Supplement to “Technical Change, Wage Inequality, and Optimal Taxes in An Assignment Model”

BEEN-LON CHEN
Academia Sinica

FEI-CHI LIANG
National Chengchi University

APPENDIX B (ONLINE)

B.1 *Optimal Policy and Constrained Efficient Allocation in the Two-talent-two-task model*

Social planning problem ($\mathbf{P_2}$). Suppose that the social planner attaches Pareto weight g_i to agents of talent $i \in \{L, H\}$, with $\mathrm{g}_L + \mathrm{g}_H = 1$, and the government's spending is G_t. For $i \in \{H, L\}$, the agent's lifetime utility is $U(i) = \sum_{t=1}^{T} \beta^{t-1} \left[u\left(c_t(i)\right) - h\left(e_t(i)\right)\right]$, and the social planner maximizes the social welfare given by

$$W = \mathrm{g}_L U\left(L\right) + \mathrm{g}_H U(H),$$

subject to resource constraints in period $t = 1, 2, \ldots, T$,

$$(\chi_t) : Y_t \geq G_t + \sum_{i=L,H} c_t\left(i\right) f\left(i\right) + \sum_{v=\underline{v},\bar{v}} \left[k_{t+1}\left(v\right) - \left(1 - \delta_k\right) k_t\left(v\right) + \mathcal{C}_q\left(q_t\left(v\right)\right) + \mathcal{C}_n\left(n_t\left(v\right)\right)\right],$$

and the incentive compatible constraint (hereafter, ICC),

$$(\eta) : U(H) \geq \sum_{t=1}^{T} \beta^{t-1} \left[u\left(c_t(L)\right) - h\left(\frac{w_t(L)e_t(L)}{w_t(H)}\right)\right],$$

where χ_t and η are the shadow price of the resource constraints in period $t = 1, 2, \ldots, T$ and the shadow price of the ICC, respectively. [1]

Solving the above planning problem and referring to the definition of wedges in (4a)-(4e), the following Proposition A1 shows the four wedges in this simple two-talent-two-task model. We let $\Delta_t \equiv h'\left(\frac{w_t(L)e_t(L)}{w_t(H)}\right)\left(\frac{w_t(L)e_t(L)}{w_t(H)}\right)$ and $\Psi_t(i) \equiv \frac{u'(c_t(i))}{f(i)h'(e_t(i))}$, and the indicator function $\mathbf{1}_i^L$ be one if $i = L$, and zero if $i = H$. We are ready to characterize

---

[1] A well-known implication of the Spence-Mirrlees single crossing condition is that non-local incentive constraints do not bind. See Milgrom and Shannon (1994). Moreover, agents are usually assumed to have incentives to underreport their types in the standard Mirrlees model. Hence, in the two-type model here, we focus on binding local downward incentive constraints and thus, only impose an incentive constraint to prevent high types from reporting as low types.

the wedges.

PROPOSITION A1. Optimal allocations in the two-type, two-task model are characterized as follows:

(i) **Capital wedge** for task $v \in \{\overline{v},\underline{v}\}$ is

$$\frac{\tau_i^{k_t}(v)}{1-\tau_i^{k_t}(v)} = \mathcal{W}_t^k\left(v\right)\kappa_v^t + \mathcal{M}_t^k\left[R_t(v) - \frac{\partial Y_t}{\partial k_t(v)}\right] \text{ for any } i \in \{L,H\} \tag{A21}$$

where $\mathcal{W}_t^k\left(v\right) \equiv \frac{\beta^{t-1}\eta\Delta_t}{\chi_{t-1}k_t(v)} > 0$ and $\mathcal{M}_t^k = \frac{\chi_t}{\chi_{t-1}} > 0$.

(ii)**Labor wedge** for type $i \in \{H,\ L\}$ is

$$\frac{\tau^{l_t}\left(i\right)}{1-\tau^{l_t}\left(i\right)} = \mathcal{N}_t\left(i\right)\mathbf{1}_i^L + \mathcal{W}_t^l\left(i\right)\phi_i^t + \mathcal{M}_t^l\left(i\right)\left[f\left(i\right)w_t(i) - \frac{\partial Y_t}{\partial e_t(i)}\right], \tag{A22}$$

where $\mathcal{N}_t\left(i\right) \equiv \frac{\beta^{t-1}\eta\Psi_t(i)}{\chi_t}\left[h'(e_t(i)) - \frac{\Delta_t}{e_t(i)}\right] > 0$, $\mathcal{W}_t^l\left(i\right) \equiv \frac{\beta^{t-1}\eta\Delta_t\Psi_t(i)}{\chi_t e_t(i)} > 0$ and $\mathcal{M}_t^l\left(i\right) \equiv \Psi_t\left(i\right) > 0$.

(iii) **Corporate wedge** for task $v \in \{\overline{v},\underline{v}\}$ is

$$\tau^{n_t}\left(v\right) = \sum_{s=t}^{\sum_{s=t}} T\mathcal{W}_{s,t}^n\left(v\right)\varphi_v^s - \sum_{s=tr}^{T}\mathcal{M}_{s,t}^n\left(v\right)\left[\frac{\chi_s}{\chi_t}\frac{\partial Y_s}{\partial b_s(v)} - \frac{\pi'\left(b_t\left(v\right)\right)}{\mathcal{R}_s^t\left(v\right)}\right], \tag{A23}$$

where $\mathcal{W}_{s,t}^n\left(v\right) \equiv \frac{\beta^{s-1}\eta\Delta_s}{\chi_s b_s(v)}\Gamma_t^s(v)\frac{\partial A_t^v}{\partial n_t(v)} > 0$ and $\mathcal{M}_{s,t}^n\left(v\right) = \Gamma_t^s(v)\frac{\partial A_t^v}{\partial n_t(v)} > 0$.

(iv)**R&D wedge** for task $v \in \{\overline{v},\underline{v}\}$ is

$$s^{q_t}\left(v\right) = -\sum_{s=t}^{T}\mathcal{W}_{s,t}^q\left(v\right)\varphi_v^s + \sum_{s=t}^{T}\mathcal{M}_{s,t}^q\left(v\right)\left[\frac{\chi_s}{\chi_t}\frac{\partial Y_s}{\partial b_s(v)} - \frac{\pi'\left(b_t\left(v\right)\right)}{\mathcal{R}_s^t\left(v\right)}\right], \tag{A24}$$

where $\mathcal{W}_{s,t}^q\left(v\right) \equiv \frac{\beta^{s-1}\eta\Delta_s}{\chi_s b_s(v)}\Gamma_t^s(v)\frac{\partial A_t^v}{\partial q_t(v)} > 0$ and $\mathcal{M}_{s,t}^q\left(v\right) = \Gamma_t^s(v)\frac{\partial A_t^v}{\partial q_t(v)} > 0$.

PROOF OF PROPOSITION A1.

Set the Lagrange of the social planning problem (P_2) as follows.

$$\begin{aligned}\mathcal{L} = \max\sum_{i=L,H}\mathrm{g}_i\left[\sum_{t=1}^{T}\beta^{t-1}\left[u\left(c_t(i)\right) - h\left(e_t(i)\right)\right]\right] \\ +\eta\left\{\sum_{t=1}^{T}\beta^{t-1}\left[u\left(c_t(H)\right) - h\left(e_t(H)\right)\right] - \sum_{t=1}^{T}\beta^{t-1}\left[u\left(c_t(L)\right) - h\left(\frac{w_t(L)e_t(L)}{w_t(H)}\right)\right]\right\} \\ +\sum_{t=1}^{T}\chi_t\left[Y_t + \sum_{v=\underline{v},\bar{v}}\left[\left(1-\delta_k\right)k_t\left(v\right) - k_{t+1}\left(v\right) - \mathcal{C}_q\left(q_t\left(v\right)\right) - \mathcal{C}_n\left(n_t\left(v\right)\right)\right] - G_t - \sum_{i=L,H}f\left(i\right)c_t\left(i\right)\right].\end{aligned}$$

The first-order conditions for consumption are as follows.

$$[c_t(H)] : \left(\mathrm{g}_H + \eta\right)\beta^{t-1}u'\left(c_t(H)\right) - \chi_t f(H) = 0,$$

$$[c_t(L)] : \left(\mathrm{g}_L - \eta\right)\beta^{t-1}u'\left(c_t(L)\right) - \chi_t f(L) = 0.$$

(i) Capital wedge: The first-order condition of the top task's capital demand is

$$[k_t(\bar{v})]: \eta\beta h'\left(\frac{w_t(L)e_t(L)}{w_t(H)}\right)\frac{\partial\left(\frac{w_t(L)}{w_t(H)}\right)}{\partial k_t(\bar{v})}e_t(L) - \chi_{t-1} + \chi_t\left[\frac{\partial Y_t}{\partial k_t(\bar{v})} + 1 - \delta_k\right] = 0,$$

which implies that

$$\begin{aligned}\frac{\tau_i^{k_t}(\bar{v})}{1-\tau_i^{k_t}(\bar{v})} &= \frac{\beta\left[R_t(\bar{v}) + 1 - \delta_k\right]u'(c_t(i))}{u'(c_{t-1}(i))} - 1 \\ &= \underbrace{\frac{\beta^{t-1}\eta}{\chi_{t-1}}h'\left(\frac{w_t(L)e_t(L)}{w_t(H)}\right)\left(\frac{w_t(L)e_t(L)}{w_t(H)k_t(\bar{v})}\right)\kappa_{\bar{v}}^t}_{\text{wage compression:>0}} + \underbrace{\frac{\chi_t}{\chi_{t-1}}\left[R_t(\bar{v}) - \frac{\partial Y_t}{\partial k_t(\bar{v})}\right]}_{\text{Pigouvian:<0}}.\end{aligned}$$

The above equation can be expressed as (A21) for $v = \bar{v}$, when we use the following notations

$$\mathcal{W}_t^k(\bar{v}) = \frac{\beta^{t-1}\eta}{\chi_{t-1}}h'\left(\frac{w_t(L)e_t(L)}{w_t(H)}\right)\left(\frac{w_t(L)e_t(L)}{w_t(H)k_t(\bar{v})}\right) \text{ and } \mathcal{M}_t^k = \frac{\chi_t}{\chi_{t-1}}.$$

Furthermore, the first-order condition of the bottom task's capital demand is

$$[k_t(\underline{v})]: \eta\beta h'\left(\frac{w_t(L)e_t(L)}{w_t(H)k_t(\bar{v})}\right)\frac{\partial\left(\frac{w_t(L)}{w_t(H)}\right)}{\partial k_t(\underline{v})}e_t(L) - \chi_{t-1} + \chi_t\left[\frac{\partial Y_t}{\partial k_t(\underline{v})} + (1 - \delta_k)\right] = 0,$$

which implies that

$$\begin{aligned}\frac{\tau_i^{k_t}(\underline{v})}{1-\tau_i^{k_t}(\underline{v})} &= \frac{\beta\left[R_t(\underline{v}) + 1 - \delta_k\right]u'(c_t(i))}{u'(c_{t-1}(i))} - 1 \\ &= \underbrace{\frac{\beta^{t-1}\eta}{\chi_{t-1}}h'\left(\frac{w_t(L)e_t(L)}{w_t(H)}\right)\left(\frac{w_t(L)e_t(L)}{w_t(H)}\right)\frac{\kappa_{\underline{v}}^t}{k_t(\underline{v})}}_{\text{wage compression:<0}} + \underbrace{\frac{\chi_t}{\chi_{t-1}}\left[R_t(\underline{v}) - \frac{\partial Y_t}{\partial k_t(\underline{v})}\right]}_{\text{Pigouvian:<0}} < 0.\end{aligned}$$

The above equation can be expressed as(A21) for $v = \underline{v}$, when we use the following notation.

$$\mathcal{W}_t^k(\underline{v}) = \frac{\beta^{t-1}\eta}{\chi_{t-1}}h'\left(\frac{w_t(L)e_t(L)}{w_t(H)}\right)\left(\frac{w_t(L)e_t(L)}{w_t(H)}\right) \text{ and } \mathcal{M}_t^k = \frac{\chi_t}{\chi_{t-1}}.$$

(ii) Labor wedge: The first-order condition of the high-talent agent's labor supply is

$$[e_t(H)]: \left(-g_H - \eta + \eta\frac{h'\left(\frac{w_t(L)e_t(L)}{w_t(H)}\right)}{h'(e_t(H))}\underbrace{\frac{\partial\left(\frac{w_t(L)}{w_t(H)}\right)}{\partial e_t(H)}}_{=\frac{-w_t(L)\phi_H^t}{w_t(H)e_t(H)}}e_t(L)\right)\beta^{t-1}h'(e_t(H)) + \chi_t\frac{\partial Y_t}{\partial e_t(H)} = 0,$$

which implies that

$$\frac{\tau^{l_t}(H)}{1-\tau^{l_t}(H)} = \frac{w_t(H)\,u\prime(c_t(H))}{h'(e_t(H))} - 1$$

$$= \underbrace{\frac{\beta^{t-1}u'(c_t(H))\,\eta}{\chi_t f(H)} \left(\frac{h'\left(\frac{w_t(L)e_t(L)}{w_t(H)}\right)}{h'(e_t(H))} \frac{w_t(L)e_t(L)\phi_H^t}{w_t(H)e_t(H)} \right)}_{\text{wage compression: <0}} + \underbrace{\frac{u'(c_t(H))\left[f(H)w_t(H) - \frac{\partial Y_t}{\partial e_t(H)}\right]}{h'(e_t(H))\,f(H)}}_{\text{Pigouvian <0}}$$

The above equation is expressed as (A22) for $i = H$, when we use the notation $\mathbf{1}_H^L = 0$,

$$\mathcal{W}_t^l(H) = \frac{\beta^{t-1}\eta u'(c_t(H))}{\chi_t f(H)\,h'(e_t(H))} h'\left(\frac{w_t(L)e_t(L)}{w_t(H)}\right)\left(\frac{w_t(L)e_t(L)}{w_t(H)e_t(H)}\right) \text{ and } \mathcal{M}_t^l(H) = \frac{u'(c_{H,t})}{h'(e_{H,t})\,f(H)}.$$

Next, the first-order condition of the low-talent agent's labor supply is

$$[e_t(L)]: \left(-g_L + \eta \frac{h'\left(\frac{w_t(L)e_t(L)}{w_t(H)}\right)\frac{w_t(L)}{w_t(H)}}{h'(e_t(L))} + \eta \frac{h'\left(\frac{w_t(L)e_t(L)}{w_t(H)}\right)}{h'(e_t(L))} \frac{\partial\left(\frac{w_t(L)}{w_t(H)}\right)}{\partial e_t(L)} e_t(L) \right) \beta^{t-1} h'(e_t(L)) + \chi_t \frac{\partial Y_t}{\partial e_t(L)} = 0,$$

which implies that

$$\frac{\tau^{l_t}(L)}{1-\tau^{l_t}(L)} = \frac{w_t(L)\,u\prime(c_t(L))}{h\prime(e_t(L))} - 1$$

$$= \frac{\beta^{t-1}u\prime(c_t(L))\,\eta}{\chi_t f(L)} \left[\underbrace{1 - \frac{h\prime\left(\frac{w_t(L)e_t(L)}{w_t(H)}\right)}{h\prime(e_t(L))}\left(\frac{w_t(L)}{w_t(H)}\right)}_{\text{Mirrlees:>0}} + \underbrace{\frac{h\prime\left(\frac{w_t(L)e_t(L)}{w_t(H)}\right)}{h\prime(e_t(L))}\left(\frac{w_t(L)}{w_t(H)}\right)\phi_L^t}_{\text{wage compression:>0}} \right] + \underbrace{\frac{u\prime\left(c_{L,t}\right)\left[f(L)w_t(L) - \frac{\partial Y_t}{\partial e_t(L)}\right]}{h\prime(e_t(L))\,f(L)}}_{\text{Pigouvian:<0}}$$

The above equation can be expressed as (A22) for $i = L$, when we use notation as follows.

$$\mathcal{N}_t(L) = \frac{\beta^{t-1}\eta u\prime(c_t(L))}{\chi_t f(L)h'(e_t(L))}\left[h'(e_t(L)) - h'\left(\frac{w_t(L)e_t(L)}{w_t(H)}\right)\left(\frac{w_t(L)}{w_t(H)}\right)\right],$$

$$\mathcal{W}_t^l(L) = \frac{\beta^{t-1}\eta u\prime(c_t(L))}{\chi_t f(L)h'(e_t(L))} h'\left(\frac{w_t(L)e_t(L)}{w_t(H)}\right)\left(\frac{w_t(L)}{w_t(H)}\right) \text{ and } \mathcal{M}_t^l(L) = \frac{u'(c_t(L))}{h'(e_t(L))\,f(L)}.$$

(iii) Corporate wedge: The first-order condition of the top task's R&D inputs is

$$[n_t(\bar{v})]: \eta \sum_{s=t}^{T} \beta^{s-1} h'\left(\frac{w_s(L)\,e_s(L)}{w_s(H)}\right) e_s(L) \frac{\partial\left(\frac{w_s(L)}{w_s(H)}\right)}{\partial b_s(\bar{v})} \Gamma_t^s(\bar{v}) \frac{\partial A_t^{\bar{v}}}{\partial n_t(\bar{v})}$$
$$+ \sum_{s=t}^{T} \chi_s \frac{\partial Y_s}{\partial b_s(\bar{v})} \Gamma_t^s(\bar{v}) \frac{\partial A_t^{\bar{v}}}{\partial n_t(\bar{v})} - \chi_t \mathcal{C}_n'(n_t(\bar{v})) = 0$$

which implies that

$$\tau^{n_t}(\bar{v}) \equiv \frac{\partial A^{\bar{v}}}{\partial n_t(\bar{v})}\left[\sum_{s=t}^{T} \frac{1}{\mathcal{R}_s^t(\bar{v})} \pi_s'(b_s(\bar{v}))\,\Gamma_t^s(\bar{v})\right] - \mathcal{C}_n'(n_t(\bar{v}))$$

$$= \underbrace{\frac{\eta}{\chi_t}\sum_{s=t}^{T}\beta^{s-1}h'\left(\frac{w_s(L)e_s(L)}{w_s(H)}\right)\left(\frac{w_s(L)e_s(L)}{w_s(H)b_s(\bar{v})}\right)\varphi_{\bar{v}}^{s}\Gamma_t^s(\bar{v})\frac{\partial A_t^{\bar{v}}}{\partial n_t(\bar{v})}}_{\text{wage compression}} - \underbrace{\sum_{s=t}^{T}\left(\frac{\chi_s}{\chi_t}\frac{\partial Y_t}{\partial b_t(\bar{v})} - \frac{\pi'(b_t(\bar{v}))}{\mathcal{R}_s^t(\bar{v})}\right)\Gamma_t^s(\bar{v})\frac{\partial A_t^{\bar{v}}}{\partial n_t(\bar{v})}}_{\text{Pigouvian}}.$$

The above equation can be expressed as (A23) for$v = \overline{v}$, when we use the following notation.

$$\mathcal{W}_{s,t}^{n}(\bar{v}) = \frac{\beta^{s-1}\eta}{\chi_t}h'\left(\frac{w_s(L)e_s(L)}{w_s(H)}\right)\left(\frac{w_s(L)e_s(L)}{w_s(H)b_s(\bar{v})}\right)\Gamma_t^s(\bar{v})\frac{\partial A_t^{\bar{v}}}{\partial n_t(\bar{v})} \text{ and } \mathcal{M}_{s,t}^{n}(\bar{v}) = \Gamma_t^s(\bar{v})\frac{\partial A_t^{\bar{v}}}{\partial n_t(\bar{v})}.$$

Next, the first-order condition of the bottom task's R&D investment is

$$[n_t(\underline{v})]:\eta\sum_{s=t}^{T}\beta^{s-1}h'\left(\frac{w_s(L)e_s(L)}{w_s(H)}\right)e_s(L)\frac{\partial\left(\frac{w_s(L)}{w_s(H)}\right)}{\partial b_s(\underline{v})}\Gamma_t^s(\underline{v})\frac{\partial A_t^{\underline{v}}}{\partial n_t(\underline{v})}$$
$$+\sum_{s=t}^{T}\chi_s\frac{\partial Y_s}{\partial b_s(\underline{v})}\Gamma_t^s(\underline{v})\frac{\partial A_t^{\underline{v}}}{\partial n_t(\underline{v})} - \chi_t\mathcal{C}_n'(n_t(\underline{v})) = 0,$$

which implies that

$$\tau^{n_t}(\underline{v}) \equiv \frac{\partial A^{\underline{v}}}{\partial n_t(\underline{v})}\left[\sum_{s=t}^{T}\frac{1}{\mathcal{R}_s^t(\underline{v})}\pi_s'(b_s(\underline{v}))\Gamma_t^s(\underline{v})\right] - \mathcal{C}_n'(n_t(\underline{v}))$$
$$= \underbrace{\frac{\eta}{\chi_t}\sum_{s=t}^{T}\beta^{s-1}h'\left(\frac{w_s(L)e_s(L)}{w_s(H)}\right)\left(\frac{w_s(L)e_s(L)}{w_s(H)b_s(\underline{v})}\right)\varphi_{\underline{v}}^{s}\Gamma_t^s(\underline{v})\frac{\partial A_t^{\underline{v}}}{\partial n_t(\underline{v})}}_{\text{wage compression}} - \underbrace{\sum_{s=t}^{T}\left(\frac{\chi_s}{\chi_t}\frac{\partial Y_t}{\partial b_t(\underline{v})} - \frac{\pi'(b_t(\underline{v}))}{\mathcal{R}_s^t(\underline{v})}\right)\Gamma_t^s(\underline{v})\frac{\partial A^{\underline{v}}}{\partial n_t(\underline{v})}}_{\text{Pigouvian}}.$$

The above equation can be expressed as (A23) for $v = \underline{v}$, when we use the following notation.

$$\mathcal{W}_{s,t}^{n}(\underline{v}) = \frac{\beta^{s-1}\eta}{\chi_t}h'\left(\frac{w_s(L)e_s(L)}{w_s(H)}\right)\left(\frac{w_s(L)e_s(L)}{w_s(H)b_s(\underline{v})}\right)\Gamma_t^s(\underline{v})\frac{\partial A_t^{\underline{v}}}{\partial n_t(\underline{v})} \text{ and } \mathcal{M}_{s,t}^{n}(\underline{v}) = \Gamma_t^s(\underline{v})\frac{\partial A_t^{\underline{v}}}{\partial n_t(\underline{v})}$$

(iv) R&D wedge: The first-order condition of the top task's R&D investments is

$$[q_t(\bar{v})]:\eta\sum_{s=t}^{T}\beta^{s-1}h'\left(\frac{w_s(L)e_s(L)}{w_s(H)}\right)e_s(L)\frac{\partial\left(\frac{w_s(L)}{w_s(H)}\right)}{\partial b_s(\bar{v})}\Gamma_t^s(\bar{v})\frac{\partial A_t^{\bar{v}}}{\partial q_t(\bar{v})}$$
$$+\sum_{s=t}^{T}\chi_s\frac{\partial Y_s}{\partial b_s(\bar{v})}\Gamma_t^s(\bar{v})\frac{\partial A_t^{\bar{v}}}{\partial q_t(\bar{v})} - \chi_t\mathcal{C}_q'(q_t(\bar{v})) = 0$$

which implies that

$$s^{q_t}(\bar{v}) \equiv \mathcal{C}_q'(q_t(\bar{v})) - \frac{\partial A^{\bar{v}}}{\partial q_t(\bar{v})}\left[\sum_{s=t}^{T}\frac{1}{\mathcal{R}_s^t(\bar{v})}\pi_s'(b_s(\bar{v}))\Gamma_t^s(\bar{v})\right]$$
$$= \underbrace{\frac{-\eta}{\chi_t}\sum_{s=t}^{T}\beta^{s-1}h'\left(\frac{w_s(L)e_s(L)}{w_s(H)}\right)\left(\frac{w_s(L)e_s(L)}{w_s(H)b_s(\bar{v})}\right)\varphi_{\bar{v}}^{s}\Gamma_t^s(\bar{v})\frac{\partial A_t^{\bar{v}}}{\partial q_t(\bar{v})}}_{\text{wage compression}} + \underbrace{\sum_{s=t}^{T}\left(\frac{\chi_s}{\chi_t}\frac{\partial Y_t}{\partial b_t(\bar{v})} - \frac{\pi'(b_t(\bar{v}))}{\mathcal{R}_s^t(\bar{v})}\right)\Gamma_t^s(\bar{v})\frac{\partial A^{\bar{v}}}{\partial q_t(\bar{v})}}_{\text{Pigouvian}}.$$

The above equation can be expressed as (A24) for $v=\bar{v}$, when we use the following notation.

$$\mathcal{W}^{q}_{s,t}\left(\bar{v}\right)=\frac{\beta^{s-1}\eta}{\chi_t}h'\left(\frac{w_s(L)e_s(L)}{w_s(H)}\right)\left(\frac{w_s(L)e_s(L)}{w_s(H)b_s\left(\bar{v}\right)}\right)\Gamma^s_t(\bar{v})\frac{\partial A^{\bar{v}}_t}{\partial q_t\left(\bar{v}\right)}\text{ and }\mathcal{M}^{q}_{s,t}(\bar{v})=\Gamma^s_t\left(\bar{v}\right)\frac{\partial A^{\bar{v}}_t}{\partial q_t\left(\bar{v}\right)}.$$

Next, the first-order condition of the bottom task's R&D investments is

$$\begin{aligned}\left[q_t\left(\underline{v}\right)\right]:&\eta\sum_{s=t}^{T}\beta^{s-1}h'\left(\frac{w_s(L)e_s(L)}{w_s(H)}\right)e_s(L)\frac{\partial\left(\dfrac{w_s\left(L\right)}{w_s\left(H\right)}\right)}{\partial b_s\left(\underline{v}\right)}\Gamma^s_t(\underline{v})\frac{\partial A^{\underline{v}}_t}{\partial q_t\left(\underline{v}\right)}+\sum_{s=t}^{T}\chi_s\frac{\partial Y_s}{\partial b_s\left(\underline{v}\right)}\Gamma^s_t(\underline{v})\frac{\partial A^{\underline{v}}_t}{\partial q_t\left(\underline{v}\right)}\\&-\chi_t\mathcal{C}'_q\left(q_t\left(\underline{v}\right)\right)=0,\end{aligned}$$

which implies that

$$\begin{aligned}&s^{q_t}\left(\underline{v}\right)\equiv\mathcal{C}'_q\left(q_t\left(\underline{v}\right)\right)-\frac{\partial A^{\underline{v}}}{\partial q_t\left(\underline{v}\right)}\left[\sum_{s=t}^{T}\frac{1}{\mathcal{R}^t_s\left(\underline{v}\right)}\pi'_s\left(b_s\left(\underline{v}\right)\right)\Gamma^s_t(\underline{v})\right]\\&=\underbrace{\frac{-\eta}{\chi_t}\sum_{s=t}^{T}\beta^{s-1}h'\left(\frac{w_s(L)e_s(L)}{w_s(H)}\right)\left(\frac{w_s(L)e_s(L)}{w_s(H)b_s\left(\underline{v}\right)}\right)\varphi^s_{\underline{v}}\Gamma^s_t(\underline{v})\frac{\partial A^{\underline{v}}_t}{\partial q_t\left(\underline{v}\right)}}_{\text{wage compression}}+\underbrace{\sum_{s=t}^{T}\left(\frac{\chi_s}{\chi_t}\frac{\partial Y_t}{\partial b_t\left(\underline{v}\right)}-\frac{\pi'\left(b_t\left(\underline{v}\right)\right)}{\mathcal{R}^t_s\left(\underline{v}\right)}\right)\Gamma^s_t(\underline{v})\frac{\partial A^{\underline{v}}}{\partial q_t\left(\underline{v}\right)}}_{\text{Pigouvian}}.\end{aligned}$$

The above equation can be expressed as (A24) for $v=\underline{v}$, when we use the following notation.

$$\mathcal{W}^{q}_{s,t}\left(\underline{v}\right)=\frac{\beta^{s-1}\eta}{\chi_t}h'\left(\frac{w_s(L)e_s(L)}{w_s(H)}\right)\left(\frac{w_s(L)e_s(L)}{w_s(H)b_s\left(\underline{v}\right)}\right)\Gamma^s_t(\underline{v})\frac{\partial A^{\underline{v}}_t}{\partial q_t\left(\underline{v}\right)}\text{ and }\mathcal{M}^{q}_{s,t}(\underline{v})=\Gamma^s_t(\underline{v})\frac{\partial A^{\underline{v}}_t}{\partial q_t\left(\underline{v}\right)}.$$

□

B.2 *Extension – When Agents' Types are Time-varying*

In this Appendix, we consider an extension of our model by allowing for agents' types to change over time. The wedges derived in Proposition 1 would be slightly changed due to the extended setting, but the major effects such as Mirrlees term, wage compression term and Pigouvian term which are emphasized in Proposition 1 still hold in the extension. Because the extension makes notations complicated, to lighten the notations, in this section we assume that the capital and R&D investment are fully depreciated, but none of the results in this section depend on this simplification.

When allowing for agents' types to change over time, to simplify the analysis, we study the two-type, two-sector model. In the first period, an agent's type is denoted by $i\in L,H$, and the probability of type i is $f_{i,1}$, where $f_{L,1}+f_{H,1}=1$. In the second period, the agent's type is denoted by $j\in\{L,H\}$. Thus, at the end of the second period, there are four different types of history: $ij\in\{LL,LH,HL,HH\}$, and the probability of the type of history ij is $f_{ij,2}$, where $f_{HL,2}+f_{HH,2}=f_{H,1}$ and $f_{LL,2}+f_{LH,2}=f_{L,1}$. Suppose that an agent is type i in period 1, the conditional probability of the agent's type being j in period 2 is $\Pr\left(j|i\right)=\frac{\Pr\left(i\cap j\right)}{\Pr\left(i\right)}=\frac{f_{ij,2}}{f_{i.1}}$. Then, the expected lifetime utility for an agent of type i in the first period is:

$$u\left(c_{i,1}\right)-h\left(e_{i,1}\right)+\beta\sum_{j=L,H}\left[u\left(c_{ij,2}\right)-h\left(e_{ij,2}\right)\right]\frac{f_{ij,2}}{f_{i.1}}. \tag{A25}$$

Except the types as specified above, other environment, such as agents' talents, sectors, and firm's decisions, is the same as that in Subsection 2.1.

Social planning problem (P$_3$). Suppose that the social planner attaches Pareto weight g_i to agents of talent $i \in \{L, H\}$, where $\mathrm{g}_L + \mathrm{g}_H = 1$. With U_i in (A25), the social planner maximizes the social welfare

$$W = \mathrm{g}_L U_L + \mathrm{g}_H U_H,$$

subject to a resource constraint for each period $t = 1, 2$,

$$(\chi_1): Y_1 \geq \sum_{j=L,H} c_{i,1} f_i + \sum_{v=\underline{v},\bar{v}}\left[k_2\left(v\right)+M_q\left(q_1\left(v\right)\right)+M_n\left(n_1\left(v\right)\right)\right]+G_1,$$

$$(\chi_2): Y_2 \geq \sum_{ij=LL,LH,HL,HH} c_{ij,2} f_{ij,2} + \sum_{v=\underline{v},\bar{v}}\left[M_q\left(q_2\left(v\right)\right)+M_n\left(n_2\left(v\right)\right)\right]+G_2,$$

where χ_t is the shadow price, and subject to three ICCs.

The first ICC is set to prevent a high type from reporting as a low type in the first period,

$$(\eta_1): U_H \geq u\left(c_{L,1}\right)-h\left(\frac{w_{L,1}e_{L,1}}{w_{H,1}}\right)+\beta\sum_{j=L,H}\left[u\left(c_{Lj,2}\right)-h\left(e_{Lj,2}\right)\right]\frac{f_{Hj,2}}{f_{H,1}},$$

where η_1 is the shadow price.

The second and the third ICCs are set to prevent a high type from reporting as a low type in the second period. Specifically, for agents of type $i \in L, H$ in the first period, the ICCs are

$$(\eta_{i,2}): u\left(c_{iH,2}\right)-h\left(e_{iH,2}\right) \geq u\left(c_{iL,2}\right)+h\left(\frac{w_{L,2}e_{iL,2}}{w_{H,2}}\right),$$

where $\eta_{i,2}, i = L, H,$ is the shadow price.

Based on the above social planning problem P_3, we derive the labor wedge, the capital wedge and the R&D wedge. The results are formally stated in Proposition A2.

Some notations are needed. As in Proposition 1 in the text, let $\Delta_1 \equiv h'\left(\frac{w_{L,1}e_{L,1}}{w_{H,1}}\right)\frac{w_{L,1}e_{L,1}}{w_{H,1}}$ and $\Psi_{i,1} \equiv \frac{u'(c_{i,1})}{f_{i,1}h'(e_{i,1})}$ for period 1, and $\mathbf{1}_i^L$ is the indicator function, which is 1 if $i = L$, and zero if $i = H$. Moreover, we define $\Delta_{i,2} \equiv h'\left(\frac{w_{L,2}e_{iL,t}}{w_{H,t}}\right)\left(\frac{w_{L,2}e_{iL,t}}{w_{H,2}}\right)$ and $\Psi_{ij,2} \equiv \frac{u'(c_{ij,2})}{f_{ij,2}h'(e_{ij,2})}$ for period 2.

PROPOSITION A2. Let an agent's type in the first period be denoted by $i \in \{H,\ L\}$ and the type of history in the second period be denoted by $ij \in \{HH, HL,\ LH, LL\}$. Then
(i) **Capital wedge** for task $v \in \{\overline{v}, \underline{v}\}$ and $i \in \{L, H\}$, is

$$\frac{\tau_i^{k_2}(v)}{1-\tau_i^{k_2}(v)} = \mathcal{W}_2^k(v)\,\kappa_v^t + \mathcal{M}_2^k\left[R_2(v) - \frac{\partial Y_2}{\partial k_2(v)}\right] + \underbrace{\mathcal{I}_1(i)\,R_2(v)}_{\text{insurance effect}} \tag{A26}$$

where $\mathcal{W}_2^k(v) \equiv \frac{\left[\eta_{L,2}\Delta_{L,2}+\eta_{H,2}\Delta_{H,2}\right]}{\chi_1 k_t(v)} > 0, \mathcal{M}_2^k = \frac{\chi_2}{\chi_1} > 0,$
and $\mathcal{I}_1(H) \equiv \frac{\eta_{H,2}\left[u'(c_{HL,2})-u'(c_{HL,2})\right]}{\chi_1 f_{H,1}},$
and $\mathcal{I}_1(L) \equiv \frac{u'(c_{LL,2})\left[\beta\eta_1\left[\frac{f_{HL,2}}{f_{H,1}}-\frac{f_{LL,2}}{f_{L,1}}\right]+\eta_{L,2}\right]+u'(c_{LH,2})\left[\beta\eta_1\left[\frac{f_{HH,2}}{f_{H,1}}-\frac{f_{LH,2}}{f_{L,1}}\right]-\eta_{L,2}\right]}{\chi_1 f_{L,1}}.$
(ii)**Labor wedge**

In the first period, the labor wedge for type $i \in \{H,\ L\}$ is

$$\frac{\tau_i^{l_1}}{1-\tau_i^{l_1}} = \mathcal{N}_1(i)\,\mathbf{1}_i^L + \mathcal{W}_1^l(i)\,\phi_i^1 + \mathcal{M}_1^l(i)\left[w_1(v_i) - \frac{\partial Y_1}{\partial l_1(v_i)}\right], \tag{A27}$$

where $\mathcal{N}_1(i) \equiv \frac{\eta_1\Psi_{i,1}}{\chi_1}\left[h'(e_{i,1}) - \frac{\Delta_1}{e_{i,1}}\right] > 0, \mathcal{W}_1^l(i) \equiv \frac{\eta_1\Delta_1\Psi_{i,1}}{\chi_1 e_{i,1}} > 0$
and $\mathcal{M}_1^l(i) \equiv \Psi_{i,1} f_{i,1} a_i(v_i) > 0$.
In the second period, the labor wedge for type $ij \in \{HH, HL,\ LH, LL\}$ is

$$\frac{\tau_{ij}^{l_2}}{1-\tau_{ij}^{l_2}} = \mathcal{N}_2(ij)\,\mathbf{1}_j^L + \mathcal{W}_t^l(i)\,\phi_{ij}^2 + \mathcal{M}_2^l(ij)\left[w_2(v_j) - \frac{\partial Y_2}{\partial l_2(v_j)}\right], \tag{A28}$$

where $\mathcal{N}_2(ij) \equiv \frac{\eta_(i,2)\Psi_{ij,2}}{\chi_2}\left[h'(e_{ij,2}) - \frac{\Delta_{i,2}}{e_{ij,2}}\right] > 0,$
$\mathcal{W}_2^l(ij) \equiv \frac{\Psi_{ij,2}\left[\eta_{L,2}\Delta_{L,2}+\eta_{H,2}\Delta_{H,2}\right]}{\chi_2 e_{ij,2}} > 0$ and $\mathcal{M}_2^l(ij) \equiv \Psi_{i,t} f_{ij} a_i(v_i) > 0$.

(iii) **R&D wedge** for task $v \in \{\overline{v}, \underline{v}\}$ in $t = 1,\ 2$ is

$$s^{q_t}(v) = -\sum_{s=t}^{2}\mathcal{W}_{s,t}^q(v)\,\varphi_v^s + \sum_{s=t}^{2}\mathcal{M}_{s,t}^q(v)\left[\frac{\chi_s}{\chi_t}\frac{\partial Y_s}{\partial b_s(v)} - \frac{\pi'(b_t(v))}{\mathcal{R}_s^t(v)}\right] \tag{A29}$$

where $\mathcal{W}_{1,1}^q(v) \equiv \frac{\eta_1\Delta_1}{\chi_1 b_1(v)}\frac{\partial A_1^v}{\partial q_1(v)}, \mathcal{W}_{2,1}^q(v) \equiv \frac{(\eta_{L,2}\Delta_{L,2}+\eta_{H,2}\Delta_{H,2})}{\chi_1 b_2(v)}\frac{\partial A_2^v}{\partial b_1(v)}\frac{\partial A_1^v}{\partial q_1(v)},$
$\mathcal{W}_{2,2}^q(v) \equiv \frac{(\eta_{L,2}\Delta_{L,2}+\eta_{H,2}\Delta_{H,2})}{\chi_2 b_2(v)}\frac{\partial A_2^v}{\partial q_2(v)},\ \mathcal{M}_{1,1}^q(v) = \frac{\partial A_1^v}{\partial q_1(v)}, \mathcal{M}_{2,1}^q(v) = \frac{\partial A_2^v}{\partial b_1(v)}\frac{\partial A_1^v}{\partial q_1(v)},$
and $\mathcal{M}_{2,2}^q(v) = \frac{\partial A_2^v}{\partial q_2(v)}$.

(iv) **Corporate wedge** for task $v \in \{\overline{v}, \underline{v}\}$ in $t = 1,\ 2$ is

$$\tau^{n_t}(v) = \sum_{s=t}^{2}\mathcal{W}_{s,t}^n(v)\,\varphi_v^s - \sum_{s=t}^{2}\mathcal{M}_{s,t}^n(v)\left[\frac{\chi_s}{\chi_t}\frac{\partial Y_s}{\partial b_s(v)} - \frac{\pi'(b_t(v))}{\mathcal{R}_s^t(v)}\right] \tag{A30}$$

where $\mathcal{W}^n_{1,1}(v) \equiv \frac{\eta_1 \Delta_1}{\chi_1 b_1(v)} \frac{\partial A_1^v}{\partial n_1(v)}$, $\mathcal{W}^n_{2,1}(v) \equiv \frac{(\eta_{L,2}\Delta_{L,2}+\eta_{H,2}\Delta_{H,2})}{\chi_1 b_2(v)} \frac{\partial A_2^v}{\partial b_1(v)} \frac{\partial A_1^v}{\partial n_1(v)}$, $\mathcal{W}^n_{2,2}(v) \equiv \frac{(\eta_{L,2}\Delta_{L,2}+\eta_{H,2}\Delta_{H,2})}{\chi_2 b_2(v)} \frac{\partial A_2^v}{\partial n_2(v)}$, $\mathcal{M}^n_{1,1}(v) = \frac{\partial A_1^v}{\partial n_1(v)}$, $\mathcal{M}^n_{2,1}(v) = \frac{\partial A_2^v}{\partial b_1(v)} \frac{\partial A_1^v}{\partial n_1(v)}$, and $\mathcal{M}^n_{2,2}(v) = \frac{\partial A_2^v}{\partial n_2(v)}$.

It is clear that Proposition A2 is the same as those in Proposition 1 except for two things. First, the labor wedge in period 2 now depends on the whole history of type $ij \in \{LL, LH, HL, HH\}$. Second, there is an additional insurance term in the capital wedge for the insurance purpose. These two differences are the standard results in the new dynamic public finance literature, as agents' types are time-varying.

PROOF OF PROPOSITION A2.
Based on the social planning problem (P_3), we set the Lagrange as follows.

$$\begin{aligned}
\mathcal{L} = max \sum_{i=L,H} \mathrm{g}_i & \left[u\left(c_{i,1}\right) - h\left(e_{i,1}\right) + \beta \sum_{j=L,H} \left[u\left(c_{ij,2}\right) - h\left(e_{ij,2}\right)\right] \frac{f_{ij,2}}{f_{i.1}} \right] \\
& + \chi_1 \left[Y_1 - f_{L,1}c_{L,1} - f_{H,1}c_{H,1} - \sum_{v=\underline{v},\bar{v}} \left[k_2(v) + M_q\left(q_1(v)\right) + M_n\left(n_1(v)\right)\right] - G_1 \right] \\
& + \chi_2 \left[Y_2 - \sum_{ij=LL,LH,HL,HH} c_{ij,2} f_{ij,2} - \sum_{v=\underline{v},\bar{v}} \left[M_q\left(q_2(v)\right) + M_n\left(n_2(v)\right)\right] - G_2 \right] \\
& + \eta_1 \left\{ u\left(c_{H,1}\right) - h\left(e_{H,1}\right) + \beta \sum_{j=L,H} \left[u\left(c_{Hj,2}\right) - h\left(e_{Hj,2}\right)\right] \frac{f_{Hj,2}}{f_{H,1}} - u\left(c_{L,1}\right) + h\left(\frac{w_{L,1}e_{L,1}}{w_{H,1}}\right) \right. \\
& \left. + \beta \sum_{j=L,H} \left[-u\left(c_{Lj,2}\right) + h\left(e_{Lj,2}\right)\right] \frac{f_{Hj,2}}{f_{H,1}} \right\} \\
& + \eta_{L,2} \left[u\left(c_{LH,2}\right) - h\left(e_{LH,2}\right) - u\left(c_{LL,2}\right) + h\left(\frac{w_{L,2}e_{LL,2}}{w_{H,2}}\right) \right] \\
& + \eta_{H,2} \left[u\left(c_{HH,2}\right) - h\left(e_{HH,2}\right) - u\left(c_{HL,2}\right) + h\left(\frac{w_{L,2}e_{HL,2}}{w_{H,2}}\right) \right]
\end{aligned}$$

First, we take the first-order conditions with respect to the consumption as follows:

$$\left[c_{H,1}\right] : \left(\mathrm{g}_H + \eta_1\right) u'\left(c_{H,1}\right) - \chi_1 f_{H,1} = 0, \tag{A31}$$

$$\left[c_{L,1}\right] : \left(\mathrm{g}_L - \eta_1\right) u'\left(c_{L,1}\right) - \chi_1 f_{L,1} = 0, \tag{A32}$$

$$\left[c_{LL,2}\right] : \left[\left(\mathrm{g}_L \frac{f_{LL,2}}{f_{L,1}} - \eta_1 \frac{f_{HL,2}}{f_{H,1}}\right)\beta - \eta_{L,2}\right] u'\left(c_{LL,2}\right) - \chi_2 f_{LL,2} = 0, \tag{A33}$$

$$\left[c_{LH,2}\right] : \left[\left(\mathrm{g}_L \frac{f_{LH,2}}{f_{L,1}} - \eta_1 \frac{f_{HH,2}}{f_{H,1}}\right)\beta + \eta_{L,2}\right] u'\left(c_{LH,2}\right) - \chi_2 f_{LH,2} = 0, \tag{A34}$$

$$[c_{HL,2}]:\left[\left(g_H\frac{f_{HL,2}}{f_{H,1}}+\eta_1\frac{f_{HL,2}}{f_{H,1}}\right)\beta-\eta_{H,2}\right]u'\left(c_{HL,2}\right)-\chi_2 f_{HL,2}=0, \tag{A35}$$

$$[c_{HH,2}]:\left[\left(g_H\frac{f_{HH,2}}{f_{H,1}}+\eta_1\frac{f_{HH,2}}{f_{H,1}}\right)\beta+\eta_{H,2}\right]u'\left(c_{HH,2}\right)-\chi_2 f_{HH,2}=0, \tag{A36}$$

(i) Capital wedge

Given the definition of the capital wedge $\frac{\tau_i^{k_2}(v)}{1-\tau_i^{k_2}(v)}\equiv\frac{\beta R_2(v)E_1\left[u'\left(c_{ij,2}\right)\right]}{u'\left(c_{i,1}\right)}-1$ and the fact $E_1\left[u'\left(c_{ij,2}\right)\right]=\sum_{j=L,H}u'\left(c_{ij,2}\right)\frac{f_{ij,2}}{f_{i,1}}$, the capital wedges for sector $v=\overline{v},\ \underline{v}$ and type $i=H,L$ can be derived from the first-order conditions (A31)-(A36) along with the following condition:

$$[k_2(v)]:\left[\eta_{L,2}h'\left(\frac{w_{L,2}e_{LL,2}}{w_{H,2}}\right)e_{LL,2}+\eta_{H,2}h'\left(\frac{w_{L,2}e_{HL,2}}{w_{H,2}}\right)e_{HL,2}\right]\frac{\partial\left(\frac{w_{L,2}}{w_{H,2}}\right)}{\partial k_2(v)}-\chi_1+\chi_2\frac{\partial Y_2}{\partial k_2(v)}=0,$$

Based on these first-order condition, the capital wedges for high type $i=H$ agents are

$$\frac{\tau_H^{k_2}(v)}{1-\tau_H^{k_2}(v)}=\underbrace{\frac{-\left[\eta_{L,2}h'\left(\frac{w_{L,2}e_{LL,2}}{w_{H,2}}\right)e_{LL,2}+\eta_{H,2}h'\left(\frac{w_{L,2}e_{HL,2}}{w_{H,2}}\right)e_{HL,2}\right]}{\chi_1}\frac{\partial\left(\frac{w_{L,2}}{w_{H,2}}\right)}{\partial k_2(v)}}_{\text{wage compression}}$$
$$+\underbrace{\frac{\chi_2}{\chi_1}\left[R_2(v)-\frac{\partial Y_2}{\partial k_2(v)}\right]}_{\text{Pigouvian}}+\underbrace{\frac{u'\left(c_{HL,2}\right)-u'\left(c_{HH,2}\right)}{\chi_1 f_{H,1}}\eta_{H,2}R_2(v)}_{\text{Insurance effect}}, \tag{A37}$$

while the capital wedge for low type $i=L$ agents are

$$\frac{\tau_L^{k_2}(v)}{1-\tau_L^{k_2}(v)}=\underbrace{\frac{-\left[\eta_{L,2}h'\left(\frac{w_{L,2}e_{LL,2}}{w_{H,2}}\right)e_{LL,2}+\eta_{H,2}h'\left(\frac{w_{L,2}e_{HL,2}}{w_{H,2}}\right)e_{HL,2}\right]}{\chi_1}\frac{\partial\left(\frac{w_{L,2}}{w_{H,2}}\right)}{\partial k_2(v)}}_{\text{wage compression}}+\underbrace{\frac{\chi_2\left[R_2(v)-\frac{\partial Y_2}{\partial k_2(v)}\right]}{\chi_1}}_{\text{Pigouvian}}$$
$$+\underbrace{\frac{u'\left(c_{LL,2}\right)\left[\beta\eta_1\left[\frac{f_{HL,2}}{f_{H,1}}-\frac{f_{LL,2}}{f_{L,1}}\right]+\eta_{L,2}\right]+u'\left(c_{LH,2}\right)\left[\beta\eta_1\left[\frac{f_{HH,2}}{f_{H,1}}-\frac{f_{LH,2}}{f_{L,1}}\right]-\eta_{L,2}\right]}{\chi_1 f_{L,1}}R_2(v)}_{\text{Insurance effect}} \tag{A38}$$

Using the fact that $\frac{\partial\left(\frac{w_{L,2}}{w_{H,2}}\right)}{\partial k_2(v)}=\frac{-w_{L,2}}{w_{H,2}}\frac{\kappa_v^2}{k_2(v)}$ for sector $v=\overline{v},\ \underline{v}$, equations (A37)-(A38) can be expressed as (A26) in Proposition A1.

(ii) Labor wedge:

In the first period, based on the definition of the labor wedge $\frac{\tau_i^{l_1}}{1-\tau_i^{l_1}} \equiv \frac{w_{i,1}u'\left(c_{i,1}\right)}{h'\left(e_{i,1}\right)} - 1$ and the following first-order conditions

$$\left[e_{H,1}\right]: \left(-\mathrm{g}_H - \eta_1 + \eta_1 \frac{h'\left(\frac{w_{L,1}e_{L,1}}{w_{H,1}}\right)}{h'\left(e_{H,1}\right)} \frac{\partial\left(\frac{w_{L,1}}{w_{H,1}}\right)}{\partial e_{H,1}} e_{L,1}\right) h'\left(e_{H,1}\right) + \chi_1 f_{H,1} a_H\left(\bar{v}\right) \frac{\partial Y_1}{\partial l_1\left(\bar{v}\right)} = 0,$$

$$\left[e_{L,1}\right]: \left(-\mathrm{g}_L + \eta_1 \frac{\frac{w_{L,1}}{w_{H,1}} h'\left(\frac{w_{L,1}e_{L,1}}{w_{H,1}}\right)}{h'\left(e_{L,1}\right)} + \eta_1 \frac{h'\left(\frac{w_{L,1}e_{L,1}}{w_{H,1}}\right)}{h'\left(e_{L,1}\right)} \frac{\partial\left(\frac{w_{L,1}}{w_{H,1}}\right)}{\partial e_{L,1}} e_{L,1}\right) h'\left(e_{L,1}\right)$$
$$+ \chi_1 f_{L,1} a_L\left(\underline{v}\right) \frac{\partial Y_1}{\partial l_1\left(\underline{v}\right)} = 0,$$

we derive the labor wedges in the first period for type $i = H, L$, respectively, as follows.

$$\frac{\tau_H^{l_1}}{1-\tau_H^{l_1}} = \underbrace{\frac{-\eta_1 u'\left(c_{H,1}\right)}{\chi_1 f_{H,1}} \frac{h'\left(\frac{w_{L,1}e_{L,1}}{w_{H,1}}\right)}{h'\left(e_{H,1}\right)} \frac{\partial\left(\frac{w_{L,1}}{w_{H,1}}\right)}{\partial e_{H,1}} e_{L,1}}_{\text{wage compression<0}} + \underbrace{\frac{u'\left(c_{H,1}\right) a_H\left(\bar{v}\right)}{h'\left(e_{H,1}\right)} \left(w_1\left(\bar{v}\right) - \frac{\partial Y_1}{\partial l_1\left(\bar{v}\right)}\right)}_{\text{Pigouvian<0}} \tag{A39}$$

and

$$\frac{\tau_L^{l_1}}{1-\tau_L^{l_1}} = \frac{u'\left(c_{L,1}\right)\eta_1}{\chi_1 f_{L,1}} \left[\underbrace{1 - \frac{\frac{w_{L,1}}{w_{H,1}} h'\left(\frac{w_{L,1}e_{L,1}}{w_{H,1}}\right)}{h'\left(e_{L,1}\right)}}_{\text{Mirrlees>0}} + \underbrace{\frac{-h'\left(\frac{w_{L,1}e_{L,1}}{w_{H,1}}\right)}{h'\left(e_{L,1}\right)} \frac{\partial\left(\frac{w_{L,1}}{w_{H,1}}\right)}{\partial e_{L,1}} e_{L,1}}_{\text{wage compression >0}}\right]$$
$$+ \underbrace{\frac{u'\left(c_{L,1}\right) a_L\left(\underline{v}\right)}{h'\left(e_{L,1}\right)} \left[w_1\left(\underline{v}\right) - \frac{\partial Y_1}{\partial l_1\left(\underline{v}\right)}\right]}_{\text{Pigouvian<0}}. \tag{A40}$$

Using the fact that $\frac{\partial\left(\frac{w_{L,1}}{w_{H,1}}\right)}{\partial e_{i,1}} = \frac{-w_{L,1}}{w_{H,1}} \frac{\phi_i^1}{e_{i,1}}$ for $i = H, L$, equations (A39)-(A40) can be expressed as (A27) in Proposition A1.

In the second period, based on the definition of the labor wedge $\frac{\tau_{ij}^{l_2}}{1-\tau_{ij}^{l_2}} \equiv \frac{w_{i,1}u'\left(c_{ij,2}\right)}{h'\left(e_{ij,2}\right)} - 1$ and (A33)-(A36) and the following first-order conditions

$$\left[e_{LL,2}\right]: \left[\left(\frac{-\mathrm{g}_L f_{LL,2}}{f_{L,1}} + \frac{\eta_1 f_{HL,2}}{f_{H,1}}\right)\beta + \frac{\eta_{L,2} h'\left(\frac{w_{L,2}e_{LL,2}}{w_{H,2}}\right) \frac{w_{L,2}}{w_{H,2}}}{h'\left(e_{LL,2}\right)}\right.$$
$$\left. + \frac{\partial\left(\frac{w_{L,2}}{w_{H,2}}\right)}{\partial e_{LL,2}} \frac{\left[\eta_{L,2} h'\left(\frac{w_{L,2}e_{LL,2}}{w_{H,2}}\right) e_{LL,2} + \eta_{H,2} h'\left(\frac{w_{L,2}e_{HL,2}}{w_{H,2}}\right) e_{HL,2}\right]}{h'\left(e_{LL,2}\right)}\right] h'\left(e_{LL,2}\right) + \chi_2 f_{LL,2} a_L\left(\underline{v}\right) \frac{\partial Y_2}{\partial l_2\left(\underline{v}\right)} = 0,$$

$$\left[e_{LH,2}\right]:\left[\left(\frac{-g_L f_{LH,2}}{f_{L,1}}+\frac{\eta_1 f_{HH,2}}{f_{H,1}}\right)\beta-\eta_{L,2}\right.$$
$$\left.+\frac{\partial\left(\frac{w_{L,2}}{w_{H,2}}\right)}{\partial e_{LH,2}}\frac{\left[\eta_{L,2}h'\left(\frac{w_{L,2}e_{LL,2}}{w_{H,2}}\right)e_{LL,2}+\eta_{H,2}h'\left(\frac{w_{L,2}e_{HL,2}}{w_{H,2}}\right)e_{HL,2}\right]}{h'\left(e_{LH,2}\right)}\right]h'\left(e_{LH,2}\right)+\chi_2 f_{LH,2}a_H\left(\bar{v}\right)\frac{\partial Y_2}{\partial l_2\left(\bar{v}\right)}=0,$$

$$\left[e_{HL,2}\right]:\left[\frac{\left(-g_H-\eta_1\right)\beta f_{HL,2}}{f_{H,1}}+\frac{\eta_{H,2}h'\left(\frac{w_{L,2}e_{HL,2}}{w_{H,2}}\right)\frac{w_{L,2}}{w_{H,2}}}{h'\left(e_{HL,2}\right)}\right.$$
$$\left.+\frac{\partial\left(\frac{w_{L,2}}{w_{H,2}}\right)}{\partial e_{HL,2}}\frac{\left[\eta_{L,2}h'\left(\frac{w_{L,2}e_{LL,2}}{w_{H,2}}\right)e_{LL,2}+\eta_{H,2}h'\left(\frac{w_{L,2}e_{HL,2}}{w_{H,2}}\right)e_{HL,2}\right]}{h'\left(e_{HL,2}\right)}\right]h'\left(e_{HL,2}\right)+\chi_2 f_{HL,2}a_L\left(\underline{v}\right)\frac{\partial Y_2}{\partial l_2\left(\underline{v}\right)}=0,$$

$$\left[e_{HH,2}\right]:\left[\frac{\left(-g_H-\eta_1\right)\beta f_{HH,2}}{f_{H,1}}-\eta_{H,2}\right.$$
$$\left.+\frac{\partial\left(\frac{w_{L,2}}{w_{H,2}}\right)}{\partial e_{HH,2}}\frac{\left[\eta_{L,2}h'\left(\frac{w_{L,2}e_{LL,2}}{w_{H,2}}\right)e_{LL,2}+\eta_{H,2}h'\left(\frac{w_{L,2}e_{HL,2}}{w_{H,2}}\right)e_{HL,2}\right]}{h'\left(e_{HH,2}\right)}\right]h'\left(e_{HH,2}\right)+\chi_2 f_{HH,2}a_H\left(\bar{v}\right)\frac{\partial Y_2}{\partial l_2\left(\bar{v}\right)}=0,$$

we derive the labor wedge in the second period for type $ij=LL,LH,HL,HH$, respectively, as follows.

$$\frac{\tau^{l_t}_{LL}}{1-\tau^{l_t}_{LL}}=\frac{\eta_{L,2}u'\left(c_{LL,2}\right)}{\chi_2 f_{LL,2}}\left[\underbrace{1-\frac{h'\left(\frac{w_{L,2}e_{LL,2}}{w_{H,2}}\right)\frac{w_{L,2}}{w_{H,2}}}{h'\left(e_{LL,2}\right)}}_{\text{Mirrlees}}+\underbrace{\frac{-\left[h'\left(\frac{w_{L,2}e_{LL,2}}{w_{H,2}}\right)e_{LL,2}+\frac{\eta_{H,2}}{\eta_{L,2}}h'\left(\frac{w_{L,2}e_{HL,2}}{w_{H,2}}\right)e_{HL,2}\right]}{h'\left(e_{LL,2}\right)}\frac{\partial\left(\frac{w_{L,2}}{w_{H,2}}\right)}{\partial e_{LL,2}}}_{\text{wage compression}}\right]$$
$$+\underbrace{\frac{u'\left(c_{LL,2}\right)a_L\left(\underline{v}\right)}{h'\left(e_{LL,2}\right)}\left[w_2\left(\underline{v}\right)-\frac{\partial Y_2}{\partial l_2\left(\underline{v}\right)}\right]}_{\text{Pigouvian}},\tag{A41}$$

$$\frac{\tau^{l_2}_{LH}}{1-\tau^{l_2}_{LH}}=\underbrace{\frac{-u'\left(c_{LH,2}\right)\left[\eta_{L,2}h'\left(\frac{w_{L,2}e_{LL,2}}{w_{H,2}}\right)e_{LL,2}+\eta_{H,2}h'\left(\frac{w_{L,2}e_{HL,2}}{w_{H,2}}\right)e_{HL,2}\right]}{\chi_2 f_{LH,2}h'\left(e_{LH,2}\right)}\frac{\partial\left(\frac{w_{L,2}}{w_{H,2}}\right)}{\partial e_{LH,2}}}_{\text{wage compression}}$$
$$+\underbrace{\frac{u'\left(c_{LH,2}\right)a_H\left(\bar{v}\right)}{h'\left(e_{LH,2}\right)}\left[w_2\left(\bar{v}\right)-\frac{\partial Y_2}{\partial l_2\left(\bar{v}\right)}\right]}_{\text{Pigouvian}},\tag{A42}$$

$$\frac{\tau_{HL}^{l_t}}{1-\tau_{HL}^{l_t}} = \frac{\eta_{H,2}u'\left(c_{HL,2}\right)}{\chi_2 f_{HL,2}}\left[\underbrace{1-\frac{h'\left(\frac{w_{L,2}e_{HL,2}}{w_{H,2}}\right)\frac{w_{L,2}}{w_{H,2}}}{h'\left(e_{HL,2}\right)}}_{\text{Mirrlees}} + \underbrace{\frac{-\left[\frac{\eta_{L,2}}{\eta_{H,2}}h'\left(\frac{w_{L,2}e_{LL,2}}{w_{H,2}}\right)e_{LL,2}+h'\left(\frac{w_{L,2}e_{HL,2}}{w_{H,2}}\right)e_{HL,2}\right]}{h'\left(e_{HL,2}\right)}\frac{\partial\left(\frac{w_{L,2}}{w_{H,2}}\right)}{\partial e_{HL,2}}}_{\text{wage compression}}\right]$$
$$+\underbrace{\frac{u'\left(c_{HL,2}\right)a_L\left(\underline{v}\right)}{h'\left(e_{HL,2}\right)}\left[w_1\left(\underline{v}\right)-\frac{\partial Y_1}{\partial l_1\left(\underline{v}\right)}\right]}_{\text{Pigouvian}}, \tag{A43}$$

$$\frac{\tau_{HH}^{l_2}}{1-\tau_{HH}^{l_2}} = \underbrace{\frac{-u'\left(c_{HH,2}\right)\left[\eta_{L,2}h'\left(\frac{w_{L,2}e_{LL,2}}{w_{H,2}}\right)e_{LL,2}+\eta_{H,2}h'\left(\frac{w_{L,2}e_{HL,2}}{w_{H,2}}\right)e_{HL,2}\right]}{\chi_2 f_{HH,2}h'\left(e_{HH,2}\right)}\frac{\partial\left(\frac{w_{L,2}}{w_{H,2}}\right)}{\partial e_{HH,2}}}_{\text{wage compression}}$$
$$+\underbrace{\frac{u'\left(c_{HH,2}\right)a_H\left(\bar{v}\right)}{h'\left(e_{HH,2}\right)}\left[w_2\left(\bar{v}\right)-\frac{\partial Y_2}{\partial l_2\left(\bar{v}\right)}\right]}_{\text{Pigouvian}}. \tag{A44}$$

Using the fact that $\frac{\partial\left(\frac{w_{L,2}}{w_{H,2}}\right)}{\partial e_{ij,2}} = \frac{-w_{L,2}}{w_{H,2}}\frac{\phi_i^2}{e_{ij,2}}$ for $ij = LL, LH, HL, HH$, equations (A41)-(A44) can be expressed as (A28) in Proposition A1.

(iii) R&D wedges

Based on the definition of R&D wedge (4e), and the following first-order conditions

$$\begin{aligned}
\left[q_1\left(v\right)\right]:&\eta_1 h'\left(\frac{w_{L,1}e_{L,1}}{w_{H,1}}\right)e_{L,1}\frac{\partial\left(\frac{w_{L,1}}{w_{H,1}}\right)}{\partial b_1\left(v\right)}\frac{\partial A_1^v}{\partial q_1\left(v\right)}\\
&+\left[\eta_{L,2}h'\left(\frac{w_{L,2}e_{LL,2}}{w_{H,2}}\right)e_{LL,2}+\eta_{H,2}h'\left(\frac{w_{L,2}e_{HL,2}}{w_{H,2}}\right)e_{HL,2}\right]\frac{\partial\left(\frac{w_{L,2}}{w_{H,2}}\right)}{\partial b_2\left(v\right)}\frac{\partial A_2^v}{\partial b_1\left(v\right)}\frac{\partial A_1^v}{\partial q_1\left(v\right)}\\
&+\chi_1\frac{\partial Y_1}{\partial b_1\left(v\right)}\frac{\partial A_1^v}{\partial q_1\left(v\right)}+\chi_2\frac{\partial Y_2}{\partial b_2\left(v\right)}\frac{\partial A_2^v}{\partial b_1\left(v\right)}\frac{\partial A_1^v}{\partial q_1\left(v\right)}-\chi_1 M_q'\left(q_1\left(v\right)\right)=0,\\
\left[q_2\left(v\right)\right]:&\left[\eta_{L,2}h'\left(\frac{w_{L,2}e_{LL,2}}{w_{H,2}}\right)e_{LL,2}+\eta_{H,2}h'\left(\frac{w_{L,2}e_{HL,2}}{w_{H,2}}\right)e_{HL,2}\right]\frac{\partial\left(\frac{w_{L,2}}{w_{H,2}}\right)}{\partial b_2\left(v\right)}\frac{\partial A_2^v}{\partial q_2\left(v\right)}\\
&+\chi_2\left[\frac{\partial Y_2}{\partial b_2\left(v\right)}\frac{\partial A_2^v}{\partial q_2\left(v\right)}-M_q'\left(q_2\left(v\right)\right)\right]=0,
\end{aligned}$$

we can derive the R&D wedges for each sector $v = \bar{v}, \underline{v}$, for each period $t = 1, 2$ as follows:

$$\begin{aligned}
s^{q_1}(v) &\equiv M_q'(q_1(v)) - \frac{\partial A_1^v}{\partial q_1(v)}\left[\sum_{s=1}^{\sum_{s=1}} 2\frac{1}{\mathcal{R}_s^1(v)}\pi_s^{\prime}(b_s(v))\Gamma_1^s(v)\right] \\
&= \underbrace{\frac{\eta_1 e_{L,1}}{\chi_1}h'\left(\frac{w_{L,1}e_{L,1}}{w_{H,1}}\right)\frac{\partial\left(\frac{w_{L,1}}{w_{H,1}}\right)}{\partial b_1(\bar{v})}\frac{\partial A_1^{\bar{v}}}{\partial q_1(v)} + \frac{\left[\eta_{L,2}h'\left(\frac{w_{L,2}e_{LL,2}}{w_{H,2}}\right)e_{LL,2}+\eta_{H,2}h'\left(\frac{w_{L,2}e_{HL,2}}{w_{H,2}}\right)e_{HL,2}\right]}{\chi_1}\frac{\partial\left(\frac{w_{L,2}}{w_{H,2}}\right)}{\partial b_2(v)}\frac{\partial A_2^v}{\partial b_1(v)}\frac{\partial A_1^v}{\partial q_1(v)}}_{\text{wage compression}} \\
&\quad + \underbrace{\left[\frac{\partial Y_1}{\partial b_1(v)} - \pi_1'(b_1(v))\right]\frac{\partial A_1^v}{\partial q_1(v)} + \left[\frac{\chi_2}{\chi_1}\frac{\partial Y_2}{\partial b_2(v)} - \pi_2'(b_2(v))\right]\frac{\partial A_2^v}{\partial b_1(v)}\frac{\partial A_1^v}{\partial q_1(v)}}_{\text{Pigouvian}}
\end{aligned} \tag{A45}$$

$$\begin{aligned}
s^{q_2}(v) &\equiv M_q'(q_2(v)) - \frac{\partial A_2^v}{\partial q_2(v)}\pi_2'(b_2(v)) \\
&= \underbrace{\frac{\left[\eta_{L,2}h'\left(\frac{w_{L,2}e_{LL,2}}{w_{H,2}}\right)e_{LL,2}+\eta_{H,2}h'\left(\frac{w_{L,2}e_{HL,2}}{w_{H,2}}\right)e_{HL,2}\right]}{\chi_2}\frac{\partial\left(\frac{w_{L,2}}{w_{H,2}}\right)}{\partial b_2(v)}\frac{\partial A_2^v}{\partial q_2(v)}}_{\text{wage compression}} \\
&\quad + \underbrace{\left[\frac{\partial Y_2}{\partial b_2(v)} - \pi_2'(b_2(v))\right]\frac{\partial A_2^v}{\partial q_2(v)}}_{\text{Pigouvian}}
\end{aligned} \tag{A46}$$

Using the fact that $\frac{\partial\left(\frac{w_{L,t}}{w_{H,t}}\right)}{\partial b_t(v)} = \frac{-w_{L,t}}{w_{H,t}}\frac{\varphi_v^t}{b_t(v)}$ for sector $v = \bar{v}$, $\underline{v}$, equations (A45)-(A46) can be expressed as (A29) in Proposition A1.

(iv) Corporate wedges

Based on the definition of corporate wedge (4c), and the following first-order conditions

$$\begin{aligned}
[n_1(v)]:&\eta_1 h'\left(\frac{w_{L,1}e_{L,1}}{w_{H,1}}\right)e_{L,1}\frac{\partial\left(\frac{w_{L,1}}{w_{H,1}}\right)}{\partial b_1(v)}\frac{\partial A_1^v}{\partial n_1(v)} \\
&+\left[\eta_{L,2}h'\left(\frac{w_{L,2}e_{LL,2}}{w_{H,2}}\right)e_{LL,2}+\eta_{H,2}h'\left(\frac{w_{L,2}e_{HL,2}}{w_{H,2}}\right)e_{HL,2}\right]\frac{\partial\left(\frac{w_{L,2}}{w_{H,2}}\right)}{\partial b_2(v)}\frac{\partial A_2^v}{\partial b_1(v)}\frac{\partial A_1^v}{\partial n_1(v)} \\
&+\chi_1\frac{\partial Y_1}{\partial b_1(v)}\frac{\partial A_1^v}{\partial n_1(v)} + \chi_2\frac{\partial Y_2}{\partial b_2(v)}\frac{\partial A_2^v}{\partial b_1(v)}\frac{\partial A_1^v}{\partial n_1(v)} - \chi_1 M_n'(n_1(v)) = 0, \\
[n_2(v)]:&\left[\eta_{L,2}h'\left(\frac{w_{L,2}e_{LL,2}}{w_{H,2}}\right)e_{LL,2}+\eta_{H,2}h'\left(\frac{w_{L,2}e_{HL,2}}{w_{H,2}}\right)e_{HL,2}\right]\frac{\partial\left(\frac{w_{L,2}}{w_{H,2}}\right)}{\partial b_2(v)}\frac{\partial A_2^v}{\partial n_2(v)} \\
&+\chi_2\left[\frac{\partial Y_2}{\partial b_2(v)}\frac{\partial A_2^v}{\partial n_2(v)} - M_n'(n_2(v))\right] = 0,
\end{aligned}$$

we can derive the R&D wedges for each sector $v = \bar{v}, \underline{v}$, for each period $t = 1, 2$ as follows:

$$\begin{aligned}
\tau^{n_1}(v) &\equiv \frac{\partial A_1^v}{\partial n_1(v)}\left[\sum_{s=1}^{2}\frac{1}{\mathcal{R}_s^1(v)}\pi_s'(b_s(v))\Gamma_1^s(v)\right] - M_n'(n_1(v)) \\
&= \underbrace{\frac{-\eta_1 e_{L,1}}{\chi_1}h'\left(\frac{w_{L,1}e_{L,1}}{w_{H,1}}\right)\frac{\partial\left(\frac{w_{L,1}}{w_{H,1}}\right)}{\partial b_1(\bar{v})}\frac{\partial A_1^{\bar{v}}}{\partial n_1(v)} - \frac{\left[\eta_{L,2}h'\left(\frac{w_{L,2}e_{LL,2}}{w_{H,2}}\right)e_{LL,2} + \eta_{H,2}h'\left(\frac{w_{L,2}e_{HL,2}}{w_{H,2}}\right)e_{HL,2}\right]}{\chi_1}\frac{\partial\left(\frac{w_{L,2}}{w_{H,2}}\right)}{\partial b_2(v)}\frac{\partial A_2^v}{\partial b_1(v)}\frac{\partial A_1^v}{\partial n_1(v)}}_{\text{wage compression}} \\
&+ \underbrace{\left[\pi_1'(b_1(v)) - \frac{\partial Y_1}{\partial b_1(v)}\right]\frac{\partial A_1^v}{\partial n_1(v)} + \left[\pi_2'(b_2(v)) - \frac{\chi_2}{\chi_1}\frac{\partial Y_2}{\partial b_2(v)}\right]\frac{\partial A_2^v}{\partial b_1(v)}\frac{\partial A_1^v}{\partial n_1(v)}}_{\text{Pigouvian}}
\end{aligned} \tag{A47}$$

$$\begin{aligned}
\tau^{n_2}(v) &\equiv \frac{\partial A_2^v}{\partial n_2(v)}\pi_2'(b_2(v)) - M_n'(n_2(v)) \\
&= \underbrace{\frac{-\left[\eta_{L,2}h'\left(\frac{w_{L,2}e_{LL,2}}{w_{H,2}}\right)e_{LL,2} + \eta_{H,2}h'\left(\frac{w_{L,2}e_{HL,2}}{w_{H,2}}\right)e_{HL,2}\right]}{\chi_2}\frac{\partial\left(\frac{w_{L,2}}{w_{H,2}}\right)}{\partial b_2(v)}\frac{\partial A_2^v}{\partial n_2(v)}}_{\text{wage compression}} \\
&+ \underbrace{\left[\pi_2'(b_2(v)) - \frac{\partial Y_2}{\partial b_2(v)}\right]\frac{\partial A_2^v}{\partial n_2(v)}}_{\text{Pigouvian}}
\end{aligned} \tag{A48}$$

Using the fact that $\frac{\partial\left(\frac{w_{L,t}}{w_{H,t}}\right)}{\partial b_t(v)} = \frac{-w_{L,t}}{w_{H,t}}\frac{\varphi_v^t}{b_t(v)}$ for sector $v = \overline{v}, \underline{v}$, equations (A47)-(A48) can be expressed as (A30) in Proposition A1.

□

B.3 *General function form of technology evolution*

For tractability, we follow Akcigit, Hanley and Stantcheva. (2022) and adopt the multiplicate evolution (12a) of the technology level $b_t(v)$ in our quantitative analysis. We now consider varying degrees of complementarity between R&D investment and R&D inputs by using the following general function form.

$$b_t(v) = (1-\delta_b)\, b_{t-1}(v) + \left(\frac{1}{2} q_t(v)^{1-\varrho_{qn}} + \frac{1}{2} n_t(v)^{1-\varrho_{qn}}\right)^{\frac{2}{1-\varrho_{qn}}}. \tag{A49}$$

Note that the functional form (A49) can be reduced to (12a) as $\varrho_{qn} \to 1$. In Figure A1, we demonstrate the labor wedge, the capital wedge, the R&D wedge and the corporate wedge in our model under three different complementarity values between R&D investment and R&D inputs, $\varrho_{qn} = 0.8,\ 1,\ 1.2$. As can be seen, these wedges are almost the same under these three different complementarity values. For the labor wedge and capital wedge, they are even indistinguishable to the naked eyes. This result implies that the complementarity between R&D investment and R&D inputs has little impacts on the wedges in our model.

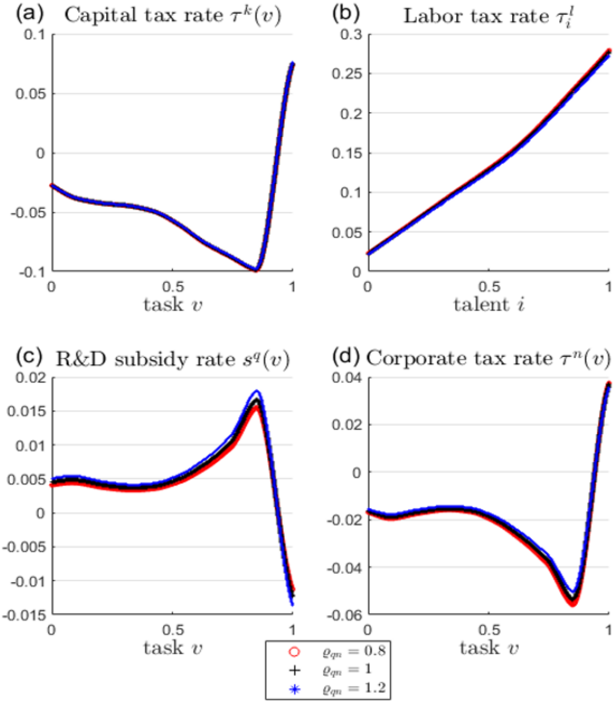

FIGURE A1. Wedges under different complementarity ϱ_{qn}.

B.4 *General final goods production form*

For simplicity, we follow Akcigit, Hanley and Stantcheva. (2022) and assume that the outputs of different sectors are perfect substitutes in the final goods production. In our quantitative analysis, we use the final goods production function described in (12b). However, in this section, we consider imperfectly substitutable goods in the general final goods production in A50.

$$Y_t = \left[\int_{\underline{v}}^{\bar{v}} \left[b_t\left(v\right)^{1-\alpha} y_t\left(v\right)^{\alpha} \right]^{\varepsilon_f} dv \right]^{1/\varepsilon_f}, \tag{A50}$$

where $\frac{1}{1-\varepsilon_f}$ is the elasticity of substitution of intermediate goods in the final goods production. When $\varepsilon_f \to 1$ the production function reduces to (12b).

In Figure A2, we show that the capital wedge, the labor wedge, the R&D wedge and the corporate wedge in our model for different degrees of substitution ε_f. As shown, these four wedges exhibit similar shapes across various degrees of substitution ε_f. The capital wedge, labor wedge and corporate wedge increase with ε_f, while the R&D wedge decreases with ε_f.

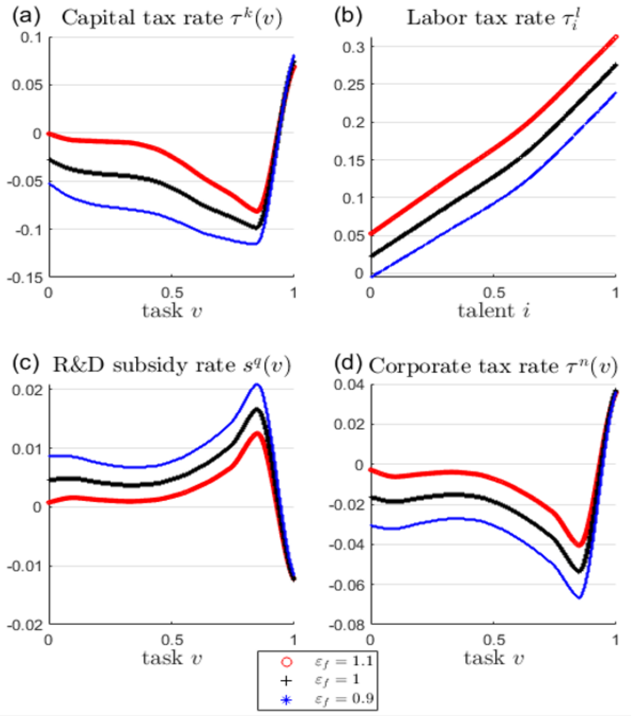

FIGURE A2. Wedges under different degrees of substitution ε_f.

B.5 *Wage growth and technology level under different shape parameter* κ

In Subsection 4.3, we demonstrate that the U-shaped patterns of wage growth and technology evolution are driven by the sectoral elasticity of substitution. To examine whether these patterns could also be a result from our assumption regarding the talent distribution, we compute wage growth and technology levels using different values of the shape parameter, κ. As shown in Figure A3, the U-shaped patterns remain robust across these alternative values of κ.

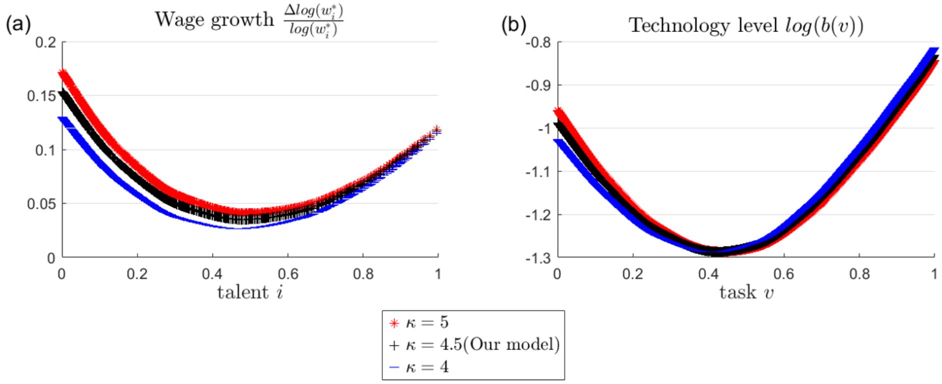

FIGURE A3. Wage growth (a) and technology level (b) with different shape parameter.

B.6 *Implementation through Personal Income Taxes, Profit Taxes and R&D Subsidies*

This section addresses how the optimal allocation implied by the wedge formula in (10a)-(10c) can be implemented by a tax system in the decentralized economy.

Suppose that $A^* = \left\{c_t^*(i), e_t^*(i), k_t^{i*}(v), l_t^*(v), y_t^*(v), q_t^*(v), n_t^*(v), b_t^*(v)\right\}$ is the optimal allocation in the second-best solution of the social planning problem in Section 3. In this section, we construct a tax system to implement the second-best solution in the decentralized economy. We must note that the tax system we construct is not a unique one that can implement the second-best solution. Other tax systems have been proposed in the existing literature; see, for example, Kocherlakota (2005), Albanesi and Sleet (2006), and Chen and Liang (2024). Different from the existing literature, our tax system incorporates not only personal income taxes but also corporate taxes and R&D subsidies, as there are firms doing innovations, which have monopoly power and positive profits.

Personal income taxes. Suppose that type i agent's net capital income and labor income are, respectively, denoted by

$$\mathbb{K}_t(i) = \int \left[(R_t(v) + 1 - \delta_k) k_t^i(v) - k_{t+1}^i(v)\right] dv$$
$$\mathbb{L}_t(i) = w_t(i) e_t(i).$$

The personal income tax $T_t(\{k_t^i(v)\},\mathbb{L}_t(i))$ is constructed as follows

$$T_t\left(\left\{k_t^i(v)\right\},\mathbb{L}_t(i)\right)\begin{cases}\mathbb{K}_t(i)+\mathbb{L}_t(i)-c_t^*(j)\,, & \text{if } \exists j \text{s.t}\, \mathbb{L}_t(i)=\mathbb{L}_t^*(j) \text{ and } k_t^i(v)=k_t^{j*}(v) \text{ for all } v,\\ \mathbb{K}_t(i)+\mathbb{L}_t(i)-c_{0,t}^*, & \text{otherwise}\end{cases} \tag{A51}$$

where $c_{0,t}^*$ satisfies $\sum_{t=1}^{T}\beta^{t-1}u(c_{0,t}^*)\le\min\{U^*(j)\ |\forall j\in[\underline{i},\bar{i}]\}$, and $\mathbb{L}_t^*(j)=w_t^*(j)\,e_t^*(j)$.

Agent's problem. Given the personal income tax T_t, talent i agent chooses $\left\{c_t(i),e_t(i),k_t^i(v)\right\}$ to maximize the following lifetime utility

$$\max U(i)=max\sum_{t=1}^{T}\beta^{t-1}\left[u\left(c_t(i)\right)-h\left(e_t(i)\right)\right], \tag{A52}$$

subject to the following budget constraint

$$c_t(i)+\int_{\underline{v}}^{\bar{v}}k_{t+1}^i(v)dv\le\int\left[\left(R_t(v)+1-\delta_k\right)k_t^i(v)\right]dv+w_t(i)\,e_t(i)-T_t\left(\left\{k_t^i(v)\right\},\mathbb{L}_t(i)\right)$$

Firm's R&D innovations, R&D subsidies, and profit taxes. As to the tax on intermediate goods producers, we construct a linear profit tax rate $\tau^f(v)$ and a nonlinear R&D subsidy $S_t^v(q_t(v),n_t(v))$:

$$\begin{aligned}S_t^v\left(q_t(v),n_t(v)\right)=&\tau^f(v)\left[\mathcal{C}_q\left(q_t(v)\right)+\mathcal{C}_n\left(n_t(v)\right)\right]+\left(1-\tau^f(v)\right)s^{q_t}(v)\,q_t(v)\\&-\left(1-\tau^f(v)\right)\tau^{n_t}(v)\,n_t(v),\end{aligned} \tag{A53}$$

where $s^{q_t}(v)$ is the R&D wedge defined in (4e), and $\tau^{n_t}(v)$ is the corporate wedge (or profit wedge) defined in (4c). Given these R&D subsidies and corporate taxes, the firm's R&D problem in (3b) becomes the following problem.

$$\max_{q_t(v),n_t(v)}\sum_{s=t}^{T}\frac{1}{\mathcal{R}_s^t(v)}\left[\left(1-\tau^f(v)\right)\pi_s\left(b_s(v)\right)-\mathcal{C}_q\left(q_s(v)\right)-\mathcal{C}_n\left(n_s(v)\right)+S_s^v\left(q_s(v),n_s(v)\right)\right], \tag{A54}$$

where $\pi_s(b_s(v))$ is in (2b) and $b_s(v)$ evolves according to (3a).

Government. Given the government expenditure G_t, and the tax and the subsidy schemes described above, the periodic government budget constraint is as follows:

$$G_t+\int_{\underline{v}}^{\bar{v}}S_t^v\left(q_t(v),n_t(v)\right)dv\le\int f(i)\left[T_t\left(\left\{k_t^i(v)\right\},\mathbb{L}_t(i)\right)\right]di+\int_{\underline{v}}^{\bar{v}}\tau^f(v)\pi_t\left(b_t(v)\right)dv \tag{A55}$$

Given the agent's optimization problem, the firm's maximization problem, and the government's budget constraint, we are ready to define the tax equilibrium as follows:

Tax equilibrium. Given the government spending G_t, a tax equilibrium consists of a personal income tax function T_t, a linear profit tax $\tau^f(v)$, a non-linear R&D subsidy $S_t^v(q_t(v),n_t(v))$, an allocation profile

$$\left\{c_t(i),e_t(i),k_t^i(v),\lambda_t(i,v),\ l_t(v),q_t(v),n_t(v)|\forall i\in[\underline{i},\bar{i}],v\in[\underline{v},\bar{v}]\right\},$$

and price and profit profiles $\{p(b_t(v),y_t(v)),w_t(v)\,,\ R_t(v),\ \pi(b_t(v))|\,\forall v\in[\underline{v},\overline{v}]\}$, such that
(i) the allocation $\{c_t(i)\,,e_t(i)\,,k_t^i(v)\}$ solves the agent's optimization problem in (A52);
(ii) the allocation $\{l_t(v)\,,k_t(v)\,,q_t(v)\,,n_t(v)\}$ solves firms' maximization problems in (2b) and (A54), with $b_t(v)$ satisfying (3a) and $k_t(v)=\int_{\underline{i}}^{\overline{i}}k_t^i(v)\,f(i)di$ for $v\in[\underline{v},\overline{v}]$;
(iii) goods markets clear: (3d) holds;
(iv) labor markets clear: (3e)-(3f) hold;
(v) the government's budget constraint (A55) holds;
(vi) price and profit profiles $\{p(b_t(v),y_t(v))\,,w_t(v)\,,R_t(v)\,,\pi(b_t(v))\}$ satisfy (2a)-(2c).

Proposition A3 below shows that these tax and subsidy polices, along with the second-best allocation, can form a tax equilibrium that satisfies all the conditions (i)-(vi) above.

PROPOSITION A3. **(Tax implemenatation)** Let the government spending be G_t and the second-best allocation of the social planning problem be

$$A^*=\{c_t^*(i),e_t^*(i)\,,k_t^*(v)\,,l_t^*(v)\,,y_t^*(v)\,,q_t^*(v)\,,n_t^*(v)\,,b_t^*(v)\}\,.$$

Then, the profiles of taxes and subsidies, including the personal income tax T_t in (A51) and the linear profit tax rate $\tau^f(v)$ and the nonlinear R&D subsidy $S_t^v(q_t(v)\,,n_t(v))$ in (A53), satisfy the following two conditions

$$\max_{q_t(v),n_t(v)}\sum_{s=t}^{T}\frac{1}{\mathcal{R}_s^t(v)}\left[\left(1-\tau^f(v)\right)\pi_s(b_s(v))-\mathcal{C}_q(q_s(v))-\mathcal{C}_n(n_s(v))+S_s^v(q_s(v)\,,n_s(v))\right], \tag{A56}$$

$$G_t+\int_{\underline{v}}^{\overline{v}}S_t^v(q_t^*(v)\,,n_t^*(v))\,dv\le\int f(i)\left[T_t\left(\left\{k_t^{i*}(v)\right\},\mathbb{L}_t^*(i)\right)\right]di+\int_{\underline{v}}^{\overline{v}}\tau^f(v)\pi_t(b_t^*(v))\,dv, \tag{A57}$$

which can implement the second-best allocation in a decentralized economy.

PROOF OF PROPOSITION A3.

First of all, the price and profit profiles $\{p(b_t^*(v)\,,y_t^*(v))\,,\omega_t^*(v)\,,R_t^*(v)\,,\pi(b_t^*(v))\}$ are set to satisfy (1c)-(2c) with respect to the second-best allocation.
Second, we need to show that given the price profile, the second-best allocation $\{c_t^*(i),e_t^*(i)\,,k_t^{i*}(v)\,|\forall i,\ v\}$ solves the agent's utility maximization problem (A52). Based on the income tax T_t defined in (A51), we can find that the best strategy for agents with type i is to restrict their labor income and capital allocation in a sector v, respectively, to be $w_t^*(j)e_t^*(j)$ and $k_t^{j*}(v)$ for some $j\in[\underline{i},\overline{i}]$. Therefore, the agent's problem (A52) can be transformed as the following problem:

$$\max_j\sum_{t=1}^{T}\beta^{t-1}\left[u(c_t^*(j))-h\left(\frac{w_t^*(j)e_t^*(j)}{w_t^*(i)}\right)\right]$$

Since the second-best allocation is incentive-compatible, then $j=i$. Hence, the second-best allocation indeed solves the agent's maximization problem.

Third, we need to show that the second-best allocation also solves the firm's profit maximization problem. Since price and profit profiles

$$\left\{p\left(b_t^*\left(v\right), y_t^*\left(v\right)\right), \omega_t^*\left(v\right), R_t^*\left(v\right), \pi\left(b_t^*\left(v\right)\right)\right\}$$

are set to satisfy (1c)-(2c) one can easily find that the second-best allocation $\left\{l_t^*\left(v\right), k_t^*\left(v\right)\right\}$ solves firms' maximization problems in (2b). We can also prove that the second-best R&D inputs $\left\{q_t^*\left(v\right), n_t^*(v)\right\}$ solves the firm's maximization problem in (A54), by the use of the R&D wedge $s^{q_t}\left(v\right)$ and corporate wedge $\tau^{n_t}\left(v\right)$ defined in (4e) and (4c), and the following two first-order conditions of the firm's maximization problem in (A54):

$$[q_t\left(v\right)] : \left(1-\tau^f\left(v\right)\right)\left(\left[\sum_{s=t}^{T}\frac{1}{\mathcal{R}_s^t\left(v\right)}\pi_s'\left(b_s\left(v\right)\right)\Gamma_t^s\left(v\right)\right]\frac{\partial A_t^v}{\partial q_t\left(v\right)}-\mathcal{C}_q'\left(q_t\left(v\right)\right)\right)+\left(1-\tau^f\left(v\right)\right)s^{q_t}\left(v\right)=0$$

$$[n_t\left(v\right)] : \left(1-\tau^f\left(v\right)\right)\left(\left[\sum_{s=t}^{T}\frac{1}{\mathcal{R}_s^t\left(v\right)}\pi_s'\left(b_s\left(v\right)\right)\Gamma_t^s\left(v\right)\right]\frac{\partial A_t^v}{\partial n_t\left(v\right)}-\mathcal{C}_n'\left(n_t\left(v\right)\right)\right)-\left(1-\tau^f\left(v\right)\right)\tau^{n_t}\left(v\right)=0.$$

The condition (A56) implies that the linear profit tax rate $\tau^f\left(v\right)$ and the nonlinear R&D subsidy $S_t^v\left(q_t^*\left(v\right), n_t^*\left(v\right)\right)$ guarantees a non-negative profit, which provides intermediate goods firms' incentives to participate in the production, and the condition (A57) implies that government 's budget constraint (A55) is met under the second-best allocation. Finally, according to the resource constraint in the planning problem, it is obvious that the second-best allocation clears the goods market. Moreover, based on Proposition 1, it is obvious that the second-best allocation also clears the labor market. Then, we complete the proof.

□

References

Albanesi, S. and C. Sleet (2006) Dynamic optimal taxation with private information. *Review of Economic Studies*, 73, 1-30. [18]

Akcigit, Ufuk, Douglas Hanley, and Stefanie Stantcheva (2022) Optimal taxation and R&D policies. *Econometrica*, 90 (2), 645-684. [16, 17]

Chen, Been-Lon. and Fei-Chi. Liang (2024) Optimal taxation in the life cycle with human capital. *Review of Economic Dynamics*, 52, 21-45. [18]

Kocherlakota, Narayana R. (2005) Zero expected wealth taxes: A Mirrlees approach to dynamic optimal taxation. *Econometrica*, 73, 1587-1621. [18]