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Technical change, wage inequality, and optimal taxes in an assignment model

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This paper studies income inequality and optimal taxation policies in a talent-to-task assignment model of self-selection. Our model considers relative capital-skill complementarities across tasks, leading to the polarization of capital and technology by task complexity, which in turn drives the polarization of job and wage growth by talent levels. Regarding optimal tax policy, the wage compression channel remains effective through the trickle-down effect of subsidizing high-wage earners and taxing low-wage earners. Yet, the wage compression channel via capital, corporate, and R&D taxes, aimed at reducing wage inequality, does not operate via a trickle-down effect. Instead, it works by taxing capital income and R&D investments in high-task-complexity sectors while subsidizing those in low-task-complex sectors. Moreover, we identify a Pigouvian effect that arises to address spillovers, which modifies the marginal tax rates on labor income, capital income, firm profits, and R&D investments.

Keywords. Assignment, technical change, job polarization, wage distribution, optimal taxation.

JEL classification. D31, H21, H23, H24, H32, J31, O33.

1. Introduction

Since the early 1980s, the US has experienced an increase in the wage gap between college and high school graduates, despite a large rise in the supply of college-educated workers. This trend has been accompanied by a polarization of wage growth and a distinct pattern of job polarization, observed not only in the US but across the EU (e.g., Acemoglu and Autor (2011)). These shifts in the earnings distribution have spurred substantial research into the underlying causes. A large body of literature attributes the relative wage increases to skill-biased technological change. For instance, Krusell, Ohanian, Ríos-Rull, and Violante (2000) introduced equipment-specific technical progress

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with capital-skill complementarity, while Acemoglu (2002) proposed skill-biased technical change driven by innovations. The canonical model, as described by Acemoglu and Autor (2011), has been a central framework in this field. It assumes two imperfectly substitutable skill groups performing two distinct jobs. This model's tractability has made it successful in explaining skill premium evolution in the US and other advanced economies. However, despite its strengths, the canonical model falls short in providing a comprehensive framework for analyzing job polarization patterns and the resulting polarization in earnings distribution. To address these limitations, the assignment model offers a promising alternative framework for tackling these complex issues.

Our study examines a talent-to-task assignment model of self-selection, where the economy has a continuum of privately known talents with imperfect substitutability and a continuum of tasks with varying complexities. In this model, individuals with higher talents have comparative advantages working in more complex tasks, resulting in assortative matching of talents to tasks. We extend existing assignment models by incorporating capital, R&D, and relative capital-skill complementarities across tasks (or sectors).[1] Our model depicts a monopolist in each task, who utilizes both labor and capital to produce intermediates, with R&D investments improving the quality of these intermediates. These intermediates are then used in final goods production. This approach allows us to explain the polarization of jobs and wage growth across different talent levels. To address the dynamic nature of self-selection, we account for spillovers between intermediate and final goods producers in the private market. We solve this dynamic mechanism design problem with spillovers by extending the two-step approach with "inner" and "outer" problems, as proposed by Rothschild and Scheuer (2013), to a dynamic setting.

Our model makes two key contributions. First, it provides insights into how the inclusion of capital and relative capital-skill complementarities drives job polarization. Next, it illustrates how incorporating capital, R&D, and relative capital-skill complementarities reshape the wage compression force, extending its influence beyond wage earners to include capital and R&D investments in taxation policy.

Our main findings are as follows. First, we observe a polarization of capital and technology across tasks, leading to a polarization of job and wage growth across different talent levels. This polarization arises from the self-selection model, which sorts low-skilled workers to low-complexity tasks and high-skilled workers to high-complexity tasks. As capital-skill complementarity increases with task complexity, capital, and routine low-skilled labor in simple tasks are substitutes, driving sizeable investment in task-automating machinery and tools in low-complexity sectors. In contrast, in high-complex tasks, capital and high-skilled, nonroutine labor are complementary, resulting in substantial investment in machinery and knowhow for skill augmentation. This leaves less investment for middle-complexity tasks, creating a polarization in capital and technological progress across tasks. Consequently, wage growth and output are higher in both low-skill and high-skill sectors compared to middle-skill sectors.

[1] Recent estimates by Herrendorf, Herrington, and Valentinyi (2015) indicate that capital-skill complementarity varies across sectors, with agriculture exhibiting lower complementarity compared to manufacturing and services.

Second, regarding optimal taxation, we find that the wage compression effect operates through capital and R&D taxes to redistribute wealth from high to low wage earners.[2] While the traditional "trickle-down effect" of subsidizing high wage earners and taxing low wage earners remains, wage compression via capital and R&D taxes works differently. It reduces wage inequality by taxing capital income and R&D investments in high-complexity sectors while subsidizing those in low-complexity sectors. This mechanism does not rely on trickle-down effects. The rationale is that wage inequality increases when more output-augmenting machinery is allocated to high-complexity sectors (complementing high-skill, nonroutine labor) and decreases when more task-automating machines are allocated to low-complexity tasks (substituting low-skill, routine labor). Additionally, we identify a Pigouvian channel that corrects for spillovers between intermediate and final goods producers in the private market.[3] This Pigouvian effect reduces marginal tax rates for labor across all talent levels, as well as for capital, R&D investments, and firms' profits across all sectors.

1.1 *Related literature*

Our paper contributes to the literature on wage distribution, particularly the rising skill premium in recent decades. Previous research has attributed this trend to increasing demand for skilled labor due to skill-biased technological change (e.g., Krusell, Ohanian, Ríos-Rull, and Violante (2000), Acemoglu (2002)). This literature highlights job polarization and wage growth polarization (Autor and Dorn (2013)), with a focus on how technological change displaces routine labor in the middle of the wage distribution (e.g., Autor, Chin, Salomons, and Seegmiller (2021)). The canonical model, rooted in ideas from Welch (1973), Katz and Murphy (1992), and Card and Lemieux (2001), has been a dominant framework for analyzing skill demands and wage inequality. It assumes skill-biased technical change with two imperfectly substitutable skill groups performing two distinct jobs. However, this model falls short in explaining the polarization of earnings and employment in the US.

Acemoglu and Autor (2011) proposed the assignment model to address these limitations.[4] They developed a Ricardian labor market model with three imperfectly substitutable skill types, where job tasks, technologies, and skill supplies interact to shape the wage structure. They derived threshold tasks that can be profitably produced using either high-skill or medium-skill workers, or medium-skill or low-skill workers. Their

[2] In multisector, self-selection models, the wage distribution is endogenous. An increase in low-skill wages compresses the relative wages of high-skill workers, which helps ease incentive compatibility constraints. This wage compression effect typically leads to lower taxes on high wage earners and higher taxes on low wage earners.

[3] While Lockwood, Nathanson, and Weyl (2017) focused on externalities in task assignments and the impact of income taxes on talent allocation, our paper does not address tax considerations arising from assignment-related externalities.

[4] The assignment model has been applied to a wide range of economic issues, including occupational allocation (Heckman (1974), Heckman and Sedlacek (1985)), schooling choices (Willis and Rosen (1979)), immigration decisions (Borjas (1987)), training program participation (Ham and Lalonde (1996)), the impact of technical change on labor markets (e.g., Costinot and Vogel (2010), Autor and Dorn (2013)), and talent allocation in the US (Hsieh, Hurst, Jones, and Klenow (2019)).

model illustrates how the introduction of machines replacing middle-skill tasks leads to a decline in wages for middle-skill workers relative to high- and low-skill workers, providing a framework for understanding job and wage polarization.

Our contribution to Acemoglu and Autor (2011) is twofold. First, we incorporate relative capital-skill complementarities across tasks, resulting in the polarization of both capital and technology, extending beyond the employment polarization in their model. Second, we generalize the assignment model with public skill types to a talent-to-task model with privately-known skill types, enabling us to analyze optimal tax policy for desirable redistribution, an aspect not explored in their work.

Our paper also contributes to the normative literature on imperfectly substitutable workers. Stiglitz (1982) extended Mirrlees (1971)'s model of perfectly substitutable skill types by incorporating two imperfectly substitutable skill types, but without occupational choice. In such models, the wage distribution is determined endogenously, where increasing low-skill wages compresses high-skill relative wages, easing incentive compatibility constraints. This wage compression effect leads to lower taxes on high wage earners and higher taxes on low wage earners, resulting in a less progressive tax system than the one in the standard Mirrlees model.

Rothschild and Scheuer (2013) further extended Stiglitz (1982)'s model by incorporating a self-selection assignment model a la Roy (1950), with multidimensional talents dependent on tasks.[5] In their two-task, two-dimensional talent economy, the optimal policy mirrors Stiglitz's: taxing low talents and subsidizing (or reducing taxes on) high talents. While Stiglitz's wage compression effect remains, job choices lead to more progressive labor income taxes than Stiglitz's model, but still less progressive than the standard Mirrlees model. Their equilibrium also exhibits intratask wage dispersion and overlapping sectoral wage distributions due to two-dimensional talent types.

Ales, Kurnaz, and Sleet (2015) analyzed the effect of technical change on labor income tax policy by extending Rothschild and Scheuer (2013)'s work to a task-to-talent assignment framework with a continuum of tasks and one-dimensional talents. In their model, the wage compression force in labor income tax depends on the elasticity of relative wages of other talent types in relation to a given talent type's effort. Unlike Rothschild and Scheuer (2013), their setup ensures no overlap in sectoral wage distributions and allows technical change to influence optimal policy. They found that technical change intensifies the wage compression effect at low-wage incomes but tempers it at high-wage incomes, with an opposite effect on the Mirrlees incentive. Quantitatively, the Mirrlees incentive effect dominates, leading to reduced marginal taxes for low earners and increased marginal taxes for high earners, which contrasts with the findings of Stiglitz (1982) and Rothschild and Scheuer (2013). However, Ales, Kurnaz, and Sleet (2015) treated technical progress parametrically, estimating technological parameters from US data that show increased technology for low and high tasks relative to middle tasks when comparing the 2000s to the 1970s.

[5] Roy (1950)'s assignment model with occupational choice is a cornerstone in labor economics. The principle of comparative advantage, recognized by Roy (1951), guides assignment, and was extended to labor markets by Sattinger (1975) and Rosen (1978).

Our model builds upon the work of Stiglitz (1982), Rothschild and Scheuer (2013), and Ales, Kurnaz, and Sleet (2015) by incorporating capital, R&D, and relative capital-skill complementarities across different tasks. While we retain the wage compression effect on labor income tax from these models, our model introduces two distinct outcomes. First, we endogenously generate polarized technical change. The polarization in technology levels and capital allocation results in polarized wage growth, with wages for low and high skills rising relative to those in the middle. This stems from the greater capital-skill complementarity found in more complex tasks. Second, without relying on the traditional trickle-down effect, we achieve a desirable redistribution effect that reduces wage inequality. This is achieved through the wage compression effect, which involves subsidizing capital income and R&D in low task-complexity sectors while taxing these elements in high task-complexity sectors. Besides, our model reveals a novel Pigouvian channel that addresses technology spillovers between intermediates and final goods producers in the private market, an aspect not explored in previous models.

Our work complements the recent study by Akcigit, Hanley, and Stantcheva (2022), who employs a dynamic mechanism design model to examine optimal corporate taxation and R&D policies from a firm perspective.[6] We integrate their innovation process into an assignment model while incorporating capital and relative capital-skill complementarities across tasks. Unlike their model, ours introduces skill and task differentials to explain job polarization, allowing us to analyze labor and capital wedges, an aspect not addressed in their study.

Slavík and Yazici (2014) and Tsai, Yang, and Yu (2022) applied Stiglitz (1982)'s logic to capital taxation. They posited that equipment capital is more complementary to skilled labor, thus increasing the marginal product of skilled labor relative to unskilled labor. Both studies identified the need for a positive tax on (equipment) capital, as the skill premium increases with capital. Our model diverges from this approach by assuming that labor with different talents complements tasks of varying complexity. Consequently, we derive a negative capital tax for low-complexity tasks. Specifically, capital-skill complementarity in our model increases with task complexity, allowing our optimal capital income tax formula to adjust accordingly, with the goal of reducing wage inequality.

Guerreiro, Rebelo, and Teles (2022), Thuemmel (2023), and Costinot and Werning (2023) examined optimal robot taxes, drawing on the frameworks of Stiglitz (1982) and Rothschild and Scheuer (2013). Despite varied applications, their rationale for taxing robots aligns with the logic for taxing capital in the works of Slavík and Yazici (2014) and Tsai, Yang, and Yu (2022): the presence of imperfectly substitutable skill types and fixed capital-skill (or robot-skill) complementarity. While their research primarily focuses on robot investments under fixed robot-skill complementarity, our study extends the analysis to capital and R&D investments, considering relative capital-skill complementarity across tasks. This distinction leads to different marginal capital income tax formulas, which depend on the relative capital-skill complementarity between tasks. Furthermore, our work uniquely addresses the polarization of technical change and wage growth, an aspect not covered in these papers.

[6]For a comprehensive and recent review of the literature on R&D policies, see Bloom, Van Reenen, and Williams (2019).

Our research also contributes to the literature on optimal nonlinear tax reform in models with imperfectly substitutable labor, exemplified by Jacobs and Thuemmel (2018), Sachs, Tsyvinski, and Werquin (2020), and Loebbing (2019). A key difference is that these papers employed a variational approach to study tax incidence, with taxes confined to a specific class, as introduced by Saez (2001) and Golosov, Tsyvinski, and Werquin (2014). In contrast, we utilize a mechanism design approach with an unrestricted tax system to explore optimal taxes. This aligns our work with the new dynamic public finance literature, which applies mechanism design tools to study dynamic income taxation under idiosyncratic risks. Related papers in this vein include Albanesi and Sleet (2006), GTWD+ (2006), and Farhi and Werning (2013). Our work is particularly close to Stantcheva (2017) and Chen and Liang (2024), who incorporated endogenous human capital investments into individual dynamic tax problems. Notably, our treatment of capital income taxation, driven by wage compression through imperfect substitution of skill types, differs from capital tax based on the insurance effect resulting from changes in skill types over time, as discussed in discussed in Golosov, Kocherlakota, and Tsyvinski (2003).

Finally, there is compelling real-world evidence supporting the concepts of observable R&D and technology polarization, as well as the varying elasticity of substitution between capital and skill across sectors. Empirical research has indicated that both high-skill (high-tech) and low-skill (low-tech) sectors attract higher levels of investment when compared to medium-tech industries. Notable studies supporting this include Ortega-Argilés, Piva, Potters, and Vivarelli (2010), Ortega-Argilés, Piva, and Vivarelli (2014), and Fontenele, Cabral, Forte, and Costa (2016).[7]

Furthermore, the elasticity of substitution between capital and labor (or skill) varies significantly across sectors. In high-tech industries, capital investments in technology tend to complement skilled labor, whereas in sectors with routine or manual tasks, capi-

[7] Using firm-level data from OECD and European enterprises from 1987 to 2005, Ortega-Argilés, Piva, Potters, and Vivarelli (2010) show that while R&D investment enhances productivity across all levels of technological intensity, low-technology sectors primarily rely on embodied technological change (i.e., investments in physical capital like machinery and equipment). Productivity gains from R&D in these sectors are comparatively modest. Ortega-Argilés, Piva, and Vivarelli (2014) confirm a similar pattern using firm-level data from the US and Europe from 1990 to 2008. Complementary evidence from a case study of Brazilian firms by Fontenele, Cabral, Forte, and Costa (2016) further highlights that high-tech firms conduct substantial internal R&D and benefit from significant tax incentives. In contrast, low-tech firms predominantly adopt external technologies through process innovation and receive substantial public support for physical capital investment. Medium-tech firms occupy an intermediate position, with limited access to either form of support and weaker associated benefits. Further research by Autor (2015) and Acemoglu and Restrepo (2018) shows that automation and technological upgrades are more prevalent in sectors where routine tasks, often performed by middle-skill workers, can be replaced by machines. This pattern supports the idea that R&D and capital investments are concentrated in high-tech innovations and low-tech automation, contributing to the polarization. Taken together, these studies provide robust evidence of polarization in R&D investment and its returns across the technological spectrum. Both low- and high-tech industries receive relatively greater R&D-related support and realize higher productivity gains, while medium-tech sectors are comparatively underinvested in R&D and exhibit weaker productivity outcomes.

tal often substitutes for lower-skilled labor. Studies by Griliches (1969), Katz and Murphy (1992), and Autor, Katz, and Krueger (1998) provide evidence for this pattern.[8]

Empirical data indicate that R&D and capital investments are unevenly distributed across sectors. High-tech industries typically display higher capital intensity, with substantial investments in R&D, software, and computer technology, while routine-intensive sectors receive comparatively lower R&D spending, as highlighted by OECD (2017) and NIPA data.[9] These patterns collectively reinforce the idea that R&D spending, technology polarization, and capital-skill substitution are observable phenomena with important implications for labor markets, wage dynamics, and sectoral investment trends.

The rest of the paper is structured as follows: In Section 2, we present the talent-to-task assignment model. The optimal tax policies are studied in Section 3. In Section 4, we provide a numerical analysis. Finally, we offer concluding remarks in Section 5.

2. A talent-to-task assignment model

Our talent-to-task assignment framework follows from the works of Costinot and Vogel (2010) and Ales, Kurnaz, and Sleet (2015). The key innovation in our model is that we incorporate capital, R&D, and relative capital-skill complementarities across different tasks. The economy is populated by a continuum of agents, each with a privately known talent type $i \in \mathcal{I} = [\underline{i}, \bar{i}]$, with the fraction of agents of talent i given by $f(i)$. We will interchange the terms "type i" and "talent i".[10] There is a continuum of tasks (or sectors) $v \in \mathcal{V} = [\underline{v}, \overline{v}]$, where tasks vary in the degree of complexity, with larger values of v corresponding to more complex tasks.

Agents Agents live for T periods and share identical preferences regardless of their talent level. Given labor income tax T_t^L and capital income tax T_t^K, each agent of type i chooses consumption $c_t(i)$ and work effort $e_t(i)$ to maximize the following separable

[8]Early studies by Griliches (1969) documented strong capital-skill complementarity in US manufacturing during the 1960s and 1970s, noting that equipment capital, in particular, tended to complement skilled labor. Recent research by Katz and Murphy (1992) and Autor, Katz, and Krueger (1998) demonstrates that skill-biased technological change increases the relative productivity of skilled labor, leading to higher returns for skilled labor. This pattern is especially evident in high-tech industries, where capital investments in technology complement skilled labor. In contrast, in industries with routine or manual tasks, capital tends to substitute for lower-skilled labor, further reinforcing the divide between skilled and unskilled labor.

[9]The OECD (2017) report highlights the uneven distribution of R&D and capital investments across sectors, with industries with greater innovation potential receiving higher levels of investment, while sectors exhibiting routine tasks invest less. The National Income and Product Accounts (NIPA), published by the Bureau of Economic Analysis (BEA) support this trend, showing that high-tech industries are typically capital-intensive, with substantial investments in equipment, structures, and intellectual property products (IPP). In contrast, routine-task-intensive sectors allocate fewer resources to IPP investments. For further details, see Fixler and De Francisco (2022).

[10]To simplify the analysis, we assume that agents' types do not change over time, as time-varying types do not affect our results except one term. A version in which agents' types change over time is studied in Online Supplement Appendix B.2 of Chen and Liang (2026), with an additional positive term in the capital wedge that serves the insurance purpose against future risks.

utility function:

$$U(i) = \sum_{t=1}^{T} \beta^{t-1}\big[u\big(c_t(i)\big) - h\big(e_t(i)\big)\big], \quad \forall i \in [\underline{i}, \bar{i}], \tag{1a}$$

subject to the following budget constraint:

$$c_t(i) + \int_{\mathcal{V}} k^i_{t+1}(v)\,dv \leq z_t(i) - T^L_t\big(z_t(i)\big) + \int_{\mathcal{V}} \big[s^i_t(v) - T^K_t\big(s^i_t(v)\big)\big]\,dv,$$

where $\beta \in (0, 1)$ is the discount factor. Labor income is defined as $z_t(i) \equiv w_t(i)e_t(i)$ and capital income from task v is given by $s^i_t(v) \equiv [R_t(v) + 1 - \delta_k]k^i_t(v)$. The wage rate $w_t(i)$ and the interest rates $R_t(v)$ are taken as given, and δ_k is the depreciation rate of capital. The utility function $u(\cdot)$ satisfies $u' > 0 > u''$ and the Inada condition, while the disutility of effort function $h(\cdot)$ satisfies $h' > 0$ and $h'' > 0$.

Talents and tasks Let $a(i, v) > 0$ be the productivity of an agent with talent i when performing task v. Following Ales, Kurnaz, and Sleet (2015), we posit that higher-talent agents have both a comparative advantage in more complex tasks and an absolute advantage in all tasks, as formalized by the following conditions.

Assumption 1. *The productivity function* $a : [\underline{i}, \bar{i}] \times [\underline{v}, \bar{v}] \to \mathbb{R}_+$ *is twice differentiable and satisfies* (*i*) *comparative advantage*: $a(i', v')a(i, v) > a(i, v')a(i', v)$ *for any* $i' > i$, *and* $v' > v$; (*ii*) *absolute advantage*: *for each* $v \in [\underline{v}, \bar{v}]$, $a(i', v) > a(i, v)$ *for any* $i' > i$.

Assumption 1 (i) states that $a(i, v)$ is strictly log-supermodular in talent and task, a condition that ensures positive assortative matching between tasks and talents in equilibrium. Although Assumption 1(ii) is not essential for our core results, it guarantees that wages are strictly increasing in talent, thereby preventing multiple talents from earning the same wage.

Let $\lambda_t(i, v)$ denote the fraction of agents with talent i who are employed in task v during period t. Consequently, the labor input for task v in period t can be expressed as

$$l_t(v) = \int_{\underline{i}}^{\bar{i}} \lambda_t(i, v)a(i, v)e_t(i)\,di.$$

Final good production As in Akcigit, Hanley, and Stantcheva (2022), the final good Y_t is produced competitively using differentiated intermediate goods $y_t(v)$ as inputs. The final good's production technology is

$$Y_t = \int_{\underline{v}}^{\bar{v}} \widetilde{Y}_t\big(b_t(v), y_t(v)\big)\,dv, \tag{1b}$$

where $\widetilde{Y}_t(b_t(v), y_t(v))$ is the contribution of the intermediate good in task v to the final good, which depends on the technology level $b_t(v)$ of the intermediate good $y_t(v)$ in task v.

Following Akcigit, Hanley, and Stantcheva (2022, Section 5), the production function of the final good is parameterized as the Cobb–Douglas form as follows.[11]

Assumption 2. $\widetilde{Y}(b_t(v), y_t(v)) = b_t(v)^{1-\alpha} y_t(v)^{\alpha}$, *where* $\alpha \in (0, 1)$.

Intermediate goods production For each task $v \in [\underline{v}, \overline{v}]$, the production technology of the intermediate good in task v is

$$y_t(v) = F^v\bigl(l_t(v), k_t(v)\bigr), \tag{1c}$$

where $l_t(v)$ is labor and $k_t(v) \equiv \int_{\underline{i}}^{\overline{i}} k_t^i(v) f(i)\, di$ is capital. The function $F^v(l, k)$ satisfies $F_x^v > 0$, $F_{xx}^v < 0$ and $F_{xx'}^v > 0$ for any $x, x' \in \{l, k\}$ and $x \neq x'$.[12]

For ease of analysis, the production technology of the intermediate goods is parameterized with the familiar constant-elasticity-of-substitution (CES) form as follows.

Assumption 3. $y_t(v) \equiv F^v(l_t(v), k_t(v)) = [(1-\rho) l_t^{\varepsilon_v}(v) + \rho k_t^{\varepsilon_v}(v)]^{\frac{1}{\varepsilon_v}}$, *where* $\rho \in (0, 1)$ *and* $\varepsilon_v < 1$.

The parameter $\rho \in (0, 1)$ is the capital intensity, while $\sigma_v \equiv \frac{1}{1-\varepsilon_v}$ is the elasticity of substitution between capital and labor in task v, where $\varepsilon_v < 1$. This specification allows for varying elasticities of substitution across different tasks (or sectors). In our subsequent quantitative analysis, we will show that this assumption is crucial in explaining the observed phenomena of wage and job polarization.

Intermediate firms' decisions The intermediate good is produced by a monopolist. The monopolist faces the demand derived from the final good producer's optimization problem, given by

$$p\bigl(b_t(v), y_t(v)\bigr) = \frac{\partial \widetilde{Y}_t}{\partial y_t(v)}, \quad \forall v \in \{\underline{v}, \overline{v}\}, \tag{2a}$$

where $p(b_t(v), y_t(v))$ is the price of the intermediate good in task v.

Given the technology level $b_t(v)$, the wage rate $\omega_t(v)$, the interest rate $R_t(v)$, the production technology in (1c), and the demand function in (2a), the producer of the intermediate good in task v chooses labor and capital to maximize the following profit:

$$\pi\bigl(b_t(v)\bigr) = \max_{l_t(v), k_t(v)} p\bigl(b_t(v), y_t(v)\bigr) y_t(v) - \omega_t(v) l_t(v) - R_t(v) k_t(v). \tag{2b}$$

[11] Acemoglu and Restrepo (2022), and Guner, Ventura, and Xu (2008) also use the Cobb–Douglas form to combine the technical level and the intermediate good into the final good.

[12] Both Cobb–Douglas and CES functions satisfy this condition.

The first-order conditions are as follows:[13]

$$R_t(v) = \frac{\partial F^v}{\partial k_t(v)}\left[\frac{\partial p}{\partial y_t} y_t(v) + p\big(b_t(v), y_t(v)\big)\right] \tag{2c}$$

$$\text{and} \quad \omega_t(v) = \frac{\partial F^v}{\partial l_t(v)}\left[\frac{\partial p}{\partial y_t} y_t(v) + p\big(b_t(v), y_t(v)\big)\right]. \tag{2d}$$

In our model, the capital return $R_t(v)$ is task-specific, and the return to effort for talent i working on task v is given by the wage rate $\omega_t(v)a(i, v)$, where $w_t(i) \equiv \max_v \omega_t(v)a(i, v)$. In the tax equilibrium, we introduce a condition on the marginal capital tax to equalize the after-tax marginal returns of capital across all tasks. This condition leads to task-specific capital wedges (implicit capital tax rates) in the constrained efficient allocation. A similar approach was employed by Slavík and Yazici (2014) in their model featuring two types of capital. They derived different capital wedges between the marginal returns to these two capital types ensuring that after accounting for capital-specific taxes, the returns to capital are equalized for both types of capital.

Technical change This paper focuses on R&D policy design, rather than intellectual property rights (IPR). Following Akcigit, Hanley, and Stantcheva (2022), we model the IPR policy as it currently is in the world, where patent protection is granted to innovators. As a result, successfully innovating firms have full monopoly rights. Therefore, one key role of policy is to address the distortion caused by monopoly powers introduced by the patent system.

Intermediate good firms invest in R&D activities to enhance the quality of their differentiated products. The technology level $b_t(v)$ of firms in task v evolves according to[14]

$$b_t(v) = A^v\big(b_{t-1}(v), q_t(v), n_t(v)\big), \tag{3a}$$

where $q_t(v)$ and $n_t(v)$ are inputs for quality improvement in period t. Similar to Akcigit, Hanley, and Stantcheva (2022), we assume that quality improvement requires both observable R&D investments $q_t(v)$ and unobservable R&D inputs $n_t(v)$.[15] The respective costs of these inputs are denoted as $\mathcal{C}_q(q_t(v))$ and $\mathcal{C}_n(n_t(v))$, with $\mathcal{C}'_q, \mathcal{C}'_n > 0$ and $\mathcal{C}''_q, \mathcal{C}''_n \geq 0$. The technology function $A^v(b_-, q, n)$ satisfies standard concavity properties: $A^v_z > 0$ and $A^v_{zz} \leq 0$, for any $z \in \{b_-, q, n\}$. Since R&D investments $q_t(v)$ are observable, the government can directly subsidize them to incentivize increased R&D activity. Conversely, R&D inputs $n_t(v)$ are unobservable, and thus cannot be taxed. Consequently, we assume that the R&D subsidy $S^q_t(\cdot)$ is based on the firm's observable R&D investment $q_t(v)$, while the corporate tax $T^c_t(\cdot)$ is based on the firm's profits π_t.

[13] If the intermediate good producer has no monopoly power, $p(b_t(v), y_t(v))$ in (2b) is a constant. Then the first-order conditions reduce to the standard forms: $\omega_t(v) = \frac{\partial F^v}{\partial l_t(v)} p(b_t(v), y_t(v))$, and $R_t(v) = \frac{\partial F^v}{\partial k_t(v)} p(b_t(v), y_t(v))$.

[14] By convention, the superscript v in $F^v(.)$ and $A^v(.)$ is used to denote the goods production technology and the quality improvement technology associated with task v, respectively.

[15] Following Akcigit, Hanley, and Stantcheva (2022), we categorize R&D activities "R&D investments" and "R&D inputs." In their framework, firms make observable R&D investments such as expenditures on laboratory equipment, and also incur unobservable R&D inputs, like organizational-level management costs that support research efforts.

R&D decisions Given R&D costs $\mathcal{C}_q(q_t(v))$ and $\mathcal{C}_n(n_t(v))$, the profit function $\pi(b_t)$ from (2b), and the evolution of the technology level from (3a), the firm chooses R&D investments $q_t(v)$ and R&D inputs $n_t(v)$ to maximize the following objective:

$$\max_{q_t(v),n_t(v)} \sum_{s=t}^{T} \frac{1}{\mathcal{R}_s^t(v)} \big[\pi_s\big(b_s(v)\big) - \mathcal{C}_q\big(q_s(v)\big) - \mathcal{C}_n\big(n_s(v)\big) - T_s^c\big(\pi_s(b_s)\big) + S_s^q\big(q_s(v)\big)\big], \tag{3b}$$

where $\mathcal{R}_s^t(v) = 1$ if $s = t$ and $\mathcal{R}_s^t(v) = \prod_{u=t+1}^{s} R_u(v)$ if $s > t$.

Tax equilibrium Given a fixed government spending G_t, a tax equilibrium consists of the following: tax and subsidy functions $\{T_t^L, T_t^K, T_t^c, S_t^q\}$, allocations $\{c_t(i), e_t(i), k_t^i(v), l_t(v), q_t(v), n_t(v)\}$, and wage rates and interest rates $\{\omega_t(v), R_t(v)\}$. These must satisfy the following conditions:

(i) The allocation $\{c_t(i), e_t(i), k_t^i(v)\}$ solves the agent's problem (1a), and the marginal returns of each task-specific capital are equal across tasks. Specifically, the following condition hold:

$$\big[R_t(v) + 1 - \delta_k\big]\big[1 - \partial T_t^K\big(s_t^i(v)\big)\big] = \big[R_t(v\prime) + 1 - \delta_k\big]\big[1 - \partial T_t^K\big(s_t^i(v')\big)\big] \tag{3c}$$

for any $v \neq v'$.

(ii) The wage rates and interest rates satisfy (2c)–(2d).

(iii) The allocation $\{q_t(v), n_t(v)\}$ solves the firm's problem (3b).

(iv) The following goods market clearing condition holds:

$$\begin{aligned} G_t + \int_{\underline{i}}^{\bar{i}} c_t(i) f(i)\, di + \int_{\underline{v}}^{\bar{v}} \big[k_{t+1}(v) - (1-\delta_k)k_t(v) + \mathcal{C}_q\big(q_t(v)\big) \\ + \mathcal{C}_n\big(n_t(v)\big)\big]\, dv \le Y_t. \end{aligned} \tag{3d}$$

(v) The labor market clearing conditions hold for each $v \in [\underline{v}, \overline{v}]$,

$$l_t(v) = \int_{\underline{i}}^{\bar{i}} \lambda_t(i, v) a(i, v) e_t(i)\, di, \tag{3e}$$

and for each $i \in [\underline{i}, \bar{i}]$,

$$f(i) = \int_{\underline{v}}^{\bar{v}} \lambda_t(i, v)\, dv. \tag{3f}$$

3. Optimal policy and constrained efficient allocation

In this section, we derive the optimal allocation in a mechanism design problem. This allocation, referred to as the constrained efficient allocation, resolves the mechanism design issue. It is characterized by wedges' implicit taxes and subsidies that gauge the deviations from a laissez-faire economy under patent protection. This model distinguishes four types of wedges.

(i) Capital wedge is an implicit capital tax, which measures the gap between the household's marginal rate of substitution in consumption across adjacent periods and the firm's marginal product of capital in the latter period (i.e., the interest rate). It is defined as

$$\tau_i^{k_t}(v) \equiv 1 - \frac{u'\big(c_{t-1}(i)\big)}{\big[R_t(v) + 1 - \delta_k\big]\beta u'\big(c_t(i)\big)}. \tag{4a}$$

If the capital income tax T_t^K in a tax equilibrium is differentiable, then according to the first-order conditions from the agent's problem in (1a), we obtain $\partial T_t^K(s_t^i(v)) = \tau_i^{k_t}(v)$. Consequently, condition (4a) is satisfied.

(ii) Labor wedge represents an implicit labor tax, which measures the gap between the marginal rate of substitution between consumption and labor and the marginal rate of transformation (i.e., the wage rate). It is defined as

$$\tau^{l_t}(i) \equiv 1 - \frac{h'\big(e_t(i)\big)}{w_t(i)u'\big(c_t(i)\big)}. \tag{4b}$$

If the labor income tax T_t^L in a tax equilibrium is differentiable, then from the first-order conditions of the agent's problem in (1a), we have $\partial T_t^L(z_t(i)) = \tau^{l_t}(i)$.

(iii) Corporate wedge, or **profit wedge** is an implicit tax on firm's profits,[16] which measures the gap between the expected discounted sum of future marginal benefits and the marginal cost of the firm's R&D inputs. It is defined as

$$\tau^{n_t}(v) \equiv \left[\sum_{s=t}^{T} \frac{1}{\mathcal{R}_s^t(v)} \pi_s'\big(b_s(v)\big)\Gamma_t^s(v)\right] \frac{\partial A_t^v}{\partial n_t(v)} - \mathcal{C}_n'\big(n_t(v)\big), \tag{4c}$$

where

$$\Gamma_t^s(\nu) \equiv \begin{cases} 1, & \text{when } s = t, \\ \displaystyle\prod_{u=t+1}^{s} \frac{\partial A_u^\nu}{\partial b_{u-1}(\nu)}, & \text{when } s = t+1, t+2, \dots, T. \end{cases} \tag{4d}$$

If the corporate tax T_t^c in a tax equilibrium is differentiable, then from the first-order conditions of the firm's problem in (3b), we find obtain: $\tau^{n_t}(v) = \frac{\partial T_t^c}{\partial \pi_t}[\frac{\partial \pi_t}{\partial b_t}\frac{\partial A_t^v}{\partial n_t}]$ in the terminal period $t = T$.[17] This implies that the corporate wedge defined in (4c) can be represented by the marginal profit tax rates, as suggested by Akcigit, Hanley, and Stantcheva (2022, p. 662).

[16] Akcigit, Hanley, and Stantcheva (2022, p. 662) explained the reason that the R&D input wedge is interchangeably called the corporate wedge or the profit wedge, since it mimics a tax on firms' profits, gross of R&D investments.

[17] In the terminal period, the optimal conditions are influenced only by the current corporate tax and are not affected by future corporate taxes, due to the persistent impact of R&D decisions in our model.

(iv) R&D investment wedge,[18] or **R&D wedge** for short, is an implicit subsidy to R&D investments $s^{q_t}(v)$, which measures the gap between the marginal cost and the expected discounted sum of future marginal benefits of firm's R&D investments. It is defined as

$$s^{q_t}(v) \equiv \mathcal{C}_q'\big(q_t(v)\big) - \left[\sum_{s=t}^{T} \frac{1}{\mathcal{R}_s^t(v)} \pi_s'\big(b_s(v)\big)\Gamma_t^s(v)\right] \frac{\partial A_t^v}{\partial q_t(v)}. \tag{4e}$$

Note that if the R&D subsidy S_t^q in a tax equilibrium is differentiable, then from the first-order conditions of the firm's problem in (3b), we have $s^{q_t}(v) = \partial S_t^q(q_t(v)) - \tau^{n_t}(v)$ in the terminal period $t = T$. This implies that the R&D wedge defined in (4e) represents the net marginal R&D subsidy rate, after accounting for the effect from marginal profit tax rate.

These wedges capture the distortions induced by informational and other failures. As in Akcigit, Hanley, and Stantcheva (2022), we interchangeably refer to these wedges as taxes or subsidies, since they effectively represent taxes or subsidies on income and taxes or subsidies on profits, gross of R&D investments.

To identify optimal policies in our talent-to-task assignment model, we employ the inner–outer method building on the approaches of Rothschild and Scheuer (2013) and Ales, Kurnaz, and Sleet (2015). This method consists of two steps. In the inner step, given the effort assignments $\{e_t(i)\}$, capital $\{k_t(v)\}$, and technology levels $\{b_t(v)\}$, we solve the task assignment problem. This step allows us to construct an indirect, microfounded production function for the final good. In the outer step, using this microfounded production function, we solve the social planning problem to determine the constrained efficient allocation and associated wedges.

3.1 *Inner problem*

The objective of the inner problem is to assign talents to tasks. Each talent i agent chooses to work in task v that maximizes the following marginal labor income, as given by

$$w_t(i) \equiv \max_v \omega_t(v) a(i, v). \tag{5}$$

In Proposition 1, we show that there exists a matching function M_t, which assigns an agent of talent i to the optimal task $v = M_t(i)$, following the proof in Lemma 1 of Costinot and Vogel (2010).

Proposition 1. *Under Assumptions 1–3, given the labor effort, capital, and technology level profiles $\{e_t(i), k_t(v), b_t(v)\}$ in a tax equilibrium, there exists a continuous and strictly increasing matching function $M_t : \mathcal{I} \to \mathcal{V}$ such that:*

(i) $\lambda_t(i, v) > 0$ if and only if $M_t(i) = v$, and

(ii) $M_t(\underline{i}) = \underline{v}$ and $M_t(\bar{i}) = \bar{v}$.

[18] These names, R&D investment and input wedges, are the same as those in Akcigit, Hanley, and Stantcheva (2022). As the technical level is set to depreciate at rate δ in Akcigit, Hanley, and Stantcheva (2022), their functional form implies $\Gamma_t^s(v) = (1 - \delta)^{s-t}$.

PROOF. See Appendix A.1. □

Proposition 1 characterizes the efficiency of the talent-to-task assignment. Condition (i) of Proposition 1 implies that $\lambda_t(i, v) = 0$ for all $v \in \{v \in \mathcal{V} | v \neq M_t(i)\}$. Based on labor market clearing condition (3f), this implies that $f(i) = \lambda_t(i, M_t(i))$. Therefore, the final goods production function in (1b) is rewritten as an indirect and microfounded production function as follows:

$$\begin{aligned} Y_t &= \int_{\underline{v}}^{\bar{v}} \widetilde{Y}_t\big(b_t(v), y_t(v)\big)\, dv \\ &= \int \widetilde{Y}_t\Big(b_t(v), F^v\big(\underbrace{a\big(M_t^{-1}(v), v\big) f\big(M_t^{-1}(v)\big) e_t\big(M_t^{-1}(v)\big)}_{=l_t(v)}, k_t(v)\big)\Big)\, dv, \end{aligned} \tag{6a}$$

where $a(M_t^{-1}(v), v)$ and $f(M_t^{-1}(v))$ are exogenously given. The production function in (6a) can be expressed as an indirect and microfounded function as follows:

$$Y_t \equiv \hat{Y}_t\big(\{e_t(i), k_t(v), b_t(v)\}\big). \tag{6b}$$

Given the function $a(i, v)$ and the fraction $f(i)$, this formulation (6b) shows that the final output depends on labor effort $\{e_t(i)\}$, capital $\{k_t(v)\}$, and technology level $\{b_t(v)\}$.

3.2 *Outer problem*

The outer problem involves solving the social planning problem using the indirect, microfounded production function $\hat{Y}_t$ obtained from the inner problem. To characterize the optimal policy, we adopt the Mirrlees approach, framing the problem within the context of mechanism design. In this approach, agents report their types, and the social planner selects an allocation contingent on the reports, which provides the correct incentives for agents to truthfully report their types. The social planner introduces an incentive compatibility constraint (ICC) as follows:

$$U(i) = \max_{i'} \sum_{t=1}^{T} \beta^{t-1} \left[u\big(c_t(i')\big) - h\left(\frac{w_t(i') e_t(i')}{w_t(i)} \right) \right]. \tag{7a}$$

This implies the following envelope condition:

$$\dot{U}(i) = \sum_{t=1}^{T} \beta^{t-1} h'\big(e_t(i)\big) e_t(i) \frac{\dot{w}_t(i)}{w_t(i)}, \tag{7b}$$

where $\dot{U}(i) \equiv \frac{\partial U(i)}{\partial i}$ and $\dot{w}_t(i) \equiv \frac{\partial w_t(i)}{\partial i}$.

Social planning problem (P_1) To obtain the constrained efficient allocation, the social planner maximizes the following Utilitarian social welfare function,[19]

$$W = \int_{\underline{i}}^{\bar{i}} U(i) f(i)\, di, \tag{8a}$$

subject to the envelope condition (7b) and the following resource constraints (8b):

$$Y_t \geq G_t + \int_{\underline{i}}^{\bar{i}} c_t(i) f(i)\, di + \int_{\underline{v}}^{\bar{v}} \big[k_{t+1}(v) - (1-\delta_k) k_t(v) + \mathcal{C}_q\big(q_t(v)\big) + \mathcal{C}_n\big(n_t(v)\big)\big]\, dv \tag{8b}$$

for $t = 1, 2, \dots, T$. Here, χ_t denotes the shadow price for the aggregate resource constraint (8b) in period t and $-\eta(i)$ denotes the costate variable associated with the envelope condition (7b) for talent i. Additionally, Y_t satisfies (6b). For brevity, we have placed the Hamiltonian of this planning problem in Appendix A.2. This Appendix also contains the following Lemma 1, which proves that $\chi_t > 0$, and $\eta(i) > 0$ for all $i \in (\underline{i}, \bar{i})$.

Lemma 1. *The shadow price for the aggregate resource constraint χ_t satisfies the following condition*:

$$\chi_t = \beta^{t-1} u'\big(c_t(\hat{i})\big) \quad \textit{for some } \hat{i} \in (\underline{i}, \bar{i}).$$

Additionally, the costate variable $\eta(i)$ can be expressed as

$$\eta(i) = \int_{\underline{i}}^{i} \left[1 - \frac{\chi_t}{\beta^{t-1} u'\big(c_t(i')\big)}\right] f\big(i'\big)\, di'.$$

Moreover, if $c_t(i)$ is monotonically increasing in i, then $\chi_t > 0$ and $\eta(i) > 0$ for all $i \in (\underline{i}, \bar{i})$.

Proof. See Appendix A.2. □

Wage inequality and cross-relative wage elasticity Before deriving the wedges from the planning problem, we introduce a key elasticity: the cross-relative wage elasticity of factor inputs, which is useful for analyzing the wedges and also appears in Ales, Kurnaz, and Sleet (2015). Let $\kappa^t_{v,j}$, $\phi^t_{i,j}$, and $\varphi^t_{v,j}$, for $i, j \in [\underline{i}, \bar{i}]$ and $v \in [\underline{v}, \bar{v}]$, denote the cross-relative wage elasticity of $x_t \in \{k_t(v), e_{i,t}, b_t(v)\}$; that is, capital of task v, labor effort of talent i, and technology progress of task v, respectively. The cross-relative wage elasticity of x_t is defined as the percent change in wage premium $\dot{w}_t(j)/w_t(j)$ for talent type j, divided by the percent change of x_t as follows:

$$E_j(x_t) \equiv \frac{x_t}{\dot{w}_t(j)/w_t(j)} \frac{\partial\big(\dot{w}_t(j)/w_t(j)\big)}{\partial x_t}, \tag{9}$$

[19]The Utilitarian social welfare function here ensures that agents have no incentive to pretend as higher talent type. A general social welfare function can be used here, but the incentive compatible constraint should be restricted to downward binding: $U(i) \geq \sum_{t=1}^{T} \beta^{t-1}[u(c_t(i')) - h(\frac{w_t(i')e_t(i')}{w_t(i)})]$ for any $i' \leq i$ in order to be consistent with the standard Mirrlees model. In Online Supplement Appendix B.1 of Chen and Liang (2026), we use a later approach in the two-talent-two-task model.

$x_t \in \{k_t(v), e_t(i), b_t(v)\}$, $i, j \in [\underline{i}, \bar{i}]$, and $v \in [\underline{v}, \bar{v}]$. Thus, $E_j(k_t(v)) = \kappa^t_{v,j}$, $E_j(e_t(i)) = \phi^t_{i,j}$, and $E_j(b_t(v)) = \varphi^t_{v,j}$ for $i, j \in [\underline{i}, \bar{i}]$ and $v \in [\underline{v}, \bar{v}]$.

Given the production function $Y_t \equiv \hat{Y}_t(\{e_t(i), k_t(v), b_t(v)\})$ in (6b) and the fact that $w_t(j) = \partial \hat{Y}_t / \partial e_t(j)$, the elasticities $\kappa^t_{v,j}$, $\phi^t_{i,j}$, $\varphi^t_{v,j}$ in (9) can be derived from the production function.[20] In Appendix A.4, we provide explicit expressions for these elasticities in a two-type example and analyze how their signs vary across different talents and sectors.

We are ready to characterize the wedges in the talent-to-task assignment model. While $\Gamma^s_t(v)$ is defined in (4d), we denote

$$\Delta_t(j) \equiv h'\big(e_t(j)\big)e_t(j)\left(\frac{\dot{w}_t(j)}{w_t(j)}\right), \quad \text{and} \quad \Psi_t(i) \equiv \frac{u'\big(c_t(i)\big)}{f(i)h'\big(e_t(i)\big)}.$$

The wedges in the talent-to-task assignment model are characterized as follows.

Proposition 2. *Optimal allocations in the talent-to-task assignment model are characterized as follows.*

(*i*) ***Capital wedge*** *for task $v \in [\underline{v}, \bar{v}]$ is independent of agents' type i and is*

$$\frac{\tau^{k_t}_i(v)}{1-\tau^{k_t}_i(v)} = \int_{\underline{i}}^{\bar{i}} \mathcal{W}^k_t(v,j)\kappa^t_{v,j}\,dj + \mathcal{M}^k_t\left[R_t(v) - \frac{\partial \hat{Y}_t}{\partial k_t(v)}\right], \tag{10a}$$

where $\mathcal{W}^k_t(v,j) \equiv \frac{\beta^{t-1}\eta(j)\Delta_t(j)}{\chi_{t-1}k_t(v)} > 0$, *and* $\mathcal{M}^k_t \equiv \frac{\chi_t}{\chi_{t-1}} > 0$;

(*ii*) ***Labor wedge*** *for type $i \in [\underline{i}, \bar{i}]$ is*

$$\frac{\tau^{l_t}(i)}{1-\tau^{l_t}(i)} = \mathcal{N}_t(i) + \int_{\underline{i}}^{\bar{i}} \mathcal{W}^l_t(i,j)\phi^t_{i,j}\,dj + \mathcal{M}^l_t(i)\left[f(i)w_t(i) - \frac{\partial \hat{Y}_t}{\partial e_t(i)}\right], \tag{10b}$$

where $\mathcal{N}_t(i) \equiv \frac{\beta^{t-1}\eta(i)\Psi_t(i)[h''(e_t(i))e_t(i)+h'(e_t(i))]}{\chi_t} > 0$, $\mathcal{W}^l_t(i,j) \equiv \frac{\beta^{t-1}\eta(j)\Delta_t(j)\Psi_t(i)}{\chi_t e_{i,t}} > 0$, *and* $\mathcal{M}^l_t(i) \equiv \Psi_t(i) > 0$;

(*iii*) ***Corporate wedge*** *for task $v \in [\underline{v}, \bar{v}]$ is*

$$\tau^{n_t}(v) = \sum_{s=t}^{T}\int_{\underline{i}}^{\bar{i}} \mathcal{W}^n_{s,t}(v,j)\varphi^s_{v,j}\,dj - \mathcal{M}^n_{s,t}(v)\left[\frac{\chi_s}{\chi_t}\frac{\partial \hat{Y}_s}{\partial b_s(v)} - \frac{\pi'\big(b_s(v)\big)}{\mathcal{R}^t_s(v)}\right], \tag{10c}$$

where $\mathcal{W}^n_{s,t}(v,j) \equiv \frac{\beta^{s-1}\eta(j)\Delta_s(j)\Gamma^s_t(v)}{\chi_t b_s(v)}\frac{\partial A^v}{\partial n_t(v)} > 0$, *and* $\mathcal{M}^n_{s,t}(v) \equiv \Gamma^s_t(v)\frac{\partial A^v}{\partial n_t(v)} > 0$;

(*iv*) ***R&D wedge*** *for task $v \in [\underline{v}, \bar{v}]$ is*

$$s^{q_t}(v) = \sum_{s=t}^{T}\int_{\underline{i}}^{\bar{i}} -\mathcal{W}^q_{s,t}(v,j)\varphi^s_{v,j}\,dj + \mathcal{M}^q_{s,t}(v)\left[\frac{\chi_s}{\chi_t}\frac{\partial \hat{Y}_s}{\partial b_s(v)} - \frac{\pi'\big(b_s(v)\big)}{\mathcal{R}^t_s(v)}\right], \tag{10d}$$

[20]As $\dot{w}_t(j) = \frac{\partial w_t(j)}{\partial j} = \lim_{\Delta\to 0}\frac{\partial \hat{Y}_t/\partial e_t(j+\Delta) - \partial \hat{Y}_t/\partial e_t(j)}{\Delta}$, the wage premium is expressed as $\frac{\dot{w}_t(j)}{w_t(j)} = \frac{1}{\partial \hat{Y}_t/\partial e_t(j)}\lim_{\Delta\to 0}\frac{\partial \hat{Y}_t/\partial e_t(j+\Delta) - \partial \hat{Y}_t/\partial e_t(j)}{\Delta}$. Based on (9), $\kappa^t_{v,j}$, $\phi^t_{i,j}$, $\varphi^t_{v,j}$ can be directly derived by the given production function $\hat{Y}_t$.

where $\mathcal{W}^q_{s,t}(v, j) \equiv \frac{\beta^{s-1}\eta(j)\Delta_s(j)\Gamma^s_t(v)}{\chi_t b_s(v)} \frac{\partial A^v}{\partial q_t(v)} > 0$, *and* $\mathcal{M}^q_{s,t}(v) \equiv \Gamma^s_t(v) \frac{\partial A^v}{\partial q_t(v)} > 0$; *with* $\chi_t = \beta^{t-1} u'(c_t(\hat{i})) > 0$ *for some* $\hat{i} \in (\underline{i}, \bar{i})$ *and* $\eta(i) = \int_{\underline{i}}^{i} [1 - \frac{\chi_t}{\beta^{t-1} u'(c_t(i'))}] f(i')\, di' > 0$.

Proof. See Appendix A.3. □

The wedge formulas in (10a)–(10d) consist of two or three terms, each measuring the distortions in the constrained efficient allocation of the planning problem relative to the laissez-faire economy. These terms are as follows:

(1) **Mirrlees term** $\{\mathcal{N}_t(i)\}$: The Mirrlees term reflects the incentive effect found in standard Mirrlees taxation models, arising from asymmetric information about agent's productivity. It increases the labor wedge to discourage underreporting of productivity, but has no effects on the other wedges.

(2) **Wage compression term** $\{\mathcal{W}^x_t, x \in \{k, l, q, n\}\}$: The wage compression effect is not present in standard Mirrlees taxation models, where workers of different types are considered perfect substitutes. In our model, however, workers of different types are imperfect substitutes, making the wage distribution endogenously influenced by tax policy. Consequently, a wage compression term emerges, influencing four key wedges. The signs of the wage compression term hinge on the signs of cross-relative wage elasticity of factor inputs $\{\kappa^t, \phi^t, \varphi^t\}$. The signs of these elasticities are ambiguous in the general model. To illustrate how they vary with talents and tasks, we will derive the signs of these elasticities for each talent and each task in a simple two-talent, two-task model in the next subsection (Proposition 3).

(3) **Pigouvian term** $\{\mathcal{M}^x_t, x \in \{k, l, q, n\}\}$: This effect is absent in the works of Stiglitz (1982), Rothschild and Scheuer (2013), and Ales, Kurnaz, and Sleet (2015). Our paper introduces R&D considerations, particularly focusing on technology spillovers between intermediate and final goods producers in the private market. To correct this distortion, we incorporate a Pigouvian term that incentivizes monopoly intermediate goods firms to allocate more resources to capital, labor, and R&D.

By considering the combined effects of the Mirrlees, wage compression, and Pigouvian terms, we gain a deeper understanding of the various components in the optimal wedge formulas from Proposition 2. First, the labor wedge for labor type $i \in [\underline{i}, \bar{i}]$ in (10b) is influenced by all these factors.

The Mirrlees effect increases the labor wedge to prevent underreporting of productivity due to unobservable talent differences. In contrast, the wage compression effect can either increase or decrease the labor wedge for a given talent type i, depending on whether the labor effort $e_t(i)$ of talent type i raises or lowers the wage premium $\frac{\dot{w}_t(j)}{w_t(j)}$ for talent type j. If $e_t(i)$ increases the wage premium $\frac{\dot{w}_t(j)}{w_t(j)}$ for talent type j, then the percentage change in the wage premium $\frac{\dot{w}_t(j)}{w_t(j)}$ divided by the percentage change in $e_t(i)$ is positive, implying a positive cross-relative wage elasticity of type i's labor effort $\phi^t_{i,j}$. In this scenario, the wage compression term increases the labor wedge for labor type i in order to decrease wage inequality. Conversely, if $e_t(i)$ decreases the wage premium, the cross-relative wage elasticity is negative, and the wage compression term decreases the labor wedge for type i, further compressing wage inequality. Lastly, the Pigouvian effect, driven by technology spillovers in the private market, pushes monopoly intermediate goods firms to hire more labor effort from talent type i, thus reducing the labor wedge for this group.

Similar to Ales, Kurnaz, and Sleet (2015), the wage compression term reduces overall wage inequality by increasing labor taxes for talent types whose labor may increase wage inequality. As lower-skilled workers increase their labor, their wages tend to fall, widening wage gaps. To counter this, the wage compression term imposes higher taxes on low earners compared to high earners, resulting in a less progressive labor income tax schedule than the one in the optimal Mirrlees (1971) model, consistent with Stiglitz (1982), Rothschild and Scheuer (2013), and Ales, Kurnaz, and Sleet (2015). Our model builds on these studies by showing how wage compression also affects capital, corporate, and R&D wedges, as discussed further below.

The capital wedge for task $v \in [\underline{v}, \overline{v}]$ in (10a) is influenced by both the wage compression and Pigouvian terms. Similar to the labor wedge, the wage compression effect can either increase or decrease the capital wedge for a given task v. If capital $k_t(v)$ for task v increases the wage premium $\frac{\dot{w}_t(j)}{w_t(j)}$ for talent types j, the percentage change in the wage premium $\frac{\dot{w}_t(j)}{w_t(j)}$ divided by the percentage change in $k_t(v)$ is positive, implying a positive cross-relative wage elasticity of task v's capital, $\kappa^t_{v,j}$. In this state, the wage compression term raises the capital wedge for task v to decrease wage inequality. Conversely, if capital $k_t(v)$ reduces the wage premium, the cross-relative wage elasticity of capital becomes negative, and the wage compression term decreases the capital wedge for task v to further compress wage inequality. Lastly, the Pigouvian effect, due to technology spillovers, encourages monopoly intermediate goods firms to allocate more capital to task v, reducing the capital wedge for that task.

Additionally, the corporate wedge for task $v \in [\underline{v}, \overline{v}]$ in (10c) is influenced by both the wage compression and the Pigouvian terms. Similar to the capital and labor wedges, the wage compression effect either increases or decreases the corporate wedge for task v, depending on whether R&D inputs $n_t(v)$ increase or decrease the wage premium $\frac{\dot{w}_t(j)}{w_t(j)}$ for talent types j. This corresponds to the sign of the cross-relative wage elasticity of task v's technology, $\varphi^t_{v,j}$. The Pigouvian effect reduces the corporate wedge for task v by incentivizing intermediate goods firms to invest more in R&D inputs due to technology spillovers.

Finally, the R&D wedge for task $v \in [\underline{v}, \overline{v}]$ in (10d) is similarly influenced by both the wage compression and Pigouvian effects. The wage compression effect can increase or decrease the R&D wedge for task v, depending on whether R&D investments $q(v)$ raise or lower the wage premium $\frac{\dot{w}_t(j)}{w_t(j)}$ for talent types j, which determines the sign of the cross-relative wage elasticity of task v's technology, $\varphi^t_{v,j}$. The Pigouvian effect increases the R&D wedge for task v, as firms are encouraged to invest more in R&D due to spillover effects.

In summary, Mirrlees terms are positive in labor wedges, while Pigouvian terms are negative in labor, capital, and corporate wedges but positive in R&D wedges. The signs of the wage compression terms in these four wedges depend on the signs of the cross-relative wage elasticities $\kappa^t_{v,j}$, $\phi^t_{i,j}$, and $\varphi^t_{v,j}$. To explore how these cross-relative wage elasticities vary across different talent and task types, we will analyze a simplified two-talent, two-task model in the next subsection, which allow us to derive the signs of these elasticities under specific conditions.

3.3 *An example: The signs of optimal policy in a two-talent, two-task model*

This subsection examines a simplified model with two talents and two tasks to analyze the signs of cross-relative wage elasticities. For clarity, we define the talents as low ($i = L$) and high ($i = H$), and the talents as the bottom sector ($v = \underline{v}$) and the top sector ($v = \bar{v}$). We assume that high-talent agents are assigned to the top sector, while low-talent agents work in the bottom sector. Based on Assumptions 2–3, the final goods technology in this simplified model is given by the following form:

$$Y_t = b_t(\bar{v})^{1-\alpha}\big[(1-\rho)l_t^{\varepsilon_{\bar{v}}}(\bar{v}) + \rho k_t^{\varepsilon_{\bar{v}}}(\bar{v})\big]^{\frac{\alpha}{\varepsilon_{\bar{v}}}} + b_t(\underline{v})^{1-\alpha}\big[(1-\rho)l_t^{\varepsilon_{\underline{v}}}(\underline{v}) + \rho k_t^{\varepsilon_{\underline{v}}}(\underline{v})\big]^{\frac{\alpha}{\varepsilon_{\underline{v}}}}. \quad (11a)$$

Based on (11a), we derive the signs of the cross-relative wage elasticities in Proposition 3.

Proposition 3. *Suppose that Assumptions 2–3 hold, in the two-talent and two-task model, the cross-relative wage elasticities satisfy the following conditions*:

(*i*) *The sign of κ_v^t depends on the elasticity of substitution between labor and capital*: $\kappa_{\bar{v}}^t \gtreqless 0$ *if and only if* $\varepsilon_{\bar{v}} \lesseqgtr \alpha$, *and* $\kappa_{\underline{v}}^t \lesseqgtr 0$ *if and only if* $\varepsilon_{\underline{v}} \lesseqgtr \alpha$.

(*ii*) $\phi_H^t < 0$ *and* $\phi_L^t > 0$.

(*iii*) $\varphi_{\bar{v}}^t > 0$ *and* $\varphi_{\underline{v}}^t < 0$.

Proof. See Appendix A.4. □

The economic implications of Proposition 3 can be summarized as follows.

(i) When $\varepsilon_v < \alpha$, capital and labor are complements in a task (or sector). In this case, an increase in capital raises wages within the corresponding sector. Specifically, in the top sector $\bar{v}$, additional capital raises the wage of high-talent workers, thereby widening wage inequality. Conversely, in the bottom sector $\underline{v}$, more capital increases the wage of low-talent workers, thus reducing wage inequality. As a result, $\kappa_{\bar{v}}^t > 0$ and $\kappa_{\underline{v}}^t < 0$ as $\varepsilon_v < \alpha$. Alternatively, when $\varepsilon_v > \alpha$, capital and labor are substitutes. In this case, an increase in capital reduces labor productivity and wages. In the top sector $\bar{v}$, capital accumulation decreases the wage of high-talent workers, narrowing inequality. In the bottom sector $\underline{v}$, additional capital decreases wages of low-talent workers, widening inequality. Thus, $\kappa_{\bar{v}}^t < 0$ and $\kappa_{\underline{v}}^t > 0$ as $\varepsilon_v > \alpha$. We highlight the scenario where $\varepsilon_v < \alpha$ in (11a), which reflects complementarity between labor and capital. In this case, more capital in the top sector raises the wage of high-skilled workers $w_t(H)$, thereby increasing wage inequality, while more capital in the bottom sector raises the wage of low-skilled workers $w_t(L)$, reducing inequality. These effects warrant a positive marginal capital tax rate in the top sector and a negative rate in the bottom sector to help compress wage disparities.

(ii) With respect to labor effort, an increase in high-talent labor effort reduces the wage of high-talent workers, while an increase in low-talent labor effort similarly lowers the wage of low-talent workers. These dynamics affect wage inequality in opposite directions: increased high-talent effort compresses wage inequality, whereas increased low-talent effort amplifies it. Formally, $\phi_H^t < 0$ and $\phi_L^t > 0$.

(iii) The impact of R&D on wage inequality depends on the task sector in which it is directed. R&D investment enhances the technology associated with a given task, thereby increasing labor productivity. Since high-talent workers hold a comparative advantage in the top task, R&D targeted to that sector disproportionately boosts their productivity and wages, leading to greater wage inequality ($\varphi_{\bar{v}}^t > 0$). In contrast, low-talent workers have a comparative advantage in the bottom task. R&D in that sector disproportionately benefits them, raising their productivity and wages, and thus reducing wage inequality ($\varphi_{\underline{v}}^t < 0$).

In Online Supplement Appendix B.1 of Chen and Liang (2026), we present the social planner's problem for the simplified two-talent, two-task model. Proposition A1 derives the capital, labor, corporate, and R&D wedges within this framework, serving as a condensed counterpart to Proposition 2. The signs of these four wedges are summarized in the fifth and sixth rows of Table 1, allowing for direct comparison with results from the existing literature.

First, in the standard Mirrlees model, where labor is perfectly substitutable, only the Mirrlees term $\mathcal{N}_t$ appears. The labor wedge for low-type agents is positive ($\tau^{l_t}(L) > 0$), while the labor wedge for high-type agents is zero ($\tau^{l_t}(H) = 0$). Moreover, with nonstochastic talent types (as in our model), there is no need for insurance against future income uncertainty.[21] Capital taxes are zero for agents in both task sectors ($\tau_i^{k_t}(\bar{v}) = 0$ and $\tau_i^{k_t}(\underline{v}) = 0$). Furthermore, the standard model does not include endogenous R&D, so R&D subsidies and corporate tax are not applicable.

Table 1 provides a comprehensive summary of the wedge structures across different models, highlighting the novel features introduced in our approach.[22]

Table 1. The wedges in the two-type, two-sector model.

Wedges	$\tau_i^{l_t}(H)$	$\tau_i^{l_t}(L)$	$\tau_i^{k_{t_i}}(\bar{v})$	$\tau_i^{k_{t_i}}(\underline{v})$	$s^{q_t}(\bar{v})$	$s^{q_t}(\underline{v})$	$\tau_i^{n_t}(\bar{v})$	$\tau_i^{n_t}(\underline{v})$
Mirrlees ($\mathcal{N}_t$)	0	+	0	0	NA	NA	NA	NA
Ales, Kurnaz, and Sleet (2015) ($\mathcal{N}_t + \mathcal{W}_t^x$)	−	+	NA	NA	NA	NA	NA	NA
Slavík and Yazici (2014) ($\mathcal{N}_t + \mathcal{W}_t^x$)	−	+	+/0	+/0	NA	NA	NA	NA
Akcigit, Hanley, and Stantcheva (2022) ($\mathcal{M}_t^x$)	NA	NA	NA	NA	+	+	−	+/−
Our model (no IPR) ($\mathcal{N}_t + \mathcal{W}_t^x$)	−	+	+	−	−	+	+	−
Our model ($\mathcal{N}_t + \mathcal{W}_t^x + \mathcal{M}_t^x$)	↓	↓	↓	↓	↑	↑	↓	↓

Note: NA means not available. The effects in our model are compared with our model (no IPR), for which the effects on $\tau_i^{kt_i}(\bar{v})$ and $\tau_i^{kt_i}(\underline{v})$ are under the condition $\varepsilon_{\bar{v}} < \alpha$ and $\varepsilon_{\underline{v}} < \alpha$. Slavík and Yazici (2014) divide capital into equipment capital and structure capital, and their capital wedge for equipment is positive, while the capital wedge for structure is zero.

[21]Online Supplement Appendix B.2 of Chen and Liang (2026) examines a setting in where agents' types evolve over time. Proposition A2 in this Appendix shows that, although an additional insurance term appears in the capital wedge, all the effects identified in Proposition A1 continue to hold in this time-varying context.

[22]The "no IPR" version of our model in Table 1 is identical to the baseline model except that it excludes IPR policy. As a result, intermediate goods firms operate under perfect competition.

When labor from different talent types is imperfectly substitutable, a wage compression effect $\mathcal{W}_t$ emerges, as noted by Ales, Kurnaz, and Sleet (2015) and Slavík and Yazici (2014). This effect is summarized in the second and third rows of Table 1. In these models, wage compression increases the positive labor wedge for low-talent agents ($\tau^{l_t}(L) > 0$), while decreasing it for high-talent agents, resulting in a negative labor wedge ($\tau^{l_t}(H) < 0$).

Both Slavík and Yazici (2014) and our model incorporate a capital wedge, in contrast to the zero capital wedge in the standard Mirrlees setting. In Slavík and Yazici (2014), wage compression results in a positive capital wedge for equipment capital, due to its complementarity with skilled labor, and a zero capital wedge for structure capital. Conversely, in our model, the capital-labor complementarity varies across sectors: capital in the top sector is more complementary to high-talent agents, while capital in the bottom sector is more complementary to low-talent agents. As a result, the wage compression effect $\mathcal{W}_t$ generates a positive capital wedge in the top sector ($\tau_i^{k_t}(\overline{v}) > 0$) and a negative capital wedge in the bottom sector ($\tau_i^{k_t}(\underline{v}) < 0$). Intuitively, with relative capital-skill complementarity, increasing capital in the top sector raises high talents' wage, and thus increases inequality, whereas increasing capital in the bottom sector raises low talents' wage and reduces inequality. Therefore, it is optimal to increase the capital wedge in the top sector and decrease it in the bottom sector.

Moreover, our model incorporates R&D investments and inputs. In the absence of IPR, the wage compression term $-\mathcal{W}_{s,t}^q$ leads to a negative R&D wedge in the top sector and a positive R&D wedge in the bottom sector ($s^{q_t}(\overline{v}) < 0$, $s^{q_t}(\underline{v}) > 0$). Simultaneously, the wage compression term $\mathcal{W}_{s,t}^n$ results in a positive corporate wedge in the top sector and a negative corporate wedge in the bottom sector ($\tau^{n_t}(\overline{v}) > 0$, $\tau^{n_t}(\underline{v}) < 0$). These results are summarized in Table 1 under our model without IPR policy.

Patent protection is granted to technology monopolists in our model, which gives rise to the Pigouvian term $\mathcal{M}_t$ in Table 1. This term reflects the need to correct market distortions by encouraging intermediate goods producers to hire more labor, use more capital, and invest further in R&D. Specifically, the Pigouvian term reduces capital wedges $\tau_i^{k_t}(v)$ (via term $\mathcal{M}_t^k$) and corporate wedges $\tau^{n_t}(v)$ (via term $\mathcal{M}_{s,t}^n$) in both the top and the bottom sectors, and decreases labor wedges $\tau^{l_t}(i)$ (via term $\mathcal{M}_t^l$) for both low- and high-talent workers. Moreover, via term $\mathcal{M}_{s,t}^q$, it increases subsidies to R&D investments. As a result, a previously negative R&D wedge in the top sector under no IPR protection ($s^{q_t}(\overline{v}) < 0$) may become less negative or even turn positive. Meanwhile, a previously positive R&D wedge in the bottom sector ($s^{q_t}(\underline{v}) > 0$) is further amplified, remaining positive.

As the Akcigit, Hanley, and Stantcheva (2022) model also incorporates the Pigouvian effect, Table 1 also compares their R&D and corporate wedges with ours. In their framework, R&D wedges (or subsidies to R&D investments) are positive for both low- and high-productivity sectors ($s^{q_t}(\overline{v}) > 0$, $s^{q_t}(\underline{v}) > 0$). In contrast, our model also yields positive a R&D wedge in the bottom sector ($s^{q_t}(\underline{v}) > 0$), but due to the wage compression effect, the wedge in the top sector may be negative. Furthermore, in Akcigit, Hanley, and Stantcheva (2022), corporate wedges (or taxes on firms' profits) are negative for high-productivity firms ($\tau^{n_t}(\overline{v}) < 0$), while they are ambiguous for low-productivity firms ($\tau^{n_t}(\underline{v}) \gtrless 0$). The ambiguity stems from the fact that firm types are private information, introducing incentive constraints that push taxes up, while Pigouvian motives push them down. In contrast, our model assumes firm types are publicly observable, eliminating incentive concerns. As a result, corporate taxes are unambiguously negative for low-productivity firms.

Notably, a negative capital wedge arises only in our model, a result not found in the existing literature summarized in Table 1. This unique feature is a consequence of our model's structure, where different types of labor are complementary to different types of capital. To elucidate the mechanisms driving this result, we next compare our final goods production function (11a) with that in Krusell, Ohanian, Ríos-Rull, and Violante (2000). Their model incorporates two types of labor and two types of capital, and can be interpreted within the general framework used by Slavík and Yazici (2014).

Comparison with the production technology in Krusell, Ohanian, Ríos-Rull, and Violante (2000) Krusell, Ohanian, Ríos-Rull, and Violante (2000) develop a production function that features two types of labor and two types of capital as follows:

$$\mathcal{F}(k_s, k_e, L_s, L_u) = k_s^{\alpha_*}\left\{\mu_* L_u^{\sigma_*} + (1-\mu_*)\left[(1-\lambda_*)L_s^{\rho_*} + \lambda_* k_e^{\rho_*}\right]^{\frac{\sigma_*}{\rho_*}}\right\}^{\frac{1-\alpha_*}{\sigma_*}}, \tag{11b}$$

where $\rho_* < \sigma_* < 1$. Here, L_s and L_u denote skilled and unskilled labor, while k_s and k_e represent structure capital and equipment capital. The condition $\rho_* < \sigma_* < 1$ implies that skilled workers are more complementary to equipment capital than unskilled workers.[23]

In contrast, under the production technology in (11a), our model shows that capital taxation in sector v depends on the elasticity of substitution between labor and capital, given by $\sigma_v \equiv \frac{1}{1-\varepsilon_v}$. As the two types of capital used in the top and bottom sectors, $k_t(\bar{v})$ and $k_t(\underline{v})$, affect wage inequality differently, the resulting capital tax structure in our model deviates from those in Slavík and Yazici (2014) and Tsai, Yang, and Yu (2022). Figure 1 illustrates how wage inequality responds to capital inputs in each sector within our model and compares these effects with the roles of capital in Krusell, Ohanian, Ríos-Rull, and Violante (2000).

First, under the technology (11b) specified in Krusell, Ohanian, Ríos-Rull, and Violante (2000), the two types of labor L_s and L_u are both complementary to equipment capital k_e with degrees given by $1-\rho_*$ and $1-\sigma_*$, respectively. As illustrated in Figure 1(a), increases in equipment capital k_e raise the productivity of both labor types L_s and L_u. Moreover, condition $\sigma_* > \rho_*$ implies that skilled workers are more complementary to equipment capital than unskilled workers, implying that equipment capital k_e enhances the efficiency of skilled labor L_s more than unskilled labor L_u. Consequently, equipment capital k_e raises wage inequality, as captured by the ratio $w_s/w_u = \mathcal{F}_3/\mathcal{F}_4$.[24] Building on this framework, Slavík and Yazici (2014) and Tsai, Yang, and Yu (2022) concluded that to ease wage inequality, it is optimal to impose a positive tax on equipment

[23]To distinguish between the parameters used in our production technology and those in Krusell, Ohanian, Ríos-Rull, and Violante (2000), we denote the parameters from Krusell, Ohanian, Ríos-Rull, and Violante (2000) with an asterisk subscript.

[24]$\mathcal{F}_i$ denotes the derivative of $\mathcal{F}$ with respect to the ith argument. A similar notation is used for F_i below.

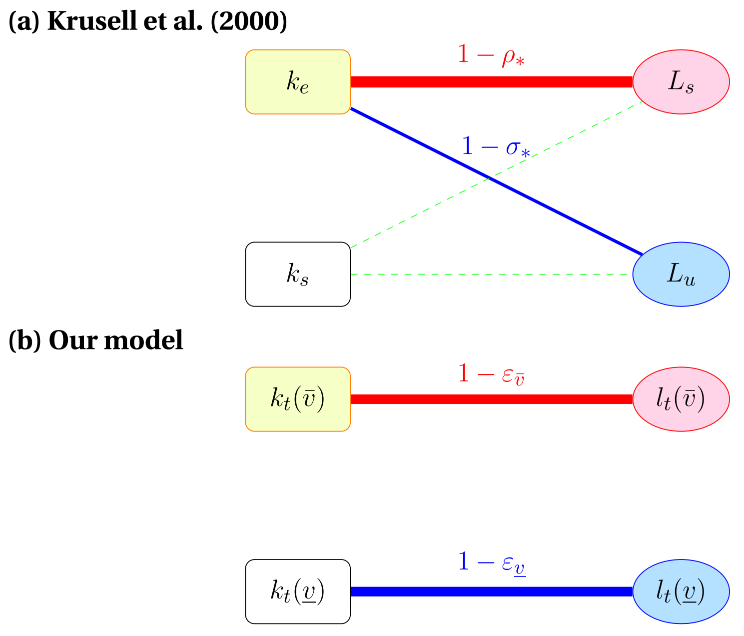

Figure 1. Capital and wage inequality in (a) Krusell, Ohanian, Ríos-Rull, and Violante (2000) and (b) our models. **Note:** 1. In the Krusell, Ohanian, Ríos-Rull, and Violante (2000) model, increases in equipment capital k_e raise the wage rates of both skilled and the unskilled workers, w_s and w_u. When $1-\rho_* > 1-\sigma_*$, increases in equipment capital k_e enlarge wage inequality $\frac{w_s}{w_u}$. Structural capital k_s does not affect wage inequality. 2. In our model, increases in capital $k_t(\bar{v})$ in the top task raise the wage rate for skilled workers $w_t(H)$, thereby widening wage inequality $\frac{w_t(H)}{w_t(L)}$. Conversely, increases in capital $k_t(\underline{v})$ in the bottom task raise the wage rate for unskilled workers $w_t(L)$, thereby reducing wage inequality $\frac{w_t(H)}{w_t(L)}$.

capital. Additionally, as the wage ratio $\mathcal{F}_3/\mathcal{F}_4$ is unaffected by structure capital k_s, the optimal tax on structure capital is zero.

In contrast, our model adopts a different technology (11a), in which labor and capital are complementary when $\varepsilon_v < \alpha$, and relative capital-skill complementarities vary across sectors. The complementarity between high-talent labor $L_t(\bar{v})$ and top-sector capital $k_t(\bar{v})$ is characterized by $1-\varepsilon_{\bar{v}}$, while that between low-talent labor $L_t(\underline{v})$ and bottom-sector capital $k_t(\underline{v})$ is given by $1-\varepsilon_{\underline{v}}$. Thus, capital in both sectors affects wage inequality. As shown in Figure 1(b), capital in the top sector increases wage inequality but capital in the bottom sector decreases it. When $\varepsilon_v < \alpha$, it is therefore optimal to impose a positive capital tax in the top sector and a negative one in the bottom sector in order to reduce wage inequality. This diverges from the positive capital equipment tax policy in Slavík and Yazici (2014) and Tsai, Yang, and Yu (2022), which assume fixed capital-skill complementarity.[25]

[25]Please refer to the third and fourth rows of Table 1. While Slavík and Yazici (2014) used an implicit final goods technology, they incorporated the equipment capital-skill complementarity framework from Krusell, Ohanian, Ríos-Rull, and Violante (2000).

Finally, we emphasize that R&D investments, combined with relative capital-skill complementarities, lead to a polarized allocation of capital across sectors, giving rise to a U-shaped pattern in wage growth. Consequently, the optimal capital wedge formulas across tasks, as expressed equation (10a), rely on sufficient statistics involving κ_v^t, which captures the influence of relative capital-skill complementarities across tasks. In the next section, we quantitatively assess our model to simulate the dynamics of technology evolution, job polarization, and wage growth. We also evaluate the design of optimal tax policies to mitigate wage inequality.

4. The model at work: Quantitative analysis

This section provides quantitative support for our theoretical model by calibrating it and simulating the patterns of job polarization and the optimal tax and R&D policy designs. We explore how the relative elasticity of substitution between labor and capital across sectors contributes to the job-polarization phenomenon and derive the corresponding optimal tax policy. Our quantitative results are based on a production technology with three key features: (i) imperfect substitutability of labor types, (ii) relative capital-skill complementarities between sectors, and (iii) endogenous technological progress with IPR protection. These features enable us to analyze the dynamics of job polarization and simulate labor taxes for different talent levels, as well as capital taxes, R&D subsidies, and corporate profit taxes across tasks affected by job polarization. We begin by specifying the functional forms for preferences and technology. Then we construct a decentralized economy that reflects the current US tax system, which serves as our status quo benchmark model. Following Stantcheva (2017), we set the linear labor income tax rate at $dT_t^L = 0.13$, and the linear capital tax rate at $dT_t^K = 0.25$. In line with Akcigit, Hanley, and Stantcheva (2022), we assume a linear corporate tax rate of $dT_t^c = 0.23$ and a linear R&D subsidy rate of $dS_t^q = 0.19$. Using this benchmark economy, we calibrate five key parameters: a shape parameter governing the agent's type distribution, a parameter capturing agents' comparative advantage, a scale parameter, and two cost parameters related to R&D technology. Finally, we solve the social planner's problem and simulate both the pattern of job polarization and the corresponding optimal tax policy.

4.1 *Calibration*

We assume that agents' talents i are drawn from a log-Pareto distribution, $\log(i) \sim P(x_m, \kappa)$, where x_m is the scale parameter and κ is the shape parameter. For normalization, we set $x_m = 1$, while the shape parameter κ is calibrated to match the targets. Using this distribution, we simulate a population of 10,000 agents. Following Ales, Kurnaz, and Sleet (2015), we parameterize the agents' productivity as $\hat{a}(i, v) \equiv \log(a(i, v))$, with the marginal effect of talent on productivity given by $\frac{\partial \hat{a}(i,v)}{\partial i} = \theta_1 + \theta_2 v$. Here, θ_1 captures pure talent and θ_2 represents comparative advantage in task v. We normalize pure talent to $\theta_1 = 1$, while the comparative advantage parameter θ_2 is calibrated to match empirical targets.[26] The shape parameter $\kappa = 4.5$ and the comparative advantage

[26] Following Ales, Kurnaz, and Sleet (2015), we model the technology using two parameters $a(i, v)$ and $b(v)$: $a(i, v)$ for the marginal productivity across different talent levels and $b(v)$ for factor-augmenting

Table 2. Parameter values.

Definition	Symbol	Value	Source/Note
Parameter values exogenously set			
Scale parameter	x_m	1	Normalization
Pure talents	θ_1	1	Normalization
Disutility elasticity	γ	4/3	Ales, Kurnaz, and Sleet (2015)
Discount factor	β	0.72	4% annual discount rate
R&D investment cost elasticity	ζ_q	1.5	Akcigit, Hanley, and Stantcheva (2022)
R&D input cost elasticity	ζ_n	0.86	Akcigit, Hanley, and Stantcheva (2022)
Degree of monopoly power	α	0.85	Akcigit, Hanley, and Stantcheva (2022)
Capital share	ρ	1/3	Data
Capital depreciation rate	δ_k	0.65	Slavík and Yazici (2014)
Technology depreciation rate	δ_b	0.57	Akcigit, Hanley, and Stantcheva (2022)
Substitution between labor and capital	ε_v	[0.401, −0.493]	Krusell, Ohanian, Ríos-Rull, and Violante (2000)
Parameter values endogenously calibrated			
Shape parameter	κ	4.5	
Comparative advantage	θ_2	2.1	
Scale of R&D level	B	0.19	
Scale of R&D investment cost	μ_q	0.06	
Scale of R&D input cost	μ_n	0.9	

$\theta_2 = 2.1$ are jointly calibrated to replicate two key empirical moments: an income Gini coefficient of 0.56 and a variance of log income of 0.94. The calibrated parameter values are summarized in Table 2, and the corresponding target moments are reported in Table 3.

Table 3. Target statistics and model counterparts.

Moment	Model	Target	Source
M1. Income Gini	0.579	0.56	Kuhn and Ríos-Rull (2016, Table 5)
M2. Variance of log income	0.936	0.94	Kuhn and Ríos-Rull (2016, Table 5)
M3. Mean of technology level	0.32	0.32	Ales, Kurnaz, and Sleet (2015)
M4. R&D-Sale ratio	0.04	0.041	Akcigit, Hanley, and Stantcheva (2022)
M5. Wage premium	1.89	1.8	Heathcote, Perri, and Violante (2010)

Note: The mean technology level of 0.32 is taken from the data used in Ales, Kurnaz, and Sleet (2015).

progress across sectors. However, with inclusion of capital in our model, labor returns differ from those in Ales, Kurnaz, and Sleet (2015). To account for this, we calibrate agent's productivity θ_2 to match the income Gini (i.e., the variance of income). Factor-augmenting progress $b(v)$ is then endogenously driven by R&D, and the scale of R&D and its costs are calibrated to align with the R&D-sales ratio, and average technology progress, and the wage premium.

While our model builds on the framework developed by Ales, Kurnaz, and Sleet (2015), it introduces three new features. First, the production function incorporates labor, capital, and the technology level, allowing for a more dynamic representation of economic activity. To capture the evolution of capital over time, we extend the static model of Ales, Kurnaz, and Sleet (2015) into a dynamic setting with a five-period horizon ($T = 5$) for our quantitative analysis. Second, the technology level $b_t(v)$ is treated as endogenous, evolving in response to economic decisions. Finally, unlike in Ales, Kurnaz, and Sleet (2015), intermediate goods producers in our model possess monopoly powers and, therefore, have positive profits.

Agent's preference Following Ales, Kurnaz, and Sleet (2015), the agent's utility function is specified as

$$U(i) = \sum_{t=1}^{T} \beta^{t-1}\left[\log\bigl(c_t(i)\bigr) - \frac{\bigl(e_t(i)\bigr)^{1+\gamma}}{1+\gamma}\right].$$

We set the Frisch elasticity for labor supply to $1/\gamma = 0.75$, consistent with Ales, Kurnaz, and Sleet (2015). In this five-period model, each period represents 8 years, corresponding to an agent's working life from age 20 to 60. An annual discount rate of 4% corresponds to a discount factor of $\beta = 0.72$ over an 8-year period.[27]

R&D The technology level $b_t(v)$ evolves according to (3a). Following Akcigit, Hanley, and Stantcheva (2022), the technology level is parameterized by[28]

$$b_t(v) = (1-\delta_b)b_{t-1}(v) + Bq_t(v)n_t(v), \tag{12a}$$

and the costs of R&D investment $q_t(v)$ and R&D inputs $n_t(v)$ are parameterized by $\mathcal{C}_q(q_t(v)) = \frac{\mu_q(q_t(v))^{1+\zeta_q}}{1+\zeta_q}$ and $\mathcal{C}_n(n_t(v)) = \frac{\mu_n(n_t(v))^{1+\zeta_n}}{1+\zeta_n}$. Following Akcigit, Hanley, and Stantcheva (2022), the parameters of the R&D cost are set at $\zeta_q = 1.5$, $\zeta_n = 0.86$, and the depreciation rate $\delta_b = 0.57$.[29] The scale parameters for R&D $B = 0.19$, the R&D investment costs $\mu_q = 0.06$ and the R&D input costs $\mu_n = 0.9$ are jointly calibrated to match three empirical targets: the average technology level 0.32, the R&D-to-sales ratio 0.041, and the wage premium 1.8. Following Stantcheva (2017), the wage premium is defined as the ratio of the average wage of the top 42.7% workers to that of the bottom 42.7%.

[27] With an annual discount rate of 4%, the discount factor over 8 years is $\beta = 1/(1/0.96)^8 = 0.72$.

[28] According to Akcigit, Hanley, and Stantcheva (2022), $b_t(v) = (1-\delta_b)b_{t-1}(v) + (\frac{1}{2}q_t(v)^{1-\varrho_{qn}} + \frac{1}{2}n_t(v)^{1-\varrho_{qn}})^{2/(1-\varrho_{qn})}$ is more general than (12a), which reduces to (12a) if $\varrho_{qn} \to 1$. Akcigit, Hanley, and Stantcheva (2022) note that, given the data, it is difficult to empirically discipline ϱ_{qn}, so $\varrho_{qn} = 1$ is specified for tractability, which we follow in (12a). In Online Supplement Appendix B.3 of Chen and Liang (2026), we also compute the wedges when ϱ_{qn} is different from unity by setting $\varrho_{qn} = 0.8$ and $\varrho_{qn} = 1.2$. We find that the wedges are almost the same as those under (12a). See Figure A1 in Online Supplement Appendix B.3 of Chen and Liang (2026).

[29] According to Akcigit, Hanley, and Stantcheva (2022), the annual depreciation rate of innovation is 0.1. Since our baseline model consists of five periods, with each period corresponding to 8 years, the adjusted depreciation rate is calculated as $\delta_b = 1-(1-0.1)^8 = 0.57$.

Goods production Based on Assumptions 1 and 2, the production technologies for final goods and intermediate goods are, respectively, given by

$$Y_t = \int_{\underline{v}}^{\bar{v}} b_t(v)^{1-\alpha} y_t(v)^{\alpha}\, dv, \tag{12b}$$

$$y_t(v) = \left\{(1-\rho)\left[l_t(v)\right]^{\varepsilon_v} + \rho\left[k_t(v)\right]^{\varepsilon_v}\right\}^{\frac{1}{\varepsilon_v}}. \tag{12c}$$

In (12b), $b_t(v)$ and $y_t(v)$ are combined using a Cobb–Douglas function. While Ales, Kurnaz, and Sleet (2015) treated $b_t(v)$ as exogenous and used data to estimate the parameter, we consider $b_t(v)$ to be endogenous, with its evolution determined by R&D investments and R&D inputs as specified in (12a).[30]

The task space $[\underline{v}, \bar{v}]$ is normalized to the interval $[0, 1]$. The capital share is set at $\rho = 1/3$, consistent with standard macroeconomic calibration. We set $\alpha = 0.85$, in line with the mark-up rate of the intermediate goods producers, as used in Akcigit, Hanley, and Stantcheva (2022). The elasticity of substitution between capital and labor is $\sigma_v \equiv \frac{1}{1-\varepsilon_v}$. Existing studies (e.g., Krusell, Ohanian, Ríos-Rull, and Violante (2000) and Herrendorf, Herrington, and Valentinyi (2015)) document relative capital-skill complementarities across sectors. These findings suggest that capital-labor elasticity is higher in less complex, routine tasks and lower in more complex, manual tasks. Accordingly, the elasticity σ_v is modeled as decreasing over the task space, reflecting increasing task complexity across $v \in [0, 1]$. Based on Krusell, Ohanian, Ríos-Rull, and Violante (2000), we set σ_v to decrease from 1.67 to 0.67 across $v \in [0, 1]$. Given the relationship $\sigma_v = \frac{1}{1-\varepsilon_v}$, this implies that the value of ε_v decreases from 0.401 to -0.493 over the task space.[31]

The law of motion for capital is $k_t(v) = (1-\delta_k)k_{t-1}(v) + I_t(v)$, where the depreciation rate is set at $\delta_k = 0.65$ for 8 years, following the estimate for annual equipment capital depreciation 0.124 in Slavík and Yazici (2014).

Price functions With monopoly power, an intermediate goods producer sets prices above marginal cost, and pays wages and interest rates below their marginal products.

[30] In Online Supplement Appendix B.4 of Chen and Liang (2026), we show that our tax wedges are robust when considering a more general setting that incorporates the production function for final goods $Y_t = [\int_{\underline{v}}^{\bar{v}} [b_t(v)^{1-\alpha} y_t(v)^{\alpha}]^{\varepsilon_f}\, dv]^{1/\varepsilon_f}$, with varying degrees of the elasticity of substitution intermediate goods in different sectors, $1/(1-\varepsilon_f)$.

[31] While Krusell, Ohanian, Ríos-Rull, and Violante (2000) analyzes capital-skill complementarities by distinguishing between skilled and unskilled labor, their framework can be naturally extended to capture task-level heterogeneity. An expanding body of empirical literature documents the presence of capital-task complementarities across different types of tasks. Autor, Levy, and Murnane (2003) and Autor, Katz, and Kearney (2006) developed a task-based model of capital, categorizing tasks into abstract, routine, and manual (or nonroutine manual) types. Their findings indicate that capital, particularly in the form of computer technologies, tends to substitute for routine tasks while complementing abstract tasks, thereby boosting productivity in the latter. The relationship between capital and manual tasks is less definitive, partly due to labor reallocation dynamics; however, the overall evidence suggests that capital exhibits strong complementarities with abstract tasks but weaker, or even substitutive, effects with routine tasks. More recently, Haslberger (2022) refined the measurement of tasks and confirmed these patterns. His work shows that abstract tasks continue to display strong complementarities with capital, routine tasks experience declining returns as automation progresses, and manual tasks remain relatively unaffected by technological change.

With the production technology for final goods in (12b), the demand function faced by the intermediate goods producer in sector v in (2a) gives the following price for goods:

$$p_t(y_t(v)) = \alpha b_t(v)^{1-\alpha} y_t(v)^{\alpha-1}. \tag{13a}$$

Furthermore, the factor demand functions of the intermediate goods producer in sector v, as given in (2c)–(2d), yields the following wage and interest rates:

$$\begin{aligned}
\omega_t(v) &= \alpha \cdot \mathrm{MPL}_v = \alpha^2(1-\rho)b_t(v)^{1-\alpha} \\
&\quad \times \left\{(1-\rho)\left[l_t(v)\right]^{\varepsilon_v} + \rho\left[k_t(v)\right]^{\varepsilon_v}\right\}^{\frac{\alpha-\varepsilon_v}{\varepsilon_v}}\left[l_t(v)\right]^{\varepsilon_v-1}, \qquad (13b)\\
R_t(v) &= \alpha \cdot \mathrm{MPK}_v = \alpha^2 \rho b_t(v)^{1-\alpha} \\
&\quad \times \left\{(1-\rho)\left[l_t(v)\right]^{\varepsilon_v} + \rho\left[k_t(v)\right]^{\varepsilon_v}\right\}^{\frac{\alpha-\varepsilon_v}{\varepsilon_v}}\left[k_t(v)\right]^{\varepsilon_v-1}, \qquad (13c)
\end{aligned}$$

where $\mathrm{MPL}_v \equiv \frac{\partial Y_t}{l_t(v)} = \frac{\alpha(1-\rho)b_t(v)^{1-\alpha}[y_t(v)]^{\alpha-\varepsilon_v}}{[l_t(v)]^{1-\varepsilon_v}}$ and $\mathrm{MPK}_v \equiv \frac{\partial Y_t}{k_t(v)} = \frac{\alpha\rho b_t(v)^{1-\alpha}[y_t(v)]^{\alpha-\varepsilon_v}}{[k_t(v)]^{1-\varepsilon_v}}$.

Thus, the intermediate goods producer in sector v pays a fraction α of the marginal product of labor (MPL) for the wage and the marginal product of capital (MPK) as the interest rate. As a result, the intermediate goods producer generates positive profits. Substituting the price, wage, and interest rates from (13a)–(13c) into (2b) shows that the profit is positive, expressed as follows:

$$\pi(b_t(v)) = \alpha(1-\alpha)b_t(v)^{1-\alpha}\left[y_t(v)\right]^{\alpha} > 0. \tag{14a}$$

Moreover, using (2b) and (13a), the marginal revenue is $MR(y_t(v)) = \alpha p_t(y_t(v))$. The amount of intermediate goods $y_t(v)$ is determined by the condition $MC(y_t(v)) = MR(y_t(v))$, which gives

$$\frac{p_t(y_t(v))}{MC(y_t(v))} = \frac{1}{\alpha} > 1, \tag{14b}$$

indicating a gross markup pricing strategy. The markup $1/\alpha$ is the reciprocal of the curvature in the final goods production function with respect to intermediate inputs (cf. (12b)). This markup over marginal cost is consistent with models of monopolistic competition (see, e.g., Chugh (2015, Chapter 21)).

4.2 *Empirical evidence of U-shaped wage growth and technology change*

In this subsection, we compare the evolution of wages and technology in our status quo model with existing empirical findings. The polarization of wage in the United States over the past few decades has been extensively documented. Acemoglu and Autor (2011) highlighted a U-shaped wage growth in 1988–2008, indicating a decline in middle-skill occupations while wages and employment increasingly concentrated at both the low-skill and high-skill ends. Autor and Dorn (2013) also observed a similar U-shaped wage growth trend from 1980–2005.

Ales, Kurnaz, and Sleet (2015) addressed occupational and wage polarization using an assignment model that endogenizes the relationship between workers' talents and

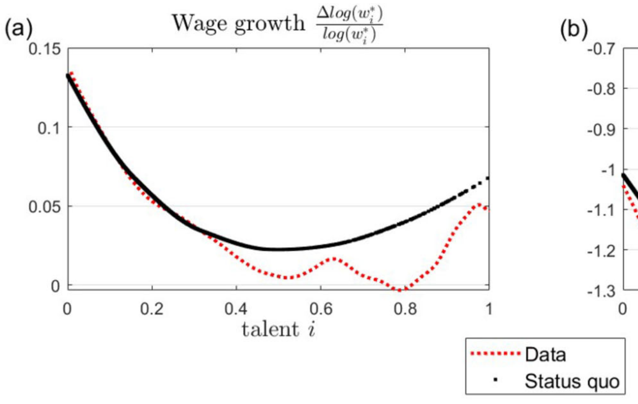

FIGURE 2. (a) Evolution of wage growth, and (b) evolution of technology level. **Data source:** Ales, Kurnaz, and Sleet (2015).

their occupations. They estimate exogenously given technical change parameters across different tasks and find a U-shaped technological progress, with mid-level talent workers experiencing the slowest wage growth during the 2000s.

In contrast to the exogenous technical changes considered by Ales, Kurnaz, and Sleet (2015), our model endogenizes technological progress and incorporates capital and relative capital-skill complementarities across sectors, leading to polarization patterns in employment. We calibrate our model using the capital-labor elasticity of substitution estimates from Krusell, Ohanian, Ríos-Rull, and Violante (2000), setting the parameter ε_v in (12c) to decrease with task complexity v.[32]

Using these calibrated parameters, we compare the evolution of wages by talent and technology levels by task in the status quo economy with the corresponding data used in Ales, Kurnaz, and Sleet (2015). As shown in Figure 2, our calibrated model closely mirrors the empirical U-shaped patterns of wage growth and technology progress.[33]

The U-shaped evolution of technology derived from our model aligns with the parametrically estimated evolution for (smoothed) log technology by job tasks in the 2000s from Ales, Kurnaz, and Sleet (2015). Specifically, it shows a slight increase for low-skill tasks, a large rise for high-skill tasks, and a decline for intermediate-skill tasks. Moreover, our theoretically derived wage growth polarization is consistent with the empirical values of (smoothed) log wage by talent types, calculated from the CPS data in Ales, Kurnaz, and Sleet (2015). From the 1980s to the 2000s, wages for low-talent workers caught up with those of mid-talent workers, while wages for mid-talent workers lagged behind those of high-talent workers.[34]

[32]We acknowledge the work by Aum, Lee, and Shin (2018), who estimated capital-skill complementarities across tasks in 10 sectors, with a particular focus on the computer capital-labor elasticity. While their estimates provide valuable insights into the role of the computer industry in polarization, we chose not to adopt their values, as our focus is on the broader impact of capital and R&D investments on polarization, rather than solely on computer capital.

[33]For Figures 2–6 in our 5-period model, we use the middle period $t = 3$, for illustration.

[34]The results in Figure 2 align with the findings of Aum (2018) and Aum, Lee, and Shin (2018). Aum's empirical analysis revealed that from 1980 to 2010, the US experienced faster wage growth for high-skilled cog-

4.3 *Reasoning of job polarization and the direction of technical change*

In this subsection, we highlight the role of relative capital-skill complementarities in job polarization.[35] To illustrate this, we simulate an alternative version of the model where the capital-labor elasticity of substitution is fixed across all tasks. For this simulation, we use an aggregate elasticity value of 0.84 as estimated by Herrendorf, Herrington, and Valentinyi (2015), and set $\varepsilon_v = -0.19$ for all tasks v.

Figure 3 presents the results of our simulations. The loci marked with "+" represent outcomes from the model with relative capital-skill complementarities, while those marked with "-" correspond to the model with a fixed capital-labor elasticity of substitution (referred to as aggregate elasticity). In the model with relative elasticities, we observe a distinct U-shaped wage growth patter across talents and a U-shaped technology distribution across tasks.

By contrast, the model with aggregate elasticity shows a monotone increase in both wage growth across talents and technology across tasks. The U-shaped technology evolution in the relative elasticity model indicates that technological progress is faster for low- and high-skill tasks, relative to middle-skill tasks, which mirrors the pattern of job polarization. Larger technological changes in Figure 3, seen in more complex tasks where higher-skilled talents are employed, point to a skill-biased direction of technical change, aligning with prior evidence (e.g., Acemoglu and Autor (2011)).

The polarization of wage growth and the salient pattern of job polarization observed in the US over recent decades can be explained when capital-skill complementarity varies across tasks. In contrast, using a fixed capital-labor elasticity results in a monotone increase in wage growth across talents and a steady rise in technology across tasks.

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nitive labor and low-skilled manual labor compared to middle-skilled routine labor (see their Figure 6.1). Additionally, the employment shares of both cognitive and manual workers increased (see their Table 5.1 and Figures 6.2 and 6.3). Furthermore, when R&D investments are proxied by computer usage, the polarization of R&D investments in low- and high-skilled tasks in Figure 2 mirrors the findings of Aum, Lee, and Shin (2018). Using data on computer investment from the 2010 NIPA, Aum, Lee, and Shin (2018) found that industries with more cognitive-task intensity (high-wage occupations) tend to invest more computer hardware and software, while industries with more manual-task intensity (low-wage occupations) primarily rely more on computer hardware compared to industries with routine-task intensity (middle-wage occupations) (see their Figure 2).

[35] There is a rich body of research examining the evolution capital skill complementarity over time. Early studies (e.g., Griliches (1969)) documented strong capital-skill complementarity in US manufacturing during the 1960s and 1970s and particularly equipment capital tended to complement skilled labor. Using US data from 1963 to 1992, Krusell, Ohanian, Ríos-Rull, and Violante (2000) found elasticity of substitution between equipment and skilled labor of around 1.67, indicating significant complementarity. Following this, research by Acemoglu (2002) and Autor, Goldin, and Katz (2020) suggests that technology became even more skill-biased during the 1990s, with new technologies designed to complement educated workers. This intensified capital skill complementarity and contributed to wage inequality. Strobel (2014) provided cross country and sectoral evidence showing that as ICT capital expanded during the 1990s and 2000s, its complementarity with skilled labor became more pronounced across industrialized economies. More recently, Ohanian, Orak, and Shen (2023) revisited Krusell, Ohanian, Ríos-Rull, and Violante (2000) with updated data. They found that capital-skill complementarity increased in high-tech sectors, especially those with high levels of ICT capital. They also demonstrated that capital deepening in ICT-intensive sectors led to a larger increase in the skill premium, a hallmark of high capital skill complementarity. Thus, capital skill complementarity has increased from the 1970s–1980s to the post-1990 period.

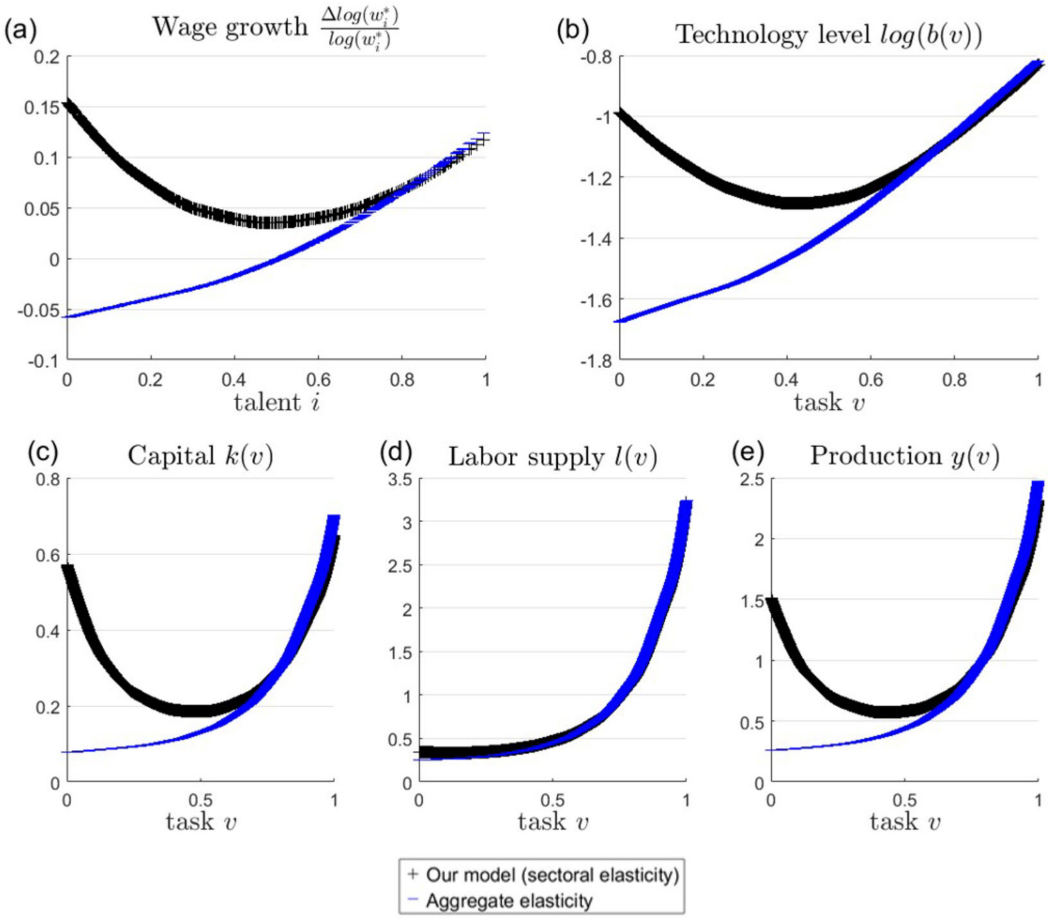

FIGURE 3. Allocation in models with and without relative capital-skill complementarity (a) Evolution of wage growth (b) evolution of technology level (c) capital (d) labor (e) output. **Note:** 1. Relative capital-skill complementarities between different sectors can well characterize the pattern of task polarization: U-shaped wage growth and U-shaped technical change. 2. The technology level $b(v)$ is endogenously accumulated by R&D investments $q(v)$ and R&D inputs $n(v)$ through equation (12a).

These findings suggest that the U-shaped wage growth and technology evolution can be justified in the model with relative capital-skill complementarities, but not in the model with fixed complementarity.[36]

To understand why our model with relative capital-skill complementarities leads to U-shaped wage growth and technological evolution, let us compare the allocation of capital and labor across tasks in both models. The results are reported in Figure 3. The

[36]To demonstrate that the U-shaped patterns of wage growth and technology evolution are robust to the assumption on talent distribution, Online Supplement Appendix B.5 of Chen and Liang (2026) considers different values of the shape parameter κ. The U-shaped patterns in both wage growth and technology levels remain present under these alternative values of κ.

allocation of capital differs substantially between the two models, while the allocation of labor remains almost identical. In the model with a fixed complementarity, capital increases monotonically across tasks. By contrast, in the model with relative capital-skill complementarities, capital is more heavily allocated to low- and high-skilled tasks compared to middle-skill tasks, resulting in a U-shaped pattern of capital allocation.[37]

Finally, let us explain why the model with relative capital-skill complementarities leads to the polarization of capital allocation. On the one hand, task-automating machines serve as substitutes for low-skilled labor in less complex routine tasks, prompting significant capital allocation to these tasks in order to automate and boost production. As shown in Figure 3(d), sectors with low task complexity employ a relatively small workforce. Even for the simplest tasks, where capital-skill complementarity is minimal, the increased substitutability between capital and low-skilled labor does not result in capital displacing workers. Rather, it suggests that augmenting capital inputs improves production efficiency more effectively than increasing labor inputs. This accounts for the higher capital investments in low task-complexity sectors compared to those in medium task-complexity sectors, as shown in Figure 3(c). With more capital directed toward simpler tasks than toward more complex middle-skill tasks, the marginal productivity of low-skilled workers increases more sharply than that of middle-skilled workers, leading to greater wage growth for low-skilled workers relative to middle-skilled workers.

On the other hand, output-augmenting machines complement nonroutine and manual occupations. As a result, a large portion of capital is allocated to more complex tasks to enhance the productivity of highly skilled workers. This leaves minimal capital allocation in the middle of the skill distribution, resulting in polarized capital allocation. This capital allocation pattern drives job polarization, which in turn causes output polarization. Output is higher in low- and high-skill tasks compared to middle-skill tasks, as illustrated in panel (e) of Figure 3. The output polarization then drives the polarization of wage growth, shown in panel (a) of Figure 3. Our analysis complements the empirical findings of Dixon, Hong, and Wu (2021), who examined data from Canadian businesses between 2000 and 2015. Their research found that investments in robotics are linked to employment growth for both low- and high-skilled workers, while reducing opportunities for middle-skilled workers. Our study complements this empirical research by demonstrating that both wage and technology polarization can be attributed to the task-specific capital-labor elasticity of substitution.

[37]Existing studies suggest that capital investments are generally higher in high-tech (high-skilled) and low-tech (low-skilled) sectors, with relatively lower investments in medium-tech (medium-skilled) sectors. For instance, Bertoni, Colombo, and Grilli (2011) show that high-tech sectors attract substantial capital investment due to their need for large upfront R&D and infrastructure investments and high potential returns. Low-tech sectors, such as textiles and food processing, also require significant capital for machinery and equipment despite low R&D intensity. Fontenele, Cabral, Forte, and Costa (2016) find that low-tech Brazilian firms devote about half of their innovation funds to equipment purchases, whereas high-tech firms focus more on R&D. By contrast, medium-tech sectors (e.g., automotive, machinery, chemicals) tend to receive relatively lower capital investment. Dolfsma and Leydesdorff (2008) discover that medium-tech manufacturing contributes to economic synergy but demands less capital intensity than high-tech (R&D-heavy) or low-tech (infrastructure-heavy) industries. See also Autor, Levy, and Murnane (2003) and OECD (2017).

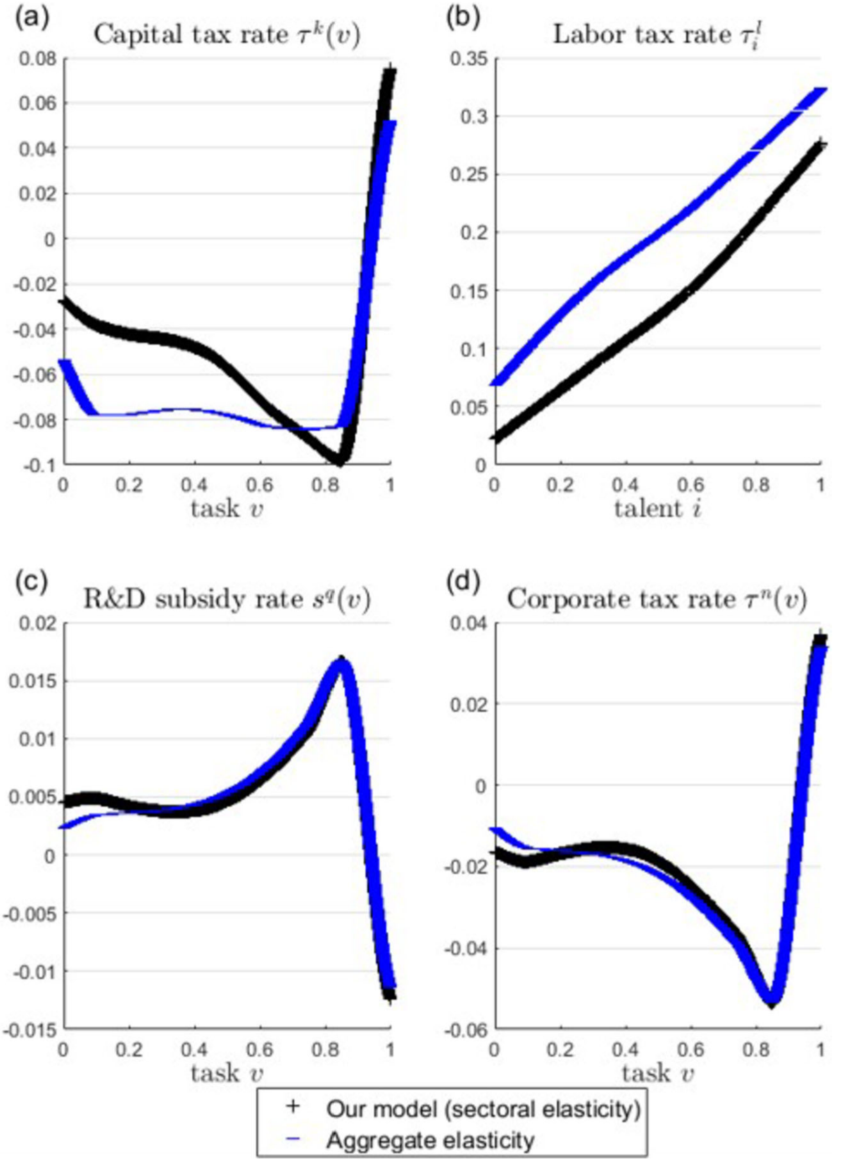

FIGURE 4. Optimal taxation and R&D policies in models with and without relative capital-skill complementarity.

4.4 *Results of optimal taxation and R&D policies*

We now quantify the optimal taxation and R&D policies by simulating the evolution of labor taxes based on talent levels, and capital taxes, R&D subsidies, and corporate taxes based on task complexity. Figure 4 illustrates the optimal tax structure in our model with relative capital-skill complementarities (marked with "+"). Key findings are as follows.

First, labor income taxes are positive and progressive.[38] Second, marginal capital income taxes are negative for low-complexity tasks and positive and progressive for high-complexity tasks. Moreover, R&D subsidy rates are positive for low-complexity tasks, negative for high-complexity tasks, and regressive with respect to task complexity. Finally, corporate tax rates are negative for low-complexity tasks, positive for high-complexity tasks, and progressive as task complexity increases. Our optimal R&D policies differ from those of Akcigit, Hanley, and Stantcheva (2022), who subsidized R&D investments for both low- and high-complexity firms while taxing only low-profit firms.

To highlight the role of relative capital-skill complementarities, Figure 4 also shows optimal taxes in a model with a fixed aggregate elasticity (marked with "-"). While labor income tax, R&D subsidy, and corporate tax rates are similar across both models, the capital income tax rate differs significantly for low-complexity tasks. The discrepancy reflects the divergent capital allocation patterns shown in Figure 3(c).

This divergence can be explained by Proposition 3. The capital-labor elasticity of substitution ε_v affects the cross-relative wage elasticity of capital, κ_v^t, but does not impact the elasticities of labor or technical progress, ϕ_i^t and φ_v^t. The cross-relative wage elasticity of capital, in turn, influences the capital wedge via the wage compression term. For high-complexity tasks, a low capital-labor elasticity of substitution implies that capital complements high-talent labor, driving up high-skilled wages and increasing inequality. This justifies a positive and progressive capital tax to mitigate rising wage inequality. For low-complexity tasks, increased capital raises low-skilled wages, thereby reducing wage inequality. As a result, capital income subsidies are warranted. As task complexity increases from very low levels, decreasing capital-labor elasticity of substitution means that capital becomes less substitutable for labor, necessitating larger subsidies to address the reduced wage inequality. This contrasts with the fixed-elasticity model, where capital subsidies remain relatively stable across tasks, except for very high-complexity tasks. In our model, capital taxes are optimized based on the relative capital-labor substitution elasticity. As seen in Figure 4(a), this leads to increasing subsidies up to a certain complexity threshold, after which taxing capital becomes optimal.

As outlined in Proposition 2, the optimal taxes and subsidies are shaped by three key factors: the Mirrlees term, the wage compression term, and the Pigouvian term. To illustrate the impact of each effect on the taxes and subsidies, Figure 5 breaks down each tax component according to these terms.

First, under the assumption of perfectly substitutable labor, the Mirrlees effect addresses the need to correct workers' incentives due to unobservable differences in labor types. The Mirrlees term only affects the labor income tax rate, leading to a positive tax rate for all talent types, as shown in Figure 5 (labeled with "*").

Next, with imperfectly substitutable labor, the wage increase for low-skilled workers compresses the relative wages of high-skilled workers. Consequently, the wage compression term calls for taxing low-skilled workers to ease the incentive constraints on

[38]Our simulation reveals a progressive labor income tax structure, which contrasts with the hump-shaped tax observed in Ales, Kurnaz, and Sleet (2015), due to the unbounded log-Pareto distribution of agent types in our model, as opposed to the bounded agent types in their model.

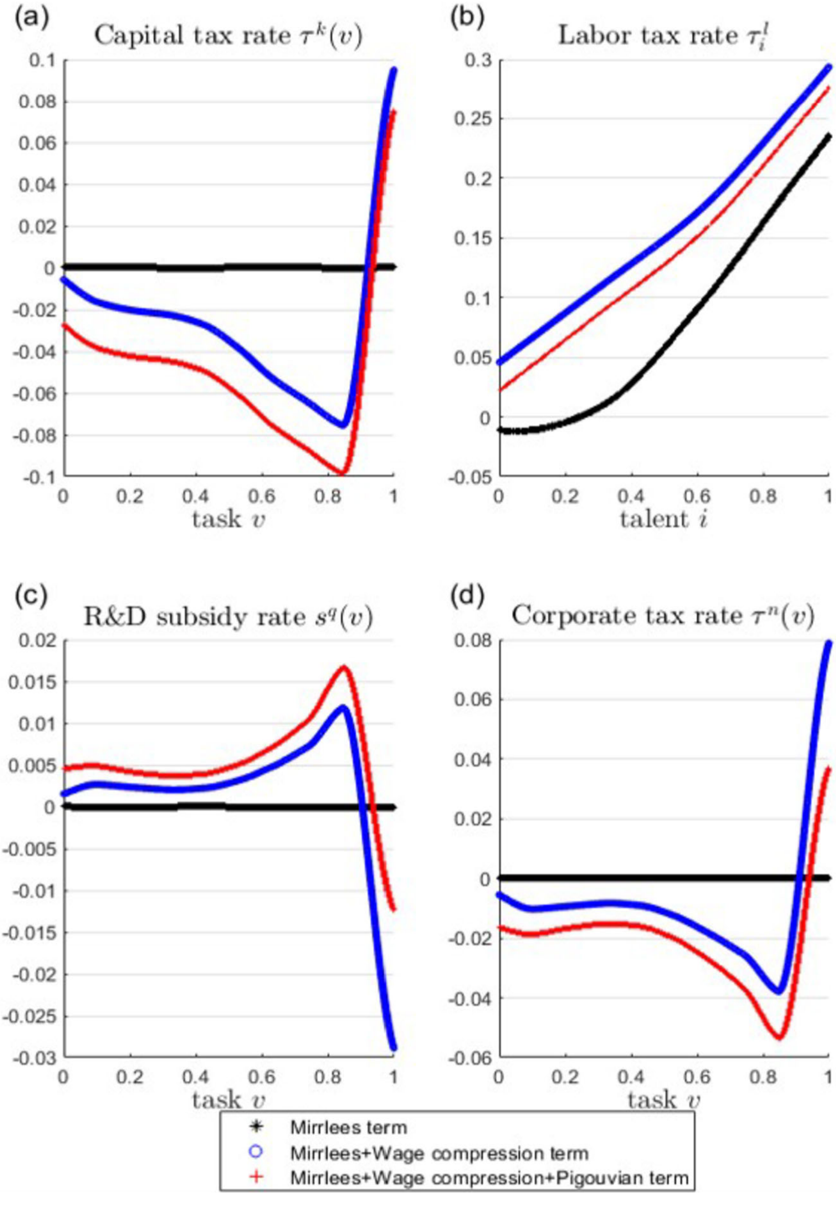

FIGURE 5. Decomposition of optimal taxes and R&D policies.

high-skilled workers. This is reflected in an increased marginal labor tax rate, as indicated by the loci marked with "o" in Figure 5. However, the wage compression effect influences capital taxes differently than labor taxes. As capital-skill complementarity increases with task complexity, the wage compression effect leads to subsidies for capital income in low-complexity tasks (where low-skilled workers are employed), while high-complexity tasks (which employ high-skilled workers) face progressively higher capital income to reduce wage inequality. Similarly, the wage compression effect calls for subsidizing R&D investments and inputs (or increasing the R&D wedge and decreasing the corporate wedge) in low-complexity tasks, while taxing both R&D investments and corporate profits in high-complexity tasks where high-skilled workers are employed.

Finally, the Pigouvian term corrects distortions caused by technology spillovers between intermediate and final goods producers in the private market. The Pigouvian effect lowers the labor tax rate, as indicated by the loci marked with "+" in Figure 5. This results in a marginal labor tax rate that is lower than the rate influenced by the Mirrlees and wage compression effects. Moreover, the Pigouvian effect leads to a decrease in the marginal tax rate for capital income, meaning the capital income tax rate for all tasks is lower than what would be derived solely from the wage compression effect. The Pigouvian effect also increases the marginal subsidy rate for R&D investments and decreases the marginal tax rate on corporate profits. Consequently, the marginal subsidy rate for R&D investments is higher, and our marginal tax rate on corporate profits is lower than the values influenced by the wage compression effect.

4.5 *Simpler tax instruments and welfare gains*

The nonlinear marginal tax rates and R&D subsidy rates presented in Section 4.4 are derived under unrestricted tax forms. While this yields the highest possible welfare, the resulting tax policies may be too complex for practical implementation in a decentralized economy.[39] In this subsection, we explore simpler, more implementable tax schemes by imposing functional restrictions and evaluating their welfare performance relative to our second-best benchmark.

To impose tractable restrictions, we draw on prior work. Saez (2001) and Sachs, Tsyvinski, and Werquin (2020) employ a variational approach based on small perturbations to tax rates, while Jonsson and Klein (1996) introduce a parametric nonlinear tax function of the form $T(x) = x - \lambda_0 x^{1-\tau_0}$, referred to as the JK tax. Heathcote, Storesletten, and Violante (2017) adopt this JK specification and show that the resulting welfare loss, relative to the fully optimal Mirrleesian tax schedule, is relatively modest. Along similar lines, Akcigit, Hanley, and Stantcheva (2022) compare their unrestricted mechanism to the JK tax form and find that nonlinearity in R&D subsidies yields greater welfare than nonlinearity in corporate taxation.

Building on these insights, we evaluate the welfare effects of applying both linear and nonlinear JK-form tax policies to various fiscal instruments' labor income, capital income, R&D subsidies, and corporate taxes relative to the unrestricted tax forms. By comparing each policy configuration to the status quo, we identify which forms of tax nonlinearity are most important for enhancing welfare. The results are reported in Table 4, expressed as equivalent gains in consumption.

Panel A presents welfare under current US policies, including a 13% linear labor income tax, a 25% linear capital income tax, a 23% linear corporate tax, and a 19% linear R&D subsidy. These status quo policies achieve 56.89% of the welfare level relative to

[39] In Online Supplement Appendix B.6 of Chen and Liang (2026), we construct a tax system comprising personal income taxes, corporate taxes, and R&D subsidies to implement our second-best solution in a decentralized economy.

TABLE 4. Welfare from simpler tax schemes.

Policy Type	Welfare Relative to the Second-Best	Welfare Gains Over the Status Quo
A. Current US policy (status quo policy) $dT_t^L = 0.13$, $dT_t^K = 0.25$, $dT_t^C = 0.23$, $dS_t^q = 0.19$	56.89%	0%
B. Optimal linear taxes $dT_t^L = \tau_{l0}$, $dT_t^K = \tau_{k0}$, $dT_t^C = \tau_{c0}$, $dS_t^q = \tau_{q0}$	69.7%	0.462%
C. JK tax on profit $dT_t^L = \tau_{l0}$, $dT_t^K = \tau_{k0}$, $T_t^C(\pi_t) = \pi_t - \tau_{c1}\pi_t^{1-\tau_{c2}}$, $dS_t^q = \tau_{q0}$	69.82%	0.468%
D. JK tax on R&D subsidy $dT_t^L = \tau_{l0}$, $dT_t^K = \tau_{k0}$, $dT_t^C = \tau_{c0}$, $S_t^q(q_t) = q_t - \tau_{q1}q_t^{1-\tau_{q2}}$	75.46%	0.735%
E. JK tax on profit and R&D subsidy $dT_t^L = \tau_{l0}$, $dT_t^K = \tau_{k0}$, $T_t^C(\pi_t) = \pi_t - \tau_{c1}\pi_t^{1-\tau_{c2}}$, $S_t^q(q_t) = q_t - \tau_{q1}q_t^{1-\tau_{q2}}$	75.6%	0.743%
F. JK tax on capital income $dT_t^L = \tau_{l0}$, $T_t^K(s_t) = s_t - \tau_{k1}s_t^{1-\tau_{k2}}$, $dT_t^C = \tau_{c0}$, $dS_t^q = \tau_{q0}$	76.3%	0.779%
G. JK tax on labor income $T_t^L(z_t) = z_t - \tau_{l1}z_t^{1-\tau_{l2}}$, $dT_t^K = \tau_{k0}$, $dT_t^C = \tau_{c0}$, $dS_t^q = \tau_{q0}$	80.92%	1.041%
H. JK tax on labor income and capital income $T_t^L(z_t) = z_t - \tau_{l1}z_t^{1-\tau_{l2}}$, $T_t^K(s_t) = s_t - \tau_{k1}s_t^{1-\tau_{k2}}$, $dT_t^C = \tau_{c0}$, $dS_t^q = \tau_{q0}$	86.43%	1.4%
I. Nonlinear JK taxes for all $T_t^L(z_t) = z_t - \tau_{l1}z_t^{1-\tau_{l2}}$, $T_t^K(s_t) = s_t - \tau_{k1}s_t^{1-\tau_{k2}}$, $T_t^C(\pi_t) = \pi_t - \tau_{c1}\pi_t^{1-\tau_{c2}}$, $S_t^q(q_t) = q_t - \tau_{q1}q_t^{1-\tau_{q2}}$	94.38%	2.04%
J. Second-best economy	100%	2.6%

our second-best economy. Panel B demonstrates that optimal linear tax policies reach 69.7% of the welfare relative to our second-best economy, offering a modest 0.462% improvement over the status quo policy.

Panels C through I examine various combinations of linear and nonlinear JK tax policies. Panel C shows that implementing a nonlinear JK corporate tax, while keeping other taxes linear, achieves 69.82% of the welfare relative to our second-best economy, yielding a 0.468% welfare improvement over the status quo policy. Panel D reveals that implementing a nonlinear JK R&D subsidy, while keeping other taxes linear, achieves 75.46% of the welfare relative to our second-best economy, generating a 0.735% improvement over the status quo policy. This aligns with Akcigit, Hanley, and Stantcheva (2022), which indicates that the nonlinearity of R&D subsidies has a stronger welfare effect than the nonlinearity of corporate taxes. Panel E shows that the combination of nonlinear JK corporate taxes and R&D subsidies, while maintaining other taxes linear, achieves 75.6% of the welfare relative to our second-best economy, yielding a 0.743% improvement over the status quo policy, only marginally better than policies with solely nonlinear R&D subsidies.

Panel F demonstrates that implementing a nonlinear JK capital income tax, while keeping other taxes linear, while maintaining other taxes linear, achieves 76.3% of the welfare relative to our second-best economy, providing a 0.779% welfare gain over the status quo policy—outperforming the combination of nonlinear JK corporate taxes and R&D subsidies. Panel G illustrates that a nonlinear JK labor income tax, with other taxes remaining linear, achieves 80.92% of the welfare relative to our second-best economy, yielding a 1.041% welfare gain over the status quo policy, slightly exceeding the performance of the nonlinear capital income tax.

Panel H shows that combining nonlinear JK labor and capital income taxes, while keeping other taxes linear, achieves 86.43% of the welfare relative to our second-best economy, generating a 1.4% welfare gain over the status quo policy—substantially outperforming the combination of nonlinear JK R&D subsidies and corporate taxes in panel E. Panel I reveals that implementing nonlinear JK policies for all taxes and subsidies achieves 94.38% of the welfare relative to our second-best economy, yielding a 2.04% improvement over the status quo policy, surpassing the combination in Panel H. Finally, panel J demonstrates that the tax policy in our second-best economy achieves a 2.6% welfare gain relative to the status quo policy.

Our results indicate that introducing nonlinearity in labor income taxation generates the largest welfare gains, followed by nonlinearity in capital income taxation. In contrast, making R&D subsidies and corporate taxes nonlinear proves less consequential. As illustrated in Figure 4, this pattern arises because the variations in capital and labor income tax rates, 0.18 and 0.25, respectively, are substantially larger than those observed for R&D subsidy and corporate tax rates. Consequently, maintaining linear structures in R&D subsidy and corporate taxation leads to considerably smaller welfare loss than doing so in linear labor and capital income taxation.

We now discuss how different tax systems influence the polarization of wage growth. Figure 6(a) compares wage growth patterns across four tax systems. The first two represent a decentralized economy with no taxes and with the status quo tax (loci marked with "." and "+," respectively). The latter two correspond to the tax structures of the second-best and first-best economies (loci marked with "*" and "-," respectively).

The results show that wage growth patterns in the first two systems are parallel, with the no-tax system exhibiting lower wage growth than the status quo tax system. Similarly, the second-best and first-best economies also show parallel wage growth patterns, with the second-best economy delivering higher wage growth. These results suggest that different tax systems affect the level of wage growth for each worker but do not change the shape of the wage growth distribution. Specifically, the U-shaped pattern remains intact.

In Figure 6(b), we compare wage growth across the status quo economy, the second-best economy, and a simple nonlinear tax system (loci marked with "o"). Given that our welfare analysis underscores the importance of labor income tax nonlinearity, the simple nonlinear tax system implements the nonlinear JK labor income tax, while keeping the capital tax, R&D subsidy and corporate tax linear.

The results show that high-skill agents experience much higher wage growth in our second-best economy compared to the status quo economy. As a result, the U-shaped

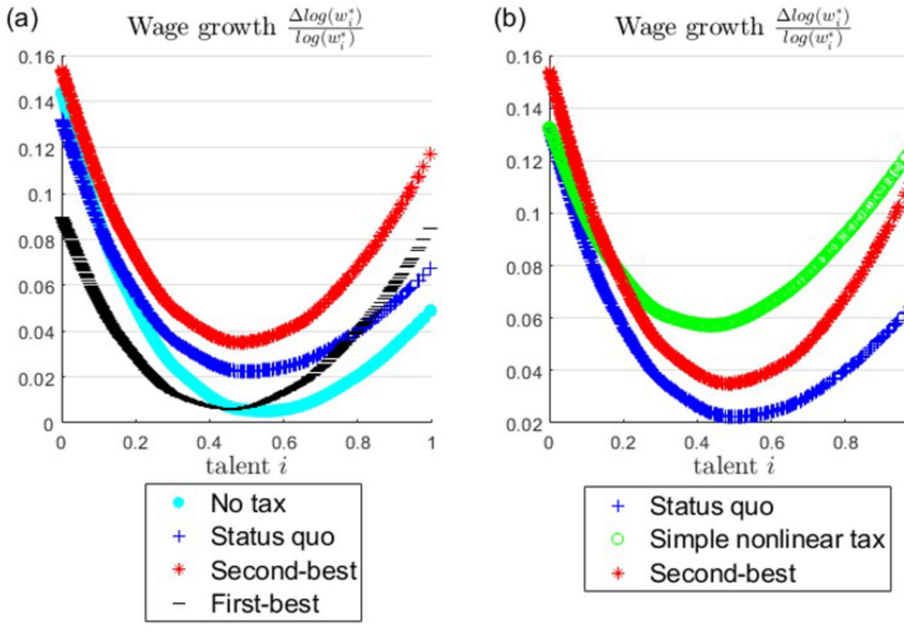

FIGURE 6. Wage growth under different tax systems.

wage growth pattern is more pronounced in the second-best economy. Notably, the simple nonlinear tax system closely replicates this U-shaped pattern, aligning closely with the outcome in the second-best economy. This consistency reinforces our welfare findings: the JK nonlinear tax policy entails only a small welfare loss relative to the unrestricted tax policy in our second-best economy.

5. Concluding remarks

This paper develops a talent-to-task assignment model of self-selection, featuring a continuum of imperfectly substitutable talents and a continuum of tasks. In our framework, more talented individuals have a comparative advantage in performing more complex tasks. We extend the model to incorporate capital, R&D, and relative capital-skill complementarities across tasks. The model allows us to capture the polarization of capital and technology, as well as the resulting polarization in output and wage growth. We then use the model to evaluate optimal tax policies aimed at mitigating wage inequality.

In our model, high-talent individuals are more complementary to capital (or machines) in high-complexity task sectors, whereas low-talent individuals exhibit weaker complementary with capital in low-complexity task sectors. Consequently, output-augmenting R&D investments are directed towards high-complexity sectors, where machines complement professional work, while task-replacing R&D investments are concentrated in low-complexity sectors, where automatic machines substitute for routine and clerical labor. This divergence leads to a polarized allocation of capital and technology across sectors, contributing to patterns of job and output polarization and ultimately resulting in a U-shaped wage growth pattern across the talent distribution.

We explore optimal policy responses to address this inequality. Our analysis reveals that the traditional wage compression mechanism, via trickle-down effects from subsidizing high earners and taxing low earners, remains active. However, capital and R&D taxes policies generate different dynamics. Rather than supporting trickle-down redistribution, our model suggests subsidizing capital income in low-complexity sectors while progressively taxing capital income in high-complexity sectors.

As observable R&D investments and unobservable R&D inputs complement labor, the wage compression rationale supports subsidizing observable R&D investments and firm profits in low-complexity sectors, as well as taxing them in high-complexity sectors. Finally, our model highlights a Pigouvian channel that corrects for spillovers between intermediate and final goods producers, underscoring the role of policy in addressing coordination failures in private markets.

Appendix A

A.1 *Proof of Proposition* 1

This proof is similar to the proof of Lemma 1 in Costinot and Vogel (2010), where they demonstrate the existence of an increasing match function. Unlike their model, which uses only labor inputs in production, our model incorporates both capital inputs and R&D inputs in goods production. Throughout the proof, we denote $\mathcal{I}_t(v) \equiv \{i \in \mathcal{I} | \lambda_t(i, v) > 0\}$ and $\mathcal{V}_t(i) \equiv \{v \in \mathcal{V} | \lambda_t(i, v) > 0\}$. Before proceeding with the proof, we first establish some useful conditions that will be needed throughout. First, based on Assumptions 2–3, the conditions (2a) and (2d) imply that

$$\omega_t(v) = \frac{(1-\rho)\alpha^2 b_t(v)^{1-\alpha} y_t(v)^{\alpha-\varepsilon_v}}{\left[l_t(v)\right]^{1-\varepsilon_v}}. \tag{A1}$$

Second, the inner problem (5) implies that

$$\begin{aligned} w_t(i) &\geq \omega_t(v) a(i, v) \quad \text{for all } i \in \mathcal{I}, \\ w_t(i) &= \omega_t(v) a(i, v) \quad \text{for all } i \in \mathcal{I}_t(v) \equiv \{i \in \mathcal{I} | \lambda_t(i, v) > 0\}. \end{aligned} \tag{A2}$$

Third, the labor market clearing conditions (3e) and (3f) state that for each $v \in [\underline{v}, \bar{v}]$

$$l_t(v) = \int_{\underline{i}}^{\bar{i}} \lambda_t(i, v) a(i, v) e_t(i)\, di \tag{A3}$$

and for each $i \in [\underline{i}, \bar{i}]$

$$f(i) = \int_{\underline{v}}^{\bar{v}} \lambda_t(i, v)\, dv. \tag{A4}$$

The conditions (A1)–(A4) will be used in this proof. To prove the Proposition 1, we proceed in five steps.

Step 1: $\mathcal{I}_t(v) \neq \emptyset$ for all $v \in \mathcal{V}$ and $\mathcal{V}_t(i) \neq \emptyset$ for all $i \in \mathcal{I}$.

Since $f(i) > 0$ for all i, condition (A4) directly implies $\mathcal{V}_t(i) \neq \emptyset$ for all $i \in \mathcal{I}$. Next, we use a contradiction to prove that $\mathcal{I}_t(v) \neq \emptyset$ for all $v \in \mathcal{V}$. Suppose there exists v' such that $\mathcal{I}_t(v') = \emptyset$ and thus $l_t(v') = 0$. However, since $\mathcal{V}_t(i) \neq \emptyset$ for all $i \in \mathcal{I}$, there exists v'' such that $\mathcal{I}_t(v'') \neq \emptyset$, and thus $l_t(v'') > 0$. By condition (A1), the fact that $l_t(v') = 0$ and $l_t(v'') > 0$ implies that $\omega_t(v'')/\omega_t(v') = 0$. By condition (A2), for $i \in \mathcal{I}_t(v'')$ we have $0 = \frac{\omega_t(v'')}{\omega_t(v')} \geq \frac{a(i,v')}{a(i,v'')} > 0$, which is a contradiction.

Step 2: (i) For any $v \in \mathcal{V}$, $\mathcal{I}_t(v)$ is a nonempty interval of $[\underline{i}, \bar{i}]$; (ii) for any $v' > v$ if $i' \in \mathcal{I}_t(v\prime)$ and $i \in \mathcal{I}_t(v)$, then $i' \geq i$.

We use a contradiction to show that $\mathcal{I}_t(v)$ is an interval. Suppose that there exists a task v and three talent types $i_1 < i_2 < i_3$ such that $i_1, i_3 \in \mathcal{I}_t(v)$ but $i_2 \notin \mathcal{I}_t(v)$. By Step 1, $\mathcal{V}_t(i_2) \neq \emptyset$, so there must be $v' \neq v$ such that $i_2 \in \mathcal{I}_t(v')$. Without loss of generality, we set $v' > v$. (The case for $v > v'$ can be proved by replacing i_3 by i_1 in the following arguments.) Then by condition (A2), we have

$$w_t(i_2) = \omega_t\big(v'\big)a\big(i_2, v'\big) \geq \omega_t(v)a(i_2, v),$$
$$w_t(i_3) = \omega_t(v)a(i_3, v) \geq \omega_t\big(v'\big)a\big(i_3, v'\big).$$

Combining the above two inequalities, we get $a(i_2, v')a(i_3, v) \geq a(i_2, v)a(i_3, v')$, which contradicts to the comparative advantage of $a(i, v)$ in Assumption 1(i). Thus, $\mathcal{I}_t(v)$ must be an interval.

To prove step 2(ii), we use a contradiction again. Suppose that there exist $v' > v$ and $i' > i$ such that $i' \in \mathcal{I}_t(v)$ and $i \in \mathcal{I}_t(v')$. Then using the same argument in step 2(i), condition (A2) implies that $a(i, v')a(i', v) \geq a(i, v)a(i', v')$, which contradicts the comparative advantage of $a(i, v)$ in Assumption 1(i).

Step 3: $\mathcal{I}_t(v)$ is a singleton for all but a countable subset of $\mathcal{V}$.

By step 2(i), $\mathcal{I}_t(v)$ is a nonempty interval, implying that $\mathcal{I}_t(v)$ is Lebesgue measurable for any v. The Lebesgue measure over $\mathbb{R}$ is denoted by μ, and let $\mathcal{V}^0$ be the subset of $\mathcal{V}$ such that $\mu(\mathcal{I}_t(v)) > 0$. That is, $\mathcal{V}^0 \equiv \{v \in \mathcal{V} | \mu(\mathcal{I}_t(v)) > 0\}$. Before proving step 3, we first prove that $\mathcal{V}^0$ is a countable set as follows. Let $\underline{i}_t(v) \equiv \inf \mathcal{I}_t(v)$ and $\bar{i}_t(v) \equiv \sup \mathcal{I}_t(v)$. The fact that $\mu(\mathcal{I}_t(v)) > 0$ for any $v \in \mathcal{V}^0$ implies $\underline{i}_t(v) < \bar{i}_t(v)$ for any $v \in \mathcal{V}^0$. Thus, there exists $j_0 \in \mathbb{N}$ such that $\bar{i}_t(v) - \underline{i}_t(v) \geq (\bar{i} - \underline{i})/j_0$ for any $v \in \mathcal{V}^0$. Let $\mathcal{V}_j^0 \equiv \{v \in \mathcal{V} | (\bar{i}_t(v) - \underline{i}_t(v)) \geq (\bar{i} - \underline{i})/j\}$ for any $j \in \mathbb{N}$. By step 2(ii), $\mu(\mathcal{I}_t(v) \cap \mathcal{I}_t(v')) = 0$ for any $v \neq v'$. Thus, there can be at most j elements in $\mathcal{V}_j^0$. To see why, we take $j = 2$ as an example. If there exist three different tasks v_1, v_2, v_3 in $\mathcal{V}_2^0$, then by step 2(ii), the three sets $\mathcal{I}_t(v_1), \mathcal{I}_t(v_2), \mathcal{I}_t(v_3)$ that are mutually μ-almost disjoint, and thus the union of $\mathcal{I}_t(v_1), \mathcal{I}_t(v_2), \mathcal{I}_t(v_3)$ would exceed the boundary of $[\underline{i}, \bar{i}]$, which is implausible. Thus, there must be at most j elements in $\mathcal{V}_j^0$, which implies that $\mathcal{V}_j^0$ is a countable set. By construction, we have $\mathcal{V}^0 = \mathcal{V}_{j_0}^0$, and thus $\mathcal{V}^0$ is a countable set. By the definition of $\mathcal{V}^0 \equiv \{v \in \mathcal{V} | \mu(\mathcal{I}_t(v)) > 0\}$, we have $\mu(\mathcal{I}_t(v)) = 0$ for any $v \notin \mathcal{V}^0$, and along with step 2(i), we prove that $\mathcal{I}_t(v)$ is singleton for all but a countable set $\mathcal{V}^0 \subset \mathcal{V}$.

Step 4: $\mathcal{V}_t(i)$ is a singleton for all but a countable subset of $\mathcal{I}$.

This follows from a similar argument as presented in step 2 and step 3.

Step 5: $\mathcal{I}_t(v)$ is a singleton for all $v \in \mathcal{V}$.

To prove step 5, we use a contradiction. Suppose that there exists $v \in \mathcal{V}$ such that $\mathcal{I}_t(v)$ is not a singleton. By step 2(i), this implies $\mu(\mathcal{I}_t(v)) > 0$. By step 4, we know that $\mathcal{V}_t(i) = v$ for μ-almost all $i \in \mathcal{I}_t(v)$. Hence, condition (A4) implies that for μ-almost all $i \in \mathcal{I}_t(v)$,

$$\lambda_t(i, v) = f(i)\delta[1 - \mathbf{1}_{\mathcal{I}_t(v)}], \tag{A5}$$

where δ is a Dirac delta function. By step 3, we also know that there exist $v' \in \mathcal{V}$ such that $\mathcal{I}_t(v') = \{i'\}$, and thus condition (A4) implies that

$$\lambda_t(i', v') \le f(i')\delta[1 - \mathbf{1}_{\mathcal{I}_t(v')}]. \tag{A6}$$

Combining conditions (A3) and (A5)–(A6), the fact that $\mu(\mathcal{I}_t(v)) > 0$ implies that $l_t(v) = \infty$ and $l_t(v\prime) < \infty$. Using condition (A1), this implies $\frac{\omega_t(v)}{\omega_t(v')} = 0$. Since $i \in \mathcal{I}_t(v)$, the condition (A2) implies that $0 = \frac{\omega_t(v)}{\omega_t(v\prime)} \ge \frac{a(i,v\prime)}{a(i,v)} > 0$, which is a contradiction. Hence, we complete the proof of step 5.

Step 5 implies that there exists a function $H_t : \mathcal{V} \to \mathcal{I}$ such that $\lambda_t(i, v) > 0$ if and only if $H_t(v) = i$. By steps 2(ii) and step 4, we know that H_t must be strictly increasing. By step 1, we know that $\mathcal{V}_t(i) \neq \emptyset$ for all $i \in \mathcal{I}$, and thus H_t must be continuous and satisfy $H_t(\underline{v}) = \underline{i}$ and $H_t(\bar{v}) = \bar{i}$. The proof of Proposition 1 is completed by setting $M_t \equiv H_t^{-1}$.

A.2 *The Hamilton of planning problem in Section* 3.2

Set the Hamiltonian of the social planning problem (P_1) as follows:

$$\begin{aligned}\mathcal{H}(i) = \max \{f(i)U(i)\} & \\ + \sum_{t=1}^{T-1} \Big\{ & \hat{Y}_t - G_t - c_t(i)f(i) \\ & + \int_{\underline{v}}^{\bar{v}} \big[-k_{t+1}(v) + (1-\delta_k)k_t(v) - \mathcal{C}_q\big(q_t(v)\big) - \mathcal{C}_n\big(n_t(v)\big)\big]\,dv \Big\} \\ + \chi_T \Big\{ & \hat{Y}_T - G_T - c_T(i)f(i) + \int_{\underline{v}}^{\bar{v}} \big[(1-\delta_k)k_T(v) - \mathcal{C}_q\big(q_T(v)\big) - \mathcal{C}_n\big(n_T(v)\big)\big]\,dv \Big\} \\ - \int \eta(j) & \left\{ \sum_{t=1}^{T} \beta^{t-1} h'\big(e_t(j)\big) e_t(j) \frac{\dot{w}_t(j)}{w_t(j)} \right\} dj.\end{aligned}$$

Using Equation (1a) to replace $c_T(i)$ by

$$u^{-1}\left\{ \frac{U(i)}{\beta^{T-1}} + h\big(e_T(i)\big) - \sum_{t=1}^{T-1} \beta^{t-T}\big[u\big(c_t(i)\big) - h\big(e_t(i)\big)\big] \right\}$$

then the above Hamiltonian can be rewritten as follows:

$$\mathcal{H}(i) = \max \{f(i)U(i)\}$$

$$
\begin{aligned}
&+\sum_{t=1}^{T-1}\Bigg\{\hat{Y}_t - G_t - c_t(i)f(i) \\
&+\int_{\underline{v}}^{\bar{v}}\big[-k_{t+1}(v)+(1-\delta_k)k_t(v)-\mathcal{C}_q\big(q_t(v)\big)-\mathcal{C}_n\big(n_t(v)\big)\big]\,dv\Bigg\} \\
&+\chi_T\Bigg\{\hat{Y}_T - G_T - f(i)\Bigg[u^{-1}\Bigg\{\frac{U(i)}{\beta^{T-1}}+h\big(e_T(i)\big)-\sum_{t=1}^{T-1}\beta^{t-T}\big[u\big(c_t(i)\big)-h\big(e_t(i)\big)\big]\Bigg\}\Bigg] \\
&+\int_{\underline{v}}^{\bar{v}}\big[(1-\delta_k)k_T(v)-\mathcal{C}_q\big(q_T(v)\big)-\mathcal{C}_n\big(n_T(v)\big)\big]\,dv\Bigg\} \\
&-\int \eta(j)\Bigg\{\sum_{t=1}^{T}\beta^{t-1}h'\big(e_t(j)\big)e_t(j)\frac{\dot{w}_t(j)}{w_t(j)}\Bigg\}\,dj.
\end{aligned}
$$

The first-order conditions of the above Hamilton are as follows:

$$
\big[c_t(i)\big]: f(i)\Bigg[-\chi_t+\chi_T\frac{\beta^{t-T}u'\big(c_t(i)\big)}{u'\big(c_T(i)\big)}\Bigg]=0 \quad \text{for } t=1,2,\dots,T-1, \tag{A7}
$$

$$
\begin{aligned}
\big[k_t(v)\big]:\ &\chi_t\Bigg[\frac{\partial\hat{Y}_t}{\partial k_t(v)}+(1-\delta)\Bigg]-\chi_{t-1} \\
&-\beta^{t-1}\int \eta(j)h'\big(e_t(j)\big)e_t(j)\frac{\partial\big(\dot{w}_t(j)/w_t(j)\big)}{\partial k_t(v)}\,dj=0,
\end{aligned} \tag{A8}
$$

$$
\begin{aligned}
\big[e_t(i)\big]:\ &\chi_t\frac{\partial\hat{Y}_t}{\partial e_t(i)}-\chi_T\frac{\beta^{t-T}h'\big(e_t(i)\big)}{u'\big(c_T(i)\big)}f(i)-\eta(i)\beta^{t-1}\frac{\dot{w}_t(i)}{w_t(i)}\big[h''\big(e_t(i)\big)e_t(i)+h'\big(e_t(i)\big)\big] \\
&-\beta^{t-1}\int \eta(j)h'\big(e_t(j)\big)e_t(j)\frac{\partial\big(\dot{w}_t(j)/w_t(j)\big)}{\partial e_t(i)}\,dj=0,
\end{aligned} \tag{A9}
$$

$$
\begin{aligned}
\big[n_t(v)\big]:\ &\sum_{s=t}^{T}\chi_s\frac{\partial\hat{Y}_s}{\partial b_s(v)}\Gamma_t^s(v)\frac{\partial A_t^v}{\partial n_t(v)}-\chi_t\mathcal{C}_n'\big(n_t(v)\big) \\
&-\sum_{s=t}^{T}\beta^{s-1}\int \eta(j)h'\big(e_s(j)\big)e_s(j)\frac{\partial\big(\dot{w}_s(j)/w_s(j)\big)}{\partial b_s(v)}\Gamma_t^s(v)\frac{\partial A_t^v}{\partial n_t(v)}\,dj=0,
\end{aligned} \tag{A10}
$$

$$
\text{where } \Gamma_t^s(\nu)\equiv\begin{cases}1, & \text{when } s=t,\\ \displaystyle\prod_{u=t+1}^{s}\frac{\partial A_u^\nu}{\partial b_{u-1}(\nu)}, & \text{when } s=t+1,t+2,\dots,T.\end{cases}
$$

$$
\big[q_t(v)\big]: \sum_{s=t}^{T}\chi_s\frac{\partial\hat{Y}_s}{\partial b_s(v)}\Gamma_t^s(v)\frac{\partial A_t^v}{\partial q_t(v)}-\chi_t\mathcal{C}_q'\big(q_t(v)\big)
$$

$$-\sum_{s=t}^{T}\beta^{s-1}\int \eta(j)h'\big(e_s(j)\big)e_s(j)\frac{\partial\big(\dot{w}_s(j)/w_s(j)\big)}{\partial b_s(v)}\Gamma_t^s(v)\frac{\partial A_t^v}{\partial q_t(v)}\,dj=0, \tag{A11}$$

$$\big[U(i)\big]:\frac{d\mathcal{H}}{dU(i)}=\frac{d\eta(i)}{di}=f(i)-\frac{\chi_T}{\beta^{T-1}u'\big(c_T(i)\big)}f(i). \tag{A12}$$

Using the condition (A12) and the boundary conditions $\eta(\underline{i})=\eta(\bar{i})=0$, we prove Lemma 1.

Proof of Lemma 1. Based on the condition (A7), we have $\frac{\chi_T}{\beta^{T-1}u'(c_T(i))}=\frac{\chi_t}{\beta^{t-1}u'(c_t(i))}$. Thus, the condition (A12) can be rewritten as follows:

$$\frac{d\eta(i)}{di}=\left[1-\frac{\chi_t}{\beta^{t-1}u'\big(c_t(i)\big)}\right]f(i). \tag{A13}$$

From the integral version of (A13) along with the boundary condition $\eta(\underline{i})=\eta(\bar{i})=0$, we get

$$\eta(i)=\int_{\underline{i}}^{i}\left[1-\frac{\chi_t}{\beta^{t-1}u'\big(c_t(i')\big)}\right]f(i')\,di'.$$

By Rolle's theorem, the boundary conditions $\eta(\underline{i})=\eta(\bar{i})=0$ also imply that there exists $\hat{i}\in(\underline{i},\bar{i})$ such that $\frac{d\eta(\hat{i})}{d\hat{i}}=0$, and thus by equation (A13), we obtain

$$\chi_t=\beta^{t-1}u'\big(c_t(\hat{i})\big)>0. \tag{A14}$$

When $c_t(i)$ is monotonically increasing in i, and $u'>0>u''$, and thus (A14) implies that

$$\chi_t<\beta^{t-1}u'\big(c_t(i)\big)\quad\text{for any } i<\hat{i}\quad\text{and}\quad\chi_t>\beta^{t-1}u'\big(c_t(i)\big)\quad\text{for any } i>\hat{i}.$$

From the integral version of (A13), we get

$$\eta(i)=\int_{\underline{i}}^{i}\left[1-\frac{\chi_t}{\beta^{t-1}u'\big(c_t(i')\big)}\right]f(i')\,di'>0\quad\text{for any } i\le\hat{i}\quad\text{and}$$

$$\eta(i)=\int_{i}^{\bar{i}}\left[\frac{\chi_t}{\beta^{t-1}u'\big(c_t(i')\big)}-1\right]f(i')\,di'>0\quad\text{for any } i>\hat{i}.$$

Therefore, $\eta(i)>0$ for any $i\in(\underline{i},\bar{i})$. □

A.3 *Proofs of Proposition 2*

(i) **Capital wedges:** From equation (A8), we have

$$\frac{\chi_t}{\chi_{t-1}}\big[(1-\delta)\big]-1=\frac{\chi_t}{\chi_{t-1}}\frac{\beta^{t-1}}{\chi_{t-1}}\int \eta(j)h'\big(e_t(j)\big)e_t(j)\frac{\partial\big(\dot{w}_t(j)/w_t(j)\big)}{\partial k_t(v)}\,dj-\frac{\chi_t}{\chi_{t-1}}\left[\frac{\partial\hat{Y}_t}{\partial k_t(v)}\right].$$

Adding the term $\frac{\chi_t}{\chi_{t-1}} R_t(v)$ to both sides of the above equation, we get

$$\frac{\chi_t}{\chi_{t-1}}\big[R_t(v)+(1-\delta)\big]-1=\frac{\chi_t}{\chi_{t-1}}\frac{\beta^{t-1}}{\chi_{t-1}}\int \eta(j)h'\big(e_t(j)\big)e_t(j)\frac{\partial\big(\dot{w}_t(j)/w_t(j)\big)}{\partial k_t(v)}\,dj - \frac{\chi_t}{\chi_{t-1}}\left[\frac{\partial \hat{Y}_t}{\partial k_t(v)}-R_t(v)\right].$$

Based on equation (A7), we have $\frac{\chi_t}{\chi_{t-1}}=\frac{\beta u'(c_t(i))}{u'(c_{t-1}(i))}$, and based on the definition (4a), the above equation can be written as follows:

$$\frac{\tau_i^{k_t}(v)}{1-\tau_i^{k_t}(v)}=\frac{\chi_t}{\chi_{t-1}}\frac{\beta^{t-1}}{\chi_{t-1}}\int \eta(j)h'\big(e_t(j)\big)e_t(j)\frac{\partial\big(\dot{w}_t(j)/w_t(j)\big)}{\partial k_t(v)}\,dj - \frac{\chi_t}{\chi_{t-1}}\left[\frac{\partial \hat{Y}_t}{\partial k_t(v)}-R_t(v)\right]. \tag{A15}$$

Using the notation $\kappa_{v,j}^t=\frac{k_t(v)}{\dot{w}_t(j)/w_t(j)}\frac{\partial(\dot{w}_t(j)/w_t(j))}{\partial k_t(v)}$, $\mathcal{W}_t^k(v,j)\equiv\frac{\beta^{t-1}\eta(j)h'(e_t(j))e_t(j)(\frac{\dot{w}_t(j)}{w_t(j)})}{\chi_{t-1}k_t(v)}$, and $\mathcal{M}_t^k\equiv\frac{\chi_t}{\chi_{t-1}}$, equation (A15) implies (10a).

(ii) **Labor wedges:** From equation (A9), we have

$$\frac{\partial \hat{Y}_t}{\partial e_t(i)}-\frac{\chi_T\beta^{t-T}h'\big(e_t(i)\big)f(i)}{\chi_t u'\big(c_T(i)\big)}=\frac{\eta(i)\beta^{t-1}\big[h''\big(e_t(i)\big)e_t(i)+h'\big(e_t(i)\big)\big]}{\chi_t}\frac{\dot{w}_t(i)}{w_t(i)} + \frac{\beta^{t-1}}{\chi_t}\int \eta(j)h'\big(e_t(j)\big)e_t(j)\frac{\partial\big(\dot{w}_t(j)/w_t(j)\big)}{\partial e_t(i)}\,dj.$$

Based on equation (A7), we have $\frac{\chi_T\beta^{t-T}}{\chi_t u'(c_T(i))}=\frac{1}{u'(c_t(i))}$, and thus the above equation can be written as

$$\frac{\partial \hat{Y}_t}{\partial e_t(i)}-\frac{h'\big(e_t(i)\big)f(i)}{u'\big(c_t(i)\big)}=\frac{\eta(i)\beta^{t-1}\big[h''\big(e_t(i)\big)e_t(i)+h'\big(e_t(i)\big)\big]}{\chi_t}\frac{\dot{w}_t(i)}{w_t(i)} + \frac{\beta^{t-1}}{\chi_t}\int \eta(j)h'\big(e_t(j)\big)e_t(j)\frac{\partial\big(\dot{w}_t(j)/w_t(j)\big)}{\partial e_t(i)}\,dj.$$

Multiplying the term $\frac{u'(c_t(i))}{h'(e_t(i))f(i)}$ to both sides of the above equation, we get

$$\frac{u'\big(c_t(i)\big)}{h'\big(e_t(i)\big)f(i)}\frac{\partial \hat{Y}_t}{\partial e_t(i)}-1=\frac{\eta(i)\beta^{t-1}u'\big(c_t(i)\big)\big[h''\big(e_t(i)\big)e_t(i)+h'\big(e_t(i)\big)\big]\frac{\dot{w}_t(i)}{w_t(i)}}{\chi_t h'\big(e_t(i)\big)f(i)} + \frac{\beta^{t-1}u'\big(c_t(i)\big)\int \eta(j)h'\big(e_t(j)\big)e_t(j)\frac{\partial\big(\dot{w}_t(j)/w_t(j)\big)}{\partial e_t(i)}\,dj}{\chi_t h'\big(e_t(i)\big)f(i)}.$$

Adding the term $\frac{u'(c_t(i))}{h'(e_t(i))f(i)}[w_t(i)f(i) - \frac{\partial \hat{Y}_t}{\partial e_t(i)}]$ to both sides of the above equation, and referring to the definition (4b), we have

$$\begin{aligned}\frac{\tau^{l_t}(i)}{1-\tau^{l_t}(i)} &= \frac{u'\big(c_t(i)\big)w_t(i)}{h'\big(e_t(i)\big)} - 1 \\ &= \frac{\eta(i)\beta^{t-1}u'\big(c_t(i)\big)\left[\frac{h''\big(e_t(i)\big)}{h'\big(e_t(i)\big)}e_t(i)+1\right]\frac{\dot{w}_t(i)}{w_t(i)}}{\chi_t f(i)} \\ &\quad + \frac{\beta^{t-1}u'\big(c_t(i)\big)\int \eta(j)h'\big(e_t(j)\big)e_t(j)\frac{\partial\big(\dot{w}_t(j)/w_t(j)\big)}{\partial e_t(i)}\,dj}{\chi_t h'\big(e_t(i)\big)f(i)} \\ &\quad + \frac{u'\big(c_t(i)\big)\left[w_t(i)f(i) - \frac{\partial \hat{Y}_t}{\partial e_t(i)}\right]}{h'\big(e_t(i)\big)f(i)}.\end{aligned} \tag{A16}$$

Using the notation $\phi^t_{i,j} = \frac{e_t(j)}{(\dot{w}_t(j)/w_t(j))}\frac{\partial(\dot{w}_t(j)/w_t(j))}{\partial e_t(i)}$, $\mathcal{N}_t(i) \equiv \frac{\beta^{t-1}\eta(i)u'(c_t(i))}{\chi_t f(i)}[\frac{h''(e_t(i))}{h'(e_t(i))}e_t(i) + 1]$, $\mathcal{W}^l_t(i,j) \equiv \frac{\beta^{t-1}\eta(j)u'(c_t(i))e_t(j)(\frac{\dot{w}_t(j)}{w_t(j)})}{\chi_t e_{i,t} f(i)}$, and $\mathcal{M}^l_t(i) \equiv \frac{u'(c_t(i))}{f(i)h'(e_t(i))}$, equation (A16) implies (10b).

(iii) **Corporate wedges:** From equation (A10), we have

$$\begin{aligned}&\sum_{s=t}^{T}\frac{\chi_s}{\chi_t}\frac{\partial \hat{Y}_s}{\partial b_s(v)}\Gamma^s_t(v)\frac{\partial A^v_t}{\partial n_t(v)} - \mathcal{C}'_n\big(n_t(v)\big) \\ &\quad = \frac{1}{\chi_t}\sum_{s=t}^{T}\beta^{s-1}\int \eta(j)h'\big(e_s(j)\big)e_s(j)\frac{\partial\big(\dot{w}_s(j)/w_s(j)\big)}{\partial b_s(v)}\Gamma^s_t(v)\frac{\partial A^v_t}{\partial n_t(v)}\,dj.\end{aligned}$$

Adding $\sum_{s=t}^{T}[\frac{\pi'_s(b_s(v))}{\mathcal{R}^t_s(v)} - \frac{\chi_s}{\chi_t}\frac{\partial \hat{Y}_s}{\partial b_s(v)}]\Gamma^s_t(v)\frac{\partial A^v_t}{\partial n_t(v)}$ to both sides and referring to the definition (4c), the above equation implies

$$\begin{aligned}\tau^{n_t}(v) &\equiv \sum_{s=t}^{T}\frac{\pi'_s\big(b_s(v)\big)}{\mathcal{R}^t_s(v)}\Gamma^s_t(v)\frac{\partial A^v_t}{\partial n_t(v)} - \mathcal{C}'_n\big(n_t(v)\big) \\ &= \frac{1}{\chi_t}\sum_{s=t}^{T}\beta^{s-1}\int \eta(j)h'\big(e_s(j)\big)e_s(j)\frac{\partial\big(\dot{w}_s(j)/w_s(j)\big)}{\partial b_s(v)}\Gamma^s_t(v)\frac{\partial A^v_t}{\partial n_t(v)}\,dj \\ &\quad + \left[\frac{\pi'_s\big(b_s(v)\big)}{\mathcal{R}^t_s(v)} - \frac{\chi_s}{\chi_t}\frac{\partial \hat{Y}_s}{\partial b_s(v)}\right]\Gamma^s_t(v)\frac{\partial A^v_t}{\partial n_t(v)}.\end{aligned} \tag{A17}$$

Using the notation $\varphi^s_{v,j} = \frac{b_s(v)}{(\dot{w}_s(j)/w_s(j))}\frac{\partial(\dot{w}_s(j)/w_s(j))}{\partial b_s(v)}$, $\mathcal{W}^n_s(v,j) \equiv \frac{\beta^{s-1}\eta(j)h'(e_s(j))e_t(j)(\frac{\dot{w}_t(j)}{w_t(j)})\Gamma^s_t(v)}{\chi_t b_s(v)}\frac{\partial A^v_t}{\partial n_t(v)}$, and $\mathcal{M}^n_{s,t}(v) \equiv \Gamma^s_t(v)\frac{\partial A^v_t}{\partial n_t(v)}$, equation (A17) implies (10c).

(iv) **R&D wedge:** From equation (A11), we have

$$\sum_{s=t}^{T}\frac{\chi_s}{\chi_t}\frac{\partial\hat{Y}_s}{\partial b_s(v)}\Gamma_t^s(v)\frac{\partial A_t^v}{\partial q_t(v)}-\mathcal{C}_q'\big(q_t(v)\big)$$
$$=\frac{1}{\chi_t}\sum_{s=t}^{T}\beta^{s-1}\int\eta(j)h'\big(e_s(j)\big)e_s(j)\frac{\partial\big(\dot{w}_s(j)/w_s(j)\big)}{\partial b_s(v)}\Gamma_t^s(v)\frac{\partial A_t^v}{\partial q_t(v)}\,dj.$$

Adding $\sum_{s=t}^{T}[\frac{\pi_s'(b_s(v))}{\mathcal{R}_s^t(v)}-\frac{\chi_s}{\chi_t}\frac{\partial\hat{Y}_s}{\partial b_s(v)}]\Gamma_t^s(v)\frac{\partial A_t^v}{\partial q_t(v)}$ to both sides and referring to the definition (4e), the above equation implies

$$\begin{aligned}s^{q_t}(v)&\equiv\mathcal{C}_q'\big(q_t(v)\big)-\left[\sum_{s=t}^{T}\frac{1}{\mathcal{R}_s^t(v)}\pi_s'\big(b_s(v)\big)\Gamma_t^s(v)\right]\frac{\partial A_t^v}{\partial q_t(v)}\\&=\frac{-1}{\chi_t}\sum_{s=t}^{T}\beta^{s-1}\int\eta(j)h'\big(e_s(j)\big)e_s(j)\frac{\partial\big(\dot{w}_s(j)/w_s(j)\big)}{\partial b_s(v)}\Gamma_t^s(v)\frac{\partial A_t^v}{\partial q_t(v)}\,dj\\&\quad-\left[\frac{\pi_s'\big(b_s(v)\big)}{\mathcal{R}_s^t(v)}-\frac{\chi_s}{\chi_t}\frac{\partial\hat{Y}_s}{\partial b_s(v)}\right]\Gamma_t^s(v)\frac{\partial A_t^v}{\partial q_t(v)}.\end{aligned}\tag{A18}$$

Using the notation $\varphi_{v,j}^s=\frac{b_s(v)}{(\dot{w}_s(j)/w_s(j))}\frac{\partial(\dot{w}_s(j)/w_s(j))}{\partial b_s(v)}$, $\mathcal{W}_s^q(v,j)\equiv\frac{\beta^{s-1}\eta(j)h\prime(e_s(j))e_t(j)(\frac{\dot{w}_t(j)}{w_t(j)})\Gamma_t^s(v)}{\chi_t b_s(v)}\frac{\partial A_t^v}{\partial q_t(v)}$, and $\mathcal{M}_{s,t}^q(v)\equiv\Gamma_t^s(v)\frac{\partial A_t^v}{\partial q_t(v)}$, equation (A18) implies (10d).

A.4 *Poof of Proposition 1: Signs of elasticities in simple two-talent, two-task model*

In the simple two-talent, two-task model, the cross-relative wage elasticity defined in (9) should be revised as follows:

$$\begin{aligned}E(x_t)&\equiv\frac{x_t}{w_t(H)/w_t(L)}\frac{\partial\big(w_t(H)/w_t(L)\big)}{\partial x_t},\\&x_t\in\big\{k_t(v),e_t(i),b_t(v)\big\},\ i\in\{L,H\}\text{ and }v\in\{\underline{v},\overline{v}\}.\end{aligned}\tag{A19}$$

Thus, $E(k_t(v))=\kappa_v^t$, $E(e_t(i))=\phi_i^t$, and $E(b_t(v))=\varphi_v^t$ for $i\in\{L,H\}$ and $v\in\{\underline{v},\overline{v}\}$. Based on (2a) and (2d), Assumptions 2 and 3 imply that the wage rate in task v is

$$\begin{aligned}\omega_t(v)&=\frac{\partial F^v}{\partial l_t(v)}\left[\frac{\partial p}{\partial y_t}\cdot y_t(v)+p\big(b_t(v),y_t\big)\right]=\frac{\partial F^v}{\partial l_t(v)}\left[\frac{\partial^2\widetilde{Y}}{\partial y_t^2}\cdot y_t(v)+\frac{\partial\widetilde{Y}}{\partial y_t}\right]\\&=(1-\rho)\alpha^2 b_t(v)^{1-\alpha}\big[l_t(v)\big]^{\varepsilon_v-1}\big\{(1-\rho)\big[l_t(v)\big]^{\varepsilon_v}+\rho\big[k_t(v)\big]^{\varepsilon_v}\big\}^{\frac{\alpha-\varepsilon_v}{\varepsilon_v}}.\end{aligned}$$

Thus, the wage rates for agents of high talent and low talent are, respectively,

$$w_t(H)=a(H,\bar{v})\omega_t(\bar{v})$$

$$= \frac{a(H,\bar{v})(1-\rho)\alpha^2 b_t(\bar{v})^{1-\alpha}}{\big[f(H)a(H,\bar{v})e_t(H)\big]^{1-\varepsilon_{\bar{v}}}}\big\{(1-\rho)\big[f(H)a(H,\bar{v})e_t(H)\big]^{\varepsilon_v}+\rho\big[k_t(\bar{v})\big]^{\varepsilon_{\bar{v}}}\big\}^{\frac{\alpha-\varepsilon_{\bar{v}}}{\varepsilon_{\bar{v}}}},$$

$$w_t(L) = a(L,\underline{v})\omega_t(\underline{v})$$

$$= \frac{a(L,\underline{v})(1-\rho)\alpha^2 b_t(\underline{v})^{1-\alpha}}{\big[f(L)a(L,\underline{v})e_t(L)\big]^{1-\varepsilon_{\underline{v}}}}\big\{(1-\rho)\big[f(L)a(L,\underline{v})e_t(L)\big]^{\varepsilon_{\underline{v}}}+\rho\big[k_t(\underline{v})\big]^{\varepsilon_{\underline{v}}}\big\}^{\frac{\alpha-\varepsilon_{\underline{v}}}{\varepsilon_{\underline{v}}}}.$$

Hence, the wage premium is

$$\frac{w_t(H)}{w_t(L)} = \frac{\big[a(H,\bar{v})\big]^{\varepsilon_{\bar{v}}} b_t(\bar{v})^{1-\alpha}\big[f(H)\big]^{\varepsilon_{\bar{v}}-1}\big[\psi(\bar{v})\big]^{(\alpha-\varepsilon_{\bar{v}})/\varepsilon_{\bar{v}}}}{\big[a(L,\underline{v})\big]^{\varepsilon_{\underline{v}}} b_t(\underline{v})^{1-\alpha}\big[f(L)\big]^{\varepsilon_{\underline{v}}-1}\big[\psi(\underline{v})\big]^{(\alpha-\varepsilon_{\underline{v}})/\varepsilon_{\underline{v}}}}, \tag{A20}$$

where $\psi(\bar{v}) = \frac{(1-\rho)[f(H)a(H,\bar{v})]^{\varepsilon v}}{\{[e_t(H)]^{(1-\alpha)\varepsilon_{\bar{v}}/(\alpha-\varepsilon_{\bar{v}})}\}} + \frac{\rho[k_t(\bar{v})]^{\varepsilon_{\bar{v}}}}{\{[e_t(H)]^{(1-\varepsilon_{\bar{v}})\varepsilon_{\bar{v}}/(\alpha-\varepsilon_{\bar{v}})}\}}$ and $\psi(\underline{v}) = \frac{(1-\rho)[f(L)a(L,\underline{v})]^{\varepsilon_{\underline{v}}}}{\{[e_t(L)]^{(1-\alpha)\varepsilon_{\underline{v}}/(\alpha-\varepsilon_{\underline{v}})}\}} + \frac{\rho[k_t(\underline{v})]^{\varepsilon_{\underline{v}}}}{\{[e_t(L)]^{(1-\varepsilon_{\underline{v}})\varepsilon_{\underline{v}}/(\alpha-\varepsilon_{\underline{v}})}\}}$.

(i) To derive the cross-relative wage elasticity with respect to capital by taking the derivatives of (A20) with respect to $k_t(\bar{v})$ and $k_t(\underline{v})$, respectively, we get

$$\kappa^t_{\bar{v}} = (\alpha-\varepsilon_{\bar{v}})\frac{\rho\big[k_t(\bar{v})\big]^{\varepsilon_{\bar{v}}-1}}{\big[e_t(H)\big]^{(1-\varepsilon_{\bar{v}})\varepsilon_{\bar{v}}/(\alpha-\varepsilon_{\bar{v}})}}$$

$$\times\frac{\big[a(H,\bar{v})\big]^{\varepsilon_{\bar{v}}} b_t(\bar{v})^{1-\alpha}\big[f(H)\big]^{\varepsilon_{\bar{v}}-1}\big[\psi(\bar{v})\big]^{(\alpha-2\varepsilon_{\bar{v}})/\varepsilon_{\bar{v}}}}{\big[a(L,\underline{v})\big]^{\varepsilon_{\underline{v}}} b_t(\underline{v})^{1-\alpha}\big[f(L)\big]^{\varepsilon_{\underline{v}}-1}\big[\psi(\underline{v})\big]^{(\alpha-\varepsilon_{\underline{v}})/\varepsilon_{\underline{v}}}}\frac{k_t(\bar{v})}{\big(w_t(H)/w_t(L)\big)}$$

$$= (\alpha-\varepsilon_{\bar{v}})\frac{\rho\big[k_t(\bar{v})\big]^{\varepsilon_{\bar{v}}}}{\psi(\bar{v})\big[e_t(H)\big]^{(1-\varepsilon_{\bar{v}})\varepsilon_{\bar{v}}/(\alpha-\varepsilon_{\bar{v}})}} \gtreqless 0 \quad \text{when } \varepsilon_{\bar{v}} \lesseqgtr \alpha,$$

and

$$\kappa^t_{\underline{v}} = (\varepsilon_{\underline{v}}-\alpha)\frac{\rho\big[k_t(\underline{v})\big]^{\varepsilon_{\underline{v}}-1}}{\big[e_t(L)\big]^{(1-\varepsilon_{\underline{v}})\varepsilon_{\underline{v}}/(\alpha-\varepsilon_{\underline{v}})}}$$

$$\times\frac{\big[a(H,\bar{v})\big]^{\varepsilon_{\bar{v}}} b_t(\bar{v})^{1-\alpha}\big[f(H)\big]^{\varepsilon_{\bar{v}}-1}\big[\psi(\bar{v})\big]^{(\alpha-\varepsilon_{\bar{v}})/\varepsilon_{\bar{v}}}}{\big[a(L,\underline{v})\big]^{\varepsilon_{\underline{v}}} b_t(\underline{v})^{1-\alpha}\big[f(L)\big]^{\varepsilon_{\underline{v}}-1}\big[\psi(\underline{v})\big]^{\alpha/\varepsilon_{\underline{v}}}}\frac{k_t(\underline{v})}{\big(w_t(H)/w_t(L)\big)}$$

$$= (\varepsilon_{\underline{v}}-\alpha)\frac{\rho\big[k_t(\underline{v})\big]^{\varepsilon_{\underline{v}}}}{\psi(\underline{v})\big[e_t(L)\big]^{(1-\varepsilon_{\underline{v}})\varepsilon_{\underline{v}}/(\alpha-\varepsilon_{\underline{v}})}} \lesseqgtr 0 \quad \text{when } \varepsilon_{\underline{v}} \lesseqgtr \alpha.$$

(ii) To derive the cross-relative wage elasticity with labor effort, if we take the derivatives of (A20) with respect to $e_t(H)$ and $e_t(L)$, respectively, we obtain

$$\phi^t_H = -\Bigg[\frac{(1-\alpha)(1-\rho)\big[f(H)a(H,\bar{v})\big]^{\varepsilon_v}}{\big[e_t(H)\big]^{\alpha(1-\varepsilon_{\bar{v}})/(\alpha-\varepsilon_{\bar{v}})}} + \frac{(1-\varepsilon_{\bar{v}})\rho\big[k_t(\bar{v})\big]^{\varepsilon_{\bar{v}}}}{\big[e_t(H)\big]^{(\alpha-\varepsilon^2_{\bar{v}})/(\alpha-\varepsilon_{\bar{v}})}}\Bigg]$$

$$\times \frac{\left[a(H,\bar{v})\right]^{\varepsilon_{\bar{v}}} b_t(\bar{v})^{1-\alpha}\left[f(H)\right]^{\varepsilon_{\bar{v}}-1}\left[\psi(\bar{v})\right]^{\frac{\alpha-2\varepsilon_{\bar{v}}}{\varepsilon_{\bar{v}}}}}{\left[a(L,\underline{v})\right]^{\varepsilon_{\underline{v}}} b_t(\underline{v})^{1-\alpha}\left[f(L)\right]^{\varepsilon_{\underline{v}}-1}\left[\psi(\underline{v})\right]^{\frac{\alpha-\varepsilon_{\underline{v}}}{\varepsilon_{\underline{v}}}}} \frac{e_t(H)}{\left(w_t(H)/w_t(L)\right)}$$

$$= -\left[(1-\alpha)\frac{(1-\rho)\left[f(H)a(H,\bar{v})\right]^{\varepsilon_v}}{\left[e_t(H)\right]^{\alpha(1-\varepsilon_{\bar{v}})/(\alpha-\varepsilon_{\bar{v}})}} + (1-\varepsilon_{\bar{v}})\frac{\rho\left[k_t(\bar{v})\right]^{\varepsilon_{\bar{v}}}}{\left[e_t(H)\right]^{(\alpha-\varepsilon_{\bar{v}}^2)/(\alpha-\varepsilon_{\bar{v}})}}\right]\frac{e_t(H)}{\psi(\bar{v})} < 0,$$

and

$$\phi_L^t = \left[\frac{(1-\alpha)(1-\rho)\left[f(L)a(L,\underline{v})\right]^{\varepsilon_{\underline{v}}}}{\left[e_t(L)\right]^{\alpha(1-\varepsilon_{\underline{v}})/(\alpha-\varepsilon_{\underline{v}})}} + \frac{(1-\varepsilon_{\underline{v}})\rho\left[k_t(\underline{v})\right]^{\varepsilon_{\underline{v}}}}{\left[e_t(L)\right]^{(\alpha-\varepsilon_{\underline{v}}^2)/(\alpha-\varepsilon_{\underline{v}})}}\right]$$

$$\times \frac{\left[a(H,\bar{v})\right]^{\varepsilon_{\bar{v}}} b_t(\bar{v})^{1-\alpha}\left[f(H)\right]^{\varepsilon_{\bar{v}}-1}\left[\psi(\bar{v})\right]^{\frac{\alpha-\varepsilon_{\bar{v}}}{\varepsilon_{\bar{v}}}}}{\left[a(L,\underline{v})\right]^{\varepsilon_{\underline{v}}} b_t(\underline{v})^{1-\alpha}\left[f(L)\right]^{\varepsilon_{\underline{v}}-1}\left[\psi(\underline{v})\right]^{\frac{\alpha}{\varepsilon_{\underline{v}}}}} \frac{e_t(L)}{\left(w_t(H)/w_t(L)\right)}$$

$$= \left[(1-\alpha)\frac{(1-\rho)\left[f(L)a(L,\underline{v})\right]^{\varepsilon_{\underline{v}}}}{\left[e_t(L)\right]^{\alpha(1-\varepsilon_{\underline{v}})/(\alpha-\varepsilon_{\underline{v}})}} + (1-\varepsilon_{\underline{v}})\frac{\rho\left[k_t(\underline{v})\right]^{\varepsilon_{\underline{v}}}}{\left[e_t(L)\right]^{(\alpha-\varepsilon_{\underline{v}}^2)/(\alpha-\varepsilon_{\underline{v}})}}\right]\frac{e_t(L)}{\psi(\underline{v})} > 0.$$

(iii) Finally, we derive the cross-relative wage elasticity with respect to R&D investments by taking derivatives on (A20) with respect to $b_t(\bar{v})$ and $b_t(\underline{v})$, and we get

$$\varphi_{\bar{v}}^t = \frac{(1-\alpha)\left[a(H,\bar{v})\right]^{\varepsilon_{\bar{v}}}\left[f(H)\right]^{\varepsilon_{\bar{v}}-1}\left[\psi(\bar{v})\right]^{\frac{\alpha-\varepsilon_{\bar{v}}}{\varepsilon_{\bar{v}}}}}{\left[a(L,\underline{v})\right]^{\varepsilon_{\underline{v}}} b_t(\underline{v})^{1-\alpha}\left[f(L)\right]^{\varepsilon_{\underline{v}}-1}\left[\psi(\underline{v})\right]^{\frac{\alpha-\varepsilon_{\underline{v}}}{\varepsilon_{\underline{v}}}}} \frac{b_t(\bar{v})^{1-\alpha}}{\left(w_t(H)/w_t(L)\right)} > 0,$$

$$\varphi_{\underline{v}}^t = \frac{-(1-\alpha)\left[a(H,\bar{v})\right]^{\varepsilon_{\bar{v}}} b_t(\bar{v})^{1-\alpha}\left[f(H)\right]^{\varepsilon_{\bar{v}}-1}\left[\psi(\bar{v})\right]^{\frac{\alpha-\varepsilon_{\bar{v}}}{\varepsilon_{\bar{v}}}}}{\left[a(L,\underline{v})\right]^{\varepsilon_{\underline{v}}}\left[f(L)\right]^{\varepsilon_{\underline{v}}-1}\left[\psi(\underline{v})\right]^{\frac{\alpha-\varepsilon_{\underline{v}}}{\varepsilon_{\underline{v}}}}} \frac{b_t(\underline{v})^{\alpha-1}}{\left(w_t(H)/w_t(L)\right)} < 0.$$

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All authors assume responsibility for all aspects of the paper.