Journal of Macroeconomics 50 (2016) 273–292

Contents lists available at ScienceDirect

Journal of Macroeconomics

journal homepage: www.elsevier.com/locate/jmacro

Relation between growth and unemployment in a model with labor-force participation and adverse labor institutions[☆]

Been-Lon Chen[a,*], Mei Hsu[b,1], Chih-Fang Lai[c,2]

[a] Institute of Economics, Academia Sinica, 128 Academia Road Section 2, Taipei 11529, Taiwan
[b] College of Management, National Taiwan Normal University, 162 He-Ping E. Road Section 1, Taipei 106, Taiwan
[c] Department of Economics, Soochow University, 56 Kuei-Yang Street Section 1, Taipei 100, Taiwan

ARTICLE INFO

Article history:
Received 18 November 2015
Revised 28 July 2016
Accepted 26 October 2016
Available online 2 November 2016

JEL classification:
E24
J64
O41

Keywords:
Labor force
Unemployment
Economic growth

ABSTRACT

Based on labor search models with an exogenous labor force, existing papers have found a negative relation between long-run economic growth and unemployment. Motivated by the fact that the labor force participation has changed substantially across OECD countries, this paper revisits the long-run relation by taking account of endogenous labor-force participation. We find that, via the effects on employment, changes in labor market institutions may increase or decrease long-run economic growth. Moreover, depending upon the effects on the labor force and employment, these labor market institutions may increase or decrease unemployment rates in the long run. Thus, changes in labor market institutions lead to a non-monotone relation between long-run economic growth and unemployment that is consistent with the data.

1. Introduction

Is there a tradeoff between unemployment and economic growth in the long run? The simultaneous slowdown of economic growth and a rise in unemployment in industrial countries in the late 1970s has led numerous economists to believe that there is a negative relation between the two economic variables. Some authors have offered econometric evidence that estimated the effects of total factor productivity growth on unemployment in the long run.[3]

To the best of our knowledge, Bean and Pissarides (1993) was the first theoretical paper that studied the link between unemployment and economic growth in the long run. Using an overlapping-generations model modified to allow for sustainable growth and labor search, their paper found that adverse labor market institutions such as increases in unemployment compensation, vacancy posting costs, and workers' bargaining power all raise unemployment and lower employment and economic growth, and thus there is a negative relation between long-run economic growth and unemployment in the

☆ We have benefited from comments by an anonymous referee. Earlier versions have benefited from discussion with Roger Farmer and suggestions offered from seminar participants at the National Taiwan University and conference participants at the Allied Social Sciences Association Meeting held in San Francisco.

* Corresponding author. Fax: 886 2 2785-3946.
E-mail addresses: bchen@econ.sinica.edu.tw (B.-L. Chen), mhsu@ntnu.edu.tw (M. Hsu), cflai@scu.edu.tw (C.-F. Lai).
1 Fax: 886 2 23648372.
2 Fax: 886 2 23822001.

3 See Pissarides and Vallanti (2007) for the empirical evidence.

http://dx.doi.org/10.1016/j.jmacro.2016.10.004
0164-0704/

Table 1
Labor market and GDP growth rate in selected OECD countries, 1971–74 and 1997–2000.

	LF participation rate			Unemployment rate			Employment rate			GDP growth rate		
	71–74	97–00	diff.	71–74	97–00	diff.	71–74	97–00	diff.	71–74	97–00	diff.
Belgium	60.83	64.80	3.96	2.31	9.96	7.65	59.43	58.36	−1.07	4.83	3.22	−1.61
Denmark	74.55	79.41	4.86	1.65	5.47	3.83	73.32	75.05	1.73	2.53	2.86	0.33
Finland	70.50	73.26	2.76	2.21	11.03	8.82	68.94	65.18	−3.76	5.08	5.12	0.04
France	65.70	68.57	2.87	2.49	9.78	7.29	64.07	61.87	−2.20	5.29	3.13	−2.16
Germany	67.91	70.99	3.08	1.20	8.87	7.66	67.09	64.69	−2.40	3.28	2.13	−1.14
Ireland	63.74	66.82	3.08	6.05	7.18	1.13	59.88	62.07	2.19	4.74	10.46	5.72
Italy	57.30	59.38	2.08	5.93	11.49	5.56	53.90	52.56	−1.34	4.53	2.10	−2.43
Netherlands	56.72	73.05	16.33	2.17	4.06	1.90	55.49	70.09	14.60	4.22	4.21	−0.01
Spain	60.31	63.30	2.99	2.76	17.24	14.48	58.65	52.42	−6.23	6.55	4.53	−2.02
Sweden	75.62	77.15	1.53	2.42	7.92	5.50	73.79	71.04	−2.74	2.60	4.01	1.41
UK	71.52	74.43	2.92	2.55	6.16	3.61	69.69	69.84	0.15	2.84	3.80	0.96
EU	64.99	68.48	3.49	2.81	9.55	6.75	63.16	61.94	−1.22	4.24	3.17	−1.07
US	66.30	75.94	9.65	5.51	4.42	−1.09	62.65	72.59	9.94	3.42	4.47	1.05

Sources: OECD (2014a, 2014b).
Note: The labor force participation rate is the number of the labor force divided by the number of the population aged 15–64. The unemployment rate is the number of unemployed divided by the number of the labor force. The employment rate is the number of employed divided by the population aged 15–64. The GDP growth rate comes directly from OECD (2014b). The EU's GDP growth rate is the population weighted average of the 11 EU countries listed in the table.

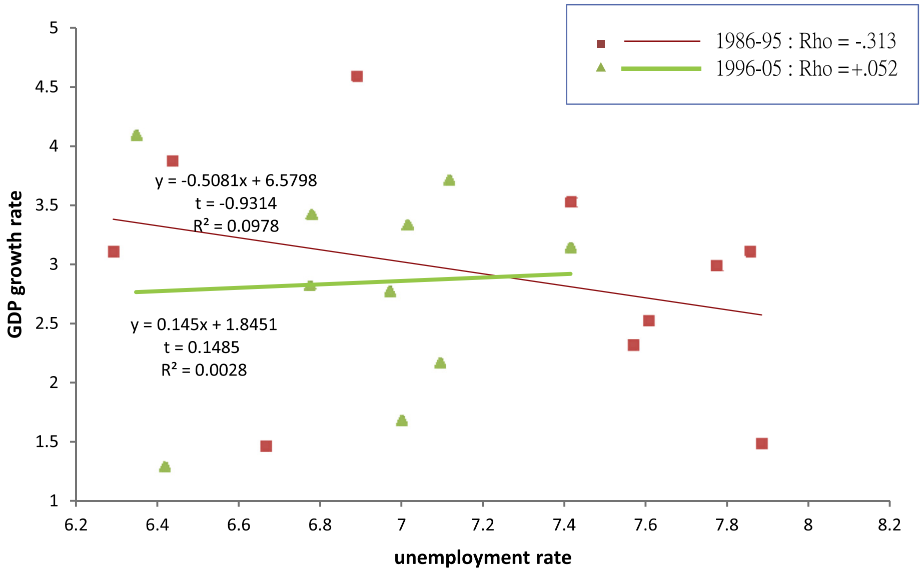

Fig. 1. Relation between economic growth and unemployment in OECD countries. (Sources: OECD (2014a, 2014b))

long run. The same result was obtained in a model which was otherwise the same except for infinitely lived households (Eriksson, 1997).[4]

All the existing related work above assumed a fixed labor force, and thus all agents are either employed or unemployed. Then, changes in adverse labor institutions that increase unemployment will decrease employment, which reduces economic growth. However, the data in the OECD indicate that the labor force is not fixed across countries, but rather has increased substantially. See Table 1, which also suggests that the unemployment rates of all OECD countries, except for the US, have increased. Moreover, while some countries gained GDP growth, other countries lost GDP growth, with GDP growth moving in the same direction as employment. Nevertheless, the data suggest little evidence of a robust connection between long-run economic growth and unemployment of either sign. See Fig. 1, which indicates a statistically insignificant negative link between economic growth and unemployment in 1986–1995 and a statistically insignificant positive one in 1996–2005.[5]

[4] The same long-run negative relation was obtained in models with semi-endogenous growth (Irmen, 2009) and human capital accumulation (Chen et al., 2011).

[5] For relations found in early periods, see Figures 1 and 2 in Bean and Pissarides (1993) which point to a statistically insignificant, negative link in 1955-1965 and a statistically insignificant, positive one in both 1965-1975 and 1975-1985.

Thus, the data offer an ambiguous and not a negative relation between economic growth and unemployment in the long run.

In this paper, we extend the models of Bean and Pissarides (1993) and Eriksson (1997), and revisit the long-run relation between economic growth and unemployment that takes account of the endogenous labor force. Although several papers have analyzed models with endogenous labor forces, a special feature of our paper is that the labor-force participation is modeled as a control variable, as opposed to a state variable as in the existing literature.[6] This modeling strategy has an advantage in that an endogenous labor force is easily introduced into the framework within a representative household, which simplifies the analysis. We investigate changes in adverse labor market policies studied by Bean and Pissarides (1993) and Eriksson (1997) that characterize some of the differences in labor market institutions between the EU and the US. We investigate the relation between economic growth and unemployment in the long run from one steady state to another steady state by analyzing the effects of changes in adverse labor market policies on the labor force, unemployment, employment and economic growth. These effects are compared in the models with and without endogenous labor-force participation.

In the model in which the labor-force participation is fixed, we find that because these adverse labor market policies decrease the firms' net marginal value of employment, unemployment increases and employment decreases, which reduces economic growth in the long run. Thus, as in Bean and Pissarides (1993) and Eriksson (1997), there is a negative relation between long-run economic growth and unemployment. By contrast, in the model with endogenous labor-force participation, the labor force is enlarged by the increase in unemployment compensation, which in turn increases employment and economic growth. Yet, the effect on unemployment is ambiguous, as unemployment increases if the positive labor force effect dominates the positive employment effect, but decreases otherwise. In the case of increases in vacancy posting costs and in workers' bargaining power, the labor force is reduced, which lowers employment and economic growth. Unemployment is also ambiguous in that it increases if the negative employment effect dominates the negative labor force effect, but decreases if otherwise.[7] Therefore, these adverse labor market policies generate a non-monotone link between long-run economic growth and unemployment that is consistent with the data.

We must note that in models with a fixed labor force and technological progress, Aghion and Howitt (1994) and Mortensen and Pissarides (1998) have obtained a non-monotone relation between productivity growth and unemployment that depends on either the creation and capitalization effect or the renovation and the updating cost effect. In our model with an endogenous labor force and labor institutional factors, the non-monotone relation depends on the relative effect between employment and the labor force. Thus, our non-monotone relation based on changes in labor market institutions may be viewed as complementary to the models based on technological progress.[8]

A conceptual roadmap follows. In Section 2, we set up a model wherein the non-employed choose between participating and not participating in the labor force, and the unemployed search for jobs. Individuals' optimizations are analyzed in this section. The balanced growth path is studied in Section 3. In Section 4, we study the effects of adverse labor market policies on labor supply and economic growth. Finally, concluding remarks are offered in Section 5.

2. A simple endogenous growth model with labor search

Our model is based on Bean and Pissarides (1993) and Eriksson (1997), and may be thought of as an integration of the endogenous growth models of Romer (1986) into the labor search models of Merz (1995) and Andolfatto (1996). We extend the model to allow for an endogenous labor force. The economy consists of a continuum of households and firms with a passive fiscal authority.

2.1. The basic economic environment

There is a representative large household which consists of a continuum of family members of unit mass. The setup of a large household is convenient in that family members are homogeneous, equally contributing to and enjoying fam-

[6] Existing theoretical papers have studied different effects of changes in labor market institutions in models with endogenous labor force participation. Early analyses include Burdett et al. (1984) and Andolfatto and Gomme (1996). Pissarides (2000, Ch. 7) developed a general equilibrium matching model with labor force participation wherein there were no flows in and out of the labor market. Garibaldi and Wasmer (2005), Pries and Rogerson (2009) and Krusell et al. (2011) extended this model to generate flows into and out of the labor market. In these models, participation is a state with exogenous random arrival rates in which the participation decision is a binary choice. Compared to these papers, in our study the participation decision is a control variable that trades off between the marginal benefit of non-participation and that of participation made by non-employed people.

[7] Parallel to this literature are business-cycle models that study the effects of adverse labor market institutions in a transition to a steady state, as opposed to studying the effect from a steady state to another steady state as in our model. The standard business-cycle model can generate either sufficiently large cyclical fluctuations in unemployment, or a sufficiently small response of unemployment to adverse labor market institutions, but it cannot do both. See survey papers by Rogerson et al. (2005) and Hornstein et al. (2005). Moreover, variable search and separation, finite unemployment benefit duration, efficiency wages, and capital all fail to resolve this puzzle. In a recent paper, Costain and Reiter (2008) have found that either sticky wages or match-specific productivity shocks can improve the model's performance by making the firm's flow of surplus more procyclical, which makes hiring more procyclical.

[8] In a Schumpeterian model, Aghion and Howitt (1994) studied the effect of an increase in long-run productivity growth, via the introduction of new technology, on unemployment, and found a positive effect when the creation effect is strong and a negative effect when the capitalization effect is strong. In a vintage model, Mortensen and Pissarides (1998) showed that higher productivity growth, via an increase in productivity at the technology frontier, induced lower unemployment when renovation costs are low, but switched to higher unemployment when the cost of updating existing technology is high.

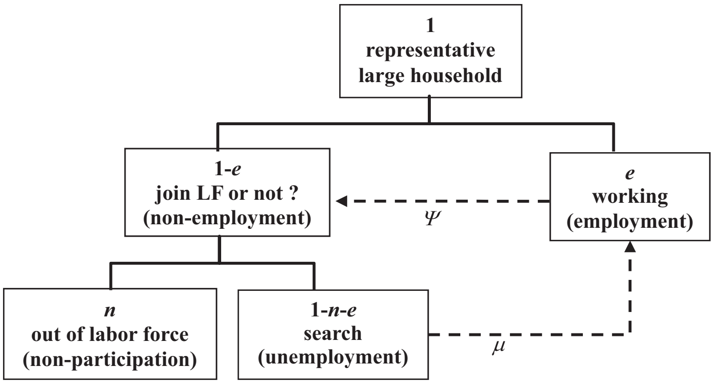

Fig. 2. Labor allocation for the large household.

ily resources regardless of their labor market status. This useful method of modeling perfect consumption insurance in general-equilibrium search models has been common since Merz (1995), Andolfatto (1996) and Chen and Lai (2015). Family members in the household are either employed or non-employed. Denote e as the fraction of employed members in the large household, and then $(1 - e)$ is the fraction of non-employed members. Non-employed members decide whether to participate in the labor force or not. If n is the fraction of members engaging in non-market activities (referred to as non-participants), then $(1 - n - e)$ is the fraction of unemployed members. See Fig. 2 for the labor allocation in the representative large household.

The change in employment from the household's perspective is

$$e_{t+1} - e_t = \mu_t(1 - n_t - e_t) - \psi e_t, \tag{1a}$$

where μ_t is the (endogenous) job finding rate in period t, and ψ is the (exogenous) job separation rate. Thus, the increase in employment in the next period is equal to the inflow of job searchers into the employment pool ($\mu_t(1 - n_t - e_t)$) net of the outflow as a result of job separation (ψe_t).

Denote c_t as consumption and k_t as capital with δ as its depreciation rate. Furthermore, denote w_t and r_t as the wage rate and the rental rate, respectively. The large household's budget constraint is

$$k_{t+1} = w_t e_t + (1 - \delta + r_t)k_t + \pi_t + B_t(1 - n_t - e_t) - T_t - c_t, \tag{1b}$$

where T_t is lump-sum taxes and B_t is unemployment compensation. To be consistent with a perpetual growth framework, we assume that unemployment compensation is proportional to the wage: $B_t = bw_t$, where $b \in (0, 1)$. Unemployed members $(1 - n_t - e_t)$ receive unemployment compensation, and members outside the labor force do not.[9] Households also receive profits π_t remitted from firms as they own firms. The budget constraint stipulates that disposable income is allocated to consumption and savings.

All family members obtain utility from consumption. Moreover, a member obtains a leisure utility when he is outside the labor force or unemployed, with the utility level being χ_1 and χ_2, respectively. The representative large household's utility is simply the sum of utilities over all its members given by:

$$h(c_t, n_t, e_t) = e_t[u(c_t)] + (1 - n_t - e_t)[u(c_t) + \chi_2] + n_t[u(c_t) + \chi_1], u'(c_t) > 0 > u''(c_t). \tag{2}$$

Following Garibaldi and Wasmer (2005), Pries and Rogerson (2009) and Krusell et al. (2011), we use different constant values of leisure utility for members outside the labor force and for members in the labor force searching for jobs. As in these studies, we restrict $\chi_1 > \chi_2$ in order to allow for a non-degenerated fraction of members outside the labor force.[10] In a departure from the linear utility of consumption used in Garibaldi and Wasmer (2005) and Pries and Rogerson (2009), we follow Krusell et al. (2011) and employ an increasing and concave utility of consumption such that the implied intertemporal elasticity of substitution (henceforth IES) is not infinite.

It is well known that in a perpetual growth framework with a utility of leisure, in order to be consistent with the balanced growth path, it is required that the utility of consumption exhibit a constant IES. We thus use the form: $u(c_t) = \frac{1}{1-\sigma}(c_t^{1-\sigma} - 1)$, where $\sigma > 0$ is the reciprocal of the IES.

The production side of the economy features a representative large firm in the sense that it operates many jobs and consequently has many individual workers attached to it through those jobs. The firm creates job vacancies, which entail

[9] There is an issue of moral hazard for unemployment compensation, as it is difficult for the government to find out who actually does job searching (e.g., Shavell and Weiss, 1979; Hopenhayn and Nicolini, 1997). In this paper we simplify the analysis by assuming that the government knows who does job searching, and that only job seekers obtain unemployment compensation.

[10] The assumption $\chi_1 > \chi_2$ captures the notion that because of searching for jobs, an agent has a lower leisure utility than one who does not search for jobs. See Pissarides (2000, Ch7) who also assumed that the leisure utility of an unemployed worker is smaller than that of a non-participant.

costs. The firm rents capital and hires labor to produce the final good y with the production technology given by

$$y_t = f(k_t, e_t) = A_t {e_t}^{\alpha} k_t^{1-\alpha}, \alpha \in (0, 1) \tag{3}$$

In order for the model to exhibit perpetual economic growth, we follow Bean and Pissarides (1993) and Eriksson (1997) and assume the technology level is $A_t = A\bar{k}_t^{\alpha} > 0$, where $A > 0$ is a coefficient and $\bar{k}_t$ is economy-wide average capital in t, which is taken as given by the representative firm. In equilibrium, $\bar{k}_t$ is endogenous and equals k_t.

The production technology (3) features a scale effect in that economic growth rates rise with employment rates. We use the production technology based on the following reasons. First, since our model is based on Bean and Pissarides (1993) and Eriksson (1997), we maintain the same production technology used by these authors. Next, we note that a scale effect remains assumed in several recent papers. For example, in a model of accounting with modern growth theory, Fernald and Jones (2014) have assumed that scale (the population size of countries producing new ideas) matters for idea-based economies. Moreover, the notion that economic growth rises with employment rates is consistent with data in Table 1 wherein, except for Finland, the Netherlands and Sweden, countries gaining economic growth from the early 1970s–2000 are those with rising employment rates.

The large firm creates and maintains multiple job vacancies v_t in order to hire workers. As in Fang and Rogerson (2009), the firm's vacancy posting has an up-front cost λ_t. In order to be consistent with a perpetual growth setup, we assume that vacancy posting cost is in proportion to average capital in the economy: $\lambda_t = \lambda_0 \bar{k}_t$, where $\lambda_0 > 0$ is a coefficient. This setup is natural the more the economy uses capital, the more the firms compete for resources and the greater vacancy posting cost will be. The profit flow of a firm is

$$\pi_t = A_t {e_t}^{\alpha} k_t^{1-\alpha} - w_t e_t - r_t k_t - \lambda_t v_t. \tag{4}$$

The evolution of employment from the perspective of the firm in the economy is

$$e_{t+1} - e_t = \eta_t v_t - \psi e_t, \tag{5}$$

where η_t is the (endogenous) recruitment rate. Thus, the change in employment is equal to the new recruitment ($\eta_t v_t$) net of the outflow (ψe_t).

Finally, there is a passive government. The government levies lump-sum taxes and pays unemployment compensation so as to meet the following budget constraint

$$T_t = B_t(1 - n_t - e_t). \tag{6}$$

It is worth noting that our model includes the following special cases. In the case where $\chi_1 = 0$ in (2), the labor force n_t is exogenous. With an exogenous labor-force participation rate, our model degenerates to an otherwise standard matching model of endogenous growth as analyzed by Bean and Pissarides (1993) and Eriksson (1997).

2.2. Optimization of households and firms

We now analyze the optimization conditions. Denote ρ as the time preference rate. The representative household maximizes its discounted lifetime utility $\sum_{t=0}^{\infty} (\frac{1}{1+\rho})^t h(c_t, n_t, e_t)$subject to the constraints (1a) and (1b). Denote $U(k_t, e_t)$ as the value of the household's discounted lifetime utility when capital is k_t and employment is e_t at the beginning of period t. The first-order conditions with respect to c_t and n_t and the Benveniste–Scheinkman conditions for k_t and e_t are

$$u'(c_t) = \tfrac{1}{1+\rho} U_k(k_{t+1}, e_{t+1}), \tag{7a}$$

$$\chi_1 = \left[u'(c_t)B_t + \chi_2\right] + \left[\tfrac{\mu_t}{1+\rho} U_e(k_{t+1}, e_{t+1})\right], \tag{7b}$$

$$U_k(k_t, e_t) = u'(c_t)(1 - \delta + r_t), \tag{7c}$$

$$U_e(k_t, e_t) = \left[u'(c_t)w_t + \tfrac{1-\psi}{1+\rho} U_e(k_{t+1}, e_{t+1})\right] - \left[u'(c_t)B_t + \chi_2 + \tfrac{\mu_t}{1+\rho} U_e(k_{t+1}, e_{t+1})\right]. \tag{7d}$$

Eqs. (7a) and (7c) give the standard consumption Euler equation between periods t and $t+1$: $u'(c_t) = u'(c_{t+1}) \frac{1-\delta+r_{t+1}}{1+\rho}$. Without taking into account the labor-force participation, (7d) is the marginal value of employment, which is the difference in the marginal value between working and searching for a job. Thus, unemployment compensation B_t is an opportunity cost of employment.

However, if labor-force participation is endogenously chosen, things are different. Condition (7b) trades off participating versus not participating in the labor force. The marginal utility of not participating in the labor force is the leisure utility outside the labor force, χ_1. The marginal benefit of participating in the labor force includes unemployment compensation and the leisure utility when searching for a job (the first brackets in (7b)) as well as the expected discounted future marginal value of employment (the second brackets in (7b)). In particular, (7b) and (7d) together give

$$U_e(k_t, e_t) = \left[u'(c_t)w_t + \tfrac{1-\psi}{1+\rho} U_e(k_{t+1}, e_{t+1})\right] - [\chi_1]. \tag{7e}$$

Thus, with endogenous labor participation, the marginal value of employment is the difference in the marginal value between employment and non-participation. Unlike (7d), unemployment compensation B_t does not affect the marginal value of employment in (7e). Intuitively, as a member could choose to switch from not participating to participating in the labor force and searching for a job, the opportunity cost of employment involves the leisure outside the labor force and does not include the benefit of unemployment.

Next, we envisage the firm's optimization condition. As employment is a state variable, the firm's problem is an optimal control problem. The firm maximizes the discounted sum of profits in (4) subject to the production technology in (3) and the evolution of employment in (5). However, as the economy features sustainable growth, the capital stock grows unboundedly, which causes the profit flow to not be concave such that the firm's problem is not well-defined. To resolve the stationarity problem, we follow Chen et al. (2011) and transform the firm's profit flow π_t into an effective unit by dividing it by the social capital,$\bar{k}_t$. The social capital does not affect the firm's optimization, while it ensures a bounded discounted sum of profits such that the firm's problem is well defined.

Let $\Pi(e_t)$ denote the bounded value of the firm's discounted sum of profits when its employment level is e_t in t. The first-order conditions with respect to k_t and v_t and the Benveniste–Scheinkman condition for e_t are, respectively,

$$(1-\alpha)A\bar{k}_t^{\alpha}e_t^{\alpha}k_t^{-\alpha}=r_t, \tag{8a}$$

$$\frac{\eta_t}{1+\xi_t}\Pi_e(e_{t+1})=\frac{\lambda_t}{\bar{k}_t}=\lambda_0, \tag{8b}$$

$$\Pi_e(e_t)=\frac{1}{\bar{k}_t}(\alpha A\bar{k}_t^{\alpha}(\frac{k_t}{e_t})^{1-\alpha}-w_t)+\frac{1-\psi}{1+\xi_t}\Pi_e(e_{t+1}), \tag{8c}$$

where $\frac{1}{1+\xi_t}\equiv\frac{1}{1+\rho}\frac{u'(c_{t+1})}{u'(c_t)}$ is the firm's discount factor because households are the ultimate owners of firms.[11] While condition (8a) is standard, (8b) indicates that the firm creates the number of vacancies up to the margin when the expected discounted marginal value of recruitment in the next period equals the marginal cost of vacancies. The firm's marginal value of recruitment in this period is given by (8c), which is the sum of the marginal product of labor net of the wage and the discounted marginal value of recruitment in the next period.

2.3. Labor matching and bargaining

The labor market exhibits search frictions with the aggregate flow of matches depending on the masses of job seekers and vacancies. Following Diamond (1982), the matching technology takes the constant-return form: $M_t=m(1-n_t-e_t)^{\beta}(v_t)^{1-\beta}$, where $m>0$ measures the degree of matching efficacy and $\beta\in(0,1)$ is the contribution of job seekers in matching. The matching function facilitates the endogenous determination of job finding rates and recruitment rates.

A household's surplus from a successful match is evaluated by the marginal value of employment, which is $U_e(k_t, e_t)$. A firm's surplus is evaluated by the marginal value of recruitment, which is $\Pi_e(e_t)$. Notice that both the household's surplus and the firm's surplus have already taken into account the outside options. According to (7e), the household's surplus is the discounted sum of wage in unit of utilities $u'(e_t)w_t$ minus the outside option of employment, which is the leisure utility of being outside the labor force, χ_1. According to (8c), the firm's surplus is the discounted sum of the marginal product of labor net of the wage. Since the opportunity cost of a vacancy is the marginal cost of vacancies, (8b) dictates that the firm creates vacancies up to the margin when the discounted marginal value of recruitment in the next period equals the marginal cost of vacancies.

Following the conventional wisdom, the wage is determined by a matched worker-job pair through a cooperative bargaining game. In the game, the following joint surplus is maximized: $[U_e(k_t, e_t)]^{\gamma}[\Pi_e(e_t)]^{1-\gamma}$, where $\gamma\in(0,1)$ is the workers' bargaining power. In solving the wage bargaining problem, the worker-job pair treats as given the matching rates (μ_t and η_t), the beginning-of-period level of employment (e_t), and the market interest rate (r_t). The worker also takes as given the wage of all others. The first-order condition of the bargaining game is

$$\frac{\gamma}{U_e(k_t,e_t)}\frac{dU_e(k_t,e_t)}{dw_t}+\frac{1-\gamma}{\Pi_e(e_t)}\frac{d\Pi_e(e_t)}{dw_t}=0. \tag{9}$$

Thus, the wage is determined on the margin wherein the effect of changes in the wage on the marginal value of employment and the effect on the marginal value of recruitment are summed to zero.

2.4. The aggregate resources and equilibrium

The economy faces an aggregate goods constraint which, using (1b), (4) and (6), is

$$c_t+k_{t+1}-(1-\delta)k_t=r_tk_t+w_te_t+\pi_t=A(e_t)^{\alpha}k_t-\lambda_0v_tk_t. \tag{10}$$

[11] Using (7a) and (7c), the discount factor is thus $\frac{1}{1+\xi_t}=\frac{1}{1+(r_{t+1}-\delta)}$.

A search equilibrium is households' choices $\{c_t, n_t, k_{t+1}, e_t\}$, firms' choices $\{v_t, k_t, e_t\}$, prices $\{w_t, r_t\}$, matching rates $\{M_t, \mu_t, \eta_t\}$ and transfers $\{T_t\}$, such that: (i) households optimize; (ii) firms optimize; (iii) the employment evolution conditions hold; (iv) labor-market matching and wage bargaining conditions are met; (v) the government budget is balanced; and (vi) the goods market clears.

A long-run search equilibrium is a balanced growth path (henceforth, BGP) along which the rental rate r, employment e, the labor force $(1-n)$, vacancies v, and matching rates M, μ and η are all constant, and consumption c, capital k and the wage rate w all grow at the same rate. In order to analyze the BGP, we will transform the perpetually growing variables of consumption, capital and wage into the great ratios of c/k and w/k.

3. The balanced growth path

In a BGP, the labor market satisfies the matching relations (the Beveridge curve) given by $m(1-n-e)^{\beta}(v)^{1-\beta}=\mu(1-n-e)=\eta v=\psi e$. The number of matched pairs equals the employment inflow both from the household side, $\mu(1-n-e)$, and from the firm side, ηv, and in the long run, is equal to the employment outflow. These relations enable us to solve the job finding rate, the recruiting rate and equilibrium vacancies as functions of e and n.

$$\mu = \mu(e,n) \equiv \frac{\psi e}{(1-n-e)}, \tag{11a}$$

$$\eta = \eta(e,n) \equiv \left[m\left(\frac{1-n-e}{\psi e}\right)^{\beta}\right]^{\frac{1}{1-\beta}}, \tag{11b}$$

$$v = v(e,n) \equiv \left[\frac{\psi e}{m(1-n-e)^{\beta}}\right]^{\frac{1}{1-\beta}}. \tag{11c}$$

As more employment (a higher e) decreases but more labor-force participation (a higher $1-n$ and thus, lower n) increases the number of job seekers, the job finding rate and the job vacancies are increasing in employment and decreasing in participation, while the recruitment rate is decreasing in employment and increasing in participation. These relations give $\frac{\eta}{\mu} = \frac{1-e-n}{v}$ which measures the degree of the labor market tightness.

Along the BGP, (8a) yields the rental rate $r=(1-\alpha)Ae^{\alpha} \equiv r(e)$. Then, using (7a) and (7c), the discount rate of the firm is $\xi=r(e)-\delta$. Moreover, (7a) and (7c) yield $(1+g_{t+1})^{\sigma} = \frac{u'(c_t)}{u'(c_{t+1})} = \frac{1+r_{t+1}-\delta}{1+\rho}$, where g_t is the economic growth rate in t. Along the BGP, $g = (\frac{1+(1-\alpha)Ae^{\alpha}-\delta}{1+\rho})^{\frac{1}{\sigma}} - 1 \equiv g(e)$. The economic growth rate is positive if the technology level A is sufficiently large.[12] The goods market clearing condition (9) gives the following consumption to capital ratio.

$$\frac{c}{k} = [Ae^{\alpha} - \lambda_0 v(e,n) - \delta - g(e)] \equiv z(e,n;\lambda_0). \tag{12a}$$

In (12a), higher employment has a positive direct effect on consumption due to the resulting increases in output, but it also has a negative indirect effect on consumption due to a higher vacancy cost. As proposed by Fang and Rogerson (2009), the direct effect dominates the indirect effect. Thus, consumption is increasing in employment.

Furthermore, (8c) is the marginal value of recruitment. In a BGP, it is

$$\Pi_e = \frac{1+r(e)-\delta}{\psi+r(e)-\delta}(MPL - \frac{w}{k}), \tag{12b}$$

where $MPL=\alpha Ae^{-(1-\alpha)}$.

To analyze the BGP, we will simplify the equilibrium conditions in terms of two relations. One is the vacancy and employment condition which trades off between the marginal benefit of recruitment and the marginal cost of vacancy creation. The other is the labor participation condition of the household's tradeoff between participating and not participating in the labor force. We start by simplifying the equilibrium conditions in the model with endogenous labor-force participation, followed by the model with exogenous labor-force participation.

3.1. The model with endogenous labor-force participation

When the labor-force participation is endogenous, (7e) measures the household's surplus from a successful match. In a BGP, it is

$$U_e = \frac{1+\rho}{\rho+\psi}\left[u'(c)w - \chi_1\right]. \tag{13}$$

Using (13) and the firm's surplus from a match in (12b), (9) gives the following bargained wage

$$w = \gamma[MPL \cdot k] + (1-\gamma)\left[\frac{\chi_1}{u'(c)}\right]. \tag{14}$$

[12] The condition is imposed throughout the rest of the paper.

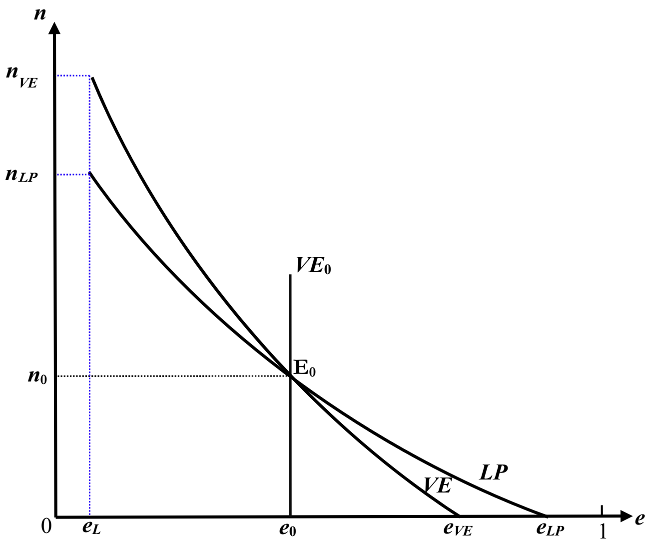

Fig. 3. Existence and uniqueness of BGP.

Obviously, the bargained wage is a weighted average of the marginal product of labor and the opportunity cost of employment. The wage is consistent with the BGP only if the utility of consumption is logarithmic such that $\frac{1}{u'(c)} = c$, which requires $\sigma = 1$. The requirement is the same as that in the two-sector, endogenous growth model put forth by Benhabib and Perli (1994).[13]

Now we are ready to derive two simplified equilibrium conditions that solve employment and the labor force. First, (8b) is the **Vacancy and Employment** (henceforth, VE) condition, which equates the firm's marginal cost of vacancies with the marginal value of recruitment. With the use of the recruitment rate (11b), the marginal value of recruitment (12b) and the firm's discount rate $\xi = r(e) - \delta$, the VE condition is $\frac{\eta(e,n)}{r(e)-\delta+\psi}[MPL - \frac{w}{k}] = \lambda_0$. With the use of (12a) and (14), the VE condition is

$$\Gamma(e, n; \gamma)[\alpha A(e)^{-(1-\alpha)} - \chi_1 z(e, n; \lambda_0)] - \lambda_0 \equiv \Omega(e, n) = 0, \tag{15}$$

where $\Gamma(e, n; \gamma) \equiv (1-\gamma)\frac{\eta(e,n)}{\psi-\delta+r(e)} > 0$.

Next, (7b) is the **Labor Participation** (henceforth, LP) condition, which equates the household's marginal value of participation with the marginal cost of participation. If we use the job finding rate (11a) and the household's marginal value of employment (13), the LP condition is $\frac{\mu(e,n)}{\rho+\psi}[\frac{w}{c} - \chi_1] + b\frac{w}{c} + \chi_2 = \chi_1$, which can be rewritten as $\frac{\mu(e,n)}{\rho+\psi}[\frac{w}{k} - \chi_1\frac{c}{k}] + b\frac{w}{k} + \chi_2\frac{c}{k} = \chi_1\frac{c}{k}$. By using (11b) and (12a) and (14), the LP condition is

$$\left(\tfrac{\mu(e,n)}{\rho+\psi} + b\right)\gamma\alpha A(e)^{-(1-\alpha)} + \chi_2 z(e, n) - \left(\tfrac{\mu(e,n)}{\rho+\psi}\gamma - b(1-\gamma) + 1\right)\chi_1 z(e, n) \equiv \Lambda(e, n) = 0. \tag{16}$$

Both the VE and LP conditions give relations between employment and labor participation. In the (e, n) plane, they are referred to as Locus VE and Locus LP, respectively. To determine the BGP, we analyze the slope of the two loci.

In the Appendix, we have shown that in the VE locus, more employment (a higher e) decreases the marginal value of recruitment because of the resulting decreases in the recruitment rate and increases in the effective discount rate. Moreover, under a sufficiently large technology level A, a smaller labor-force participation rate (a higher n) also decreases the marginal value of recruitment due to the resulting decreases in the recruitment rate and increases in the outside option. Hence, the VE locus is negatively sloping in the (e, n) plane. See Fig. 3. Intuitively, as increases in employment lower the marginal value of recruitment, labor participation will increase in order to increase the marginal value of recruitment.

In the Appendix, we have also shown that, in the LP locus, under a sufficiently large technology level A, more employment increases the net marginal value of participation because of the resulting increases in the job finding rate. Moreover, when the time-preference rate ρ is sufficiently small, then because of the resulting increases in the job finding rate, smaller labor-force participation and thus larger non-participation increases the net marginal value of participation. Hence, the LP locus is negatively sloping in the (e, n) plane. Intuitively, if employment is increased, the net marginal value of participation is enlarged. Then, labor participation will decrease in order to reduce the net marginal value of participation.

[13] In a human capital-based endogenous growth model with a complete labor market, Benhabib and Perli (1994) showed that if there is utility of leisure in intensive margins, the utility of consumption must be logarithmic in order to be consistent with the BGP. Although our model does not involve human capital, with the utility of leisure in both extensive margins and participation margins our model also requires a logarithmic utility of consumption in order to be consistent with the BGP.

With two downward-sloping loci, the VE locus may not intersect the LP locus. In the Appendix, we have shown that the VE locus intersects the LP locus once, and thus there exists a steady state. The two curves need to have relative slopes that yield a new equilibrium when parameters are changed. This requires that the Locus LP be flatter than the Locus VE at each intersection.[14] The requirement ensures a unique steady state. The unique BGP is illustrated by E_0 in Fig. 3 in which the pair is (e_0, n_0).

With employment and the labor force in the BGP, we can solve for other variables. In particular, unemployment is $(1\text{-}e_0\text{-}n_0)$ and the long-term economic growth rate is $g_0 = \frac{(1-\alpha)Ae_0^{\alpha}-\delta-\rho}{1+\rho}$.

3.2. The model with exogenous labor-force participation

If the labor-force participation is exogenous, $n = \bar{n}$. The model then degenerates to existing matching models with exogenous labor-force participation studied by Bean and Pissarides (1993), Eriksson (1997) and others. The Locus LP is not an equilibrium condition. Moreover, the VE locus changes because the outside option of employment is unemployment, rather than non-participation. With a given labor force, the household's surplus from a successful match is (7d). In a BGP, it is

$$U_e = \frac{1+\rho}{\rho+\psi+\mu}\Big[u'(c)w - [u'(c)B + \chi_2]\Big]. \tag{17}$$

In comparison with (13), two remarks are in order. First, compensation to the unemployed B decreases the household's surplus in (17) but does not affect (13). Next, under an exogenous labor force, the value of unemployment includes the prospect of employment whose value is increasing in the job finding rate μ. As a higher job finding rate increases the value of unemployment, this in turn reduces the household's surplus from a job match in (17), as opposed to a zero effect of a higher job finding rate on the household's surplus under endogenous participation in (13).

The household's surplus from a match in (17), along with the firm's surplus from a match in (12b), gives the following bargained wage.[15]

$$w = \gamma[MPL\cdot k] + (1-\gamma)[B + \tfrac{\chi_2}{u'(c)}] = \tfrac{\gamma}{1-(1-\gamma)b}[MPL\cdot k] + \tfrac{1-\gamma}{1-(1-\gamma)b}[\tfrac{\chi_2}{u'(c)}], \tag{18}$$

As in (14), the bargained wage is a weighted average of the marginal product of labor and the opportunity cost of employment. Unlike (14), the opportunity cost of employment includes an unemployment payment and leisure utilities in unemployment χ_2. The feasibility requires that $(1-\gamma)b < 1$, which is clearly met, given $\gamma < 1$ and $b < 1$.

Using (18), along with (11b), (11c) and (12a), the VE condition under an exogenous labor force is

$$\tfrac{\Gamma(e;\bar{n},\gamma)}{1-(1-\gamma)b}[(1-b)\alpha A(e)^{-(1-\alpha)} - \chi_2 z(e,\bar{n};\lambda_0)] - \lambda_0 \equiv \Phi(e;\bar{n}) = 0, \tag{19}$$

where $\Gamma(e,\bar{n};\gamma) \equiv (1-\gamma)\frac{\eta(e,\bar{n})}{\psi-\delta+r(e)} > 0$.

In comparison with (15), (19) is otherwise the same except for two differences. First, as the outside option of employment is unemployment, a higher leisure utility in unemployment χ_2 reduces the marginal value of employment in (19). Next, a higher unemployment compensation b increases the bargained wage and thus reduces the firm's marginal value of employment in (19). Suppose that n is fixed at n_0 in Fig. 3. Then, (19) uniquely determines a vertical VE_0 locus at $e = e_0$.

4. Relation between unemployment and economic growth

In this section, we study the relation between long-run economic growth and unemployment. Like Bean and Pissarides (1993) and Eriksson (1997), we explore the following three types of adverse labor market policies: increases in unemployment compensation, vacancy posting costs, and workers' bargaining power. These characterize some of the differences in labor market institutions between the EU and the US. The effects of these changes in labor market policies help us understand the relation between long-run economic growth and unemployment.

We separate the effects in the model with and without endogenous labor-force participation. We begin our analysis by analyzing the model with exogenous labor-force participation. The comparative-statics analysis is relegated to the Appendix.

4.1. The model with exogenous Labor-force participation

4.1.1. Unemployment compensation

First, we envisage the effects of increases in unemployment compensation (higher b). Suppose that the initial BGP is (e_0, n_0) at E_0 in Fig. 4. When the labor force is exogenous, the initial BGP at E_0 may be thought of as being determined at the intersection of the vertical VE_0 locus $e = e_0$ and the horizontal line $n = n_0$.

[14] To see this, it is expected that a higher leisure utility of unemployment χ_2 attracts labor force participation and thus decreases n. Moreover, a higher leisure utility of unemployment shifts the Locus LP downward without shifting the Locus VE. However, should the Locus LP be steeper than the Locus VE, the labor force participation would then decrease rather than increase.

[15] The second equality in (18) follows by substituting in unemployment compensation which is in proportion to the wage, $B = bw$.

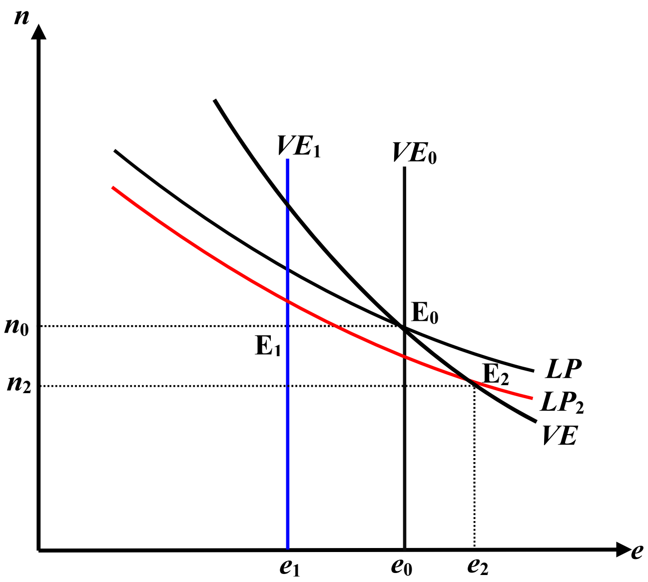

Fig. 4. Effects of increases in unemployment compensation.

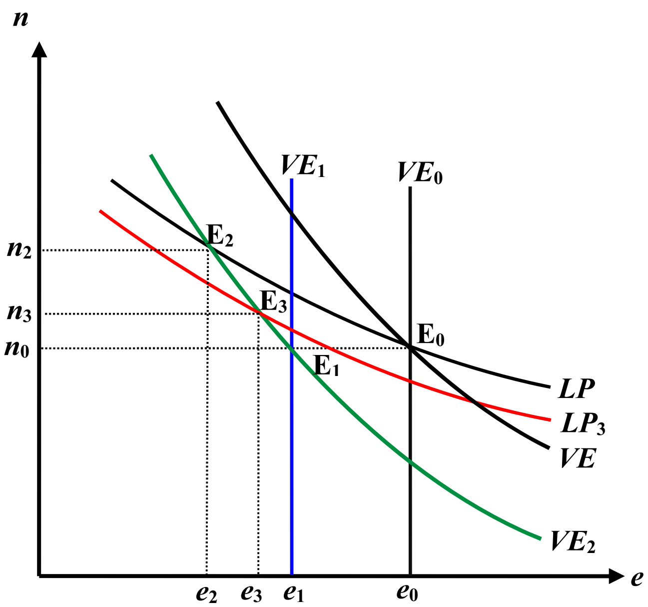

Fig. 5. Effects of increases in vacancy posting costs and workers' bargaining power.

Now, without a choice of labor-force participation, the outside option of employment is unemployment. Then, increases in unemployment compensation raise the opportunity cost of employment and lower the firms' marginal value of recruitment. As a result, employment is decreased, thus increasing the marginal value of recruitment, thereby shifting the vertical VE$_0$ locus leftward to VE$_1$. The BGP is at E$_1$ and employment falls to e_1. Consequently, the economic growth rate declines from $g_0 = \frac{(1-\alpha)Ae_0^{\alpha}-\delta-\rho}{1+\rho}$ to $g_1 = \frac{(1-\alpha)Ae_1^{\alpha}-\delta-\rho}{1+\rho}$. Since the labor force is fixed at 1-n_0, unemployment increases from $(1-n_0-e_0)$ to $(1-n_0-e_1)$. Therefore, there is a negative relation between long-run economic growth and unemployment.

4.1.2. Vacancy posting costs and workers' bargaining shares

Next, we envisage the effects of increases in vacancy posting costs (a higher λ_0) and workers' bargaining power (a higher γ). Suppose that the initial BGP is at E$_0$ in Fig. 5. With an exogenous labor force, the initial BGP at E$_0$ is at the intersection of the vertical VE$_0$ locus $e = e_0$ and the horizontal line $n = n_0$.

Now, increases in vacancy posting costs directly amplify the marginal cost of recruitment, but with a decrease in the bargained wage, they also indirectly raise the firms' marginal value of recruitment. In the Appendix we have shown that under a large productivity A and thus a large marginal product of labor, the direct effect dominates the indirect effect and the firms' net marginal value of employment declines, thereby shifting the vertical VE_0 locus leftward (cf. VE_1 in Fig. 5). The new BGP is E_1. As a result, employment falls from e_0 to e_1 and the economic growth rate declines from g_0 to $g_1 = \frac{(1-\alpha)Ae_1^{\alpha}-\delta-\rho}{1+\rho}$. Since the labor force is fixed at $1-n_0$, unemployment increases from $(1-n_0-e_0)$ to $(1-n_0-e_1)$. Moreover, increases in workers' bargaining power raise that bargained wage, which decreases the marginal value of recruitment, thus shifting the VE_0 locus leftward (cf. VE_1 in Fig. 5).[16] Thus, the effects are similar to those of increases in vacancy posting costs. Then, there is a negative relation between economic growth and unemployment.

To summarize the effects in the model with an exogenous labor force, we obtain

Proposition 1. *In a matching model of endogenous growth with an exogenous labor force, as unemployment compensation, vacancy posting costs and workers' bargaining power increase, employment and economic growth decrease and unemployment increases, thereby resulting in a negative relation between long-run economic growth and unemployment.*

4.2. The model with endogenous Labor-force participation

4.2.1. Unemployment compensation

First, the effects of increases in unemployment compensation are illustrated in Fig. 4. Here, the initial BGP at E_0 is determined by the intersection of Loci VE and LP in the figure. With an active participation margin, the outside option of employment is non-employment. Then, unemployment compensation paid to the unemployed is not an opportunity cost of employment. Hence, unlike in the model with an exogenous labor force, the VE locus does not shift. Instead, increases in unemployment compensation augment the household's marginal value of participation, which shifts Locus LP downward to LP_2. Thus, the labor force increases in size.

Now, the Locus VE is not vertical but is negatively sloping. Increases in the size of the labor force raise the firm's marginal value of recruitment, and so employment increases. The new BGP is E_2. Thus, the size of the labor force increases from $(1-n_0)$ to $(1-n_2)$ and employment increases from e_0 to e_2.[17] Because of higher employment, economic growth increases from g_0 to $g_2 = \frac{(1-\alpha)Ae_2^{\alpha}-\delta-\rho}{1+\rho}$, as opposed to a decrease to g_1 under a fixed labor force.

Yet, owing to the positive effect on both the labor force and employment, the effect on unemployment $(1-n_2-e_2)$ may be larger or smaller than the initial level $(1-n_0-e_0)$. If the positive effect on the labor force dominates, unemployment increases; otherwise, unemployment decreases.

It should be noted that Sattinger (1995) and Garibaldi and Wasmer (2005) have studied the effects of increases in unemployment compensation in models with an endogenous labor force. In Sattinger (1995), increases in unemployment compensation, financed by distortionary wage taxes or output taxes, have an ambiguous effect on employment but unambiguously raise unemployment. In Garibaldi and Wasmer (2005), in a partial equilibrium setup with a fixed job-finding rate, increases in unemployment compensation raise participation entries and have ambiguous effects on exits. Our model is different from these two studies. First, in our model, unemployment compensation is financed by lump-sum taxes so as to isolate it from the effects of distortionary taxes. Thus, unlike Sattinger (1995), there is an ambiguous effect on unemployment, but employment is raised unambiguously. Next, although our model is like that of Garibaldi and Wasmer (2005) wherein increases in unemployment compensation enlarge the labor force, our result is obtained in a general equilibrium setup as opposed to a partial equilibrium setup as in Garibaldi and Wasmer (2005). More importantly, these two papers did not study the relation between long-run economic growth and unemployment.

4.2.2. Vacancy posting costs and workers' bargaining shares

Next, we analyze the effects of increases in vacancy posting costs and workers' bargaining power. The effects are illustrated in Fig. 5 with the initial BGP being at E_0. As in the model with an exogenous labor force, increases in vacancy posting costs and workers' bargaining power reduce the firm's marginal value of recruitment and shift the VE locus leftward (cf. VE_2 in Fig. 5). With a negatively sloping Locus LP, even if the locus does not shift, the new BGP would be at E_2. Then, the size of the labor force would decrease considerably from $(1-n_0)$ to $(1-n_2)$ such that employment declines substantially from e_0 to e_2.

Now, the Locus LP also shifts. Firstly, increases in vacancy posting costs reduce both the leisure utility of non-participation and the leisure utility of unemployment in units of consumption. We have shown that under a large productivity A, there is a large marginal value of participation, and then the former effect dominates and thus the labor force increases. As a result, the Locus LP shifts downward to LP_3 in Fig. 5. Moreover, increases in the workers' wage bargaining power raise the bargained wage and thus increase the marginal value of participation. Thus, the labor force increases. Hence, the Locus LP

[16] Although we draw the shift of the Locus VE to VE_1 in Figure 5, this is done for ease of illustration in one figure, and readers must keep in mind that this does not literally mean that the levels of these shifts are the same.

[17] Our result is consistent with the estimates obtained by Barnichon and Figura (2015). These authors used matched CPS micro-data to estimate an empirical model of nonparticipants' propensity to want a job, and found that increases in the provision of welfare and social insurance raised the labor force participation.

shifts downward (cf. LP_3 in Fig. 5).[18] Therefore, the BGP moves from E_2 to E_3, so that employment is reduced by less.[19] Yet, because of a decrease in the labor force from $(1-n_0)$ to $(1-n_3)$, the new employment level e_3 is still less than the employment e_1 under a fixed labor force. As a result, economic growth decreases from g_0 to $g_3 = \frac{(1-\alpha)Ae_3^{\alpha}-\delta-\rho}{1+\rho}$, which is less than g_1 under an exogenous labor force.

Unemployment changes to $(1-n_3-e_3)$ which, because of the negative effect on both the labor force and employment, may be larger or smaller than the initial level $(1-n_0-e_0)$. If the negative effect on employment dominates, then unemployment increases; otherwise, unemployment decreases.

To summarize our main results, we obtain

Proposition 2. *In a matching model of endogenous growth with an endogenous labor force,*

(i) *when unemployment compensation increases, the labor force, employment and long-run economic growth all increase, but unemployment is ambiguous, increasing if the positive effect on the labor force dominates the positive employment effect and decreasing if otherwise*;

(ii) *when vacancy posting costs and the workers' wage bargaining power increase, the labor force, employment and economic growth all decline, but unemployment is ambiguous, increasing if the negative employment effect dominates the negative labor force effect and decreasing if otherwise*.

To recap the theoretical findings, in the model with an exogenous labor force, adverse labor market institutions lead to a negative relation between long-run economic growth and unemployment. Conversely, when the labor force is endogenous, these adverse labor market institutions yield a non-monotone relation between long-run economic growth and unemployment.

4.3. Quantitative analysis

In this subsection, we offer simple quantitative exercises to understand the effects of adverse labor market policies on employment, unemployment, economic growth, and the relation between long-run economic growth and unemployment. Most parameters follow the usual practice and are set to match the US annual data in 1971–2000.[20]

4.3.1. Calibration

First, calculating the US data in 1971–2000 yields the average fraction of employment in the working-age population at 69.01%, the average unemployment rate at 6.47% and the annual per capita real economic growth rate at 3.34%. The data give $e=0.6901$, $(1-n-e)/(1-n)=6.47\%$ and $g=3.34\%$. These data give $n=0.2622$, and thus the labor-force participation rate is $1\text{-}n=0.7378$. Based on the Census Population Survey in the US, Shimer (2012) constructed a time series dataset of the quarterly job finding rate with the average of 57.41% in 1971–2000.[21] We go along with this rate and translate it into an annual rate of $\mu=1\text{-}(1\text{–}0.5741)^4=0.9671$. Then, from the matching relations, the annual separation rate is computed at $\psi=\mu(1-n-e)/e=0.0669$.[22] Moreover, following Shimer (2005), we normalize the steady-state market tightness $(1-n-e)/v$ to unity, which gives $v=0.0477$.[23] Then, we use the matching relations to calibrate the recruitment rate and matching efficacy $\eta=m=0.9671$.

Next, as in Andolfatto (1996), we choose the share of capital at $1\text{-}\alpha=0.36$. As in Kydland and Prescott (1991), we use $\rho=4\%$ as the annual rate of time preference. Chen et al. (2011) employed 2% as the quarterly rate of the capital depreciation, which we follow and thus set $\delta=0.08$. Then, we calibrate the rental rate to target the annual per capita real economic growth rate of $g=3.34\%$, which gives $r=0.1547$. Then, we use (8a) to calibrate $A=0.5450$. Using the production function, the annual capital-output ratio is $k/y=2.3265$.

Finally, the consumption-output ratio of the U.S. economy is around 0.7.[24] We utilize the data and (12a) to calibrate $\lambda_0=0.3258$ and then apply (14) to compute the bargained wage to capital ratio $w/k=0.3509$. We follow Shimer (2005) to set the ratio of unemployment compensation to the wage at $b=40\%$. Moreover, we follow Ljungqvist and Sargent (2007) and Garibaldi and Wasmer (2005) to place the job seeker's contribution in matching at $\beta=50\%$. Then, the Hosios (1990) condition allows us to pin down the workers' bargaining power at $\gamma=\beta$. Finally, when labor participation is endogenous, we

---

[18] We draw the shift of the Locus LP to LP_3 in Fig. 5, but this is done for ease of illustration in one figure and readers must keep in mind that this does not literally mean that the levels of these shifts are the same.

[19] Based on our numerical results in the next section, we rule out the case wherein the Locus LP in Figure 5 shifts downward so much that the intersection of the new LP locus and the Locus VE_2 results in an employment level larger than e_1.

[20] Alternatively, we can calibrate our model to match the US quarterly data. We find that the results are similar. As our focus is a long-run relationship, we here report the results when annual data are matched.

[21] This dataset is available in http://home.uchicago.edu/~shimer/data/flows/.

[22] Based on Shimer (2012), the quarterly average separation rate is 3.8% in 1971-2000 and the corresponding annual rate is $\psi=1\text{-}(1\text{-}0.0380)^4=0.1436$. We are aware that the calibrated value of ψ is lower than the value implied by the data. However, since we have used the data of the employment, we lost the degree of freedom and must calibrate the value of the separation rate in order to be consistent with the Beveridge curve relationship.

[23] We will carry out robustness checks later and confirm that different values of market tightness give the same quantitative results.

[24] The data of the economic growth rate and the consumption-output ratio in the US are obtained from the **Penn World Table** (http://pwt.econ.upenn.edu/php_site/pwt_index.php**)**.

Table 2
Benchmark parameter values and calibration.

Benchmark parameters & observables	Variables	Annually	Source
fraction of employment	e	0.6901	US Data
unemployment rate	$\frac{1-n-e}{1-n}$	0.0647	US Data
unemployment-vacancy ratio	$\frac{1-n-e}{v}$	1.0000	Shimer (2005)
job finding rate	μ	0.9671	US Data
time preference rate	ρ	0.0400	Kydland and Prescott (1991)
depreciation rate of capital	δ	0.0800	Chen et al. (2011)
capital's share	$1-\alpha$	0.3600	Andolfattot (1996)
growth rate	g	0.0334	US Data
consumption-output ratio	c/y	0.7000	US Data
unemployment compensation rate	b	0.4000	Shimer (2005)
labor's share in matching function	β	0.5000	Ljungqvist and Sargent (2007)
labor's bargaining power	γ	0.5000	Hosios rule
Calibration			
fraction of non-participants	n	0.2622	
labor-force participation rate	L^f	0.7378	
vacancy creation	v	0.0477	
job separation rate	ψ	0.0669	
employee recruitment rate	η	0.9671	
rate of return to capital	r	0.1547	
coefficient of production technology	A	0.5450	
capital-output ratio	k/y	2.3265	
consumption-capital ratio	z	0.3009	
unit cost of vacancy posting	λ_0	0.3258	
wage-capital ratio	w/k	0.3509	
coefficient of matching function	m	0.9671	
leisure utility of unemployed (exog. n)	χ_2	0.5413	
leisure utility of unemployed (endo. n)	χ_2	−0.8933	
leisure utility of non-participants	χ_1	1.0078	

Table 3
Numerical results: baseline (%).

	$(1-n)$	$\Delta(1-n)$	e	Δe	$(1-n-e)$	$\Delta(1-n-e)$	g	Δg
Benchmark	73.779	0	69.007	0	4.772	0	3.340	0
Exog. LF								
$b \uparrow 40\%$	73.779	0	62.135	−6.87	11.644	6.87	2.374	−0.97
$\lambda_0 \uparrow 40\%$	73.779	0	67.645	−1.36	6.134	1.36	3.151	−0.19
$\gamma \uparrow 40\%$	73.779	0	66.083	−2.92	7.696	2.92	2.933	−0.41
Endog. LF								
$b \uparrow 40\%$	75.138	1.36	69.965	0.96	5.172	0.40	3.472	0.13
$\lambda_0 \uparrow 40\%$	71.991	−1.79	66.517	−2.49	5.474	0.70	2.994	−0.35
$\gamma \uparrow 40\%$	73.544	−0.24	66.520	−2.49	7.024	2.25	2.995	−0.35

Note: all results are expressed in %. Δx means changes in x from the benchmark value. $(1-n)$ is the labor-force participation rate; e is the employment rate; $(1-n-e)$ is the unemployment rate.

utilize (13) to calibrate the leisure utility of the non-participating $\chi_1 = 1.0078$ and (16) to calibrate the leisure utility of the unemployed $\chi_2 = -0.8933$. When labor participation is exogenous, there is no χ_1, and (17) calibrates the leisure utility of the unemployed $\chi_2 = 0.5413$. The baseline parameter values, observables and calibrated values are in Table 2. Under the baseline parameter values, we obtain a unique BGP.

4.3.2. Quantifying the effects

Now, we quantify the effects of more adverse labor market policies on the labor allocation and economic growth in the long run. We carry out the exercise by increasing each of unemployment compensation (b), vacancy posting costs (λ_0) and workers' bargaining power (γ) by 40%.[25] The quantitative results in the baseline parameter values are demonstrated in Table 3.

First, the results indicate that, under an exogenous labor force, all these changes in labor market policies decrease employment and long-run economic growth ($\Delta e < 0$, $\Delta g < 0$) and increase unemployment ($\Delta(1\text{-}n\text{-}e) > 0$). See the top panel in

[25] We use a 40% increase in unemployment compensation because, according to the OECD (1999, Table 2.2), the population weighted average unemployment payment rate in the EU in the late 1990s was 69.72% which is roughly 40% higher than the 50% average unemployment payment rate in the US. Although the differences in the two latter types of labor market institutions in the EU from the US may not be 40%, for simplicity we conduct exercises where we increase them by 40%. The results are the same if different percentages are used.

Table 4
Numerical results when $\frac{1-n-e}{v} = 0.5$ (%).

	$(1-n)$	$\Delta(1-n)$	e	Δe	$(1-n-e)$	$\Delta(1-n-e)$	g	Δg
Benchmark	73.779	0	69.007	0	4.772	0	3.340	0
Exog. LF								
b ↑40%	73.779	0	62.135	−6.87	11.644	6.87	2.374	−0.97
λ_0↑40%	73.779	0	67.645	−1.36	6.134	1.36	3.151	−0.19
γ ↑40%	73.779	0	66.083	−2.92	7.696	2.92	2.933	−0.41
Endog. LF								
b ↑40%	75.138	1.36	69.965	0.96	5.172	0.40	3.472	0.13
λ_0↑40%	71.991	−1.79	66.517	−2.49	5.474	0.70	2.994	−0.35
γ ↑40%	73.544	−0.24	66.520	−2.49	7.024	2.25	2.995	−0.35

Note: See notes in Table 3. All parameters are the same as the parameter table except for $v = 0.0954$, $\lambda_0 = 0.1629$ and $m = 0.6838$.

Table 5
Numerical results when $\frac{1-n-e}{v} = 2$ (%).

	$(1-n)$	$\Delta(1-n)$	e	Δe	$(1-n-e)$	$\Delta(1-n-e)$	g	Δg
Benchmark	73.779	0	69.007	0	4.772	0	3.340	0
Exog. LF								
b ↑40%	73.779	0	62.135	−6.87	11.644	6.87	2.374	−0.97
λ_0↑40%	73.779	0	67.645	−1.36	6.134	1.36	3.151	−0.19
γ ↑40%	73.779	0	66.083	−2.92	7.696	2.92	2.933	−0.41
Endog. LF								
b ↑40%	75.138	1.36	69.965	0.96	5.172	0.40	3.472	0.13
λ_0↑40%	71.991	−1.79	66.517	−2.49	5.474	0.70	2.994	−0.35
γ ↑40%	73.544	−0.24	66.520	−2.49	7.024	2.25	2.995	−0.35

Note: See notes in Table 3. All parameters are the same as the parameter table except for $v = 0.0239$, $\lambda_0 = 0.6515$ and $m = 1.3677$.

Table 3. Thus, there is a negative relation between long-run economic growth and unemployment when the labor force is exogenous.

Next, with an endogenous labor force, in the case of increases in unemployment compensation (b), the size of the labor force is enlarged, which enhances employment and economic growth. Conversely, in the case of increases in vacancy posting costs (λ_0) and increases in workers' bargaining power (γ), the labor force is shrunk in size, which dampens employment and economic growth. In all these three more adverse labor market policies, as the size of the labor force changes in the same direction as employment, the change in unemployment is ambiguous, and depends on whether the labor force effect or the employment effect dominates. In the case of increases in unemployment compensation, the increase in the size of the labor force dominates the increase in employment. Thus, unemployment increases, which leads to a positive relation between long-run economic growth and unemployment. In the case of increases in vacancy posting costs and increases in workers' bargaining power, the decrease in employment dominates the decrease in the size of the labor force. Unemployment also increases, which results in a negative relation between long-run economic growth and unemployment.

Our quantitative analysis above has followed Shimer (2005), normalizing the steady-state market tightness $(1-n-e)/v$ to unity and calibrating the value of the recruitment rate η. Here, we carry out three kinds of robustness checks to see whether the quantitative results are affected by such normalization. First, we lower the normalization of the steady-state market tightness $(1-n-e)/v$ from unity to 0.5. Next, we increase the normalization to 2. The value of v is increased by 100% in the former case and reduced by 50% in the latter case. Then, we recalibrate the model in these two cases. In the former case when the steady-state market tightness is reduced to 0.5, we find that all other parameter values are unchanged except for both λ_0 and m, which are decreased to 0.1629 and 0.6838, respectively. In the latter case when the steady-state market tightness is increased to 2, all other parameter values are unchanged except for both λ_0 and m, which are increased to 0.6515 and 1.3677, respectively. The quantitative results are in Tables 4 and 5, respectively. It is clear that the results are exactly the same as Table 3.

Finally, the above has normalized the number of vacancies v and calibrated the value of the recruitment rateη. By contrast, we may try to find the data to calculate the number of vacancies v and then calibrate the recruitment rateη. In the Federal Reserve Economic Data (FRED) compiled by the St. Louis Fed, there are the data of the hiring rate which are available from the year 2000. The hiring rate is the number of new hires during the entire period as a percent of total employment. Although not all vacancies posted are filled, the number of hires may be viewed as a lower bound of the number of vacancies posted in a period. The monthly hiring rate in the FRED before 2008 is 3.8% and thus the annual hiring rate is 45.6%, which serves a lower bound of $\frac{v}{e}$. As $e = 0.6901$, then $v = 0.3147$ which is much larger than the normalized value above. As such, $\eta = 0.1467$, which is much lower than the calibrated value above. In this case, all other parameter values are unchanged except for $\lambda_0 = 0.0494$ and $m = 0.3766$. The quantitative results are in Table 6, and are exactly the same as those in Table 3.

Table 6
Numerical results when $\frac{v}{e} = 0.456$ (%).

	$(1-n)$	$\Delta(1-n)$	e	Δe	$(1-n-e)$	$\Delta(1-n-e)$	g	Δg
Benchmark	73.779	0	69.007	0	4.772	0	3.340	0
Exog. LF								
b ↑40%	73.779	0	62.135	−6.87	11.644	6.87	2.374	−0.97
λ_0↑40%	73.779	0	67.645	−1.36	6.134	1.36	3.151	−0.19
γ ↑40%	73.779	0	66.083	−2.92	7.696	2.92	2.933	−0.41
Endog. LF								
b ↑40%	75.138	1.36	69.965	0.96	5.172	0.40	3.472	0.13
λ_0↑40%	71.991	−1.79	66.517	−2.49	5.474	0.70	2.994	−0.35
γ ↑40%	73.544	−0.24	66.520	−2.49	7.024	2.25	2.995	−0.35

Note: See notes in Table 3. All parameters are the same as the parameter table except for $v = 0.3147$, $\eta = 0.1467$, $\lambda_0 = 0.0494$ and $m = 0.3766$.

Table 7
Effects of adverse labor market institutions on unemployment.

	1976–85	1986–95	1996–05	1976–05
Constant	7.5638***	10.864***	5.0412***	8.6282***
	(5.4921)	(6.3424)	(2.0526)	(8.9572)
UR	0.3178	−1.6032	4.5949	−0.0009
	(0.1592)	(−0.6265)	(1.2216)	(−0.0007)
R^2	0.0003	0.0033	0.0127	1E-09
N	82	119	118	319
	1976–85	1986–95	1996–05	1976–05
Constant	12.043***	12.155***	8.634***	10.591***
	(12.3407)	(14.7120)	(14.5391)	(22.9460)
UD	−0.091***	−0.0548***	−0.0152	−0.0457***
	(−4.6921)	(−3.2366)	(−1.1789)	(−4.7801)
R^2	0.2137	0.0815	0.0116	0.0665
N	83	120	120	323

Note: Parenthesis are t-values with *, ** and *** being statistically significant at the 90%, 95% and 99% level. N indicates the number of observations.

To summarize, with an exogenous labor force, the quantitative relation between long-run economic growth and unemployment is unambiguously negative. With an endogenous labor force, the quantitative relation between long-run economic growth and unemployment is non-monotone and depends on the types of changes in adverse labor market institutions. The relation is quantitatively positive when unemployment compensation is increased, but is quantitatively negative when vacancy posting costs and workers' bargaining power are increased.

4.4. Cross-country estimation

So far, we have offered calibration exercises. Alternatively, it is interesting to carry out econometric exercises to estimate and test the effects of adverse labor market institutions across OECD countries. However, employment, unemployment, the labor force and economic growth are all endogenous in our model. Thus, if we place all these variables in regressions to estimate and test the effects of adverse labor market institutions on all these variables, there are econometric issues that are beyond the scope of this paper. Given that the effect of adverse labor market institutions on unemployment is the most important in our study, here we provide simple estimations and test the effect of adverse labor market institutions on unemployment. The following econometric model is estimated.

$$u_{jt} = \alpha_0 + \alpha_1 x_{jt} + \varepsilon_{jt}, \tag{20}$$

where u_{jt} and x_{jt} are, respectively, country j's unemployment rates and adverse labor market institutions in period t. Error term ε_{jt} is assumed to be independent and identically distributed with the normal distribution.

In our paper, the labor market policy variables include the unemployment compensation, vacancy posting cost and labor's bargaining power. There are data for the unemployment replacement rate (UR) and union density (UD) in OECD countries, which are used as proxies of unemployment compensation and the labor's bargaining power, respectively. Yet, there are no data for vacancy posting cost. Thus, we can only estimate the effects of unemployment compensation and labor's bargaining power on the unemployment rate. Data for the unemployment replacement rate are taken from van Vliet and Caminada (2012), the union density from Visser (2011), and the unemployment rate from OECD (2010). The data are annual for the period of 1976–2005 for the OECD countries listed in Table 1.[26] The time series for UR is constructed from the fraction of

[26] See Appendix Table 1 which lists the years when the data are available in the period under study.

current income which the social unemployment benefit system provides to a person if he or she does not work. The UD is constructed from the net union membership as a proportion of wage and salary earners in employment. The estimation is executed in 10-year subsample periods of 1976–1985, 1986–1995 and 1995–2005 and the full sample period of 1976–2005. The estimation results are in Table 7.

The results indicate that, no matter whether in a subsample period or in a full sample period, the effects of unemployment compensation (UR) on the unemployment rate may be positive or negative, but they are all statistically insignificant. By contrast, the effects of labor bargaining power (UD) on the unemployment rate are all negative and, except the subsample period 1996–2005, all are statistically significant. Thus, higher labor's bargaining power tends to decrease unemployment.

In sum, our estimated results indicate that the effects of adverse labor market institutions on unemployment are not necessarily positive, which is different from what the model with an exogenous labor force predicts. In particular, the statistically and significantly negative effects of labor bargaining power (UD) on the unemployment rate implicitly indicate that the negative labor force effect dominates the negative employment effect, which lends support to a model with an endogenous labor force.

5. Concluding remarks

Under models with a fixed-size labor force, existing studies have found that adverse labor market policies raise unemployment and lower employment and long-run economic growth, thereby giving rise to a negative relation between long-run economic growth and unemployment. However, the data in OECD suggest that the labor force has changed substantially across countries, and there is a non-monotone relation between long-run economic growth and unemployment.

In this paper we build a model with endogenous labor-force participation and adverse labor market institutions. A special feature of our model is that the labor-force participation is modeled as a control variable, as opposed to a state variable, which simplifies our analysis. We analyze the relation between long-run economic growth and unemployment by investigating the effects of changes in adverse labor market institutions on the labor force, unemployment, employment and economic growth. When unemployment compensation is increased, we find that the incentive to participate in the labor force is increased, which in turn enlarges employment and long-run economic growth. As the size of the labor force and employment both increase, unemployment is ambiguous and depends on the relative effect between the labor force and employment. On the other hand, when vacancy posting costs and workers' bargaining power are increased, the size of the labor force is reduced, which decreases employment and economic growth. As the size of the labor force and employment both decrease, unemployment is also ambiguous, and depends on the relative effect between the labor force and employment. Our calibration exercises and estimation exercises both indicate that the relation between long-run economic growth and unemployment is not monotone, and depends on the types of changes in adverse labor market institutions.

Appendix

1. The VE locus when the labor force participation is endogenous

$$(1-\gamma)\frac{\eta(e,n)}{r(e)-\delta+\psi}[\alpha A e^{-(1-\alpha)} - \chi_1 z(e,n)] - \lambda_0 \equiv \Omega(\underset{(-)}{e},\underset{(-)}{n}) = 0. \tag{15}$$

Differentiating (15) yields

$$\Omega_e de + \Omega_n dn = -\Omega_\lambda d\lambda - \Omega_\gamma d\gamma - \Omega_b db,$$

where $\Omega_e = \frac{(r-\delta+\psi)\eta_e - \eta r_e}{(r-\delta+\psi)^2}(1-\gamma)[\alpha A e^{\alpha-1} - \chi_1 z] - \frac{\eta(1-\gamma)}{r-\delta+\psi}[\alpha(1-\alpha)Ae^{\alpha-2} + \chi_1 z_e] < 0,$

$$\Omega_n = \underbrace{\frac{\eta_n(1-\gamma)}{r-\delta+\psi}\left[\alpha A e^{\alpha-1} - \chi_1 z\right]}_{-} \underbrace{-\frac{\eta(1-\gamma)}{r-\delta+\psi}\chi_1 z_n}_{+} = \frac{\beta}{1-\beta}\frac{\lambda_0}{1-n-e}\left(\frac{\eta(1-\gamma)}{r-\delta+\psi}\chi_1\nu - 1\right) < 0$$

if $\frac{\eta(1-\gamma)}{r-\delta+\psi}\chi_1\nu < 1 \Rightarrow (1-\gamma)\chi_1\psi e < r-\delta+\psi$,

$$\Omega_\lambda = \underbrace{-\frac{\eta(1-\gamma)}{r-\delta+\psi}\chi_1 z_\lambda}_{+} - 1 = \frac{\eta(1-\gamma)}{r-\delta+\psi}\chi_1\nu - 1 < 0 \text{ if } (1-\gamma)\chi_1\psi e < r-\delta+\psi,$$

$$\Omega_\gamma = -\frac{\eta}{r-\delta+\psi}[\alpha A e^{\alpha-1} - \chi_1 z] = -\frac{\lambda_0}{1-\gamma} < 0,$$

$$\Omega_b = 0.$$

Thus, if $(1-\gamma)\chi_1\psi e < r-\delta+\psi$, the slope of the VE locus is negative: $\frac{dn}{de} = -\frac{\Omega_e}{\Omega_n} < 0$. The condition $(1-\gamma)\chi_1\psi e < r-\delta+\psi$ is rewritten as $e[\frac{(1-\alpha)}{e^{1-\alpha}}A - (1-\gamma)\chi_1\psi] > \delta - \psi$, which is easily met if A is sufficiently large.

2. The LP locus

$$\frac{\mu(e,n)}{\rho+\psi}\gamma\left[\alpha Ae^{\alpha-1} - \chi_1 z(e,n)\right] + b\left[\gamma\alpha Ae^{\alpha-1} + (1-\gamma)\chi_1 z(e,n)\right] - (\chi_1-\chi_2)z(e,n) \equiv \Lambda(\underset{+}{e},\underset{+}{n}) = 0. \tag{16}$$

Differentiating (16) yields

$$\Lambda_e de + \Lambda_n dn = -\Lambda_\lambda d\lambda - \Lambda_\gamma d\gamma - \Lambda_b db,$$

where $\Lambda_e = \frac{\gamma}{\rho+\psi}[\mu_e[\alpha Ae^{\alpha-1} - \chi_1 z] - \mu[\alpha(1-\alpha)Ae^{\alpha-2} + \chi_1 z_e]] - b\gamma\alpha(1-\alpha)Ae^{\alpha-2} + z_e[\chi_2 - b\gamma\chi_1]$

$$= \underbrace{\alpha Ae^{\alpha-2}\left[\frac{\mu\gamma}{\rho+\psi}\left(\frac{e}{1-n-e}+\alpha\right) - b\gamma(1-\alpha)\right]}_{+} \underbrace{-\chi_1\left[\frac{\mu\gamma}{\rho+\psi}\left(\frac{1}{e}\frac{1-n}{1-n-e}z - z_e\right)\right]}_{-} + \underbrace{z_e[\chi_2 - b\gamma\chi_1]}_{-} > 0,$$

if $A > \chi_1 \frac{e^{1-\alpha}}{\alpha} \frac{(1-n)z-(1-n-e)ez_e}{[(1-\alpha)e+\alpha(1-n)] - \frac{\rho+\psi}{\mu}b(1-\alpha)(1-n-e)}$.

$$\Lambda_n = \underbrace{\frac{\mu_n\gamma}{\rho+\psi}\left[\alpha Ae^{\alpha-1} - (\chi_1-0)z\right]}_{+} \underbrace{-\frac{\mu\gamma}{\rho+\psi}(\chi_1-0)z_n + b(1-\gamma)(\chi_1-0)z_n}_{+\text{if}\,\rho<\gamma\mu/[(1-\gamma)b]-\psi} \underbrace{-(\chi_1-\chi_2)z_n}_{+} > 0,$$

$$\Lambda_\lambda = \left[\left(-\frac{\mu\gamma}{\rho+\psi} + b(1-\gamma)\right)\chi_1 - (\chi_1-\chi_2)\right]z_\lambda > 0 \,\text{if}\, b < \frac{\gamma}{1-\gamma}\frac{\mu}{\rho+\psi},$$

$$\Lambda_\gamma = \left(\frac{\mu\gamma}{\rho+\psi} + b\right)\left[\alpha Ae^{\alpha-1} - \chi_1 z\right] > 0,$$

$$\Lambda_b = \left[\gamma\alpha Ae^{\alpha-1} + (1-\gamma)\chi_1 z\right] = \tfrac{w}{k} > 0.$$

Thus, if $A > \chi_1 \frac{e^{1-\alpha}}{\alpha} \frac{(1-n)z-(1-n-e)ez_e}{[(1-\alpha)e+\alpha(1-n)] - \frac{\rho+\psi}{\mu}b(1-\alpha)(1-n-e)}$ and $\rho < \frac{\gamma\mu}{(1-\gamma)b} - \psi$, the slope of the LP locus is negative: $\frac{dn}{de} = -\frac{\Lambda_e}{\Lambda_n} < 0$. The first condition is easily met if the technology level A is sufficiently large. The second condition is easily met if the time preference rate is sufficiently small.

3. The VE locus when the labor force participation is exogenous

$$\frac{\eta(e,n)}{r(e)-\delta+\psi}\frac{1-\gamma}{1-(1-\gamma)b}\left[(1-b)\alpha Ae^{-(1-\alpha)} - \chi_2 z(e,n)\right] - \lambda \equiv \Phi(\underset{(-)}{e};\bar{n}) = 0. \tag{19}$$

Differentiating (19) yields

$$\Phi_e de = -\Phi_\lambda d\lambda - \Phi_\gamma d\gamma - \Phi_b db,$$

where $\Phi_e = \frac{(r-\delta+\psi)\eta_e - \eta r_e}{(r-\delta+\psi)^2}\frac{1-\gamma}{1-(1-\gamma)b}[(1-b)\alpha Ae^{\alpha-1} - \chi_2 z] - \frac{\eta}{r-\delta+\psi}\frac{1-\gamma}{1-(1-\gamma)b}[(1-b)\alpha(1-\alpha)Ae^{\alpha-2} + \chi_2 z_e] < 0,$

$$\Phi_\lambda = \underbrace{-\frac{\eta}{r-\delta+\psi}\frac{1-\gamma}{1-(1-\gamma)b}\chi_2 z_\lambda}_{+} - 1 = \frac{\eta}{r-\delta+\psi}\frac{1-\gamma}{1-(1-\gamma)b}\chi_2 \upsilon - 1 < 0,$$

$$\text{if}\, 1 > \frac{\eta}{r-\delta+\psi}\frac{1-\gamma}{1-(1-\gamma)b}\chi_2\upsilon \Rightarrow \tfrac{(1-\gamma)\chi_2\psi e}{1-(1-\gamma)b} < (r-\delta+\psi),$$

$$\Phi_\gamma = -\frac{\eta}{r-\delta+\psi}\frac{1-\gamma}{(1-(1-\gamma)b)^2}[(1-b)\alpha Ae^{\alpha-1} - \chi_2 z] < 0,$$

$$\Phi_b = \underbrace{\frac{\eta}{r-\delta+\psi}\left(\frac{1-\gamma}{1-(1-\gamma)b}\right)^2\left[(1-b)\alpha Ae^{\alpha-1} - \chi_2 z\right]}_{+} \underbrace{-\frac{\eta}{r-\delta+\psi}\frac{1-\gamma}{1-(1-\gamma)b}\alpha Ae^{\alpha-1}}_{-}$$

$$= -\frac{\eta}{r-\delta+\psi}\frac{1-\gamma}{1-(1-\gamma)b}\frac{w}{k} < 0.$$

The condition $\frac{(1-\gamma)\chi_2\psi e}{1-(1-\gamma)b} < (r-\delta+\psi)$ is rewritten as $e[\frac{(1-\alpha)}{e^{1-\alpha}}A - \chi_2\frac{(1-\gamma)\psi}{1-(1-\gamma)b}] > (\delta-\psi)$ which is easily met if the technology coefficient A is large.

4. Existence of steady state in a range of employment with a positive labor force

Appendices 1 and 2 show that the VE and the LP loci are both negatively sloping. Now, we prove the existence of steady state by showing that the VE locus is steeper than the LP locus in a feasible range of employment that is consistent with a positive labor force. We will obtain $e_{VE} < e_{LP}$ and $n_{VE} > n_{LP}$, as seen in Fig. 3. To do this, we rearrange (15) and (16) to yield, respectively,

$$\left[m\left(\frac{1-n-e}{\psi e}\right)^{\beta}\right]^{\frac{1}{1-\beta}}\frac{1}{(1-\alpha)Ae^{\alpha}-\delta+\psi}(1-\gamma)[\alpha A(e)^{-(1-\alpha)}-\chi_1 z(e,n)] = \lambda_0 \tag{A1}$$

$$\frac{\psi e}{1-n-e}\frac{\gamma}{\rho+\psi}\left[\alpha Ae^{\alpha-1}-\chi_1 z(e,n)\right]+b\left[\gamma\alpha Ae^{\alpha-1}+(1-\gamma)\chi_1 z(e,n)\right]=(\chi_1-\chi_2)z(e,n), \tag{A2}$$

where $z(e,n) = (\frac{\rho+\delta}{1+\rho})Ae^{\alpha} + \frac{\rho(1-\delta)}{1+\rho} - \lambda_0[\frac{\psi e}{m(1-e-n)^{\beta}}]^{\frac{1}{1-\beta}}$.

First, we will show that when $n\to 0$, z goes to the highest value $\bar{z}(e)$ and the Locus VE and the Locus LP approach e_{VE} and e_{LP}, respectively, with their associated value of $\bar{z}(e)$ being given by $\bar{z}_{VE} \equiv \bar{z}(e_{VE})$ and $\bar{z}_{LP} \equiv \bar{z}(e_{LP})$, respectively. Specifically, when $n\to 0$, $z(e,0) = (\frac{\rho+\delta}{1+\rho})Ae^{\alpha} + \frac{\rho(1-\delta)}{1+\rho} - \lambda_0[\frac{\psi e}{m(1-e)^{\beta}}]^{\frac{1}{1-\beta}} \equiv \bar{z}(e)$ and then (A1) and (A2) are, respectively,

$$\left[m\left(\frac{1-e}{\psi e}\right)^{\beta}\right]^{\frac{1}{1-\beta}}\frac{1}{(1-\alpha)Ae^{\alpha}-\delta+\psi}(1-\gamma)\left[\alpha Ae^{\alpha-1}-\chi_1\bar{z}(e)\right] = \lambda_0, \tag{A3}$$

$$\frac{\psi e}{1-e}\frac{\gamma}{\rho+\psi}\left[\alpha Ae^{\alpha-1}-\chi_1\bar{z}(e)\right]+b\left[\gamma\alpha Ae^{\alpha-1}+(1-\gamma)\chi_1\bar{z}(e)\right]=(\chi_1-\chi_2)\bar{z}(e). \tag{A4}$$

Rearranging (A3) and (A4) gives

$$\frac{\psi e_{VE}}{1-e_{VE}} = m^{\frac{1}{\beta}}\left(\frac{(1-\gamma)\left(\alpha Ae_{VE}^{\alpha-1}-\chi_1\bar{z}_{VE}\right)}{\lambda_0\left((1-\alpha)Ae_{VE}^{\alpha}-\delta+\psi\right)}\right)^{\frac{1-\beta}{\beta}}, \tag{A5}$$

$$\frac{\psi e_{LP}}{1-e_{LP}}=(\rho+\psi)\frac{(\chi_1-\chi_2)\bar{z}_{LP}-b\left[\gamma\alpha Ae_{LFP}^{\alpha-1}+(1-\gamma)\chi_1\bar{z}_{LP}\right]}{\gamma\left(\alpha Ae_{LP}^{\alpha-1}-\chi_1\bar{z}_{LP}\right)}, \tag{A6}$$

which yield, respectively, e_{VE} and e_{LP}, where $\bar{z}_{VE} \equiv \bar{z}(e_{VE})$ and $\bar{z}_{LP} \equiv \bar{z}(e_{LP})$. Recall that $\bar{z}(e)$ is increasing in e. If $e_{VE} < e_{LP}$, then $\bar{z}_{VE} < \bar{z}_{LP}$. The condition for $e_{VE} < e_{LP}$ is that the right-hand side of (A5) is smaller than the right-hand side of (A6); that is,

$$m^{\frac{1}{\beta}}\left(\frac{(1-\gamma)\left(\alpha Ae_{VE}^{\alpha-1}-\chi_1\bar{z}_{VE}\right)}{\lambda_0\left((1-\alpha)Ae_{VE}^{\alpha}-\delta+\psi\right)}\right)^{\frac{1-\beta}{\beta}} < (\rho+\psi)\left(\frac{[(1-b)\chi_1-\chi_2]\bar{z}_{LP}}{\gamma\left(\alpha Ae_{LP}^{\alpha-1}-\chi_1\bar{z}_{LP}\right)}-b\right), \tag{A7}$$

which may be rewritten as

$$\frac{m(1-\gamma)^{1-\beta}}{(\rho+\psi)^{\beta}{\lambda_0}^{1-\beta}} < \left[\frac{[(1-b)\chi_1-\chi_2]\bar{z}_{LP}}{\gamma[MPL(e_{LP})-\chi_1\bar{z}_{LP}]}-b\right]^{\beta}\left[\frac{r(e_{VE})-\delta+\psi}{MPL(e_{VE})-\chi_1\bar{z}_{VE}}\right]^{1-\beta}. \tag{A8}$$

Recall that $r(e)$ is increasing in e and MPL is decreasing in e. Thus, $e_{VE} < e_{LP}$ leads to $r(e_{VE}) < r(e_{LP})$ and $MPL(e_{VE}) > MPL(e_{LP})$, which in turn indicates that the inequality in (A8) always holds if the following condition is met:

$$\frac{m(1-\gamma)^{1-\beta}}{(\rho+\psi)^{\beta}{\lambda_0}^{1-\beta}} < \left[\frac{[(1-b)\chi_1-\chi_2]\bar{z}_{LP}}{\gamma[MPL(e_{LP})-\chi_1\bar{z}_{LP}]}-b\right]^{\beta}\left[\frac{r(e_{LP})-\delta+\psi}{MPL(e_{LP})-\chi_1\bar{z}_{VE}}\right]^{1-\beta}. \tag{A9}$$

Next,we will show that when e declines to e_L, the consumption-capital ratio z goes to the lowest value $\tilde{z}(n)$ and the Locus VE and Locus LP approach n_{VE} and n_{LP}, respectively, with their associated values of $\tilde{z}(n)$ being given by $\tilde{z}_{VE} \equiv \tilde{z}(n_{VE})$ and $\tilde{z}_{LP} \equiv \tilde{z}(n_{LP})$, respectively. Specifically, the value of e cannot go to 0 as then $\mu = \eta = 0$ and the Beveridge curve relations

Table A.1
Data used in Table 7.

	Unemployment replacement rate	Union density	Unemployment Rate
	Years lacking data	Years lacking data	Years lacking data
Belgium	1976 1978 1980 1994 1996 1998	No	1976 1977 1978 1978 1980 1981 1982
Denmark	1978 1982	No	1976 1977 1978 1978 1980 1981 1982
Finland	No	No	No
France	No	No	1976 1977 1978 1978 1980 1981 1982
Germany	No	No	No
Ireland	No	No	1976 1978 1980 1982
Italy	No	No	No
Netherlands	No	No	No
Spain	1976 1977 1978 1980 1982	1976 1977 1978 1979	No
Sweden	No	No	No
UK	No	No	1976 1977 1978 1978 1980 1981 1982 1983
US	No	No	No

Sources: van Vliet and Caminada (2012) and Visser (2011).
Note: This table lists the years when the data are not available. No means that the data are available for all years in 1976–2005, in which case the number of observations is 30.

in (10) do not hold. There exists a lowest value of $e > 0$, denoted by e_L. Then, when $e \to e_L$, $z(e_L, n) = (\frac{\rho+\delta}{1+\rho})A{e_L}^{\alpha} + \frac{\rho(1-\delta)}{1+\rho} - \lambda_0[\frac{\psi e_L}{m(1-e_L-n)^{\beta}}]^{\frac{1}{1-\beta}} \equiv \tilde{z}(n)$ and (15) and (16) are, respectively,

$$n_{VE} = 1 - e_L - \psi e_L m^{-\frac{1}{\beta}} \left(\frac{(1-\gamma)\left(\alpha A e_L^{\alpha-1} - \chi_1 \tilde{z}_{VE}\right)}{\lambda_0\left((1-\alpha)A e_L^{\alpha} - \delta + \psi\right)} \right)^{-\frac{1-\beta}{\beta}}, \tag{A10}$$

$$n_{LP} = 1 - e_L - \frac{\psi e_L \gamma \left(\alpha A e_L^{\alpha-1} - \chi_1 \tilde{z}_{LP}\right)}{(\rho+\psi)\left[(\chi_1 - \chi_2)\tilde{z}_{LP} - b\left[\gamma \alpha A e_L^{\alpha-1} + (1-\gamma)\chi_1 \tilde{z}_{LP}\right]\right]}, \tag{A11}$$

which yield, respectively, n_{VE} and n_{LP}. Recall that $z(e, n)$ is decreasing in n. If $n_{VE} > n_{LP}$, then $\tilde{z}_{VE} < \tilde{z}_{LP}$. The condition for $n_{VE} > n_{LP}$ is that the right-hand side of (A10) is larger than the right-hand side of (A11); that is,

$$(\rho+\psi)\left(\frac{[(1-b)\chi_1 - \chi_2]\tilde{z}_{LP}}{\gamma\left(\alpha A e_L^{\alpha-1} - \chi_1 \tilde{z}_{LP}\right)} - b \right) < m^{\frac{1}{\beta}} \left(\frac{(1-\gamma)\left(\alpha A e_L^{\alpha-1} - \chi_1 \tilde{z}_{VE}\right)}{\lambda_0\left((1-\alpha)A e_L^{\alpha} - \delta + \psi\right)} \right)^{\frac{1-\beta}{\beta}}, \tag{A12}$$

which is rewritten as

$$\left[\frac{[(1-b)\chi_1 - \chi_2]\tilde{z}_{LP}}{\gamma[MPL(e_L) - \chi_1 \tilde{z}_{LP}]} - b \right]^{\beta} \left[\frac{r(e_L) - \delta + \psi}{MPL(e_L) - \chi_1 \tilde{z}_{VE}} \right]^{1-\beta} < \frac{m(1-\gamma)^{1-\beta}}{(\rho+\psi)^{\beta} {\lambda_0}^{1-\beta}}. \tag{A13}$$

Denote $\Gamma(e, z_{VE}, z_{LP}) \equiv \frac{((\chi_1-\chi_2)z_{LP})^{\beta}(r(e)-\delta+\psi)^{1-\beta}}{(MPL(e)-\chi_1 z_{LP})^{\beta}(MPL(e)-\chi_1 z_{VE})^{1-\beta}}$. Consider

Condition E: $\Gamma(e_L, \tilde{z}_{VE}, \tilde{z}_{LP}) < \frac{m(1-\gamma)^{1-\beta}}{(\rho+\psi)^{\beta}{\lambda_0}^{1-\beta}} < \Gamma(e_{LP}, \tilde{z}_{VE}, \tilde{z}_{LP})$.

By combining the conditions (A9) and (A13), Condition E then gives $e_{VE} < e_{LP}$ and $n_{VE} > n_{LP}$, as seen in Fig. 3, and thus there exists a steady state.

References

Aghion, P., Howitt, P., 1994. Growth and unemployment. Rev. Econ. Stud. 61, 477–494.
Andolfatto, D., 1996. Business cycles and labor-market search. Am. Econ. Rev. 86, 112–132.
Andolfatto, D., Gomme, P., 1996. Unemployment insurance and labor market activity in Canada. Carnegie Rochester Ser. Public Policy 44, 47–82.
Barnichon, R. and A. Figura, 2015, Declining desire to work and downward trends in unemployment and participation, *NBER Working Paper* No. 21252.
Bean, C., Pissarides, C.A., 1993. Unemployment, consumption and growth. Eur. Econ. Rev. 37, 837–859.
Benhabib, J., Perli, R., 1994. Uniqueness and indeterminacy: On the dynamics of endogenous growth. J. Econ. Theor. 63, 113–142.
Burdett, K., Kiefer, N., Mortensen, D., Neumann, G., 1984. Earnings, unemployment and the allocation of time over time. Rev. Econ. Stud. 51, 559–578.
Chen, B.-L., Chen, H.-J., Wang, P., 2011. Labor-market frictions, human capital accumulation, and long-run growth: positive analysis and policy evaluation. Int. Econ. Rev. 52, 131–160.
Chen, B.-L., Lai, C.-F., 2015. Effects of labor taxes and unemployment compensation on labor supply in a search model with an endogenous labor force. J. Macroecon. 43, 300–317.
Costain, J.S., Reiter, M., 2008. Business cycles, unemployment insurance, and the calibration of matching models. J. Econ. Dyn. Control 32, 1120–1155.
Diamond, P.A., 1982. Wage determination and efficiency in search equilibrium. Rev. Econ. Stud. 49, 217–227.
Eriksson, C., 1997. Is there a trade-off between employment and growth. Oxford Econ. Paper 49, 77–88.
Fang, L., Rogerson, R., 2009. Policy analysis in a matching model with intensive and extensive margins. Int. Econ. Rev. 50, 1153–1168.
Fernald, J.G., Jones, C.I., 2014. The Future of U.S. Economic Growth. Federal Reserve Bank of San Francisco *Working Paper* 2014-02.

Garibaldi, P., Wasmer, E., 2005. Equilibrium search unemployment, endogenous participation, and labor market flows. J. Eur. Econ. Assoc. 3, 851–882.
Hopenhayn, H., Nicolini, J.P., 1997. Optimal unemployment insurance. J. Political Econ. 105, 412–438.
Hornstein, A., Krusell, P., Violante, G., 2005. The replacement problem in frictional economies: a near equivalence result. J. Eur. Econ. Assoc. 3, 1007–1057.
Hosios, A.J., 1990. On the efficiency of matching and related models of search and unemployment. Rev. Econ. Stud. 57, 279–298.
Irmen, A., 2009. Frictional unemployment, labor market institutions, and endogenous economic growth. Econ. Bull. 29, 1127–1138.
Krusell, P., Mukoyama, T., Rogerson, R., Sahin, A., 2011. A three state model of worker flows in general equilibrium. J. Econ. Theor. 146, 1107–1133.
Kydland, F.E., Prescott, E.C., 1991. Hours and employment variation in business cycle theory. Econ. Theor. 1, 63–81.
Ljungqvist, L., Sargent, T.J., 2007. Understanding European unemployment with matching and search-island models. J. Monetary Econ. 54, 2139–2179.
Merz, M., 1995. Search in the labor market and the real business cycle. J. Monetary Econ. 36, 269–300.
Mortensen, D.T., Pissarides, C.A., 1998. Technological progress, job creation, and job destruction. Rev. Econ. Dyn. 1, 733–753.
OECD, 1999. Benefit Systems and Work Incentives. OECD, Paris, France.
OECD, 2010. Labour productivity growth. OECD Productivity Statistics (database) doi: 10.1787/data-00493-en.
OECD, 2014a. Labour Force Statistics: Summary tables. OECD Employment and Labour Market Statistics (database) doi:10.1787/data-00286-en.
OECD, 2014b. National Accounts OECD Countries 2. doi:10.1787/na_ma_dt-v2014-2-en.
Pissarides, C.A., 2000. Equilibrium Unemployment Theory. 2nd, Basil Blackwell, Oxford.
Pissarides, C., Vallanti, G., 2007. The impact of TFP growth on steady-state unemployment. Int. Econ. Rev. 48, 607–640.
Pries, M., Rogerson, R., 2009. Search frictions and labor market participation. Eur. Econ. Rev. 53, 568–587.
Rogerson, R., Shimer, R., Wright, R., 2005. Search-theoretic models of the labor market: a survey. J. Econ. Lit. 43, 959–988.
Romer, P., 1986. Increasing returns and long-run growth. J. Political Econ. 94, 1002–1037.
Sattinger, M., 1995. General equilibrium effects of unemployment compensation with labor force participation. J. Labor Econ. 13, 623–652.
Shavell, S., Weiss, L., 1979. The optimal payment of unemployment insurance benefits over time. J. Political Econ. 87, 1347–1362.
Shimer, R., 2005. The cyclical behavior of equilibrium unemployment and vacancies. Am. Econ. Rev. 95, 25–49.
Shimer, R., 2012. Reassessing the Ins and Outs of Unemployment. Rev. Econ. Dyn. 15, 127–148.
van Vliet, L., Caminada, K., 2012. Unemployment replacement rates dataset among 34 welfare states 1971-2009: An update, extension and modification of Scruggs' Welfare State Entitlements Data Set. Leiden University *NEUJOBS Special Report* No. 2.
Visser, J., 2011, Data base on institutional characteristics of trade unions, wage setting, state intervention and social pacts, 1960-2010 (ICTWSS), Amsterdam institute for advanced labour studies AIAS university of Amsterdam version 3.0.