# Effects of labor taxes and unemployment compensation on labor supply in a search model with an endogenous labor force ${ }^{2 \pi}$ 

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#### Abstract

Labor taxes and unemployment compensation were blamed for causing relative declines in labor supply in the EU to the US in the past decades. We propose a model with an endogenous labor force and compare with the model with an exogenous labor force. Because of discouraging the labor force, labor taxes decrease employment in our model less than the model with an exogenous labor force, have ambiguous effects on hours, and decrease less labor supply in our model. Due to boosting the labor force, unemployment compensation increases employment in our model and decreases in the model with an exogenous labor force, but with opposite effects on hours, labor supply is ambiguous in both models. To understand the net effect on labor supply, we feed in the data of increases in labor taxes and unemployment compensation in the EU relative to the US. We find that the model with an exogenous labor force explain excessively of decreases in employment and labor supply, with increases in hours against the data. In contrast, our model explains reasonable decreases in labor supply, with sensible decreases in employment and in hours. Thus, with an endogenous labor force, our model explains relative declines in labor supply better than the model with an exogenous labor force.


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## 1. Introduction

Average labor supply in the EU declined about one fourth relative to those in the US from the early 1970s to the early 2000s. A growing body of literature has sought to understand the relative importance of the various policies and institutional factors that have been proposed as competing explanations. In particular, two important labor market policies are blamed for causing declining labor supply in the EU relative to the US over the past 30 years. One of these is higher labor taxes that were advocated by Prescott (2002, 2004) and his followers (e.g., Ohanian et al., 2008; Jacobs, 2009; Rogerson and Wallenius, 2009) and the other is generous unemployment benefits that were stressed by Alesina et al. (2006) and Ljungqvist and

[^0]Sargent (2007a, 2008). The former studies involve only an intensive margin (working hours per worker), whereas the latter papers include only an employment margin. The only exception is Fang and Rogerson (2009) who took both margins into account. ${ }^{1}$ Thus, these existing models considered either the intensive margin, the employment margin, or both margins of labor supply.

A notable feature in the data is that differences in average labor supply in the EU relative to the US are due to differences along three margins: the intensive margin, the employment margin and the participation margin (the labor force). According to OECD (2010a, 2010b), the US added more to the labor force than the EU over the past 30 years. See Table 1. While there are models that incorporate an endogenous labor force, no paper incorporates all three margins when explaining declining labor supply in the EU relative to the US. ${ }^{2}$ In this paper we attempt to fill the gap by studying a model with all these three margins. Our paper compares the long-run effects on labor supply of increases in labor taxes and unemployment compensation in models with and without participation margins. These two policies may not fully explain the difference in labor supply between the EU and the US in the past 3 decades, because there are differences in other labor market policies and institutions. ${ }^{3}$ Yet, by considering the participation, our model serves as a first step in understanding the effects of the two major labor market policies on labor supply.

Specifically, our model is the large household model of Fang and Rogerson (2009) extended to consider the participation margin. The large household pools all resources for its members and decides between consumption and savings. Employment is a predetermined state and the employed members choose between working and leisure time. The large firm creates and maintains multiple vacancies and produces goods. Job vacancies and job seekers are brought together by the matching technology and, upon a successful match, bargain over wage and working hours. Unlike Fang and Rogerson (2009), here the nonemployed are free to choose between searching for jobs and engaging in nonmarket activities. A novel feature of our work is that the participation margin is modeled as a control variable, not a state variable, and thus can be introduced into the framework within a representative large household. Our model renders as a special case the model studied by Fang and Rogerson (2009) wherein the labor-force participation is exogenous.

In analyzing the long-run effects regarding the policies of increases in labor tax rates and unemployment compensation on labor supply, main results are as follows. First, with increases in labor taxes, due to discouraging labor-force participation, the employment in our model is decreased less than that in the model with an exogenous labor force and, with ambiguous effects on hours worked per worker in both models, labor supply is decreased by less in our model. Next, with increases in unemployment compensation, due to inducing the labor force, employment increases in our model but decreases in the model with an exogenous labor force and, with effects on hours worked per worker opposite to those on employment, the effects on labor supply are ambiguous in both models, depending on whether the effect on employment or that on hours worked per worker dominates.

To quantify the net effect on labor supply, we calibrate our model to the US economy. By feeding in the data of increases in the labor tax and unemployment compensation in the EU relative to the US from the early 1970s to the early 2000s, we find that the model with an exogenous labor force explains too much of the decreases in employment and labor supply between the EU and the US. In particular, this model predicts an increase in hours worked per worker, but the data indicates a decrease. By contrast, with an endogenous labor force, our model explains a more reasonable decrease in labor supply, with a sensible decrease in employment and a modest decrease rather than an increase in hours worked per worker in the EU relative to the US. Thus, with the participation margin, our model explains the difference in labor supply better than the model with an exogenous labor force.

We must point out that Tripier (2004) and Shimer (2011) have considered non-participation as a control wherein the nonemployed decide to be unemployed or inactive. ${ }^{4}$ Tripier (2004) used his model to quantitatively account for the allocation of time among employment, unemployment and non-participation in the US. Shimer (2011) applied his model to study countercyclical unemployment rates and persistent fluctuations in the vacancy-unemployment ratio in the US. Unlike these two papers, our paper explores the effects of increases in labor taxes and unemployment compensation on labor supply as a result from changes in hours per worker, employment and the labor force. Kim (2003) is also close to our paper in that he analyzed the effect of unemployment benefits in a search model with an endogenous labor force. However, non-participation is a state rather than a control in Kim (2003), so it is difficult to offer analytical analysis.

This paper is outlined as follow. In Section 2, we document differences in the aggregate labor supply between the US and EU along intensive, employment and participation margins. In Section 3, we set up a matching model with the three margins and then characterize the steady state equilibrium in models with and without an endogenous participation margin. In

[^1]Table 1
Labor supply in EU relative to US, 1970-1973 and 2000-2003. Sources: OECD (2010a, 2010b)

|  | Labor supply |  |  | Hours per worker |  |  | Employment rate (\%) |  |  | Participation rate (\%) |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 70-73 | 00-03 | diff. (\%) | 70-73 | 00-03 | diff. (\%) | 70-73 | 00-03 | diff. | 70-73 | 00-03 | diff. |
| EU | $\begin{aligned} & 1227 \\ & (109.4) \end{aligned}$ | $\begin{aligned} & 1034 \\ & (83.39) \end{aligned}$ | -15.71 | $\begin{aligned} & 1940 \\ & (107.7) \end{aligned}$ | $\begin{aligned} & 1613 \\ & (93.67) \end{aligned}$ | -16.84 | $\begin{aligned} & 97.4 \\ & (102.9) \end{aligned}$ | $\begin{aligned} & 92.2 \\ & (97.17) \end{aligned}$ | -5.32 | $\begin{aligned} & 65 \\ & (98.78) \end{aligned}$ | $\begin{aligned} & 69.5 \\ & (91.63) \end{aligned}$ | 7.06 |
| US | $\begin{aligned} & 1122 \\ & (100) \end{aligned}$ | $\begin{aligned} & 1240 \\ & (100) \end{aligned}$ | 10.57 | $\begin{aligned} & 1802 \\ & (100) \end{aligned}$ | $\begin{aligned} & 1722 \\ & (100) \end{aligned}$ | -4.43 | $\begin{aligned} & 94.7 \\ & (100) \end{aligned}$ | $\begin{aligned} & 94.9 \\ & (100) \end{aligned}$ | 0.23 | $\begin{aligned} & 65.8 \\ & (100) \end{aligned}$ | $\begin{aligned} & 75.9 \\ & (100) \end{aligned}$ | 15.41 |

Note: The hours per worker are annual hours of market work per worker. The employment rate is the number of the employed divided by the number in the labor force. The participation rate is the number of the labor force divided by the number of the population aged 15-64. Finally, the labor supply is annual hours of market work per capita and is calculated by the annual work hours per worker times the number of the employed divided by the working-age population. In a cell with two values, the tops are the original values and the bottoms in parenthesis are relative to the US with the value in the US normalized to $100 \%$ in both 1970-1973 and 2000-2003. The difference is a percentage difference of a value in 2000-2003 to a value in 1970-1973. The EU includes Belgium, Denmark, Finland, France, Germany, Ireland, Italy, Netherlands, Spain, Sweden and the UK wherein the data are available in both periods.

Section 4, we analyze the effects of increases in labor taxes and unemployment compensation on labor supply and then offer quantitative results. Finally, we provide concluding remarks in Section 5.

## 2. The model

Our model is based on Fang and Rogerson (2009) and extended to include an active participation margin. The economy is composed by a continuum of households and firms, and a passive fiscal authority. The details of the environment follow.

### 2.1. Households

The economy is populated by a continuum of "large" households of unit mass. The setup of large households is convenient in that all family members pool all resources regardless of their labor market status. This useful method of modeling perfect consumption insurance in general-equilibrium search models has been common since Merz (1995) and Andolfatto (1996). A representative large household consists of a continuum of family members (of measure one). Family members are either employed, by engaging in productive activities, or nonemployed, by engaging either in job search activities or in other nonmarket activities. Employment is a predetermined state in each period. If we denote $e$ as the fraction of employed members (referred to as employed workers) in the representative large household, then the fraction of nonemployed members is ( $1-e$ ). Employed members choose between working time $h$ and leisure time $(1-h)$. Nonemployed members decide to search for jobs (referred to as job seekers or unemployed workers) or to engage in other nonmarket activities (referred to as non-participants). If $n$ is the fraction of members engaging in other non-market activities, the fraction of members in the labor force is $(1-n)$ and thus $(1-e-n)$ is the fraction in the labor force not working but searching for jobs. See the labor allocation in Fig. 1. ${ }^{5}$ Given the basics of the environment, the unemployment rate is the fraction of unemployment in the labor force and is thus $\mathbf{u} \equiv(1-n-e) /(1-n)$.

Let $\mu_{t}$ denote the (endogenous) job finding rate and $\lambda$ the (exogenous) job separation rate. Then, changes of employment from the household's perspective are

$$
\begin{equation*}
e_{t+1}-e_{t}=\mu_{t}\left(1-e_{t}-n_{t}\right)-\lambda e_{t} . \tag{1}
\end{equation*}
$$

Let $w$ denote the wage per employed worker, ${ }^{6} r$ the capital rental, $\tau$ the labor income tax rate, $b$ unemployment benefits received by unemployed members, $\pi$ profits remitted from firms and $T$ lump-sum taxes. The large household's budget constraint is

$$
\begin{equation*}
c_{t}+\left[k_{t+1}-(1-\delta) k_{t}\right]=r_{t} k_{t}+(1-\tau) w_{t} e_{t}+b\left(1-e_{t}-n_{t}\right)+\pi_{t}-T_{t} \tag{2}
\end{equation*}
$$

where $c$ is consumption, $k$ physical capital and $\delta$ the depreciation rate of capital.
An agent obtains utility from consumption and leisure depending on the labor-market status. The utility of an employed member is $u\left(c_{t}\right)+\chi_{1} V\left(1-h_{t}\right)$, the utility of an unemployed member is $u\left(c_{t}\right)+\chi_{2}$, and the utility of a member outside the labor force is $u\left(c_{t}\right)+\chi_{3}$, where $\chi_{1}, \chi_{2}$ and $\chi_{3}$ are parameters. The representative household's utility simply sums up utilities over its members and is thus

$$
u\left(c_{t}\right)+e_{t} \chi_{1} V\left(1-h_{t}\right)+\left(1-e_{t}-n_{t}\right) \chi_{2}+n_{t} \chi_{3} .
$$

[^2]

Fig. 1. Labor allocation for the large household

Some remarks about the utility of the representative household follow. Following Garibaldi and Wasmer (2005), Pries and Rogerson (2009) and Krusell et al. (2011), we use a linear utility of leisure for members outside the labor force as well as members in the labor force not working but actively searching for a job. Moreover, as these studies, we restrict $\chi_{3}>\chi_{2}$ in order to allow for a non-degenerated fraction of members outside the labor force. ${ }^{7}$ Different from the linear utility of consumption adopted by Garibaldi and Wasmer (2005) and Pries and Rogerson (2009), which implicitly impose assumptions on income and substitution effects that govern labor supply that are not consistent with standard specifications, we follow Krusell et al. (2011) and employ a concave utility of consumption. Unlike Krusell et al. (2011) wherein an employed worker devotes all the time endowment to work so $h=1$, we follow Fang and Rogerson (2009) to assume a concave utility of leisure for an employed worker so as to give interior work and leisure time. To ease the analysis, we follow Andolfatto (1996) and use the parametric forms of utility given by

$$
\begin{equation*}
u(c)=\ln c \quad \text { and } \quad V(1-h)=\frac{(1-h)^{1-\sigma}}{1-\sigma} \tag{3}
\end{equation*}
$$

in which $\sigma>0$ is the reciprocal of the elasticity of leisure. These forms of utility are consistent with a balanced growth path.
The household chooses a path for consumption $c_{t}$ and a path for non-participation $n_{t}$ to maximize its lifetime utility subject to the flow budget constraint (2), taking as given the law of motion for employment (1) as well as the job-finding rate, the capital rental rate, the wage rate, the income tax rate and the unemployment benefit. Let $U\left(k_{t}, e_{t}\right)$ be the lifetime value of the representative household. The household's optimization problem is written as

$$
U\left(k_{t}, e_{t}\right)=\max _{\left\{k_{t+1}, n_{t}\right\}}\left\{\left[u\left(c_{t}\right)+e_{t} \chi_{1} V\left(1-h_{t}\right)+\left(1-e_{t}-n_{t}\right) \chi_{2}+n_{t} \chi_{3}\right]+\frac{1}{1+\rho} U\left(k_{t+1}, e_{t+1}\right)\right\},
$$

subject to the constraints (1) and (2), where $\rho>0$ is the time preference rate. The first-order conditions with respect to $c_{t}$ and $n_{t}$ and the Benveniste-Scheinkman conditions for $k_{t}$ and $e_{t}$ are

$$
\begin{align*}
& u^{\prime}\left(c_{t}\right)=\frac{1}{1+\rho} U_{k}\left(k_{t+1}, e_{t+1}\right),  \tag{4a}\\
& u^{\prime}\left(c_{t}\right) b+\chi_{2}+\frac{1}{1+\rho} \mu_{t} U_{e}\left(k_{t+1}, e_{t+1}\right)=\chi_{3},  \tag{4b}\\
& U_{k}\left(k_{t}, e_{t}\right)=u^{\prime}\left(c_{t}\right)\left(1-\delta+r_{t}\right),  \tag{4c}\\
& U_{e}\left(k_{t}, e_{t}\right)=\left[u^{\prime}\left(c_{t}\right)(1-\tau) w_{t}+\chi_{1} V\left(1-h_{t}\right)+\frac{1}{1+\rho}(1-\lambda) U_{e}\left(k_{t+1}, e_{t+1}\right)\right]-\left[u^{\prime}\left(c_{t}\right) b+\chi_{2}+\frac{1}{1+\rho} \mu_{t} U_{e}\left(k_{t+1}, e_{t+1}\right)\right] . \tag{4d}
\end{align*}
$$

In these conditions, (4c) is the marginal value of capital and, with the use of (4a), we obtain the standard consumption Euler equation

$$
\begin{equation*}
u^{\prime}\left(c_{t}\right)=\frac{1-\delta+r_{t+1}}{1+\rho} u^{\prime}\left(c_{t+1}\right) . \tag{5a}
\end{equation*}
$$

[^3]Condition (4d) gives the marginal value of employment which is the difference in the marginal utility between employment and unemployment. If the labor force is exogenous, (4d) indicates that a higher unemployment benefit $b$ increases the marginal utility of unemployment which gives a smaller marginal value of employment. Conversely, if the labor force is endogenous, (4b) is the condition which states that, in optimum, the marginal utility of unemployment is equal to the marginal utility of non-participation. In this case, if we replace the last brackets in (4d) by terms in (4b), we obtain

$$
\begin{equation*}
U_{e}\left(k_{t}, e_{t}\right)=\left[u^{\prime}\left(c_{t}\right)(1-\tau) w_{t}+\chi_{1} V\left(1-h_{t}\right)+\frac{1}{1+\rho}(1-\lambda) U_{e}\left(k_{t+1}, e_{t+1}\right)\right]-\left[\chi_{3}\right] . \tag{5b}
\end{equation*}
$$

Thus, with an endogenous labor force, a higher unemployment benefit $b$ does not affect the marginal value of employment. Intuitively, because the marginal utility of unemployment is equal to the fixed marginal utility of non-participation, a higher unemployment benefit cannot affect the marginal utility of unemployment and thus the marginal value of employment is not changed.

### 2.2. Firms

The production side of the economy includes a continuum of representative firms that creates job vacancies, rent capital and hire labor in order to produce final goods. The representative firm is "large" in the sense that it operates many jobs and consequently has many individual workers attached to it through those jobs. The production technology is neoclassical, represented by

$$
\begin{equation*}
y_{t}=A k_{t}^{\alpha}\left(e_{t} h_{t}\right)^{1-\alpha} \tag{6}
\end{equation*}
$$

where $A>0$ is a productivity parameter and $\alpha \in(0,1)$ is capital's income share.
From the firm's perspective, employment is increased by the inflow of employment and decreased by the outflow due to separation.

$$
\begin{equation*}
e_{t+1}-e_{t}=\eta_{t} v_{t}-\lambda e_{t} \tag{7}
\end{equation*}
$$

where $\eta_{t}$ is the (endogenous) recruitment rate and $v_{t}$ is the number of job vacancies.
As in Fang and Rogerson (2009), we assume that creating and maintaining one vacant job has a constant up-front cost of $\phi>0$. Hence, firm's flow profits in $t$ equal the output net of the costs of capital, labor, and vacancy creation; i.e.,

$$
\begin{equation*}
\pi_{t}=A k_{t}^{\alpha}\left(e_{t} h_{t}\right)^{1-\alpha}-r_{t} k_{t}-w_{t} e_{t}-\phi v_{t} \tag{8}
\end{equation*}
$$

The firm maximizes its value taking as given the law of motion for employment as well as the recruitment rate, the capital rental rate and the wage rate. Let $\Pi\left(e_{t}\right)$ denote the firm's lifetime value and $\frac{1}{1+\tilde{\xi}_{t}} \equiv \frac{1}{1+\rho} \frac{u^{\prime}\left(c_{t+1}\right)}{u^{\prime}\left(c_{t}\right)}$ denote its discount factor. ${ }^{8}$ The Bellman equation associated with the firm is

$$
\Pi\left(e_{t}\right)=\max _{\left\{k_{t}, v_{t}\right\}}\left[\pi_{t}+\frac{1}{1+\xi_{t}} \Pi\left(e_{t+1}\right)\right]
$$

subject to the constraint (7). The first-order conditions are

$$
\begin{align*}
& \alpha A\left(\frac{k_{t}}{e_{t} h_{t}}\right)^{\alpha-1}=r_{t}  \tag{9a}\\
& \phi=\frac{\eta_{t}}{1+\xi_{t}} \Pi_{e}\left(e_{t+1}\right)  \tag{9b}\\
& \Pi_{e}\left(e_{t}\right)=\left[(1-\alpha) A\left(\frac{k_{t}}{e_{t} h_{t}}\right)^{\alpha} h_{t}-w_{t}\right]+\frac{1-\lambda}{1+\xi_{t}} \Pi_{e}\left(e_{t+1}\right) \tag{9c}
\end{align*}
$$

Condition (9a) determines the demand for capital which gives the effective capital-labor ratio as $q_{t} \equiv \frac{k_{t}}{e_{t} h_{t}}=\left(\frac{\alpha A}{r_{t}}\right)^{\frac{1}{1-\alpha}}$. Condition (9b) shows that the firm creates the number of vacancies up to the margin when the marginal cost of vacancies equals the expected discounted marginal value of recruitment in the next period. The marginal value of recruitment in this period given by (9c) is the sum of the marginal product of labor net of the wage and the discounted marginal value of recruitment in the next period.

[^4]
### 2.3. Labor matching and bargaining

Following Diamond (1982), we assume pair-wise random matching. The matching technology takes the following con-stant-returns form: $M_{t}=m\left(1-e_{t}-n_{t}\right)^{\gamma}\left(v_{t}\right)^{1-\gamma}$, where $m>0$ measures the degree of matching efficacy and $\gamma \in(0,1)$ the contribution of job seekers in random matching. Aggregate job seekers ( $1-e_{t}-n_{t}$ ) and unfilled vacancies $v_{t}$ behave like two inputs in the matching function and the output is aggregate matched pairs $M_{t}$. The matching function facilitates the endogenous determination of job finding rates and recruitment rates.

A job seeker's surplus from a successful match is evaluated by the marginal value of employment $U_{e}$ in ( 5 b), whereas a vacant job's gain from a successful match is gauged by the marginal value of recruitment $\Pi_{e}$ in (9c). In a frictionless Walrasian world, taking the wage as given, the household maximizes $U_{e}$ and the firm maximizes $\Pi_{e}$ in order to decide their supply of and demand for labor. There is implicitly an auctioneer in the labor market which sets an equilibrium wage so as to equate labor supply to labor demand. Yet, there is no auctioneer in a frictional labor market and a job seeker would meet at most one unfilled job one time and similarly, an unfilled job would meet at most one job seeker one time. This situation creates a bilateral monopoly.

Following conventional wisdom, the wage is determined by a matched pair through a Nash bargaining game. Like Fang and Rogerson (2009), a worker does not devote all the time endowment to work in our model and thus the pair of a successful match also bargains over working hours. In a cooperative bargaining game, the following joint surplus is maximized: $\left[U_{e}\left(k_{t}, e_{t}\right)\right]^{\beta}\left[\Pi_{e}\left(e_{t}\right)\right]^{1-\beta}$, where $\beta \in(0,1)$ measures a labor's bargaining power. In solving the bargaining problem, the worker-job pair treats as given matching rates ( $\mu_{t}$ and $\eta_{t}$ ), the beginning-of-period level of employment $e_{t}$, and the market interest rate $r_{t}$. The worker also takes as given the wage and working hours of all others. The first-order conditions with respect to the wage and working hours are such that the resulting changes in the marginal value of employment and the marginal value of recruitment are summed to zero. ${ }^{9}$

### 2.4. The government

The government's behavior is passive. The government levies labor income taxes in order to pay unemployment benefits that satisfy the following budget constraint.

$$
\begin{equation*}
T_{t}+\tau w_{t} e_{t}=b\left(1-e_{t}-n_{t}\right) \tag{10}
\end{equation*}
$$

In order to isolate the effects of policy changes carried out later, we include lump-sum taxes $T_{t}$. When the labor tax rate is increased, with unemployment benefits being held constant, lump-sum taxes will change accordingly in order to balance the government budget. Similarly, when unemployment compensation is increased, with the labor tax rate being held constant, lump-sum taxes will adjust accordingly to balance the budget.

### 2.5. Equilibrium

A search equilibrium is a tuple of individual quantity variables, $\left\{e_{t}, h_{t}, n_{t}, v_{t}, c_{t}, k_{t}, y_{t}\right\}$, a pair of aggregate quantities, $\left\{M_{t}, T_{t}\right\}$, a pair of matching rates, $\left\{\mu_{t}, \eta_{t}\right\}$, and a pair of prices, $\left\{w_{t}, r_{t}\right\}$, such that: (i) all households and firms optimize; (ii) all employment evolutions hold, (iii) labor-market matching and wage and hours bargaining conditions are met; (iv) the government budget is balanced; and (v) the goods market clears. We focus on a steady state which is search equilibrium when all variables do not change over time. In a steady state, the consumption Euler equation (5a) gives $r=\rho+\delta$, and thus $\xi=r-\delta=\rho$. With this result, the effective capital-labor ratio in a steady state is $q \equiv \frac{k}{e h}=\left(\frac{\alpha A}{\rho+\delta}\right)^{\frac{1}{1-\alpha}}$ which is constant. If we use the household's budget (2) and the firm's flow profit (8), along with the government's budget (10), the goods market clearing condition in a steady state is

$$
\begin{equation*}
y=c+\delta k+\phi v \tag{11}
\end{equation*}
$$

Moreover, in a steady state the labor market satisfies the following matching relationships (Beveridge curve) given by $\mu(1-e-n)=\eta v=m(1-e-n)^{\gamma}(v)^{1-\gamma}=\lambda e$. These relationships enable us to solve matching rates and equilibrium vacancies as functions of $e$ and $n$.

$$
\begin{align*}
\mu & =\frac{\lambda e}{(1-n)-e} \equiv \mu(e, 1-n),  \tag{12a}\\
v & =\left[\frac{\lambda e}{m(1-n-e)^{\gamma}}\right]^{\frac{1}{1-\gamma}} \equiv v(\underset{+}{(e, 1-n),} \tag{12b}
\end{align*}
$$

[^5]\[

$$
\begin{equation*}
\eta=\frac{\lambda e}{v(e, 1-n)}=\left[m\left(\frac{1-n-e}{\lambda e}\right)^{\gamma}\right]^{\frac{1}{1-\gamma}} \equiv \eta(e, 1-n) . \tag{12c}
\end{equation*}
$$

\]

Intuitively, more employment $e$ decreases and higher labor-force participation $(1-n)$ increases job seekers. Thus, in these relationships, the job finding rate and the equilibrium vacancy are increasing in the number of employment and decreasing in the number of participants, while the firm's recruitment rate is decreasing in the number of employment and increasing in the number of participants. These relationships give $\frac{\mu}{\eta}=\frac{v}{1-n-e}$ which indicates that the ratio of job finding rates to recruitment rates is equal to the ratio of job vacancies to job seekers, a measure of the labor market tightness.

With the parametric forms of utility in (3), the steady-state conditions are as follows. First, (11) and (12b) give consumption as a function of employment, labor-force participation and work hours. ${ }^{10}$

$$
\begin{equation*}
c=\left(A q^{\alpha}-\delta q\right) e h-\phi v(e, 1-n) \equiv c(e, 1-n, h), \quad c_{e}>0, \quad c_{1-n}>0, \quad c_{h}>0 \tag{13}
\end{equation*}
$$

where higher employment and working hours increase output available for consumption. Moreover, higher labor-force participation reduces vacancies and hence more output is available for consumption.

The firm's gain from a successful match is (9c) and in a steady state, with $\xi=\rho$, is

$$
\begin{equation*}
\Pi_{e}=\frac{1+\rho}{\rho+\lambda}(M P L \cdot h-w) \tag{14}
\end{equation*}
$$

where MPL $\equiv(1-\alpha) A q^{\alpha}$ is the marginal product of labor which is fixed in a steady state. The firm's gain is the capitalized value of working hours-augmented marginal product of labor net of the wage.

## 3. Two models

If the labor-force participation is exogenous, $n_{t}$ is exogenously given by $\bar{n}$. Then, it is the model studied by Fang and Rogerson (2009). We will first study the steady state of the Fang and Rogerson model and then our model with an endogenous labor force.

### 3.1. Model with exogenous labor-force participation

First, as the labor force is exogenously given at $1-\bar{n}$, the consumption function $c$ in (13) can be expressed as $c(e, h ; 1-\bar{n})$. In a steady state, the household's surplus from a match in (4d) is

$$
\begin{equation*}
U_{e}=\frac{1+\rho}{\rho+\lambda+\mu}\left\{\left[u^{\prime}(c)(1-\tau) w+\chi_{1} V(1-h)\right]-\left[u^{\prime}(c) b+\chi_{2}\right]\right\} . \tag{15}
\end{equation*}
$$

Thus, the household's surplus from a match is the capitalized value of the difference in the marginal value between employment and unemployment.

By using the firm's gain from a match in (14) and the household's gain from a match in (15), we maximize the joint surplus of a match to determine the bargained wage as follows. ${ }^{11}$

$$
\begin{equation*}
w=\beta[M P L \cdot h]+(1-\beta)\left[\frac{1}{1-\tau}\left[\left(\chi_{2}-\chi_{1} \frac{(1-h)^{1-\sigma}}{1-\sigma}\right) c(e, h ; 1-\bar{n})+b\right]\right] \equiv w(e, h ; \tau, b, 1-\bar{n}), \tag{16}
\end{equation*}
$$

where $w_{e}>0, w_{h}>0, w_{\tau}>0$ and $w_{b}>0$. The wage is the weighted average of the marginal product of labor and the taxadjusted opportunity cost of employment. In addition to unemployment compensation, the opportunity cost is $M R S^{s e}(e, h ; 1-\bar{n}) \equiv\left(\chi_{2}-\chi_{1} \frac{(1-h)^{1-\sigma}}{1-\sigma}\right) c(e, h ; 1-\bar{n})>0$, the difference in the marginal rate of substitution (MRS) between leisure and consumption from searching a job to being employed. ${ }^{12}$ In view of the $c$ function in (13), the bargained wage is increasing in employment $e$, working hours per worker $h$ and the labor force $1-\bar{n}$. Moreover, a higher labor tax rates $\tau$ and a higher unemployment benefit $b$ increase the opportunity cost and thus raises the wage.

Moreover, we maximize the joint surplus to attain the condition for hours worked per worker. Dividing this condition by the first-order condition for the bargained wage gives

$$
\begin{equation*}
\chi_{1}(1-h)^{-\sigma} c(e, h ; 1-\bar{n})=(1-\tau) M P L \tag{17a}
\end{equation*}
$$

where the marginal cost of working hours is an employee's MRS between leisure and consumption and the marginal benefit of working hours is the after-tax MPL. The condition relates hours worked per worker $h$ to employment $e$ and exogenous factors in the way as follows.

[^6]\[

$$
\begin{equation*}
h=h(e ; 1-\bar{n}, \tau), \tag{17b}
\end{equation*}
$$

\]

where $h_{e}<0, h_{1-n}<0$ and $h_{\tau}<0$. These signs emerge because working hours increase the marginal cost of working hours, while employment increases the marginal cost of working hours and the labor force and the labor tax rate both decrease the marginal benefit of working hours.

Finally, we use $r=\rho+\delta$ and the $\Pi_{e}$ in (14) to rewrite the vacancy creation condition in (9b) as $\frac{\eta(e, 1-\bar{n})}{\rho+\lambda}(M P L \cdot h-w)=\phi$. The condition equates the marginal cost $\phi$ to the firm's capitalized value of the marginal product of recruitment net of the wage. Notice that a higher recruitment rate $\eta$ increases the capitalized value. By using the $w$ function in (16), this condition is rewritten as

$$
\begin{equation*}
\frac{\eta(e ; 1-\bar{n})}{\rho+\lambda}(1-\beta)\left[M P L \cdot h-\frac{1}{1-\tau}\left(M R S^{s e}(e, h ; 1-\bar{n})+b\right)\right]=\phi \tag{18}
\end{equation*}
$$

where the terms in the brackets are the flow gain from a match of which the firm's share is $1-\beta$. The left-hand side is the firm's capitalized value of the gain from a match and thus, the firm's marginal benefit of employment. With a given labor force $1-\bar{n}$, the condition relates employment $e$ to hours worked per worker $h$.

Eqs. (17b) and (18) are the most simplified conditions in the model with an exogenous labor force. They determine a unique pair of employment $e$ and hours worked per worker $h$ in the steady state. With employment, if we use (12a) and (12b), the ratio of vacancy to unemployment is determined. As we will focus on a simultaneous determination of employment and labor force in the next section, here we substitute hours worked per worker in (17b) into (18) in order to obtain an expression that relates employment as a function of labor force as follows.

$$
\begin{equation*}
\frac{\eta(e ; 1-\bar{n})}{\rho+\lambda}(1-\beta)\left[M P L \cdot h(\underset{-}{e} ; \tau, 1-\bar{n})-\frac{1}{1-\tau}\left(M R S^{\text {se }}(e, h(\underset{-}{e} ; \tau, 1-\bar{n}) ; 1-\bar{n})+b\right)\right]-\phi=0 . \tag{19}
\end{equation*}
$$

Thus, given $n$, (19) determines employment in the steady state. As indicated in Fig. 2, with $\bar{n}=n_{0}$ (19) determines the steady state $Q_{0}$ with unique employment given by the level $e_{0}$. A different value of $n$ would give a different level of employment. In particular, an increase in $n$ (and thus a decrease in the labor force) raises the firm's capitalized value of the marginal product of recruitment net of the wage. Then, employment will decrease so as to decrease the firm's capitalized value of the marginal product of recruitment net of the wage. Thus, $n$ and $e$ are inversely related in (19). As a result, we may perceive (19) as a negatively-sloping locus in the $(e, n)$ plane which, for convenience, is referred to as Locus E (employment).

### 3.2. Our model with endogenous labor-force participation

Next, we study our model. Now, the household's surplus from a successful match is not (4d) but is (5b). In a steady state, (5b) gives

$$
\begin{equation*}
U_{e}=\frac{1+\rho}{\rho+\lambda}\left\{\left[u^{\prime}(c)(1-\tau) w+\chi_{1} V(1-h)\right]-\left[\chi_{3}\right]\right\} \tag{20}
\end{equation*}
$$

which is the capitalized value of the difference in the marginal value between employment and non-labor force.
Note that the job finding rate $\mu$ reduces the household's surplus from a match in (15) but not here. The reason is that without choices of labor-force participation in (15), the outside option of employment is unemployment. The value of unemployment includes the prospect of employment which is increasing in job finding rates. However, with choices of labor-force participation here, the outside option of employment is non-employment and unemployment benefits are not a value of non-employment.

Next, by using the firm's gain from a match in (14) and the household's surplus from a match in (20), we maximize the joint surplus of a match and obtain the following bargained wage.

$$
\begin{equation*}
w=\beta[M P L \cdot h]+(1-\beta)\left[\frac{1}{1-\tau}\left(\chi_{3}-\chi_{1} \frac{(1-h)^{1-\sigma}}{1-\sigma}\right) c(e, 1-n, h)\right] \equiv w(e, 1-n, h ; \tau), \tag{21}
\end{equation*}
$$

where $w_{e}>0, w_{1-n}>0, w_{h}>0$ and $w_{\tau}>0$. The bargained wage is otherwise the same as (16) except for the opportunity cost of employment, $\operatorname{MRS}^{n e}(e, 1-n, h) \equiv\left(\chi_{3}-\chi_{1} \frac{(1-h)^{1-\sigma}}{1-\sigma}\right) c(e, 1-n, h)>0$. Because of choices of labor-force participation, the opportunity cost here is the difference in the MRS from non-employment to employment and thus, unemployment benefits do not directly affect the bargaining game here. In view of the $c$ function in (13), like (16), the bargained wage is increasing in employment $e$, labor force $1-n$ and hours worked per worker $h$. Clearly, a higher labor tax rates $\tau$ leads to an increase in the wage.

For the condition determining hours worked per worker, even though the household's surplus from a match is (20) instead of (15), except for $n$ being endogenous, the condition is still the same as (17b) and thus, $h=h(e, 1-n ; \tau)$.

As the bargained wage is (21), the vacancy creation condition is not (19) but is

$$
\begin{equation*}
\frac{\eta(e, 1-n)}{\rho+\lambda}(1-\beta)\left[M P L \cdot h(\underset{-}{e}, 1-n ; \tau)-\frac{1}{1-\tau} M_{-} S^{n e}\left(e, 1-n, h\left(\underset{-}{e}, 1_{-}-n ; \tau\right)\right)\right]-\phi=0 . \tag{22}
\end{equation*}
$$



Fig. 2. Existence of steady state

Like (19), the condition equates the marginal benefit of employment to the marginal cost. Yet, the marginal benefit is the capitalized value of the gain of a match from non-labor force to employment, rather than from unemployment to employment. The condition determines employment $e$ as a function of labor force $(1-n)$, also referred to as Locus $E$ in the ( $e, n$ ) plane. Like (19), it is downward sloping in the ( $e, n$ ) plane as seen in Fig. 2. The reasons are that more employment (a higher $e)$ decreases recruitment rates and increases employees' outside options $M R S^{n e}$, so the net marginal benefit of employment is increased. Moreover, a smaller labor force (a higher $n$ ) decreases both recruitment rates and outside options so the effect on the net marginal benefit is ambiguous, but it also increases work hours and decreases the net marginal benefit of employment which dominates the other ambiguous effects. ${ }^{13}$

Besides, there is a labor-force participation condition. By using $r=\rho+\delta$, the $\mu$ function in (12a), the $h$ function in (17b), the $U_{e}$ function in (20) and the $w$ function in (21), the participation condition (4) is rewritten as

$$
\begin{equation*}
\left\{\frac{\mu(e, 1-n)}{\rho+\lambda} \beta\left[M P L \cdot h(e, 1-n ; \tau)-\frac{1}{1-\tau} M R S^{n e}(e, 1-n, h(e, 1-n ; \tau))\right]+b\right\}-\left[\frac{1}{1-\tau} M^{M S} S^{n s}(e, 1-n, h(e, 1-n ; \tau))\right]=0 . \tag{23}
\end{equation*}
$$

In the condition above, the terms in the large brackets are the marginal benefit of labor-force participation which includes the capitalized value of the gain of a match from non-labor force to employment and unemployment benefits. The marginal cost of participation is the tax-adjusted loss in leisure utilities from non-labor force to search, $\operatorname{MRS}^{n s}(e, 1-n, h) \equiv\left(\chi_{3}-\chi_{2}\right) c(e, 1-n, h)>0$. The condition determines the labor force $1-n$ as a function of employment $e$. In the $(e, n)$ plane, the condition is referred to as Locus P.

To study the slope of Locus P, with given work hours, a smaller labor force (a higher $n$ ) directly increases the net marginal gain of participation as the resulting higher job finding rate $\mu$ and lower employees' outside option $M R S^{n e}$ increase the gain of a match from non-labor force to employment and the resultant smaller loss in leisure utilities from non-participation to participation $M R S^{n s}$ decreases the marginal cost of participation. By increasing work hours, a smaller labor force indirectly exerts a negative effect, but the effect is dominated by the positive direct effect, so the net marginal gain of participation is increased. Moreover, with given work hours, more employment (a higher $e$ ) has an ambiguous direct effect on the net marginal gain of participation as it increases both the marginal gain from a match (via a higher job finding rate) and the marginal cost of participation. By reducing work hours per worker, more employment indirectly brings in a positive effect that dominates the ambiguous direct effect, so the net marginal gain of participation is increased. Thus, Locus P is downward sloping in the $(e, n)$ plane.

Although Loci E and P are both downward sloping, the two loci intersect only once. We have shown that as $n$ goes to $0, h$ goes to the lowest value $h_{L}$ and Locus E and Locus P approach to the $e$ axis at $e_{E}$ and $e_{P}$ in Fig. 2, respectively. The value of $e$ cannot go to 0 , as then $h$ does not exist. ${ }^{14}$ There is thus a minimum value $e_{L}$ to which $e$ can approach. As $e$ decreases to $e_{L}, h$ goes to the highest value $h_{H}$ and Locus E and Locus P approach to $n_{E}$ and $n_{P}$, respectively. We also show that under a minor condition, $e_{E}<e_{P}$ and $n_{E}>n_{P}$ and thus, there exists an intersection of Loci E and P . We then show that if $A$ is larger, the value $h_{L}$ is smaller and the value $h_{H}$ is larger, so that it is easier to meet the condition that assures $e_{E}<e_{P}$ and $n_{E}>n_{P}$. Furthermore, it is required that Locus $P$ be flatter than Locus $E$ in each intersection. ${ }^{15}$ Therefore, there exists a unique steady state.

In Fig. 2, the unique steady state is $Q_{0}$, with the employment at $e_{0}$ and the labor force at $1-n_{0}$. With unemployment at $\left(1-e_{0}-n_{0}\right)$, the unemployment rate is $u_{0}=\left(1-e_{0}-n_{0}\right) /\left(1-n_{0}\right)=1-e_{0} /\left(1-n_{0}\right)$. Substituting $e_{0}$ and $n_{0}$ into (17b) gives working hours per worker $h_{0}$. Thus, labor supply in the economy, or equivalently hours worked per person, is $L^{s}{ }_{0}=e_{0} h_{0}$.

[^7]

Fig. 3. Steady-state effects of increases in wage taxes ( $\tau \uparrow$ )

## 4. Policy analysis

This section analyzes two policies of pervasive interest to compare the long-run effect on labor supply in our model and the model with an exogenous labor force. These two policies are a tax on the employed which is proportional to labor income and is used to make a lump-sum transfer; and a benefit to the unemployed which is proportional to labor income as financed by a lump-sum tax. We start with the analysis of increases in labor income taxes, followed by increases in unemployment compensation. Here, we offer graphical illustrations. ${ }^{16}$

### 4.1. Effects of labor taxes

### 4.1.1. Model with exogenous labor-force participation

Now, we analyze the effects of increases in the labor tax rate (higher $\tau$ ). When the labor-force participation is exogenously given, Locus E is the only relevant equilibrium condition. In Fig. 3, the intersection of the Locus E with the horizontal line $n=n_{0}$ determines the initial steady state $Q_{0}$. When the labor tax is increased, with given working hours, a higher labor tax decreases the net marginal benefit of employment, thereby shifting Locus E leftward. Yet, by way of reducing working hours per worker, a higher labor tax generates an indirect effect that increases the net marginal benefit of employment. Because the indirect effect is dominated by the direct effect, Locus E in Fig. 3 is shifted leftward to Locus $\mathrm{E}_{1}$. Thus, that employment is reduced from $e_{0}$ to $e_{1}$. Then, unemployment is increased from $\left(1-n_{0}-e_{0}\right)$ to $\left(1-n_{0}-e_{1}\right)$ and, with a given labor force, the unemployment rate is increased from $\mathbf{u}_{0}=\left(1-e_{0}-n_{0}\right) /\left(1-n_{0}\right)$ to $\mathbf{u}_{1}=\left(1-n_{0}-e_{1}\right) /\left(1-n_{0}\right)$.

### 4.1.2. Our model with endogenous labor-force participation

With endogenous labor-force participation, the initial steady state $Q_{0}$ is the intersection of Locus E with Locus P in Fig. 3. Like the model with an exogenous labor force above, a higher labor tax decreases the net marginal benefit of employment here. In order to increase the net marginal benefit of employment, given a labor force level, employment will decrease. Thus, the Locus E shifts leftward toward Locus $\mathrm{E}_{2}$. When labor-force participation is endogenous, the Locus E is also affected by labor force. With a sufficiently large value of $b$, the Locus E here is shifted leftward toward Locus $\mathrm{E}_{2}$ that is less than Locus $\mathrm{E}_{1}$.

Moreover, as the household chooses labor-force participation, a higher labor tax affects Locus P. With given working hours, a higher labor tax directly shifts Locus P upward, because the marginal benefit of participation is decreased which shrinks labor force. Yet, by reducing hours worked per worker, a higher labor tax generates an indirect effect that increases the marginal benefit of participation. As the direct effect dominates the indirect effect, Locus $P$ is shifted upward to Locus $\mathrm{P}_{2}$.

The new steady state is at $Q_{2}$ in Fig. 3. In this steady state, the labor force is decreased from ( $1-n_{0}$ ) to ( $1-n_{2}$ ), so employment is decreased from $e_{0}$ to $e_{2}$, a level less than that in the model with an exogenous labor force. As unemployment is ( $1-n_{2}-e_{2}$ ), the unemployment rate is $\mathrm{u}_{2}=\left(1-e_{2}-n_{2}\right) /\left(1-n_{2}\right)$ which may decrease or increase that depends on whether labor force $\left(1-n_{2}\right)$ are decreased more or less than employment $\left(e_{2}\right)$.

With labor force and employment, hours worked per worker are in turn determined. When the labor force is exogenous, the effect of a higher labor tax rate on hours worked per worker is ambiguous, because, with unemployment compensation, the indirect positive effect from much lower employment may offset the direct adverse effect. ${ }^{17}$ When the labor force is endogenous, the effects on hours worked per worker are still ambiguous because the indirect positive effects from smaller labor

[^8]

Fig. 4. Steady-state effects of increases in unemployment compensation ( $b \uparrow$ )
force and lower employment both may offset the direct adverse effects. Nevertheless, as the adverse effect on employment is strong, no matter whether the labor force is endogenous or not, the labor supply ( $L^{s}=e h$ ) is likely to decrease. Yet, in the model with an exogenous labor force, as employment is reduced by more, labor supply is reduced by more.

To summarize the effects of a higher labor tax rate, we obtain
Proposition 1. When the labor tax is increased, because of a decrease in the labor force, the employment in the model with an endogenous labor force is reduced less than the model with an exogenous labor force and, with ambiguous effects on hours per worker in both models, labor supply in the model with an endogenous labor force is reduced less than that in the model with an exogenous labor force.

### 4.2. Effects of unemployment compensation

### 4.2.1. Model with exogenous labor-force participation

Next, we envisage the effects of increases in unemployment compensation (a higher b). Suppose that the initial steady state is $Q_{0}$ in Fig. 4. With an exogenous labor force, the outside option of employment is unemployment. With given working hours, a higher unemployment benefit directly raises the opportunity cost from unemployment to employment which decreases the net marginal benefit of employment. Thus, Locus $E$ is shifted leftward which reduces employment. Besides, by increasing working hours per worker, the net marginal benefit of employment is decreased further, thereby generating an indirect effect to shift Locus E leftward even more (cf. Locus $\mathrm{E}_{1}$ ). With a given labor force ( $1-n_{0}$ ), the new steady state is $Q_{1}$ and employment is decreased from $e_{0}$ to $e_{1}$. Unemployment is increased from $\left(1-n_{0}-e_{0}\right)$ to ( $\left.1-n_{0}-e_{1}\right)$ which causes the unemployment rate to increase from $\mathbf{u}_{0}=\left(1-e_{0}-n_{0}\right) /\left(1-n_{0}\right)$ to $\mathbf{u}_{1}=\left(1-e_{1}-n_{0}\right) /\left(1-n_{0}\right)$.

### 4.2.2. Our model with endogenous labor force-participation

When the labor-force participation is endogenous, a higher unemployment benefit does not affect the gain of a match and thus does not shift Locus E. Yet, a higher unemployment benefit increases the gain from non-participation to participation. With given working hours, a higher unemployment benefit raises the marginal benefit of participation which increases the labor force and thus shifts Locus P downward. In addition, by decreasing working hours per worker, a larger labor force generates two further effects. First, the marginal benefit of participation is increased which in turn induces more labor force and thus shifts Locus P downward more (cf. Locus $\mathrm{P}_{2}$ ). Second, a lower working hour per worker makes Locus E flatter which is rotated counter-clockwise (cf. Locus $\mathrm{E}_{2}$ ).

As a result, the labor force is increased from $\left(1-n_{0}\right)$ to $\left(1-n_{2}\right)$ and the employment is increased from $e_{0}$ to $e_{1}$. Moreover, it is clear that unemployment $\left(1-e_{2}-n_{2}\right)$ and the unemployment rate $\mathbf{u}_{1}=\left(1-e_{2}-n_{2}\right) /\left(1-n_{2}\right)$ are both ambiguous; both may decrease or increase depending on whether labor force ( $1-n_{2}$ ) are increased more or less than employment ( $e_{2}$ ).

Unemployment compensation has no direct effect on hours worked per worker. When the labor force is exogenous, higher unemployment compensation decreases employment which indirectly increase hours worked per worker. Thus, the effect on labor supply is ambiguous. When the labor force is endogenous, higher unemployment compensation increases both the labor force and employment which indirectly reduce hours worked per worker. As a result, the effect on labor supply is also unambiguous, depending on whether the effect on employment or the effect on hours worked per worker dominates.

To summarize the effects of higher unemployment benefits, we obtain
Proposition 2. A higher unemployment benefit decreases employment in the model with an exogenous labor force but increases both labor force and employment in the model with an endogenous labor force. As the effects on hours worked per worker are
opposite to those of the effects on employment and labor force, the effects on labor supply are ambiguous under both models with and without an endogenous labor force.

### 4.3. Quantitative analysis

We have analyzed the effects of changes in labor taxes and unemployment compensation on labor supply in our model and the model without an endogenous labor force. Yet, as the theoretical effects on hours worked per worker are ambiguous or opposite to those of the effects on employment and labor force, the net effects on labor supply are ambiguous. In this subsection, we carry out quantitative exercises so as to pin down the net effects. In the quantitative exercise, we assume that the economic structure in the EU and the US is the same except for labor tax rates and unemployment benefits.

### 4.3.1. Calibration

We start by calibrating parameters and variables in a quarterly frequency. First, the productivity coefficient is normalized to unity $(A=1)$ and the capital share is set at $\alpha=0.36$. With the annual time preference rate of $4 \%$ in the US data, we set the quarterly time preference rate to $\rho=0.01$. Then, we calibrate the capital depreciation rate to target a quarterly capital-output ratio of $k / y=12$. We obtain $\delta=0.02$ which is in the range of a $3-8 \%$ annual depreciation rate of capital. These values give the quarterly interest rate equal to $r=0.03$ and the effective capital-labor ratio equal to $q=48.5535$.

Next, we use the employment rate and the labor force participation rate in the US to compute the fraction of employment in the working-age population and the average unemployment rate. We obtain $e=72.03 \%$ and $\mathrm{u}=5.1 \%$, respectively, the former value close to the value $71.9 \%$ calculated by Alesina et al. (2006) and the latter the same as the value obtained by Krusell et al. (2011). These values give $n=0.2410$ and thus a labor force participation rate of $1-n=0.7590$, a value close to the data in the OECD. Then, we calibrate $h=0.3471$ in order to target a $25 \%$ productive time allocated to the market, $L=e h$ (Prescott, 2006). These parameter values yield an output level of $y=1.0115$.

According to Shimer (2005), the monthly job finding rate is 0.45 . We go along with this rate and translate it into a quarterly value of $\mu=1-(1-0.45)^{3}=0.8336$. Then, we employ the matching relationships to compute the quarterly separation rate $\lambda=\mu(1-n-e) / e=0.0448$. Moreover, we calibrate the steady-state vacancies $v=0.0387$ in order to target a unit degree of the labor market tightness in a steady state as proposed by Shimer (2005). This value in turn gives a quarterly recruitment rate at $\eta=\mu(1-n-e) / v=0.8336$.

By using (13), we calibrate a unit vacancy creation cost of $\phi=1.5679$ in order to target a $70 \%$ consumption-output ratio (c) $y=0.7$ ). Then, we use (14), together with (9b), to compute and obtain the wage per worker of $w=0.7957$. Shimer (2005) estimated a $40 \%$ unemployment replacement rate. We go along with Shimer (2005) and calibrate $b$ to target the $40 \%$ unemployment replacement rate. We get $b=0.3183$ which is set as the baseline unemployment compensation. Utilizing the data compiled by McDaniel (2007), Rogerson (2008) used the labor tax in Belgium, France, Germany, Italy, and the Netherlands to represent the tax in the EU. We follow this invention and compute the population-weighted average effective tax rate on labor income for these five European countries in 1970-1973. The average effective tax rate is $\tau=0.3982$ which is set as the baseline labor tax rate.

In our utility function, the parameter $\sigma$ governs the intertemporal elasticity of leisure for the employed and is negatively related to the Frisch labor supply elasticity (LSE): $(1-h) /(\sigma h)$. The LSE for men estimated by MaCurdy (1981) ranged from 0.1 to 0.5 and that for women was higher. Conversely, Greenwood et al. (1988) suggested that $L S E=1.7$ was reasonable. For the present purpose, we go along with Andolfatto (1996) and set $L S E=1$ which is within the estimates above. This gives $\sigma=1.8812 .{ }^{18}$ We then calibrate $\chi_{1}=0.987$ in order to be consistent with the hour bargaining condition.

Finally, following Blanchard and Diamond (1989), we set the search worker's contribution in matching at $\gamma=0.4$. By assuming that Hosios' rule holds (Hosios, 1990), we pin down labor's bargaining share at $\beta=\gamma$. Then, from the matching relationships we can compute $m=0.8336$. With the values above, we compute other parameters in the utility of leisure. When the labor force is endogenous, we compute and obtain $\chi_{3}=-1.0129$ and $\chi_{2}=-2.3507$ so as to be consistent with the participation condition in (23) and the bargained wage condition in (21). By contrast, when the labor force is exogenous, we compute and obtain $\chi_{2}=-1.4624$ so as to be consistent with the bargained wage condition in (16).

The benchmark parameter values, observables and calibrated values are listed in Table 2. Under the benchmark parameter values, we obtain a unique steady state.

### 4.3.2. Quantifying the effects of increases in tax rates and unemployment compensation

To quantify the effects of increases in tax rates and unemployment compensation, we start by measuring changes in labor taxes and unemployment compensation in the EU relative to the US from the early 1970s to the early 2000s.

For labor taxes, based on the data in McDaniel (2007), we follow Rogerson (2008) and calculate the population-weighted average effective tax rate on labor income in Belgium, France, Germany, Italy, and the Netherlands in 2000-2003. We obtain

[^9]Table 2
Benchmark parameter values and calibration quarters.

| Benchmark parameters and observables | Variables | Quarterly | Source |
| :--- | :--- | :--- | :--- |
| Coefficient of production technology | $A$ | 1.0000 | Normalization |
| Capital's share | $\alpha$ | 0.3600 | Kydland and Prescott (1982, 1991) |
| Time preference rate | $\rho$ | 0.0100 | Kydland and Prescott (1991) |
| Fraction of employment | $e$ | 0.7203 | OECD (2010b) |
| Unemployment rate | $\mathbf{u}$ | 0.0510 | OECD (2010b) |
| Fraction of non-participants | $n$ | 0.2410 | OECD (2010b) |
| Job finding rate | $\mu$ | 0.8336 | Shimer (2005) |
| Labor tax rate | $\tau$ | 0.3982 | McDaniel (2007) |
| Labor's share in matching function | $\gamma$ | 0.4000 | Blanchard and Diamond (1989) |
| Calibration |  |  | Target |
| Depreciation rate of capital | $\delta$ | 0.0200 | Capital-output ratio $=12$ |
| Hours worked per worker | $h$ | 0.3471 | Hours of work per person $=25 \%$ |
| Job separation rate | $\lambda$ | 0.0448 | Matching relationship |
| Vacancy creation | $v$ | 0.0387 | Vacancy-search worker ratio $=1$ |
| Unit cost of vacancy creation | $\phi$ | 1.5679 | Consumption-output ratio $=0.7$ |
| Unemployment compensation | $b$ | 0.3183 | Unemploy. replacement rate $=40 \%$ |
| The intertemporal elasticity of leisure | $\sigma$ | 1.8812 | Frisch labor supply elasticity $=1$ |
| Coefficient of worker's leisure | $\chi_{1}$ | 0.9870 | Bargaining hour condition |
| Leisure utility of unemployed (endo. $n)$ | $\chi_{2}$ | -2.3507 | Bargained wage condition |
| Leisure utility of non-participants | $\chi_{3}$ | -1.0129 | Participation condition |
| Leisure utility of unemployed (exog. $n$ ) | $\chi_{2}$ | -1.4624 | Bargained wage condition |
| Labor's bargaining power | $\beta$ | 0.4000 | Hosios' rule |
| Coefficient of matching function | $m$ | 0.8336 | Matching technology |

the tax rate 0.5168 . Together with the data that the effective labor tax rate increased a little bit in the US in the past 30 years, this indicates about a $30 \%$ increase in the labor tax rate in the EU relative to the US from the early 1970s to the early 2000s. ${ }^{19}$

For unemployment compensation, based upon the dataset compiled by van Vliet and Caminada (2012), we calculate the net unemployment replacement rate for one earner couple in Belgium, France, Germany, Italy, and the Netherlands in the EU and the US in 1971 and 2001. We find that the ratio of the net unemployment replacement rate between the EU and the US was increased from 0.85 in 1971 to 1.24 in 2001 which indicates about a $40 \%$ increase from 1971 to $2001 .^{20}$

Given these data, we quantify the effects of an increase in the value of $\tau$ by $30 \%$ and an increase in the value of $b$ by $40 \%$ from their baselines. In each exercise, the government balances the budget in each period by adjusting lump-sum taxes or transfers. First, the effects of an increase in the labor tax by $30 \%$ are reported in the top panel of Table 3. In the model with an exogenous labor force, the labor force is fixed at the baseline level of $1-n=0.7590$. In this model, employment is reduced largely by 18.9 percentage points; thus, unemployment is increased largely. Hours worked per worker change little, though it is increased due to the dominance effect of lower employment. Because of a large decrease in employment, labor supply is decreased by 6.38 percentage points which, if we normalize the baseline value to $100 \%$, amounts to a reduction by $25.51 \%$ as seen in the parenthesis.

By contrast, in our model with an endogenous labor force, as an increase in the labor tax rate by 30\% also reduces the labor force by 13.71 percentage points, employment is reduced less that is by 13.22 percentage points. Because the labor force and employment are reduced by about the same size, unemployment changes little. As the effect from small labor force offsets the effects from lower employment, hours worked per worker also changes little. Because of a decrease in the labor force, labor supply in our model is decreased less than the model with an exogenous labor force.

Next, the effects of an increase in unemployment compensation by $40 \%$ are demonstrated in the middle panel of Table 3. When the labor force is exogenously fixed at $1-n=0.7590$, employment is decreased largely by 29.67 percentage points; thus, unemployment is increased largely. Because of a large decrease in employment, as a substitute, hours worked per worker are increased substantially by 8 percentage points. As the employment effect dominates, labor supply is decreased by 6.86 percentage points which means a reduction by $27.45 \%$ from the baseline.

By contrast, in our model with an endogenous labor force, as an increase in unemployment compensation by 40\% enhances the labor force, employment is increased slightly. Because the effect from larger labor force offsets the effects from higher employment, unemployment changes little and so do hours worked per worker and labor supply.

Moreover, we quantify the total effect by simultaneously increasing the tax rate by $30 \%$ and unemployment compensation by $40 \%$. See the results in bottom panel of Table 3. In the model with an exogenous labor force, employment is reduced very largely by 54.18 percentage points and as a result, the unemployment rate is increased by 71.38 percentage points.

[^10]Table 3
Effects of increases in labor tax rate and unemployment compensation (\%).

| Benchmark | $e$ |  | $1-n$ |  | $\mathbf{u}=(1-n-e) /(1-n)$ |  | $h$ |  | $L^{s}=e h$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 72.03 | 0.00 * | 75.90 | 0.00 | 5.10 | 0.00 * | 34.71 | 0.00 ${ }^{\text {* }}$ | 25.00 | 0.00 |
| $\tau \uparrow 30 \%$ |  |  |  |  |  |  |  |  |  |  |
| exog LF | 53.09 | $\begin{aligned} & -18.94 \\ & (-26.29) \end{aligned}$ | 75.90 | $\begin{aligned} & 0.00 \\ & (0.00) \end{aligned}$ | 30.06 | +24.96 | 35.08 | $\begin{aligned} & +0.37 \\ & (+1.07) \end{aligned}$ | 18.62 | $\begin{aligned} & -6.38 \\ & (-25.51) \end{aligned}$ |
| endo LF | 58.81 | $\begin{aligned} & -13.22 \\ & (-18.35) \end{aligned}$ | 62.20 | $\begin{aligned} & -13.71 \\ & (-18.06) \end{aligned}$ | 5.45 | +0.35 | 34.34 | $\begin{aligned} & -0.37 \\ & (-1.07) \end{aligned}$ | 20.19 | $\begin{aligned} & -4.81 \\ & (-19.23) \end{aligned}$ |
| $b \uparrow 40 \%$ |  |  |  |  |  |  |  |  |  |  |
| exog LF | 42.36 | $\begin{aligned} & -29.67 \\ & (-41.19) \end{aligned}$ | 75.90 | $\begin{aligned} & 0.00 . \\ & (0.00) \end{aligned}$ | 44.19 | +39.09 | 42.81 | $\begin{aligned} & +8.11 \\ & (+23.37) \end{aligned}$ | 18.14 | $\begin{aligned} & -6.86 \\ & (-27.45) \end{aligned}$ |
| endo LF | 74.26 | $\begin{aligned} & +2.23 \\ & (+3.10) \end{aligned}$ | 78.74 | $\begin{aligned} & +2.84 \\ & (+3.74) \end{aligned}$ | 5.69 | +0.59 | 34.09 | $\begin{aligned} & -0.62 \\ & (-1.79) \end{aligned}$ | 25.32 | $\begin{aligned} & +0.32 \\ & (+1.28) \end{aligned}$ |
| $\tau \uparrow$ and $b \uparrow$ |  |  |  |  |  |  |  |  |  |  |
| exog LF | 17.85 | $\begin{aligned} & -54.18 \\ & (-75.22) \end{aligned}$ | 75.90 | $\begin{aligned} & 0.00 \\ & (0.00) \end{aligned}$ | 76.48 | +71.38 | 53.90 | $\begin{aligned} & +19.19 \\ & (+55.29) \end{aligned}$ | 9.62 | $\begin{aligned} & -15.38 \\ & (-61.51) \end{aligned}$ |
| endo LF | 61.31 | $\begin{aligned} & -10.72 \\ & (-14.88) \end{aligned}$ | 65.49 | $\begin{aligned} & -10.42 \\ & (-13.73) \end{aligned}$ | 6.37 | +1.27 | 33.50 | $\begin{aligned} & -1.21 \\ & (-3.49) \end{aligned}$ | 20.54 | $\begin{aligned} & -4.46 \\ & (-17.84) \end{aligned}$ |

Note: All changes under columns with " are in percentage points from the baseline except for those in the parenthesis which are percent changes from their baseline values that are normalized to $100 \%$.

Hours worked per worker are also increased largely. As the employment effect dominates, labor supply is reduced by 15.38 percentage points which amounts to a reduction by $61.51 \%$ from the baseline.

By contrast, in our model with an endogenous labor force, because the favorable effect of increases in unemployment compensation lessens the adverse effect of a higher tax on the labor force, the employment is decreased by less and the labor force and employment are both decreased by about 10 percentage points. The unemployment rate is slightly increased and hours worked per worker are slightly decreased. As a result, labor supply is reduced by 4.46 percentage points which amounts to a decrease of $17.84 \%$ from the baseline.

We find that these results above are robust for different values of LSE. Moreover, these results hold even when the Hosios' rule does not hold. Specifically, we have fixed the labor's contribution in search at $\gamma=0.4$ and varied the labor's bargaining share $\beta$ to take alternative values $\{0.235,0.54,0.72\}$ used by Hall (2005), Hall and Milgrom (2008) and Shimer (2005), respectively. To save the space, we do not report these robustness analyses.

Overall, we find that the model with an exogenous labor force explains too much of the decrease in employment and the decrease in labor supply in the data in the EU relative to the US. In particular, in response to these two important sources of labor market regulation, the model with an exogenous labor force predicts an increase in hours worked per worker as opposed to a decrease in the data. By contrast, our model takes account of endogenous changes in the labor force, so it explains a reasonable $17 \%$ decrease in labor supply in the UE relative to the US, which is close to a $26 \%$ decrease in the data. Our model explains 10 percentage-point decreases in both employment and labor force which is also close to the data, along with a decrease rather than an increase in hours worked per worker. Because of other differences in labor market characteristics and regulations between the EU and the US, our model cannot explain all the difference in labor supply in the EU relative to the US. Yet, our model explains the difference in labor supply better than the model with an exogenous labor force.

### 4.3.3. Effects when the form of the matching function is different

The previous analysis uses the matching technology a la Diamond (1982). The trouble with this type of technology is that it does not guarantee that the number of matched pairs is smaller than the number of job seekers and the number of unfilled vacancies so that matching probabilities could exceed the unity. This problem may lead to unstable numerical results when parameter values are changed. In this subsection, we adopt an alternative matching technology proposed by Den Haan et al. (2000). We envisage whether our findings remain hold true under the alternative matching technology.

Following Den Haan et al. (2000), the matching technology takes the following form: $M_{t}=\frac{\left(1-n_{t}-e_{t}\right)\left(v_{t}\right)}{\left[\left(1-n_{t}-e_{t}\right)^{l}\left(v_{t}\right)^{1 / l}\right.}$, where $l$ is a parameter. ${ }^{21}$ This functional form imposes that job seekers and firms have a symmetric contribution to a match. With the functional form, the equilibrium vacancy and the firm's recruitment rate in the steady state are $v=\frac{\lambda e(1-n-e)}{\left[(1-n-e)^{1}-(i e)\right]^{1 / / /}} \equiv v(e, 1-n)$ and $\eta=\frac{\lambda e}{\nu(e, 1-n)}=\left[1-\left(\frac{\lambda e}{1-n-e}\right)^{l}\right]^{1 / l} \equiv \eta(e, 1-n)$. The model is the same except for the matching technology. Now, the effects of changes in employment and those of changes in the labor force on the equilibrium vacancy and the firm's recruitment have

[^11]Table 4
Policy effects under alternative matching technology (\%).

| Benchmark | $e$ |  | $1-n$ |  | $\mathbf{u}=(1-n-e) /(1-n)$ |  | $h$ |  | $L^{s}=e h$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 72.03 | 0.00 * | 75.90 | 0.00 | 5.10 | 0.00 * | 34.71 | 0.00 * | 25.00 | 0.00 * |
| $\tau \uparrow 30 \%$ |  |  |  |  |  |  |  |  |  |  |
| exog LF | 30.19 | $\begin{aligned} & -41.84 \\ & (-58.09) \end{aligned}$ | 75.90 | $\begin{aligned} & 0.00 \\ & (0.00) \end{aligned}$ | 60.23 | +55.13 | 45.62 | $\begin{aligned} & +10.91 \\ & (+31.43) \end{aligned}$ | 13.77 | $\begin{aligned} & -11.23 \\ & (-44.92) \end{aligned}$ |
| endo LF | 58.91 | $\begin{aligned} & -13.12 \\ & (-18.21) \end{aligned}$ | 62.29 | $\begin{aligned} & -13.62 \\ & (-18.14) \end{aligned}$ | 5.41 | +0.31 | 34.30 | $\begin{aligned} & -0.41 \\ & (-1.18) \end{aligned}$ | 20.21 | $\begin{aligned} & -4.79 \\ & (-19.17) \end{aligned}$ |
| $b \uparrow 40 \%$ |  |  |  |  |  |  |  |  |  |  |
| exog LF | 27.52 | $\begin{aligned} & -44.51 \\ & (-41.19) \end{aligned}$ | 75.90 | $\begin{aligned} & 0.00 . \\ & (0.00) \end{aligned}$ | 63.74 | +58.64 | 51.01 | $\begin{aligned} & +16.30 \\ & (+46.96) \end{aligned}$ | 14.04 | $\begin{aligned} & -10.96 \\ & (-43.84) \end{aligned}$ |
| endo LF | 74.31 | $\begin{aligned} & +2.28 \\ & (+3.17) \end{aligned}$ | 78.78 | $\begin{aligned} & +2.88 \\ & (+3.79) \end{aligned}$ | 5.68 | +0.58 | 34.08 | $\begin{aligned} & -0.63 \\ & (-1.82) \end{aligned}$ | 25.33 | $\begin{aligned} & +0.33 \\ & (+1.32) \end{aligned}$ |
| $\tau \uparrow$ and $b \uparrow$ |  |  |  |  |  |  |  |  |  |  |
| exog LF | 12.57 | $\begin{aligned} & -59.46 \\ & (-82.55) \end{aligned}$ | 75.90 | $\begin{aligned} & 0.00 \\ & (0.00) \end{aligned}$ | 83.44 | +78.34 | 60.19 | $\begin{aligned} & +25.48 \\ & (+73.41) \end{aligned}$ | 7.57 | $\begin{aligned} & -17.43 \\ & (-69.72) \end{aligned}$ |
| endo LF | 60.74 | $\begin{aligned} & -11.29 \\ & (-15.67) \end{aligned}$ | 65.03 | $\begin{aligned} & -10.87 \\ & (-14.32) \end{aligned}$ | 6.60 | +1.50 | 33.69 | $\begin{aligned} & -1.02 \\ & (-2.94) \end{aligned}$ | 20.46 | $\begin{aligned} & -4.54 \\ & (-18.16) \end{aligned}$ |

Note: All changes under columns with * are in percentage points from the baseline except for those in the parenthesis which are percent changes from their baseline values that are normalized to $100 \%$.
the same signs as those in (12b) and (12c). It follows that the theoretical results of increases in income tax rates and unemployment compensation on labor supply in Propositions 1 and 2 remain unchanged.

To study the quantitative effects of policy changes, we recalibrate our model. The new matching function does not change our calibration procedure. Here, the values of $m$ and $\gamma$ are no longer needed. Moreover, we find that all other parameter values listed in Table 2 remain unchanged. With these parameter values, we use the new matching relationship to compute $l=3.8085$.

The quantitative results are presented in Table 4, with the effects of an increase in income tax rates being at the top panel, the effects of an increase in unemployment compensation at the middle panel and the joint effects of increases in income tax rates and unemployment compensation at the bottom panel. It is clear to see that the effects are similar to those in Table 3, except for being due to a large number of $l$ in the matching technology wherein the effects in the model with an exogenous labor force are a little bit different from those in Table 3. As a result, our finding is robust for different matching technology.

### 4.3.4. A decomposition analysis

The previous analysis indicates that our model with an endogenous labor force explains the difference in labor supply better than the model with an exogenous labor force. In this subsection, we carry out a decomposition analysis in order to understand that, in response to the two policy changes under study, how much each different margin of labor supply explains the decline in labor supply. To shed light the difference, we also illustrate the model with an exogenous labor force.

In the model with an exogenous labor force, labor supply $\left(L^{s}\right)$ is equal to hours worked per worker ( $h$ ) multiplied by employment (e): $L^{s}=h e$. The effects of changes in policies work via their effects on the intensive margin (hours per worker) and the extensive margin (employment). To decompose the effects on labor supply into the two margins of labor supply, we denote $x$ as a relevant policy. Then, the effects of changes in $x$ on labor supply are

$$
\begin{equation*}
\frac{d L^{s}}{d x}=e_{0} \frac{d h}{d x}+h_{0} \frac{d e}{d x} \tag{24}
\end{equation*}
$$

where the effects are evaluated at a steady-state level denoted by a subscript ${ }_{0}$. The first term on the right-hand side is the effect of changes in policy via responses in hours per worker and the second term is the effect via responses in employment rates.

By contrast, in our model with an endogenous labor force, the extensive margin is equal to the employment margin (the employment rate in the labor force, $1-\mathbf{u})$ multiplied by the participation margin (the labor force, $1-n): e=(1-\mathbf{u})(1-n)$. Then, the effects of changes in policies on labor supply are

$$
\begin{equation*}
\frac{d L^{s}}{d x}=\left(1-\mathbf{u}_{0}\right)\left(1-n_{0}\right) \frac{d h}{d x}+h_{0}\left(1-n_{0}\right) \frac{d(1-\mathbf{u})}{d x}+h_{0}\left(1-\mathbf{u}_{0}\right) \frac{d(1-n)}{d x}, \tag{25}
\end{equation*}
$$

where, as in (24), the effects are evaluated at a steady-state level denoted by a subscript ${ }_{0}$. Thus, the effects of change in policies on labor supply are decomposed into the effects on the three margins. The first term on the right-hand side is the effect via changes in hours per worker, the second term is the effect via changes in the employment rate in the labor force, and the third term is the effect via changes in the labor force.

Table 5
Decomposition of changes in labor supply (\%).

|  | Changes in hours per worker | Changes in employment | Changes in labor force labor force $(1-n)$ |
| :--- | :---: | :---: | :--- |
| $\tau \uparrow 30 \%$ |  |  |  |
| exog LF | 3.65 | -103.65 | 0 |
| endo LF | -5.06 | -1.72 | -93.23 |
| $b \uparrow 40 \%$ |  |  |  |
| exog LF | 84.01 | -184.01 | 0 |
| endo LF | 141.47 | 49.50 | -290.97 |
| $\tau \uparrow$ and $b \uparrow$ | 145.38 | -245.38 | 0 |
| exog LF | -18.04 | -6.86 | -75.10 |
| endo LF |  |  |  |

Note: The calculation is based on Eqs. (24) and (25) evaluated at the average of the initial and the new steady-state values.

To offer a quantitative analysis of the decomposition, it should be noted that the effects may be different when the evaluation it taken at the initial steady-state values or the new steady-state values. To avoid the results being affected by the steady-state values being evaluated, we compute the results when the effects are evaluated at the average of both the initial and the new steady-state values. We use the effects of policy changes demonstrated in Table 3 to compute the decomposition analysis. ${ }^{22}$ The results are reported in Table 5.

In the table, the top panel is the decomposition of the effects on different margins of labor supply when the income tax rate is increased by $30 \%$. In the case when the participation margin is exogenous, because hours per worker are increased, a more than $100 \%$ of the decline in labor supply is explained by the decrease in employment. By contrast, when the participation margin is endogenous, because labor taxes reduce hours per worker, the decrease in employment explains only $1.72 \%$ of the decline in labor supply, with $92.23 \%$ being explained by the decrease in the labor force.

The middle panel is the decomposition when unemployment compensation is increased by $40 \%$. When the participation margin is exogenous, because there is a large increase in hours per worker, the reduction in employment explains more than $180 \%$ of the decline in labor supply. When the participation margin is endogenous, there is an increase in employment which is about $49 \%$ of the change in labor supply. Moreover, there is an even larger increase in hours per worker which is about $141 \%$ of the change in labor supply. As a result, the reduction in employment explains more than $290 \%$ of the decline in labor supply.

Finally, the bottom panel is the decomposition when both the income tax rate and unemployment compensation are increased. When the participation margin is exogenous, because both policies increase hours worked per worker, there is a larger increase in hours per worker and thus, the decrease in employment explains $245 \%$ of the decline in labor supply which is larger than those in the top and the middle panels. By contrast, when the participation margin is endogenous, both policy changes together decrease all of the three margins of labor supply. The results indicate that of the declining labor supply in the EU relative to the US, $18.04 \%$ comes from the decrease in hours per worker, $6.86 \%$ from the decrease in employment, and $75.10 \%$ from the decrease in the labor force.

## 5. Concluding remarks

The labor supply in the EU declined on average about one fourth relative to that in the US from the early 1970s to the early 2000s. The existing papers have used increases in labor taxes and unemployment benefits to explain declining labor supply in the EU relative to the US. These existing models include the intensive margin, the employment margin, or both margins of labor supply, but they did not take into account the participation margin. Our article extends the existing model to the one with the participation margin. We compare the long-run effects on labor supply of increases in labor taxes and unemployment benefits in the models with and without the participation margin.

We find that with increases in labor taxes, thanks to discouraging the labor force, the employment in our model is reduced less than that in the model without an endogenous labor force and, with ambiguous effects on hours worked per worker, labor supply is decreased by less in our model. In the case of increases in unemployment benefits, due to inducing the labor force, employment increases in our model but decreases in the model with an exogenous labor force and, with the effect on hours worked per worker being opposite to that on employment, the effect on labor supply is ambiguous in both models, depending on whether the effect on employment or that on hours worked per worker dominates.

To quantify the net effect on labor supply, we calibrate our model to the US economy. By feeding in the data of increases in the labor tax and unemployment compensation, we find that the model without an endogenous labor force explains too much of the decreases in employment and labor supply in the EU relative to the US from the early 1970s to the early 2000s.

[^12]In particular, the model without an endogenous labor force predicts an increase in hours worked per worker which is at odd with the data. By contrast, due to the endogenous labor force, our model explains a reasonable decrease in labor supply, along with a reasonable decrease in employment and a moderate decrease rather than an increase in hours worked per worker. Overall, because of considering the endogenous labor force, our model explains the difference in labor supply better than the model with an exogenous labor force.

Finally, we should mention that differences in labor-force participation may come from older and younger workers and female labor-force participation. Moreover, differences in labor supply may also reflect differences in workweeks, full and part-time jobs, holidays and vacation days. Our model and the models studied by Ljungqvist and Sargent (2007a, 2007b), Fang and Rogerson (2009) and Shimer (2011) consider neither life-cycle elements nor female and male labor-force participation, because these models are aimed at understanding differences in the labor supply or employment for a representative agent with full-time employment instead of the choice of part-time versus full-time and female versus male employment. Although there are some variations in the EU relative to the US, the key pattern these existing papers wish to emphasize is that the very large differences in average labor supply per person in the past decades are due to large differences in hours work per worker and employment. Our model adds value to these existing studies in that by taking account of an endogenous labor force, it explains the difference in labor supply in the EU relative to the US better than the model with an exogenous labor force.

## Appendix A. Supplementary material

Supplementary data associated with this article can be found, in the online version, at http://dx.doi.org/10.1016/j.jmacro. 2014.12.005.

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[^1]:    ${ }^{1}$ Fang and Rogerson (2009) is the Andolfatto (1996) model that abstracted from capital but allowed for an employee to choose between working time and leisure time. Their paper analyzed the implications of increases in the labor tax and increases in the cost of job creation on labor supply of the intensive and extensive margin in a steady state.
    ${ }^{2}$ There are existing papers that studied different topics with an endogenous labor force. Early theoretic analyses of labor force participation include Burdett et al. (1984) and Andolfatto and Gomme (1996). Pissarides (2000, Ch. 7) developed a general equilibrium matching model with labor force participation wherein there were no flows in and out of the labor market. Garibaldi and Wasmer (2005), Pries and Rogerson (2009) and Krusell et al. (2011) extended this model to generate flows into and out of the labor market. These models did not analyze changes in an average labor supply. Moreover, in these papers the participation margin is a state with exogenous random arrival rates such that the participation decision is a discrete, binary choice.
    ${ }^{3}$ Other policies and institutions that were argued to cause declining labor supply in the EU include working-time regulation and employment protection (Causa, 2008), home production (Ngai and Pissarides, 2008; Olovsson, 2009) and preferences (Blanchard, 2004; Azariadis et al., 2013).
    ${ }^{4}$ Tripier (2004) and Shimer (2011) are large household models a la Merz (1995) with standard preferences and technologies.

[^2]:    ${ }^{5}$ Our model does not allow for a direct transit from out of labor force to employment, because the direct flows from out of labor force to employment in the data are due to misclassification problems in a time aggregation bias, as argued by Garibaldi and Wasmer (2005) and others.
    ${ }^{6}$ The wage per worker $w$ equals the wage per hour $\omega$ multiplied by working hours per worker $h: w_{t}=\omega_{t} h_{t}$. The pair of a successful match bargains over the wage and working hours. No matter whether the wage is paid in terms of per worker or per hour, our results are the same.

[^3]:    ${ }^{7}$ See Pissarides (2000, Ch. 7, p. 167) who also assumed that the leisure utility of non-participants is demonstrably greater than that of unemployed workers. Note that implicit in the assumption $\chi_{3}>\chi_{2}$ is the notion that because of a job search, an unemployed worker has a lower utility of leisure than one who does not search for a job.

[^4]:    ${ }^{8}$ This is the discount factor of firms because households are the ultimate owners of firms. Using (5a), the discount factor is $\frac{1}{1+\xi_{t}}=\frac{1}{1-\delta+t+1}$.

[^5]:    ${ }^{9}$ The conditions are $\frac{\beta}{U_{e}\left(k_{t}, e_{t}\right)} \frac{d U_{e}\left(k_{t}, e_{t}\right)}{d x_{t}}+\frac{1-\beta}{\Pi_{e}\left(e_{t}\right)} \frac{d \Pi_{e}\left(e_{t}\right)}{d x_{t}}=0, x_{t}=w_{t}, h_{t}$.

[^6]:    ${ }^{10} c_{e}>0$ if $\phi$ is not too large.
    ${ }^{11}$ The derivations concerning the signs of the expressions discussed here and below are relegated to the online Appendix A.
    ${ }^{12}$ To ensure a loss of leisure utilities from unemployment to employment, we assume $\chi_{2}>\chi_{1}(1-h)^{1-\sigma} \frac{1}{1-\sigma}$ near the steady state, so the leisure utility of unemployed workers is larger than that of employed workers. See also Cheron and Langot (2004) for the same assumption.

[^7]:    ${ }^{13}$ See the online Appendix A.
    ${ }^{14}$ See the online Appendix A.
    ${ }^{15}$ Suppose that the unit cost of vacancies increases. A higher unit cost of vacancies reduces the net marginal benefit of vacancy and shifts Locus E in Fig. 2 leftward without shifting Locus P. It is reasonable to expect that job vacancies decrease and thus employment decreases. However, should Locus P of Fig. 2 be steeper than Locus E, employment would have had increased.

[^8]:    ${ }^{16}$ The comparative-static analysis is in the online Appendix A.
    ${ }^{17}$ When unemployment compensation is zero, the effect of a higher labor tax rate on hours worked per worker is negative. Thus it is the situation studied in Fang and Rogerson (2009) and Chen et al. (2015).

[^9]:    ${ }^{18}$ Our results remain unchanged for a large rage of the value of $L S E$.

[^10]:    ${ }^{19}$ Based on the data in McDaniel (2007), the effective labor tax rate (on household income and payroll) in the US increased from 0.1775 in 1970-1973 to 0.22475 in 2000-2003.
    ${ }^{20}$ Based on the data in van Vliet and Caminada (2012), the net unemployment replacement rate for one earner couple was 0.5001 for these five countries in the EU and 0.59 in the US in 1971. In 2001, the corresponding rate was 0.6813 for these five countries in the EU and 0.55 in the US.

[^11]:    ${ }^{21}$ With this functional form, the job finding rate is $\mu_{t}=\frac{M_{t}}{1-n_{t}-e_{t}}=\frac{v_{t}}{\left[\left(1-n_{t}-e_{t}\right)^{l}+\left(v_{t}\right)^{1 / / /}\right.}$ and the recruitment rate is $\eta_{t}=\frac{M_{t}}{v_{t}}=\frac{1-n_{t}-e_{t}}{\left[\left(1-n_{t}-e_{t}\right)^{l}+\left(v_{t}\right)\right]^{1 / / 2}}$. Their values lie between
    zero and one.

[^12]:    ${ }^{22}$ The decomposition of changes in labor supply based on Table 4 is available upon request.

