

WELFARE IMPLICATIONS AND EQUILIBRIUM INDETERMINACY IN A TWO-SECTOR GROWTH MODEL WITH CONSUMPTION EXTERNALITIES

BEEN-LON CHEN

Academia Sinica

YU-SHAN HSU

National Chung Cheng University

KAZUO MINO

Institute of Economic Research, Kyoto University

In one-sector neoclassical growth models, consumption externalities lead to an inefficient allocation in a steady state and indeterminate equilibrium toward a steady state only if there is a labor–leisure trade-off. This paper shows that in a two-sector neoclassical growth model, even without a labor–leisure trade-off, consumption spillovers easily lead to an inefficient allocation in a steady state and indeterminate equilibrium toward a steady state. Negative consumption spillovers that yield over-accumulation of capital in a one-sector model may lead to underaccumulation or overaccumulation of capital in two-sector models, depending on the relative capital intensity between sectors. Moreover, a two-sector model economy with consumption externalities is less stabilized than an otherwise identical one-sector model economy.

Keywords: Two-Sector Model, Consumption Externalities, Efficiency, Indeterminacy

1. INTRODUCTION

Economists have paid a great deal of attention to consumption externalities since the inception of modern economic thought.¹ Some empirical and experimental studies provide convincing support for the significance of consumption externalities associated with social comparison: see, for example, Clark and Oswald (1996), Oswald (1997), and Luttmer (2005).² Many macro models specify preferences for which an average consumption level of the economy affects individuals' felicity.

We thank Roger Farmer, Xavier Raurich, Jang-Ting Guo, two anonymous referees of this journal, and participants at the 2011 Asian Meeting of the Econometrics Society held in Seoul (Korea) for valuable suggestions on earlier drafts. Chen acknowledges the financial support of the Taiwan National Science Council through Grant NSC 101–2410-H-001–003–MY3. Address correspondence to: Been-Lon Chen, Institute of Economics, Academia Sinica, 128 Academia Road Section 2, Taipei 11529, Taiwan; e-mail: bchen@econ.sinica.edu.tw.

In particular, this departure from standard preference specifications has been presented in growth models in order to account for some empirical phenomena that cannot be explained under more traditional specifications.

Generally speaking, we can classify consumption externalities as the “keeping up with the Joneses” (hereinafter, KIJ) effect and the “catching up with the Joneses” (henceforth, CIJ) effect. The KIJ effect emerges when *contemporary* average consumption in the economy makes a marginal increase of personal current consumption more valuable, whereas the CIJ effect arises when *past* average consumption in the economy makes a marginal increase of personal current consumption more valuable. Many studies found the KIJ effect to be of central importance in accounting for asset prices [Gali (1994)], capital accumulation [Fisher and Hof (2000); Dupor and Liu (2003); Liu and Turnovsky (2005); Arrow and Dasgupta (2009)], consumption across generations [Abel (2005)], business cycles [Chen and Hsu (2007); Alonso-Carrera et al. (2008a)], and dynamic altruism [Alonso-Carrera et al. (2008b)]. Moreover, a number of analyses revealed the CIJ effect upon asset prices [Abel (1990); Campbell and Cochrane (1999)], business cycles [Ljungqvist and Uhlig (2000)], capital accumulation [Turnovsky and Monteiro (2007)], and long-term balanced growth [Alvarez-Cuadrado et al. (2004); Chen (2007); Doi and Mino (2008)] to be crucial.³

In one-sector neoclassical growth models with constant time preference rates, only if there is a labor–leisure trade-off, contemporary consumption externalities can lead to an inefficient allocation in a steady state and generate indeterminate dynamic equilibrium paths toward a steady state. The reason that the labor–leisure trade-off plays such a role can be explained by using a negative consumption externality as an illustrative example. First, Liu and Turnovsky (2005) and Turnovsky and Monteiro (2007) show that negative spillover leads to overconsumption that is associated with inefficiently less leisure time and more labor supply and, with fixed capital intensity in the long run, capital is thus overaccumulated.⁴ Moreover, Alonso-Carrera et al. (2008a) show that, if negative consumption spillovers give the Frisch labor supply a certain shape, the dynamic equilibrium toward a steady state is indeterminate.⁵

This paper shows that in a two-sector neoclassical growth model, even without a labor–leisure trade-off, contemporary consumption spillovers can yield not only an inefficient allocation in a steady state but also indeterminate dynamic equilibrium paths toward a steady state. We show these results in a two-sector model where the general goods sector produces goods that are used as consumption and investment and the consumption goods sector produces goods that are used only as consumption. The reasons for having these results lie in relative prices of these two goods and factor allocation between the two sectors. In a producer’s optimization, the price of general goods relative to consumption goods equals the marginal rate of transformation (hereinafter, MRT), the ratio of the marginal product in the consumption sector to the marginal product in the general goods sector. In a consumer’s optimization, the relative price of general goods equals the marginal rate of substitution (hereinafter, MRS), the ratio of the marginal utility of general

goods to that of consumption goods. The presence of consumption externalities distorts the relationship between MRS and MRT, which may generate inefficiency and indeterminacy.

Our two-sector setting has two advantages over the foregoing studies with regard to the role of consumption externalities in dynamic macroeconomics. First, our model is more general than the models found in the extant literature. In effect, when the consumption goods sector does not use capital, the goods are the consumption of leisure and our model is reduced to a one-sector model with elastic leisure, which was also studied by Liu and Turnovsky (2005) and Alonso-Carrera et al. (2008a). The second advantage of our modeling is that, unlike the case of homogeneous-good models, we can treat commodity-specific externalities. We show that asymmetric external effects may play a key role in equilibrium determinacy. It should be noted that Ravn et al. (2006), Doi and Mino (2008) and Hori (2011) studied commodity-specific consumption externalities. In analyzing models with monopolistic competition and a variety of consumption goods, those authors assume that consumers put the same importance on all types of consumption of other households. In such a situation, a consumer would be concerned equally about the consumption of food and autos by other consumers. However, recent studies in behavioral economics have offered evidence that consumers place different importance on different types of consumption by other consumers. See, for example, Solnick and Hemenway (1998), Alpizar et al. (2005), and Carlsson et al. (2007). They investigated the positionality degree for different goods based on empirical or experimental approaches. Our formulation follows such a research agenda.

In this paper we first explore the relation between consumption externalities and efficiency of the steady-state equilibrium. To see how inefficiency emerges, we consider as an example a negative externality of general goods consumption. Because there is no production externality, the MRT in the market is equal to the MRT in a centrally planned economy, thereby indicating the same capital intensity in a market and a centrally planned economy. The negative externality leads to a lower individual marginal utility of general goods consumption; thus, the MRS in a market economy is lower than the MRS in a centrally planned economy. With concave utility, a lower market MRS indicates an equilibrium general goods consumption that is too high and equilibrium consumption goods that are too low in the long run compared with their respective social optimum. As a result, the allocation in the market equilibrium is different from the efficient level.

Next, we explain the indeterminacy in terms of the KUJ effect of general goods consumption. Indeterminate equilibrium emerges if self-fulfilling expectations can be supported as an equilibrium. Suppose that the representative agent expects an increase in the relative price of general goods. The agent will reallocate input factors from the consumption goods sector to the general goods sector, which increases the MRT. Yet more production of general goods increases general goods consumption. If general goods consumption has no KUJ effect, more general goods consumption lowers the marginal utility of general goods consumption, so the

MRS is lower than the MRT. Thus, self-fulfilling expectations of a higher relative price of general goods cannot be supported as an equilibrium. However, when general goods consumption has a KIJ effect, more general goods consumption can increase the MRS to keep pace with the higher MRT and the higher relative price of general goods. As a result, self-fulfilling expectations about higher relative prices of general goods can be supported as an equilibrium.

We should note that in one-sector growth models, when production externalities establish indeterminacy, this requires that labor supply be elastic and in particular that the labor supply and demand curves cross with the “wrong slopes” [e.g., Benhabib and Farmer (1994); Farmer and Guo (1994)]. Moreover, when consumption externalities create indeterminacy in one-sector growth models, a labor–leisure trade-off is still necessary so that the externality can cause the Frisch labor supply to have a certain shape, even though the labor supply need not cross the labor demand with the “wrong slopes” [Alonso-Carrera et al. (2008a)]. In our two-sector model, general goods consumption externalities produce indeterminacy even when there is no labor–leisure trade-off.⁶

Our primary findings are as follows. First, even with a negative spillover of general goods consumption, capital is over- (resp. under-) accumulated if the general goods sector is more (resp. less) capital-intensive than the consumption sector. This result is in sharp contrast to the one-sector model studied by Liu and Turnovsky (2005) and Turnovsky and Monteiro (2007), in which a negative externality of general goods consumption must lead to an overaccumulation of capital. Although consumption externalities distort the allocation of capital in a two-sector model, we find that no active capital taxes are necessary once the sources of consumption distortions are identified and corrected by consumption taxes.

Next, when the general goods sector is more capital-intensive, each of general goods consumption externalities and consumption externalities can easily establish indeterminacy, and it is easier for the consumption externality in the consumption goods sector to produce indeterminacy. The consumption good is reduced to leisure if capital is not an input in this sector. This foregoing result is contrary to existing one-sector models studied by Benhabib and Farmer (2000) and Weder (2004), who found that the leisure externality itself cannot create indeterminacy. Further, if there are symmetric degrees of consumption externalities from both goods, the utility is homothetic and the competitive equilibrium is efficient in a steady state. However, we still find indeterminate equilibrium because symmetric externalities produce different shadow prices of capital between a market and a socially planned economy, which cause market failures in transitions. The result is different from the one-sector growth model studied by Alonso-Carrera et al. (2008a) in which consumption externalities in a homothetic utility do not lead to indeterminacy. Finally, it does not matter whether consumption externalities are from general goods, consumption goods, or both goods; we find that it is much easier for a two-sector growth model to trigger indeterminacy than for an otherwise identical one-sector growth model with elastic leisure to do so.

We organize this paper as follows. We define a two-sector model with consumption externalities in Section 2. In Section 3, we study welfare properties. In Section 4, we investigate the dynamic properties of equilibrium. Finally, concluding remarks are found in Section 5.

2. THE BASIC MODEL

The economy is populated by a representative firm and a representative household. There are two sectors: the general goods (y_1) and consumption goods sectors (y_2). The general goods sector produces goods that are used as consumption and investment and the consumption goods sector produces pure consumption goods only. We will also refer to general goods as good 1 and to consumption goods as good 2.⁷ The representative firm hires labor and rents capital in order to produce goods in the two sectors. The representative household has a fixed supply of labor, which is normalized to unity, and chooses savings and consumption of both goods.

2.1. Technology

The production function is

$$y_i = f^i(k_i, l_i), \quad i = 1, 2, \quad (1)$$

where k_i and l_i are capital and labor allocated to sector i .

We assume that the function f^i is twice continuously differentiable and is homogeneous of degree one with respect to both inputs. Moreover, the function is strictly increasing and strictly concave in inputs and satisfies the Inada condition.⁸ Our basic assumption is that sector 1 is more capital intensive than sector 2, but we also consider the opposite case.

2.2. Preference

The representative household supplies all its labor to work, and there is thus no leisure activity. The household's utility is affected not only by personal consumption but also by average consumption in the society. Let $\rho > 0$ denote the time preference rate, c_i denote personal consumption of goods $i = 1, 2$, and \bar{c}_i denote average consumption of goods i in the society. The agent's lifetime utility is represented by

$$U = \int_0^\infty e^{-\rho t} u(c_1, c_2, \bar{c}_1, \bar{c}_2) dt. \quad (2)$$

We assume that the instantaneous utility function is twice continuously differentiable and is strictly increasing and strictly concave in c_1 and c_2 . The effect of \bar{c}_i may be positive or negative. It is said that the household is "admiring" in good c_i if $\partial u / \partial \bar{c}_i > 0$ and "jealous" in good c_i if $\partial u / \partial \bar{c}_i < 0$ [e.g., Dupor and Liu (2003); Liu and Turnovsky (2005)]. Moreover, the consumption activity is described as "KUI" if $\partial^2 u / (\partial c_i \partial \bar{c}_i) > 0$ [e.g., Gali (1994); Ljungqvist and Uhlig (2000)].

2.3. Resource Constraints and Markets

Denote k as total capital (per capita) in the economy at a point in time and k_i as capital allocated to sector $i = 1, 2$. The resource constraints in the economy at a point in time are given by

$$k = k_1 + k_2 = sk + (1 - s)k, \tag{3a}$$

$$1 = l_1 + l_2 = l + (1 - l). \tag{3b}$$

With a fixed labor supply normalized at unity, y_i in (1) is output per capita produced in sector i . Let the endogenous fraction of total capital in the economy allocated to sector 1 be denoted by $s \in (0,1)$; then $k_1 = sk$. Thus, the fraction of total capital going to sector 2 is $1-s$, so that $k_2 = (1-s)k$. We denote the endogenous fraction of labor allocated to sector 1 as $l \in (0,1)$, with the remaining fraction going to sector 2 being $(1-l)$.

Finally, the goods market clearance conditions in the economy are

$$\dot{k} = f^1(sk, l) - c_1 - \delta k, \tag{4a}$$

$$c_2 = f^2((1-s)k, 1-l), \tag{4b}$$

where δ is the rate of depreciation of capital.

Note that when y_2 does not use capital, c_2 provides only leisure services. In this case, our model is reduced to a one-sector neoclassical growth model with leisure and leisure externalities. If leisure exhibits no externalities, this is the model studied by Liu and Turnovsky (2005) and Alonso-Carrera et al. (2008a).

3. INEFFICIENCY OF ALLOCATION IN A STEADY STATE

3.1. Allocation in a Decentralized Economy and in a Socially Planned Economy

In a competitive market economy, the representative agent takes \bar{c}_i as given by the society. By substituting (4b) into (2), the representative agent's problem is to choose c_1, s, l , and k in order to maximize (2) subject to (4a). The optimal conditions for c_1, s, l , and k are

$$u_1(c_1, c_2, \bar{c}_1, \bar{c}_2) = \lambda, \tag{5a}$$

$$\frac{\lambda}{u_2(c_1, c_2, \bar{c}_1, \bar{c}_2)} = \frac{f_2^2((1-s)k, 1-l)}{f_2^1(sk, l)} = p, \tag{5b}$$

$$\frac{f_2^2((1-s)k, 1-l)}{f_2^1(sk, l)} = \frac{f_1^2((1-s)k, 1-l)}{f_1^1(sk, l)}, \tag{5c}$$

$$\frac{\dot{\lambda}}{\lambda} = (\rho + \delta) - f_1^1(sk, l), \tag{5d}$$

where λ is the shadow price of capital and p is the relative price of general goods in terms of consumption goods.

In these optimal conditions, (5a) equates the marginal utility of general goods consumption to the shadow price of capital, which determines general goods consumption. In (5b), the MRS between two types of consumption is equal to the MRT between two sectors and is also equal to the relative price of general goods. In (5c), the MRT between the two sectors must equal for both capital and labor. Finally, (5d) is the Euler equation that governs how the shadow price of capital changes over time.

In a socially planned economy, the planner internalizes the consumption externality \bar{c}_i . Thus, the optimal conditions for c_1 , s , l , and k are

$$u_1(c_1, c_2, \bar{c}_1, \bar{c}_2) + u_3(c_1, c_2, \bar{c}_1, \bar{c}_2) = \lambda^p, \quad (6a)$$

$$\frac{\lambda^p}{u_2(c_1, c_2, \bar{c}_1, \bar{c}_2) + u_4(c_1, c_2, \bar{c}_1, \bar{c}_2)} = \frac{f_2^2[(1-s)k, 1-l]}{f_2^1(sk, l)}, \quad (6b)$$

$$\frac{f_2^2[(1-s)k, 1-l]}{f_2^1(sk, l)} = \frac{f_1^2[(1-s)k, 1-l]}{f_1^1(sk, l)}, \quad (6c)$$

$$\frac{\dot{\lambda}^p}{\lambda^p} = (\rho + \delta) - f_1^1(sk, l), \quad (6d)$$

where λ^p is the shadow price of capital.

3.2. Efficiency of Allocation in a Decentralized Economy

The equilibrium conditions in a competitive market include (4a) and (4b) and (5a)–(5d), along with $\bar{c}_i = c_i$. The efficient allocation conditions in a socially planned economy are (4a) and (4b) and (6a)–(6d) with $\bar{c}_i = c_i$. In a steady state when $\dot{k} = \dot{\lambda} = \dot{\lambda}^p = 0$, these two sets of conditions are the same except for (5a) and (5b) in a decentralized economy and (6a) and (6b) in a socially planned economy. Combining (5a) and (5b) gives

$$\text{MRS} \equiv \frac{u_1(c_1, c_2, \bar{c}_1, \bar{c}_2)}{u_2(c_1, c_2, \bar{c}_1, \bar{c}_2)} = \frac{f_2^2[(1-s)k, 1-l]}{f_2^1(sk, l)} \equiv \text{MRT}, \quad (7a)$$

and combining (6a) and (6b) yields

$$\text{MRS}^p \equiv \frac{u_1(c_1, c_2, \bar{c}_1, \bar{c}_2) + u_3(c_1, c_2, \bar{c}_1, \bar{c}_2)}{u_2(c_1, c_2, \bar{c}_1, \bar{c}_2) + u_4(c_1, c_2, \bar{c}_1, \bar{c}_2)} = \frac{f_2^2[(1-s)k, 1-l]}{f_2^1(sk, l)} \equiv \text{MRT}^p. \quad (7b)$$

Because the MRT in (7a) is the same as the MRT^p in (7b), the allocation in a decentralized economy is the same as the allocation in a socially planned economy if the MRS in (7a) is the same as the MRS^p in (7b). We obtain the following result.

PROPOSITION 1. *In a neoclassical growth model with general goods and consumption goods, the competitive equilibrium allocation is efficient in a steady*

state if and only if

$$\frac{u_1(c_1, c_2, \bar{c}_1, \bar{c}_2)}{u_2(c_1, c_2, \bar{c}_1, \bar{c}_2)} = \frac{u_3(c_1, c_2, \bar{c}_1, \bar{c}_2)}{u_4(c_1, c_2, \bar{c}_1, \bar{c}_2)} \tag{8}$$

for all feasible c_1 and c_2 , where $\bar{c}_1 = c_1$ and $\bar{c}_2 = c_2$.

Condition (8) stipulates that the MRS between personal consumption of goods 1 and 2 must be equal to the MRS^p between the *social* consumption of goods 1 and 2. Only when this condition is met is the equilibrium allocation of consumption, labor, and capital in a competitive market efficient in the long run; otherwise, the equilibrium allocation is inefficient.

A typical specification of the utility function that satisfies condition (8) is a function that is multiplicatively separable between (c_1, c_2) and (\bar{c}_1, \bar{c}_2) , such that

$$u(c_1, c_2, \bar{c}_1, \bar{c}_2) = v(c_1, c_2)h[v(\bar{c}_1, \bar{c}_2)], \tag{9a}$$

where the $h(\cdot)$ is a monotonically increasing or decreasing function.

Another functional form satisfying (8) is one in which $u(\cdot)$ is weakly separable between (c_1, \bar{c}_1) and (c_2, \bar{c}_2) , such that

$$u(c_1, c_2, \bar{c}_1, \bar{c}_2) = V[h(c_1, \bar{c}_1)h(c_2, \bar{c}_2)], \tag{9b}$$

where $h(\cdot)$ is a homothetic function. In this case, (8) is written as $\frac{h_1(c_1, \bar{c}_1)}{h_1(c_2, \bar{c}_2)} = \frac{h_2(c_1, \bar{c}_1)}{h_2(c_2, \bar{c}_2)}$. Because $h(\cdot)$ is homothetic, we obtain $h_1(\frac{c_1}{\bar{c}_1}, 1)h_2(\frac{c_2}{\bar{c}_2}, 1) = h_1(\frac{c_2}{\bar{c}_2}, 1)h_2(\frac{c_1}{\bar{c}_1}, 1)$, which always holds when $\bar{c}_i = c_i$.

When condition (8) fails to hold, a competitive equilibrium allocation is inefficient in a steady state. Then the consumption externality can lead to an inefficient allocation even though there is no leisure choice. In the one-sector neoclassical growth model studied by Liu and Turnovsky (2005), the consumption externality produces an inefficient allocation in a steady state only when there is a labor–leisure trade-off. In particular, in a one-sector growth model, a jealous (i.e., negative) consumption externality leads to an overaccumulation of capital and an admiring (i.e., positive) consumption externality results in an underaccumulation of capital. In contrast, in our two-sector growth model, a jealous (or admiring) general goods consumption externality may cause an overaccumulation or underaccumulation of capital depending on relative capital intensities. To see this, we use a parametric version of our model with the following utility function with a constant elasticity of substitution and the Cobb-Douglas production function

$$u(c_1, c_2, \bar{c}_1, \bar{c}_2) = \left[\gamma \left(\frac{c_1}{\bar{c}_1^{\theta_1}} \right)^{\frac{\epsilon-1}{\epsilon}} + (1-\gamma) \left(\frac{c_2}{\bar{c}_2^{\theta_2}} \right)^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}}, \quad 0 < \gamma \leq 1, \tag{10a}$$

$$f^i(k_i, l_i) = A_i k_i^{a_i} l_i^{1-a_i}, \quad A_i > 0, 0 \leq a_i < 1, \quad i = 1, 2, \tag{10b}$$

where α_i is the capital intensity in sector i . Parameter $\gamma > 0$ is the share of good 1 relative to good 2 in utility, and θ_i measures the degree of the external consumption effect arising from \bar{c}_i . A negative (i.e., jealous) consumption externality emerges if $\theta_i > 0$, whereas a positive (i.e., admiring) consumption externality occurs if $\theta_i < 0$. We assume that $\theta_i < 1$ to keep a positive social level of the marginal utility of each good. Parameter $\varepsilon > 0$ is the elasticity of substitution (hereinafter, ES) between two goods. Consumption i displays a KIJ effect if $\theta_i(\varepsilon - 1) < 0$.

Several special cases are worthy of mention:

1. If $\varepsilon = 1$ and $\gamma = 1$, goods 2 is not demanded and (10a) becomes $u = \log[c_1(\bar{c}_1)^{-\theta_1}]$. Our model is reduced to a one-sector growth model with consumption externalities and an inelastic labor supply. Thus, the competitive equilibrium allocation is efficient in the long run.
2. If $\varepsilon = 1$ and $\theta_1 = \theta_2 = \theta$, (10a) becomes $u = \log[(c_1^\gamma c_2^{1-\gamma})(\bar{c}_1^\gamma \bar{c}_2^{1-\gamma})^{-\theta}]$, which satisfies (9a). Thus (8) is satisfied and a competitive equilibrium allocation is efficient in the long run.
3. If $\varepsilon \neq 1$ and $\theta_1 = \theta_2 = \theta$, then (10a) is homothetic and satisfies (9b). Thus (8) is met and a competitive equilibrium allocation is efficient in the long run.
4. If $\varepsilon \neq 1$ and $\theta_1 \neq \theta_2$, the degree of externalities is asymmetric. Thus (8) fails to hold and a competitive equilibrium allocation is inefficient in a steady state. Condition (8) fails to be satisfied because

$$\frac{u_3(\cdot)}{u_4(\cdot)} = \frac{\theta_1 u_1(\cdot)}{\theta_2 u_2(\cdot)} \neq \frac{u_1(\cdot)}{u_2(\cdot)} = \frac{\gamma}{1 - \gamma} \left[\frac{(c_1)^{-1 - \theta_1(\varepsilon - 1)}}{(c_2)^{-1 - \theta_2(\varepsilon - 1)}} \right]^{\frac{1}{\varepsilon}} \quad \text{if } \varepsilon \neq 1, \theta_1 \neq \theta_2.$$

In what follows we focus on the case $\varepsilon \neq 1$ and $\theta_1 \neq \theta_2$. The following proposition characterizes a steady-state competitive equilibrium allocation for the parametric version of our model.

PROPOSITION 2. *Suppose $\varepsilon \neq 1$. Then, in a steady state, capital is overaccumulated if (i) $\alpha_1 > \alpha_2$ and $\theta_1 > \theta_2$ or (ii) $\alpha_1 < \alpha_2$ and $\theta_1 < \theta_2$ and underaccumulated if (i) $\alpha_1 > \alpha_2$ and $\theta_1 < \theta_2$ or (ii) $\alpha_1 < \alpha_2$ and $\theta_1 > \theta_2$.*

To obtain the properties in Proposition 2, we rewrite (3a) as

$$lx_1 + (1 - l)x_2 = k, \tag{11a}$$

where $x_1 = \frac{k_1}{l_1} = \frac{sk}{l}$ and $x_2 = \frac{k_2}{l_2} = \frac{(1-s)k}{1-l}$ are the capital intensities in sectors 1 and 2, respectively. Then the production function in (1) becomes

$$y_1 = lg_1(x_1), y_2 = (1 - l)g_2(x_2),$$

where $g_i(x_i) \equiv f^i(x_i, 1) = \frac{f^i(k_i, l_i)}{l_i}$. Thus, the relative price satisfies $p = \frac{f_1^2(\cdot)}{f_1^1(\cdot)} = \frac{g_2'(x_2)}{g_1'(x_1)}$ and (5c) becomes

$$\frac{g_2(x_2) - x_2 g_2'(x_2)}{g_2'(x_2)} = \frac{g_1(x_1) - x_1 g_1'(x_1)}{g_1'(x_1)}. \tag{11b}$$

In a steady state, $\varrho + \delta = f_1^l(x_1, 1) = g_1'(x_1)$, and, in view of (11b), the steady state levels of capital intensity, x_1 and x_2 , are uniquely determined and are independent of the presence of consumption externalities. The relative price p is thus uniquely determined. The steady state expression of (7a) in terms of the utility function (10a) is

$$\begin{aligned} \text{MRS} &\equiv \frac{u_1(\cdot)}{u_2(\cdot)} = \frac{\gamma}{1 - \gamma} \left[\frac{(c_2)^{1+\theta_2(\varepsilon-1)}}{(c_1)^{1+\theta_1(\varepsilon-1)}} \right]^{\frac{1}{\varepsilon}} \\ &= \frac{A_2(1 - a_2) \left[\frac{(1-s)k}{1-l} \right]^{a_2}}{A_1(1 - a_1) \left(\frac{sk}{l} \right)^{a_1}} = \frac{g_2'(x_2)}{g_1'(x_1)} \equiv \text{MRT} = p, \end{aligned} \tag{12a}$$

When the external effects are internalized, (7b) is expressed as

$$\begin{aligned} \text{MRS}^p &\equiv \frac{u_1(\cdot) + u_3(\cdot)}{u_2(\cdot) + u_4(\cdot)} = \frac{\gamma}{1 - \gamma} \left(\frac{1 - \theta_1}{1 - \theta_2} \right) \left[\frac{(c_2^p)^{1+\theta_2(\varepsilon-1)}}{(c_1^p)^{1+\theta_1(\varepsilon-1)}} \right]^{\frac{1}{\varepsilon}} \\ &= \frac{g_2'(x_2^p)}{g_1'(x_1^p)} \equiv \text{MRT}^p = p. \end{aligned} \tag{12b}$$

With no production externalities, (5d) and (6d) determine the same long-run capital–labor ratio $x_1^p = x_1 = \left(\frac{A_1 a_1}{\varrho + \delta} \right)^{1/(1-\alpha_1)}$ for a social optimum and an equilibrium and thus the MRT in (12a) and the MRT^p in (12b) are equal. As a result, the MRS in (12a) and the MRS^p in (12b) are equal. Under $\theta_1 = \theta_2 = \theta$, there are symmetric consumption externalities and the utility is homothetic. Because of the homothetic utility, the same allocation in a competitive equilibrium and in a social optimum leads to an equalization of the planner’s MRS^p in (12b) and the market’s MRS in (12a). As a result, an allocation in a market economy is the same as an allocation in a centrally planned economy and is thus efficient in the long run. Conversely, under $\theta_1 \neq \theta_2$ and thus $\frac{1-\theta_1}{1-\theta_2} \neq 1$, the equalization of a planner’s MRS^p in (12b) and a market’s MRS in (12a) indicates that a competitive equilibrium allocation is different from a social optimum allocation and thus is inefficient.

To see how the relative consumption externalities interact with relative capital intensities between the two sectors and determine whether capital is over- or underaccumulated, note that the equilibrium shares and the social optimum shares of capital and labor between the two sectors satisfy $\frac{1-l^p}{l^p} \frac{s^p}{1-s^p} = \frac{1-l}{l} \frac{s}{1-s} = \frac{a_1(1-a_2)}{a_2(1-a_1)}$. When this relationship is used, the equilibrium consumption good is

$$\begin{aligned} y_2 = c_2 &= A_2 \left(\frac{sk}{l} \right)^{a_2} \left(\frac{l}{1-l} \frac{1-s}{s} \right)^{a_2} (1-l) \\ &= A_2 \left(\frac{A_1 a_1}{\varrho + \delta} \right)^{\frac{a_2}{1-a_1}} \left[\frac{a_2(1-a_1)}{a_1(1-a_2)} \right]^{a_2} (1-l), \end{aligned} \tag{13a}$$

which is linear in the fraction of equilibrium labor allocated to the consumption goods sector.

When $\theta_1 > \theta_2 > 0$ and thus $(1 - \theta_1)/(1 - \theta_2) < 1$, the concave utility indicates that long-run equilibrium consumption c_1 is too high and equilibrium consumption c_2 is too low compared to the social optimum c_1^P and c_2^P , respectively. From (13a), c_2 is proportional to $1-l$ and thus $1-l$ is too low compared to the social optimum $1 - l^P$. A fixed labor supply in the economy indicates that equilibrium labor l is overvalued; thus $dl > 0$. The equilibrium capital stock k is higher than the social optimum k^P if the increase in l is larger than the increase in s . Specifically, a fixed $\frac{sk}{l}$ gives⁹

$$\frac{dk}{k} = \frac{dl}{l} - \frac{ds}{s} = \frac{dl}{l} - \frac{dl}{l} \frac{1-s}{1-l} > 0 \text{ if } s > l. \tag{13b}$$

If the capital intensity is the same in both sectors (i.e., $\alpha_1 = \alpha_2$), the fraction of capital and the fraction of labor in the economy allocated to sector 1 are the same ($s = l$). Thus, (13b) suggests that, despite consumption externalities, the equilibrium capital stock k is the same as the social optimum k^P . Conversely, if sector 1 is more capital-intensive than sector 2 (i.e., $\alpha_1 > \alpha_2$), then the fraction of capital allocated to sector 1 is larger than the fraction of labor allocated to sector 1 (i.e., $s > l$); if sector 1 is less capital-intensive than sector 2 (i.e., $\alpha_1 < \alpha_2$), then the fraction of capital allocated to sector 1 is smaller than the fraction of labor allocated to sector 1 (i.e., $s < l$). Thus, (13b) indicates that the steady-state equilibrium capital is overaccumulated if sector 1 is more capital-intensive ($\alpha_1 > \alpha_2$) and underaccumulated if sector 1 is less capital-intensive ($\alpha_1 < \alpha_2$).

Alternatively, when $\theta_2 > \theta_1 > 0$ and thus $(1 - \theta_1)/(1 - \theta_2) > 1$, steady-state equilibrium consumption c_1 is too low and equilibrium consumption c_2 is too high compared to the social optimum c_1^P and c_2^P , respectively. Overvalued c_2 indicates that the equilibrium $1-l$ is too high compared to the social optimum $1 - l^P$. Thus, l is undervalued, so that $dl < 0$. As a result, (13b) indicates that the steady-state capital is underaccumulated if sector 1 is more capital-intensive than sector 2 ($\alpha_1 > \alpha_2$) and overaccumulated if sector 1 is less capital-intensive ($\alpha_1 < \alpha_2$)

This completes the proof of Proposition 2.

Finally, it should be noted that that under $\alpha_2 = 0$, $c_2 = (1 - l)$ is leisure. If $\theta_2 = 0$, then our model is reduced to the one-sector growth model with elastic leisure studied by Liu and Turnovsky (2005). In this case, if the consumption of general goods is jealous (i.e., $\theta_1 > 0$), capital is overaccumulated in a steady state in a one-sector growth model. In addition to jealousy consumption, now we introduce jealousy leisure ($\theta_2 \neq 0$) into a one-sector growth model, which was not analyzed by Liu and Turnovsky (2005). In particular, if $\theta_2 > \theta_1 > 0$, because $\alpha_1 > \alpha_2 = 0$, equilibrium consumption c_1 is too low and equilibrium leisure $c_2 = 1 - l$ is too high compared to the social optima c_1^P and $c_2^P = 1 - l^P$, respectively. Thus, equilibrium labor supply l is undervalued, so that $l < l^P$. With the same capital-labor ratio for both the market and the social optimum in a steady state, the equilibrium of capital stock k is lower than the social optimum k^P .

As consumption externalities distort resource allocation in the long run, it follows that the allocation in the short run must be inefficient.¹⁰ There is an opportunity for government tax policy to improve efficiency. In the next subsection, we analyze the optimal tax policy.

3.3. Optimal Tax Policy

Consider again a decentralized economy. Let τ_k , τ_{c1} , and τ_{c2} be denoted as the tax rates on capital and the consumption of good 1 and good 2, respectively, and let T denote lump-sum transfers (or taxes).¹¹ The representative household maximizes the lifetime utility (2), subject to the budget constraint

$$\dot{k} = (1 - \tau_k)rk - (1 + \tau_{c1})c_1 - (1 + \tau_{c2})\frac{c_2}{p} - \delta k + T, \tag{14a}$$

where $r = f_1^1(sk, l) = \frac{1}{p} f_1^2[(1-s)k, 1-l]$ is the return to capital. The government maintains a balanced budget, rebating all tax revenues to the households in a lump-sum fashion:

$$\tau_k rk + \tau_{c1}c_1 + \tau_{c2}\frac{c_2}{p} = T. \tag{14b}$$

The objective is to characterize an optimal tax structure such that the decentralized economy mimics the dynamic equilibrium path of the centrally planned economy in (6a)–(6d). To achieve this, we allow the tax rates τ_k , τ_{c1} , and τ_{c2} to be time-varying. First, we derive the optimal conditions of a decentralized economy with taxes. Then, we replicate the allocation in a decentralized economy with that in a centralized economy. Replication involves setting time-varying tax rates such that the allocation of capital and consumption is equal in a decentralized economy and in a centralized economy, which requires $\frac{\dot{\lambda}}{\lambda} = \frac{\dot{\lambda}^p}{\lambda^p}$, or equivalently $\chi \lambda = \lambda^p$, where $\chi > 0$ is an arbitrary constant. The main results are as follows (see the Appendix).

PROPOSITION 3. *In a neoclassical economy with general goods and consumption goods, the entire time path of the optimal resource allocation can be obtained by setting tax rates at each instant of time in accordance with*

$$\begin{aligned} \tau_k = \bar{\tau}_k; 1 + \tau_{c2}(t) &= \xi \frac{u_2(c_1, c_2, c_1, c_2)}{u_2(c_1, c_2, c_1, c_2) + u_4(c_1, c_2, c_1, c_2)}; \\ \frac{1 + \tau_{c1}(t)}{1 + \tau_{c2}(t)} &= \frac{\frac{u_1(c_1, c_2, c_1, c_2)}{u_1(c_1, c_2, c_1, c_2) + u_3(c_1, c_2, c_1, c_2)}}{\frac{u_2(c_1, c_2, c_1, c_2)}{u_2(c_1, c_2, c_1, c_2) + u_4(c_1, c_2, c_1, c_2)}}, \end{aligned}$$

where $\bar{\tau}_k$ and ξ are arbitrary constants.

Intuitively, as there are consumption externalities and no production externalities, optimal consumption taxes are needed. If there are negative (or positive) consumption externalities of good $i = 1, 2$ and thus $u_i < 0$ (or $u_i > 0$), the

consumption of good i is more (or less) than an efficient level. Then, the government should tax (or subsidize) the consumption of good i and therefore, $\tau_{ci} > 0$ (or $\tau_{ci} < 0$). Although both consumption externalities distort the allocation of capital, no active capital taxes are necessary once the sources of consumption distortions are identified and corrected by consumption taxes. The capital income tax can thus be set to zero.

In a special case where good 2 is leisure, the tax on the consumption of good 2 is a tax on leisure; thus, it is a negative labor income tax. This is the case analyzed in Liu and Turnovsky (2005) where leisure has no externalities and thus $u_4 = 0$. In this case of $u_4 = 0$, the labor income tax is $\tau_w(t) = -\tau_{c2}(t)$ and is an arbitrary constant. Setting an arbitrary constant tax on labor income at $\bar{\tau}_w$ and identifying $\xi = (1 - \bar{\tau}_w)$ would give the following optimal tax rate on the consumption of goods 1:

$$\frac{1 + \tau_{c1}(t)}{1 - \bar{\tau}_w} = \frac{u_1(c_1, c_2, c_1, c_2)}{u_1(c_1, c_2, c_1, c_2) + u_3(c_1, c_2, c_1, c_2)}, \tag{14c}$$

which is identical to that in Liu and Turnovsky (2005, p. 1117).

In a more general case when good 2 is leisure and leisure has externalities ($u_4 \neq 0$), the optimal tax rate on labor income varies over time, given by $\tau_w \equiv -\tau_{c2} = \xi \frac{u_4}{u_2 + u_4}$. In particular, in the most general case, where the production of good 2 requires both capital and labor, the tax on the consumption of good 2 is different from the tax on labor income. In these general cases, the tax rate on the consumption of good 1 is the expression in Proposition 3, which is different from the expression in (14c).

4. INDETERMINACY IN TRANSITIONAL DYNAMICS

Consumption externalities not only cause an inefficient allocation in a steady state, but also generate indeterminate equilibrium paths toward a steady state. This section analyzes the conditions of indeterminacy.

4.1. Conditions of Indeterminacy

Dynamic equilibrium conditions in a competitive market are summarized by (4a) and (4b) and (5a)–(5d) with six variables: $c_1, c_2, s, l, \lambda,$ and k . Differently from a one-sector model, our model involves two goods, and it is difficult to simplify these equations to a dynamic system with a state vector $\{k, c_1\}$. We will simplify them to a system with a state vector $\{k, \lambda\}$ as follows.¹²

First, with the help of (4b) and (5a)–(5c), we use (5d) to obtain the Keynes–Ramsey condition,

$$\dot{\lambda} = \lambda\{(\varrho + \delta) - f_1^1[s(\lambda, k)k, l(\lambda, k)]\} \equiv J_1(\lambda, k). \tag{15a}$$

Next, with (4b) and (5a)–(5c), we rewrite the general goods market clearance condition (4a),

$$\dot{k} = f^1[s(\lambda, k)k, l(\lambda, k)] - c_1(\lambda, k) - \delta k \equiv J_2(\lambda, k). \tag{15b}$$

Equations (15a) and (15b) constitute a simplified dynamic system. The steady state is determined by $\dot{k} = 0$ and $\dot{\lambda} = 0$. The dynamic property of equilibrium is analyzed if we take Taylor's linear expansion of (15a) and (15b) around the steady state (k, λ) . The expansion gives

$$\begin{bmatrix} \dot{\lambda} \\ \dot{k} \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \begin{bmatrix} \lambda(t) - \lambda \\ k(t) - k \end{bmatrix}. \tag{15c}$$

The dynamic system (15c) includes a state variable whose initial value is given at $k(0)$. There are two roots associated with the Jacobian matrix in (15c), denoted by J . The steady state is a saddle if there is only one root with a negative real part and a sink if there are two roots with negative real parts. If the steady state is a sink, the equilibrium path toward the steady state is indeterminate.

Before analyzing dynamic properties, we investigate the existence and uniqueness of a steady state in the case without consumption externalities ($\theta_1 = \theta_2 = 0$). First, for the slope of the $\dot{k}^* = 0$ locus in the (k, λ) plane,¹³ we find $J_{21}^* > 0$ and $J_{22}^* > 0$ when k is small and $J_{22}^* < 0$ when k is large. The sign of J_{21}^* is positive, because a higher λ (a higher shadow price of capital) attracts more capital and labor to the general goods sector and thus increases general goods production. It also reduces general goods consumption, because of an increased cost of general goods consumption relative to consumption goods. Moreover, for the sign of J_{22}^* , when k is small, because of a high marginal product of capital, larger capital increases production of general goods more than consumption of general goods. When k is very large, because of a diminishing marginal product of capital, the production of general goods is increased less than the increase in the consumption of general goods. Thus, $J_{22}^* > 0$ when k is small and $J_{22}^* < 0$ when k is large. As a result, the $\dot{k}^* = 0$ locus is downward-sloping when k is smaller than a threshold and upward-sloping when k is larger than the threshold, just like the $\dot{k}^* = 0$ locus in the standard one-sector growth model. See Figure 1. Moreover, it is easy to show that $\dot{k}^* = -c_1 < 0$ when $k = 0$ and $\dot{k}^* = -\infty < 0$ when $k = \infty$, implying that the $\dot{k}^* = 0$ locus approaches $\lambda = \infty$ on both ends of $k = 0$ and $k = \infty$.

Next, for the slope of the $\dot{\lambda}^* = 0$ locus, we find $J_{11}^* < 0$ and $J_{12}^* > 0$. For a given k , a higher λ decreases the consumption of general goods, which increases the marginal utility of general goods consumption and leads to a higher MRS between c_1 and c_2 in (5b). In optimum, the marginal product of labor and capital in general goods relative to that in consumption goods in (5b) and (5c) needs to increase. Thus, $J_{11}^* < 0$. Further, larger capital has two effects. It decreases the marginal product of capital, which directly increases the shadow price of capital. As Sector 1 is more capital-intensive than Sector 2 under construction, the Rybczynski theorem stipulates that larger capital and labor shares are allocated to Sector 1. A larger labor share increases the marginal product of capital, which indirectly decreases the shadow price of capital. In general, the direct effect dominates the indirect effect and thus $J_{12}^* > 0$. As a result, the $\dot{\lambda}^* = 0$ locus is upward-sloping in the (λ, k) plane. See A1 and A2 in Figure 1. Moreover, it is clear that $\dot{\lambda}^* = -\infty < 0$

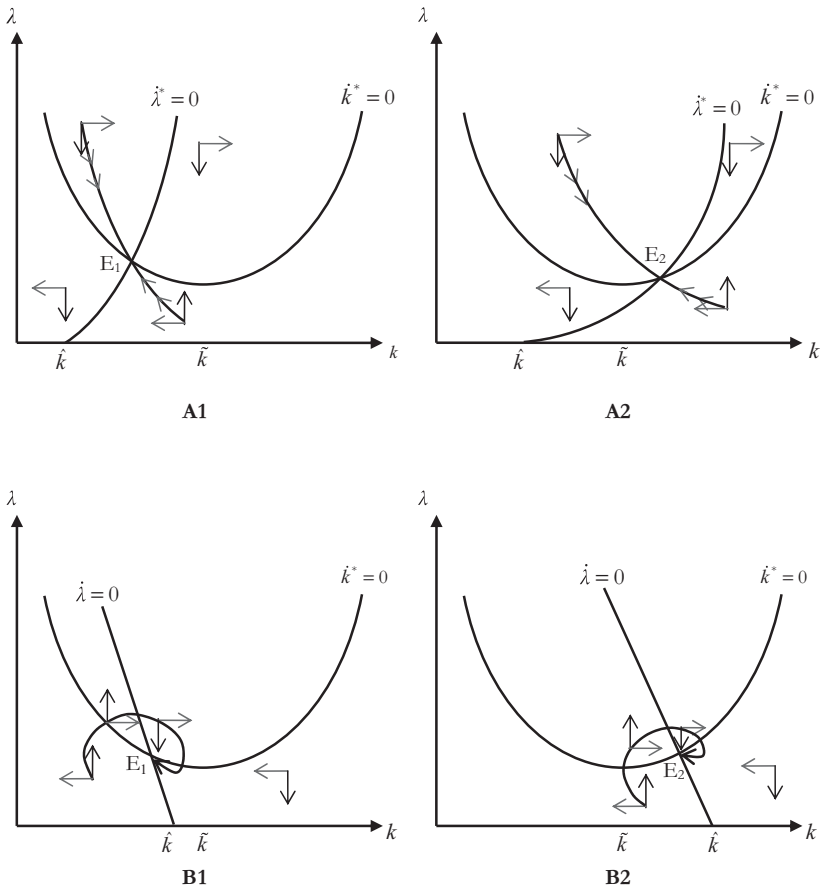


FIGURE 1. Two-sector growth model with a sufficiently large KUJ effect: a sink.

at $(k, \lambda) = (0, 0)$, as the Inada condition implies an infinite marginal product of capital. Thus, the $\dot{\lambda}^* = 0$ locus will start from a finite value of k , so that at $\lambda = 0$, the marginal product of capital can equal the sum of the discount rate and the depreciation rate.

The shape of the two loci indicates that the $\dot{\lambda}^* = 0$ locus intersects the $\dot{k}^* = 0$ locus only once, and there is a unique steady state (k, λ) . The $\dot{\lambda}^* = 0$ locus may intersect the $\dot{k}^* = 0$ locus at the downward (A1)- or upward (A2)-sloping segment, as illustrated in Figure 1.

In the two-sector growth model without consumption externalities, the steady state is a saddle. This indicates that there is only one root with a negative real part and the condition is $\text{Det}(J^*) = J_{11}^* J_{22}^* - J_{21}^* J_{12}^* < 0$. Under $J_{21}^* > 0$ and $J_{11}^* < 0$, $\text{Det}(J^*) < 0$ indicates that $(-J_{12}^*/J_{11}^*) > (-J_{22}^*/J_{21}^*) > (\text{or} <) 0$. Thus, the positive slope of the $\dot{\lambda}^* = 0$ locus is greater than the slope of the $\dot{k}^* = 0$. See the steady

states E_1 and in A1 and A2, Figure 1, where the intersection is, respectively, at the negative- and positive-slope sections of $\dot{k}^* = 0$.

However, if consumption exhibits externalities, the steady state may be a sink. Consider the KIJ effect as follows.

Condition KIJ. $\partial^2 u / (\partial c_i \partial \bar{c}_i) > 0$.

The KIJ effect may lead to $J_{11} > 0$, so the locus $\dot{\lambda} = 0$ is negatively sloping. To illustrate this point, suppose that general goods consumption exhibits a KIJ effect. Then, when the KIJ effect is sufficiently large, a higher λ increases rather than decreases general goods consumption, which reduces the MRS between c_1 and c_2 . At the optimum, the marginal product of labor and capital in the general goods sector relative to that in the consumption goods sector in (5b) and (5c) needs to decrease. Thus, $J_{11} > 0$, so the $\dot{\lambda} = 0$ locus is negatively sloping.

When the $\dot{\lambda} = 0$ locus is negatively sloping, the dynamic property of equilibrium may change. In particular, when the negatively sloping $\dot{\lambda} = 0$ is steeper than locus $\dot{k} = 0$, as illustrated in B1 and B2 of Figure 1, the steady state is a sink. This requires two roots with negative real parts, and the conditions are $\text{Det}(J) = J_{11}J_{22} - J_{21}J_{12} > 0$ and $\text{Tr}(J) = J_{11} + J_{22} < 0$, which are equivalent to $(-J_{12}/J_{11}) < (-J_{22}/J_{21}) > (\text{or} <) 0$. Thus, the negatively sloping $\dot{\lambda} = 0$ is steeper than locus $\dot{k} = 0$.

4.2. A Parametric Version

For ease of exposition, in this subsection we use the parametric version of the utility function (10a) and the production function (10b) to illustrate the dynamic properties of equilibrium. The utility function stipulates that if $[-\theta_i(\varepsilon - 1)] > 0$, then goods i consumption exhibits the KIJ effect. The 2×2 dynamical equations in (15a) and (15b) are derived as follows.

First, with the production function (10b), factor allocation between sectors in (5c) leads to

$$l = l(s) \equiv \left[1 + \frac{a_1(1-a_2)}{a_2(1-a_1)} \left(\frac{1}{s} - 1 \right) \right]^{-1}, \tag{16a}$$

where $l'(s) = \frac{a_1(1-a_2)}{a_2(1-a_1)} \frac{l^2}{s^2} > 0$. The positive sign is due to the complementarity of capital and labor. Feasibility of l restricts $s < \frac{a_1(1-a_2)}{a_2(1-a_1)}$. Note that $\alpha_1 > \alpha_2$ implies $\frac{a_1(1-a_2)}{a_2(1-a_1)} \frac{l}{s} > 1$ and thus $l'(s) > \frac{l}{s}$.

Next, the production function (9b) and the consumption goods equilibrium (4b) indicate

$$c_2 = c_2(s, k) \equiv c_2(s, k)A_2[(1-s)k]^{a_2}[1-l(s)]^{1-a_2}, \tag{16b}$$

where

$$\frac{\partial c_2}{\partial s} = -c_2 \left[\frac{a_2}{1-s} + \frac{1-a_2}{1-l} l'(s) \right] < 0,$$

$$\frac{\partial c_2}{\partial k} = \frac{a_2 c_2}{k} > 0.$$

Intuitively, a smaller share of capital allocated to Sector 2 (i.e., a larger s) reduces the production and consumption of good 2. Moreover, larger capital in the economy indicates more capital allocated to Sector 2, which increases the production and consumption of good 2.

Further, with the utility (10a), (5a) is $\gamma c_1^{\frac{(\varepsilon-1)(1-\theta_1)}{\varepsilon}} + (1-\gamma)c_2^{\frac{(\varepsilon-1)(1-\theta_2)}{\varepsilon}} = [\lambda c_1^{\frac{1+\theta_1(\varepsilon-1)}{\varepsilon}} \gamma^{-1}]^{\varepsilon-1}$, which gives

$$c_1 = c_1(\lambda, c_2), \quad (16c)$$

where

$$\begin{aligned} \frac{\partial c_1}{\partial \lambda} &= \frac{\varepsilon c_1}{\lambda} \frac{1+p(c_1/c_2)}{B} < 0 \text{ if } \theta_1 = 0; \text{ ambiguous if otherwise;} \\ \frac{\partial c_1}{\partial c_2} &= -(1-\theta_2) \frac{c_1}{c_2} \frac{1}{B} > 0 \text{ if } \theta_1 = \theta_2 = 0; \text{ ambiguous if otherwise;} \\ B &= -1 + [-\theta_1(\varepsilon-1)][1+p(c_1/c_2)\varepsilon/(\varepsilon-1)] < 0 \text{ if } \theta_1 = 0; \text{ ambiguous if otherwise.}^{14} \end{aligned}$$

Thus, if $\theta_1 = \theta_2 = 0$, $(\frac{\partial c_1}{\partial \lambda})^* = -\varepsilon \lambda^{-1} c_1 (1+p c_1/c_2) < 0$. Intuitively, it follows that without consumption externalities, for a given c_2 , a higher shadow price of capital must decrease c_1 in order to increase the marginal utility of c_1 . Moreover, $(\frac{\partial c_1}{\partial c_2})^* = \frac{c_1}{c_2} > 0$ if $\theta_1 = \theta_2 = 0$. This result comes because given λ , c_1 and c_2 are complementary in utility. However, with externalities from \bar{c}_1 , a higher shadow price of capital may increase c_1 if the KIJ effect $([-\theta_1(\varepsilon-1)] > 0)$ is sufficiently large. In this case, c_1 and c_2 are negatively related.

Furthermore, using (4b), (5a), and (16a)–(16c), the utility function (10a), and the production function (10b), the equalization of the MRS to the MRT in (5b) is rewritten as

$$\frac{\gamma c_2(s, k)^{\frac{1+\theta_2(\varepsilon-1)}{\varepsilon}}}{(1-\gamma)c_1(\lambda, c_2)^{\frac{1+\theta_1(\varepsilon-1)}{\varepsilon}}} = \frac{A_2(1-a_2)}{A_2(1-a_1)} \frac{(1-s)^{a_2}[1-l(s)]^{a_2}}{(s)^{a_1}[l(s)]^{a_1}} \frac{1}{k^{(a_1-a_2)}},$$

where the KIJ effect affects the MRS between goods 2 and 1 via $c_1(\lambda, c_2)$. This condition gives

$$s = s(\lambda, k), \quad (16d)$$

where

$$\begin{aligned} \frac{\partial s}{\partial \lambda} &= [1 + \theta_1(\varepsilon-1)] \frac{1}{\varepsilon} \frac{1}{c_1} \frac{\partial c_1}{\partial \lambda} \frac{1}{\Xi} > 0 \text{ if } \theta_1 = \theta_2 = 0; \text{ ambiguous if otherwise;} \\ \frac{\partial s}{\partial k} &= \left[\frac{1+\theta_1(\varepsilon-1)}{\varepsilon} \frac{c_2}{c_1} \frac{\partial c_1}{\partial c_2} - \frac{1+\theta_2(\varepsilon-1)}{\varepsilon} \right] \frac{a_2}{k} \frac{1}{\Xi} - \frac{a_1-a_2}{k} \frac{1}{\Xi} > 0 \text{ if } \theta_1 = \theta_2 = 0; \text{ ambiguous if otherwise;} \\ \Xi &= \left[\frac{a_2}{1-s} - \frac{a_2}{1-l} l'(s) \right] + \left[\frac{a_1}{s} - \frac{a_1}{l} l'(s) \right] + \left[\frac{1+\theta_1(\varepsilon-1)}{\varepsilon} \frac{c_2}{c_1} \frac{\partial c_1}{\partial c_2} - \frac{1+\theta_2(\varepsilon-1)}{\varepsilon} \right] \left[\frac{a_2}{1-s} + \frac{1-a_2}{1-l} l'(s) \right] < 0 \\ &\text{ if } \theta_1 = \theta_2 = 0; \text{ ambiguous if otherwise.} \end{aligned}$$

Note that $\frac{c_2}{c_1} (\frac{\partial c_1}{\partial c_2})^* = 1$ under $\theta_1 = \theta_2 = 0$, which gives $\Xi^* = \left[\frac{a_2}{1-s} - \frac{a_2}{1-l} l'(s) \right] + \left[\frac{a_1}{s} - \frac{a_1}{l} l'(s) \right] < 0$. Thus, under $\theta_1 = \theta_2 = 0$, $(\frac{\partial s}{\partial \lambda})^* = -\frac{1+p(c_1/c_2)}{\lambda^*} \frac{1}{\Xi^*} > 0$ and $(\frac{\partial s}{\partial k})^* = -(\frac{a_1}{a_2} - 1) \frac{a_2}{k} \frac{1}{\Xi^*} > 0$. Intuitively, it follows that without consumption externalities, for a given k , a higher shadow price of capital must decrease c_1 in order to increase the *MRS* between c_1 and c_2 . Optimally, a larger share of capital

needs to be allocated to the general goods sector (i.e., s increases) in order to decrease the marginal product of capital in general goods relative to consumption goods to increase the MRT. For a given λ , if capital increases, because Sector 1 is more capital-intensive than Sector 2, the Rybczynski theorem stipulates that more capital is allocated to Sector 1 and thus s increases. However, with consumption externalities, these relationships are ambiguous. In particular, with the KUJ effect, a higher shadow price of capital decreases the share of capital allocated to the general goods sector.

Finally, with the use of (16a)–(16d), the dynamic equations in (15a) and (15b) are

$$\dot{\lambda} = J_1(\lambda, k) \equiv \lambda \left((\varrho + \delta) - A_1 a_1 \left\{ \frac{l[s(\lambda, k)]}{s(\lambda, k)} \right\}^{1-a_1} \frac{1}{k^{1-a_1}} \right), \tag{17a}$$

$$\dot{k} = J_2(\lambda, k) \equiv A_1 s(\lambda, k)^{a_1} k^{a_1} l(s(\lambda, k))^{1-a_1} - c_1(\lambda, c_2(s(\lambda, k), k)) - \delta k. \tag{17b}$$

To envisage the dynamic property, by differentiating (17a) and (17b) around the steady state, with the use of (16a), we obtain the elements in the Jacobian matrix in (15c) as follows:

$$J_{11} = -\lambda(\varrho + \delta)(1 - a_1) \frac{(a_1 - a_2)l}{a_2(1 - a_1)s} \frac{\partial s}{\partial \lambda}, \tag{18a}$$

$$J_{12} = \lambda(\varrho + \delta)(1 - a_1) \left[\frac{1}{k} - \frac{(a_1 - a_2)l}{a_2(1 - a_1)s} \frac{\partial s}{\partial k} \right], \tag{18b}$$

$$J_{21} = \left\{ (c_1 + k) \left[\frac{a_1}{s} + \frac{1 - a_1}{l} l'(s) \right] + c_2 \left[\frac{a_2}{1 - s} + \frac{1 - a_2}{1 - l} l'(s) \right] \frac{\partial c_1}{\partial c_2} \right\} \frac{\partial s}{\partial \lambda} - \frac{\partial c_1}{\partial \lambda}, \tag{18c}$$

$$J_{22} = \left\{ (c_1 + \delta k) \left[\frac{a_1}{s} + \frac{1 - a_1}{l} l'(s) \right] + c_2 \left[\frac{a_2}{1 - s} + \frac{1 - a_2}{1 - l} l'(s) \right] \frac{\partial c_1}{\partial c_2} \right\} \frac{\partial s}{\partial k} + \frac{c_1}{k} \left[a_1 - a_2 \frac{c_2}{c_1} \frac{\partial c_1}{\partial c_2} \right] - (1 - a_1)\delta. \tag{18d}$$

In the case of $\theta_1 = \theta_2 = 0$ and thus the two-sector growth model without consumption externalities, (16d) indicates that $\frac{\partial s}{\partial \lambda} = \left(\frac{\partial s}{\partial \lambda}\right)^* > 0$ and $\left(\frac{\partial s}{\partial k}\right)^* > 0$. Thus, $J_{11}^* < 0$. As Sector 1 is more capital-intensive than Sector 2 under construction, an expansion of capital increases capital (sk) and labor (l) allocated to Sector 1; moreover, capital sk is increased proportionally more than the proportional increase of labor l , so that $J_{12}^* > 0$. As a result, the $\dot{\lambda}^* = 0$ locus is positive-sloping: $\frac{d\lambda}{dk} \Big|_{\dot{\lambda}=0} = -J_{12}^*/J_{11}^* > 0$.

Further, the slope of the $\dot{k}^* = 0$ locus is $\frac{d\lambda}{dk} \Big|_{\dot{k}=0} = -J_{22}^*/J_{21}^*$. Under $\theta_1 = \theta_2 = 0$, (16c) indicates $\frac{\partial c_1}{\partial \lambda} = \left(\frac{\partial c_1}{\partial \lambda}\right)^* < 0$ and thus $J_{21}^* > 0$. The sign of J_{22}^* depends on the threshold of k , denoted by \tilde{k} , where the marginal product of capital minus the effect

of capital on consumption is equal to the depreciation rate, and $J^*_{22} > (\text{resp. } <) 0$ when $k < (\text{resp. } >) \hat{k}$.¹⁵ Thus, $\dot{k}^* = 0$ is a U-shaped locus, as illustrated in Figure 1.

In a two-sector growth model without consumption externalities, the steady state is a saddle. The condition is $\text{Det}(J^*) < 0$ and is equivalent to $-J^*_{12}/J^*_{11} > -J^*_{22}/J^*_{21}$. The condition requires the positive slope of the $\dot{\lambda}^* = 0$ locus to be greater than the slope of the $\dot{k}^* = 0$ locus, as seen in A1 and A2, Figure 1. The $\dot{\lambda}^* = 0$ locus needs to start at a positive and finite value of $k = \hat{k} > 0$ for the marginal product of capital to equal the sum of the discount rate and the depreciation rate at $\lambda = 0$.¹⁶ With consumption externalities, the dynamic property of equilibrium determinacy may change.

Based on the analytical relations derived in the preceding, we investigate the indeterminacy conditions for special cases. When discussing calibrated models in these special cases that follow, we use the model without consumption externalities as the benchmark case. Although authors such as Maurer and Meier (2008) and Ravina (2007) have estimated the degree of external consumption effects in models with one type of consumption goods,¹⁷ their estimates correspond to the value of $\theta_1(\varepsilon - 1)$ if the two sectors are interpreted as general goods and home goods, or to an average over the two values of $\theta_1(\varepsilon - 1)$ and $\theta_2(\varepsilon - 1)$ if the two sectors are interpreted as manufacture and services. These estimates are not θ_1 and θ_2 . In fact, it is difficult to specify the magnitudes of θ_1 and θ_2 by using the standard estimation strategy based on the Euler equation of the representative household. Therefore, in what follows, we use the model with empirically plausible parameter values and $\theta_1 = \theta_2 = 0$ as the benchmark setting. Then, with all other values in the baseline parameterization remaining unchanged, we examine the levels of θ_1 and θ_2 with variations of ε and γ in preference specifications under which indeterminacy may emerge. As to the factor-intensity ranking between the two sectors, we focus mainly on the case where the general good sector is more capital intensive than the pure consumption good sector. We also briefly consider the opposite case.

Only good 1 has consumption externalities. First, we consider the externality arising only from general goods consumption; that is, $\theta_1 \neq 0$ and $\theta_2 = 0$. This is the type of externality analyzed in existing one-sector models studied by Gali (1994), Dupor and Liu (2003), Liu and Turnovsky (2005) and Alonso-Carrera et al. (2008a).

With general goods consumption externalities, when there is a KUI effect, self-fulfilling expectations can be supported as an equilibrium. To explain the reasons, we use (5a)–(5c) to obtain

$$\text{MRS} = \frac{\lambda}{u_2} = \frac{\gamma}{1 - \gamma} \left[\frac{c_2}{(c_1)^{1 - [\theta_1(\varepsilon - 1)]}} \right]^{\frac{1}{\varepsilon}} = \text{MRT} = \frac{f_2^2[(1 - s)k, (1 - l)]}{f_2^1(sk, l)} = p. \tag{19}$$

Suppose that there are sunspot expectations that the relative price of general goods in terms of consumption goods will increase (higher p). This raises the MRT between general goods and consumption goods. Thus, the agent allocates more

capital and labor to the general goods sector (thus sk and l are increased), which will lower the marginal product in the general goods sector and raise the marginal product in the consumption goods sector. Yet more capital and labor in the general goods sector increases the production of general goods, which increases general goods consumption. If there is no consumption externality ($\theta_1 = 0$), (19) indicates a lower MRS between c_1 and c_2 , which will not equal the MRT. As a result, anticipations of higher prices of general goods relative to consumption goods cannot be supported as an equilibrium. Suppose instead that $\theta_1 \neq 0$ and there is a KIJ effect, $[-\theta_1(\varepsilon - 1)] > 0$. If the KIJ effect is sufficiently large and is within a proper range, then the increase in general goods consumption can raise the MRS to equal the MRT. In this situation, self-fulfilling expectations can be supported as an equilibrium.

To see how a sufficiently large KIJ effect leads to indeterminacy, with good 1 consumption externalities, the elements of the Jacobian matrix in (18a)–(18d) are $J_{11}^{\theta_1}$, $J_{12}^{\theta_1}$, $J_{21}^{\theta_1}$, and $J_{22}^{\theta_1}$.¹⁸ The change in the dynamic property of equilibrium comes mainly from the change in the sign from $J_{11}^* < 0$ to $J_{11}^{\theta_1} > 0$ as a result of the KIJ effect. When $\theta_1 \neq 0$, $(\frac{\partial s}{\partial \lambda})$ in (18a) is modified as $(\frac{\partial s}{\partial \lambda})^{\theta_1}$ and the sign of $J_{11}^{\theta_1} > 0$ is opposite to the sign of $(\frac{\partial s}{\partial \lambda})^{\theta_1}$ given by

$$\left(\frac{\partial s}{\partial \lambda}\right)^{\theta_1} = \frac{1 - (-\theta_1(\varepsilon - 1))}{\varepsilon} \frac{1}{c_1} \left(\frac{\partial c_1}{\partial \lambda}\right)^{\theta_1} \frac{1}{\Xi^{\theta_1}}, \tag{20}$$

where

$$\left(\frac{\partial c_1}{\partial \lambda}\right)^{\theta_1} = -\frac{\varepsilon c_1}{\lambda} \frac{1 + p(c_1/c_2)}{1 - [-\theta_1(\varepsilon - 1)][1 + p(c_1/c_2)\varepsilon/(\varepsilon - 1)]},$$

$$\begin{aligned} \Xi^{\theta_1} &= \left[\frac{a_2}{1-s} - \frac{a_2}{1-l} l'(s) \right] + \left[\frac{a_1}{s} - \frac{a_1}{l} l'(s) \right] \\ &+ \left\{ \frac{[1 - (-\theta_1(\varepsilon - 1))]}{\varepsilon} \frac{c_2}{c_1} \left(\frac{\partial c_1}{\partial c_2}\right)^{\theta_1} - \frac{1}{\varepsilon} \right\} \left(\frac{a_2}{1-s} + \frac{1-a_2}{1-l} l'(s) \right) \\ &\equiv \Xi^* + \Lambda^{\theta_1}, \end{aligned}$$

$$\Lambda^{\theta_1} = \left\{ \frac{[1 - (-\theta_1(\varepsilon - 1))]}{\varepsilon} \frac{c_2}{c_1} \left(\frac{\partial c_1}{\partial c_2}\right)^{\theta_1} - \frac{1}{\varepsilon} \right\} \left(\frac{a_2}{1-s} + \frac{1-a_2}{1-l} l'(s) \right),$$

$$\left(\frac{\partial c_1}{\partial c_2}\right)^{\theta_1} = \frac{c_1}{c_2} \frac{1}{1 - [-\theta_1(\varepsilon - 1)][1 + p(c_1/c_2)\varepsilon/(\varepsilon - 1)]}.$$

Thus, if $(\frac{\partial s}{\partial \lambda})^{\theta_1} < 0$, then $J_{11}^{\theta_1} > 0$. It is clear that the KIJ effect leads to $\frac{c_2}{c_1} (\frac{\partial c_1}{\partial c_2})^{\theta_1} > 1$ and $\frac{[1 - (-\theta_1(\varepsilon - 1))]}{\varepsilon} < \frac{1}{\varepsilon}$. Moreover, the KIJ effect can make $\left\{ \frac{[1 - (-\theta_1(\varepsilon - 1))]}{\varepsilon} \frac{c_2}{c_1} (\frac{\partial c_1}{\partial c_2})^{\theta_1} - \frac{1}{\varepsilon} \right\} > 0$, which gives $\Lambda^{\theta_1} > 0$. Further, the KIJ effect

causes $\Lambda^{\theta_1} > -\Xi^* > 0$, which gives $\Xi^{\theta_1} > 0$. Finally, the KIJ effect maintains $(\frac{\partial c_1}{\partial \lambda})^{\theta_1} < 0$. With $\Xi^{\theta_1} > 0$ and $(\frac{\partial c_1}{\partial \lambda})^{\theta_1} < 0$, then $(\frac{\partial s}{\partial \lambda})^{\theta_1} < 0$ and thus $J_{11}^{\theta_1} > 0$.

The same reasoning also indicates $(\frac{\partial s}{\partial k})^{\theta_1} < 0$ which gives $J_{12}^{\theta_1} > 0$ and $J_{21}^{\theta_1} > 0$. As a result, the slope of $\dot{\lambda} = 0$ locus is negative: $-J_{12}^{\theta_1}/J_{11}^{\theta_1} < 0$. When the negative slope of the $\dot{\lambda} = 0$ locus is less than the slope of the $\dot{k} = 0$ locus, as illustrated in B1 and B2, Figure 1, $\text{Det}(J^{\theta_1}) > 0$ and $\text{Tr}(J^{\theta_1}) < 0$ and the steady state is a sink.

To obtain the required KIJ effect under which the steady state is a sink, in the Appendix we have shown that the relative slope condition gives

$$\varphi_1[-\theta_1(\varepsilon - 1)]^2 + b_1[-\theta_1(\varepsilon - 1)] + \text{Det}(J^*) > 0, \tag{21}$$

where φ_1 and b_1 are coefficients that are functions of consumption and the shadow price of capital evaluated at the steady state. This inequality gives the KIJ effect in which the steady state is a sink. Recall that when $\alpha_2 = 0$, $c_2 = A_2(1 - l)$ is leisure.¹⁹ With $\theta_2 = 0$, our model in this subsection is reduced to the one-sector growth model with elastic labor supply studied by Alonso-Carrera et al. (2008a). In the model of Alonso-Carrera et al. (2008a), indeterminacy arises only if consumption externalities cause the Frisch labor supply to have certain shape. Differing from Alonso-Carrera et al. (2008a), in our two-sector model, general goods consumption externalities lead to indeterminacy even though the labor supply is inelastic.

It is interesting to explore whether the conditions of indeterminacy are quantitatively easy to meet under proper preference specifications summarized by parameters θ_1 , ε , and γ . Without estimated values of θ_1 and θ_2 , our quantitative strategy is as follows. We calibrate our model to the baseline economy without consumption externalities (i.e., $\theta_1 = \theta_2 = 0$). Then, with all other values in the baseline parameterization remaining unchanged, we vary the values of the preference parameters θ_1 , ε , and γ that satisfy the equilibrium conditions and calculate the required smallest degree of the KIJ effect (i.e., $\theta_1(\varepsilon - 1)$) that causes indeterminacy. If the required smallest value of the KIJ effect is smaller than the foregoing estimates found by Ravin (2007) and Maurer and Meier (2008), then local indeterminacy is plausible.

To calibrate the model, the capital share in the general goods sector is set at $\alpha_1 = 0.32$, following Herrendorf and Valentinyi (2008). The consumption goods sector is thought of as the service sector, which includes restaurants, transportation services, and financial and retail services. Lee and Wolpin (2006) provided estimates of the share of labor earnings in the service sector in selected years using data coming from the Bureau of Economic Analysis in the United States. Following their estimation, the labor share in the consumption sector is set equal to 72%, which is the average share of labor earnings in the service sector from 1985 to 2000. Hence, the implied value of α_2 is 0.28. Moreover, these two authors pointed out that as a fraction of total employment, service-sector employment grew from 57 to 75% between 1950 and 2000. We take this number and set $1 - l = 0.75$. For the value of the ES between two consumption goods, we choose $\varepsilon = 1.25$ as

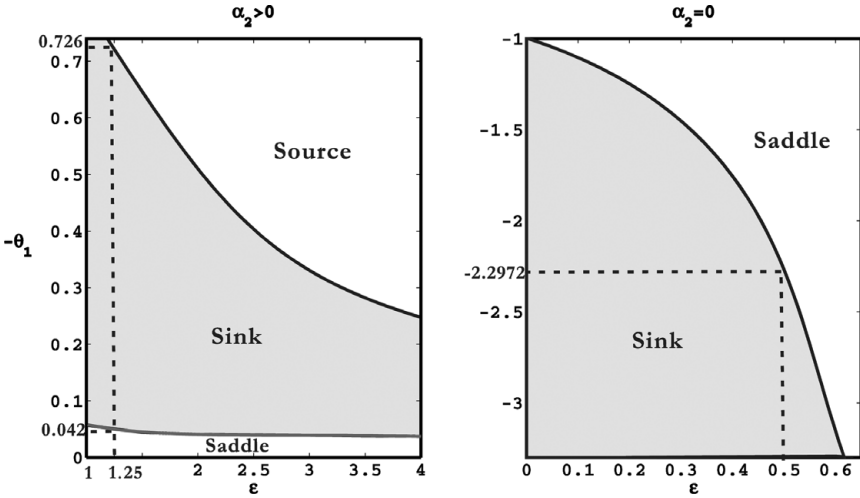


FIGURE 2. $\theta_1 \neq 0$. Baseline parameters: $\alpha_1 = 0.32, \alpha_2 = 0.28, \delta = 0.05, \rho = 0.04, \gamma = 0.1654, \theta_1 = \theta_2 = 0$.

our baseline value, which is in the range estimated by Ogaki and Reinhart (1998). For these values, we use (5b) to calculate $\gamma = 0.1654$. Then, according to (5c), we calculate $s = 0.2874$. If we set the depreciation rate to equal $\delta = 0.05$ and the discount rate to equal $\rho = 0.04$, as conventionally suggested, we can use the steady state condition in (14a) to compute $k^* = 5.6179$. Finally, we use (4a) and (7a) to obtain $c_1^* = 0.1732$ and $c_2^* = 1.1987$. We find that the steady state is a saddle. Then, with all other parameter values in the baseline remaining unchanged, we change the preference parameters θ_1, ε , and γ that cause local indeterminacy.

To start, we consider a special model of $\alpha_2 = 0$, the one-sector growth model with elastic leisure studied by Alonso-Carrera et al. (2008a). We hold $\alpha_2 = 0$ and all other parameter values unchanged and adjust the values of θ_1, ε , and γ that meet equilibrium conditions. In the right diagram of Figure 2, the shaded area is the region of $(\varepsilon, -\theta_1)$ under which the steady state is a sink. In this case, indeterminacy arises when $\varepsilon < 1$ and $-\theta_1 < 0$. Thus, when the labor supply is elastic, the general goods consumption externality generates indeterminacy only when the consumption externality is negative. The estimates in Zabalza et al. (1980) indicates that the ES between general goods and leisure is around $\varepsilon = 0.5$.²⁰ At $\varepsilon = 0.5$, indeterminacy is established at the smallest value of $\theta_1 = 2.2798$.²¹ The required smallest degree of the KIJ effect is $[(-\theta_1)(\varepsilon - 1)] = 1.1399$, which is larger than the estimates obtained by Ravina (2007) and Maurer and Meier (2008). The result suggests that it is difficult for consumption externalities to create indeterminacy in a one-sector model.

Now, we quantify our general two-sector growth model, and thus y_2 is consumption goods. We hold $\alpha_2 = 0.28$ and all other parameter values unchanged and vary the values of θ_1, ε , and γ that meet equilibrium conditions. The results are

illustrated in the left diagram of Figure 2. In the diagram, the shaded area is the region of $(\varepsilon, -\theta_1)$ under which the steady state is a sink. Here, indeterminacy arises only when $\varepsilon > 1$ and $-\theta_1 > 0$. Thus, with the consumption goods sector, general goods consumption externalities generate indeterminacy only when the externality is positive. At the empirically plausible value of $\varepsilon = 1.25$ estimated by Ogaki and Reinhart (1998), the smallest absolute value of $-\theta_1$ where indeterminacy can be established is $-\theta_1 = 4.2\%$.²² The required smallest degree of the KUI effect is $(-\theta_1)(\varepsilon - 1) = 0.0105$, which is smaller than the estimates obtained by Ravina (2007) and Maurer and Meier (2008). Indeed, the requirement is only 1% as large as the smallest degree required in a one-sector model. The results imply that it is easy for consumption externalities to establish indeterminacy in a two-sector model.

Only good 2 has consumption externalities ($\theta_1 = 0, \theta_2 \neq 0$) Next, we consider the externality arising only from consumption goods. To see why the KUI effect of consumption goods can generate indeterminacy, we use (5a)–(5c) to obtain

$$MRS = \frac{\lambda}{u_2} = \frac{\gamma}{1-\gamma} \left[\frac{(c_2)^{1-[-\theta_2(\varepsilon-1)]}}{c_1} \right]^{\frac{1}{\varepsilon}} = MRT = \frac{f_2^2[(1-s)k, (1-l)]}{f_2^1(sk, l)} = p. \quad (22)$$

To see that sunspot expectations equilibrium can emerge, suppose that the representative agent expects that the price of general goods relative to consumption goods is increasing (higher p). This raises the MRT between general goods and consumption goods. Thus, the agent allocates more input to the general goods sector and less input to the consumption goods sector, which reduces the marginal product in the general goods sector, increases the marginal product in the consumption goods sector, and reduces the production of consumption goods. When the KUI effect of consumption goods ($[-\theta_2(\varepsilon - 1)] > 0$) is sufficiently large, consumption goods are consumed less, to increase the MRS and equal the MRT. Thus, self-fulfilling expectations can be supported as an equilibrium.

In this case, the elements of the Jacobian matrix in (18a)–(18d) are $J_{11}^{\theta_2}, J_{12}^{\theta_2}, J_{21}^{\theta_2}$ and $J_{22}^{\theta_2}$. The KUI effect may affect the sign of $J_{11}^{\theta_2}$ and $J_{12}^{\theta_2}$. To see how the KUI effect works when $\theta_2 \neq 0$, $(\frac{\partial s}{\partial \lambda})$ and $(\frac{\partial s}{\partial k})$ in (18a) and (18b) become, respectively,

$$\left(\frac{\partial s}{\partial \lambda} \right)^{\theta_2} = \frac{1}{\varepsilon} \frac{1}{c_1} \left(\frac{\partial c_1}{\partial \lambda} \right)^{\theta_2} \frac{1}{\Xi^{\theta_2}}, \quad (23a)$$

$$\left(\frac{\partial s}{\partial k} \right)^{\theta_2} = -\theta_2 \frac{a_2}{k} \frac{1}{\Xi^{\theta_2}} - \frac{a_1 - a_2}{k} \frac{1}{\Xi^{\theta_2}}, \quad (23b)$$

where

$$\begin{aligned} \left(\frac{\partial c_1}{\partial \lambda} \right)^{\theta_2} &= -\frac{\varepsilon c_1 [1 + p(c_1/c_2)]}{\lambda} < 0, \\ \Xi^{\theta_2} &= \left[\frac{a_2}{1-s} - \frac{a_2}{1-l} l'(s) \right] + \left[\frac{a_1}{s} - \frac{a_1}{l} l'(s) \right] - \theta_2 \left[\frac{a_2}{1-s} + \frac{1-a_2}{1-l} l'(s) \right] \equiv \Xi^* + \Lambda^{\theta_2}, \\ \Lambda^{\theta_2} &= -\theta_2 \left[\frac{a_2}{1-s} + \frac{1-a_2}{1-l} l'(s) \right]. \end{aligned}$$

Because $(\frac{\partial c_1}{\partial \lambda})^{\theta_2} < 0$, the KIJ effect has to give $\Xi^{\theta_2} > 0$ in order to obtain $(\frac{\partial s}{\partial \lambda})^{\theta_2} < 0$. With $\Xi^* < 0$, this is possible only if $\Lambda^{\theta_2} > 0$, which requires $\theta_2 < 0$ and, under the KIJ effect, $\varepsilon > 1$. Moreover, if $-\theta_2 > 0$ is sufficiently large so that $\Xi^{\theta_2} > 0$, then $(\frac{\partial s}{\partial \lambda})^{\theta_2} < 0$. and thus $J_{11}^{\theta_2} > 0$. However, the value of $-\theta_2 > 0$ cannot be too high, because a large $-\theta_2 > 0$ gives a large $(\frac{\partial s}{\partial k})^{\theta_2} > 0$ so that $J_{12}^{\theta_2} < 0$ and the slope of the $\dot{\lambda}^{\theta_2} = 0$ locus is positive. Thus, it is required that $[-\theta_2(\varepsilon - 1)]$ lie within a range such that the slope of locus $\dot{\lambda}^{\theta_2} = 0$ is negative, as illustrated in B1 and B2, Figure 1.

Thus, a sink arises if $\text{Det}(J^{\theta_2}) > 0$ and $\text{Tr}(J^{\theta_2}) < 0$, which requires that the slope of locus $\dot{\lambda}^{\theta_2} = 0$ be smaller than the slope of locus $\dot{k}^{\theta_2} = 0$. In the Appendix, we have shown that the relative slope condition is met under

$$\varphi_2[-\theta_2(\varepsilon - 1)]^2 + b_2[-\theta_2(\varepsilon - 1)] + \text{Det}(J^*) > 0, \tag{24}$$

where φ_2 and b_2 are coefficients that are functions of consumption and the shadow price of capital evaluated at a steady state. Then, we obtain the range of the KIJ effect where the steady state is a sink. It is worth noting that when $\alpha_2 = 0$, with $\theta_2 \neq 0$ our model is reduced to a one-sector growth model with a leisure externality. Benhabib and Farmer (2000) and Weder (2004) have studied the role of positive leisure externalities in establishing indeterminacy.²³ These authors showed that positive leisure externalities help establish indeterminacy, as leisure externalities make it easier for the Frisch labor supply curve to slope down as a function of the real wage. However, they both found that it is difficult for the leisure externality alone to generate indeterminacy.

To quantify the conditions of indeterminacy in our model with the externality arising only from consumption goods, we offer the results in the left diagram of Figure 3. The shaded area in the diagram is the region of $(\varepsilon, -\theta_2)$ under which the steady state is a sink. At $\varepsilon = 1.25$, indeterminacy can be established at the smallest absolute value of $-\theta_2 = 0.6\%$.²⁴ The required smallest degree of the KIJ in consumption goods is $(-\theta_2)(\varepsilon - 1) = 0.0015$. This required degree is only 10% as large as the required smallest KIJ effect of general goods consumption externalities at 0.0105 in Figure 2. The results indicate that indeterminacy emerges even more easily under consumption goods externalities than under general goods consumption externalities.

We also quantify the case under $\alpha_2 = 0$. The right diagram of Figure 3 offers the region of $(\varepsilon, -\theta_2)$ that leads to indeterminacy. The results indicate that indeterminacy can arise under both positive and negative leisure externalities. At the empirically plausible ES between general goods and leisure at $\varepsilon = 0.5$, indeterminacy can be established at the smallest value of $\theta_2 = 8.0002$.²⁵ This indicates that the required smallest degree of the KIJ effect in leisure externalities is 4.0001. This required smallest KIJ effect is almost four times as large as the required smallest KIJ effect in general goods in a one-sector model (cf. 1.1399) in the right diagram of Figure 2. The result thus confirms the conclusions found by

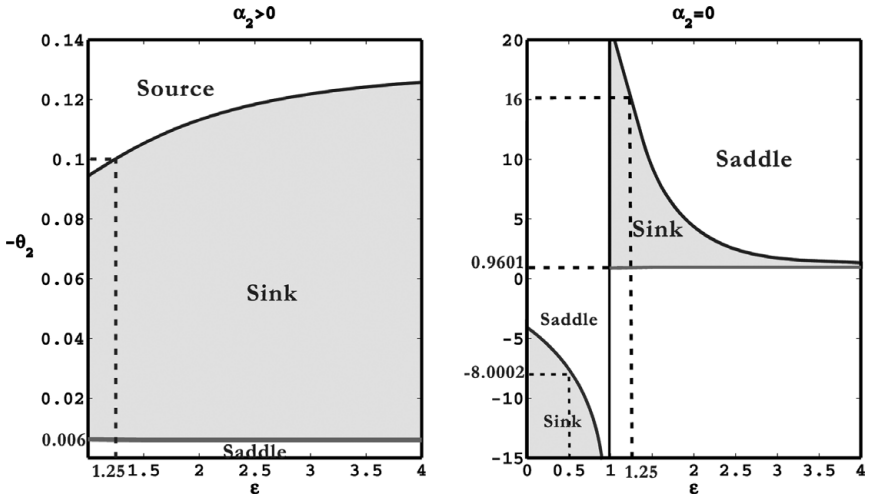


FIGURE 3. $\theta_2 \neq 0$. Baseline parameters: same as Figure 2.

Benhabib and Farmer (2000) and Weder (2004) in that it is difficult for the leisure externality alone to generate indeterminacy in one-sector growth models.

Goods 1 and 2 exhibit symmetric consumption externalities ($\theta_1 \neq 0, \theta_2 \neq 0$). We have derived the conditions under which general goods consumption externalities and consumption goods externalities each can be a source of indeterminacy. If we combine these conditions, externalities in both types of consumption together can establish indeterminacy.

A more appealing case is $\theta_1 = \theta_2 = \theta \neq 0$, when consumption externalities from the two goods are symmetric. In this case, we have shown in Section 3 that if $\epsilon \neq 1$, the utility is homothetic and the market equilibrium is efficient in a steady state. Yet symmetric consumption externalities can cause inefficiency in transitions, as the shadow price of capital in a market is no longer a fixed proportion of the shadow price of capital in a socially planned economy. As we will see in the following, indeterminacy arises when the symmetric KUJ effect is in a proper range.

To see how this works, we use (5a)–(5c) to obtain

$$MRS = \frac{\lambda}{u_2} = \frac{\gamma}{1 - \gamma} \left[\frac{c_2}{c_1} \right]^{\frac{1 - [-\theta(\epsilon - 1)]}{\epsilon}} = MRT = \frac{f_2^2[(1 - s)k, (1 - l)]}{f_2^1(sk, l)} = p. \tag{25}$$

To see that sunspot expectations equilibrium can emerge, suppose that the representative agent expects a higher price of general goods relative to consumption goods (higher p). This raises the MRT between general and consumption goods. Thus, the agent allocates more input to general goods, which decreases the marginal product in general goods and increases the marginal product in

consumption goods. Thus, there is more production of general goods and less production of consumption goods. When these have symmetric KUI effects (i.e., $(-\theta(\varepsilon - 1)) > 0$), general goods may be consumed more and consumption goods may be consumed less, to increase the MRS and equal the MRT. In this situation, self-fulfilling expectations can be supported as an equilibrium.

To derive the conditions, the elements of the Jacobian matrix in (18a)–(18d) are J_{11}^θ , J_{12}^θ , J_{21}^θ , and J_{22}^θ , which are combinations of those where only good 1 or only good 2 has consumption externalities. To see how the KUI effect works when $\theta_1 = \theta_2 = \theta \neq 0$, $(\frac{\partial s}{\partial \lambda})^\theta$ and $(\frac{\partial s}{\partial k})^\theta$ are, respectively,

$$\left(\frac{\partial s}{\partial \lambda}\right)^\theta = \{1 - [-\theta(\varepsilon - 1)]\} \frac{1}{\varepsilon} \frac{1}{c_1} \left(\frac{\partial c_1}{\partial \lambda}\right)^\theta \frac{1}{\Xi^\theta}, \tag{26a}$$

$$\left(\frac{\partial s}{\partial k}\right)^\theta = \frac{1 - [-\theta(\varepsilon - 1)]}{\varepsilon} \left[\frac{c_2}{c_1} \left(\frac{\partial c_1}{\partial c_2}\right)^\theta - 1 \right] \frac{a_2}{k} \frac{1}{\Xi^\theta} - \frac{a_1 - a_2}{k} \frac{1}{\Xi^\theta}, \tag{26b}$$

where

$$\begin{aligned} \left(\frac{\partial c_1}{\partial \lambda}\right)^\theta &= -\frac{\varepsilon c_1}{\lambda} \frac{1+p(c_1/c_2)}{1-[-\theta(\varepsilon-1)][1+p(c_1/c_2)\varepsilon/(\varepsilon-1)]}, \\ \left(\frac{\partial c_1}{\partial c_2}\right)^\theta &= \frac{c_1}{c_2} \frac{1-\theta}{1-[-\theta(\varepsilon-1)][1+p(c_1/c_2)\varepsilon/(\varepsilon-1)]}, \\ \Xi^\theta &= \left[\frac{a_2}{1-s} - \frac{a_2}{1-l} l'(s)\right] + \left[\frac{a_1}{s} - \frac{a_1}{l} l'(s)\right] + \frac{1-[-\theta(\varepsilon-1)]}{\varepsilon} \left[\frac{c_2}{c_1} \left(\frac{\partial c_1}{\partial c_2}\right)^\theta - 1\right] \left[\frac{a_2}{1-s} + \frac{1-a_2}{1-l} l'(s)\right] \equiv \\ &\Xi^* + \Lambda^\theta, \\ \Lambda^\theta &= \frac{1-[-\theta(\varepsilon-1)]}{\varepsilon} \left[\frac{c_2}{c_1} \left(\frac{\partial c_1}{\partial c_2}\right)^\theta - 1\right] \left[\frac{a_2}{1-s} + \frac{1-a_2}{1-l} l'(s)\right]. \end{aligned}$$

Similarly to the case with only general goods consumption externalities (cf. (20)), here the KUI effect $(-\theta(\varepsilon - 1))$ appears in a multiplicative term in $(\frac{\partial s}{\partial \lambda})^\theta$ and $(\frac{\partial s}{\partial k})^\theta$. If the KUI effect lies within a range,²⁶ it is possible to obtain $(\frac{\partial s}{\partial \lambda})^\theta < 0$ and $(\frac{\partial s}{\partial k})^\theta < 0$. Then $J_{11}^\theta > 0$, $J_{12}^\theta > 0$ and $J_{21}^\theta < 0$. The required condition is $\text{Det}(J^\theta) > 0$ and $\text{Tr}(J^\theta) < 0$, which gives a negative slope of $\dot{\lambda} = 0$ that is steeper than the slope of $\dot{k} = 0$. In the Appendix, we have shown that the relative slope condition is met if

$$\varphi_3[-\theta(\varepsilon - 1)]^3 + b_3[-\theta(\varepsilon - 1)]^2 + d_3[-\theta(\varepsilon - 1)] + \text{Det}(J^*) > 0, \tag{27}$$

where φ_3 , b_3 , and d_3 are coefficients that are functions of consumption and the shadow price of capital evaluated at a steady state. Thus, we obtain the range of the KUI effect in which the steady state is a sink.

Our analysis indicates that in a two-sector growth model, symmetric consumption externalities lead to indeterminacy even when the utility is homothetic. The result is different from that in the one-sector growth model provided by Alonso-Carrera et al. (2008a) in which consumption externalities do not lead to indeterminacy when the utility is homothetic. The difference arises because there is a relative price of the two goods in our model.

Figure 4 offers quantitative results about the region of $(\varepsilon, -\theta)$ in which the steady state is a sink (see left diagram). At the empirically plausible value of

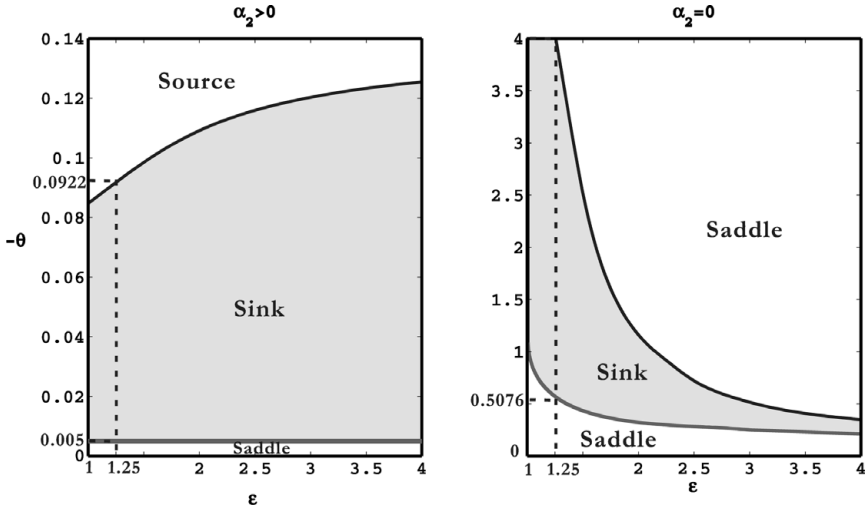


FIGURE 4. $\theta_1 = \theta_2 = \theta \neq 0$. Baseline parameters: same as Figure 2.

$\epsilon = 1.25$, indeterminacy is established at the smallest absolute value of $-\theta = 0.5\%$. The required smallest KIJ effect of symmetric consumption externalities is $-\theta(1 - \epsilon) = 0.00125$, which is smaller than the required smallest KIJ effect of general goods consumption (0.00105). Hence, the consumption externality helps the general goods consumption externality establish indeterminacy.

When $\alpha_2 = 0$, $\theta_2 > 0$ is a leisure externality.²⁷ With $\theta_1 = \theta_2 = \theta$, quantitative results are shown in the right diagram of Figure 4. Notice that this case may be thought of as the case of the general goods consumption externality with an additional symmetric leisure externality. Thus, similarly to the left diagram of Figure 2 with only the general goods consumption externality, indeterminacy here arises only when externalities are positive ($-\theta_1 > 0$). However, with additional symmetric, positive leisure externalities $-\theta_2 = -\theta > 0$, the required degree of general goods consumption externalities here is much higher than that in Figure 2. For example, at $\epsilon = 1.25$, indeterminacy can be established at the smallest absolute value of $-\theta = 50.76\%$ here in Figure 4, as opposed to $-\theta_1 = 4.2\%$ in Figure 2. Thus, a positive leisure externality does not help the general goods consumption externality to establish indeterminacy in a one-sector model. This result is different from the findings of Benhabib and Farmer (2000) and Weder (2004) in which the leisure externality helps establish indeterminacy in a model with both money and externalities in production.

Good 2 is more capital-intensive than good 1. We have so far assumed the plausible case that the general goods sector is more capital-intensive than the pure consumption goods sector, $\alpha_1 > \alpha_2$. Nevertheless, from a theoretical point of view, it might be interesting to consider the opposite case where $\alpha_1 < \alpha_2$.

As shown in Section 3, with the jealousy effect of general goods consumption, the case $\alpha_1 < \alpha_2$ produces underaccumulation, rather than overaccumulation of capital in the long run. It is interesting to investigate local dynamic properties of equilibrium quantitatively under different intensities of capital.

To this end, we set $\alpha_1 = 0.28$ and $\alpha_2 = 0.32$, so the general goods sector is less capital-intensive. We recalibrate the model following the same method used when only good 1 has consumption externalities. The baseline parameter values are not different but the values of the steady state change.²⁸ We quantify the KUI effect with an adjustment of ES between the two goods to see whether the steady state is a sink. The results are as follows.

First, if the ES between the two goods is smaller than unity ($\varepsilon < 1$), the steady state is always a saddle. Second, if the ES between the two goods is larger than one ($\varepsilon > 1$), whether the steady state is a sink or not depends on the source of consumption externalities and is as follows.

Case 1. $\theta_1 \neq 0$ and $\theta_2 = 0$. In this case, only general goods consumption has externalities. We find that the steady state is either a saddle or a source, not a sink. The result is thus different from those in the case of $\alpha_1 > \alpha_2$ as illustrated in the left diagram in Figure 2.

Case 2. $\theta_1 = 0$ and $\theta_2 \neq 0$. In this case, only the consumption of consumption goods has externalities. We find that the steady state is a sink only if $1 < \varepsilon < 1.45$. See the left diagram of Figure 5. As we see from the diagram, a sink arises only if the value of $-\theta_2$ lies above a threshold. Note the difference from the corresponding left diagram of Figure 3 in which the threshold of the degree of consumption externalities is constant; the threshold here is increasing in the value of the ES between the two goods ε . Moreover, for a given value of ε , the required smallest KUI effect here is much larger. For example, at $\varepsilon = 1.25$, the required smallest KUI effect here is 5.68, as opposed to 0.0015 in Figure 3.

Case 3. $\theta_1 = \theta_2 = \theta \neq 0$. In this case, two types of consumption have symmetric externalities. We find that the steady state is a sink only if $1 < \varepsilon < 1.1$ (the right diagram of Figure 5). In this range of the ES, a sink arises if the value of $-\theta$ is higher than a high threshold. Note that differently from the corresponding left diagram of Figure 4, in which the required smallest degree of consumption externalities is constant, the threshold here is increasing in the value of the ES between the two goods. Moreover, for a given value of ε , the required degree of consumption externalities is high. For example, at $\varepsilon = 1.1$, a sink emerges at the smallest value of $-\theta = 148.6$ and thus the required smallest KUI effect is $-\theta(\varepsilon - 1) = 14.86$, which is very large as compared to 0.0005 in the left diagram of Figure 4. Even at $\varepsilon = 1.01$, a sink requires a large smallest KUI at 0.39069.

4.3. Economic Intuition for Indeterminacy Conditions

So far, we have presented a detailed analysis concerning the mathematical conditions for indeterminacy. To sum up, we have found (i) regardless of the capital intensity ranking between the two sectors, indeterminacy tends to emerge under

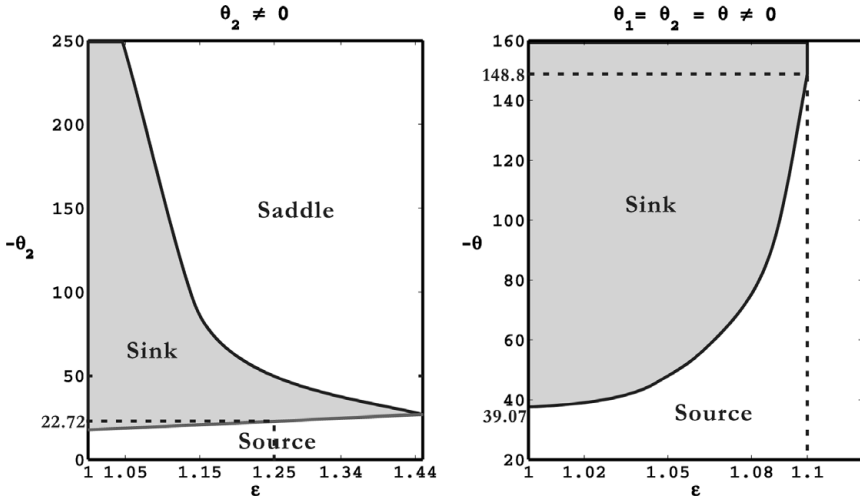


FIGURE 5. Factor intensity reversal. Baseline parameters: $\alpha_1 = 0.28, \alpha_2 = 0.32, \delta = 0.05, \rho = 0.04, \gamma = 0.1654, \theta_1 = \theta_2 = 0$.

KUJ with negative levels θ_1 and θ_2 ; (ii) if the general goods sector is more capital-intensive than the consumption goods sector, then local indeterminacy holds rather easily; and (iii) if the general goods sector is less capital-intensive than the consumption goods sector, then indeterminacy holds only when the consumption goods are associated with extremely high levels of external effects.

To find insights within those results, it is useful to focus on the relation between the relative price of the general goods p and the shadow price of capital λ . Recall that the capital-labor ratio in sector i is denoted by $x_i = \frac{k_i}{l_i}$ and output per unit of labor in sector i is $g_i(x_i) = \frac{f^i(k_i, l_i)}{l_i}$. It is well known that the two-sector model with the Cobb–Douglas production function satisfies

$$x_i = x_i(p), \text{ where } \text{sign } x'_i(p) = \text{sign}(a_2 - a_1), \tag{28a}$$

$$r = g'_1[x_1(p)] = r(p), \text{ where } \text{sign } r'(p) = \text{sign}(a_1 - a_2), \tag{28b}$$

$$y_1 = \frac{k - x_2(p)}{x_1(p) - x_2(p)} g_1[x_1(p)] = y^1(k, p),$$

$$c_2 = y_2 = \frac{x_1(p) - k}{x_1(p) - x_2(p)} g_2[x_2(p)] = y^2(k, p), \tag{28c}$$

where $\text{sign } y^1_k(k, p) = \text{sign}(a_1 - a_2), y^1_p(k, p) > 0, \text{sign } y^2_k(k, p) = \text{sign}(a_2 - a_1), y^2_p(k, p) < 0$.

First, consider the model without externalities ($\theta_1 = \theta_2 = 0$). In this case, conditions (5a) and (11b) are written, respectively, as

$$\gamma \left[\gamma + (1 - \gamma) \left(\frac{c_2}{c_1} \right)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{1}{\varepsilon-1}} = \lambda, \tag{29a}$$

$$\left(\frac{\gamma}{1 - \gamma} \right)^{\varepsilon} \left(\frac{c_2}{c_1} \right) = p^{\varepsilon}. \tag{29b}$$

These equations give the relation of p and λ such that $p = p(\lambda)$, $p'(\lambda) > 0$. The dynamic system (15a) and (15b) in the absence of consumption externalities is alternatively summarized as

$$\dot{\lambda} = \lambda \{ \varrho + \delta - r[p(\lambda)] \}, \tag{30a}$$

$$\dot{k} = y^1[k, p(\lambda)] - p(\lambda)^{-\varepsilon} y^2[k, p(\lambda)] - \delta k. \tag{30b}$$

Note that the price system (30a) is independent of the quantity system (30b). It is clear that from (28a) to (28c), in spite of the factor intensity ranking between the two sectors, it holds that

$$\text{sign} \left(\frac{\partial \dot{\lambda}}{\partial \lambda} \right) = \text{sign} (a_2 - a_1), \text{sign} \left(\frac{\partial \dot{k}}{\partial k} \right) = \text{sign} (a_1 - a_2),$$

which implies that the dynamic system exhibits a saddlepoint property.

Now consider the effect of consumption externalities. For simplicity, we focus upon the case of symmetric externalities, $\theta_1 = \theta_2 = \theta \neq 0$. Equations (5a) and (12b) are given respectively by

$$\gamma (c_1)^{\theta(\frac{1}{\varepsilon}-1)} \left[\gamma + (1 - \gamma) \left(\frac{c_2}{c_1} \right)^{\frac{(1-\theta)(\varepsilon-1)}{\varepsilon}} \right]^{\frac{1}{\varepsilon-1}} = \lambda, \tag{31a}$$

$$\left(\frac{\gamma}{1 - \gamma} \right)^{\frac{\varepsilon}{1+\theta(\varepsilon-1)}} \left(\frac{c_2}{c_1} \right) = p^{\frac{\varepsilon}{1+\theta(\varepsilon-1)}}, \tag{31b}$$

which can be summarized as

$$\begin{aligned} & \gamma \left(\frac{\gamma}{1 - \gamma} \right)^{\theta(\frac{1}{\varepsilon}-1)} \left[p^{-\frac{\varepsilon}{1+\theta(\varepsilon-1)}} y^2(k, p) \right]^{\theta(\frac{1}{\varepsilon}-1)} \\ & \times \left[\gamma + (1 - \gamma) \left(\frac{\gamma}{1 - \gamma} \right)^{\frac{(1-\theta)(\varepsilon-1)}{1+\theta(\varepsilon-1)}} p^{\frac{(1-\theta)(\varepsilon-1)}{1+\theta(\varepsilon-1)}} \right]^{\frac{1}{\varepsilon-1}} = \lambda. \end{aligned} \tag{31c}$$

Given the specification used in the numerical examples, it is assumed that $\theta < 0$, $\varepsilon < 1$, and $1 + \theta(\varepsilon - 1) > 0$. Thus, the foregoing equation gives

$$p = \pi(k, \lambda), \text{ where } \text{sign } \pi_k(k, \lambda) = \text{sign}(a_2 - a_1), \pi_{\lambda}(k, \lambda) > 0.$$

As a result, an alternative representation of the dynamic system (15a) and (15b) in the presence of symmetric consumption externalities is given by

$$\dot{\lambda} = \lambda \{ \varrho + \delta - r [\pi (k, \lambda)] \}, \tag{32a}$$

$$\dot{k} = y^1 [k, \pi (k, \lambda)] - c^1 [k, \pi (k, \lambda)] - \delta k, \tag{32b}$$

where

$$c^1 [k, \pi (k, \lambda)] = [\pi (k, \lambda)]^{-\frac{\epsilon}{1+\theta(\epsilon-1)}} y^2 [k, \pi (k, \lambda)].$$

Notice that the price system (32a) is no longer independent of the quantity system (32b). A key condition is to require a negative sign for the trace of the Jacobian matrix as follows:

$$\frac{\partial \dot{\lambda}}{\partial \lambda} + \frac{\partial \dot{k}}{\partial k} = -\lambda r' (p) \pi_{\lambda} + y_k^1 (k, p) + y_p^1 \pi_k - c_k^1 (k, p) - c_p^1 (k, p) \pi_k - \delta.$$

We see that indeterminacy tends to emerge easily when the general goods sector is more capital-intensive than the consumption goods sector ($\alpha_1 > \alpha_2$), because the shadow price of capital shows self-stabilizing behavior, $\partial \dot{\lambda} / \partial \lambda < 0$. Moreover, a negative effect of an increase in capital on the relative price of general goods p gives rise to a possibility of $\partial \dot{k} / \partial k < 0$. Thus, although an increase in capital enhances the production of general goods and reduces the consumption of general goods, a lower relative price of general goods caused by a rise in capital reduces the production and increases the consumption of general goods. This stabilizing effect never exists in the model without consumption externalities.

Conversely, when the general goods sector is less capital-intensive than the consumption goods sector, the shadow price of capital exhibits self-destabilizing behavior, $\partial \dot{\lambda} / \partial \lambda > 0$. It is thus necessary to hold $\partial \dot{k} / \partial k < 0$ for indeterminacy. In addition, the self-stabilizing effect of capital should dominate the self-destabilizing effect of the shadow price of capital. With $\alpha_2 > \alpha_1$, because the relative price of capital goods increases, a rise in capital stock reduces the production of general goods and increases their consumption, which promotes self-stabilizing behavior of capital. However, a higher k increases p , which increases the production of general goods. Therefore, if the external effects are small, the stabilizing effect of capital is not large enough to dominate the destabilizing effect of the shadow price of capital.

To sum up, indeterminacy in the presence of consumption externalities stems from interaction between the price system and the quantity system. If there is no consumption externality, the price system is independent of the quantity system. Such independence no longer holds in models with consumption externalities, even if external effects are symmetric, so that the preference is homothetic. Numerical examples shown in the preceding illustrate magnitudes of preference parameters for which the interaction between price and capital generates indeterminacy of converging paths.

5. CONCLUDING REMARKS

In one-sector neoclassical growth models, consumption externalities can produce an inefficient allocation in a steady state and create indeterminate equilibrium paths toward a steady state only if there is a labor–leisure trade-off. In our paper, we have shown that in a two-sector neoclassical growth model with general goods and consumption goods, even if there is no labor–leisure trade-off, consumption spillovers can yield inefficient allocation in a steady state and generate indeterminate equilibrium paths toward a steady state.

In our two-sector model, the factor reallocation and the consumption allocation between sectors are the mechanisms that generate these results. Consumption externalities change the MRS between sectors and affect the MRT, which gives rise to an inefficient allocation in a steady state. Moreover, equilibrium paths toward a steady state are indeterminate because these externalities generate the KUI effect, which influences the MRS and the MRT in such a way that self-fulfilling expectations about relative prices of the two goods can be supported as an equilibrium.

We find that even with negative general goods consumption externalities, capital is over- (under-) accumulated only if the general goods sector is more (less) capital-intensive than the consumption sector. Although consumption externalities distort the allocation of capital, no active capital taxes are necessary once the sources of consumption distortions are identified and corrected by consumption taxes. When the general goods sector is more capital-intensive, general goods consumption externalities generate indeterminacy more easily in a two-sector model than in a one-sector model. The consumption goods are leisure if the consumption goods sector does not use capital, but consumption externalities easily cause indeterminacy, whereas it is difficult for leisure externalities to generate indeterminacy. Finally, when there are symmetric consumption externalities, so that utility is homothetic, the allocation is efficient in a steady state but the equilibrium path may be indeterminate. As a result, it does not matter whether consumption externalities are from general goods, consumption goods, or both; it is much easier for consumption externalities to exhibit indeterminacy in a two-sector growth model than in a one-sector growth model. A two-sector model economy with consumption externalities is thus less stabilized than a one-sector model economy.

NOTES

1. The concept of consumption externalities and habits may be traced to Hume (1748 [1955]), who argued that preferences were influenced not simply by what a person did in the past, what his/her parents did, and what contemporary peers were doing, but also by the behavior of past generations of peers. Contemporary ideas date to Marshall (1898) and Veblen (1912) and were first formalized by Dusenberry (1949) as a determinant of aggregate consumption in his development of the relative income hypothesis. More recently, the Easterlin paradox, first mentioned in Easterlin (1974) and since that time further expanded upon, along with lively debate, posits that income growth does not necessarily enhance individual welfare, and instead, emphasizes the role of consumption externalities and social comparison; see for example Easterlin (2001).

2. Clark and Oswald (1996) presented some direct empirical evidence for British workers, showing that their reported satisfaction levels are inversely related to their comparison wage rates. Based on both the psychological evidence and the more fragmentary evidence in behavioral economics, Oswald (1997) concluded that both these sources support the claim that satisfaction depends upon the agent's relative position, again emphasizing the role of the externalities it generates. Luttmer (2005) presented a detailed empirical study showing that the level of earnings relative to average earnings may significantly affect individual welfare. He indicated further that the relative consumption is the most relevant proxy of unobservable earnings of other households. See also Frank (2005), Clark et al. (2008), and Maurer and Meier (2008) for further discussion.

3. External consumption habits are accumulated consumption externalities. As external habits are formed by both current and past average consumption, the habit effect is a mix of the KUJ effect and the CUJ effect [e.g., Carroll et al. (1997); Abel (1999); Turnovsky and Monteiro (2007); Chen et al. (2013)]. Conversely, if the habit is formed only by past average consumption, the habit effect involves only the CUJ effect [e.g., Campbell and Cochrane (1997); Grishchenko (2010)].

4. Liu and Turnovsky (2005) and Turnovsky and Monteiro (2007) found that consumption externalities cause long-run distortions only if there is a labor-leisure trade-off. Other than depending on personal consumption and leisure, Liu and Turnovsky (2005) assumed that an individual's preference depends on contemporary average consumption, whereas Turnovsky and Monteiro (2007) assumed that an individual's preference depends on external habits. As the habits are a weighted average of current and past average consumption, Liu and Turnovsky (2005) is a special case of Turnovsky and Monteiro (2007) in which the weights in the past are zero.

5. Alonso-Carrera et al. (2008a) extended the model used by Liu and Turnovsky (2005) with elastic leisure and studied the dynamic properties of equilibrium. These authors showed that indeterminacy cannot arise if the utility is homothetic in personal consumption and average consumption. They found that only if the utility is not homothetic can indeterminacy arise.

6. In one-sector and two-sector neoclassical growth models with productive externalities, Kim (2005) showed that the utility curvature does not matter for indeterminacy. Unlike Kim, a specific utility curvature is not our focus.

7. Our model is a variant of those posited in Whelan (1993), Rogerson (2008), and Duarte and Restuccia (2010). Good 1 may be thought of as manufactured goods and good 2 as service goods. Alternatively, good 2 may be interpreted as home goods, as in Benhabib et al. (1992) and Greenwood and Hercowitz (1991).

8. The Inada condition is $\lim_{k_i \rightarrow 0} f^i(k_i, l_i) = \infty$ and $\lim_{k_i \rightarrow \infty} f^i(k_i, l_i) = 0$.

9. Condition $\frac{1-l^*}{l^*} \frac{s^*}{1-s^*} = \frac{a_1(1-a_2)}{a_2(1-a_1)}$ is used in the second equality.

10. Liu and Turnovsky (2005) have shown that even with inelastic leisure, a one-sector growth model with consumption externalities can distort the transitional path, although it cannot distort the steady-state allocation.

11. As labor is supplied inelastically, it is clear that labor taxation will not improve the efficiency, and thus we do not consider a labor tax.

12. This method follows from that found in Benhabib and Farmer (1996), which is a two-sector growth model with one consumption good and one investment good. In their model, sector-specific externalities in production are the mechanism leading to indeterminacy.

13. In what follows, an asterisk superscript is used to denote the case of a two-sector growth model without consumption externalities.

14. In the parametric version, $p = f_2^2/f_2^1 = (1-a_2)A_2[(1-s)k/(1-l)]^{a_2}[(1-a_1)A_1(sk/l)^{a_1}]^{-1}$.

15. \bar{k} is determined by $[D \frac{\partial s}{\partial k} + a_1 A_1 \frac{s^{a_1} l^{1-a_1}}{k^{1-a_1}}] - \frac{\partial c_1}{\partial c_2} \frac{\partial c_2}{\partial k} = \delta$, where $D \equiv \{A_1 (\frac{s}{l})^{a_1} k^{a_1} [a_1 \frac{l}{s} + (1-a_1)l'(s)] - \frac{\partial c_1}{\partial c_2} \frac{\partial c_2}{\partial s}\} > 0$.

16. The value $k = \hat{k}$ is determined by $(\varrho + \delta) = A_1 a_1 \{l[s(0, \hat{k})/s(0, \hat{k})]^{1-a_1} (1/\hat{k})^{1-a_1}\}$.

17. Ravina (2007) and Maurer and Meier (2008) estimated the KUJ effect based on one-sector models. Maurer and Meier (2008) employed U.S. micro data from the Panel Study of Income Dynamics in which the KUJ effect is based on peer groups. They obtained a KUJ effect in the range [0.3, 0.45].

Ravina (2007) used California micro data of the credit card holders in which the KUJ effect is based on a geographic notion of neighborhood effects. She found a KUJ effect in the range [0.26, 0.29].

18. A superscript θ_i is used to represent the source of externalities from consumption c_i .

19. In this case, $s = 1$ and there is no relationship (16a). If we normalize $A_2 = 1$, then (16b) is changed to $c_2 = (1-l)$, which is leisure, (16c) is $c_1 = c_1(\lambda, 1-l)$, and (16d) is $l^{\alpha_1}[(1-l)(c_1)^{-1}]^{-[1+(\varepsilon-1)\theta_2]/\varepsilon} = \gamma(1-\alpha_1)A_1(1-\gamma)^{-1}k^{\alpha_1}$, which implies $l = l(\lambda, k)$, where $\frac{\partial l}{\partial \lambda} > 0$ and $\frac{\partial l}{\partial k} > 0$ when $\theta_1 = \theta_2 = 0$ but $\frac{\partial l}{\partial \theta}$ and $\frac{\partial l}{\partial k}$ are ambiguous if $\theta_1 \neq 0$ or $\theta_2 \neq 0$. Using $l = l(\lambda, k)$, (16c) becomes $c_1 = c_1[\lambda, 1 - l(\lambda, k)]$.

20. Using data in the United Kingdom, Zabalza et al. (1980) found that the ES between income and leisure is 0.25 for men and 1.30 for women. As men are the major labor force, $\varepsilon = 0.5$ is about the average of men and women in the labor force estimated by these authors.

21. In the simulation that gives rise to a sink, at $\varepsilon = 0.5$, θ_1 is larger than or equal to 2.2972. When θ_1 is larger, γ is larger. At $\theta_1 = 2.2972$, $\gamma = 0.6585$.

22. In the simulation that yields a sink, at $\varepsilon = 1.25$, $-\theta_1$ is in [0.042, 0.726], and γ is in [0.1675, 0.2036].

23. The analysis provided by Benhabib and Farmer (2000) is a one-sector model with real balances but without capital in the production function. Weder (2004) is a one-sector growth model with externalities in production. In these two models, leisure is elastic. They both allow leisure externalities in utility.

24. In the simulation a sink emerges, at $\varepsilon = 1.25$, $-\theta_2$ is in [0.006, 0.1], and γ is in [0.1655, 0.1659].

25. In the simulation that gives a sink, at $\varepsilon = 0.5$, θ_2 is larger than or equal to 8.0002. When $\theta_1 = 8.0002$, $\gamma = 0.0197$. When θ_1 is larger, γ is larger.

26. In the range of the KUJ effect, (i) the signs of $(\frac{\partial c_1}{\partial \lambda})^\theta$ and $(\frac{\partial c_1}{\partial c_2})^\theta$ do not deviate from their counterparts of $(\frac{\partial c_1}{\partial \lambda})^* < 0$ and $(\frac{\partial c_1}{\partial c_2})^* > 0$ under $\theta_1 = \theta_2 = 0$ and (ii) $\Lambda^\theta > -\Xi^* > 0$.

27. In the case under $\alpha_2 = 0$, as there is the leisure externality, our model is different from the Alonso-Carrera et al. (2008a) model which has no leisure externality.

28. The baseline parameters are $\alpha_1 = 0.28$, $\alpha_2 = 0.32$, $\rho = 0.04$, $\delta = 0.05$, and $\theta_1 = \theta_2 = 0$. Then γ and s are recalibrated to 0.1267 and 0.216, respectively. Under the parameter values we obtain $k^* = 5.5995$, $c_1^* = 0.1087$, and $c_2^* = 1.3202$.

29. Notice that $\alpha_1 > \alpha_2$ implies $\frac{\alpha_1(1-\alpha_2)}{\alpha_2(1-\alpha_1)} \frac{l}{s} > 1$.

REFERENCES

- Abel, Andrew B. (1990) Asset prices under habit formation and catching up with the Joneses. *American Economic Review* 80, 38–42.
- Abel, Andrew B. (1999) Risk premia and term premia in general equilibrium. *Journal of Monetary Economics* 43, 3–33.
- Abel, Andrew B. (2005) Optimal taxation when consumers have endogenous benchmark levels of consumption. *Review of Economic Studies* 72, 21–42.
- Alonso-Carrera, Jaime, Jordi Caballe, and Xavier Raurich (2008a) Can consumption spillovers be a source of equilibrium indeterminacy? *Journal of Economic Dynamics and Control* 32, 2883–2902.
- Alonso-Carrera, Jaime, Jordi Caballe, and Xavier Raurich (2008b) Estate taxes, consumption externalities, and altruism. *Journal of Public Economics* 92, 1751–1764.
- Alvarez-Cuadrado, Francisco, Goncalo Monteiro, and Stephen J. Turnovsky (2004) Habit formation, catching up with the Joneses, and non-scale growth. *Journal of Economic Growth* 9, 47–80.
- Alpizar, Francisco, Fredrik Carlsson, and Olof Johansson-Stenman (2005) How much do we care about absolute versus relative income and consumption? *Journal of Economic Behavior and Organization* 56, 405–421.
- Arrow, Kenneth J. and Partha S. Dasgupta (2009) Conspicuous consumption, inconspicuous leisure. *Economic Journal* 119, 497–516.

- Benhabib, Jess and Roger E.A. Farmer (1994) Indeterminacy and increasing returns. *Journal of Economic Theory* 63, 19–41.
- Benhabib, Jess and Roger E.A. Farmer (1996) Indeterminacy and sector-specific externalities. *Journal of Monetary Economics* 37, 421–443.
- Benhabib, Jess and Roger E.A. Farmer (2000) The monetary transmission mechanism. *Review of Economic Dynamics* 3, 523–550.
- Benhabib, Jess, Richard Rogerson, and Randall Wright (1992) Homework in macroeconomics: Household production and aggregate fluctuations. *Journal of Political Economy* 99, 1166–1187.
- Campbell, John Y. and John H. Cochrane (1999) By force of habit: A consumption-based explanation of aggregate stock market behavior. *Journal of Political Economy* 107, 205–251.
- Carlsson, Fredrik, Olof Johansson-Stenman, and Peter Martinsson (2007) Do you enjoy having more than others? Survey evidence of positional goods. *Economica* 74, 586–598.
- Carroll, Christopher D., Jody Overland, and David N. Weil (1997) Comparison utility in a growth model. *Journal of Economic Growth* 2, 339–367.
- Chen, Been-Lon (2007) Multiple BGPs in a growth model with habit persistence. *Journal of Money, Credit and Banking* 39(1), 25–45.
- Chen, Been-Lon and Mei Hsu (2007) Admiration is a source of indeterminacy. *Economic Letters* 95, 96–103.
- Chen, Been-Lon, Yu-Shan Hsu, and Kazuo Mino (2013) Can consumption habit spillovers be a source of equilibrium indeterminacy? *Journal of Economics* 109(3), 245–269.
- Clark, Andrew E., Paul Frijters, and Michael A. Shields (2008) Relative income, happiness, and utility: An explanation for the Easterlin paradox and other puzzles. *Journal of Economic Literature* 46, 95–144.
- Clark, Andrew E. and Andrew J. Oswald (1996) Satisfaction and comparison income. *Journal of Public Economics* 61, 359–381.
- Doi, Junko and Kazuo Mino (2008) A variety-expansion model of growth with external habit formation. *Journal of Economic Dynamics and Control* 32, 3055–3083.
- Duarte, Margarida and Diego Restuccia (2010) The role of the structural transformation in aggregate productivity. *Quarterly Journal of Economics* 125, 129–173.
- Dupor, Bill and Wen-Fang Liu (2003) Jealousy and equilibrium overconsumption. *American Economic Review* 93, 423–28.
- Dusenbery, James S. (1949) *Income, Saving, and the Theory of Consumer Behavior*. New York: Oxford University Press.
- Easterlin, Richard (1974) Does economic growth improve the human lot? Some empirical evidence. In Paul A. David and Melvin W. Reder (eds.), *Nations and Households in Economic Growth: Essays in Honor of Moses Abramovitz*, pp. 89–125. New York: Academic Press.
- Easterlin, Richard (2001) Income and happiness: Towards a unified theory. *Economic Journal* 111, 465–484.
- Farmer, Roger E.A. and Jang-Ting Guo (1994) Real business cycles and the animal spirits hypothesis. *Journal of Economic Theory* 63, 42–72.
- Fisher, Walter H. and Franz X. Hof (2000) Relative consumption, economic growth, and taxation. *Journal of Economics* 72, 241–262.
- Frank, Robert H. (2005) Positional externalities cause large and preventable welfare losses. *American Economic Review* 95, 137–141.
- Gali, Jordi (1994) Keeping up with the Joneses: Consumption externalities, portfolio choice, and asset prices. *Journal of Money, Credit and Banking* 26, 1–8.
- Greenwood, Jeremy and Zvi Hercowitz (1991) The allocation of capital and time over the business cycle. *Journal of Political Economy* 99, 1188–1214.
- Grishchenko, Olesya V. (2010) Internal vs. external habit formation: The relative importance for asset pricing. *Journal of Economics and Business* 62, 176–194.
- Herrendorf, Berthold and Akos Valentinyi (2008) Measuring factor income shares at the sectoral level. *Review of Economic Dynamics* 11, 820–835.

- Hori, Takeo (2011) The effects of consumption externalities in an R& D-based growth model with endogenous skilled and unskilled labor supply. *Journal of Economics* 102, 29–55.
- Hume, David (1748 [1955]) *An Enquiry Concerning Human Understanding*, reprinted. Indianapolis, IN: Bobbs-Merrill.
- Kim, Jinill (2005) Does utility curvature matter for indeterminacy? *Journal of Economic Behavior and Organization* 67, 421–429.
- Lee, Donghoon and Kenneth I. Wolpin (2006) Intersectoral labor mobility and the growth of the service sector. *Econometrica* 74, 1–46.
- Liu, Wen-Fang and Stephen J. Turnovsky (2005) Consumption externalities, production externalities, and long-run macroeconomic efficiency. *Journal of Public Economics* 89, 1097–1129.
- Ljungqvist, Lars and Harald Uhlig (2000) Tax policy and aggregate demand management under catching up with Joneses. *American Economic Review* 90, 356–366.
- Luttrmer, Erzo F.P. (2005) Neighbors as negatives: Relative earning and well-being. *Quarterly Journal of Economics* 120, 963–1002.
- Marshall, Alfred (1898) *Principles of Economics*, 8th ed. New York: Macmillan [reprinted 1920].
- Maurer, Jurhen and Andre Meier (2008) Smooth it like the “Joneses”? Estimating peer-group effects in intertemporal consumption choice. *Economic Journal* 118, 454–476.
- Ogaki, Masao and Carmen M. Reinhart (1998) Measuring intertemporal substitution: The role of durable goods. *Journal of Political Economy* 106, 1078–1098.
- Oswald, Andrew J. (1997) Happiness and economic performance. *Economic Journal* 107, 1815–1831.
- Ravina, Enrichetta (2007) Habit Persistence and Keeping Up with the Joneses: Evidence from Micro Data. Working paper, NYU Stern School of Business.
- Ravn, Morten, Stephanie Schmitt-Grohé, and Martin Uribe (2006) Deep habits. *Review of Economic Studies* 73, 195–218.
- Rogerson, Richard (2008) Structural transformation and the deterioration of European labor market outcomes. *Journal of Political Economy* 116, 235–259.
- Solnick, Sara J. and David Hemenway (1998) Is more always better? A survey on positional concerns. *Journal of Economic Behavior and Organization* 37, 373–383.
- Turnovsky, Stephen J. and Goncalo Monteiro (2007) Consumption externalities, production externalities, and efficient capital accumulation under time non-separable preferences. *European Economic Review* 51, 479–504.
- Veblen, Thorstein (1912) *The Theory of the Leisure Class*. New York: Macmillan.
- Weder, Mark (2004) A note on conspicuous leisure, animal spirits and endogenous cycles. *Portuguese Economic Journal* 3, 1–13.
- Whelan, Karl (2003) A two-sector approach to modeling U.S. NIPA data. *Journal of Money, Credit and Banking* 35, 627–656.
- Zabalza, Antoni, Christopher Pissarides, and Margaret Barton (1980) Social security and the choice between full-time work, part-time and retirement. *Journal of Public Economics* 14, 245–276.

APPENDIX A: OPTIMAL TAX POLICY

The household maximizes the lifetime utility (2) subject to (14a). The first-order conditions are

$$\frac{u_1(c_1, c_2, c_1, c_2)}{(1 + \tau_{c1})} = \lambda, \quad (\text{A.1a})$$

$$\frac{u_2(c_1, c_2, c_1, c_2)}{(1 + \tau_{c_2})} = \frac{\lambda}{p}, \quad (\text{A.1b})$$

$$\frac{\dot{\lambda}}{\lambda} = (\varrho + \delta) - (1 - \tau_k)r. \quad (\text{A.1c})$$

The representative firm maximizes the profit. Let w denote the wage. The first-order conditions of the representative firm in Sectors 1 and 2 are, respectively,

$$f_1^1(sk, l) = r, \quad f_2^1(sk, l) = w, \quad (\text{A.2a})$$

$$\frac{1}{p}f_1^2((1-s)k, 1-l) = r, \quad \frac{1}{p}f_2^2((1-s)k, 1-l) = w. \quad (\text{A.2b})$$

Equalization of the wage in (A.2a) and (A.2b) across sectors gives

$$\frac{f_2^2((1-s)k, 1-l)}{f_2^1(sk, l)} = p, \quad (\text{A.3a})$$

and if we substitute this expression into (A.1b), we obtain

$$\frac{\lambda(1 + \tau_{c_2})}{u_2(c_1, c_2, c_1, c_2)} = \frac{f_2^2((1-s)k, 1-l)}{f_2^1(sk, l)} = p. \quad (\text{A.3b})$$

Moreover, equalization of the rental to capital in (A.2a) and (A.2b) across sectors gives

$$\frac{f_2^2((1-s)k, 1-l)}{f_2^1(sk, l)} = \frac{f_1^2((1-s)k, 1-l)}{f_1^1(sk, l)}. \quad (\text{A.4a})$$

Finally, using the rental to capital in (A.2a), we rewrite (A.1c) as

$$\frac{\dot{\lambda}}{\lambda} = (\varrho + \delta) - (1 - \tau_k)f_1^1(sk, l). \quad (\text{A.4b})$$

The optimal conditions of the decentralized economy include (A.1a), (A.3b), (A.4a), and (A.4b).

Our objective is to determine a tax structure such that the decentralized economy replicates the dynamic equilibrium time path of the centrally planned economy, as described in (6a)–(6d). Let a variable with a superscript p denote the variable in the centralized economy. Replication involves setting time-varying tax rates such that $k = k^p$, $c_1 = c_1^p$, and $c_2 = c_2^p$, which requires $\frac{\dot{\lambda}}{\lambda} = \frac{\dot{\lambda}^p}{\lambda^p}$, or equivalently $\chi\lambda = \lambda^p$, where $\chi > 0$ is an arbitrary constant. It is clear that (A.4a) is the same as (6c).

First, replicating (A.4b) with (6d) gives

$$\tau_k = 0. \quad (\text{A.5a})$$

Next, replicating (A.3b) with (6b) gives

$$(1 + \tau_{c_2}) = \xi \frac{u_2(c_1, c_2, c_1, c_2)}{u_2(c_1, c_2, c_1, c_2) + u_4(c_1, c_2, c_1, c_2)}. \quad (\text{A.5b})$$

Finally, we substitute λ in (A.3b) into (A.1a) and λ^p in (6b) into (6a). Then replicating the resulting (A.1a) with the resulting (6a) gives

$$\frac{(1 + \tau_{c1})}{(1 + \tau_{c2})} = \frac{\frac{u_1(c_1, c_2, c_1, c_2)}{u_1(c_1, c_2, c_1, c_2) + u_3(c_1, c_2, c_1, c_2)}}{\frac{u_2(c_1, c_2, c_1, c_2)}{u_2(c_1, c_2, c_1, c_2) + u_4(c_1, c_2, c_1, c_2)}}. \tag{A.5c}$$

APPENDIX B: STABILITY CONDITIONS

Differentiating the general case in (15a) and (15b), the elements in the Jacobian matrix are

$$\begin{aligned} J_{11} &= -\frac{(1 + pc_1/c_2)(\varrho + \delta)(1 - a_1)}{s} \left[-1 + \frac{a_1(1 - a_2)l}{a_2(1 - a_1)s} \right] \frac{1 + \theta_1(\varepsilon - 1)}{B} \frac{1}{\Xi}, \\ J_{12} &= \frac{\lambda(\varrho + \delta)(1 - a_1)}{s} \left\{ \frac{s}{k} - \left[-1 + \frac{a_1(1 - a_2)l}{a_2(1 - a_1)s} \right] \right. \\ &\quad \times \left. \left[\frac{1 + \theta_1(\varepsilon - 1)}{\varepsilon} \frac{1 - \theta_2}{B} + \frac{1 + \theta_2(\varepsilon - 1)}{\varepsilon} + \frac{a_1 - a_2}{a_2} \right] \frac{a_2}{k} \frac{1}{\Xi} \right\}, \\ J_{21} &= \frac{(1 + pc_1/c_2)}{\lambda B} \{ [1 + \theta_1(\varepsilon - 1)] \frac{\Gamma}{\Xi} - \varepsilon c_1 \}, \\ J_{22} &= \left[\frac{1 + \theta_1(\varepsilon - 1)}{\varepsilon} \frac{1 - \theta_2}{B} + \frac{1 + \theta_2(\varepsilon - 1)}{\varepsilon} + \frac{a_1 - a_2}{a_2} \right] \frac{a_2}{k} \frac{\Gamma}{\Xi} \\ &\quad + \frac{a_1 c_1}{k} + \frac{(1 - \theta_2) a_2 c_1}{B k} - (1 - a_1)\delta, \end{aligned}$$

where

$$\begin{aligned} B &= -1 + [-\theta_1(\varepsilon - 1)][1 + p(c_1/c_2)(\varepsilon/\varepsilon - 1)], \\ \Xi &= -\left[\frac{1 + \theta_1(\varepsilon - 1)}{\varepsilon} \frac{1 - \theta_2}{B} + \frac{1 + \theta_2(\varepsilon - 1)}{\varepsilon} \right] \Psi + \Xi^*, \\ \Psi &= \left[\frac{a_2}{1 - s} + \frac{1 - a_2}{1 - l} l'(s) \right], \\ \Xi^* &= \left[\frac{a_2}{1 - s} - \frac{a_2}{1 - l} l'(s) \right] + \left[\frac{a_1}{s} - \frac{a_1}{l} l'(s) \right] < 0, \\ \Gamma &= (c_1 + \delta k) \left(\frac{a_1}{s} + \frac{1 - a_1}{l} l'(s) \right) - \frac{c_1(1 - \theta_2)}{B} \Psi. \end{aligned}$$

In the baseline model of the two-sector growth model without consumption externalities ($\theta_1 = \theta_2 = 0$), the elements in the Jacobean matrix are as follows:²⁹

$$\begin{aligned}
 J_{11}^* &= \frac{(1 + pc_1/c_2)(\varrho + \delta)(1 - a_1)}{s} \left[-1 + \frac{a_1(1 - a_2)l}{a_2(1 - a_1)s} \right] \frac{1}{\Xi^*} < 0, \\
 J_{12}^* &= \frac{\lambda^*(\varrho + \delta)(1 - a_1)}{s} \left\{ \frac{s}{k} - \left[-1 + \frac{a_1(1 - a_2)l}{a_2(1 - a_1)s} \right] \frac{a_1 - a_2}{k} \frac{1}{\Xi^*} \right\} > 0, \\
 J_{21}^* &= \frac{(1 + pc_1/c_2)}{\lambda^*} \left\{ \left[(c_1 + \delta k) \left(\frac{a_1}{s} + \frac{1 - a_1}{l} l' \right) + c_1 \Psi \right] \frac{1}{\Xi^*} - \varepsilon c_1 \right\} > 0, \\
 J_{22}^* &= \frac{n_1 - n_2}{k} \left[(c_1 + nk) \left(\frac{n_1}{s} + \frac{1 - n_1}{l} l' \right) + c_1 \Psi \right] \frac{1}{\Xi^*} \\
 &\quad + \frac{(n_1 - n_2)c_1}{k} - (1 - n_1)n > 0 \\
 &\quad \text{if } \frac{n_1 - n_2}{k} \left[(c_1 + nk) \left(\frac{n_1}{s} + \frac{1 - n_1}{l} l' \right) + c_1 \Psi \right] \frac{1}{\Xi^*} \\
 &\quad + \frac{(n_1 - n_2)c_1}{k} > (1 - n_1)n.
 \end{aligned}$$

B.1. $\theta_1 = \theta_2 = 0$

In a standard two-sector growth model in which $\theta_1 = \theta_2 = 0$, the steady state is a saddle, which indicates that the dynamic system has one root with a negative real part and one root with a positive real part. The required condition is $\text{Det}(J^*) = J_{11}^* J_{22}^* - J_{21}^* J_{12}^* < 0$.

B.2. The Case of $\theta_1 \neq 0$ and $\theta_2 = 0$

The elements in the Jacobian matrix are

$$\begin{aligned}
 J_{11}^{\theta_1} &= -\frac{1 + \theta_1(\varepsilon - 1)}{B} \frac{\Xi^*}{\Xi^* + \Xi_1^{\theta_1}} J_{11}^*, \\
 J_{12}^{\theta_1} &= \frac{\lambda}{\lambda^*} J_{12}^* + \frac{(\varrho + \delta)(1 - a_1)}{s} \left[1 - \frac{a_1(1 - a_2)l}{a_2(1 - a_1)s} \right] \frac{a_1 - a_2}{k} \frac{1 - D^{\theta_1}}{\Xi^*}, \\
 J_{21}^{\theta_1} &= -\frac{[1 + \theta_1(\varepsilon - 1)](1 + pc_1/c_2)}{\lambda B} \frac{\Xi^*}{\Xi^* + \Xi_1^{\theta_1}} J_{21}^* \\
 &\quad + \frac{[-\theta_1(\varepsilon - 1)][1 + p(c_1/c_2)\varepsilon/(\varepsilon - 1)]c_1\Psi}{B} - \varepsilon c_1 \Xi^* \left\{ -\frac{\varepsilon c_1(1 + pc_1/c_2)}{\lambda B} \right\}, \\
 J_{22}^{\theta_1} &= D^{\theta_1} J_{22}^* + \frac{[-\theta_1(\varepsilon - 1)][1 + p(c_1/c_2)\varepsilon/(\varepsilon - 1)]c_1}{B} \left[\frac{(a_1 - a_2)\Psi D^{\theta_1}}{k \Xi^*} + \frac{a_2}{k} \right] \\
 &\quad + \left[\frac{(a_1 - a_2)c_1}{k} - (1 - a_1)\delta \right] (1 - D^{\theta_1}),
 \end{aligned}$$

where $\Xi^{\theta_1} = -\left[\frac{1 + \theta_1(\varepsilon - 1)}{\varepsilon} \frac{1}{B} + \frac{1}{\varepsilon} \right] \Psi$ and $D^{\theta_1} = \left(1 - \frac{a_2}{a_1 - a_2} \frac{\Xi^{\theta_1}}{\Psi} \right) \frac{\Xi^*}{\Xi^* + \Xi_1^{\theta_1}}$.

The steady state is a sink if the Jacobean matrix J has two roots with negative real parts, whose conditions are $\text{Tr}(J^{\theta_1}) < 0$ and $\text{Det}(J^{\theta_1}) > 0$. These conditions require (i) $-J^*_{11} > J^*_{22}$ and (ii) the slope condition

$$\left. \frac{dc_1}{dk} \right|_{\dot{c}_1=0} = -\frac{J^{\theta_1}_{12}}{J^{\theta_1}_{11}} > \left. \frac{dc_1}{dk} \right|_{\dot{k}=0} = -\frac{J^{\theta_1}_{22}}{J^{\theta_1}_{21}}. \tag{B.1}$$

Let

$$\begin{aligned} a_1 &\equiv \frac{[1 + p(c_1/c_2)(\varepsilon/\varepsilon - 1)]\Xi^2}{(1 + pc_1/c_2)^2} [\text{Det}(J^*) + \Omega_1] - \frac{p(c_1/c_2)(1/(\varepsilon - 1))\Xi^*\Psi}{(1 + pc_1/c_2)^2} \\ &\quad \times \left[\text{Det}(J^*) + \Omega_1 + \left(1 + \frac{a_2}{a_1 - a_2} \frac{\Xi^*}{\Psi} \right) \left(\Omega_2 - \Omega_3 - \frac{J^*_{11}c_1(a_1 - a_2)}{k} \frac{\Psi}{\Xi^*} \right) \right], \\ b_1 &\equiv -\frac{[1 + p(c_1/c_2)\varepsilon/(\varepsilon - 1)]\Xi^2}{(1 + pc_1/c_2)^2} \Omega_1 - \frac{p(c_1/c_2)(1/(\varepsilon - 1))\Xi^*\Psi}{(1 + pc_1/c_2)^2} \\ &\quad \times \left[\Omega_4 - \left(1 + \frac{a_2}{a_1 - a_2} \frac{\Xi^*}{\Psi} \right) \Omega_2 \right] - \frac{[2 + p(c_1/c_2)\varepsilon/(\varepsilon - 1)]\Xi^2}{(1 + pc_1/c_2)^2} \Omega_4, \end{aligned}$$

where

$$\begin{aligned} \Omega_1 &= \frac{J^*_{12}c_1(1 + pc_1/c_2)}{\lambda^*} \left(\varepsilon - \frac{\Psi}{\Xi^*} \right) + \frac{J^*_{11}c_1(a_1 - a_2)}{k} \frac{\Psi}{\Xi^*} \left(1 + \frac{a_2}{a_1 - a_2} \frac{\Xi^*}{\Psi} \right), \\ \Omega_2 &= \frac{J^*_{11}[(a_1 - a_2)c_1 - (1 - a_1)\delta k - J^*_{22}k]}{k} - \frac{(\varrho + \delta)(1 - a_1)(a_1 - a_2)}{sk\Xi^*} \\ &\quad \times \left[1 - \frac{a_1(1 - a_2)l}{a_2(1 - a_1)s} \right] [J^*_{21}\lambda^* - \varepsilon c_1(1 + pc_1/c_2)], \\ \Omega_3 &= -(1 + pc_1/c_2)(\varrho + \delta)(1 - \alpha_1) \left[1 - \frac{a_1(1 - a_2)l}{a_2(1 - a_1)s} \right] \frac{a_1 - a_2}{sk} \frac{c_1\Psi}{\Xi^{*2}}, \\ \Omega_4 &= \text{Det}(J^*) + \frac{1}{\lambda^*} J^*_{12}\varepsilon c_1(1 + pc_1/c_2). \end{aligned}$$

Then the relative slope condition that yields a sink is rewritten as

$$\varphi_1[-\theta_1(\varepsilon - 1)]^2 + b_1[-\theta_1(\varepsilon - 1)] + \text{Det}(J^*) > 0. \tag{B.2}$$

If we set (B.2) to equal zero, we obtain two critical values for $[-\theta_1(\varepsilon - 1)]$, denoted by ζ_1 and ζ_2 :

$$\zeta_1, \zeta_2 = \frac{1}{2\varphi_1} \{-b_1n[(b_1)^2 - 4\varphi_1\text{Det}(J^*)]^{1/2}\}.$$

Let $\zeta_1 < \zeta_2$ when $\varphi_1 > 0$. Then, when $\varphi_1 < 0$, $\zeta_1 > \zeta_2$. Thus we have the following results:

- (i) If $\varphi_1 > 0$, (B.2) requires $[-\theta_1(\varepsilon - 1)] < \zeta_1$ or $[-\theta_1(\varepsilon - 1)] > \zeta_2$.
- (ii) If $\varphi_1 < 0$, (B.2) require $\zeta_2 < [-\theta_1(\varepsilon - 1)] < \zeta_1$.

If we combine Conditions KIJ and (B.2), the required conditions of a steady state that is a sink are summarized as follows:

- (i) $0 < [-\theta_1(\varepsilon - 1)] < \zeta_1$ or $[-\theta_1(\varepsilon - 1)] > \max\{\zeta_2, 0\}$ if $\varphi_1 > 0$;
 (ii) $\max\{\zeta_2, 0\} < [-\theta_1(\varepsilon - 1)] < \zeta_1$ if $\varphi_1 < 0$.

B.3. The Case of $\theta_1 = 0$ and $\theta_2 \neq 0$

In this case, the elements in the Jacobian matrix are

$$\begin{aligned} J_{11}^{\theta_2} &= \frac{\Xi^*}{\Xi^* - \theta_2 \Psi} J_{11}^*, \\ J_{12}^{\theta_2} &= \frac{\lambda}{\lambda^*} J_{12}^* + \frac{\lambda(\varrho + \delta)(1 - a_1)}{s} \left[1 - \frac{a_1(1 - a_2)l}{a_2(1 - a_1)s} \right] \frac{a_1 - a_2}{k} \frac{1 - D^{\theta_2}}{\Xi^*}, \\ J_{21}^{\theta_2} &= \frac{1}{\lambda(\Xi^* - \theta_2 \Psi)} [\Xi^* \lambda^* J_{21}^* + \theta_2 c_1 (1 + p c_1 / c_2) (1 - \varepsilon \Psi)], \\ J_{22}^{\theta_2} &= D^{\theta_2} J_{22}^* + \theta_2 c_1 \left[\frac{(a_1 - a_2) \Psi D^{\theta_2}}{k \Xi^*} + \frac{a_2}{k} \right] + \left[\frac{(a_1 - a_2) c_1}{k} - (1 - a_1) \delta \right] (1 - D^{\theta_2}), \end{aligned}$$

where $D^{\theta_2} = (1 - \frac{a_2}{a_1 - a_2} \frac{\Xi^{\theta_2}}{\Psi}) \frac{\Xi^*}{\Xi^* + \Xi^{\theta_2}}$.

To obtain two roots with negative real parts, the conditions are $\text{Tr}(J^{\theta_2}) = J_{11}^{\theta_2} + J_{22}^{\theta_2} < 0$ and $\text{Det}(J^{\theta_2}) = J_{11}^{\theta_2} J_{22}^{\theta_2} - J_{12}^{\theta_2} J_{21}^{\theta_2} < 0$, which lead to the relative slope condition that gives a sink.

Let

$$\begin{aligned} \varphi_2 &= \frac{\Psi^2}{(\varepsilon - 1)(1 + p c_1 / c_2)^2} \left[\left(1 + \frac{a_2}{a_1 - a_2} \frac{\Xi^*}{\Psi} \right) \frac{\Xi^*}{\Psi} \frac{\Omega_3}{(1 + p c_1 / c_2)} - \frac{J_{12}^* c_1}{\lambda^*} \right], \\ b_2 &\equiv \frac{\Xi^* \Psi}{(\varepsilon - 1)(1 + p c_1 / c_2)^2} \left[\text{Det}(J^*) - \frac{J_{11}^* c_1 (a_1 - a_2)}{k} \right. \\ &\quad \left. + \frac{J_{12}^* c_1 (1 + p c_1 / c_2) (1 - \varepsilon \Xi^* / \Psi)}{\lambda^*} \right. \\ &\quad \left. + \left(1 + \frac{a_2}{a_1 - a_2} \frac{\Xi^*}{\Psi} \right) \left(\Omega_2 + \frac{\varepsilon \Xi^*}{\Psi} \Omega_3 \right) - \frac{a_2 c_1 J_{11}^* \Xi^*}{k \Psi} \right]. \end{aligned}$$

Then the relative slope condition that gives a sink is rewritten as

$$\varphi_2 [-\theta_2(\varepsilon - 1)]^2 + b_2 [-\theta_2(\varepsilon - 1)] + \text{Det}(J^*) > 0. \quad (\text{B.3})$$

If we set (B.3) to equal zero, we obtain the two critical values for $[-\theta_2(\varepsilon - 1)]$, denoted by η_1 and η_2 . Let $\eta_1 < \eta_2$ when $\varphi_2 > 0$. Then when $\varphi_2 < 0$, $\eta_1 > \eta_2$. Thus, we have the following results:

- (i) If $\varphi_2 > 0$, (B.3) requires $[-\theta_2(\varepsilon - 1)] < \eta_1$ or $[-\theta_2(\varepsilon - 1)] > \eta_2$.
 (ii) If $\varphi_2 < 0$, (B.3) requires $\eta_2 < [-\theta_2(\varepsilon - 1)] < \eta_1$.

If we combine Conditions KUJ and (B.3), the required conditions of a steady state that is a sink are summarized as

- (i) $0 < [-\theta_2(\varepsilon - 1)] < \eta_1$ or $[-\theta_2(\varepsilon - 1)] > \max\{\eta_2, 0\}$ if $\varphi_2 > 0$;

(ii) $\max\{\eta_2, 0\} < [-\theta_2(\varepsilon - 1)] < \eta_1$ if $\varphi_2 < 0$.

B.4. The Case $\theta_1 = \theta_2 = \theta \neq 0$

In the case, the elements in the Jacobian matrix are

$$\begin{aligned}
 J_{11}^\theta &= -\frac{1 + \theta(\varepsilon - 1)}{B} \frac{\Xi^*}{\Xi^* + \Xi^\theta} J_{11}^*, \\
 J_{12}^\theta &= \frac{\lambda}{\lambda^*} J_{12}^* + \frac{\lambda(\varrho + \delta)(1 - \alpha_1)}{s} \left[1 - \frac{a_1(1 - a_2)}{a_2(1 - a_1)} \frac{l}{s} \right] \frac{a_1 - a_2}{k} \frac{1 - D^\theta}{\Xi^*}, \\
 J_{21}^\theta &= -\frac{[1 + \theta(\varepsilon - 1)](1 + pc_1/c_2)}{\lambda B} \frac{\Xi^*}{\Xi^* + \Xi^\theta} \left\{ \frac{\lambda^* \Xi^*}{(1 + pc_1/c_2)} J_{21}^* \right. \\
 &\quad \left. + \frac{[-\theta_1(\varepsilon - 1)][1 + p(c_1/c_2)]\varepsilon/(\varepsilon - 1)c_1\Psi - \varepsilon c_1 \Xi^*}{B} \right\} - \frac{\varepsilon c_1(1 + pc_1/c_2)}{\lambda B}, \\
 J_{22}^\theta &= D^\theta J_{22}^* + \frac{[-\theta_1(\varepsilon - 1)][1 + p(c_1/c_2)]\varepsilon/(\varepsilon - 1)c_1}{B} \\
 &\quad \times \left[\frac{(a_1 - a_2)\Psi D^\theta}{k \Xi^*} + \frac{a_2}{k} \right] + \left[\frac{(a_1 - a_2)c_1}{k} - (1 - a_1)\delta \right] (1 - D^\theta),
 \end{aligned}$$

where $\Xi^\theta = -\frac{1+\theta(\varepsilon-1)}{\varepsilon} [1 + \frac{1-\theta}{B}]\Psi$ and $D^{\theta_1} = (1 - \frac{a_2}{a_1 - a_2} \frac{\Xi^\theta}{\Psi}) \frac{\Xi^*}{\Xi^* + \Xi^\theta}$.

Let

$$\begin{aligned}
 \varphi_3 &\equiv \frac{\Xi^* \Psi}{(\varepsilon - 1)(1 + pc_1/c_2)} \left[\Omega_4 + \left(1 + \frac{a_2 \Xi^*}{a_1 - a_2} \right) \Omega_2 \right], \\
 b_3 &\equiv \frac{\Xi^{*2}}{(1 + pc_1/c_2)^2} \left[\left(1 + p \frac{c_1}{c_2} \frac{\varepsilon}{\varepsilon - 1} \right) \Omega_4 - \left(1 + \frac{a_2 \Xi^*}{a_1 - a_2} \right) \frac{\varepsilon \Omega_3 (1 + pc_1/c_2)}{\Psi} \right] \\
 &\quad + \frac{\Xi^* \Psi \varepsilon c_1}{(\varepsilon - 1)(1 + pc_1/c_2)} \left[\left(1 + \frac{a_2 \Xi^*}{a_1 - a_2} \right) \frac{J_{11}^* (a_1 - a_2)}{k} + \frac{J_{12}^* (1 + pc_1/c_2)}{\lambda^*} \right], \\
 d_3 &\equiv \frac{\Xi^{*2} (1 + p \frac{c_1}{c_2} \frac{\varepsilon}{\varepsilon - 1})}{(1 + pc_1/c_2)} \frac{J_{12}^* \varepsilon c_1}{\lambda^*} - \frac{\Xi^{*2} (2 + p \frac{c_1}{c_2} \frac{\varepsilon}{\varepsilon - 1})}{(1 + pc_1/c_2)^2} \Omega_4 + \frac{\Xi^* \Psi}{(\varepsilon - 1)(1 + pc_1/c_2)} \\
 &\quad \times \left[Det(J^*) + \left(1 + \frac{a_2 \Xi^*}{a_1 - a_2} \right) \left(\Omega_2 + \frac{\varepsilon \Xi^* \Omega_3}{\Psi} - \frac{J_{11}^* \varepsilon c_1 (a_1 - a_2)}{k} \right) \right].
 \end{aligned}$$

The relative slope condition that yields a sink is rewritten as

$$\varphi_3[-\theta(\varepsilon - 1)]^3 + b_3[-\theta(\varepsilon - 1)]^2 + d_3[-\theta(\varepsilon - 1)] + Det(J^*) > 0. \tag{B.4}$$

Define $\varphi = b_3/\varphi_3$, $b = d_3/\varphi_3$, and $d_3 = Det(J^*)/\varphi_3$. If we set (B.4) equal to 0, we obtain three critical values of $[-\theta(\varepsilon - 1)]$: $m + n - \varphi/3$, $m\omega + n\omega^2 - \varphi/3$, and $m\omega^2 + n\omega - \varphi/3$,

in which

$$\begin{aligned}
 m &= \frac{1}{3} \sqrt[3]{-\frac{27d - 9\varphi b + 2\varphi^3}{2} + \sqrt{\left(\frac{27d - 9\varphi b + 2\varphi^3}{2}\right)^2 + (3b - \varphi^2)^3}}, \\
 n &= \frac{1}{3} \sqrt[3]{-\frac{27d - 9\varphi b + 2\varphi^3}{2} - \sqrt{\left(\frac{27d - 9\varphi b + 2\varphi^3}{2}\right)^2 + (3b - \varphi^2)^3}}, \\
 \omega &= \frac{-1 + \sqrt{3i}}{2}.
 \end{aligned}$$

Denote the three critical values as l_1 , l_2 , and l_3 . Let l_3 be the largest and l_1 be the smallest value of the three critical values. Then the required conditions of a steady state that is a sink are summarized as

- (i) $\max\{0, l_1\} < [-\theta(\varepsilon - 1)] < \max\{0, l_2\}$ or $[-\theta(\varepsilon - 1)] > \max\{0, l_3\}$, if $\varphi_3 > 0$;
- (ii) $0 < [-\theta(\varepsilon - 1)] < \max\{0, l_1\}$ or $\max\{0, l_2\} < [-\theta(\varepsilon - 1)] < \max\{0, l_3\}$, if $\varphi_3 < 0$.