial shocks significantly impact global capital flows and trade. For instance, Aizenman et al. (2010) showed that the globalization of financial markets can amplify the effects of financial shocks. Similarly, Gourinchas and Obstfeld (2012) recognized the substantial consequences of financial crises on both trade and capital movements. Using monthly export data, Chor and Manova (2012) provided a detailed empirical account of the financial crisis's effects on trade, confirming that the collapse in trade flows was primarily due to severe disruptions in global financial condition.

Third, as explained in Section 3, our model predicts that an increase in the aggregate leverage ratio raises aggregate TFP. Several studies have empirically explored the relationship between leverage and firm productivity. For example, Hennessy and Whited (2007) found that higher leverage generally enhances production efficiency; however, excessive leverage can raise the cost of external finance, thus suppressing productivity. Similarly, Coricelli et al. (2012), using data on firms in Central and Eastern Europe, found that external financing could boost productivity, while high leverage may hinder productivity growth. Based on data from Chinese manufacturing firms, Chen and Guariglia (2013) found that production efficiency improves with external funding, provided that the level of borrowing remains moderate. Although these studies focused on firm-level productivity, their findings suggest that leverage, unless excessive, may positively influence productivity at the macroeconomic level.

However, because precise data on aggregate leverage are difficult to obtain, empirical studies often use the external debt-to-GDP ratio as a proxy for the macro-level leverage ratio. This ratio is commonly treated as an indicator of financial market development. Several empirical studies (Buera et al., 2011; Greenwood et al., 2013; Beyene and Kotosz, 2021; Li, 2022) have shown that financial development can reduce capital misallocation stemming from financial frictions, thereby increasing aggregate TFP. Nevertheless, as Pattillo et al. (2004) emphasized in their analysis of heavily indebted countries, excessive external debt can constrain productivity growth. Although these studies do not directly validate our model, they suggest that as long as macroeconomic leverage is not exceedingly high, our model's implications for TFP are broadly consistent with empirical observations.

1.1.2. Theoretical studies

This section focuses on theoretical investigations that explore international capital flows under financial constraints, aiming to clarify the mechanisms that shape capital flow patterns in open economies.[3] Past contributions include that of Gertler and Rogoff (1990), whose

[2] As to the macroeconomic implication of earning-based borrowing constraints, see, for example, S. and Uribe (2021) and Drechsel (2023).

[3] The another stand of literature on open economies with imperfect financial markets explores the effects of financial shocks on open economies, Many studies, such as Mendoza (2010) and Korinek and Mendoza (2014), explore how financial shocks affect short-run business cycles in small open economies. However, Faia (2007), Devereux and Yetman (2010), Dedola and Lombardo (2012), Yao (2019), and Pintus et al. (2019) quantitatively characterize the effects of financial frictions on international business cycles in models featuring two large countries.

seminal work employed a two-period model to examine capital movements between developed and developing countries. Matsuyama (2005) proposed a similar two-period model to elucidate the structure of trade and capital flows. Furthermore, Antras and Caballero (2009) analyzed a dynamic Heckscher–Ohlin model with infinitely lived agents, discussing the connection between real trade and capital flows under financial constraints. Mendoza et al. (2009) developed a multi-country endowment economy model that incorporated idiosyncratic endowment shocks and household borrowing constraints.[4] In a related but distinct context, Wang et al., (2017) constructed a dynamic model with infinitely lived agents to show how real and financial capital can move in opposite directions in a two-country setting.[5] Our study contributes to this strand of literature by focusing on the long-run determinants of international capital flows in the presence of financial frictions.

From an analytical standpoint, our research is closely related to the work of Itskhoki and Moll (2019), who developed a small open economy model in which the world interest rate is treated exogenously. They incorporated financial frictions and firm heterogeneity into the neoclassical growth framework for a small open economy and investigated a range of optimal policies.[6] We extend their framework to a global economy model consisting of two large countries, in which the world interest rate is endogenously determined. In the absence of financial constraints, our model reduces to a two-country neoclassical growth framework with unrestricted capital mobility: a setting that has been extensively studied in international macroeconomics.[7]

It is to be noted that there are two additional differences between our study and that of Itskhoki and Moll (2019). First, although we adopt the baseline model of Itskhoki and Moll (2019), assuming that idiosyncratic productivity shocks are independently and identically distributed (i.i.d.) over time, their numerical experiments consider the case in which these shocks are persistent. Second, while Itskhoki and Moll (2019) assumed that workers save, we assume workers behave as hand-to-mouth consumers. In Section 5, we discuss how our key results might change under general settings.

Although our study overlaps with the theoretical studies cited above, it diverges from the existing literature in three important aspects. First, we provide a clearer analytical characterization of the global economy's long-run equilibrium, while previous studies focused primarily on numerical analyses. Second, the tractability of our model enables us to analyze multiple asymmetries between countries within a unified framework. Third, we can evaluate the relative importance of the key determinants of steady-state equilibrium—an issue that remains underexplored in the literature. Nevertheless, the simplicity of our model constrains its capacity to address certain relevant questions, such as the complex interaction between real and financial transactions across borders. These issues have been more thoroughly examined in prior studies (Matsuyama 2005, Antras and Caballero 2009, Jin 2010; Wang et al., (2017)). Thus, our study should be considered complementary to, rather than a replacement for, existing contributions.

The remainder of this study is organized as follows. Section 2 introduces the basic model. Section 3 analyzes the behavior of the global economy. Section 4 conducts comparative statics within the steady-state equilibrium and provides numerical illustrations of the model. In Section 5, we explore how our main findings change when key assumptions for analytical tractability are relaxed. Finally, Section 6 concludes the study constraints.

2. Model

2.1. Setup

The world comprises two countries, namely, home and foreign. Both countries produce homogeneous final goods. In each country, homogeneous workers and heterogeneous entrepreneurs exist.[8] This section focuses on the home country, while Section 2.3 describes the behavior of the foreign country.

2.1.1. Workers

There is a continuum of identical workers with a unit mass. Their behavior is straightforward: they are myopic and do not engage in saving. The representative worker at each moment solves the following optimization problem:

$$\max_{C_{w,t},\, N_t} U_{w,t} = \max_{C_{w,t}, N_t} \log C_{w,t} - \frac{N_t^{1+\gamma}}{1+\gamma},$$

subject to $C_{w,t} = w_t N_t$, where $C_{w,t}$ is the consumption of a representative worker, N_t is labor supply the representative worker, and w_t is real wage. The optimal consumption and labor supply are given by the following[9]:

$$C_{w,t} = w_t, \quad N_t = 1. \tag{1}$$

2.1.2. Entrepreneurs

Entrepreneurs constitute a continuum with a unit mass. Each entrepreneur owns a firm. The production technology of a firm is

$$y_t = A\left(zk_t\right)^{\alpha} n_t^{1-\alpha}, \qquad A > 0, \quad 0 < \alpha < 1, \tag{2}$$

where y_t, k_t, and n_t denote the output, capital, and labor demand of a firm, respectively. Here, A stands for the efficiency of the production technology, which is common to all firms in the home country. We assume that the efficiency of capital, denoted by z, is heterogeneous among firms. This specification indicates that each entrepreneur employs the same production function, differing only in the efficiency of capital. Here, z is a stochastic variable follows a stationary Pareto distribution specified by its cumulative distribution function:

$$F(z) = 1 - z^{-\phi}, \quad \phi > 1, \quad z \geq 1, \tag{3}$$

where the shape parameter, ϕ, expresses the degree of heterogeneity in production efficiency; a lower value of ϕ means a higher level of heterogeneity in production technology among firms. We interpret z as an idiosyncratic technological shock that hits each firm at every

[4] Mendoza et al. (2009) focus on the global imbalance problem using a multi-country model.

[5] See Furusawa and Yanagawa (2013), Coeurdacier et al. (2015), Matsuyama (2014), Ghironi (2018), and Brooks and Dovis (2020) for further investigations on the relationship between real and financial trade.

[6] Itskhoki and Moll (2019) examine an open economy version of the neoclassical growth model with financial constraints developed by Moll (2014) and Buera and Moll (2015). Goméz and Neto (2016) also use Itskhoki and Moll's setting to examine a small open economy with financial constraints.

[7] See, for example, Chapter 6 in Turnovsky (1997). As Hu and Mino (2013) reveal, this result also holds in a two-sector economy in which one of the two goods are not traded and international lending and borrowing are allowed.

[8] This setting has been frequently used in the literature on macrodynamic models with financial frictions: see Kiyotaki and Moore (1997) and Liu and Wang (2014).

[9] If the workers save, their optimization problem is as follows:

$$\max \int_0^{\infty} e^{-\eta t}\left(\log C_{w,t} - \frac{N_t^{1+\gamma}}{1+\gamma}\right) dt$$

subject to $\dot{S}_t = r_t S_t + w_t N_t - C_{w,t}$, where η (> 0) is the time discount rate of workers and S_t denotes workers' asset holdings. As pointed out by Moll (2014) (Footnote 19), if the time discount rate is high enough to hold $\eta > r_t$ in the steady state and if the workers cannot borrow ($S_t \geq 0$ for all $t \geq 0$), then workers' optimal choice yields $C_{w,t} = w_t N_t$ in the long run. In this case, the hand-to-mouth behavior of workers reflects their optimal decision, at least in the long run.

moment. According to Liu and Wang (2014) and Itskhoki and Moll (2019), we assume that z is identically and independently distributed (i.i.d.) over time as well as across agents. Consequently, owing to the law of large numbers, the proportion of firms facing a particular z remains constant over time.

We posit a global financial market in which international bonds are actively traded. Entrepreneurs in both countries can lend to or borrow from domestic and foreign entrepreneurs through these financial channels. However, our model acknowledges frictions in this financial system, wherein each entrepreneur faces financial constraints. Denoting an entrepreneur's net debt as d_t, the borrowing constraint is given by

$$d_t \leq \lambda k_t, \quad 0 \leq \lambda \leq 1. \tag{4}$$

This constraint reflects that borrowing is limited by the entrepreneur's capital stock, which acts as collateral.[10] Thus, a portion of the physical capital is financed by borrowing. Denoting the entrepreneur's net worth by $x_t = k_t - d_t$, (4) is expressed as

$$k_t \leq \theta x_t, \quad \theta = \frac{1}{1-\lambda} \geq 1. \tag{5}$$

Here, the capital-to-net worth ratio, k_t/x_t, should be less than θ.[11] Thus, if $\theta = 1$ $(\lambda = 0)$, the entrepreneur's investment must be self-financed, whereas there is no financial friction if $\theta = +\infty$ $(\lambda = 1)$.

We begin by formulating the entrepreneur's decision-making process for employing labor and capital as a static optimization problem. In their role as producers, each firm hires labor and rents capital from households. The profit of the firm is given by $\pi_t = y_t - w_t n_t - r_t k_t$, where r_t denotes the rental rate of capital. For simplicity, we ignore capital depreciation. Given entrepreneurs' access to the international financial market, the non-arbitrage condition ensures that the rental rate, r_t equals the real interest rate on bonds. Thus, r_t serves as both the rental rate of capital and the real interest rate on bonds. The firm maximizes π_t by choosing n_t and k_t subject to the production technology constraint (2) and the financial constraint (5) . The first-order condition governing the optimal choice of labor input is

$$(1-\alpha) A \left(\frac{z k_t}{n_t}\right)^{\alpha} = w_t. \tag{6}$$

From (6) , the firm's profit is expressed as follows:

$$\pi_t = \alpha A z k_t \left[\frac{w_t}{(1-\alpha) A}\right]^{-\frac{1-\alpha}{\alpha}} - r_t k_t.$$

Each entrepreneur selects k_t to maximize π_t under $0 \leq k_t \leq \theta x_t$. The optimal choice of k_t is given by

$$k_t = \begin{cases} 0, & \text{for } z < \underline{z}_t, \\ \theta x_t, & \text{for } z \geq \underline{z}_t. \end{cases} \tag{7}$$

where $\underline{z}_t$ denotes the cutoff level of capital efficiency at period t, which fulfills the zero-profit condition, $\pi_t = 0$. Thus, $\underline{z}_t$ is determined by

$$\underline{z}_t = \frac{r_t}{\alpha A}\left[\frac{w_t}{(1-\alpha) A}\right]^{\frac{1-\alpha}{\alpha}}. \tag{8}$$

Because firm heterogeneity is characterized by production efficiency, z, and net worth, x_t, the profit function of the firm indexed by (x_t, z) is expressed as follows:

$$\pi(x_t, z) = \begin{cases} 0, & \text{if } z < \underline{z}_t, \\ \hat{\pi}(z, w_t, r_t)\,\theta x_t, & \text{if } z \geq \underline{z}_t, \end{cases} \tag{9}$$

where

$$\hat{\pi}(z, w_t, r_t) = z\alpha A\left[\frac{w_t}{(1-\alpha) A}\right]^{\frac{\alpha-1}{\alpha}} - r_t.$$

As a consumer, an entrepreneur maximizes a discounted, expected sum of utilities

$$U_{e,t} = E_0 \int_0^{\infty} e^{-\rho t} \log c_{e,t} dt, \quad \rho > 0,$$

subject to the flow budget constraint:

$$\dot{x}_t = r_t x_t + \pi(x_t, z) - c_{e,t}, \tag{10}$$

together with the non-Ponzi-game condition: $\lim_{t\to\infty} \exp\left(-\int_0^t r_s ds\right) x_t \geq 0$. Here, $c_{e,t}$ is the consumption of an entrepreneur and $\pi(x_t, z)$ is given by (9). The optimal consumption for the entrepreneur also follows the transversality condition: $\lim_{t\to\infty} e^{-\rho t} x_t / c_{e,t} = 0$.

To derive the optimal consumption of the entrepreneur, we define the value function such that $v_t(x_t, z) = \max E_t \int_t^{\infty} e^{\rho(t-s)} \log c_{e,s} ds$. Then, the Bellman equation is given by the following[12]:

$$\rho v_t(x_t, z) = \max_{c_{e,t}} \left\{\log c_{e,t} + \frac{1}{dt} E_t d v_t(x_t, z)\right\},$$

where x_t changes according to

$$d x_t = \left\{[r_t + \hat{\pi}(z, r_t, w_t)\theta] x_t - c_{e,t}\right\} dt.$$

As shown by Itskhoki and Moll (2019), the optimal consumption of an active entrepreneur is given by the following[13]:

$$c_{e,t} = \rho x_t. \tag{11}$$

2.2. Aggregation

2.2.1. Production function of final goods

In our formulation, each entrepreneur is characterized by its asset holdings, x, and the production efficiency, z. As we assumed z is i.i.d. across agents as well as over time, the distributions of z and x are independent each other.[14] Therefore, the aggregate levels of capital, hours worked, and output are, respectively, expressed as

$$K_t = \int_x \int_{z \geq \underline{z}_t} k_t(x, z)\, dG_t(x)\, dF(z), \qquad N_t = \int_x \int_{z \geq \underline{z}_t} n_t(x, z)\, dG_t(x)\, dF(z),$$
$$Y_t = \int_x \int_{z \geq \underline{z}_t} y_t(x, z)\, dG_t(x)\, dF(z).$$

Here, $G_t(x)$ denotes the cumulative distribution function of x at time t, so the entrepreneurs' aggregate wealth is

$$X_t = \int_x x\, dG_t(x).$$

[10] Financial constraints given by (4) represent one of the simplest forms of stock-based collateral constraints on firms. Midrigan and Xu (2014) and Lian and Ma (2021) present extensive empirical investigations on the forms of financial constraints on firms.

[11] If we define the leverage ratio as the external debt-to-net worth ratio, d_t/x_t, (4) means that there is an upper bound of the leverage ratio.

[12] Note that the value function is not stationary because it involves r_t and w_t. When $dt \to 0$, this equation becomes the Hamilton–Jacobi equation, which is the continuous-time counterpart of the Bellman equation.

[13] Suppose the value function takes the form $v_t(x_t, z) = M \log x_t + \mu \chi_t(z)$, where M and μ are undetermined constants. This specification yields $E_t d v_t(x_t, z) = M(dx_t/x_t) + \mu E_t d\chi_t(z)$. Then, using the flow budget constraint, the Bellman equation can be written as

$$\rho\mu\chi_t(z) + \rho M \log x_t$$
$$= \max_{c_{e,t}} \left\{\log c_{e,t} + \frac{M}{x_t}\left[r_t + \hat{\pi}(z, r_t, w_t)\theta x_t - c_{e,t}\right] + \mu \frac{1}{dt} E_t d\chi_t(z)\right\}$$

Based on the first-order condition, $1/c_{e,t} = M/x_t$, and the guess and verify approach, it is derived that $\rho M = 1$ and hence $c_{e,t} = \rho x_t$.

[14] In Section 5.1, we discuss how our results can be modified, if the idiosyncratic productivity shocks to firms are persistent over time, so we cannot separate the distribution of wealth among entrepreneurs from the distribution of productivity.

Note that the firms with $z \geq \underline{z}_t$ employ capital and are subject to financial constraints; thus, from (3) and $k_t(x,z) = \theta x_t$, the aggregate capital is expressed as

$$K_t = \int_x \int_{z \geq \underline{z}_t} k_t(x,z)\, dG_t(x)\, dF(z) = \theta X_t \int_{z \geq \underline{z}} dF(z) = \theta X_t \underline{z}_t^{-\phi}. \tag{12}$$

This means that $\underline{z}_t$ is related to aggregate capital and wealth in the following manner:

$$\underline{z}_t = \left(\frac{\theta X_t}{K_t}\right)^{\frac{1}{\phi}}. \tag{13}$$

Using (6) , each firm's output is expressed as

$$y_t = Azk_t \left[\frac{w_t}{(1-\alpha)A}\right]^{\frac{\alpha-1}{\alpha}}.$$

Thus, the aggregate output is determined by

$$Y_t = \int_x \int_{z \geq \underline{z}_t} Azk_t(x,z) \left[\frac{w_t}{(1-\alpha)A}\right]^{\frac{\alpha-1}{\alpha}} dG_t(x)\, dF(z)$$

$$= A \left[\frac{w_t}{(1-\alpha)A}\right]^{\frac{\alpha-1}{\alpha}} \theta X_t \frac{\phi}{\phi-1} \underline{z}_t^{1-\phi}. \tag{14}$$

Because the optimization condition (6) is expressed as $w_t n_t = (1-\alpha) y_t$, aggregating both sides of this relation gives

$$w_t = (1-\alpha)\frac{Y_t}{N_t}. \tag{15}$$

Then, substituting (15) into (14) and solving it for Y, we obtain

$$Y_t = A\left(\frac{\phi}{\phi-1}\underline{z}_t\right)^{\alpha} K_t^{\alpha} N_t^{1-\alpha}. \tag{16}$$

As the average productivity of the firms with production efficiency is greater than $\underline{z}_t$ is given by $\int_{z \geq \underline{z}_t} z\, dF(z) = \frac{\phi}{\phi-1}\underline{z}_t$, the above expression implies that the TFP of the aggregate technology depends positively on the average productivity of the active firms.

Finally, substituting (13) into (16) yields

$$Y = A\left(\frac{\phi}{\phi-1}\right)^{\alpha} \theta^{\frac{\alpha}{\phi}} \left(\frac{X_t}{K_t}\right)^{\frac{\alpha}{\phi}} K_t^{\alpha} N_t^{1-\alpha}. \tag{17}$$

This expression indicates that the TFP of the final goods production is given by

$$\text{TFP}_t = \frac{Y_t}{K_t^{\alpha} N_t^{1-\alpha}} = A\left(\frac{\phi}{\phi-1}\right)^{\alpha} \theta^{\frac{\alpha}{\phi}} \left(\frac{X_t}{K_t}\right)^{\frac{\alpha}{\phi}}. \tag{18}$$

Hence, given α, ϕ, and θ, the TFP of the home country increases with the ratio of aggregate net worth to capital stock. If we denote the aggregate external debt as D_t, from $K_t = X_t + D_t$, we obtain $X_t/K_t = 1/\left(1 + D_t/X_t\right)$. Hence, a decrease in the aggregate leverage ratio, $D_t//K_t$ (an increase in X_t/K_t) raises the cutoff z given by (13). This mitigates misallocation of capital by allocating a larger amount of capital to more efficient firms, which leads to a rise in the TFP. Conversely, with a given level of X_t/K_t, the TFP is higher, if firm heterogeneity is larger (i.e. ϕ is smaller) or if financial constraints are weaker (i.e. θ is larger). Note that if firms are homogeneous (i.e. $\phi = \infty$), then (17) becomes the following:

$$Y_t = AK_t N_t^{1-\alpha},$$

which is the aggregate production function with homogeneous firms in which A represents the aggregate TFP.

2.2.2. Rate of return to capital and aggregate excess profit

Alongside (13), $\underline{z}_t$ also satisfies (8). Thus, using (8) and (16), we find

$$Y_t = \left(\frac{\phi r_t}{(\phi-1)\alpha A}\left[\frac{w_t}{(1-\alpha)A}\right]^{\frac{1-\alpha}{\alpha}}\right)^{\alpha} K_t^{\alpha} N_t^{1-\alpha}.$$

Substituting (15) into the above equation and solving it for Y_t, we obtain the following relation:

$$r_t = \frac{\phi-1}{\phi}\alpha\frac{Y_t}{K_t}. \tag{19}$$

If firms are homogeneous, the competitive net rate of return to capital is $r_t = \alpha Y_t/K_t$. Therefore, $\frac{\phi-1}{\phi}\ (<1)$ represents an efficiency wedge.[15]

Because national income consists of factor incomes and excess profits, it holds that $Y_t = w_t N_t + r_t K_t + \Pi_t$. Consequently, from (15) and (19) the aggregate excess profit is given by

$$\Pi_t = \frac{\alpha}{\phi} Y_t. \tag{20}$$

2.3. Foreign firms and households

The foreign country's preference and production structure may mirror that of the home country. Foreign variables and parameters are marked with an asterisk. We assume that $\alpha = \alpha^*$ and $\phi = \phi^*$ to retain analytical tractability.[16] However, it may hold that $A \neq A^{\varepsilon}$, $\theta \neq \theta^*$, and $\rho \neq \rho^*$. For example, if the home country is a developing economy and the foreign country is an advanced economy, entrepreneurs in the home country may access less efficient technology for final goods production and face more stringent government regulations on financial activities than their foreign counterparts. This situation implies $A < A^*$ and $\theta < \theta^*$.

As with the home country, the efficiency cutoff, aggregate production function, and factor prices in the foreign country are given by the following:

$$\underline{z}_t^* = \left(\frac{\theta^* X_t^*}{K_t^*}\right)^{\frac{1}{\phi}}, \tag{21a}$$

$$Y_t^* = A^*\left(\frac{\phi}{\phi-1}\underline{z}_t^*\right)^{\alpha} K_t^{*\alpha} N_t^{*1-\alpha}, \tag{21b}$$

$$r_t = \alpha\left(\frac{\phi-1}{\phi}\right)\frac{Y_t^*}{K_t^*}, \tag{21c}$$

$$w_t^* = (1-\alpha)\frac{Y_t^*}{N_t^*}. \tag{21d}$$

Notably, foreign entrepreneurs also may have access to the international bond market; hence, the rental rate of capital in the foreign country equals the real interest rate on bonds, r_t.

The optimal consumption and labor supply of foreign workers are expressed as

$$C_{w,t}^* = w_t^*, \quad N_t^* = 1. \tag{21e}$$

Finally, the optimal consumption of each foreign entrepreneur is

$$c_{e,t}^* = \rho^* x_t^*. \tag{21f}$$

3. Behavior of the global economy

3.1. The link between capital and wealth

As discussed, owing to the non-arbitrage condition between real and financial capital, the rental rate of physical capital in each country

[15] As $(1-\phi)/\phi$ increases with ϕ, a higher heterogeneity among firms (i.e. a smaller ϕ) indicates a higher degree of distortion.

[16] The income share of capital (α and α^*) may not show significant international differences, but the shape parameters (ϕ and ϕ^* that determine firm size distribution in each country could vary considerably. However, if $\phi \neq \phi^*$, we cannot manipulate our model analytically, meaning that our entire analysis must rely on numerical examples. Although long-run comparative statics in Section 4 can be conducted quantitatively, we assume $\phi = \phi^*$to derive clear analytical results.

equals the real interest rate of the international bond. Hence, it holds that $r_t = r_t^*$ for all $t \geq 0$, meaning that from (19), and (21c) we obtain

$$\frac{Y_t}{K_t} = \frac{Y_t^*}{K_t^*}. \tag{22}$$

We assume that labor cannot cross the borders; thus, the labor market equilibrium conditions in both countries are $N_t = 1$ and $N_t^* = 1$. As a result, (22) gives

$$\frac{K_t^*}{K_t} = \left(\frac{A^*}{A}\right)^{\frac{\phi}{(1-\alpha)\phi+\alpha}} \left(\frac{\theta^*}{\theta}\right)^{\frac{\alpha}{(1-\alpha)\phi+\alpha}} \left(\frac{X_t^*}{X_t}\right)^{\frac{\alpha}{(1-\alpha)\phi+\alpha}}. \tag{23}$$

If $\phi = +\infty$, then firms are homogeneous, so (23) reduces to $K_t^*/K_t = (A^*/A)^{\frac{1}{1-\alpha}}$,. and allocation of capital stock between the two countries depends only on the relative TFPs of the representative firms in both countries. In contrast to the prototype model with homogeneous firms, the global allocation of capital in our model is not stationary as long as relative wealth changes during the transition.[17] Now, we express (23) as

$$K_t^* = \Gamma \left(\frac{X_t^*}{X_t}\right)^{\psi} K_t, \tag{24}$$

where

$$\Gamma \equiv \left(\frac{A^*}{A}\right)^{\frac{\phi}{(1-\alpha)\phi+\alpha}} \left(\frac{\theta^*}{\theta}\right)^{\frac{\alpha}{(1-\alpha)\phi+\alpha}} > 0 \ , \ \psi \equiv \frac{\alpha}{(1-\alpha)\phi+\alpha} \in (0,\alpha). \tag{25}$$

The equilibrium condition of the world financial market is given by

$$K_t + K_t^* = X_t + X_t^*. \tag{26}$$

Hence, from (24) and (26), we obtain

$$K_t = \Lambda\left(\frac{X_t^*}{X_t}\right) X_t, \quad K_t^* = \Lambda^*\left(\frac{X_t^*}{X_t}\right) X_t^*, \tag{27}$$

where

$$\Lambda\left(\frac{X_t^*}{X_t}\right) = \frac{1}{1+\Gamma\left(\frac{X_t^*}{X_t}\right)^{\psi}} \left(1+\frac{X_t^*}{X_t}\right),$$

$$\Lambda^*\left(\frac{X_t^*}{X_t}\right) = \frac{\Gamma\left(\frac{X_t^*}{X_t}\right)^{\psi-1}}{1+\Gamma\left(\frac{X_t^*}{X_t}\right)^{\psi}} \left(1+\frac{X_t^*}{X_t}\right).$$

Eqs. (27) imply that the aggregate leverage ratio of the home and foreign countries, K_t/X_t and K_t^*/X_t^*, can be determined by the relative wealth share between the two countries, X_t^*/X_t. Observe that $\Lambda'\left(X_t^*/X_t\right) > 0$ unless X_t^*/X_t is sufficiently small, while $\Lambda^{*\prime}\left(X_t^*/X_t\right) < 0$, unless we X_t^*/X_t is sufficiently large. Appendix A discusses the detailed properties of $\Lambda(.)$ and $\Lambda^*(.)$ functions.

The compiled results leads to the following proposition:

Proposition 1. *The aggregate production function of each country can be expressed as*

$$Y_t = A\left(\frac{\phi}{\phi-1}\right)^{\alpha} \theta^{\frac{\alpha}{\phi}} X_t^{\alpha} \left[\Lambda\left(\frac{X_t^*}{X_t}\right)\right]^{\alpha\left(1-\frac{1}{\phi}\right)}, \tag{28a}$$

$$Y_t^* = A^*\left(\frac{\phi}{\phi-1}\right)^{\alpha} \theta^{*\frac{\alpha}{\phi}} X_t^{*\alpha} \left[\Lambda^*\left(\frac{X_t^*}{X_t}\right)\right]^{\alpha\left(1-\frac{1}{\phi}\right)}. \tag{28b}$$

Proof. From (13) and (27), we obtain

$$\underline{z}_t = \left(\frac{\theta X_t}{K_t}\right)^{\frac{\alpha}{\phi}} = \theta^{\frac{\alpha}{\phi}} \left[\Lambda\left(\frac{X_t^*}{X_t}\right)\right]^{-\frac{\alpha}{\phi}}, \tag{29a}$$

$$\underline{z}_t^* = \left(\frac{\theta^* X_t^*}{K_t^*}\right)^{\frac{\alpha}{\phi}} = \theta^{*\frac{\alpha}{\phi}} \left[\Lambda^*\left(\frac{X_t^*}{X_t}\right)\right]^{-\frac{\alpha}{\phi}}. \tag{29b}$$

Substituting the above equations into (16) and (21b), respectively, and using (27) again, we obtain (28a) and (28b). ■

Due to the properties of the $\Lambda(.)$ and $\Lambda^*(.)$ functions, an increase in X_t^*/X_t decreases the cutoff level of capital efficiency in the home country and raises it in the foreign country. As (16) shows, a decrease in the cutoff lowers the average productivity of active firms in the home country, leading to a reduction the aggregate TFP in the home country. In contrast, a rise in X_t^*/X_t increases the aggregate productivity in the foreign country.[18]

3.2. Dynamic system

By aggregating the budget constraints of entrepreneurs yields the following dynamic equation:

$$\dot{X}_t = r_t X_t + \Pi_t - C_{e,t},$$

where $C_{e,t}$ is the aggregate consumption of entrepreneurs. Combining (19) and (27) , we find that the above equation can be expressed as

$$\begin{aligned}\dot{X}_t = {} & \alpha A\left(\frac{\phi}{\phi-1}\right)^{\alpha-1} \theta^{\frac{\alpha}{\phi}} X_t^{\alpha} \left[\Lambda\left(\frac{X_t^*}{X_t}\right)\right]^{\alpha-\frac{\alpha}{\phi}-1} \\ & + \frac{\alpha}{\phi} A\left(\frac{\phi}{\phi-1}\right)^{\alpha} \theta^{\frac{\alpha}{\phi}} X_t^{\alpha} \left[\Lambda\left(\frac{X_t^*}{X_t}\right)\right]^{\alpha-\frac{\alpha}{\phi}} - \rho X_t.\end{aligned} \tag{30a}$$

Similarly, the dynamic behavior of $\dot{X}_t^*$ is described by

$$\begin{aligned}\dot{X}_t^* = {} & \alpha A^*\left(\frac{\phi}{\phi-1}\right)^{\alpha-1} \theta^{*\frac{\alpha}{\phi}} X_t^{*\alpha} \left[\Lambda^*\left(\frac{X_t^*}{X_t}\right)\right]^{\alpha-\frac{\alpha}{\phi}-1} \\ & + \frac{\alpha}{\phi} A^*\left(\frac{\phi}{\phi-1}\right)^{\alpha} \theta^{*\frac{\alpha}{\phi}} X_t^{*\alpha} \left[\Lambda^*\left(\frac{X_t^*}{X_t}\right)\right]^{\alpha-\frac{\alpha}{\phi}} - \rho^* X_t^*.\end{aligned} \tag{30b}$$

Eqs. (30a) and (30b) form a complete dynamic system with respect to X_t and X_t^*.

When analyzing (30a) and (30b) , it is useful to express the dynamic system in the following manner. Letting $X_t^*/X_t = m_t$, (30a) and (30b) can be rewritten as

$$\begin{aligned}\dot{X}_t &= d\left(m_t\right) X_t^{\alpha} - \rho X_t, \\ \dot{X}_t^* &= d^*\left(m_t\right) X_t^{*\alpha} - \rho^* X_t^*,\end{aligned}$$

---

[17] Note that the equilibrium characterization of the world economy in our model depends on the joint assumptions of financial frictions and firm heterogeneity. To illustrate this, suppose firms are heterogeneous, but there are no financial constraints on firms. We also assume that the capital efficiency, z, has a maximum value, $\bar{z}$ (> 1), and the cumulative distribution function of z is

$$F(z) = \frac{1-z^{-\phi}}{1-\bar{z}^{-\phi}}.$$

In this case, as there are no financial constraints, less efficient firms with $z < \bar{z}$ cannot survive in competitive markets. As a result, our model reduces to the standard representative firm model. Conversely, suppose firms are homogeneous but financial frictions remain. In this case, if financial constraints are effective, every firm's investment is subject to borrowing constraints. Therefore, the aggregate stocks of capital and net worth in each country satisfy $K = \theta X$ and $K = \theta^* X^*$. However, because $\theta > 1$ and $\theta^* > 1$, these conditions contradict the equilibrium condition for the world financial markets given by (26). This means that in equilibrium, all firms are free from borrowing constraints, and, hence, the model reverts to the prototype two-country model with perfect capital markets.

[18] If both countries are in a state of financial autarky, $X_t = K_t$ and $X_t^* = K_t^*$. for all $t \geq 0$. Consequently, from (16), (21b), (13), and (21a), the aggregate production function of each country reduces to $Y_t = A\left(\frac{\phi}{\phi-1}\right)^{\alpha} \theta^{\frac{\alpha}{\gamma}} K_t^{\alpha} N_t^{1-\alpha}$, $Y_t^* = A\left(\frac{\phi}{\phi-1}\right)^{\alpha} \theta^{*\frac{\alpha}{\alpha}} K_t^{\alpha} N_t^{1-\alpha}$. Hence, the TFP of the aggregate production function for each country remains constant even outside the steady-state equilibrium.

where

$$d(m_t) = \alpha A \left(\frac{\phi}{\phi-1}\right)^{\alpha-1} \theta^{\frac{\alpha}{\phi}} \left[\Lambda\left(m_t\right)\right]^{\alpha-\frac{\alpha}{\phi}-1} + \frac{\alpha}{\phi} A \left(\frac{\phi}{\phi-1}\right)^{\alpha} \theta^{\frac{\alpha}{\phi}} \left[\Lambda\left(m_t\right)\right]^{\alpha-\frac{\alpha}{\phi}},$$

$$d^*(m_t) = \alpha A^* \left(\frac{\phi}{\phi-1}\right)^{\alpha-1} \theta^{*\frac{\alpha}{\phi}} \left[\Lambda^*\left(m_t\right)\right]^{\alpha-\frac{\alpha}{\phi}-1} + \frac{\alpha}{\phi} A^* \left(\frac{\phi}{\phi-1}\right)^{\alpha} \theta^{*\frac{\alpha}{\phi}} \left[\Lambda^*\left(m_t\right)\right]^{\alpha-\frac{\alpha}{\phi}}.$$

Hence, we obtain an alternative dynamic system with respect to X_t and m_t :

$$\dot{X}_t = d(m_t)X_t^{\alpha} - \rho X_t, \tag{31a}$$

$$\dot{m}_t = m_t X_t^{\alpha-1}\left[m_t^{\alpha-1}d^*(m_t) - d(m_t) - (\rho^*-\rho)X_t^{1-\alpha}\right]. \tag{31b}$$

3.3. Steady-state equilibrium

The steady-state equilibrium is established when $\dot{m}_t = \dot{X}_t = 0$ in (31a) and (31b) , so that X_t and X_t^* remain constant over time. In the following, we omit the time suffix from time-dependent endogenous variables to express their steady-state values. The steady-state values of m_t and X_t satisfy the following:

$$d(m) = \rho X^{1-\alpha}, \tag{32a}$$

$$m^{\alpha-1}d^*(m) = d(m) + \left(\rho^*-\rho\right)X^{1-\alpha}. \tag{32b}$$

Because $\Lambda(m)$ and $\Lambda^*(m)$ are increasing and decreasing functions, respectively, we define

$$\underline{z} = \theta^{\frac{\alpha}{\phi}}\left[\Lambda(\bar{m})\right]^{-\frac{\alpha}{\phi}} = 1, \quad \underline{z}^* = \theta^{*\frac{\alpha}{\phi}}\left[\Lambda^*\left(\underline{m}\right)\right]^{-\frac{\alpha}{\phi}} = 1,$$

where $\underline{z}$ and $\underline{z}^*$ represent the steady-state levels of the cutoff efficiency. Specifically, $\underline{m}$ and $\bar{m}$ satisfy

$$\frac{1+\bar{m}}{1+\Gamma\bar{m}^{\psi}} = \theta, \quad \frac{\Gamma\underline{m}^{\psi-1}\left(1+\underline{m}\right)}{1+\Gamma\underline{m}^{\psi}} = \theta^*. \tag{33}$$

Thus, $\underline{z} > 1$ for $m < \bar{m}$, and $\underline{z}^* > 1$ for $m > \underline{m}$. This means that if $\underline{m} > \bar{m}$ and if the steady-state level of m_t satisfy $\underline{m} > m > \bar{m}$, then $\underline{z} < 1$ and $\underline{z}^* < 1$, which does not meet the restriction $z > 1$. To avoid this situation, we assume the following[19]:

Assumption 1. It holds that $\bar{m} > \underline{m}$, where $\bar{m}$ and $\underline{m}$ are determined by (33).

We then establish the following:

Proposition 2. *Under Assumption 1, the world economy has a unique and feasible steady-state equilibrium.*

Proof. See Appendix B. ■

Once the steady-state level of m_t is uniquely determined, X is given by (32b) , so that X^* fulfills $X^* = m\left[d(m)/\rho\right]^{\frac{1}{1-\alpha}}$. Accordingly, the steady-state values of other key variables are determined as follows:

$$K = \Lambda(m)X, \quad K^* = \Lambda^*(m)mX,$$

$$Y = A\left(\frac{\phi}{\phi-1}\right)^{\alpha}\theta^{\frac{\alpha}{\phi}}X_t^{\alpha}\left[\Lambda(m)\right]^{\alpha\left(1-\frac{1}{\phi}\right)},$$

$$Y^* = A^*\left(\frac{\phi}{\phi-1}\right)^{\alpha}\theta^{*\frac{\alpha}{\phi}}(mX)^{\alpha}\left[\Lambda^*(m)\right]^{\alpha\left(1-\frac{1}{\phi}\right)},$$

$$\underline{z} = \left[\Lambda(m)\right]^{-\frac{1}{\phi}}, \quad \underline{z}^* = \left[\Lambda^*(m)\right]^{-\frac{1}{\phi}},$$

$$r = r^* = \alpha\left(\frac{\phi-1}{\phi}\right)X^{\alpha-1}\left[\Lambda(m)\right]^{\alpha\left(1-\frac{1}{\phi}\right)-1}.$$

We must note the following:

Corollary 1. *If entrepreneurs in both countries have an identical time preference rate, t(i.e. $\rho = \rho^*$), then the steady-state level of m is given by*

$$m = \left(\frac{A^*}{A}\right)^{\frac{1}{1-\alpha}}\left(\frac{\theta^*}{\theta}\right)^{\frac{\alpha}{(1-\alpha)\phi}}, \tag{34}$$

and the financial autarky holds in the steady state, that is, $X = K$ and $X^* = K^*$.

Proof. Eq. (B.6) in Appendix B shows that if $\rho = \rho^*$, then the steady-state level of m_t satisfies $\Lambda(m) = \Lambda^*(m)$. Thus, according to the definitions of $\Lambda(m)$ and $\Lambda^*(m)$ functions, the steady-state level of m fulfills

$$\Gamma m^{\psi-1} = 1,$$

which means that m is given by

$$m = \Gamma^{\frac{1}{\psi-1}} = \left[\left(\frac{A^*}{A}\right)^{\frac{\phi}{(1-\alpha)\phi+\alpha}}\left(\frac{\theta^*}{\theta}\right)^{\frac{\alpha}{(1-\alpha)\phi+\alpha}}\right]^{1-\frac{\alpha}{(1-\alpha)\phi+\alpha}} = \left(\frac{A^*}{A}\right)^{\frac{1}{1-\alpha}}\left(\frac{\theta^*}{\theta}\right)^{\frac{\alpha}{(1-\alpha)\phi}}. \tag{35}$$

In addition, Appendix B also shows that when $\Gamma m^{\psi-1} = 1$, it holds that $\Lambda(m) = \Lambda^*(m) = 1$. Thus, (27) reveals that $X = K$ and $X^* = K^*$. ■

Therefore, the relative wealth holdings between the two countries depend primarily on the relative technological levels, A^*/A, as well as on the relative tightness of financial constraints, θ^*/θ. Furthermore, the net asset positions of both countries (i.e. the signs of $X-K$ and X^*-K^*) depend on the sign of $\rho^*-\rho$ alone. As anticipated, the following holds in the steady-state equilibrium:

$$\text{sign}\ (X-K) = -\text{sign}\ \left(X^*-K^*\right) = \text{sign}\ \left(\rho^*-\rho\right). \tag{36}$$

Consequently, if entrepreneurs in one country are less patient compared to their foreign counterparts, that country becomes a debtor in the long run. This finding in Corollary 1 is in sharp contrast to the prototype two-country model in which the households in both countries have an identical time discount rate, and firms face no financial constraints. In such a prototype model, the steady-state level of financial asset holdings by households in each country are influenced solely by their initial financial asset holdings.[20] It is important to emphasize that Corollary 1 stems from our assumption of non-saving workers. Section 5.3 explains how the conditions in (36) can be altered if workers save.

3.4. Stability

Regarding the stability of the dynamic system comprising (31a) and (31b) , we can analytically show the following result:

Proposition 3. *If entrepreneurs in both countries have an identical time preference rate, (i.e. $\rho = \rho^*$), then the steady-state equilibrium of the world economy is globally stable.*

Proof. See Appendix C. ■

In Section 4.2, we numerically confirm that stability still holds in the case of $\rho \neq \rho^*$.

[19] See Fig. 2 in Section 4.1.

[20] See, for example, Ono and Shibata (1992) and Chapter 6 in Turnovsky (1997).

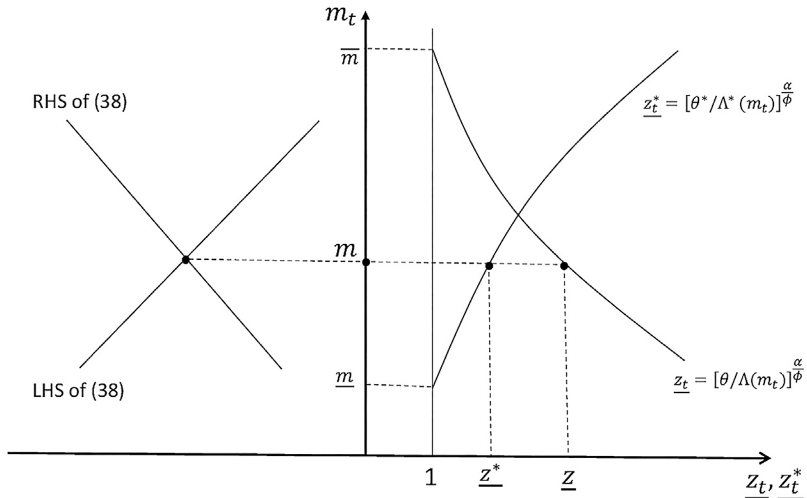

Fig. 1. Determination of the steady-state levels of m_t, $\underline{z}_t$, and $\underline{z}^*_t$.

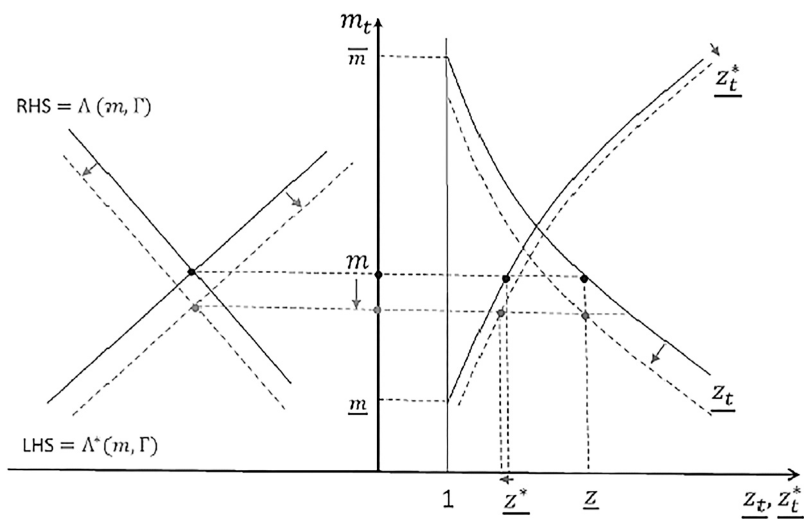

Fig. 2. Effects of a decrease in θ^* under $\rho = \rho^*$.

4. Long-run allocation of capital and wealth

Using the analytical framework presented thus far, we examine how the asymmetries between the two countries affect the international allocation of capital and wealth in the long-run equilibrium. By conducting comparative statics in the steady state, we derive the relationships between each country's capital and wealth holdings and the key parameters of the model.

4.1. Comparative statics in the steady state

To understand the international allocation of capital and wealth between the two countries, we examine the long-run relationships between capital and wealth. As stated in Section 1, we interpret permanent changes in θ and θ^* as financial regime changes, whereas permanent changes in A and A^* are regarded as real regime changes. For example, if the government of the foreign country conducts financial liberalization, θ^* rises permanently. Alternatively, if the foreign country experiences a long-lasting, negative real shock (e.g. war, pandemic, or large-scale natural disaster), A^* may fall permanently. We examine how such a permanent, unilateral parameter change affects the model's key variables in the long run. Considering the effect of changes in parameter values, we express the $\Lambda\left(m_t\right)$, $\Lambda^*\left(m_t\right)$, $d\left(m_t\right)$, and $d^*\left(m_t\right)$ functions as $\Lambda\left(m_t;\Gamma\right)$, $\Lambda^*(m_t;\Gamma)$, $d(m_t;\Gamma, A, \theta)$, and $d^*(m_t;\Gamma, A^*, \theta^*)$, respectively. These functions satisfy the following:

$$\frac{\partial \Lambda(.)}{\partial \Gamma} < 0,\quad \frac{\partial \Lambda^*(.)}{\partial \Gamma} > 0,\quad \frac{\partial d(.)}{\partial A} > 0,\quad \frac{\partial d^*(.)}{\partial A^*} > 0,\quad \frac{\partial d(.)}{\partial \theta} > 0,\quad \frac{\partial d^*(.)}{\partial \theta^*} > 0.$$

Fig. 1 illustrates the steady-state conditions of the world economy. From (B.6) in Appendix B, the steady-state level of m_t satisfies

$$\frac{\Lambda^*(m;\Gamma)}{\phi - 1} = \left(\frac{\rho^*}{\rho} - 1\right) + \left(\frac{\rho^*}{\rho}\right)\frac{\Lambda(m;\Gamma)}{\phi - 1}. \tag{37}$$

The left panel of Fig. 1 displays graphs of the left-hand side (LHS) and right-hand side (RHS) of (37) . As Appendix A shows, the graph of $\Lambda(m;\Gamma)$ starts from $\Lambda(0;\Gamma) = 1$ and it decreases first and then monotonically increases as m rises. Fig. 1 ignores the decreasing part of the graph of $\Lambda\left(m_t;\Gamma\right)$ to focus on the region where $\Lambda(.)$ monotonically increases.[21] On the other hand, the graph of LHS in (37) monotonically decreases with m. Consequently, there exists a unique steady-state value of m_t. Additionally, the right panel of Fig. 1 depicts the graphs of (29a) and (29b). Fig. 1 shows that once the steady-state level of m_t is determined, the steady-state levels of $\underline{z}_t$ and $\underline{z}^*_t$ are uniquely determined as well.

[21] Figs. 3 and 4 also ignore the decreasing part of $\Lambda\left(m_t;\Gamma\right)$ function.

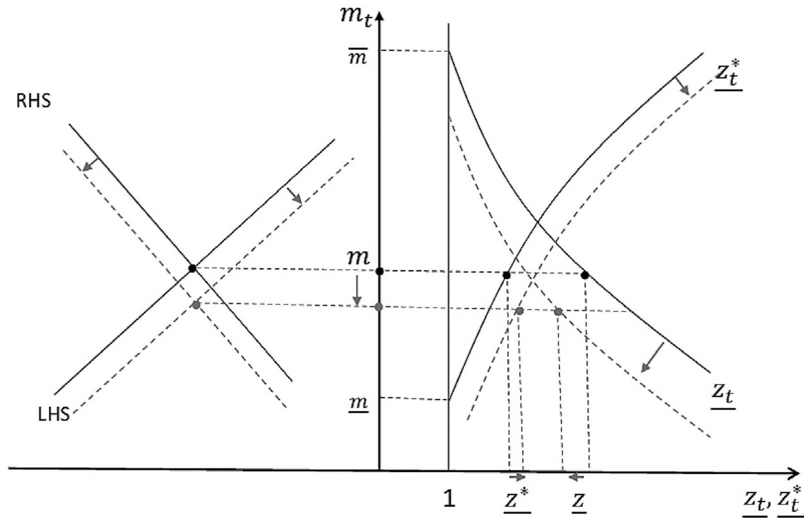

Fig. 3. Effects of a decrease in θ^* under $\rho > \rho$.

4.1.1. Financial regime change

Suppose the foreign government regulates the financial activities of domestic entrepreneurs, causing θ^* to decrease permanently. Because a decrease in θ^* reduces Γ, the graph of the LHS of (37) shifts downward, whereas that of the RHS shifts upward. First, assume that $\rho = \rho^*$. In this case, financial autarky always holds in the steady state, meaning that (37) becomes $\Lambda(m;\ \Gamma) = \Lambda^*(m;\Gamma) = 1$. As shown by Fig. 2, a decrease in θ^* (so a decrease in Γ) reduces the steady state value of m_t $\left(= X_t^*/X_t\right)$. In addition, in the case of $\rho = \rho^*$, the steady-state levels of cutoffs are given by $\underline{z} = \theta^{\frac{\alpha}{\phi}}$ and $\underline{z} = (\theta^*)^{\frac{\alpha}{\phi}}$. As a result, a fall in θ^* $\underline{z}^*$, while $\underline{z}$ remains unchanged. Therefore, a higher degree of financial constraints in the foreign country lowers the aggregate TFP in the foreign country alone. Thus, when $\rho = \rho^*$, a negative financial shock on the aggregate productivity in one country is absorbed by that country alone in the long-run equilibrium.

This outcome changes, if $\rho \neq \rho^*$. For example, suppose the foreign entrepreneurs are more impatient than the entrepreneurs in the home country, meaning that $\rho^* > \rho$. In this case, (36) leads to the following:

$$\frac{X^*}{K^*} = \Lambda^*(m;\Gamma) > 1 > \frac{K}{X} = \Lambda(m;\ \Gamma).$$

Moreover, the steady-state conditions, $\dot{X}_t = \dot{X}_t^* = 0$, determine the steady-state values of X_t and X_t^* as follows:

$$X = \left[\frac{d(m;\Gamma,A,\theta)}{\rho}\right]^{\frac{1}{1-\alpha}}, \quad X^* = \left[\frac{d^*(m;\Gamma,A^*,\theta^*)}{\rho}\right]^{\frac{1}{1-\alpha}},$$

Again, a fall in θ^* shifts the LHS's graph downward, whereas it shifts the RHS's graph upward, leading to a decrease in the steady-state value of m_t (see the left panel of Fig. 2). In addition, the right panel of Fig. 2 shows that from (29a) and (29b), the graph of $\underline{z}_t$ shifts downward and that of $\underline{z}_t^*$ shifts upward. The resulting steady-state cutoff in each country may be higher than its previous value. Intuitively, a permanent decrease in θ^* reduces investment in the foreign country, which lowers the relative asset holdings of the foreign country, X^*/X. However, a fall in X^*/X implies a rise in the relative asset position of the home country, which enhances the home country's TFP. Simultaneously, the decrease in Γ caused by stronger financial constraints in the foreign country diffuses into the home country, yielding a negative impact on the home country's TFP. As noted earlier, this diffusion effect is absent when $\rho = \rho^*$.

Next, consider a global regime change in which the governments of both countries regulate their financial markets simultaneously. As a result, both θ and θ^* decrease simultaneously. In this situation, both the graphs of the LHS and RHS shift downward. Therefore, the change in the steady-state value of m_t is relatively small. Moreover, as both $\Lambda(m;\Gamma)$ and $\Lambda^*(m;\Gamma)$ decrease, the leverage ratios in both countries, K/X and K^*/X^*, become smaller, which means that the capital imbalance, $K - X$ $(= X^* - K^*)$ shrinks. This occurs because heavier regulations of financial markets depress international capital flows, which reduces capital imbalances in the global economy.

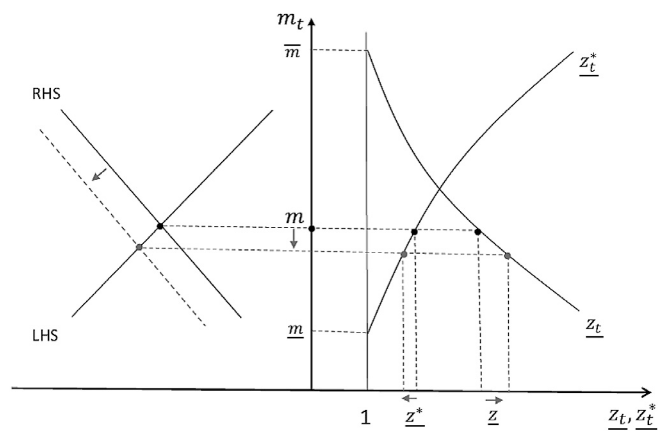

Fig. 4. Effects of a rise in ρ^*.

To summarize, we find the following:

Proposition 4. *Regardless of the sign of $\rho - \rho^*$, if the financial constraints become tighter in the foreign country, the steady-state level of wealth ratio, X^*/X, decreases. If there are simultaneous and proportional decreases in θ and θ^*, then the change in X^*/X is relatively small, but the TFPs in both countries fall.*

4.1.2. Real regime change

If a negative real regime change occurs in the foreign country, causing A^* and Γ to decrease, then its qualitative impacts on both countries are essentially the same as those of a decline in θ^*. This result also holds for simultaneous falls in A and A^*. However, the quantitative impacts of real regime changes are larger than those of financial regime changes. For example, compare the effects of simultaneous falls in θ and θ^* to those A and A^*. In the steady state, the TFPs of the aggregate production function of the home and foreign countries are, respectively, given by

$$\text{TFP} = A\left(\frac{\phi}{\phi-1}\right)^{\alpha}\theta^{\frac{\alpha}{\phi}}\left[\Lambda(m;\Gamma)\right]^{\alpha\left(1-\frac{1}{\phi}\right)}, \tag{38a}$$

Table 1
Baseline parameter values.

Parameters	Description	Home	Foreign
$1-\alpha$	income share of workers	2/3	2/3
ρ, ρ^*	the rate of time preference	0.01	0.011
A, A^*	productivity coefficient	1	1
θ, θ^*	the degree of financial constraints	1.5	4
ϕ	shape parameter of $F(z)$	1.6	1.6

$$\mathrm{TFP}^* = A^* \left(\frac{\phi}{\phi-1}\right)^{\alpha} \theta^{*\frac{\alpha}{\phi}} \left[\Lambda^*(m;\Gamma)\right]^{\alpha\left(1-\frac{1}{\phi}\right)}. \tag{38b}$$

As we have considered the case in which simultaneous negative regime changes do not affect m and Γ, simultaneous 10% decreases in A and A^* give rise to a 10% reduction in the TFPs of both countries. However, 10% decreases in θ and θ^* yield $(\alpha/\phi)\times 10\%$ falls in the TFPs of both countries. If $\alpha = 1/3$ and $\phi = 1.6$, the negative real impact on aggregate productions caused by the global real regime change is approximately three times as large.

Proposition 5. *Country-specific and global real regime changes qualitatively yield the same long-run effects as financial regime changes. However, the impact of real regime changes is greater in magnitude compared with financial regime change.*

4.1.3. Change in time discount rate

Next, consider the effects of a change in the time discount rate of entrepreneurs. Suppose the time discount rate of foreign entrepreneurs, ρ^*, rises permanently. The steady-state effects of this change are illustrated in Fig. 4. An increase in ρ^* causes the RHS graph in (37) to shift upward, which lowers the steady-state level of m_t. This means that a higher ρ^* increases foreign entrepreneurs' consumption, thereby lowering the relative asset holdings in the foreign country in the steady state. Note that the relationships between the efficiency cutoffs and m_t are independent of ρ and ρ^*, so the graphs of $\underline{z}_t$ and $\underline{z}_t^*$ will not shift. Consequently, the steady-state cutoff level in the foreign country decreases, while it increases in the home country. This occurs because the reduction in X^*/X tightens the borrowing constraints on foreign firms, leading less productive foreign firms to cease production. In the home country, the opposite outcome emerges. Thus, a negative preference shock in the foreign country increases the average productivity of active firms in the foreign country and decreases it in the home country.

Proposition 6. *If the time discount rate of foreign entrepreneurs rises, then (i) X^*/X decreases, (ii) the efficiency cutoff in the home country rises, and (iii) the efficiency cutoff in the foreign country falls.*

Recall that, from Proposition 3, if $\rho = \rho^*$, it holds that $K = X$ and $K^* = X^*$ (financial autarky) in the steady state. Now assume that $\rho = \rho^*$ initially holds, so that $\Lambda(m) = \Lambda^*(m) = 1$. Then, If ρ^* rises, from Proposition 6, $m (= X^*/X)$ falls, and thus it holds that $\Lambda(m) = K/X > 1 > \Lambda^*(m) = K^*/X^*$. In other words, the net asset position of the home country is positive, while that of the foreign country is negative. Therefore, if $\rho^* > \rho$, the home country is a creditor and the foreign country is a debtor. Conversely, if $\rho^* < \rho$, the home country is a debtor and the foreign country is a creditor.

4.2. Numerical examples

To quantitatively evaluate our analytical outcomes, we consider some numerical examples.

4.2.1. Distribution of wealth in the steady state

We specify the key parameter values in Table 1.

We set $\alpha = 1/3$, so the income share of labor, $1-\alpha$, takes a conventional value. We assume that foreign entrepreneurs are slightly more impatient than entrepreneurs in the home country. In specifying the magnitudes of θ and θ^*, we follow Moll (2014). As indicated by (4), when financial constraints bind, the debt–capital ratio of an individual firm is given by $\lambda = d_t/k_t$. If we use an aggregate debt–capital ratio as a proxy of λ, we obtain the following:

$$\lambda = 1 - \frac{1}{\theta} = \frac{D_t}{K_t} = \frac{D_t}{Y_t} \times \frac{Y_t}{K_t},$$

where D_t denotes the aggregate external debt of a country. Note that the conventional value of the income–capital ratio, Y_t/K_t, is 0.3. For example, the external debt-to-GDP ratio, D/Y, in India is about 0.5, which means that θ in India can be set as $\theta = 1/0.85 \simeq 1.2$. Similarly, D/Y in the United States is approximately 2.5, indicating that $\theta = 4.0$ in the United States. Considering these facts, if the economy is less developed, we set θ (or θ^*) = 1.5. If the economy is well developed, we set θ (or θ^*) = 4.0. Hence, in the baseline setting, we assume that the home country has less developed financial markets than the foreign country.

Regarding the shape parameter of the cumulative distribution function of the capital efficiency, z, we calibrate it using the long-run relationship between macroeconomic variables.[22] From (19) and (21c), the real interest rate satisfy $r = \frac{\phi-1}{\phi}\alpha\frac{Y}{K} = \frac{\phi-1}{\phi}\alpha\frac{Y^*}{K^*}$. Therefore, if we assume that $\alpha = 1/3$. the long-run real interest rate r is 0.02, and the output capital ratio $Y/K = 0.3$, then $\phi = 1.25$. If the real interest rate is $r = 0.04$, implying that $\phi = 1.666$. Here, we set $\phi = 1.6$ as a benchmark.[23] Note that for notational simplicity, we ignore capital depreciation. Hence, r is the gross rate of return to capital, which means that $r = 0.04$ is not implausibly high.

4.2.2. Financial and real regime changes

When conducting long-run comparative statics, we assume that the financial markets in the home country are less developed than those in the foreign country by setting $\theta = 1.5$ and $\theta^* = 4.0$. The other key parameters take their baseline values: $A = A^* = 1.0$ and $\phi = 1.6$.

(i) *Case 1*

Suppose there is a negative real regime change in the foreign country, causing A^* decrease by 10% from 1.0 to 0.9 permanently. Such a negative change can result from civil war, country-specific pandemics, natural disasters, and so on. As shown in Table 2, this negative real regime change in the foreign country lowers the steady-state values of the net worth and output by approximately 14%. Simultaneously, the long-run impacts of a fall in A^* on the net worth and output in the home country are small. As stated in Proposition 5, a lower A^* decreases the steady-state value of $m = X^*/X$. At the same time, a fall in A^* lowers Γ defined in (25). These changes decrease the value of $\Lambda^*(m)$ given in (27), which depresses the TFP in the foreign country given by (38b). In contrast, falls in m and Γ yield the opposite effects on the aggregate TFP in the home country; hence, a decrease in A^* yields minimal effects on X and Y. Similarly, if a negative real shock takes place in the home country, a 10% decrease in A can lead both X and Y to drop approximately 14% as well, whereas X^* and Y^* do not change

[22] An alternative way for calibrating ϕ is to use data on firm size distribution. Although the distribution of z indirectly affects the firm size distribution, it is not easy to calibrate the level of ϕ based on the micro data.

[23] Itskhoki and Moll (2019) set the shape parameter at 1.1. If we assume that $r = 0.015$, then ϕ is close to 1.1. However, using our notation, Liu and Wang (2014) assume that the production function of an individual firm is given by $y_t = zAk_t^{\alpha}n_t^{1-\alpha}$, where z follows a Pareto distribution with a shape parameter of 6.0. In their model. the rate of return on aggregate capital is written as

$$r_t = \left(\frac{\phi-1}{\phi}\right)^{\frac{1}{\alpha}} \alpha \frac{Y_t}{K_t}.$$

Thus, if we set $\phi = 6.0$, $\alpha = 1/3$ and $Y_y/K_y = 0.3$, we find that $r = 0.054$. Therefore, our calibrated value of ϕ is not substantially different from those used in previous studies.

Table 2
Quantitative effects of real and financial regime changes.

	X^*	X	Y^*	Y
Baseline: $A = A^* = 1,\ \theta = 1.5,\ \theta^* = 4$	409.64	368.50	13.86	10.71
Case 1 : $A = 1,\ A^* = 0.9,\ \theta = 1.5,\ \theta^* = 4$	349.042 (−14.79%)	367.59	11.84 (−14.61%)	10.71
Case 2: : $A = A^* = 1,\ \theta = 1.5,\ \theta^* = 3.6$	396.21 (−3.25%)	368.32	13.42 (−3.24%)	10.71
Case 3 : $A = A^* = 1,\ \theta = 4,\ \theta^* = 4$	408.03	498.27 (+35%)	13.87	14.54 (+36%)

much. (As we have assumed that $\rho^* > \rho$, changes in X^* and Y^* are slightly larger than the changes in X and Y in the case of a decrease in A^*.)

(ii) *Case 2*

Next, suppose there is a negative financial regime change in the foreign country, where θ^* falls by 10% from 4.0 to 3.6. This could happen if the foreign government increases regulations on domestic entrepreneurs' financial activities. In this case, the net worth and output in the foreign country decrease by approximately 3.2%, while the impacts on the net worth and output in the foreign country are negligible. A similar outcome occurs with a financial regime change in the home country. The intuition behind these outcomes is similar to that for real regime changes. As discussed previously, other things being equal, a real regime change gives rise to larger impacts on wealth and income than a financial regime change of the same scale. It is also important to note that in our examples, a local regime change in one country is almost fully absorbed by that country, and the international spillover effects on the other country are minimal, at least in the long run. In contrast, if a global negative regime change occurs, the long-run levels of wealth and income in both countries will simultaneously decrease. Moreover, global real regime changes yield substantially larger impacts than those caused by global financial regime changes.

(iii) *Case 3*

Consider a final scenario where substantial financial liberalization occurs in the home country, causing θ to increase from 1.5 to 4.0. This change aligns the home country's level of financial development with that of the foreign country. Such a substantial shift results in a 36% increase in the steady-state levels of net worth and output in the home country. However, this large increase in θ may yield relatively small effects on the net worth and output in the foreign country. In our model, the advantages of financial development in a less developed country primarily benefit the entrepreneurs and workers within that country.

Our numerical experiments demonstrate that changes in the technological efficiency and degree of financial constraints in one country yield minimal impact on the steady-state levels of wealth and income in the other country. In other words, the long-run impacts of real and financial regime changes are primarily absorbed by the country where changes occur, while their international spillovers are quantitatively negligible. The underlying reasoning for this result can be understood by considering the home country's TFP is given by $A\left(\frac{\phi}{\phi-1}\right)^{\alpha}\theta^{\frac{\alpha}{\phi}}\left[\Lambda(m;\Gamma)\right]^{\alpha\left(1-\frac{1}{\phi}\right)}$, which depends on the technological efficiency, A, the average productivity of firms, $\phi/(\phi-1)$, the degree of financial constraints, θ, and the aggregate leverage ratio represented by the function $\Lambda(m;\Gamma)$, which in turn depends on the steady-state relative wealth distribution, $m(= X^*/X)$, and a set of key parameters, Γ. For example, consider Case 2 where the degree of financial constraint in the foreign country, θ^*.decreases permanently. This change affects home country's TFP only via changes in $\Lambda(m;\gamma)$. From (25) , a decrease in θ^* lowers Γ, leading to a increase in $\Lambda(m;\Gamma)$ and a decrease in $\Lambda^*(m;\Gamma)$ under a given m. As shown by Fig. 2, these simultaneous sifts of the $\Lambda(m,\Gamma)$ and $\Lambda^*(m;\Gamma)$ functions decrease m, but the effect on international distribution of wealth are offset by the change in Γ. As a result, the aggregate leverage ratio in both countries, expressed by $\Lambda(.)$ and $\Lambda^*(.)$, does not undergo significant changes. Therefore, the impact of a decrease in θ^* is minimal on the home county's output, Y, but it reduces the foreign country's output, Y^*, through a direct negative effect caused by stronger financial constraints on the foreign entrepreneurs' investment.

4.2.3. Transition dynamics

We analyze two examples of the transition that takes place after permanent regime changes. The first example assumes that financial constraints on foreign firms become permanently stronger. We assume that the world economy initially stays in a steady state, and θ^*decreasing from 4.0 to 3.6. Fig. 5 illustrates the trajectories of X_t, X_t^*, m_t, Y_t, and Y_t^* after the financial regime change in the foreign country. As the graphs show, the net worth of the home country first declines before rising again, and it converges to the new steady-state level close to the initial level. On the other hand, the foreign country's net worth continues to fall. The wealth ratio, $m_t\left(= X_t^*/X_t\right)$, always decreases during the transition. As a result, while the home country's net worth slightly decrease in the new steady state, the foreign country's net worth declines permanently.

Fig. 5 also displays the trajectories of the aggregate incomes of both countries. As mentioned earlier, a decrease in θ^* leads to a reduction in Γ, which, under a given m_t, raises the TFP of the home country. This causes Y_t to rise initially. However, as m_t declines, Y_t eventually falls back to a level close to that of the previous steady state. In contrast, Y_t^* continues to decline due to the simultaneous decreases in Γ and m_t.

We next, consider the case of a real regime change. Again, the world economy initially stays in a steady state, and the common production efficiency of foreign firms, A^*, decreases from 1.0 to 0.9. As well as in the case of financial regime change, the net worth of the home country first declines and then increases, while the foreign country's net worth continues to fall. The behaviors of Y_t and Y_t^* are similar to those observed in the case of a financial regime change. A reduction in A^* leads to a decrease in Γ, which in turn raises the home country's TFP, causing an initial increase in Y_t. However, as m_t continues to decrease, Y_t eventually declines and stabilizes at a new steady-state value close to its original level. In contrast, due to reductions in both Γ and m_t, Y_t^* steadily decreases throughout the transition, ultimately reaching a lower value in the new steady-state equilibrium.

These numerical exercises demonstrate that, although a permanent regime change in one country generates relatively small spillover effects on another country, these effects can be substantial during the transition process toward the new steady-state equilibrium. Next, consider the case of a real regime change. Again, the world economy initially stays in a steady state, and the common production efficiency of foreign firms, A^*, decreases from 1.0 to 0.9. As well as in the case of financial regime change, the net worth of the home country first declines and then increases, while the foreign country's net worth continues to fall. The behaviors of Y_t and Y_t^* are similar to those observed in the case of a financial regime change. A reduction in A^* leads to a decrease in Γ, which in turn raises the home country's TFP, causing an initial increase in Y_t. However, as m_t continues to decrease, Y_t eventually declines and stabilizes at a new steady-state value close to its original level. In contrast, due to reductions in both Γ and m_t,

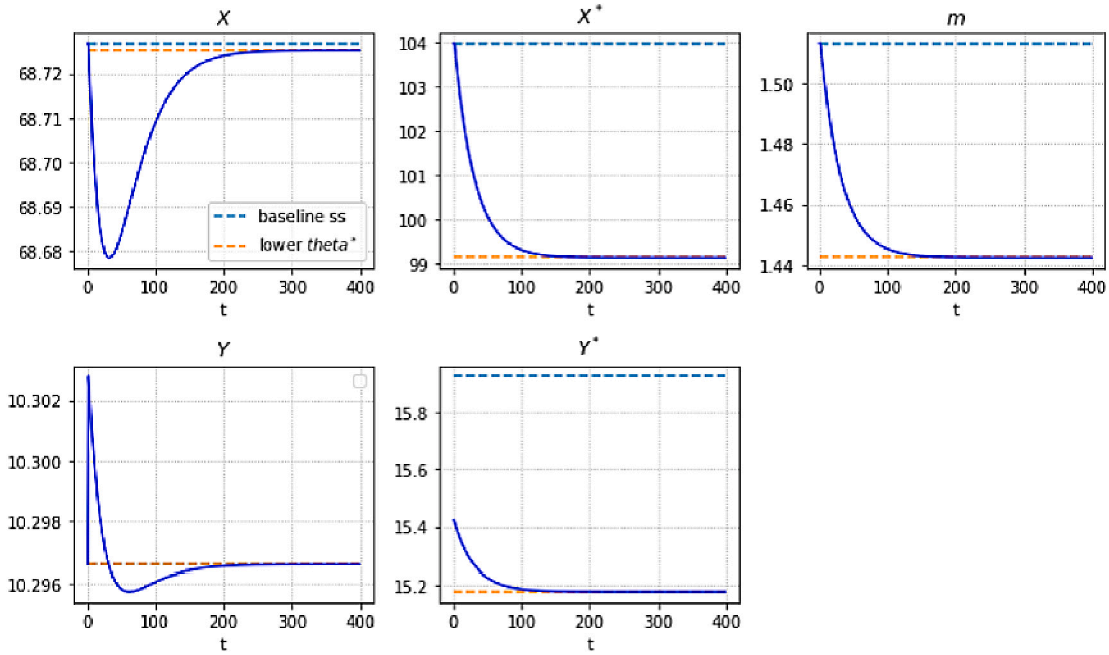

Fig. 5. Transition dynamics following a permanent decrease in θ^*.

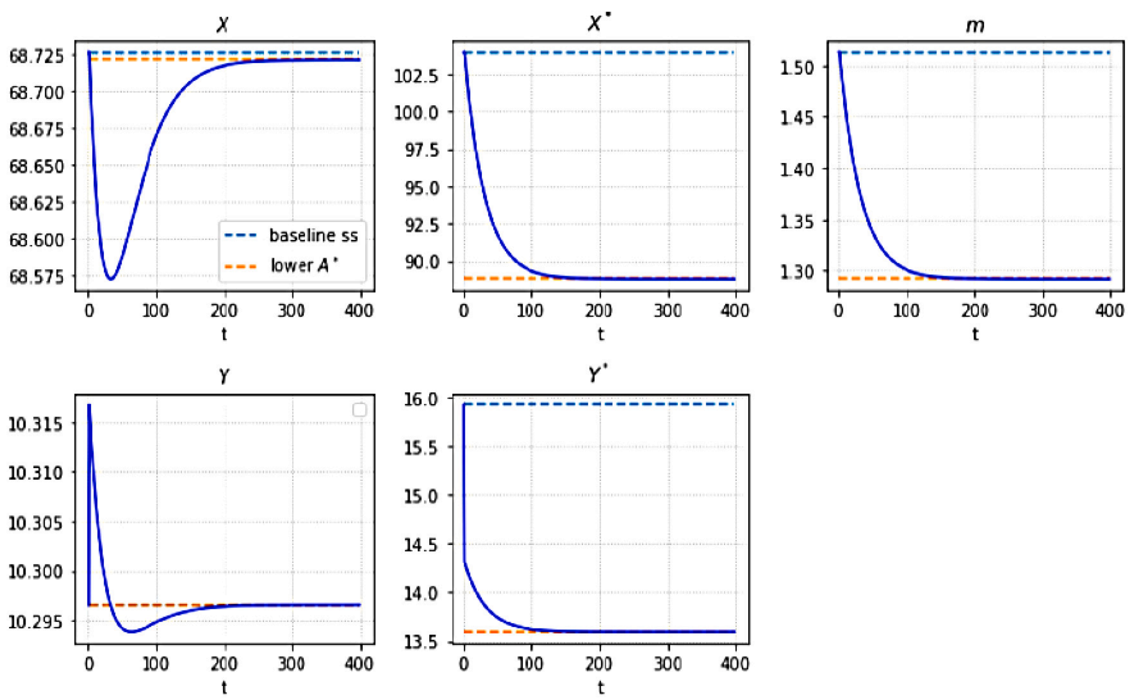

Fig. 6. Transition dynamics following a permanent decrease in A^*.

Y_t^* steadily decreases throughout the transition, ultimately reaching a lower value in the new steady-state equilibrium. (see Fig. 6).

These exercises demonstrate that, although a permanent regime change in one country generates relatively small spillover effects on another country, these effects can be substantial during the transition process toward the new steady-state equilibrium

These exercises demonstrate that, although a permanent regime change in one country generates relatively small spillover effects on another country in the long run, these effects can be substantial during the transition process toward the new steady-state equilibrium.

4.2.4. Stability under $\rho \neq \rho^*$

Given $\rho \neq \rho^*$, we numerically analyze the stability of the dynamic system defined by (31a) and (31b) under different parameter configurations. As expected, if the differences in the parameter values in both countries are relatively small such that the steady-state level of m_t is

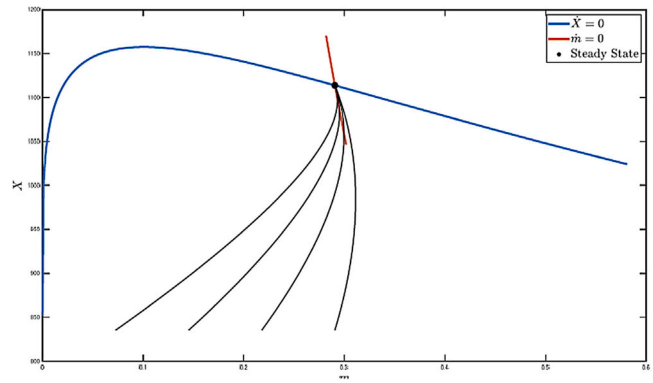

Fig. 7(a). Phase diagram under $\rho^* > \rho$.

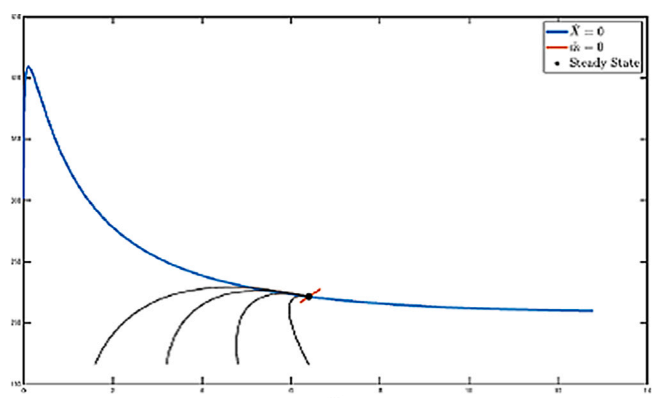

Fig. 7(b). Phase diagram under $\rho > \rho^*$.

close to 1.0, the dynamic system behaves similarly to the case where the entrepreneurs' time discount rates are identical in both countries.[24] Hence, we focus on examples with more pronounced asymmetries between the home and foreign countries, where the m substantially deviates from 1.0. 7 displays the phase diagrams of two such examples. In these diagrams, the vertical axis measures X_t and the horizontal axis measures m_t. The blue and red lines correspond to $\dot{X}_t = 0$ and $\dot{m}_t = 0$ loci, respectively, while the black lines depict converging paths toward the steady state from different initial positions. In Fig. 7(a), we set $\rho = 0.01$, $\rho^* = 0.02$, $A = 2.0$, $A^* = 2.5$, $\theta = 1.5$, and $\theta^* = 1.65$, resulting in $m \simeq 0.3$. In Fig. 7(b), we set $\rho = 0.02$ and $\rho^* = 0.01$, keeping other parameters identical to those in Panel (a). In this case, m is approximately 6.0. Both diagrams show that asymmetric steady states arising from the divergence between ρ and ρ^* exhibit global stability. Based on these numerical experiments, we can infer that our dynamic system maintains global stability across a wide range of parameter values.

5. Discussion

We have made restrictive assumptions for the sake of analytical tractability of our model. In this section, we examine how our main conclusions can be altered if we relax those key assumptions.

5.1. Persistency of idiosyncratic productivity shocks

A key assumption in our study is that idiosyncratic productivity shocks affecting each firm are i.i.d. over time. This assumption simplifies our analysis by ensuring that the distribution of wealth among entrepreneurs is independent of the distribution of productivity.[25] However, if productivity shocks are persistent, they would affect the evolution of entrepreneurs' wealth. In this case, it becomes impossible to separate the distribution of wealth among entrepreneurs from the distribution of productivity of firms. To explore the effects of persistent productivity shocks, Moll (2014), which presents the analytical basis for Itskhoki and Moll (2019), considers the case where productivity shocks follow a diffusion process given by

$$dz_t = (1/\xi)\mu\left(z_t\right)dt + \sqrt{1/\xi}\delta\left(z_t\right)dW_t, \tag{39}$$

where $\xi > 0$, and W_t follows a Weiner process. Here, ξ represents a degree of persistency of shocks, and we can verify that correlation between z_t and z_{t+s} (for $s > 0$) increases with ξ. As $\xi \to 0$, the correlation converges to zero, which corresponds to the case where shocks are i.i.d. over time. Given (39) , Moll (2014) investigates the distribution of the wealth share defied by

$$\omega(z,t) = \frac{1}{X_t}\int_0^\infty x g_t(x,z)\,dx,$$

where $\omega(z,t)$ represents the share of wealth held by the entrepreneurs with productivity z. The stationary distribution function $\omega(z)$, which is independent of time, satisfies the following Kolmogorov forward equation:

$$s(z)\omega(z) - \frac{d}{dz}\left[\mu(z)\omega(z)\right] + \frac{1}{2}\frac{d^2}{dz^2}\left[\sigma^2(z)\omega(z)\right] = 0, \tag{40}$$

where $s(z)$ denotes the savings of entrepreneurs with productivity z. Because it is not possible to obtain an analytical solution of (40) , Moll (2014) numerically approximates the $\omega(z)$ function satisfying (40) . His numerical experiments yield the following insights: If ξ increases (so persistency becomes stronger), then (i) the divergence between the distribution profiles of wealth and productivity grows, and when $\xi = 0$,the distribution profiles of wealth and productivity are identical[26]; (ii) the TFP losses caused by financial frictions decreases, leading to a higher steady-state level of aggregate output.; and (iii) the speed of convergence to the steady-state equilibrium slows down. These findings suggest that, compared to a more realistic model with persistent productivity shocks, our model with i.i.d. shocks may overstate the productivity losses arising from financial frictions and the convergence speed during the transition. Additionally, our model might not accurately capture the profile of the stationary wealth distribution among entrepreneurs.

Nonetheless, the analytical properties of Eq. (37) that determines steady-state wealth distribution between home and foreign countries remain unchanged under persistent productivity shocks. Therefore, as our primary concern is to examine the long-run patterns of wealth distribution and capital flows between these two countries, the i.i.d. assumption is not overly restrictive.

5.2. Preference structures of entrepreneurs

Proposition 3 claims that the long-run asset position of each country depends solely on the relative magnitudes of the time preference rates of entrepreneurs in both countries. Hence, if $\rho = \rho^*$, financial autarky holds in the steady state. This notable outcome stems from our joint assumption that entrepreneurs have logarithmic utility functions and

[24] For example, if $\phi = 1.1$, $\alpha = 1/3$, $A = A^* = 1$, $\theta = 1.5$, $\theta^* = 4.0$, $\rho = 0.001$, and $\rho^* = 0.011$, we obtain $m = 1.34$, and the phase diagram exhibits the global stability of the dynamic system.

[25] If productivity shocks are i.i.d. over time, shocks are unpredictable, so they do not affect the accumulation of wealth by entrepreneurs.

[26] Hence, in our model, the stationary distribution of wealth share is characterized by a Pareto distribution with a shape parameter ϕ.

workers do not save. First, suppose each entrepreneur has a CRRA-type instantaneous utility function,

$$u_{e,t} = \frac{c_{e,t}^{1-\sigma} - 1}{1-\sigma} \quad \sigma > 0,$$

where $u_{e,t} = \log c_{e,t}$ for $\sigma = 1$. In this case, the individual consumption function is given by

$$c_{e,t} = \left[\int_t^{\infty} \exp\left(-\left(1 - \frac{1}{\sigma}\right) \int_t^{v} r_\mu d\mu - \frac{\rho}{\sigma} \right) \right]^{-1} x_t.$$

As a result, unless $\sigma = 1$,the consumption-to-asset ratio is not constant during the transition process, so analysis of the transition dynamics becomes more complex than our initial discussion. However, in the steady-state equilibrium where r_t remains constant over time, the above equation is simplified as follows:

$$c_{e,t} = \left[\left(1 - \frac{1}{\sigma}\right) r + \frac{\rho}{\sigma} \right] x_t.$$

Thus, (36) is replaced by

$$\text{sign } (K - X) = -\text{ sign}\left(K^* - X^*\right) = \text{sign } \left\{ \left(\frac{1}{\sigma} - \frac{1}{\sigma^*} \right) r + \frac{\rho^*}{\sigma^*} - \frac{\rho}{\sigma} \right\}.$$

Consequently, if domestic and foreign entrepreneurs have different levels of elasticity of intertemporal substation in consumption, financial autarky will not hold even if $\rho = \rho^*$. Although households in different counties may have different shapes of utility functions, the steady-state asset position still depends on preference structures alone.

5.3. Workers' saving

Next, assume that entrepreneurs have logarithmic utility functions and that workers save. The representative worker in the home country solves the following problem:

$$\max \int_0^{\infty} e^{-\rho_w t} \frac{C_{w,t}^{1-\gamma} - 1}{1-\gamma} dt, \quad \rho_w > 0$$

subject to $\dot{S}_t = r_t S_t + w_t$, where ρ_w denotes the worker's time discount rate and S_t represents worker's asset holdings. For simplicity, we assume that the representative worker supplies one unit of labor in each moment. The representative worker in the foreign country faces a corresponding optimization problem. The assets held by workers in each country evolve according to

$$\dot{S}_t = r_t S_t + w_t - C_{w,t}, \quad \dot{S}_t^* = r_t S_t^* + w_t^* - C_{w,t}^*$$

. The equilibrium condition for the global capital market is given by

$$X_t + X_t^* + S_t + S_t^* = K_t + K_t^*.$$

The steady-state levels of aggregate assets satisfy the following:

$$\begin{aligned} \dot{X}_t &= rX_t + \frac{\alpha}{\psi} Y_t - \rho X_t = 0, \\ \dot{X}_t^* &= r_t X_t^* + \frac{\alpha}{\psi} Y_t^* - \rho X_t^* = 0, \\ \dot{S}_t &= r_t S_t + (1-\alpha) Y_t - C_{w,t} = 0, \\ \dot{S}_t^* &= r_t S_t^* + (1-\alpha) Y_t^* - \bar{c}^* N^* = 0. \end{aligned}$$

From these conditions, we can derive the following relationship:

$$\frac{X^* + S^*}{X + S} = \frac{\rho^* X^* + C_w^* - \left(\frac{\alpha}{\psi} + 1 - \alpha\right) Y^*}{\rho X + C_w - \left(\frac{\alpha}{\psi} + 1 - \alpha\right) Y\}}$$

This expression shows that even when $\rho = \rho^*$, the relative asset holdings, $(X^* + S^*)/(X + S)$, (and consequently the sign of $X + S - K$), depend on factors that influence the steady-state levels of workers' consumption and output in both countries.

The discussions in Section 5.2 and in this section emphasize that Proposition 3 depends heavily on our assumptions, which could lead to an overstatement of the role of time preference rate in determining long-run asset positions of both countries. Nonetheless, it is reasonable to claim that the patience of savers is a key determinant of imbalances in global capital markets.

5.4. Fiscal policy

Thus far, we have disregarded government behavior. Here, we briefly examine effects of fiscal policy. For simplicity, we assume that the government in each country fully consumes its tax revenue. In the presence of taxation, the flow budget constraint on entrepreneurs in each country is given by the following:

$$\begin{aligned} \dot{x}_t &= \left(1 - \tau_y\right)\left(r_t x_t + \pi_t\right) - \left(1 + \tau_c\right) \rho x_t + v_t, \\ \dot{x}_t^* &= \left(1 - \tau_k^*\right)\left(r_t x_t + \pi_t^*\right) - \left(1 + \tau_c^*\right) \rho^* x_t^* + v_t^*, \end{aligned}$$

where $\tau_y \in [0,1)$ and $\tau_y^* \in [0,1)$ denote the income tax rates in the home and foreign countries, and $\tau_c \in [0,1)$ and $\tau_c^* \in [0,1)$ express the consumption tax rates. Under this setting, the optimal consumption functions are given by

$$c_{e,t} = \frac{\rho x_t}{1 + \tau_c}, \quad c_{e,t}^* = \frac{\rho^* x_t^*}{1 + \tau_c^*}.$$

We can show that (37) is replaced by the following expression:

$$\frac{\Lambda^*\left(m; \hat{\Gamma}\right)}{\phi - 1} = \frac{\rho^*\left(1 + \tau_c\right)}{\rho\left(1 + \tau_c^*\right)} - 1 + \left[\frac{\rho^*\left(1 + \tau_c\right)}{\rho\left(1 + \tau_c^*\right)}\right] \frac{\Lambda\left(m; \hat{\Gamma}\right)}{\phi - 1},$$

where

$$\hat{\Gamma} \equiv \left[\frac{\left(1 - \tau_y^*\right) A^*}{\left(1 - \tau_y\right) A} \right]^{\frac{\phi}{(1-\alpha)\phi + \alpha}} \left(\frac{\theta^*}{\theta} \right)^{\frac{\alpha}{(1-\alpha)\phi + \alpha}}.$$

It is straightforward to verify that a rise in the income tax rate, $\tau_y \left(\text{or } \tau_y^*\right)$, has the same effects as a decrease in the technological efficiency, $A\,(\text{or } A^*)$. Similarly, a rise in the consumption tax rate, $\tau_c\,\left(\text{or } \tau_c^*\right)$, has the same effects as a fall in the time discount rate., $\rho\,(\text{or } \rho^*)$. Therefore, real regime changes may also arise from a permanent change in fiscal policy. Notice that even if entrepreneurs in both countries have the same time discount rate, financial autarky will not hold in the steady state if consumption tax rates differ between the two countries.

6. Conclusion

This study focuses on several country-specific differences including firms' production efficiency, the degree of financial constraints, savers' time discount rates, and government tax policies. Using a dynamic two-country model that incorporates financial frictions and firm heterogeneity, we examine how these international asymmetries shape each country's long-run asset position and the wealth distribution between the two countries. We conduct several comparative statics analyses in the steady state. Although the high tractability of our model allows us to obtain the main outcomes analytically, we also provide numerical examples to quantitatively assess the comparative statics outcomes. Regarding policy implications, we limit our analysis to the long-run effects of tax policies. However, our framework suggests promising avenues for future research, particularly in exploring broader policy issues such as optimal taxation and the regulation of international capital flows.

Declaration of competing interest

The authors declare no conflicts of interest associated with this manuscript.

Appendix A. Properties of the $\Lambda\left(X_t^* X_t\right)$ and $\Lambda^*\left(X_t^*/X_t\right)$ functions

Letting $X_t^*/X_t = m_t$, $\Lambda(.)$ and $\Lambda^*(.)$ functions are given by

$$\Lambda\left(m_t;\Gamma\right) = \frac{1}{1+\Gamma m_t{}^{\psi}}\left(1+m_t\right), \quad \Lambda^*\left(m_t;\Gamma\right) = \frac{\Gamma m_t^{\psi-1}}{1+\Gamma m_t{}^{\psi}}\left(1+m_t\right).$$

The above functions give

$$\begin{aligned}\frac{\partial}{\partial m}\Lambda(m;\Gamma) &= \frac{1+\Gamma m^{\psi-1}(1-\psi)\left(m-\frac{\psi}{1-\psi}\right)}{\left(1+\Gamma m_t{}^{\psi}\right)^2}\\ &= \frac{1+(1-\psi)\Gamma\left[m^{\psi}-\left(\frac{\psi}{1-\psi}\right)\frac{1}{m^{1-\psi}}\right]}{(1+\Gamma m^{\psi})^2},\end{aligned} \tag{A.1}$$

$$\begin{aligned}\frac{\partial}{\partial m}\Lambda^*(m;\Gamma) &= \frac{\Gamma m^{\psi-2}\left[m_t\left(\psi-\Gamma m^{\psi-1}\right)-(1-\psi)\right]}{(1+\Gamma m^{\psi})^2}\\ &= \frac{\Gamma m_t^{\psi-2}\left[(\psi m-\Gamma m^{\psi})-(1-\psi)\right]}{(1+\Gamma m^{\psi})^2}.\end{aligned} \tag{A.2}$$

Given Γ, it holds that $\Lambda(0;\Gamma) = 1$. As depicted by Fig. 2, we see that function $\Lambda(m;\Gamma)$ first decreases and then monotonically increases after m exceeds $\psi/(1+\psi)$. On the other hand, the expression of $\Lambda^*\left(m_t\right)$ function means that unless m_t takes an sufficiently large value to satisfy $\psi < \Gamma m_t^{\psi-1}$, it holds that $\Lambda^{*\prime}\left(m_t\right) < 0$.

Appendix B. Proof of Proposition 2

Combining (32a) and (32b), we obtain

$$\frac{\rho^*}{\rho}m^{1-\alpha} = \frac{d^*(m)}{d(m)}.$$

From the definitions of $d\left(m_t\right)$ and $d^*\left(m_t\right)$ functions given in (31a) and (31b), the above equation is expressed as

$$\frac{\rho^*}{\rho}m^{1-\alpha} = \left(\frac{A^*}{A}\right)\left(\frac{\theta^*}{\theta}\right)^{\frac{\alpha}{\phi}}\left[\frac{\Lambda^*(m)}{\Lambda(m)}\right]^{\alpha-\frac{\alpha}{\phi}-1}\left[\frac{\Lambda^*(m)+(\phi-1)}{\Lambda(m)+(\phi-1)}\right]. \tag{B.3}$$

Note that the steady-state expressions of $\Lambda\left(m_t\right)$ and $\Lambda^*\left(m_t\right)$ are respectively given by

$$\Lambda(m) \equiv \frac{1}{1+\Gamma m^{\psi}}(1+m), \quad \Lambda^*(m) = \frac{\Gamma m^{\psi-1}}{1+\Gamma m^{\psi}}(1+m). \tag{B.4}$$

Hence, (B.3) can be written as

$$\frac{\rho^*}{\rho}m^{1-\alpha} = \left(\frac{A^*}{A}\right)\left(\frac{\theta^*}{\theta}\right)^{\frac{\alpha}{\phi}}\left(\Gamma m^{\psi-1}\right)^{\alpha-\frac{\alpha}{\phi}-1}\left[\frac{\Lambda^*(m)+(\phi-1)}{\Lambda(m)+(\phi-1)}\right], \tag{B.5}$$

Because $\psi = \frac{\alpha}{(1-\alpha)\phi+\alpha}$, it holds that $(\psi-1)\left(\alpha-\frac{\alpha}{\phi}-1\right) = 1-\alpha$,.meaning that m satisfies

$$\frac{\rho^*}{\rho} = \left(\frac{A^*}{A}\right)\left(\frac{\theta^*}{\theta}\right)^{\frac{\alpha}{\phi}}\Gamma^{\alpha-\frac{\alpha}{\phi}-1}\left[\frac{\frac{\Gamma m^{\psi-1}}{1+\Gamma m^{\psi}}(1+m)+(\phi-1)}{\frac{1}{1+\Gamma m^{\psi}}(1+m)+(\phi-1)}\right].$$

Moreover, it holds that

$$\left(\frac{A^*}{A}\right)\left(\frac{\theta^*}{\theta}\right)^{\frac{\alpha}{\phi}}\Gamma^{\alpha-\frac{\alpha}{\phi}-1} = 1,$$

meaning that (35) is reduced to

$$\frac{\Gamma m^{\psi-1}(1+m)}{(1+\Gamma m^{\psi})(\phi-1)}+1 = \left(\frac{\rho^*}{\rho}\right)\left[\frac{1+m}{(\phi-1)(1+\Gamma m^{\psi})}+1\right].$$

We can express the above equation as

$$\frac{\Lambda^*(m)}{\phi-1} = \left(\frac{\rho^*}{\rho}-1\right)+\left(\frac{\rho^*}{\rho}\right)\frac{\Lambda(m)}{\phi-1}. \tag{B.6}$$

As Fig. 3 in the main text shows, $\Lambda(m) \geq 1$ for all $m \leq \bar{m}$ and $\Lambda^*(m) \geq 1$ for all $m \geq \underline{m}$. Appendix A states that unless m takes a sufficiently large value, the left-hand side in (B.6) monotonically decreases with m. On the other hand, unless m is sufficiently small, the right-hand side of (B.6) monotonically increases with m. Hence, it is easy to see that (B.6) has a unique positive solution; see the left panel in Fig. 3 in the main text.[27] ■

Appendix C. Proof of Proposition 3

We inspect the local stability of the symmetric steady state with $\rho = \rho^*$. Using one of the steady-state conditions, $X^{\alpha}d(m) = \rho X$, we find that the coefficient matrix of the dynamic system linearized at the steady state is given by

$$J = \begin{bmatrix} -(1-\alpha)\rho & X^{\alpha}d'(m) \\ m\left[(1-\alpha)(\rho-\rho^*)\frac{1}{X}\right] & X^{\alpha-1}m\left[(\alpha-1)m^{\alpha-2}d^*(m)+m^{\alpha-1}d^{*\prime}(m)-d'(m)\right]\end{bmatrix}.$$

In the above, $d'(m)$ and $d^{*\prime}(m)$ are respectively given by

$$\begin{aligned} d'(m) &= \alpha A\left(\frac{\phi}{\phi-1}\right)^{\alpha-1}\theta^{\frac{\alpha}{\phi}}\left[\Lambda(m)\right]^{\alpha-\frac{\alpha}{\phi}-2}\\ &\quad\times\Lambda'(m)\left[\frac{\alpha}{\phi-1}\left(1-\frac{1}{\phi}\right)\Lambda(m)-\left(1+\frac{\alpha}{\phi}-\alpha\right)\right],\\ d^{*\prime}(m) &= \alpha A^*\left(\frac{\phi}{\phi-1}\right)^{\alpha-1}\theta^{*\frac{\alpha}{\phi}}\left[\Lambda^*(m)\right]^{\alpha-\frac{\alpha}{\phi}-2}\\ &\quad\times\Lambda^{*\prime}(m)\left[\frac{\alpha}{\phi}\Lambda^*(m)-\left(1+\frac{\alpha}{\phi}-\alpha\right)\right].\end{aligned}$$

From Corollary 1, if $\rho = \rho^*$, it holds that $\Gamma m^{\psi-1} = 1$ and $\Lambda(m) = \Lambda^*(m) = 1$. Hence, we see the following;

$$\begin{aligned}\Lambda'(m) &= \frac{1+(1-\psi)\left(m-\frac{\psi}{1-\psi}\right)}{(1+\Gamma m^{\psi})^2}\\ &= \frac{(1-\psi)(1+m)}{(1+m)^2} = \frac{1-\psi}{1+m} > 0,\\ \Lambda^{*\prime}\left(m_t\right) &= \frac{\Gamma m_t^{\psi-1}\left[m(\psi-1)-(1-\psi)\right]}{m(1+\Gamma m^{\psi})^2} = \frac{\psi-1}{m(1+m)} < 0.\end{aligned}$$

$$\begin{aligned} d(m_t) &= \alpha A\left(\frac{\phi}{\phi-1}\right)^{\alpha-1}\theta^{\frac{\alpha}{\phi}}\left[1+\frac{1}{\phi-1}\right],\\ d^*(m_t) &= \alpha A^*\left(\frac{\phi}{\phi-1}\right)^{\alpha-1}\theta^{*\frac{\alpha}{\phi}}\left[1+\frac{1}{\phi-1}\right],\end{aligned}$$

$$\begin{aligned} d'(m) &= \alpha A\left(\frac{\phi}{\phi-1}\right)^{\alpha-1}\theta^{\frac{\alpha}{\phi}}\left[\Lambda(m)\right]^{\alpha-\frac{\alpha}{\phi}-2}\\ &\quad\times\Lambda'(m)\left[\frac{\alpha}{\phi-1}\left(1-\frac{1}{\phi}\right)\Lambda(m)-\left(1+\frac{\alpha}{\phi}-\alpha\right)\right]\\ &= \alpha A\left(\frac{\phi}{\phi-1}\right)^{\alpha-1}\theta^{\frac{\alpha}{\phi}}\left(\frac{1-\psi}{1+m}\right)\left[\frac{\alpha}{\phi}-\left(1+\frac{\alpha}{\phi}-\alpha\right)\right]\\ &= \alpha A\left(\frac{\phi}{\phi-1}\right)^{\alpha-1}\theta^{\frac{\alpha}{\phi}}\left(\frac{1-\psi}{m(1+m)}\right)(\alpha-1) < 0,\end{aligned}$$

$$\begin{aligned} d^{*\prime}(m) &= \alpha A^*\left(\frac{\phi}{\phi-1}\right)^{\alpha-1}\theta^{*\frac{\alpha}{\phi}}\left[\Lambda^*(m)\right]^{\alpha-\frac{\alpha}{\phi}-2}\\ &\quad\times\Lambda^{*\prime}(m)\left[\frac{\alpha}{\phi}\Lambda^*(m)-\left(1+\frac{\alpha}{\phi}-\alpha\right)\right]\\ &= \alpha A^*\left(\frac{\phi}{\phi-1}\right)^{\alpha-1}\theta^{*\frac{\alpha}{\phi}}\left(\frac{\psi-1}{m(1+m)}\right)(\alpha-1) > 0.\end{aligned}$$

Therefore, we find that

$$(m^{-1}X^{1-\alpha})\frac{d\dot{m}_t}{dm_t}\Big|_{m_t=m,} = (\alpha-1)m^{\alpha-2}d^*(m)+m^{\alpha-1}d^{*\prime}(m)-d'(m)$$

[27] Obviously, if $\underline{m} > \bar{m}$, a feasible steady-state equilibrium cannot exist. Therefore, we make the assumption that the values of the parameters involved in the model satisfy the conditions necessary for $\bar{m} > \underline{m}$ to hold.

$$
\begin{aligned}
&= m^{2-\alpha}(\alpha-1)\alpha A^{*}\left(\frac{\phi}{\phi-1}\right)^{\alpha-1}\theta^{*\frac{\alpha}{\phi}}\left[\frac{\phi}{\phi-1}\right]\\
&+ m^{\alpha-1}\alpha A^{*}\left(\frac{\phi}{\phi-1}\right)^{\alpha-1}\theta^{*\frac{\alpha}{\phi}}\left(\frac{\psi-1}{m(1+m)}\right)(\alpha-1)\\
&- \alpha A\left(\frac{\phi}{\phi-1}\right)^{\alpha-1}\theta^{\frac{\alpha}{\phi}}\left(\frac{1-\psi}{1+m}\right)(\alpha-1)
\end{aligned}
$$

From (34) , it holds that $m^{\alpha-1} = \left(\frac{A}{A^*}\right)\left(\frac{\theta}{\theta^*}\right)^{\frac{\alpha}{\phi}}$ and $m^{\alpha-2} = \frac{1}{m}\left(\frac{A}{A^*}\right)\left(\frac{\theta}{\theta^*}\right)^{\frac{\alpha}{\phi}}$. Substituting those relations into the above, we find:

$$
\begin{aligned}
(m^{-1}X^{1-\alpha})\frac{d\dot{m}_t}{dm_t}\Big|_{m_t=m,} &= \frac{1}{m}(\alpha-1)\alpha A\left(\frac{\phi}{\phi-1}\right)^{\alpha-1}\theta^{\frac{\alpha}{\phi}}\left[\frac{\phi}{\phi-1}\right]\\
&+ \alpha A\left(\frac{\phi}{\phi-1}\right)^{\alpha-1}\theta^{\frac{\alpha}{\phi}}\left(\frac{\psi-1}{m(1+m)}\right)(\alpha-1)\\
&- \alpha A\left(\frac{\phi}{\phi-1}\right)^{\alpha-1}\theta^{\frac{\alpha}{\phi}}\left(\frac{1-\psi}{1+m}\right)(\alpha-1)\\
&= (\alpha-1)\alpha A\left(\frac{\phi}{\phi-1}\right)^{\alpha-1}\theta^{\frac{\alpha}{\phi}}\left[\frac{\phi-(1-\psi)(\phi-1)}{m(\phi-1)}\right] < 0.
\end{aligned}
$$

Under $\rho = \rho^*$, the eigenvalues of J are $-(1-\alpha)\rho$ and $X^{\alpha-1}m\left[(\alpha-1)m^{\alpha-2}d^*(m) + m^{\alpha-1}d^{*\prime}(m) - d'(m)\right]$. Because both eigenvalues are negative, the stationary point satisfies local stability. ■

Data availability

The original data is available at: https://data.mendeley.com/datasets/f2gdxjhch4/1.

References

Aizenman, J., Chinn, M.D., Ito, H., 2010. The emerging global financial architecture: tracing and evaluating new patterns of the trilemma configuration. J. Int. Money Financ. 29, 615–641. http://dx.doi.org/10.1016/j.jimonfin.2010.01.005.

Antras, P., Caballero, R.J., 2009. Trade and capital flows: a financial frictions perspective. J. Polit. Econ 117, 701–744. http://dx.doi.org/10.1086/605583.

Beyene, S.D., Kotosz, B., 2021. The impact of external debt on total factor productivity and growth in HIPCs: non-linear regression approaches. Int. J. Dev. Issues 21, 173–194. http://dx.doi.org/10.1108/ijdi-07-2021-0145.

Brooks, W., Dovis, A., 2020. Credit market frictions and trade liberalizations. J. Monet. Econ 111, 32–47. http://dx.doi.org/10.1016/j.jmoneco.2019.01.013.

Buera, F.J., Kaboski, J.P., Shin, Y., 2011. Finance and development: a tale of two sectors. Am. Econ. Rev 101, 1964–2002. http://dx.doi.org/10.1257/aer.101.5.1964.

Buera, F.J., Moll, B., 2015. Aggregate implications of a credit crunch: the importance of heterogeneity. Am. Econ. J. Macroecon 7, 1–42. http://dx.doi.org/10.1257/mac.20130212.

Buera, F.J., Shin, Y., 2017. Productivity growth and capital flows: The dynamics of reforms. Am. Econ. J. Macroecon 9, 147–185. http://dx.doi.org/10.1257/mac.20160307.

Buiter, W.H., 1981. Time preference and international lending and borrowing in an overlapping-generations model. J. Polit. Econ 89, 769–797. http://dx.doi.org/10.1086/261002.

Chen, M., Guariglia, A., 2013. Internal financial constraints and firm productivity in China: Do liquidity and export behavior make a difference? J. Comp. Econ 41, 1123–1140. http://dx.doi.org/10.1016/j.jce.2013.05.003.

Chor, D., Manova, K., 2012. Off the cliff and back?: credit conditions and international trade during the global financial crisis. J. Int. Econ 87, 117–133. http://dx.doi.org/10.1016/j.jinteco.2011.04.001.

Coeurdacier, N., Guibaud, S., Jin, K., 2015. Credit constraints and growth in a global economy. Am. Econ. Rev 105, 2838–2881. http://dx.doi.org/10.1257/aer.20130549.

Coricelli, F., Driffield, N., Pal, S., Roland, I., 2012. When does leverage hurt productivity growth? A firm-level analysis. J. Int. Money Financ. 31, 1674–1694. http://dx.doi.org/10.1016/j.jimonfin.2012.03.006.

Dedola, L., Lombardo, G., 2012. Financial frictions, financial integration and the international propagation of shocks. Econ. Policy. 27, 319–359. http://dx.doi.org/10.1111/j.1468-0327.2012.00286.x.

Devereux, M., Yetman, J., 2010. Leverage constraints and the international transmission of shocks. J. Money Credit. Bank 41, 71–105.

Drechsel, T., 2023. Earnings-based borrowing constraints and macroeconomic fluctuations. Am. Econ. J. Macroecon 15, 1–34. http://dx.doi.org/10.1257/mac.20210099.

Faia, E., 2007. Finance and international business cycles. J. Monet. Econ 54, 1018–1034. http://dx.doi.org/10.1016/j.jmoneco.2006.04.003.

Fazzari, S., Hubbard, G.R., Petersen, C.B., Blinder, S.A., Poterba, J.M., 1988. Financing constraints and corporate investment. Brookings Pap. Eco. Ac. 1988 (141), http://dx.doi.org/10.2307/2534426.

Frenkel, J., Razin, A., Sadka, E., 1991. International Taxation in an Integrated World. MIT Press, Cambridge, MA.

Furusawa, T., Yanagawa, N., 2013. International trade and capital movement under financial imperfection. Unpublished manuscript.

Gertler, M., Rogoff, K., 1990. North-south lending and endogenous domestic capital market inefficiencies. 3932 (90), 90022. http://dx.doi.org/10.1016/0304–v.

Ghironi, F., 2018. Macro needs micro. Oxf. Rev. Econ. Policy 34, 195–218.

Goméz, E., Neto, D.-G., 2016. Financial globalization with firm heterogeneity. Unpublished manuscript.

Gordon, R., 1986. Taxation of investment and savings in a world economy: the certainty case. Am. Econ. Rev 76, 1087–1102. http://dx.doi.org/10.3386/w1723.

Gourinchas, P.-O., Obstfeld, M., 2012. Stories of the twentieth century for the twenty-first. Am. Econ. Rev 91, 1265–1284. http://dx.doi.org/10.3386/w17252.

Greenwood, J., Sanchez, J.M., Wang, C., 2013. Quantifying the impact of financial development on economic development. Rev. Econ. Dynam 16, 194–215. http://dx.doi.org/10.1016/j.red.2012.07.003.

Hennessy, C., Whited, T., 2007. How costly is external financing?: evidence from a structural estimation. J. Financ. 62, 1705–1745.

Hu, Y., Mino, K., 2013. Trade structure and belief-driven fluctuations in a global economy. J. Int. Econ 90, 414–424. http://dx.doi.org/10.1016/j.jinteco.2012.12.003.

Hubbard, R.G., Kashyap, A.K., Whited, T.M., 1995. Internal finance and firm investment. J. Money Credit. Bank 27 (683), http://dx.doi.org/10.2307/2077743.

Itskhoki, O., Moll, B., 2019. Optimal development policies with financial frictions. Econometrica 87, 139–173. http://dx.doi.org/10.3982/ecta13761.

Jin, K., 2010. International trade and international capital flows: a theoretical perspective. In: The Encyclopedia of Financial Globalization.

Kiyotaki, N., Moore, J., 1997. Credit cycles. J. Polit. Econ 105, 211–248. http://dx.doi.org/10.1086/262072.

Korinek, A., Mendoza, E.G., 2014. From sudden stops to fisherian deflation: quantitative theory and policy. Annu. Rev. Econ 6, 299–332. http://dx.doi.org/10.1146/annurev-economics-080213-041005.

Li, H., 2022. Leverage and productivity. J. Dev. Econ 154, 102752. http://dx.doi.org/10.1016/j.jdeveco.2021.102752.

Lian, C., Ma, Y., 2021. Anatomy of corporate borrowing constraints. Q. J. Econ 136, 229–291. http://dx.doi.org/10.1093/qje/qjaa030.

Liu, Z., Wang, P., 2014. Credit constraints and self-fulfilling business cycles. Am. Econ. J. Macroecon 6, 32–69. http://dx.doi.org/10.1257/mac.6.1.32.

Lucas, R., 1990. Why doesn't capital flow from rich to poor countries? Am. Econ. Rev 80, 92–96. http://dx.doi.org/10.4324/9781912281152.

Matsuyama, K., 2005. Credit market imperfections and patterns of international trade and capital flows. J. Eur. Econ. Assoc 3, 714–723. http://dx.doi.org/10.1162/jeea.2005.3.2-3.714.

Matsuyama, K., 2014. Institution-induced productivity differences and patterns of international capital flows. J. Eur. Econ. Assoc 12, 1–24. http://dx.doi.org/10.1111/jeea.12045.

Mendoza, E.G., 2010. Sudden stops. Financ. Crises, Leverage. Am. Econ. Rev 100, 1941–1966. http://dx.doi.org/10.1257/aer.100.5.1941.

Mendoza, E.G., Quadrini, V., Rios-Rull, J-V., 2009. Financial integration. Financ. Dev. Glob. Imbalances. J. Polit. Econ 117, 371–416. http://dx.doi.org/10.1086/599706.

Midrigan, V., Xu, D.Y., 2014. Finance and misallocation: evidence from plant-level data. Am. Econ. Rev 104, 422–458. http://dx.doi.org/10.1257/aer.104.2.422.

Moll, B., 2014. Productivity losses from financial frictions: can self-financing undo capital misallocation? Am. Econ. Rev 104, 3186–3221. http://dx.doi.org/10.1257/aer.104.10.3186.

Ono, Y., Shibata, A., 1992. Spill-over effects of supply-side changes in a two-country economy with capital accumulation. 1996 (92), 90053. http://dx.doi.org/10.1016/0022–m.

Pattillo, C., Poirson, H., Ricci, L., Kraay, A., Rigobon, R., 2004. Through what channels does external debt affect growth? Brookings trade forum. 22, 9–258. http://dx.doi.org/10.1353/btf.2004.0013.

Pintus, P.A., Wen, Y., Xing, X., 2019. International credit markets and global business cycles. Int. J. Econ. Theory 15, 53–75. http://dx.doi.org/10.1111/ijet.12206.

S., Schmitt-Grohé., Uribe, M., 2021. Multiple equilibria in open economies with collateral constraints. Rev. Econ. Stud. 88, 969–1001. http://dx.doi.org/10.1093/restud/rdaa023.

Turnovsky, S., 1997. International Macroeconomic Theory. MIT Press.

Wang, P., Xu, Z., Wen, Y., 2017. Two-way capital flows and global imbalaces. Econ J. 127, 2229–2269. http://dx.doi.org/10.1111/ecoj.12290.

Yao, W., 2019. International business cycles and financial frictions. J. Int. Econ 118, 283–291. http://dx.doi.org/10.1016/j.jinteco.2019.03.002.