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Optimal Education and Pension Policy in a Growth Model

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In OLG economies with physical and human capital, the growth path of the decentralized market equilibrium fails to achieve optimality, because individuals' accumulation of physical and human capital is in general different from that which maximizes welfare along the Golden Rule path. In an OLG model with the education technology being homogeneous of degree one in education input and a parental human capital externality, Del Rey and Lopez-Garcia (2013) found that education taxes with pensions can attain the Golden Rule social optimum. However, the empirical evidence suggests a weak parental human capital externality. Our paper shows that it is optimal to subsidize, rather than to tax, education, when the parental human capital externality is smaller when the education technology is homogeneous with less than one degree.

Keywords: growth model, human capital externality, education policy, intergenerational transfers
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1 Introduction

The overlapping generations (OLG hereafter) model has been a standard workhorse model for analyzing allocation of resources across generations

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since Diamond (1965). One important aspect of the OLG model is that individuals' accumulation of physical capital is generally different from the one that maximizes welfare along the Golden Rule path. When human capital is added to the OLG model, the inefficiency issue is more complicated,[1] as knowledge spillovers and education externalities are some of the mechanisms that help sustain the returns to the accumulation of human capital. These sources of inefficiency add to the potential discrepancy of physical and human capital accumulation between the decentralized allocation and the social optimum.

To analyze the social optimum, one way is that the social planner chooses the sequence of consumptions by maximizing the social welfare, which is the sum of individuals' utilities over generations with the social weight given to future generations reflecting social time preferences An alternative way is the Golden Rule of capital accumulation introduced by Phelps (1961), Diamond (1965) and Samuelson (1968). Under the Golden Rule criterion, the social planner maximizes the social welfare that treats all generations alike, while being respectful of individual utilities. More recent studies have shown that the Golden Rule criterion was robust with regard to several changes in the underlying assumptions.[2]

Docquier, Paddison, and Pestieau (2007) adopted the first standard way to study the social optimum. In their model, the human capital externalities are strong and human capital is the engine of growth. Because agents ignore the positive effect of investment in education on the future generation, the authors found that there is always under-investment in education due to the existence of the intergenerational human capital externality. For intermediate values of social weight, a mix of education subsidies and intergenerational

[1]Recent literature has stressed the role of human capital in economic growth and the consequence for human welfare. See Lucas (1988) for theoretical work, and Barro and Lee (1993) for empirical work.

[2]See Abel et al. (1989) and Zilcha (1990), who showed that the Golden Rule criterion carries over to a setting with aggregate and idiosyncratic risk in OLG models with physical capital. De la Croix and Ponthiere (2010) derived the golden rule of physical capital accumulation policy under endogenous longevity and health spending in an OLG model, and Gestsson and Zoega (2019) derived the golden rule for the optimal level of life-extending health care expenditures in a lifecycle model. See also Kuhle (2012) and Ohtaki (2013), who studied the Golden Rule efficiency and inefficiency in OLG models with heterogeneous agents, and Chen, Chien, and Yang (2017) who implemented the modified Golden Rule of an optimal Ramsey tax problem in a model of incomplete markets.

transfers can be used to reach optimality.

Recently in a three-period OLG model with both physical and human capital, Del Rey and Lopez-Garcia (2013) also assumed large parental human capital externalities so the education technology is homogeneous of degree one and the model exhibits endogenous growth. However, they studied the social optimum under the Golden Rule criterion, compared it to the decentralized allocation, and found that education taxes in combination with intergenerational transfers can reach optimality. The proposed education tax policy goes against the conventional wisdom in models with human capital externalities. The reason behind is, under the Golden Rule criterion, agents ignore the negative effect of investment in education on the growth rate of the economy rather than the positive effect on the future generation.

The purpose of our paper is to show that the optimal policy is no longer to tax education but to subsidize education, once we allow for a smaller parental human capital externality such that the education technology is homogeneous with less than one degree, as in Galor and Moav (2006) and Erosa and Koreshkova (2007). When the education technology is homogeneous with less than one degree in otherwise the same Del Rey and Lopez-Garcia (2013) model, the social planner needs not maximize the long-run welfare with a negative effect of human capital in the social optimum under the Golden Rule criterion. Under the setup, our paper shows that it is optimal to subsidize education investment

Specifically, we study a three-period OLG model of physical and human capital accumulation, as in in Del Rey and Lopez-Garcia (2013)[3] Agents are born with an endowment of knowledge inherited from their parents. Credit markets are perfect. In the first period when young, agents borrow in order to invest in their education. Inherited human capital helps transform education to human capital during middle age, and the human capital externality in our model is smaller than that in Del Rey and Lopez-Garcia (2013). In the second period, agents work, consume, repay loans, save and pay taxes or receive subsidies. In the final period, agents retire, consume and pay taxes or receive subsidies. We use this model to identify the socially optimal allocation, compare it to the decentralized allocation and identify the optimal policy. The welfare criterion is the Golden Rule, as in Del Rey and Lopez-Garcia (2013).

[3] The same model has been studied by Boldrin and Montes (2005), Docquier, Paddison, and Pestieau (2007), and Bishnu (2013).

We study the socially optimal allocation in the Golden Rule and compare it with the decentralized allocation. We show that it is optimal to subsidize education as long as the positive human capital externality is smaller than that in Del Rey and Lopez-Garcia (2013) so the education technology is homogenous with less than one degree. The reason for the education subsidy is as follow. First, the social planner no longer needs to take account the negative effect of investment in education on the growth rate under exogenous growth. Second, focusing on the long-run steady state, the social planner accounts for the social, rather than the private, marginal product of human capital when choosing education in the Golden Rule, and thus the positive effect of the human capital externality comes back. The positive intergenerational externality of human capital in the steady state exhibits like the intragenerational one. As a result, individuals underinvest in their education from the point of view of the social planner even under the Golden Rule criterion, and thus it is optimal to subsidize education in order to decentralize the Golden Rule allocation.[4] Moreover, we show that the education subsidy can be financed either by a lump-sum tax to the middle age group or to the old age group, or to both.

The only question left is the size of the human capital externality which determines whether education should be subsidized or taxed under the Golden Rule criterion. Empirical studies showed such an externality is not as strong as that in Del Rey and Lopez-Garcia (2013). For example, based on the estimates of Card (1999), private return and social return to education are very close to each other. Lange and Topel (2006) also argued that the evidence of the positive externality is very weak. In addition, Rudd (2000), Ciccone and Perri (2006), and Yamarik (2008) all shared the same view. Acemoglu and Angrist (2000) had the same conclusion but argued that if human capital externality exists, a magnitude of 1–3% external return is sufficient to justify public subsidies in education. Our finding of an optimal education subsidy is valuable, as the result is consistent with the generally accepted viewpoint of a subsidy to education in the presence of a positive education externality. Indeed, according to Heckman (2000), a policy implication of substantial human capital externalities is that education and job training should be subsidized.

We organize the remainder of this paper as follows. In Section 2, we

[4] One type of the education subsidies is to offer the public education. See Futagami and Yanagihra (2008).

set up a three-period OLG model and study the decentralized market equilibrium. In Section 3, we analyze the Del Rey and Lopez-Garcia (2013) model and the optimal policies to restore optimality. Section 4 studies our model and the optimal policies to restore optimality. Finally, we offer some concluding remarks in Section 5

2 The model

As in Del Rey and Lopez-Garcia (2013) and Erosa and Koreshkova (2007), there are overlapping generations, and individuals live for three periods. Individuals are born with an endowment of knowledge inherited from parents. When young, they borrow to invest in their education but do not consume. In middle age, they work, consume, reimburse the borrowing for their education, and save for retirement. In their older age, they retire and consume. We refer to agents working in period t (i.e., born in $t-1$) as generation t, indexed by t The size of generation t is N_t, which grows at the rate n, i.e., $N_t = (1+n)N_{t-1}$.

An individual of generation t borrows e_{t-1} from the capital market in $t-1$ and invests in education. The education investment then transforms into the human capital in t given by

$$h_t = \Phi\left(e_{t-1}, h_{t-1}\right), \tag{1a}$$

where h_{t-1} is human capital of parents, and thus, an externality.

Function $\Phi(e,h)$ satisfies the Inada condition and is increasing and strictly concave in e and h, which implies $\Phi_e > 0 > \Phi_{ee}$ and $\Phi_h > 0 > \Phi_{hh}$. Del Rey and Lopez-Garcia (2013) assumed that the parental externality is sufficiently large, so $\Phi(e,h)$ is homogeneous of degree one If we let $\alpha \in (0,1)$ be the elasticity of human capital formation with respect to e_{t-1}, then $(1-\alpha)$ is the elasticity with respect to h_{t-1} in Del Rey and Lopez-Garcia (2013). By contrast, following Galor and Moav (2006) and Erosa and Koreshkova (2007) we assume a smaller parental externality, such that $\Phi(e,h)$ is homogeneous of degree $1-\varepsilon$ where $0<\varepsilon<1-\alpha$. With $\alpha \in (0,1)$ being the elasticity of human capital formation with respect to e_{t-1}, then $(1-\alpha-\varepsilon)$ is the elasticity with respect to h_{t-1} in our model.[5]

[5]Using a Cobb-Douglas function as an example of the education technology, the function in Del Rey and Lopez-Garcia (2013) is $\Phi(e_{t-1}, h_{t-1}) = e_{t-1}^{\alpha} h_{t-1}^{1-\alpha}$, while our function is $\Phi(e_{t-1}, h_{t-1}) = e_{t-1}^{\alpha} h_{t-1}^{1-\alpha-\varepsilon}$, $0<\varepsilon<1-\alpha$.

Let $\tilde{e}_{t-1} \equiv (e_{t-1}/h_{t-1})$. In an intensive form, our education technology in (1a) is rewritten as

$$\frac{h_t}{h_{t-1}} = \frac{1}{h_{t-1}^{\varepsilon}} \Phi\left(\frac{e_{t-1}}{h_{t-1}}, \frac{h_{t-1}}{h_{t-1}}\right) \equiv \frac{1}{h_{t-1}^{\varepsilon}} \varphi\left(\tilde{e}_{t-1}\right), \tag{1b}$$

where $\varphi(\tilde{e}_{t-1})$ is positive, increasing and strictly concave in $\tilde{e}_{t-1}$ and satisfies the Inada condition. The case of Del Rey and Lopez-Garcia (2013) is $\varepsilon = 0$, so their education technology is $(h_t/h_{t-1}) = \varphi(\tilde{e}_{t-1})$.

We will show that the education tax policy in Del Rey and Lopez-Garcia (2013) changes dramatically to the education subsidy, if ε increases from a zero value, so $\varepsilon > 0$, no matter how small the increase is.

The lifetime utility function of an individual agent of generation t is given by

$$u_t = u\left(c_t, d_{t+1}\right), \tag{2}$$

where c_t and d_{t+1} are consumption at middle and old ages, respectively. Following Del Rey and Lopez-Garcia (2013), the utility is assumed to be strictly increasing, strictly concave and homogeneous of degree $b > 0$ in its two arguments. Moreover, the utility function is assumed to satisfy the Inada conditions: $u_c(0, d) = \infty$ and $u_d(c, 0) = \infty$.

There is a single final good Y_t produced by aggregate physical capital K_t and aggregate human capital H_t according to the neoclassical technology $Y_t = F(K_t, H_t)$, which is of constant returns to scale and satisfies the Inada condition. By construction, only the middle-aged work, and they supply one unit of labor inelastically, so per capita human capital is $h_t = (H_t/N_t)$. If we let $k_t = (K_t/N_t)$ be the physical capital per unit of labor, then $\tilde{k}_t \equiv (K_t/H_t) = (k_t/h_t)$ is the physical capital to human capital ratio. Thus, we can describe the final good technology in terms of $Y_t = H_t f(\tilde{k}_t)$, where $f(\tilde{k}) > 0$ satisfies the Inada condition with $f'(\tilde{k}) > 0 > f''(\tilde{k})$. Physical capital is assumed to depreciate totally after one period.

2.1 The decentralized market with the government

We assume that the capital market is perfect. At the young age in period $t-1$, an individual of generation t borrows e_{t-1} to get education. During the middle age, the individual works to earn labor income pays back the education loan, consumes, saves, and pay taxes or receives subsidies. In the old age, the individual receives the return to savings, consumes and pays taxes or receives subsidies.

The budget constraints for an individual of generation t during middle and old age are, respectively,

$$w_t h_t - R_t e_{t-1} (1 - \theta_t) - z_t^m = c_t + s_t, \tag{3a}$$

$$d_{t+1} = R_{t+1} s_t - z_{t+1}^o, \tag{3b}$$

where s_t is savings in period t, w_t is the wage rate of raw labor in period t, and R_t is the interest factor (one plus the interest rate) in period t. The notation $\theta_t > 0$ (resp.< 0) is a subsidy (resp. tax) to the education loan in period t, and $z_t^m > 0$ (resp. < 0) and $z_{t+1}^o > 0$ (resp. < 0) is lump-sum taxes (resp. transfers) when agents are middle-aged in period t and old in period $t+1$, respectively.

The problem of the representative agent of generation t is to maximize (2) subject to (1a), (3a) and (3b), given k_0 and h_0, and taking as given w_t and R_{t+1} for all t. The first-order conditions are as follows.

$$s_t : \frac{u_c\left(\tilde{c}_t, \tilde{d}_{t+1}\right)}{u_d\left(\tilde{c}_t, \tilde{d}_{t+1}\right)} = R_{t+1}, \tag{4a}$$

$$e_{t-1} : w_t \Phi_e (e_{t-1}, h_{t-1}) = (1 - \theta_t) R_t. \tag{4b}$$

While (4a) is the standard consumption Euler equation, (4b) is the optimal education condition, which equalizes the marginal return and the marginal cost, the latter of which is the gross cost of borrowing. With both $u_c(c_t, d_{t+1})$ and $u_d(c_t, d_{t+1})$ being homogeneous of degree $b - 1$, $[u_c(c_t, d_{t+1})]/[u_d(c_t, d_{t+1})] = [u_c(\tilde{c}_t, \tilde{d}_{t+1})]/[u_d(\tilde{c}_t, \tilde{d}_{t+1})]$ in the left-hand side of (4a), where in intensive forms $\tilde{c}_t \equiv (c_t/h_t)$ and $\tilde{d}_{t+1} \equiv (d_{t+1}/h_t)$ are consumption per unit of human capital at middle and old age, respectively.

Factor markets are competitive. Firms choose capital and labor, and the first-order conditions are

$$R_t = R\left(\tilde{k}_t\right) \equiv f'\left(\tilde{k}_t\right), R'\left(\tilde{k}_t\right) < 0, \tag{5a}$$

$$w_t = w\left(\tilde{k}_t\right) \equiv f\left(\tilde{k}_t\right) - \tilde{k}_t f'\left(\tilde{k}_t\right), w'\left(\tilde{k}_t\right) > 0. \tag{5b}$$

The government finances education subsidies by the revenue obtained from taxing the middle age group or the old age group. The government budget constraint in period t is

$$z_t^m N_t + z_t^o N_{t-1} = \theta_t R_t e_{t-1} N_t. \tag{6}$$

2.2 Decentralized market equilibrium

To close the model, as capital depreciates completely after one period, aggregate savings in t are used for the education investment in this period and the accumulation of capital in the next period. Thus, equilibrium in the market for physical capital is: $s_t = (1+n)(e_t + k_{t+1})$. Denote $\tilde{s}_t \equiv (s_t/h_t)$ aggregate savings per unit of human capital. Then, the market equilibrium condition is

$$\tilde{s}_t = (1+n)\left[\tilde{e}_t + \tilde{k}_{t+1}\frac{\Phi\left(e_t, h_t\right)}{h_t}\right], \tag{7}$$

where, using the aggregate savings s_t in (3a), aggregate savings per unit of human capital is given by $\tilde{s}_t = w(\tilde{k}_t) - f'(\tilde{k}_t)\tilde{e}_{t-1}(h_{t-1}/h_t)(1-\theta_t) - \tilde{z}_t^m - \tilde{c}_t$.

Definition 1. For given h_0 and k_0, a competitive equilibrium is the path $\{c_t, d_t, k_t, s_t, e_t, h_t, w_t, R_t\}_{t=0}^{\infty}$ that satisfies agents' budget constraints and optimization conditions (3a) – (3b) and (4a) – (4b), firms' optimization conditions (5a) – (5b), human capital accumulation (1a), the government budget constraint (6), and the market equilibrium condition (7).

We characterize the allocation in the decentralized market equilibrium in the following proposition.

Proposition 1. The allocation in the decentralized market equilibrium is characterized by

$$\frac{u_c\left(\tilde{c}_t^{ce}, \tilde{d}_{t+1}^{ce}\right)}{u_d\left(\tilde{c}_t^{ce}, \tilde{d}_{t+1}^{ce}\right)} = f'\left(\tilde{k}_{t+1}^{ce}\right), \tag{8a}$$

$$w\left(\tilde{k}_t^{ce}\right)\Phi_e\left(e_{t-1}^{ce}, h_{t-1}^{ce}\right) = (1-\theta_t)\, f'\left(\tilde{k}_t^{ce}\right), \tag{8b}$$

$$w\left(\tilde{k}_t^{ce}\right) - f'\left(\tilde{k}_t^{ce}\right)\tilde{e}_{t-1}^{ce}\frac{h_{t-1}^{ce}}{h_t^{ce}}(1-\theta_t) - \tilde{z}_t^m - \tilde{c}_t^{ce}$$
$$= (1+n)\left[\tilde{e}_t^{ce} + \tilde{k}_{t+1}^{ce}\frac{\Phi\left(e_t^{ce}, h_t^{ce}\right)}{h_t^{ce}}\right]. \qquad (8c)$$

where superscript ce stands for the competitive equilibrium, $\tilde{z}_t^m \equiv (z_t^m / h_t^{ce})$ and $w(\tilde{k}_t^{ce}) = f(\tilde{k}_t^{ce}) - (\tilde{k}_t^{ce}) f'(\tilde{k}_t^{ce})$.

Decentralized market equilibrium conditions (8a) – (8c) are derived from (4a) – (4b) and (7), with the use of (1a), (3a), and (5a) – (5b). The allocation in the competitive laissez-faire equilibrium is obtained by setting $\theta_t = \tilde{z}_t^m = \tilde{z}_t^o = 0$ in these conditions, where $\tilde{z}_t^o = (z_t^o / h_t)$.

3 The golden rule social optimum in Del Rey and Lopez-Garcia (2013)

This section studies the social optimum in Del Rey and Lopez-Garcia (2013). These authors solved the social optimum in terms of the Golden Rule of capital accumulation of Phelps (1961) and Diamond (1965). They assumed that the social planner purposively wants to treat all generations alike, while being respectful with individual utilities. They search for the long-run path that maximizes the lifetime welfare of a representative agent subject to the constraint that the welfare of the representative individual of every other generation is fixed at the same level.

The social planner's resource constraint in period t is

$$h_t f\left(\tilde{k}_t\right) = c_t + \frac{d_t}{1+n} + (1+n)\left(e_t + h_{t+1}\tilde{k}_{t+1}\right). \qquad (9a)$$

In an intensive form, (9a) is $f(\tilde{k}_t) = \tilde{c}_t + (d_t/(1+n)h_t) + (1+n)[\tilde{e}_t + (h_{t+1}/h_t)\tilde{k}_{t+1}]$, which, with the use of (1b), is rewritten as

$$f\left(\tilde{k}_t\right) = \tilde{c}_t + \frac{\tilde{d}_t}{(1+n)\varphi\left(\tilde{e}_{t-1}\right)\left(h_{t-1}\right)^{-\varepsilon}}$$
$$+ (1+n)\left[\tilde{e}_t + \tilde{k}_{t+1}\varphi\left(\tilde{e}_t\right)\left(h_t\right)^{-\varepsilon}\right]. \qquad (9b)$$

This section analyzes the case of $\varepsilon = 0$, which is the model of Del Rey and Lopez-Garcia (2013). The case of $\varepsilon > 0$ will be analyzed in the next section.

3.1 The social optimum in Del Rey and Lopez-Garcia (2013)

The model of Del Rey and Lopez-Garcia (2013) is the case $\varepsilon = 0$. In this case, the education technology is homogeneous of degree one in education investment and the parental externality. Then, there is endogenous growth with the economic growth rate $g_t \equiv (h_t/h_{t-1}) - 1 = \varphi(\tilde{e}_{t-1}) - 1$. Del Rey and Lopez-Garcia (2013) maximize the representative individual's lifetime welfare given by

$$u\left(\tilde{c}_t, \tilde{d}_{t+1}\right), \tag{10}$$

where $u(\tilde{c}_t, \tilde{d}_{t+1}) = u[(c_t/h_t), (d_{t+1}/h_t)] = [1/(h_t)]u(c_t, d_{t+1})$,[6] subject to the resource constraint (9b) and the constraint that the welfare of the representative individual of every other generation is fixed at the same level along the BGP Note that the long-run lifetime welfare $u(\tilde{c}_*, \tilde{d}_*) = (1/h^b)u(c_*, d_*)$ indicates that, with other things being equal, larger individual's human capital lowers the individual's welfare.

When $\varepsilon = 0$, the resource constraint (9b) reduces to

$$f\left(\tilde{k}_t\right) = \tilde{c}_t + \frac{\tilde{d}_t}{(1+n)\varphi\left(\tilde{e}_{t-1}\right)} + (1+n)\left[\tilde{e}_t + \tilde{k}_{t+1}\varphi\left(\tilde{e}_t\right)\right].$$

Using a subscript asterisk to denote the social optimum of the variables along the optimal BGP as in Del Rey and Lopez-Garcia (2013), ($\tilde{c}_*$, $\tilde{d}_*$, $\tilde{k}_*$, $\tilde{e}_*$) are constant for all t. The resource constraint is

$$f\left(\tilde{k}_*\right) = \tilde{c}_* + \frac{\tilde{d}_*}{(1+n)\varphi\left(\tilde{e}_*\right)} + (1+n)\left[\tilde{e}_* + \varphi\left(\tilde{e}_*\right)\tilde{k}_*\right], \tag{11}$$

where $\varphi(\tilde{e}_*) = 1 + g_*$ is the gross growth rate of human capital in the BGP, and thus, the growth rate of the economy in the BGP.

[6]The lifetime utility of the representative individual indicates that, with other things being equal, a higher level of the individual's human capital h_t lowers the utility. While one wonder that the setup may be inconsistent with most papers with sustainable economic growth (e.g., Romer (1986); Lucas (1988); Ortigueira and Santos (1997)), this subsection maintains their setup in order to duplicate their result. In particular, we must note that, a higher individual's human capital, partly due to a higher parental human capital externality, lowers the individual's lifetime welfare, and thus, there is a negative externality on the lifetime welfare of the representative individual.

The social planner chooses $(\tilde{c}_*, \tilde{d}_*, \tilde{k}_*, \tilde{e}_*)$ that maximizes $u(\tilde{c}_*, \tilde{d}_*)$, subject to the resource constraint in (11). The socially optimal conditions are as follows.

$$\frac{u_c\left(\tilde{c}_*, \tilde{d}_*\right)}{u_d\left(\tilde{c}_*, \tilde{d}_*\right)} = (1+n)\varphi\left(\tilde{e}_*\right), \tag{12a}$$

$$f'\left(\tilde{k}_*\right) = (1+n)\varphi\left(\tilde{e}_*\right), \tag{12b}$$

$$\varphi'\left(\tilde{e}_*\right)\left[\frac{a_*}{\left[(1+n)\varphi\left(\tilde{e}_*\right)\right]^2} - \tilde{k}_*\right] = 1. \tag{12c}$$

Condition (12a) equates the marginal rate of substitution in consumption per unit of human capital between middle age and old age (i.e., $\tilde{c}_*$ and $\tilde{d}_*$) to the economy's growth rate, while (12b) equates the marginal product of capital per unit of human capital (i.e., $\tilde{k}_*$) to the economy's growth rate. Condition (12c) is for the choice of output per unit of human capital devoted to education per unit of human capital (i.e., $\tilde{e}_*$), and the optimal choice requires that the marginal benefit of education be equal to the marginal cost of education.

The Golden Rule thus consists of the conditions $(1+g_*) = \varphi(\tilde{e}_*)$, (11), and (12a) – (12c) that need to be satisfied simultaneously. These conditions simultaneously determine the optimal allocation in the Golden Rule: $(g_*, \tilde{c}_*, \tilde{d}_*, \tilde{k}_*, \tilde{e}_*)$.

To simplify (12a) – (12c), using (12b), we can rewrite (12a) as

$$\frac{u_c\left(\tilde{c}_*, \tilde{d}_*\right)}{u_d\left(\tilde{c}_*, \tilde{d}_*\right)} = f'\left(\tilde{k}_*\right). \tag{13a}$$

Moreover, using (11) and (12b), we can rewrite (12c) as

$$\varphi'\left(\tilde{e}_*\right)\left[f\left(\tilde{k}_*\right) - \tilde{k}_* f'\left(\tilde{k}_*\right) - \Lambda_*\left(\tilde{k}_*, \tilde{e}_*\right)\right] = f'\left(\tilde{k}_*\right), \tag{13b}$$

where $\Lambda_*(\tilde{k}_*, \tilde{e}_*) \equiv (1+n)\varphi(\tilde{e}_*)\tilde{k}_* + (1+n)\tilde{e}_* + \tilde{c}_*$.

3.2 Optimal public policy

Along the BGP, the decentralized market allocation in the economy of Del Rey and Lopez-Garcia (2013) $(\tilde{c}^{ce}, \tilde{d}^{ce}, \tilde{k}^{ce}, \tilde{e}^{ce})$ is constant for all time t.

In this economy, the education technology $\Phi(e_{t+1}, h_{t+1})$ in (1a) is homogeneous of degree one, so $\Phi_e(e_{t+1}, h_{t+1})$ is homogeneous of degree zero and equal to $\varphi'(\tilde{e}_{t-1})$. Along the BGP, the gross economic growth in (1b) is $1 + \tilde{g}^{ce} \equiv (h_t/h_{t-1}) = \varphi(\tilde{e}^{ce})$, which is constant, and the marginal product of education $\varphi'(\tilde{e}^{ce})$ is also constant. The conditions (8a) – (8c) of the decentralized market equilibrium along the BGP, under $\varepsilon = 0$, is as follows.

$$\frac{u_c\left(\tilde{c}^{ce}, \tilde{d}^{ce}\right)}{u_d\left(\tilde{c}^{ce}, \tilde{d}^{ce}\right)} = f'\left(\tilde{k}^{ce}\right), \tag{14a}$$

$$\varphi'\left(\tilde{e}^{ce}\right) w\left(\tilde{k}^{ce}\right) = f'\left(\tilde{k}^{ce}\right)(1-\theta), \tag{14b}$$

$$\begin{aligned} w\left(\tilde{k}^{ce}\right) - f'\left(\tilde{k}^{ce}\right)\tilde{e}^{ce}\frac{1}{\varphi\left(\tilde{e}^{ce}\right)}(1-\theta) - \tilde{z}^m - \tilde{c}^{ce} \\ = (1+n)\left[\tilde{e}^{ce} + \tilde{k}^{ce}\varphi\left(\tilde{e}^{ce}\right)\right]. \end{aligned} \tag{14c}$$

where $w(\tilde{k}^{ce}) = f(\tilde{k}^{ce}) - \tilde{k}^{ce} f'(\tilde{k}^{ce})$.

Del Rey and Lopez-Garcia (2013) showed that, by setting $\theta = \tilde{z}^m = \tilde{z}^o = 0$ in these conditions above, the laissez-faire equilibrium cannot possibly support the Golden Rule BGP. They then discuss the optimal policy $\{\theta, \tilde{z}^m, \tilde{z}^o\}$ that allows conversion of the laissez-faire equilibrium into the optimal Golden Rule BGP.

To replicate the long-run decentralized allocation $(\tilde{c}^{ce}, \tilde{d}^{ce}, \tilde{k}^{ce}, \tilde{e}^{ce})$ with the socially optimal allocation $(\tilde{c}_*, \tilde{d}_*, \tilde{k}_*, \tilde{e}_*)$ in the Golden Rule BGP, we compare (14b) with (13b). Rewriting (13b) gives

$$\varphi'\left(\tilde{e}_*\right)\left[f\left(\tilde{k}_*\right) - \tilde{k}_* f'\left(\tilde{k}_*\right)\right] = f'\left(\tilde{k}_*\right)\left[1 + \frac{\varphi'\left(\tilde{e}_*\right)\Lambda_*\left(\tilde{k}_*, \tilde{e}_*\right)}{f'\left(\tilde{k}_*\right)}\right]. \tag{15}$$

By restricting the allocation $(\tilde{c}^{ce}, \tilde{d}^{ce}, \tilde{k}^{ce}, \tilde{e}^{ce})$ in (14b) to be identical to the allocation $(\tilde{c}_*, \tilde{d}_*, \tilde{k}_*, \tilde{e}_*)$ in (15), the left-hand side of (14b) is equal to the left-hand side of (15). Then, the right-hand side of (14b) and (15) must be equal, which yields

$$\theta_* = -\frac{\varphi'\left(\tilde{e}_*\right)\Lambda_*\left(\tilde{k}_*, \tilde{e}_*\right)}{f'\left(\tilde{k}_*\right)} < 0. \tag{16}$$

Thus, decentralizing the Golden Rule BGP entails a negative subsidy to education; that is, an optimal education tax. As a result of the education tax, Del Rey and Lopez-Garcia (2013) used positive pensions to the elderly to balance of the government budget. With the education tax, Del Rey and Lopez-Garcia (2013) proposed to transfer the tax revenue in lump sums as pensions for those of the old age.

We remark that the reason for the optimal education tax in Del Rey and Lopez-Garcia (2013) is due to the formulation that the social planner maximizes the lifetime welfare that is decreasing in the human capital, but individuals neglect the negative effect when they choose education investment. If the education technology is homogeneous with less than one degree, the social planner maximizes the lifetime welfare that is independent of the human capital. The neglect effect on the welfare does not emerge when individuals choose education investment. This is the analysis in the next section.

4 The golden rule social optimum in our model

In the case of our economy, $\varepsilon > 0$. Then, the education technology $h_t = \Phi(e_{t-1}, h_{t-1})$ in (1a) is homogeneous of degree $1 - \varepsilon$, which is less than degree one. Thus, the economy has only exogenous growth and there is a steady state. Using a superscript asterisk to denote the social optimum of variables in the steady state, $(c^*, d^*, \tilde{k}^*, h^*, e^*)$ are constant for all t. In this economy without endogenous growth, the lifetime welfare $u(c^*, d^*)$ is a function of consumption in the middle age and the old age, as opposed to a function of consumption in unit of human capital. Thus, one can easily impose the constraint that the individual's lifetime welfare be fixed at the same level for every other generation in the Golden Rule.

4.1 The social optimum in our economy with exogenous growth

The education technology in the steady state is $h^* = \Phi(e^*, h^*)$, which is rewritten as

$$h^* = \psi\left(e^*\right), \tag{17}$$

where $\psi(e^*)$ is homogeneous of degree $\gamma \equiv (\alpha/\alpha + \varepsilon) < 1$.

In the steady state, using (17), the social planner's resource constraint in

(9a) is rewritten as

$$\psi\left(e^{*}\right) f\left(\tilde{k}^{*}\right)=c^{*}+\frac{d^{*}}{1+n}+(1+n)\left[e^{*}+\psi\left(e^{*}\right) \tilde{k}^{*}\right]. \tag{18}$$

Subject to the constraint that the lifetime welfare of the individual of every other generation is fixed at the same level, the social planner chooses $(c^{*}, d^{*}, \tilde{k}^{*}, e^{*})$ that maximizes $u(c^{*}, d^{*})$ subject to the resource constraint in (18). The socially optimal conditions are

$$\frac{u_{c}\left(c^{*}, d^{*}\right)}{u_{d}\left(c^{*}, d^{*}\right)}=1+n, \tag{19a}$$

$$f^{\prime}\left(\tilde{k}^{*}\right)=1+n, \tag{19b}$$

$$\psi^{\prime}\left(e^{*}\right)\left[\frac{f\left(k^{*}\right)}{1+n}-\tilde{k}^{*}\right]=1, \tag{19c}$$

where $1+n$ is the economy's (gross) natural rate of growth.

These socially optimal conditions are otherwise identical to those of (12a) – (12c) in Section 3, except in terms of consumption, output and education investment, as opposed to consumption, output and education investment in unit of human capital in (12a) – (12c). Equations (18) and (19a) – (19c) simultaneously determine the socially optimal allocation in the Golden Rule $(c^{*}, d^{*}, \tilde{k}^{*}, e^{*})$.

First, using (19b), we can rewrite (19a) as

$$\frac{u_{c}\left(c^{*}, d^{*}\right)}{u_{d}\left(c^{*}, d^{*}\right)}=f^{\prime}\left(\tilde{k}^{*}\right). \tag{20a}$$

As $u(c, d)$ is homogeneous, in an intensive form the left-hand side of (20a) is $[u_{c}(\tilde{c}^{*}, \tilde{d}^{*})/u_{d}(\tilde{c}^{*}, \tilde{d}^{*})]$. Then, the expression of (20a) is the same as that of (13a), which is the combination of (12a) and (12b) in the social optimal conditions in the model of Del Rey and Lopez-Garcia (2013).

Next, using (19b), we rewrite (19c) as

$$\psi^{\prime}\left(e^{*}\right)\left[f\left(\tilde{k}^{*}\right)-f^{\prime}\left(\tilde{k}^{*}\right) \tilde{k}^{*}\right]=f^{\prime}\left(\tilde{k}^{*}\right), \tag{20b}$$

in which the left-hand side of (20b) does not involve a term like $\Lambda_{*}(\tilde{k}_{*}, \tilde{e}_{*})$ and thus, is different from that of (13b), which is the combination of (12b) and (12c), along with (11) in the model of Del Rey and Lopez-Garcia (2013).

4.2 Optimal public policy

In our model, the decentralized allocation $(c^{ce}, d^{ce}, \tilde{k}^{ce}, h^{ce}, e^{ce})$ is constant in the steady state. Conditions (8a) – (8c) of the decentralized market equilibrium in the steady state are rewritten as follows.

$$\frac{u_c\left(c^{ce}, d^{ce}\right)}{u_d\left(c^{ce}, d^{ce}\right)} = f'\left(\tilde{k}^{ce}\right), \tag{21a}$$

$$w\left(\tilde{k}^{ce}\right)\Phi_e\left(e^{ce}, h^{ce}\right) = f'\left(\tilde{k}^{ce}\right)(1-\theta), \tag{21b}$$

$$\begin{aligned} h^{ce} w\left(\tilde{k}^{ce}\right) - f'\left(\tilde{k}^{ce}\right) e^{ce}(1-\theta) - z^m - c^{ce} \\ = (1+n)e^{ce} + (1+n)h^{ce}\tilde{k}^{ce}, \end{aligned} \tag{21c}$$

where $w(\tilde{k}^{ce}) = f(\tilde{k}^{ce}) - \tilde{k}^{ce} f'(\tilde{k}^{ce})$, and $h^{ce} = \Phi(e^{ce}, h^{ce})$, which, using (17), is expressed by

$$h^{ce} = \psi\left(e^{ce}\right), \tag{21d}$$

where $\psi(e^{ce})$ is homogeneous of degree $\gamma = (\alpha/\alpha + \varepsilon) < 1$.

As long as $0 < \varepsilon < 1 - \alpha$, it is easy to show that the laissez-faire allocation, determined by conditions (21a) – (21d) with $\theta = z^m = z^o = 0$, cannot attain the social optimum in the Golden Rule. Thus, we turn to the optimal public policy in order for the decentralized allocation to attain the Golden Rule optimum.

These decentralized conditions (21a) – (21c) are compared with the socially optimal conditions (19a) – (19c). To see what optimal policies $\{\theta, z^m, z^o\}$ allows us to convert the decentralized allocation in the long run $(c^{ce}, d^{ce}, \tilde{k}^{ce}, h^{ce}, e^{ce})$ into the optimal allocation $(c^*, d^*, \tilde{k}^*, h^*, e^*)$ in the Golden Rule, it serves to compare the socially optimal condition (20b) with the decentralized condition (21b). Rewriting (20b) and (21b) and evaluating at the Golden Rule allocation yields, respectively,

$$f\left(\tilde{k}^*\right) - f'\left(\tilde{k}^*\right)\tilde{k}^* = f'\left(\tilde{k}^*\right)\frac{1}{\psi'\left(e^*\right)}, \tag{22a}$$

$$f\left(\tilde{k}^*\right) - f'\left(\tilde{k}^*\right)\tilde{k}^* = f'\left(\tilde{k}^*\right)\frac{1-\theta}{\Phi_e\left(e^*, h^*\right)}. \tag{22b}$$

As the left-hand sides of (22a) and (22b) are the same, their right-hand sides must be equal. This yields the optimal education policy as follows.

$$\theta^* = 1 - \frac{\Phi_e(e^*, h^*)}{\psi'(e^*)} > 0.^7 \tag{23}$$

Therefore, it is optimal to subsidize the loan for education investment. This allows us to state the following proposition.

Proposition 2. When the education function is homogeneous of degree $1 - \varepsilon < 1$, decentralizing the Golden Rule allocation entails an education subsidy.

Thus, different from Del Rey and Lopez-Garcia (2013), decentralizing the Golden Rule allocation entails an education subsidy, rather than an education tax, when the education function is homogeneous with less than one degree.

Intuitively, when determining the amount of education in the Golden Rule, the social planner takes the positive externality into account, whose marginal product is captured by the term $\psi'(e^*)$. However, when deciding the amount of the education in the decentralized market, individuals neglect the positive externality of their human capital upon the next generation, whose private marginal product is captured by the term $\Phi_e(e^*, h^*)$. As $\Phi_e(e^*, h^*) < \psi'(e^*)$, individuals underinvest in education. Therefore, it is optimal to subsidize education loans in order to increase individuals' education investment.

The subsidy to education may be financed by a lump-sum taxes to the middle age or the old age groups. The government budget constraint is in (6). In the steady state, evaluating at the social optimum, the government budget constraint is

$$\theta^* R\left(\tilde{k}^*\right) e^* = \frac{z^{o*}}{1+n} + z^{m*}. \tag{24}$$

Thus, the subsidy to education loans is financed either by a lump-sum tax to the middle age group $z^{m*} > 0$ or to the old age group $(z^{o*}/1+n) > 0$, or to both. To see how the lump sums are taxed, evaluating at the social

[7] Since in the steady state $h^* = \psi(e^*) = \Phi(e^*, \psi(e^*))$, $\psi'(e^*) = \Phi_e + \Phi_h \psi'(e^*)$. Thus, $[\Phi_e(e^*, h^*)/\psi'(e^*)] < 1$.

optimum, the good market clearing condition in (21c) gives the following optimal z^{m*}.

$$z^{m*} = \left[h^* w\left(\tilde{k}^*\right) - R\left(\tilde{k}^*\right) e^*(1-\theta) - c^*\right] - (1+n)\left[e^* + h^*\tilde{k}^*\right]. \tag{25}$$

The right-hand side of (25) includes two square brackets. The first bracket is the wage income at middle age after paying the education loan and consumption.[8] The second bracket is the sum of the education investment and the physical capital accumulation for the next generation. While the first bracket is this generation's aggregate savings, the second bracket is the usage of the aggregate savings for the next generation. If the amount of this generation's aggregate savings exceeds the usage for the next generation, then $z^{m*} > 0$ and it is optimal to tax the middle age group a lump sum; if otherwise, then $z^{m*} < 0$ and it is optimal to transfer the middle age group a lump sum.

As for the lump-sum tax or subsidy to the old age, substituting (25) into (24) yields z^{o*} as follows.

$$\frac{z^{o*}}{1+n} = -\left[h^* w\left(\tilde{k}^*\right) - R\left(\tilde{k}^*\right) e^* - c^*\right] + (1+n)\left[e^* + h^*\tilde{k}^*\right]. \tag{26}$$

The right-hand side of (26) includes two square brackets. These square brackets are otherwise those in (26) with opposite signs, except that the first bracket in (26) involves no education subsidy for the old age, while the first bracket in (25) involves an education subsidy for the middle age, since the education subsidy was given at the middle age. Thus, in the case when $z^{m*} < 0$ it is $z^{o*} > 0$ for sure. However, as the education subsidy is given at the middle age, in the case when $z^{m*} > 0$, both $z^{o*} > 0$ and $z^{o*} < 0$ are possible. As a result, the education subsidy is financed either by a lump-sum tax to the middle age group or to the old age group, or to both.

5 Concluding remarks

In this paper, we study a three-period OLG model with physical and human capital accumulation. There are intergenerational human capital externali-

[8] In the wage income, the multiplication of h^* transforms the wage income value per unit of human capital to the wage income value.

ties in transforming education to human capital in the next period. We consider degrees of human capital externalities. When the education technology is homogeneous of degree less than one, the model exhibits exogenous, not endogenous, growth.

In our OLG economy with exogenous growth, the growth path of the decentralized market equilibrium fails to achieve optimality, because individuals' accumulation of physical and human capital is different from the one that maximizes welfare along the Golden Rule path. We propose to recover the Golden Rule of physical and human capital accumulation. We study the social optimum, compare it to the decentralized allocation, and characterize the optimal policy to decentralize the Golden Rule path. We show that the optimal policy involves subsidies to education loans, as the subsidy helps increase the incentive to invest in education. Moreover, we show that the education subsidy can be financed by either a lump-sum tax to the middle age group, to the old age group, or to both.

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成長模型的最適教育與退休金政策

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在同時具有實體資本和人力資本的疊代經濟體中, 因爲個人的實體資本和人力資本累積偏離了黃金法則路徑 (Golden Rule path), 故市場經濟的成長軌跡無法達到最適。Del Rey and Lopez-Garcia (2013) 發現, 當教育技術在自身教育投入和父母人力資本外部性之間爲一階齊次時, 對教育課稅融通退休金的政策可幫助達成黃金法則路徑。然而實證證據顯示父母的人力資本外部性十分薄弱。本篇研究展示一旦父母人力資本外部性較弱, 使得教育技術的階次小於一時, 達成黃金法則路徑的最適政策爲對教育補貼而非課稅。

關鍵詞: 成長模型, 人力資本外部性, 教育政策, 跨代移轉
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