# Optimal Bank Reserve Remuneration and Capital Control Policy*

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**Abstract**

A central prediction of open economy models with a pecuniary externality due to a collateral constraint is that the unregulated economy overborrows relative to what occurs under optimal policy. A maintained assumption in this literature is that households borrow directly from foreign lenders. This paper shows that if foreign lending is intermediated by domestic banks and the government can pay interest on bank reserves and impose capital controls, the unregulated economy underborrows. The optimal bank reserve policy is countercyclical. By increasing bank reserves during contractions, the government acts as a lender of last resort to collateral-constrained households.

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# 1 Introduction

A central question in open economy macroeconomics is whether, when left to their own devices, countries overborrow. This question has been largely analyzed in the context of models in which households are subject to a collateral constraint, whereby debt is limited by a fraction of income or the value of an asset (Bianchi, 2011; Korinek, 2011; Benigno et al. 2013; Dávila and Korinek, 2018; Jeanne and Korinek, 2019; among others). Collateral constraints of this type create a pecuniary externality because individual agents take as given the prices of objects that they pledge as collateral but in the aggregate these prices are determined endogenously. A key result in this literature is that the unregulated economy overborrows from the rest of the world relative to what it would borrow under optimal capital control policy.

A common feature of this class of model is the assumption that private agents borrow directly from foreign lenders. In reality, however, individual agents seldom borrow directly from foreign lenders. Instead, capital inflows are intermediated by banks operating in domestic markets. A natural question is whether this simplification has consequences for the main predictions of this class of model. This paper revisits the question of overborrowing in the context of a model that builds on the collateral-constraint framework by adding a bank-intermediation channel. The formulation of the banking channel follows Cúrdia and Woodford (2011).

The paper studies an open economy with a collateral constraint by which household debt is limited by a fraction of income. A banking sector receives deposits from foreign investors and lends them to households. This intermediation activity is costly. Banks can mitigate the cost of originating loans by holding reserves at the central bank.

As in the related literature on macroprudential policy in open-economy models with collateral constraints, the government can impose capital control taxes. With the introduction of a banking sector the interest rate on reserves emerges as an additional policy instrument that the government may use jointly with capital controls to achieve an allocation that improves upon the one associated with the unregulated competitive equilibrium. Thus, relative to the standard overborrowing model, the present environment features an additional friction, bank intermediation, and an additional policy instrument, interest on bank reserves. The fiscal expenditures (revenues) stemming from capital controls, interest payments on bank reserves, and operating a reserve facility are assumed to be financed by income taxes (transfers).

We show that a social planner with access to capital controls and interest on reserves as policy instruments can implement an allocation in which households internalize the pecuniary externality and banks internalize the social costs and benefits of reserve provision. Specifically, the social planner maximizes the household's lifetime utility subject to the economy's resource constraint, the bank's balance sheet, and the household's collateral constraint evaluated at the equilibrium price of collateral.

The central result of the paper is that under plausible parameterizations the economy with a bank intermediation channel underborrows. That is, the distribution of external debt in the competitive equilibrium in which the government neither remunerates reserves nor taxes capital flows lies to the left of the one associated with the equilibrium in which the interest rate on bank reserves and capital control taxes are set optimally.

To understand the intuition behind this result note first that in a small open economy consumption smoothing requires that negative income shocks be financed by increases in the external debt. If households borrow directly from foreign lenders, a binding collateral constraint that forces them to deleverage after the negative income shock can prevent them from smoothing consumption. That is, household deleveraging implies economy wide deleveraging. By contrast, in the economy with banks, by the bank's balance sheet, foreign deposits (i.e., the country's external debt) are allocated either to loans to households or to bank reserves at the central bank. When the economy suffers a negative shock that causes the household's collateral constraint to bind, the government finds it optimal to step in and raise the interest rate on bank reserves. In turn, the increase in the interest rate on reserves induces banks to deposit at the central bank the funds that are no longer demanded by financially constrained households. The increased holdings of bank reserves by the central bank make their way into the budget constraints of households via fiscal policy (reductions in income taxes) providing liquidity when households need it. With this intervention the economy continues to borrow from abroad despite a binding constraint at the household level. In this way, the consolidated government acts as a lender of last resort to the non-financial sector. The prediction of the model that during a crisis financial intermediation by the central bank increases is in line with the observed behavior of central bank balance sheets during the global financial crisis (see, for example, Allen and Moessner, 2013, for evidence from the euro area).

The presence of a collateral constraint and a banking channel opens the question of whether the collateral constraint should be placed at the level of the bank or at the level of the non-bank private sector. Following the literature on overborrowing cited above, we adopt the latter option (the collateral constraint limits household debt). This modeling choice has consequences for the predicted behavior of the bank lending spread around financial crises. We show that when the collateral constraint is imposed at the level of the household, the lending spread on bank loans tends to fall during financial crises because a binding collateral constraint represents a decline in the demand for bank credit. We also show that when the collateral constraint is imposed at the bank level, the lending spread tends to rise because in this case a financial crisis represents a fall in the supply of credit. However, the equilibrium real allocation (including the volume of borrowing) is independent of these two alternative formulations.

A macroprudential instrument closely related to reserve remuneration is reserve requirements. A question of interest is whether the equilibrium outcomes a policymaker can attain with one of these instruments can also be achieved or improved upon with the other. The paper shows that there is a clear ranking between these two policy tools: bank reserve remuneration strictly welfare dominates reserve requirements. Intuitively, by paying interest on reserves, the central bank controls the price of this component of the bank's asset side but allows the quantity (bank reserves) to be determined endogenously. On the other hand, a reserve requirement with no interest on reserves represents a restriction on both the quantity and the price of bank reserves, therefore reducing the set of real allocations that it can support as competitive equilibria.

This paper is related to two strands of literature, one on overborrowing in open-economy models with collateral constraints in the nonfinancial sector and one on closed-economy models with a banking channel. Open economy models with collateral constraints at the household level are studied in Mendoza (2002), Uribe (2006), Korinek (2011), Bianchi (2011), Benigno et al. (2013), Dávila and Korinek (2018), Jeanne and Korinek (2019), Schmitt-Grohé and Uribe (2021), Arce, Bengui, and Bianchi (2022), and Davis, Devereux, and Yu (2022), among others. The banking model follows Cúrdia and Woodford (2011) and Eggertsson et al. (2019). Uribe and Yue (2006) introduce bank intermediation along the lines of the model considered in this paper to create a spread between the domestic and the world interest rates. However, their formulation does not contemplate a role for bank reserves. The present paper builds upon these two bodies of work by combining a collateral constraint at the household level and a banking sector in the context of an open economy.

Exceptions to the overborrowing result in the related literature are Benigno et al. (2013), Schmitt-Grohé and Uribe (2021), and Drechsel and Kim (2022). In these papers, the unregulated economy underborrows. However, the reasons for underborrowing in these papers are different from the ones stressed here. In Benigno et al. underborrowing stems from introducing production in the nontradable sector or distortionary sectoral taxation. In the present environment the bank intermediation channel produces underborrowing even without production or distortionary taxation in the nontradable sector. Schmitt-Grohé and Uribe (2021) show that the canonical open-economy model with a flow collateral constraint exhibits multiple equilibria under plausible calibrations. In this environment, underborrowing is the consequence of excess saving caused by the possibility of self-fulfilling financial crises. In the present paper the banking channel leads to underborrowing even under parameterizations for which the equilibrium is unique. Drechsel and Kim (2022) consider earnings-based collateral constraints on firms. In their framework, underborrowing arises because firms fail to internalize the effect of their own borrowing on equilibrium wages and hence earnings. In the present model, the introduction of a banking channel causes the economy to underborrow even in the absence of labor or capital as factor inputs in production. Finally, the paper is related to a class of models in which a collateral constraint is placed on banks rather than on households. Bocola and Lorenzoni (2020) and Céspedes and Chang (2020) represent examples of such formulations in the context of an open economy.

The remainder of the paper is organized as follows. Section 2 develops the model, introduces the policy instruments, and derives the constrained optimal allocation. Section 3 performs the quantitative analysis and presents the main results. Section 4 studies two extensions, placing the collateral constraint at the level of banks and replacing interest on reserves with reserve requirements. Section 5 concludes.

# 2 The Model

In this section, we present a model of an open economy in which banks serve as intermediaries between foreign investors, who supply funds, domestic households, who demand bank loans, and the domestic government, who operates a reserve facility. The specification of the bank lending channel follows Cúrdia and Woodford (2011). The banking sector is embedded into a standard open economy model with a flow collateral constraint, whereby household debt is limited by a fraction of income, along the lines of Mendoza (2002), Bianchi (2011), and Korinek (2011). The collateral constraint introduces a pecuniary externality because the relative price of nontradable goods, which determines the value of collateral, is taken as exogenous by individual borrowers but is endogenous to the economy. After presenting the model and the definition of a competitive equilibrium, the section characterizes the constrained optimal allocation attainable by a government that uses as policy instruments interest on bank reserves and capital controls.

## 2.1 Banks

We assume that the economy has a large number of identical, perfectly competitive financial intermediaries, which we will refer to as banks. Each period, banks issue loans, $l_t$, hold reserves, $r_t$, capture deposits, $d_t$, and distribute dividends, $\pi_t$. Banks face intermediation costs, denoted $\Gamma_t$. This cost is meant to capture expenses such as those related to loan monitoring and management. The sequential budget constraint of a bank is

$$\pi_t + l_t + r_t + (1 + i^d_{t-1})d_{t-1} + \Gamma_t = (1 + i^l_{t-1})l_{t-1} + (1 + i^r_{t-1})r_{t-1} + d_t, \tag{1}$$

where $i^d_{t-1}$ is the interest rate paid by the bank on deposits held from period $t-1$ to period $t$, $i^l_{t-1}$ is the interest rate charged by the bank on loans made in period $t-1$ and due in period $t$, and $i^r_{t-1}$ is the interest rate the bank earns on reserves deposited at the central bank from period $t-1$ to period $t$. The right-hand side of the sequential budget constraint represents the bank's sources of funds and the left-hand side the uses of funds.

The intermediation cost is assumed to depend on the volume of loans and bank reserves,

$$\Gamma_t = \Gamma(l_t, r_t). \tag{2}$$

We introduce the following assumptions about this function:

**Assumption 1** (Intermediation Cost Function)**.** *The function* $\Gamma(\cdot,\cdot)$ *satisfies: (i)* $\Gamma(\cdot,\cdot) \geq 0$*; (ii)* $\Gamma_l(\cdot,\cdot) \geq 0$ *and* $\Gamma_r(\cdot,\cdot) \leq 0$*; (iii)* $\Gamma_{ll} \geq 0$, $\Gamma_{rr} \geq 0$, $\Gamma_{lr} < 0$, *and* $\Gamma_{ll}\Gamma_{rr} - \Gamma^2_{rl} \geq 0$*; (iv)* $\Gamma(0,\cdot) = \Gamma_l(0,\cdot) = \Gamma_r(0,\cdot) = 0$ *and* $\Gamma(l,\cdot) > 0$ *for* $l > 0$, *and (v) there exists a finite level of reserves,* $\bar{r} > 0$, *such that* $\Gamma_r(\cdot, r) = 0$ *for all* $r \geq \bar{r}$.

Assumptions (i)-(iii) are standard. In particular, the assumption that the cost function is nondecreasing in loans is meant to capture administrative and default costs of originating bank credit to the private sector, the assumption that it is nonincreasing in bank reserves is meant to capture that bank reserves contribute to reducing default risk and possible maturity mismatches between bank liabilities and assets, and the assumption of a negative cross derivative makes reserves complementary in the production of loans. Assumption (iv) is meant to capture the idea that the central bank has zero default risk, so, aside from interest differentials, it is costless for banks to park funds there in the form of reserves. As will become apparent shortly, this assumption will play a role in determining the optimal interest-on-reserve policy. Assumption (v) is common in models with a formulation of the banking sector of the type studied here (Cúrdia and Woodford, 2011; Eggertsson et al., 2019). It says that there exists a satiation level of reserves above which reserves cease to lower the intermediation costs of loans. At the satiation point, however, intermediation costs need not vanish.

Following the related literature, we assume that banks distribute as dividends their beginning-of-period net worth,

$$\pi_t = (1 + i^l_{t-1})l_{t-1} + (1 + i^r_{t-1})r_{t-1} - (1 + i^d_{t-1})d_{t-1}. \tag{3}$$

In the context of the literature on the banking channel, the rationale for assuming an ad-hoc dividend rule is that if banks were assumed to choose the dividend stream to maximize the lifetime welfare of the representative household, they could fully neutralize the financial frictions. The assumption that banks do not act on behalf of the representative household is justified by the observation that in reality banks are owned by a small fraction of households with potentially different pricing kernels than the rest of the population. Modeling this source of heterogeneity explicitly is beyond the scope of this paper.

Combining the bank's sequential budget constraint (1), the intermediation cost function (2), and the dividend policy function (3) evaluated in periods $t$ and $t+1$, one can write

$$\frac{\pi_{t+1}}{1+i_t^d} = \frac{i_t^l - i_t^d}{1+i_t^d} l_t + \frac{i_t^r - i_t^d}{1+i_t^d} r_t - \Gamma(l_t, r_t). \tag{4}$$

This expression provides an alternative interpretation of the dividend policy. Banks distribute at the beginning of period $t+1$ all of the operating profits of period $t$. Thus, we refer to $\pi_{t+1}$ as profits or dividends interchangeably. By Assumption 1 profits vanish at $l_t = r_t = 0$. Thus, a profit maximizing bank would never distribute negative dividends.

Banks choose $l_t \geq 0$ and $r_t \geq 0$ to maximize (4), taking as given $i_t^l$, $i_t^d$, and $i_t^r$. Profit maximization implies the following first-order conditions with respect to $l_t$ and $r_t$:

$$\frac{i_t^l - i_t^d}{1+i_t^d} \leq \Gamma_l(l_t, r_t), \quad l_t \geq 0, \quad \left[\frac{i_t^l - i_t^d}{1+i_t^d} - \Gamma_l(l_t, r_t)\right] l_t = 0, \tag{5}$$

and

$$\frac{i_t^r - i_t^d}{1+i_t^d} \leq \Gamma_r(l_t, r_t), \quad r_t \geq 0, \quad \left[\frac{i_t^r - i_t^d}{1+i_t^d} - \Gamma_r(l_t, r_t)\right] r_t = 0. \tag{6}$$

Optimality condition (5) says that when the volume of loans is positive, $l_t > 0$, the marginal net revenue of originating a loan, given by the lending spread $(i_t^l - i_t^d)/(1+i_t^d)$, must equal the marginal cost of originating a loan $\Gamma_l(l_t, r_t)$. When bank reserves are positive, optimality condition (6) says that the bank holds bank reserves so as to equate their marginal benefit, $-\Gamma_r(l_t, r_t)$, to their marginal cost, $-(i_t^r - i_t^d)/(1+i_t^d)$. Since $\Gamma_l(\cdot,\cdot)$ is nonnegative, optimality condition (5) implies that when the volume of loans is positive, the deposit rate is the lower bound of the loan rate. Similarly, because $\Gamma_r(\cdot,\cdot)$ is nonpositive, optimality condition (6) implies that the deposit rate is the upper bound of the reserve rate.

The bank's sequential budget constraint (1) and the dividend rule (3) imply that

$$l_t + r_t + \Gamma(l_t, r_t) - d_t = 0, \tag{7}$$

which says that deposits are used to fund loans and reserves and to cover intermediation costs. With some abuse of terminology, we refer to this expression as the balance sheet of the bank at the end of the period.

## 2.2 Households

Households have preferences for consumption described by the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \tag{8}$$

where $c_t$ denotes consumption, $\beta \in (0,1)$ is the subjective discount factor, and $u(\cdot)$ is an increasing and strictly concave period utility function. Consumption is a composite of tradable and nontradable goods,

$$c_t = A(c_t^T, c_t^N), \tag{9}$$

where $c_t^T$ and $c_t^N$ denote consumption of tradables and nontradables and $A(\cdot,\cdot)$ is an increasing, concave, and linearly homogeneous aggregator function. Each period, households are endowed with $y_t^T$ units of tradable goods and $y_t^N$ units of nontradable goods, receive dividends $\pi_t$ from the ownership of banks, pay income taxes at the rate $\tau_t$, and can borrow from banks at the rate $i_t^l$. Their sequential budget constraint is

$$c_t^T + p_t c_t^N + (1 + i_{t-1}^l) l_{t-1} = (1 - \tau_t)[y_t^T + p_t y_t^N + \pi_t] + l_t, \tag{10}$$

where $p_t$ is the relative price of nontradables in terms of tradables.

Loans face a collateral constraint, which depends on the value of income in units of tradable goods as follows,

$$l_t \leq \kappa(y_t^T + p_t y_t^N), \tag{11}$$

where $\kappa > 0$ is a parameter. We use this specification of collateral to be in line with the related literature (e.g., Bianchi, 2011) and for analytical tractability. An alternative plausible but less tractable specification is one in which collateral is proportional to disposable income, including after-tax profits.

Households choose processes $c_t$, $c_t^T$, $c_t^N$, and $l_t$ to maximize the lifetime utility function (8) subject to the aggregation technology (9), the sequential budget constraint (10), and the collateral constraint (11), taking as given $p_t$, $i_t^l$, $\pi_t$, $\tau_t$, $y_t^T$, and $y_t^N$. The first-order conditions associated with this problem are

$$u'(A(c_t^T, c_t^N)) A_1(c_t^T, c_t^N) = \lambda_t, \tag{12}$$

$$\frac{A_2(c_t^T, c_t^N)}{A_1(c_t^T, c_t^N)} = p_t, \tag{13}$$

$$\lambda_t(1-\mu_t) = \beta(1+i_t^l)E_t\lambda_{t+1}, \tag{14}$$

$$\mu_t \geq 0, \tag{15}$$

and

$$\mu_t[\kappa(y_t^T + p_t y_t^N) - l_t] = 0, \tag{16}$$

where $\beta^t\lambda_t$ and $\beta^t\lambda_t\mu_t$ are the Lagrange multipliers associated with the sequential budget constraint (10) and the collateral constraint (11), respectively.

## 2.3 Foreign Lenders

Banks capture deposits from foreign lenders at the world interest rate $i_t^*$ and pay capital control taxes at the rate $\tau_t^c$, which introduces a wedge between the world interest rate and the interest rate effectively paid by domestic banks on deposits. Specifically, $i_t^d$ and $i_t^*$ are linked by the relationship

$$1 + i_t^d = (1+\tau_t^c)(1+i_t^*). \tag{17}$$

The capital control tax rate can take positive or negative values. When $\tau_t^c < 0$, the government subsidizes capital inflows and when $\tau_t^c > 0$, it taxes them.

## 2.4 The Government

The government levies income taxes and capital control taxes, $\tau_t$ and $\tau_t^c$. It also operates a bank reserve facility by setting the interest rate on bank reserves, $i_t^r$, and standing ready to accept any amount of reserves, $r_t$, offered by banks. As in Cúrdia and Woodford (2011), we assume that operating the bank-reserve facility is costly. Specifically, we assume that the central bank faces the cost $\Gamma^r(r_t)$ with the following properties:

**Assumption 2** (Bank-Reserve Cost Function)**.** *The function* $\Gamma^r(\cdot)$ *satisfies: (i)* $\Gamma^r(\cdot)$ *is increasing and convex; and (ii)* $\Gamma^r(0) = 0$ *and* $\Gamma^r(r) > 0$ *for* $r > 0$.

The need to introduce a cost of providing bank reserves is twofold. First, it is empirically realistic. In section 3.1, we document that central banks' unit operating costs are nonnegligible and in fact higher than those of private banks. For emerging countries, the cost function could also capture expenses of a financial nature, for example, if public borrowing (in this case in the form of bank reserves) raises the risk of sovereign default (Arce, 2023).

Second, central bank operating costs have real consequences. In the absence of reserve provision costs, the government can circumvent the banking friction and the household collateral constraint. The proof of this claim is in Appendix A.[1]

The government's budget constraint is then

$$\tau_t(y_t^T + p_t y_t^N + \pi_t) + \tau_{t-1}^c(1 + i_{t-1}^*)d_{t-1} + r_t = (1 + i_{t-1}^r)r_{t-1} + \Gamma^r(r_t). \tag{18}$$

We assume that the government does not play Ponzi games.

## 2.5 Competitive Equilibrium

In equilibrium, the market for nontradable goods must clear,

$$c_t^N = y_t^N. \tag{19}$$

The budget constraint of the bank (1), the budget constraint of the household (10), the interest-rate parity condition (17), the budget constraint of the government (18), and the market clearing condition in the nontraded sector (19) imply the following economy-wide resource constraint:

$$c_t^T + (1 + i_{t-1}^*)d_{t-1} = y_t^T - \Gamma(l_t, r_t) - \Gamma^r(r_t) + d_t. \tag{20}$$

We are now ready to define a competitive equilibrium.

**Definition 1** (Competitive Equilibrium)**.** *A competitive equilibrium is a set of processes $l_t$, $r_t$, $d_t$, $i_t^l$, $i_t^d$, $c_t^T$, $p_t$, $\lambda_t$, and $\mu_t$ satisfying (5)-(7), (11)-(17), and (20), for $t \geq 0$, given a reserve remuneration policy $i_t^r$, a capital control tax rate $\tau_t^c$, exogenous processes $i_t^*$, $y_t^T$, and $y_t^N$, and the initial condition $(1 + i_{-1}^*)d_{-1} > 0$.*

Following the open economy literature with collateral constraints, we consider an environment in which agents are impatient in the sense that they discount future period utilities at a higher rate than the one at which the world financial market discounts future payments. Formally, we assume that

$$\beta(1 + i_t^*) < 1.$$

[1] The result that in the absence of costly reserve provision the central bank can neutralize the banking channel friction appears in Cúrdia and Woodford (2011) in the context of a closed economy. The proof in the context of the present open-economy framework has value added for two reasons. First, in addition to the banking channel, the model features an occasionally binding collateral constraint. Second, the proof shows that capital controls are redundant ($\tau_t^c = 0$) and that the optimal interest rate on reserves is the world interest rate ($i_t^r = i_t^*$).

In the related literature this condition is assumed to be strong enough to ensure an equilibrium in which the country is a net external debtor at all times. Throughout the present analysis, we assume that this is indeed the case.

## 2.6 The Constrained Optimal Allocation

Consider a social planner who aims to achieve the following real allocation:

**Definition 2** (Constrained Optimal Allocation). *The constrained optimal allocation is a set of processes $c_t^T$, $d_t$, $l_t \geq 0$, and $r_t \geq 0$ that solves the problem*

$$\max E_0 \sum_{t=0}^{\infty} \beta^t u(A(c_t^T, y_t^N))$$

*subject to the bank's balance sheet* (7), *the economy-wide resource constraint* (20), *and to the loan constraint*

$$l_t \leq \kappa \left[ y_t^T + \frac{A_2(c_t^T, y_t^N)}{A_1(c_t^T, y_t^N)} y_t^N \right], \tag{21}$$

*taking as given the processes $i_t^*$, $y_t^T$, and $y_t^N$, and the initial condition $(1 + i_{-1}^*)d_{-1} > 0$.*

The three constraints of the social planner's problem are a subset of the complete set of equilibrium conditions listed in Definition 1. It follows immediately that the constrained optimal allocation delivers at least as much welfare as any competitive equilibrium.

The constrained optimal allocation can be supported by an appropriate combination of bank-reserve remuneration policy $i_t^r$ and capital control policy $\tau_t^c$. To see this, in the definition of a competitive equilibrium (Definition 1) set $\mu_t = 0$, which ensures that conditions (15) and (16) hold, $\lambda_t$ to satisfy (12), $p_t$ to satisfy (13), $i_t^l$ to satisfy the Euler equation (14), $i_t^d$ to satisfy (5), $i_t^r$ to satisfy (6), and $\tau_t^c$ to satisfy (17). The paths of bank profits ($\pi_t$) and of the income tax rate ($\tau_t$) associated with this equilibrium can be constructed residually to satisfy equations (3) and (18), respectively. We summarize this result in the following proposition.

**Proposition 1** (Interest on Reserves, Capital Controls, and the Constrained Optimal Allocation). *There exists a pair of processes $i_t^r$ and $\tau_t^c$ that support the constrained optimal allocation described in Definition 2 as the competitive equilibrium described in Definition 1.*

In words, the optimal policy addresses the two externalities present in the model: (i) It makes households internalize the pecuniary externality created by the collateral constraint. That is, households act as if they understood that the relative price of nontradables equals their own marginal rate of substitution of tradable for nontradable goods. This property is also present in a version of the model without a banking sector (e.g., Bianchi, 2011). And (ii) the optimal policy makes banks internalize that bank reserves provide liquidity to households and that they are costly to produce. That is, banks act as if they understood that when they increase their bank reserves they allow the government to make transfers to households, which can be particularly welfare increasing when the collateral constraint is binding, and that the cost of doing so is dictated by the function $\Gamma^r(r_t)$. Both this externality and its treatment by the social planner represent a novel contribution to the open-economy related literature on macroprudential policy and financial frictions.

An important theme of the quantitative analysis we conduct in what follows is that under the optimal policy, when the collateral constraint binds, domestic loans and bank reserves move in opposite directions. That is, bank reserves provide liquidity when households are borrowing constrained. Here, we show analytically that when the collateral constraint does not bind, under the optimal policy loans and bank reserves move in the same direction. To see this, note that the optimization problem in Definition 2 implies that when the loan constraint (21) holds with strict inequality, for any value of $d_t$, the social planner chooses $l_t$ and $r_t$ to minimize the resource cost of loan and reserve provision subject to the bank's balance sheet. Specifically, when the collateral constraint is slack, $l_t$ and $r_t$ solve the problem

$$\min_{\{l_t, r_t\}} \Gamma(l_t, r_t) + \Gamma^r(r_t)$$

subject to (7), given $d_t$. When $l_t$ and $r_t$ are both positive, the first-order condition associated with this problem is

$$\frac{\Gamma_l(l_t, r_t)}{1 + \Gamma_l(l_t, r_t)} = \frac{\Gamma_r(l_t, r_t) + \Gamma^{r\prime}(r_t)}{1 + \Gamma_r(l_t, r_t)}. \tag{22}$$

Roughly speaking, this optimality condition says that when the collateral constraint is slack, the social planner equates the private marginal cost of originating loans to the central bank's marginal cost of reserve provision net of the private bank's marginal benefit of holding reserves. Under relatively weak conditions, namely, $1 + \Gamma_r(l_t, r_t) > 0$, and $\Gamma^{r\prime}(r_t) < 1$, this optimality condition implies that $l_t$ and $r_t$ move in the same direction. This means that when the collateral constraint is slack under the optimal policy, movements in the desired level of external debt, $d_t$, are achieved by moving the volume of loans and bank reserves in the same direction, so that $d_t$, $l_t$, and $r_t$ comove positively. The economic significance of this prediction is that when the collateral constraint is slack the planner does not find it optimal to compensate credit contractions (falls in $l_t$) with liquidity injections (increases in $r_t$). As it will become apparent in the quantitative analysis that follows, the picture is quite different when the collateral constraint binds for the planner. In such circumstances, $l_t$ and $r_t$ move in opposite directions, reflecting the fact that the central bank substitutes reserves for loans to preserve liquidity.

# 3 Quantitative Analysis

The question we wish to address next is how the economy behaves on average and around financial crises under the constrained optimal equilibrium and under the laissez-faire equilibrium, where the latter is defined as the equilibrium resulting under the policy $i_t^r = \tau_t^c = 0$, for all $t$. To this end, we turn to a quantitative characterization of the model's dynamics under these two policy regimes. Appendix B describes the numerical methods used to approximate the equilibrium dynamics with and without government intervention.

## 3.1 Calibration by Simulated Method of Moments

The period utility function takes the CRRA form

$$u(c_t) = \frac{c_t^{1-\sigma} - 1}{1-\sigma},$$

with $\sigma > 0$. The aggregator function of tradable and nontradable consumption takes the CES form

$$A(c_t^T, c_t^N) = \left[a {c_t^T}^{1-1/\xi} + (1-a){c_t^N}^{1-1/\xi}\right]^{1/(1-1/\xi)}, \tag{23}$$

with $\xi > 0$ and $a \in (0, 1)$.

The financial intermediation cost function of banks takes the form

$$\Gamma(l_t, r_t) = A l_t^{1+\alpha}[1 + \phi(r_t - \bar{r})^2 I(r_t < \bar{r})], \tag{24}$$

with $A, \alpha, \phi, \bar{r} > 0$. The operating cost function of the central bank takes the form

$$\Gamma^r(r_t) = B r_t^{1+\alpha}. \tag{25}$$

The specification of the two cost functions assumes that the volume elasticity, $1 + \alpha$, is the same for the central bank and the commercial bank. The purpose of this assumption is to economize on parameters. It implies that the administrative and monitoring costs of loans and of reserve provision are similarly sensitive to the scale of operation. Because the coefficients $A$ and $B$ can in principle be different from each other and are determined to match actual data, the assumed parameterization allows for the total, the average, and the marginal intermediation costs to differ across the two types of bank. The specifications of the cost functions $\Gamma(l, r)$ and $\Gamma^r(r)$ satisfy Assumptions 1 and 2, respectively.

Table 1: Calibration

| Parameter | Value | Description |
|---|---|---|
| | | Structural Parameters |
| $\sigma$ | 2 | Inverse of intertemporal elasticity of consumption |
| $a$ | 0.31 | Parameter of CES aggregator |
| $\xi$ | 0.83 | Elasticity of substitution between tradables and nontradables |
| $i^*$ | 0.04 | World interest rate |
| $\beta$ | 0.8667 | Subjective discount factor |
| $\kappa$ | 0.2892 | Parameter of collateral constraint |
| $A$ | 0.0258 | Parameter of intermediation cost function $\Gamma(l, r)$ |
| $\alpha$ | 1.5385 | Parameter of the intermediation cost functions $\Gamma(l, r)$ and $\Gamma^r(r)$ |
| $\phi$ | 23.6933 | Parameter of intermediation cost function $\Gamma(l, r)$ |
| $\bar{r}$ | 0.1116 | Parameter of intermediation cost function $\Gamma(l, r)$ |
| $B$ | 1.7488 | Parameter of intermediation cost function $\Gamma^r(r)$ |
| | | Discretization of State Space |
| $n_{y^T}$ | 13 | Number of grid points for $\ln y_t^T$, equally spaced |
| $n_{y^N}$ | 13 | Number of grid points for $\ln y_t^N$, equally spaced |
| $n_d$ | 800 | Number of grid points for $d_t$, equally spaced |
| $\left[\ln \underline{y}^T, \ln \overline{y}^T\right]$ | [-0.1093, 0.1093] | Range for logarithm of tradable output |
| $\left[\ln \underline{y}^N, \ln \overline{y}^N\right]$ | [-0.1328, 0.1328] | Range for logarithm of nontradable output |
| $[\underline{d}, \overline{d}]$ | [0.4, 1.05] | Debt range unregulated economy |

Note. The time unit is a year.

Table 2: Empirical and Predicted Targeted Moments

| | Moment | Formula | Observed | Predicted |
|---|---|---|---|---|
| (1) | Lending spread | $\frac{i^l - i^d}{1+i^d}$ | 0.0499 | 0.0501 |
| (2) | Reserve-to-deposit ratio | $\frac{r}{d}$ | 0.0644 | 0.0629 |
| (3) | Debt-to-output ratio | $\frac{d}{y^T + py^N}$ | 0.2900 | 0.2835 |
| (4) | Intermediation-cost-to-deposit ratio | $\frac{\Gamma(l,r)}{d}$ | 0.0175 | 0.0181 |
| (5) | Central-bank-operating-cost-to-reserve ratio | $\frac{\Gamma^r(r)}{r}$ | 0.0205 | 0.0209 |
| (6) | Frequency of binding collateral constraint | | 0.0500 | 0.0548 |

Notes. Lines 1, 2, and 5 are cross-country medians of cross-time medians. The definition of an emerging country and the countries included follow Uribe and Schmitt-Grohé (2017). Data Sources: Lending spread, International Financial Statistics (IFS); reserve-to-deposit ratio, Bankscope data for commercial banks; debt-to-output ratio, Bianchi (2011); intermediation-cost-to-deposit ratio, Philippon (2015) and Bazot (2018); central-bank-operating-cost-to-bank-reserve ratio, Bankscope data for central banks. Appendix C provides details on data sources, sample periods, and country coverage.

The calibration of the parameters of the model is summarized in Table 1. The time unit is meant to be one year. Following Bianchi (2011), we set $\sigma = 2$, $a = 0.31$, $\xi = 0.83$, and the world interest rate $i_t^*$ at a constant value of 4 percent per annum. We also follow Bianchi (2011) in setting the degree of relative impatience, $\beta(1 + i^l)$, where $i^l$ denotes the average interest rate on bank loans in the laissez-faire equilibrium. Bianchi's model does not include bank intermediation, so loans to households originate directly from foreign lenders, and relative impatience equals $\beta^B(1 + i^*)$, where $\beta^B = 0.91$ denotes the subjective discount factor in Bianchi (2011). We therefore impose the restriction $\beta(1 + i^l) = \beta^B(1 + i^*)$, which implies that $\beta = \beta^B[(1+i^*)/(1+i^l)]$. The object $(1+i^l)/(1+i^*)$ is the average gross lending spread in the laissez-faire economy, which we set to 1.0499 to match the median value observed in a sample of 38 emerging economies over the period 1985-2016 (see Table 2). This yields $\beta = 0.8667$.

The sources of uncertainty are the endowments of tradable and nontradable goods, $y_t^T$ and $y_t^N$. The natural logarithms of these two variables are assumed to follow a bivariate AR(1) process. The parameters of this process are set to the values used in Bianchi (2011).

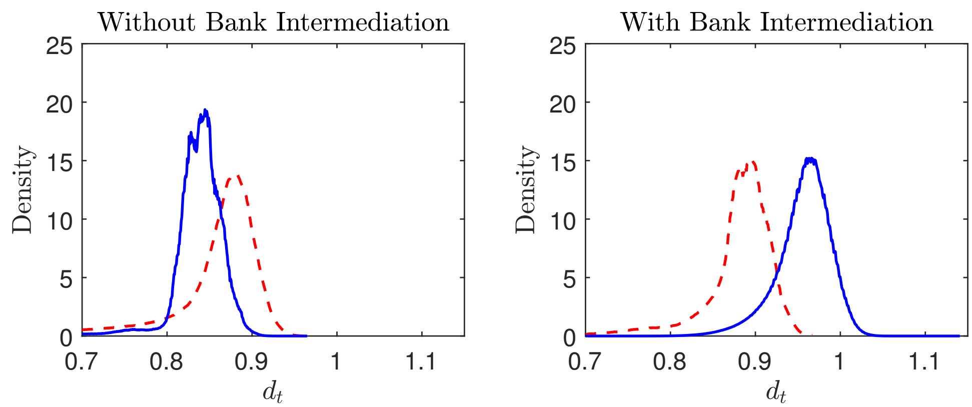
The remaining parameters, $A$, $\alpha$, $\phi$, $\bar{r}$, $B$, and $\kappa$, pertain to the financial side of the economy. They are calibrated by simulated method of moments (SMM) to jointly match six moments for which the unregulated model economy can produce precise predictions: (1)

The lending spread $((i^l - i^d)/(1 + i^d))$. Using data from the IMF's International Financial Statistics (IFS), we estimate the average value of this spread across emerging countries to be 4.99 percent per year. Recalling that the world interest rate is set at 4 percent, this estimate implies that in the model economy banks lend to the domestic private sector at a rate more than twice as high as the rate at which they borrow from international lenders. (2) The reserve-to-deposit ratio $(r/d)$. We estimate this ratio using data on commercial banks from Bankscope. On average, banks in emerging countries hold 6.44 percent of their deposits in the form of reserves at the central bank. This ratio is more than three times the one observed in rich countries (1.98 percent). (3) The debt-to-output ratio $(d/(y^T + py^N))$. In the model, $d_t$ is both the amount of bank deposits and the country's net foreign debt position. This is because, being a representative-agent economy, the model does not feature deposits by domestic agents (all households are borrowers). For this reason, one must take a stance on whether to calibrate the ratio $d/(y^T + py^N)$ to match the observed deposit-to-output ratio or to match the observed net-foreign-debt-to-output ratio. We pick the latter option to keep in line with calibrations in the related literature. Specifically, following Bianchi (2011), we set $d$ to be 29 percent of output.[2] (4) The bank-operating-cost-to-deposit ratio $(\Gamma(l, r)/d)$. We set this ratio to 0.0175. This calibration lies in the middle of the range estimated by Philippon (2015) and Bazot (2018), who estimate that bank unit costs range from 1.5 and 2 percent in the United States (first author) and in a set of 20 countries (second author). The sample in Bazot (2018) contains mostly developed countries. However, it includes four countries that during the sample period of his study, 1970-2014, can be considered emerging economies, namely, China, Portugal, South Korea, and Spain. The average unit cost across these four countries is 1.99 percent, which is close to the value assigned in the calibration. (5) The central-bank-operating-cost-to-reserve ratio $(\Gamma^r(r)/r)$. Using data from Bankscope for central banks, we estimate this ratio to be on average 2.05 percent in emerging countries. This value is slightly lower than the one observed across rich countries, 2.55 percent in the same database. (6) The probability of a binding collateral constraint. We set this moment to 5 percent (or one sudden stop every 20 years on average), which is a value within the range used in the related literature.[3] Appendix C provides more information on the data sources for the empirical moments related to the banking channel, namely, (1), (2), and (5).

[2]This argument does not apply to the calibration of the reserve-to-deposit ratio, $r/d$, because both $r$ and $d$ are expected to be proportionally affected by the presence of domestic deposits.

[3]In some of the related literature the definition of a sudden stop includes the additional requirement of a concurrent current account improvement of a certain size. For example, Bianchi (2011) requires an improvement in the current account of at least one standard deviation. As it turns out, in the present model eighty percent of the episodes of a binding collateral constraint in the unregulated economy satisfy this additional criterion.

Figure 1: Unconditional Distributions of Debt With and Without a Bank Intermediation Channel




Notes. The left panel corresponds to an economy without a bank intermediation channel and the right panel to an economy with a bank intermediation channel. Parameters take the values shown in Table 1 when applicable. The debt densities associated with constrained optimal allocations are shown with solid lines and debt densities of unregulated economies with broken lines. The figure shows that in the absence of a banking channel there is overborrowing, whereas in the presence of the bank intermediation friction there is underborrowing.

Table 2 reports the six targeted moments and the corresponding predictions of the unregulated economy. The relevant steady state for the calibration is the stochastic steady state rather than the deterministic one, as in the latter the collateral constraint binds at all times. For this reason the calibration is computationally demanding, and exact matches are not possible in general. However, as a comparison of the last two columns of Table 2 suggests, the match is quite close.

## 3.2 Underborrowing

A well-known result that arises in a version of the present model without a bank intermediation channel is that the unregulated economy overborrows (e.g., Bianchi, 2011). Specifically, when households can borrow directly from foreign lenders, the equilibrium density function of external debt is located to the right of the one associated with the constrained optimal allocation that can be supported with capital control taxes. The economy without banks is a special case of the one analyzed here in which $r = 0$ and $\Gamma(l, r) = \Gamma^r(r) = 0$, for all $l$. The left panel of Figure 1 plots the equilibrium distribution of debt in the economy without banks for the unregulated and constrained optimal cases. All parameters of the model other than those pertaining to the bank and central bank cost functions take the values shown in Table 1. The resulting economy is identical to the one analyzed in Bianchi (2011) except for the values of $\beta$ and $\kappa$, which are slightly different (0.8667 versus 0.91 and 0.2892 versus 0.32, respectively), and a finer grid of the exogenous state variables (146 versus 16 distinct pairs $(y^T, y^N)$). The debt density under optimal capital control policy lies to the left of the one associated with the unregulated economy. Thus, the plot shows that the model without banks reproduces the standard overborrowing result.

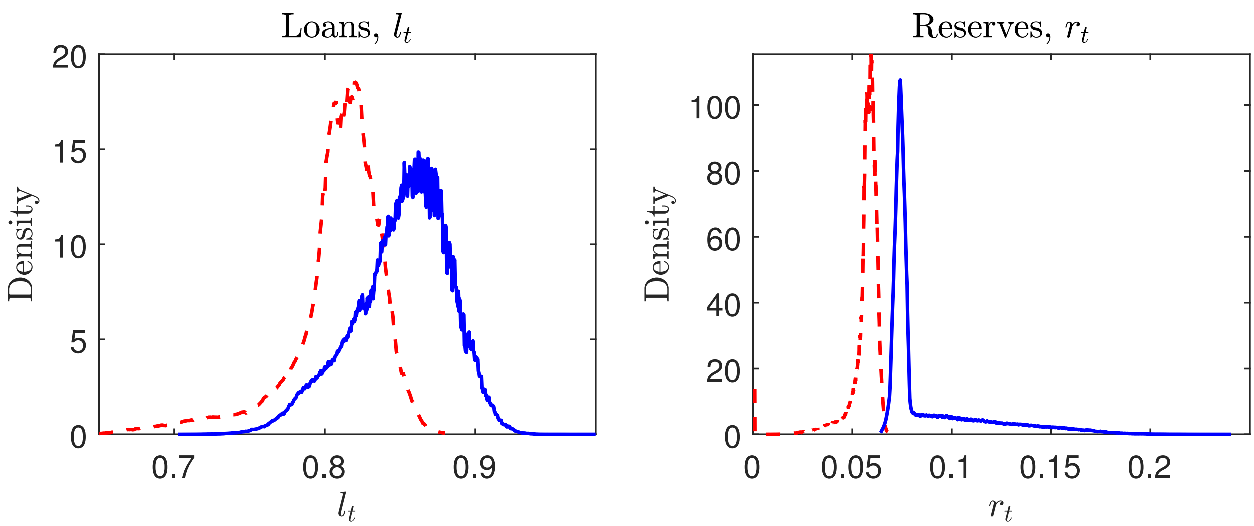
The picture is quite different when household borrowing from foreign lenders is intermediated by banks. The result is shown in the right panel of Figure 1. Now the distribution of debt in the unregulated economy lies to the left of the corresponding distribution under the constrained optimal equilibrium, suggesting that in the economy with a banking channel the unregulated economy underborrows. On average, external debt is 10 percent larger in the regulated economy than in the unregulated economy. Section 4.4 shows that the underborrowing result is robust to perturbing the parameters pertaining to the banking channel.

The intuition for why the economy with a banking channel underborrows is that interest on reserves appropriately applied dampen the negative macroeconomic consequences of credit crunches at the household level. Essentially, bank reserves introduce a cushion between debt and private loans. This can be seen from the balance sheet of the bank, equation (7), which, up to the resource cost of producing loans, says that $l_t + r_t = d_t$. So when $l_t$ is restricted by the collateral constraint, the government can achieve the desired level of external debt by making up the shortfall in loans with reserve creation. The increase in bank reserves finds its way into the budget constraint of households through a relaxation of fiscal policy.

Under the optimal policy, the government finds it particularly useful to expand bank reserves when the private sector is borrowing constrained. Suppose, for example, that the economy faces a negative tradable endowment shock that results in a binding collateral constraint. Since the endowment process is mean reverting, the intertemporal approach to the current account dictates that the economy should finance the negative shock by borrowing from international lenders (i.e., by increasing $d_t$). However, the fall in output tightens the collateral constraint, making banks reluctant to extend loans to households. In this case, the central bank can induce an increase in bank reserves by offering a higher interest rate on this type of financial asset. Thus, through an increase in bank reserves, the economy as a whole can increase external debt, $d_t$, even though $l_t$ is impeded to expand by the binding collateral constraint. The government assists households in smoothing consumption by transferring via a reduction in the income tax rate, $\tau_t$, the resources it raises in the form of bank reserves.

Figure 2 displays the distributions of loans and reserves in the economy with banks in the unregulated and constrained optimal equilibria. Under optimal policy the economy has a larger volume of both loans and bank reserves. The reason why the equilibrium volume

Figure 2: Unconditional Distributions of Loans and Reserves




Note. The densities associated with the constrained optimal allocation are shown with a solid line and the densities of the unregulated economy ($i_t^r = \tau_t^c = 0$) with a broken line.

of loans is higher under the optimal policy than under laissez-faire is that in the former the government plays the role of lender of last resort to households, which induces households to reduce precautionary saving (i.e., operate closer to the debt limit). Consistent with this intuition, the optimal distribution of bank reserves has a fat right tail, which serves to limit the magnitude of macroeconomic deleveraging (falls in $d_t$) when the household's collateral constraint binds. By contrast, in the unregulated economy the distribution of bank reserves not only lacks a fat right tail but displays a mass concentration at zero. The mass at zero arises because there are sudden stops that imply so severe a decline in the demand for loans that banks have no use for reserves. Recall that in the laissez-faire economy bank reserves are not remunerated, so their sole purpose is to reduce the marginal cost of initiating loans. The lower the volume of loans is, the smaller the marginal benefit of reserves will be because $\Gamma_{lr} < 0$.

Table 3 reports the unconditional correlation of reserves with loans and output. Consistent with the above narrative, the correlations of bank reserves with loans and output are positive under laissez-faire but negative under optimal policy. This difference in comovement is a reflection of the fact that in the laissez-faire economy the demand for reserves by banks is dictated only by the amount of loans the bank can originate. On the other hand, in the constrained optimal allocation the government raises the interest rate on reserves during bad times and in this way induces banks to hold more reserves even when they

Table 3: Unconditional Correlations of Reserves with Loans and Output

| | Laissez-faire | Constrained optimal |
|---|---|---|
| $\text{corr}(r_t, l_t)$ | 0.99 | $-0.48$ |
| $\text{corr}(r_t, y_t^T + p_t y_t^N)$ | 0.54 | $-0.79$ |

Note. The column labeled laissez-faire corresponds to the competitive equilibrium with $i_t^r = \tau_t^c = 0$ and the column labeled constrained optimal to the competitive equilibrium with $i_t^r$ and $\tau_t^c$ chosen optimally.

make fewer loans. In those circumstances, the correlation between loans and reserves can become negative, which is the case as the table shows under the present parameterization of the model. Section 4.4 shows that this result is robust to perturbations in the values of the parameters defining the banking channel.

## 3.3 Welfare Gains of Regulation With and Without Banks

In the economy with a banking channel, the welfare gains of optimal regulation through bank reserve remuneration and capital controls are larger than in the economy without banks. We calculate the welfare cost of living in the unregulated economy instead of in the economy with optimal regulation as the percentage change in the stream of consumption in the former that makes households indifferent between living in one or the other. This welfare cost measure, which we denote $\lambda(y_t^T, y_t^N, d_{t-1})$, is state dependent and therefore takes transitional dynamics into account. Formally, $\lambda(y_t^T, y_t^N, d_{t-1})$ is defined as

$$E_t \sum_{k=0}^{\infty} \beta^k \frac{\left[\left(1 + \frac{\lambda(y_t^T, y_t^N, d_{t-1})}{100}\right) c_{t+k}^u\right]^{1-\sigma} - 1}{1-\sigma} = E_t \sum_{k=0}^{\infty} \beta^k \frac{{c_{t+k}^o}^{1-\sigma} - 1}{1-\sigma},$$

where $c_t^u$ and $c_t^o$ denote consumption in the unregulated and regulated economies in period $t$. Letting $v^u(y_t^T, y_t^N, d_{t-1}) \equiv E_t \sum_{k=0}^{\infty} \beta^k \frac{{c_{t+k}^u}^{1-\sigma}-1}{1-\sigma}$ and $v^o(y_t^T, y_t^N, d_{t-1}) \equiv E_t \sum_{k=0}^{\infty} \beta^k \frac{{c_{t+k}^o}^{1-\sigma}-1}{1-\sigma}$ denote lifetime welfare in the unregulated and regulated economies conditional on the current state, we can solve the above expression for $\lambda(y_t^T, y_t^N, d_{t-1})$ to get

$$\lambda(y_t^T, y_t^N, d_{t-1}) = \left\{ \left[ \frac{v^o(y_t^T, y_t^N, d_{t-1})(1-\sigma)(1-\beta) + 1}{v^u(y_t^T, y_t^N, d_{t-1})(1-\sigma)(1-\beta) + 1} \right]^{\frac{1}{1-\sigma}} - 1 \right\} \times 100.$$

To obtain a summary measure of the welfare cost of no regulation, we compute the expected value of $\lambda(y_t^T, y_t^N, d_{t-1})$ using the ergodic density of $(y_t^T, y_t^N, d_{t-1})$ induced by the unregulated economy.

We find that the average welfare cost of living in the unregulated economy is 0.31 percent of lifetime consumption. When we eliminate the banking channel, we find that the welfare cost is only 0.10 percent of lifetime consumption. Thus, the domestic banking channel triples the welfare cost of laissez-faire.[4] The banking channel amplifies the welfare cost of no intervention because it adds a second externality to the model, namely, a discrepancy between the private and social costs of bank reserve provision. This externality is absent when domestic households are allowed to borrow directly from foreign lenders. The finding that incorporating a banking sector triples the welfare cost of no policy intervention suggests that the banking friction is at least as relevant as the collateral constraint stressed in the overborrowing literature.

## 3.4 Optimal Bank Reserve Policy During Sudden Stops

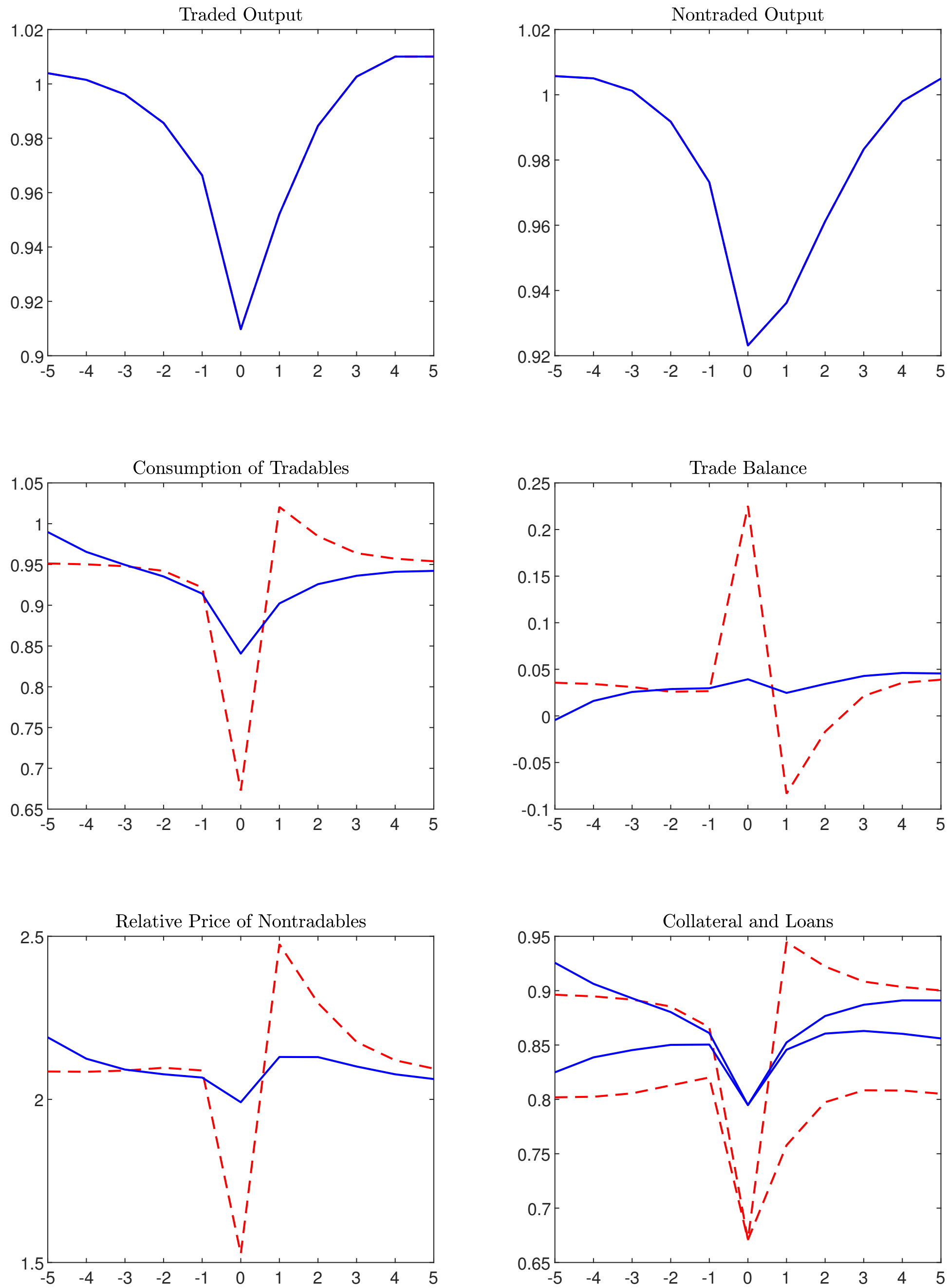
To understand how the social planner manages a sudden stop in the presence of the bank intermediation friction, we examine equilibrium dynamics in the unregulated economy and in the constrained optimal equilibrium around a typical episode in which the collateral constraint binds in the unregulated economy. To this end, we simulate the unregulated economy for 1 million periods and extract all windows of eleven years containing a binding collateral constraint in the middle. This yields 54,842 sudden stop episodes, which is consistent with the targeted frequency of a binding collateral constraint of 5 percent (see Table 2). For each variable, we compute the average across the sudden stop episodes. The result is shown with broken lines in Figures 3 and 4. The period in which the collateral constraint binds is normalized to 0, so time runs from period -5 to period 5.

To compare the sudden stop dynamics in the unregulated economy with those in the economy with optimal capital-control and reserve-remuneration policies, for each sudden stop episode in the former economy, we compute the equilibrium dynamics implied by the constrained optimal equilibrium assuming that in period -5 (five years prior to the sudden stop) the unregulated and regulated economies are in the same state $(y^T_{-5}, y^N_{-5}, d_{-6})$. We then hit the regulated economy with the same sequence of endowment shocks that buffeted the unregulated economy between periods -5 and 5. The results are shown with solid lines in Figures 3 and 4.

In the unregulated economy, the typical sudden stop occurs when the economy suffers a string of negative shocks to the endowments of tradable and nontradable goods (top row

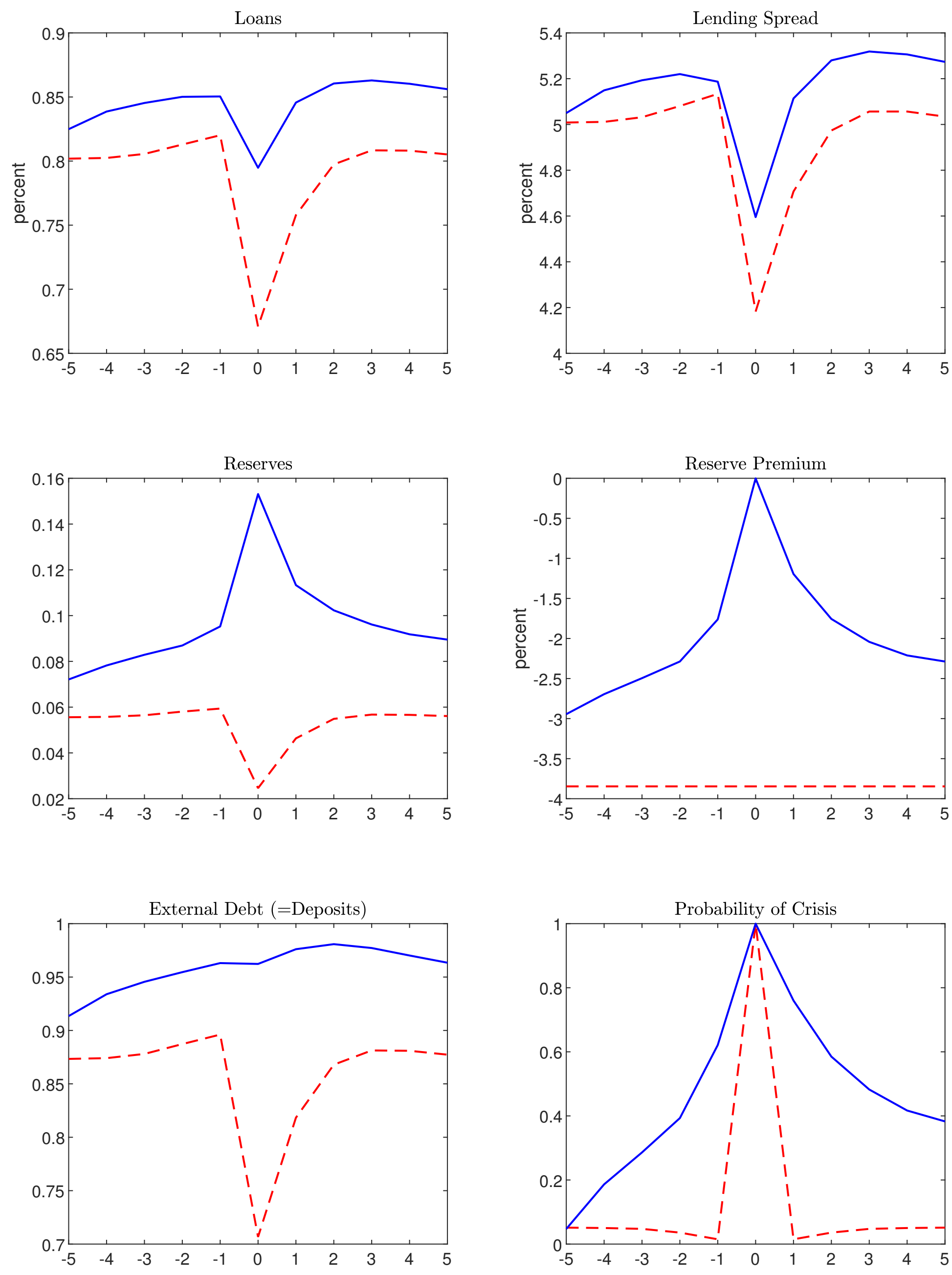
[4]The welfare cost in the absence of banks, 0.10 percent, is similar to the one reported in Bianchi (2011), 0.14 percent. The discrepancy is due to the slight differences in the calibration of $\beta$ and $\kappa$ and in the discretization of the exogenous state variables discussed in section 3.2.

Figure 3: The Typical Sudden Stop Episode




Notes. The sudden stop associated with the constrained optimal allocation is shown with a solid line and the sudden stop associated with the unregulated economy ($i_t^r = \tau_t^c = 0$) with a broken line. In the bottom right panel the value of collateral is shown with the same line type as the level of loans. The two variables are identifiable because the former is never below the latter.

Figure 4: The Typical Sudden Stop Episode (continued)




Note. The sudden stop associated with the constrained optimal allocation is shown with a solid line and the sudden stop associated with the unregulated economy ($i_t^r = \tau_t^c = 0$) with a broken line.

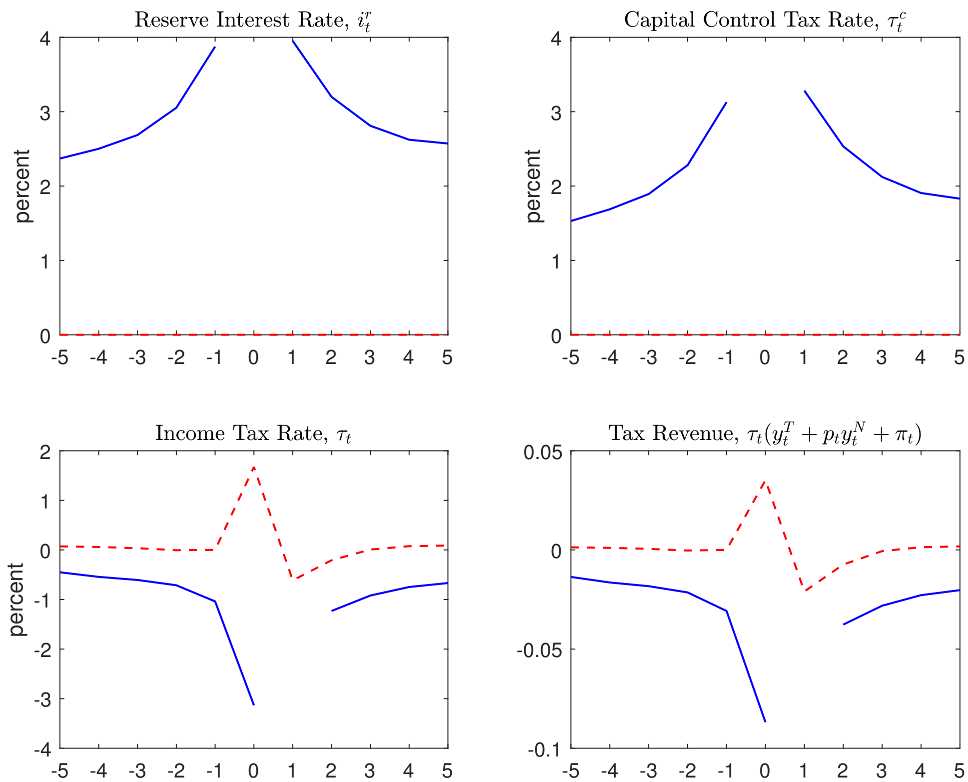
of Figure 3). Both endowments fall by more than 8 percent between periods -5 and 0. By construction, the path of the two endowments is the same in the regulated and unregulated economies. Given the relative price of nontradables, the fall in output causes a decline in the value of collateral. When the collateral constraint binds in the unregulated economy, consumption of tradables falls by more than tradable output (middle left panel). This is because the economy, forced to deleverage, must run a trade balance surplus (middle right panel). The fall in aggregate demand depresses the relative price of nontraded goods, that is, the real exchange rate depreciates (bottom left panel). This happens in spite of the fact that the endowment of nontraded goods also experiences a large contraction. The fall in the relative price of nontradables further tightens the collateral constraint, a phenomenon known as a Fisherian deflation (bottom right panel).

By contrast, in the economy with optimal capital controls and optimal bank-reserve remuneration, the contraction in the demand for nontradables and the real depreciation are milder. The reason is not that in the constrained optimal equilibrium the collateral constraint does not bind. In fact, in the regulated economy households are often borrowing constrained before, during, and after the sudden stop (see the bottom right panel of Figure 4). Instead, the reason why in the regulated economy aggregate demand and the real exchange rate are less affected by the contraction in the endowments is that under the optimal macroprudential policy the economy as a whole continues to have access to international credit. This is apparent from the bottom left panel of Figure 4, which shows that debt falls sharply in the unregulated economy but is little changed in the regulated one. In fact, in the regulated economy debt increases slightly during the entire episode. This is because, although by construction the level of debt at the beginning of period -5 (i.e., $d_{-6}$, not shown) is the same in the unregulated and regulated economies, the unconditional average level of debt is higher in the latter than in the former. So over the entire time window, the regulated economy is transitioning to a higher level of debt.

The key difference between the sudden stop in the unregulated and regulated economies lies in the behavior of bank reserves. The sudden stop causes household deleveraging (a decline in loans) in both, the unregulated and the regulated economies (top left panel of Figure 4). However, the social planner manages to avoid aggregate deleveraging (a decline in deposits) by raising bank reserves (middle left panel of Figure 4). By contrast, in the unregulated economy, the decline in loans is accompanied by a decline in bank reserves, which exacerbates macroeconomic deleveraging. As explained earlier, the reason why banks have less use for (unremunerated) reserves is that they are complementary in the production of loans, which in turn experience a sharp decline during the sudden stop.

For the constrained optimal allocation to be supported as a competitive equilibrium, the

Figure 5: The Typical Sudden Stop Episode (concluded)




Note. The sudden stop associated with the constrained optimal allocation is shown with a solid line and the sudden stop associated with the unregulated economy ($i_t^r = \tau_t^c = 0$) with a broken line.

social planner must create incentives to ensure that banks choose the optimal quantities of loans and bank reserves. The middle right panel of Figure 4 displays the behavior of the reserve spread, $(i_t^r - i_t^d)/(1 + i_t^d)$, during the typical sudden stop. In the unregulated equilibrium the reserve spread is constant at all times because central bank reserves are unremunerated and because the deposit rate equals the (constant) world interest rate, as the government does not impose capital controls. Under the optimal policy, the reserve spread increases during the sudden stop, which incentivizes banks to elevate their reserve holdings.

The top right panel of Figure 4 displays the behavior of the lending spread, $(i_t^l - i_t^d)/(1+i_t^d)$. When the collateral constraint binds (period 0), the lending spread falls in the unregulated and the regulated economies. The reason is that the sudden stop represents a decline in the demand for loans rather than a decline in the supply of loans. (Section 4.1 expands this intuition by studying the dynamics of the lending spread when the collateral constraint is placed on the bank's side, in which case deleveraging disturbs the supply of loans.)

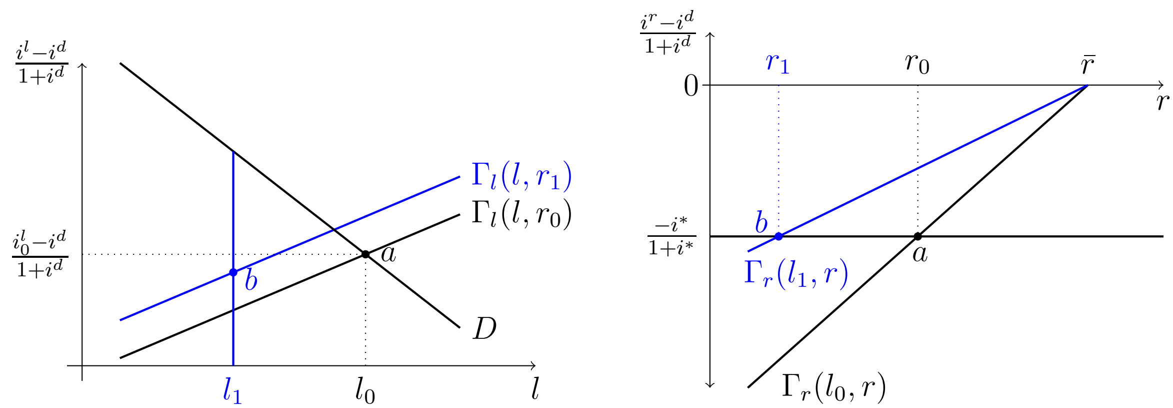
Figure 5 displays the levels of the interest rate on bank reserves ($i_t^r$), the capital control tax rate ($\tau_t^c$), the income tax rate ($\tau_t$), and income tax revenue, $\tau_t(y_t^T + p_t y_t^N + \pi_t)$. As is well known, in models with a collateral constraint of the type considered in this paper, the level of the capital control tax rate $\tau_t^c$ is not uniquely pinned down when the collateral constraint is binding (see, for example, Schmitt-Grohé and Uribe, 2017). In this case there is a whole range of capital control tax rates consistent with the optimal competitive equilibrium. This indeterminacy, which does not spill over to the real allocation or to interest rate spreads, does extend to the levels of interest rates and the income tax rate ($i_t^r$, $i_t^l$, $i_t^d$, and $\tau_t$). Consequently, the figure does not display values for these variables when they are indeterminate. The top left panel of the figure shows that in the run-up to the crisis, the government raises the interest rate it pays on bank reserves. It does so to induce banks to become more liquid. In turn, the increased stock of reserves held by banks at the central bank finds its way into the households' budget constraints through income tax cuts (more precisely, through an increase in income-based transfers, see the bottom panels of the figure). Thus, both bank reserve remuneration and transfers are countercyclical around sudden stops. By contrast, as in the related literature without a bank intermediation channel, the capital control tax rate is procyclical (i.e., $\tau_t^c$ increases as the economy approaches the crisis). The reason why the social planner tightens capital controls near the crisis is that, although reserve remuneration softens the consequences of sudden stops, they do not make them costless, so, in an uncertain environment, it still pays for the benevolent government to try to avoid a binding collateral constraint by discouraging external borrowing.

### 3.5 A Graphical Explanation of the Mechanism

To understand the behavior of quantities and prices of loans and bank reserves during a sudden stop consider the following graphical explanation. The left panel of Figure 6 depicts the loan market. The supply of loans is given by the marginal cost of bank intermediation, $\Gamma_l(l, r)$ (efficiency condition (5)). Holding bank reserves constant, the loan supply schedule is increasing in $l$. When the collateral constraint is slack, the demand for loans is downward sloping and stems from the household's Euler equation. The higher is the interest on loans, the lower the demand for loans will be, as households have an increased incentive to postpone consumption. The initial equilibrium is at point $a$, where the supply and demand for loans intersect. The volume of loans is $l_0$ and the lending spread is $(i_0^l - i^d)/(1 + i^d)$ (recall that in the unregulated equilibrium $i^d$ is constant and equal to $i^*$).

Consider now the market for bank reserves, which is depicted in the right panel of Figure 6. The supply of reserves is perfectly elastic at the constant spread $(i^r - i^d)/(1 + i^d) =$

Figure 6: The Loan and Reserve Markets During a Sudden Stop in the Unregulated Economy




$-i^*/(1+i^*)$, as the central bank stands ready to supply any amount of reserves to private banks at a zero interest rate ($i^r = 0$). Holding the volume of loans constant, the demand for bank reserves is given by the marginal benefit of reserve holdings by private banks, $\Gamma_r(l_0, r)$ (efficiency condition (6)). The demand schedule is upward sloping in the range $0 < r < \bar{r}$, which is the relevant one for the present analysis. Equilibrium in the bank reserve market occurs at point $a$, where the demand for reserves meets the (horizontal) supply of reserves.

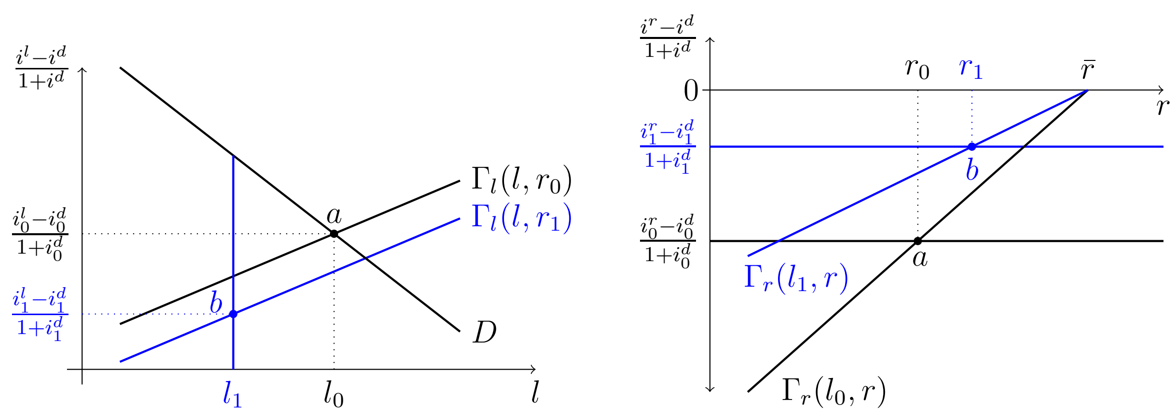
Consider now the effect of a sudden stop on the markets for loans and reserves. Suppose that the economy suffers a negative endowment shock that makes the collateral constraint bind, forcing households to deleverage. Suppose that the volume of loans demanded after the negative shock is $l_1 < l_0$. In the market for loans (left panel of Figure 6), this is represented by a kink in the demand for loans. For simplicity, we assume that the new demand for loans is given by the original one for $l < l_1$. At $l = l_1$, the new demand schedule is vertical. In the reserve market (right panel of Figure 6), the fall in the volume of loans shifts the demand schedule up and to the left from $\Gamma_r(l_0, r)$ to $\Gamma_r(l_1, r)$ (recall that $\Gamma_{rl} < 0$). The supply schedule of reserves is unchanged. The new equilibrium is at point $b$. The equilibrium level of reserves falls from $r_0$ to $r_1 < r_0$. In turn, the fall in the stock of bank reserves shifts the loan supply schedule up and to the left from $\Gamma_l(l, r_0)$ to $\Gamma_l(l, r_1)$. The new equilibrium in the loan market is at point $b$. (For expositional convenience we describe these effects as occurring sequentially but in fact they occur simultaneously.)

Comparing the initial equilibrium, points a in both panels, with the equilibrium after the shock, points b, suggests that the sudden stop causes a fall in both the volume of loans and the stock of bank reserves. The effect on the lending spread is the result of two opposing forces. The contraction in the demand for loans pushes the lending spread down, whereas the contraction in bank reserve holdings pushes it up. This second effect does not dominate because it is second order (triggered by the fall in loans itself), but it does attenuate the fall in the loan spread. This intuition is consistent with the relatively small decline displayed by the lending spread in the unregulated economy over the sudden stop episode in the calibrated model (broken line in the top right panel of Figure 4).

The intuitive explanation for why the lending spread does not display a hike during a sudden stop is that the decline in the equilibrium volume of loans is a consequence of the contraction in the demand for loans by private households. By contrast, in models in which the collateral constraint is placed at the level of the bank as opposed to at the level of the household, a sudden stop represents a contraction in the supply of loans and hence is associated with an increase in the lending spread (as, for example, in the model of Céspedes and Chang, 2020).

The situation is quite different when the government intervenes. The adjustment to a negative endowment shock is illustrated in Figure 7. Initially, the markets for loans and bank

Figure 7: The Loan and Reserve Markets of the Regulated Economy During a Sudden Stop




reserves are in equilibrium at point $a$. The equilibrium levels of loans and bank reserves are $l_0$ and $r_0$ and the lending and reserve spreads are $(i^l_0 - i^d_0)/(1 + i^d_0)$ and $(i^r_0 - i^d_0)/(1 + i^d_0)$. As in the unregulated economy, the sudden stop causes a kink in the demand schedule for loans at $l_1 < l_0$ (left panel), and a shift up and to the left in the demand schedule for bank reserves from $\Gamma_r(l_0, r)$ to $\Gamma_r(l_1, r)$ (right panel). Now, unlike in the unregulated economy, to avoid a collapse in the bank-reserve market, the government increases the banks' incentive to hold reserves by raising the reserve spread from $(i^r_0 - i^d_0)/(1 + i^d_0)$ to $(i^r_1 - i^d_1)/(1 + i^d_1) > (i^r_0 - i^d_0)/(1 + i^d_0)$. Thus, the horizontal supply of reserves shifts up in a parallel fashion (right panel of Figure 7). If the increase in the reserve spread is large enough, the new equilibrium level of reserves can be larger than before the sudden stop. This is the case illustrated in the right panel of Figure 7, where at the new equilibrium, given by point $b$, the level of bank reserves is $r_1 > r_0$. The central bank has an incentive to act aggressively because, to avoid a contraction in the level of external debt, the fall in the volume of loans must be compensated by an increase in the holdings of bank reserves. In the loan market (left panel of Figure 7), the increase in the stock of reserves shifts the loan supply schedule down and to the right from $\Gamma_l(l, r_0)$ to $\Gamma_l(l, r_1)$. The new equilibrium is at point $b$, where the lending spread has fallen from $(i_0^l - i_0^d)/(1 + i_0^d)$ to $(i_1^l - i_1^d)/(1 + i_1^d)$.

In sum, the intuition derived from Figure 7 is that if the government intervention raises the reserve spread sufficiently, then the sudden stop is associated with a fall in the volume of loans, an increase in the stock of bank reserves, and a fall in the lending spread. These qualitative effects are consistent with the predictions of the calibrated model under optimal reserve remuneration and capital control policies shown in Figure 4.

# 4 Extensions and Sensitivity

This section presents three extensions: First, an analysis of how the lending spread behaves during sudden stops depending on whether disruptions in the loan market stem from the demand for loans as in the baseline model or from the supply of loans. To this end it starts by studying a variation of the model in which the collateral constraint is placed on the side of the bank. In this case, contrary to what happens when the collateral constraint is placed on the side of the household, the lending spread rises during sudden stops. However, in the context of the model studied in this paper, the real allocation (including the volume of loans) is independent of where the collateral constraint is placed. The second extension is a variation of the model with reserve requirements instead of interest on bank reserves. It establishes that the latter welfare dominates the former as a policy tool. The third extension establishes that absent a collateral constraint (i.e., in a version of the model featuring the banking channel as the sole financial friction) reserve remuneration suffices to induce the constrained optimal allocation and capital controls are redundant.

## 4.1 The Lending Spread During Financial Crises: Demand or Supply Driven?

The baseline model assumes a collateral constraint at the household level. Under this formulation a binding collateral constraint represents a contraction in the demand for loans. An implication of this specification is that during a sudden stop the lending spread falls (top right panel of Figure 4). If instead the model were to feature a lending limit at the bank level, a financial crisis would trigger an increase in the lending spread.

To see this, consider a variation of the model in which the collateral constraint continues to be given by equation (11) but assume that it is a constraint of the bank and not of the household. As before the problem of the bank consists in maximizing profits, given in equation (4). However, now the bank must satisfy the loan limit constraint (11). Formally, banks pick $l_t$ and $r_t$ so as to maximize

$$\frac{i_t^l - i_t^d}{1 + i_t^d} l_t + \frac{i_t^r - i_t^d}{1 + i_t^d} r_t - \Gamma(l_t, r_t)$$

subject to

$$l_t \leq \kappa \left[ y_t^T + p_t y_t^N \right],$$

taking as given $i_t^l$, $i_t^d$, $i_t^r$, $y_t^T$, $p_t$, and $y_t^N$. Let $\mu_t^B \geq 0$ denote the Lagrange multiplier on the bank's loan limit. Then the first-order conditions associated with the optimal choice of $l_t$ and $\mu_t^B$ are

$$\frac{i_t^l - i_t^d}{1 + i_t^d} \leq \Gamma_l(l_t, r_t) + \mu_t^B, \quad l_t \geq 0, \quad \left[ \frac{i_t^l - i_t^d}{1 + i_t^d} - \Gamma_l(l_t, r_t) - \mu_t^B \right] l_t = 0, \tag{26}$$

$$\mu_t^B \geq 0, \tag{27}$$

$$l_t \leq \kappa \left( y_t^T + p_t y_t^N \right),$$

and

$$\mu_t^B \left[ \kappa \left( y_t^T + p_t y_t^N \right) - l_t \right] = 0. \tag{28}$$

Comparing the present setting with the one in which the collateral constraint is placed at the household level, we have that equation (26) replaces equilibrium condition (5) and equations (27) and (28) replace equilibrium conditions (15) and (16). The first-order condition with respect to $r_t$ continues to be equation (6), and the balance sheet constraint of the bank continues to be (7).

Now the household does not face the collateral constraint (11). Thus, its problem consists in choosing $c_t$, $c_t^T$, $c_t^N$, and $l_t$ to maximize the utility function (8) subject to the aggregator (9), the sequential budget constraint (10), and some borrowing limit that prevents it from engaging in Ponzi schemes, taking as given $p_t$, $i_t^l$, $\pi_t$, $\tau_t$, $y_t^T$, and $y_t^N$. The first-order conditions of this problem are (12), (13), and

$$\lambda_t = \beta(1 + i_t^l) E_t \lambda_{t+1}. \tag{29}$$

The difference between this optimization problem and that in which the household does face a collateral constraint is that condition (29) replaces condition (14).

The assumption that the borrowing limit is placed at the level of the bank rather than at the level of the household affects neither the interest rate parity condition (17) nor the economy wide resource constraint (20). A competitive equilibrium in the economy with the collateral constraint at the bank level can then be defined as follows:

**Definition 3** (Competitive Equilibrium in the Economy with the Collateral Constraint at the Bank Level)**.** *A competitive equilibrium in the economy with a collateral constraint at the level of the bank is a set of processes* $l_t$, $r_t$, $d_t$, $i^l_t$, $i^d_t$, $c^T_t$, $p_t$, $\lambda_t$, *and* $\mu^B_t$ *satisfying* (6), (7), (11), (12), (13), (17), (20), *and* (26)-(29), *for* $t \geq 0$, *given a reserve remuneration policy* $i^r_t$, *a capital control tax rate* $\tau^c_t$, *exogenous processes* $i^*_t$, $y^T_t$, *and* $y^N_t$, *and the initial condition* $(1 + i^*_{-1})d_{-1} > 0$.

Appendix D proves the following proposition:

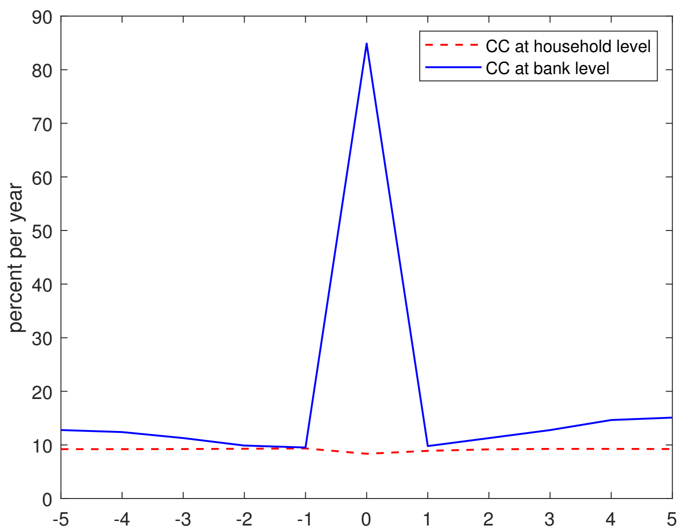
**Proposition 2** (Equivalence of Equilibrium with Collateral Constraints at the Bank or Household Level)**.** *Suppose the set of processes* $l_t$, $r_t$, $d_t$, $i^l_t$, $i^d_t$, $c^T_t$, $p_t$, *and* $\lambda_t$ *is a competitive equilibrium in the economy with a collateral constraint at the household level (i.e., satisfies Definition 1). Then this set of processes also represents a competitive equilibrium of the economy with the collateral constraint at the bank level (i.e., satisfies Definition 3) except for the loan rate,* $i^l_t$, *in states in which the collateral constraint binds. In these states the loan rate is strictly larger in the economy with the collateral constraint at the bank level than in the economy with the collateral constraint at the household level.*

According to this proposition all variables except for the lending rate behave identically in the economies with the collateral constraint at the level of the household and at the level of the bank. The key difference between the two formulations is that when the collateral constraint binds the lending spread increases in the economy with the collateral constraint at the bank level but falls in the economy with the collateral constraint at the household level.

This difference can be quantitatively significant. Figure 8 displays the behavior of the lending rate around the typical sudden stop under laissez-faire in the economies with the collateral constraint at the bank and household levels. The figure is produced using the parameter values shown in Table 1. When the collateral constraint binds (period 0) the loan rate skyrockets to 73 percent per year in the economy with a collateral constraint at the bank level, but remains flat (in fact falls by 0.45 percentage points) in the economy with the collateral constraint at the household level.

The main message conveyed by Figure 8 is that the observed behavior of the lending rate can provide information on whether disruptions in financial markets stem from the financial

Figure 8: Behavior of the Lending Rate Around a Sudden Stop in the Unregulated Economy




Notes. CC stands for collateral constraint. The figure shows that in a sudden stop, the lending rate increases sharply when the collateral constraint enters at the bank level, but falls slightly when the collateral constraint enters at the household level.

or the nonfinancial sector. Being able to make this distinction is not inconsequential for policymakers. Historically, the presumption that a key symptom of a financial crisis is a sharp increase in the lending rate has led to the misdiagnosis of major financial crises. A case in point is the Great Depression of 1929 to 1933. This episode featured a lack of a spike in the lending rate. Rockoff (2021) shows that the call money rate—the interest rate charged by banks to stock brokers on collateralized loans—actually fell between June 1930 and June 1931. According to Rockoff, the lack of a hike in this interest rate was a key reason why Oliver M. W. Sprague, the major authority on financial crises at the time and an economic advisor of the Bank of England and the Roosevelt administration, failed to recognize a financial crisis in the economic developments that unfolded during this period. Rockoff further speculates that "had he [Sprague] diagnosed a banking panic and called for an aggressive response by the Federal Reserve, it might have made a difference; but he did not."

There is also some evidence that during the great recession of 2008 demand for loans fell in emerging countries. For example, De la Torre, Pería, and Schmukler (2010) examine the effects of the great recession on bank lending to small and medium enterprises in Argentina, Chile, and Colombia using the World Bank's on-site survey of bank business and risk managers as well as other survey sources. They find that no bank reported having a shortage of funds to lend. Further, they document that a sizable share of banks (70 percent in Argentina, 33 percent in Chile, and 63 percent in Colombia) reported experiencing a significant decline in the demand for loan products.

## 4.2 Non-Equivalence of Reserve Remuneration and Reserve Requirements

A macroprudential instrument that is sometimes used in emerging countries is reserve requirements. Here we ask whether the constrained optimal allocation with reserve remuneration and capital control taxes can also be supported by an appropriate combination of reserve requirements and capital controls. We also ask whether reserve remuneration welfare dominates reserve requirements as a macroprudential tool. The analysis that follows establishes that in the present theoretical framework the policy maker can achieve a better outcome by using a combination of interest on reserves and capital controls than by using a combination of reserve requirements and capital controls.

Suppose the central bank does not pay interest on bank reserves, $i_t^r = 0$, but imposes a reserve requirement

$$r_t \geq \delta_t d_t,$$

where $\delta_t \in [0, 1)$ is a policy instrument. In addition, the government continues to have access to capital control taxes. Combining the reserve requirement with the bank's balance sheet constraint (7) yields

$$r_t \geq \delta_t[l_t + r_t + \Gamma(l_t, r_t)]. \tag{30}$$

Then the problem of a bank consists in choosing $l_t \geq 0$ and $r_t \geq 0$ so as to maximize profits,

$$\frac{i_t^l - i_t^d}{1 + i_t^d} l_t + \frac{i_t^r - i_t^d}{1 + i_t^d} r_t - \Gamma(l_t, r_t),$$

subject to the reserve requirement (30), taking as given $i_t^l$, $i_t^r$, $i_t^d$, and $\delta_t$. Letting $\eta_t$ denote the Lagrange multiplier on (30), the first-order conditions of the bank's problem are

$$r_t \geq \delta_t[l_t + r_t + \Gamma(l_t, r_t)], \quad \eta_t \geq 0, \quad \eta_t \{r_t - \delta_t[l_t + r_t + \Gamma(l_t, r_t)]\} = 0 \tag{31}$$

$$\frac{i_t^l - i_t^d}{1 + i_t^d} \leq \Gamma_l(l_t, r_t) + \eta_t \delta_t [1 + \Gamma_l(l_t, r_t)], \quad l_t \geq 0, \quad l_t \left\{ \frac{i_t^l - i_t^d}{1 + i_t^d} - \Gamma_l(l_t, r_t) - \eta_t \delta_t [1 + \Gamma_l(l_t, r_t)] \right\} = 0 \tag{32}$$

and

$$\frac{-i_t^d}{1 + i_t^d} - \Gamma_r(l_t, r_t) - \eta_t [1 - \delta_t - \delta_t \Gamma_r(l_t, r_t)] = 0, \tag{33}$$

where the last first-order condition uses the fact that reserves are unremunerated ($i_t^r = 0$).

**Definition 4** (Competitive Equilibrium with Reserve Requirements and Capital Controls)**.** *A competitive equilibrium with reserve requirements and capital controls is a set of processes* $c_t^T$*,* $d_t$*,* $p_t$*,* $i_t^l$*,* $i_t^d$*,* $\lambda_t$*,* $\mu_t$*,* $l_t$*,* $r_t$*, and* $\eta_t$ *satisfying* (7)*,* (11)*-*(17)*,* (20)*, and* (31)*-*(33) *for* $t \geq 0$*, given a reserve requirement policy* $\delta_t$*, a capital control tax rate* $\tau_t^c$*, exogenous processes* $i_t^*$*,* $y_t^T$*, and* $y_t^N$*, and the initial condition* $(1 + i_{-1}^*)d_{-1} > 0$*.*

All constraints in Definition 2 are equilibrium conditions of the economy with reserve requirements (Definition 4). It follows immediately that the best competitive equilibrium with interest on reserves and capital controls yields at least as much welfare as the best competitive equilibrium attainable with reserve requirements and capital controls. Furthermore, Appendix E proves the following claim:

**Claim 1.** *[Non-Equivalence of Interest on Reserves and Reserve Requirements] In general, the constrained optimal allocation of Definition 2 does not satisfy the competitive equilibrium conditions of the economy with reserve requirements and capital controls listed in Definition 4.*

Claim 1 shows that the constrained optimal allocation attainable with interest on reserves and capital controls defined in Definition 2 cannot be supported as a competitive equilibrium with reserve requirements, Definition 4. We thus have that the best competitive equilibrium with reserve remuneration and capital controls strictly welfare dominates the best competitive equilibrium with reserve requirements and capital controls. We summarize this result in the following proposition.

**Proposition 3** (Welfare Dominance of Reserve Remuneration Over Reserve Requirements)**.** *The constrained optimal allocation attainable with a combination of interest on reserves and capital controls* $(i_t^r, \tau_t^c)$ *strictly dominates in welfare the constrained optimal allocation attainable with a combination of reserve requirements and capital controls* $(\delta_t, \tau_t^c)$*.*

## 4.3 Redundancy of Capital Controls in the Absence of a Collateral Constraint

Here, we consider an economy in which household borrowing from banks is not subject to a collateral constraint. We show that in this case the optimal level of capital control taxes is zero ($\tau_t^c = 0$). The intuition behind this result is that the lack of a collateral constraint eliminates the pecuniary externality that the presence of a relative price in the collateral constraint creates. As a result, the social planner does not need a policy instrument dedicated to making households internalize this externality. The remaining policy instrument, namely, reserve remuneration, is still necessary because it serves the purpose of making banks internalize the net social benefits (and costs) of holding reserves.

A competitive equilibrium in the economy without the collateral constraint (11) is defined as follows:

**Definition 5** (Competitive Equilibrium in the Economy without a Collateral Constraint)**.** *A competitive equilibrium in the economy without a collateral constraint on loans is a set of processes* $l_t$, $r_t$, $d_t$, $i_t^l$, $i_t^d$, $c_t^T$, $c_t^N$, $p_t$, *and* $\lambda_t$, *satisfying* (5)-(7), (12), (13), (17), (19), (20), *and*

$$\lambda_t = \beta(1 + i_t^l)E_t\lambda_{t+1}, \tag{34}$$

*for* $t \geq 0$, *given a reserve remuneration policy* $i_t^r$, *a capital control tax rate* $\tau_t^c$, *exogenous processes* $i_t^*$, $y_t^T$, *and* $y_t^N$, *and the initial condition* $(1 + i_{-1}^*)d_{-1} > 0$.

The constrained optimal allocation of a social planner with access to interest on bank reserves ($i_t^r$) and capital control taxes ($\tau_t^c$) now takes the form:

**Definition 6** (Constrained Optimal Allocation in the Economy without a Collateral Constraint)**.** *The constrained optimal allocation is a set of processes* $c_t^T$, $d_t$, $l_t \geq 0$, *and* $r_t \geq 0$ *that solves the problem*

$$\max E_0 \sum_{t=0}^{\infty} \beta^t u(A(c_t^T, y_t^N))$$

*subject to the bank's balance sheet* (7) *and the economy-wide resource constraint* (20), *taking as given the processes* $i_t^*$, $y_t^T$, *and* $y_t^N$, *and the initial condition* $(1 + i_{-1}^*)d_{-1} > 0$.

The Lagrangian associated with the social planner's optimization problem is

$$\begin{aligned}\mathcal{L} \;=\; & E_0 \sum_{t=0}^{\infty} \beta^t \left\{ u(A(c_t^T, y_t^N)) + \lambda_t^r \left[ y_t^T + d_t - c_t^T - (1 + i_{t-1}^*)d_{t-1} - \Gamma(l_t, r_t) - \Gamma^r(r_t) \right] \right. \\ & \left. + \lambda_t^r \mu_t^r \left[ l_t + r_t + \Gamma(l_t, r_t) - d_t \right] \right\}.\end{aligned}$$

It can be shown that in the constrained optimal allocation loans are positive, $l_t > 0$ (see Appendix F). Then, the first-order conditions with respect to $c_t^T$, $d_t$, and $l_t$ are

$$u'(A(c_t^T, y_t^N))A_1(c_t^T, y_t^N) = \lambda_t^r, \tag{35}$$

$$1 - \mu_t^r = (1 + i_t^*)\beta E_t \frac{\lambda_{t+1}^r}{\lambda_t^r}, \tag{36}$$

and

$$\mu_t^r = \frac{\Gamma_l(l_t, r_t)}{1 + \Gamma_l(l_t, r_t)}. \tag{37}$$

Suppose $\{c_t^T, d_t, l_t, r_t\}$ is the constrained optimal allocation. To see that this allocation satisfies the competitive equilibrium conditions listed in Definition 5, proceed as follows. Set $c_t^N$ to satisfy (19), set $\lambda_t = \lambda_t^r$, so it satisfies (12), and set $p_t$ to satisfy (13). Set $i_t^l$ so that (34) holds. This yields $(1 + i_t^l) = \lambda_t^r/\beta/E_t\lambda_{t+1}^r$. Combing this expression with the social planner's first-order condition (36), we have

$$1 + i_t^l = \frac{1 + i_t^*}{1 - \mu_t^r}.$$

Next set $i_t^d$ so that it satisfies (5) holding with equality. Combining the resulting expression with the social planner's first-order condition (37) yields

$$1 + i_t^l = \frac{1 + i_t^d}{1 - \mu_t^r}.$$

The above two expressions imply that

$$i_t^d = i_t^*.$$

Set $i_t^r$ to satisfy (6). Finally, choose $\tau_t^c$ to satisfy (17). Because $i_t^d = i_t^*$, the capital control taxes that support the constrained optimal equilibrium must be zero at all times ($\tau_t^c = 0$). We summarize this result in the following proposition:

**Proposition 4** (Redundancy of Capital Controls in the Absence of a Collateral Constraint)**.** *The constrained optimal allocation in the absence of a collateral constraint given in Definition 6 can be supported as a competitive equilibrium by an appropriate reserve-remuneration policy* $i_t^r$ *and the capital control policy* $\tau_t^c = 0$.

## 4.4 Sensitivity

This section presents an analysis of the sensitivity of three central results of the paper to variations in the parameters defining the banking friction. Table 4 shows how predicted moments change as the parameters $A$, $B$, $\alpha$, $\phi$, and $\bar{r}$ take values 25 percent higher or lower than in the baseline calibration. The first column of the table compares the mean debt-to-output ratio in the unregulated and constrained optimal economies. For all parameter values considered, the constrained optimal economy holds on average a higher level of debt per unit of output than the unregulated economy, suggesting that the underborrowing result stressed in the paper is a robust prediction of the model. We were able to detect overborrowing for extreme values (not shown) of the parameter $B$ governing the central bank's cost of reserve provision. Specifically, the debt-to-output ratio becomes higher in the unregulated economy

Table 4: Sensitivity to Changes in Parameters of the Banking Channel

| Parameter | | | corr$(r_t, l_t)$ | | corr$(r_t, y_t^T + p_t y_t^N)$ | |
|---|---|---|---|---|---|---|
| Name | Value | $\frac{(d_t/y_t)^o}{(d_t/y_t)^u}$ | Unregulated | Regulated | Unregulated | Regulated |
| Baseline | – | 1.10 | 0.99 | -0.48 | 0.54 | -0.79 |
| High $A$ | 0.03 | 1.08 | 0.97 | -0.26 | 0.45 | -0.76 |
| Low $A$ | 0.02 | 1.12 | 0.98 | -0.65 | 0.57 | -0.82 |
| High $B$ | 2.19 | 1.09 | 0.99 | -0.45 | 0.54 | -0.77 |
| Low $B$ | 1.31 | 1.13 | 0.99 | -0.52 | 0.54 | -0.81 |
| High $\alpha$ | 1.92 | 1.16 | 0.99 | -0.50 | 0.50 | -0.81 |
| Low $\alpha$ | 1.15 | 1.04 | 0.98 | -0.46 | 0.57 | -0.75 |
| High $\phi$ | 29.62 | 1.09 | 0.98 | -0.47 | 0.55 | -0.78 |
| Low $\phi$ | 17.77 | 1.12 | 0.98 | -0.49 | 0.48 | -0.79 |
| High $\bar{r}$ | 0.14 | 1.08 | 0.98 | -0.43 | 0.52 | -0.78 |
| Low $\bar{r}$ | 0.08 | 1.13 | 0.99 | -0.53 | 0.51 | -0.80 |

Notes. All moments are computed unconditionally. The objects $(d_t/y_t)^u$ and $(d_t/y_t)^o$ denote the mean debt-to-output ratio in the unregulated and constrained optimal equilibria. The first line in the body of the table, labeled Baseline, corresponds to the baseline parameterization of the model given in Table 1.

than in the regulated economy for values of $B$ above 7 times the baseline value. This range is empirically implausible because it induces values of the cost of reserve provision per unit of bank reserve, $\Gamma^r(r)/r$, of over 14.5 percent, whereas the observed value—targeted in the calibration—is only 2.2 percent.

Columns 3 and 4 of the table compare the correlation between bank reserves and bank loans in the unregulated and constrained optimal equilibria. In the former, this correlation is consistently positive, whereas in the latter it is consistently negative. This result points to the robustness of the prediction of the model that through the optimal reserve remuneration policy the central bank acts as a lender of last resort by boosting bank reserves at times when bank loans are low. Columns 5 and 6 compare the correlation between bank reserves and output in the unregulated and constrained optimal equilibria. For all parameterizations considered the same pattern as observed in the baseline calibration emerges: in the unregulated economy reserves are procyclical (they increase with output) whereas in the constrained optimal equilibrium reserves are countercyclical. This suggests that the result that the government finds it optimal to raise reserves in contractions to ensure that deleveraging at the household level does not spill over to the aggregate level is also a robust prediction of the model.

# 5 Conclusion

This paper contributes to a literature on open economy models with a pecuniary externality due to an occasionally binding collateral constraint faced by private agents. A central result in this literature is that the economy overborrows from the rest of the world. The innovation of the present study is to replace the assumption that households borrow directly from foreign lenders with the more realistic assumption that foreign lending is intermediated by banks operating in domestic financial markets. The paper characterizes constrained optimal equilibria attainable by a policymaker with access to capital controls and interest on bank reserves.

An important finding of the paper is that in the model with a collateral constraint and a bank lending channel, the unregulated equilibrium displays underborrowing. External debt, private borrowing, and bank reserves are all lower in the laissez-faire equilibrium than in the constrained optimal equilibrium. In the constrained optimal allocation the country can borrow more than under laissez-faire because in the former the government acts as a lender of last resort to borrowing-constrained households. During contractions in which households are collateral constrained, the government raises the interest rate on bank reserves to induce banks to deposit more reserves at the central bank. The government then channels these resources to households through a more relaxed fiscal policy, thereby alleviating their liquidity needs. In this way, deleveraging at the household level does not spill over into economy wide deleveraging. By contrast, in the laissez-faire economy household deleveraging causes economy wide deleveraging. This is so because when the demand for loans collapses, so does the demand for bank reserves. In turn, bank reserves fall because they are a complementary input in the production of bank loans. Without government intervention, the banking channel amplifies macroeconomic deleveraging because the demand for foreign deposits declines not only because households demand fewer loans but also because banks demand fewer reserves.

An important question in macroprudential banking policy is whether reserve requirements and interest on bank reserves are equivalent policy instruments. In the context of the open economy model studied in this paper, this is not the case. As it turns out, reserve remuneration strictly dominates reserve requirements in a welfare sense. The reason is that reserve remuneration controls the price of reserves but lets its quantity be determined endogenously. By contrast, reserve requirements without interest on reserves amounts to fixing both the quantity and the price of reserves.

The present analysis focuses on variations in household income as the shifter of the household demand for loans. This choice was made to keep the results of the paper comparable with key papers in the related literature. It is an interesting avenue for future research to investigate the consequences of introducing additional sources of uncertainty affecting the demand for or the supply of loans. Another relevant shock affecting the demand for loans is variations in the upper bound of the leverage ratio of households ($\kappa$ in the notation of the paper). This shock is of interest because it can capture contractions stemming from failures in the domestic financial sector. A shock that affects the supply of loans to emerging countries is disturbances to the world interest rate or to the country interest rate premium ($i_t^*$ in the notation of the paper). Introducing this shock is of interest in light of the extensive literature suggesting that variations in the cost of external credit represent an important driver of aggregate fluctuations in emerging countries.

In the paper, the government is assumed to raise liquidity from the domestic banking system. The implicit assumption is that the government maintains a more fluid relationship with domestic banks than with foreign lenders. So in responding to short-run funding needs of liquidity constrained households, the government finds it easier to go through the domestic banking system. It is an interesting question for future research to relax this assumption and explore an environment in which the government uses a mix of domestic and foreign sources of funds, by, for example, introducing differentiated transactions costs. Finally, because the present study considers an endowment economy, financial frictions do not affect employment, investment, or aggregate activity. It would be of interest to extend the analysis to a production economy to shed light on these important dimensions.

# Appendix

## A Optimal Policy When Bank Reserve Provision Is Costless

This appendix shows that when $\Gamma^r(r) = 0$, the optimal interest-on-reserve policy achieves the first-best allocation. It further shows that in this case capital controls are redundant, that is, the first best allocation can be supported with the policy $\tau_t^c = 0$ for all $t$. Define the first-best allocation as the one that solves the problem of a social planner who is constrained only by the sequential resource constraint and the prohibition to play Ponzi schemes. That is, the social planner is neither subject to the collateral constraint nor to the bank intermediation friction. The following definition provides a formal statement:

**Definition A1** (First-Best Allocation)**.** *The first-best allocation is a pair of processes of tradable consumption and foreign deposits, $\tilde{c}_t^T$ and $\tilde{d}_t$, that solves the problem*

$$\max E_0 \sum_{t=0}^{\infty} \beta^t u(A(c_t^T, y_t^N))$$

*subject to the sequential resource constraint*

$$c_t^T + (1 + i_{t-1}^*) d_{t-1} = y_t^T + d_t \tag{A1}$$

*and to a no-Ponzi-game constraint, taking as given the processes $i_t^*$, $y_t^T$, and $y_t^N$, and the initial condition $(1 + i_{-1}^*) d_{-1} > 0$.*

The first-order condition of this maximization problem with respect to holdings of foreign deposits, $d_t$, is the Euler equation

$$\tilde{\lambda}_t = \beta (1 + i_t^*) E_t \tilde{\lambda}_{t+1},$$

with

$$\tilde{\lambda}_t = u'(A(\tilde{c}_t^T, y_t^N)) A_1(\tilde{c}_t^T, y_t^N).$$

To show that the first-best allocation can be supported as a competitive equilibrium by an appropriate interest-on-reserve policy and no capital controls, one must show that there exists a process for $i_t^r$ such that all conditions in Definition 1 are satisfied with $\tau_t^c = 0$. are satisfied when evaluated at the processes for consumption and foreign deposits associated with the first-best allocation. To this end, start by setting $c_t^T = \tilde{c}_t^T$ and $d_t = \tilde{d}_t$. Set $l_t = 0$.

Then the sequential resource constraint in the competitive equilibrium (20) becomes identical to the resource constraint of the social planner (A1), since $\Gamma(0, r_t) = 0$ by Assumption 1. Next, set $\mu_t = 0$. It follows immediately that equilibrium conditions (15) and (16) are satisfied. To satisfy (12), set $\lambda_t = \tilde{\lambda}_t$. Next, set $i_t^l = i_t^*$, which guarantees that equilibrium condition (14) is satisfied. Pick $p_t$ residually to satisfy (13). Because $l_t = 0$ for all $t$, the collateral constraint (11) holds. Set $r_t = \tilde{d}_t$. This guarantees the satisfaction of equilibrium condition (7). Set $i_t^d = i_t^*$. This ensures that equilibrium condition (17) is satisfied because $\tau_t^c = 0$. Equilibrium condition (5) holds because $l_t = 0$, $i_t^l = i_t^d = i_t^*$, and $\Gamma_l(\cdot, \cdot) \geq 0$. To ensure that (6) is satisfied, set the policy rate $i_t^r$ equal to $i_t^d (= i_t^*)$ and invoke Assumption 1. This completes the proof that when the provision of bank reserves is costless ($\Gamma^r(r_t) = 0$) the first-best allocation can be supported as a competitive equilibrium by an appropriate interest-on-reserve policy ($i_t^r = i_t^*$) and no capital controls ($\tau_t^c = 0$). The following proposition summarizes this result.

**Proposition A1** (Optimal Policy When Reserve Provision is Costless)**.** *Suppose that* $\Gamma^r(\cdot) = 0$ *and that Assumption 1 holds. In an economy with equilibrium conditions given by Definition 1, the first best allocation, given in Definition A1, can be supported as a competitive equilibrium by the interest-on-reserve policy* $i_t^r = i_t^*$ *and no capital control taxes,* $\tau_t^c = 0$.

The optimal bank reserve remuneration policy $i_t^r = i_t^*$ eliminates households' need to borrow from banks, $l_t = 0$. All of the financial intermediation occurs between banks and the government. Banks take deposits from international lenders and deposit them entirely at the central bank in the form of reserves. In effect, the government borrows from banks at the world interest rate and transfers resources to the private sector via income taxes or subsidies, as needed. From the point of view of the household, these taxes or subsidies are exogenous. In equilibrium households endogenously become hand-to-mouth agents. They have access to bank loans at the world interest rate and have collateral to back them, but nevertheless choose not to use this credit facility. Cúrdia and Woodford (2011) show, in the context of a closed economy, that, as in the present environment, the optimal reserve remuneration policy allows agents to completely circumvent the bank intermediation friction. Here, the optimal policy also allows agents to completely circumvent the financial friction arising from the collateral constraint, which makes it feasible to attain the first-best allocation.

# B Computation

The computation employs global methods over a discretized state space. Specifically, it uses 13 equally spaced points for each of the exogenous driving forces, $\ln y^T$ and $\ln y^N$. The transition probability matrix of the vector $(\ln y^T, \ln y^N)$ is computed using the simulation approach described in Schmitt-Grohé and Uribe (2014). The endogenous state, external debt $d$, is discretized using 800 equally spaced points. The lower panel of Table 1 provides more details.

The unregulated or laissez-faire equilibrium ($i_t^r = \tau_t^c = 0$) is approximated using an Euler equation iteration procedure over the discretized state space. The constrained optimal competitive equilibrium, that is, the equilibrium in which the policymaker optimally sets $i_t^r$ and $\tau_t^c$, is computed using a value function iteration approach over the discretized state space.

Unlike in a version of the present model without a banking channel (e.g., Bianchi, 2011; Schmitt-Grohé and Uribe, 2021), the computation of the constrained optimal equilibrium turns out to be more involved than that of the unregulated economy. The reason is that in the unregulated economy, given a value for $d_t$, the volume of loans and reserves, $l_t$ and $r_t$, are determined by solving equations (6) and (7), evaluated at $i_t^r = 0$ and $i_t^d = i_t^*$, which is a relatively simple numerical problem. With $r_t$ and $l_t$ in hand, consumption of tradables, $c_t^T$, is found residually by solving the sequential resource constraint (20). This consumption level and the associated candidate value for $d_t$ are feasible if the equilibrium collateral constraint (21) is satisfied, which is also a simple condition to check. By contrast, the social planner is not constrained by equation (6), since she can pick $i_t^r$. As a result, given $d_t$, the values of $l_t$, $r_t$, and $c_t^T$ are jointly determined as the solution to the problem of maximizing $c_t^T$ subject to (7), (20), and (21). Thus, the social planner solves a static optimization problem for each candidate choice of $d_t$ and for each state $(y_t^T, y_t^N, d_{t-1})$. In turn, this static optimization problem is nested in the dynamic optimization problem of choosing the debt policy function, $d_t$.

# C Data Sources

In this appendix, we provide an overview of the data sources used to analyze targeted moments in Table 2. These moments include the lending spread, the reserve-to-deposit ratio, and the central-bank-operating-cost-to-reserve ratio. The classification of rich and emerging countries adheres to the criteria set forth in Uribe and Schmitt-Grohé (2017).

1. [Lending Spread]

   (a) Date source: IMF International Financial Statistics December 2017 and August 2021 archive.

(b) Definition: The lending spread is defined as the deposit rate minus the lending rate. The deposit rate is *Financial, Interest Rates, Deposit, Percent per annum* (series identifier: FIDR_PA). The lending rate is *Financial, Interest Rates, Lending Rate, Percent per annum* (series identifier: FILR_PA).

(c) Data frequency and coverage: annual data from 1985 to 2016.

(d) Country list (38 emerging countries and 16 rich countries)

i. Emerging countries: Albania, Algeria, Argentina, Bahrain, Bolivia, Botswana, Brazil, Bulgaria, Chile, Colombia, Costa Rica, Cyprus, Dominican Republic, Ecuador, Egypt, El Salvador, Gabon, Greece, Guatemala, Hungary, Iran, Jordan, South Korea, Malaysia, Mexico, Morocco, Namibia, New Zealand, Panama, Paraguay, Peru, Portugal, Spain, Syrian Arab Republic, Thailand, Trinidad and Tobago, Uruguay, and Venezuela.

ii. Rich countries: Austria, Belgium, Canada, Finland, France, Germany, Hong Kong, Ireland, Italy, Japan, Netherlands, Singapore, Sweden, Switzerland, United Kingdom, and United States.

2. [Reserve-to-deposit ratio]

(a) Date source: Bankscope.

(b) Definition: The reserve-to-deposit ratio is defined as bank reserves in USD divided by bank deposit in USD. Bank reserves is *Due From Central Banks* (series identifier: data38390) and bank deposit is *Total Deposits, Money Market and Short-term Funding* (series identifier: data11580).

(c) Data frequency and coverage: annual data from 1985 to 2016.

(d) Country list (34 emerging countries and 17 rich countries)

i. Emerging countries: Albania, Argentina, Bahrain, Bolivia, Botswana, Brazil, Bulgaria, Chile, Colombia, Costa Rica, Cyprus, Dominican Republic, Ecuador, Egypt, El Salvador, Greece, Guatemala, Hungary, Jordan, South Korea, Mexico, Namibia, New Zealand, Panama, Paraguay, Peru, Portugal, South Africa, Spain, Syrian Arab Republic, Trinidad and Tobago, Turkey, Uruguay, and Venezuela.

ii. Rich countries: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Hong Kong, Ireland, Italy, Netherlands, Norway, Singapore, Sweden, Switzerland, and United Kingdom.

3. [Central-bank-operating-cost-to-reserve ratio]

(a) Date source: Bankscope

(b) Definition: The central-bank-operating-cost-to-reserve ratio is defined as the central bank's operating cost in USD divided by reserves held in central banks in USD. The central bank's operating cost is calculated as the sum of *Total Non-Interest Expenses* (series identifier: data10170), *Loan Impairment Charge* (series identifier: data10200), and *Equity-accounted Profit/ Loss - Operating* (series identifier: data10180). Reserves held in central banks is *Total Deposits, Money Market and Short-term Funding* (series identifier: data11580).

(c) Data frequency and coverage: annual data from 1985 to 2016.

(d) Country list (36 emerging countries and 18 rich countries)

i. Emerging countries: Albania, Argentina, Bahrain, Bolivia, Botswana, Brazil, Bulgaria, Chile, Colombia, Costa Rica, Cyprus, Dominican Republic, Ecuador, El Salvador, Greece, Guatemala, Hungary, Iran, Israel, Jordan, South Korea, Malaysia, Morocco, Namibia, New Zealand, Paraguay, Peru, Portugal, South Africa, Spain, Thailand, Trinidad and Tobago, Tunisia, Turkey, Uruguay, and Venezuela.

ii. Rich countries: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Hong Kong, Ireland, Italy, Japan, Netherlands, Norway, Singapore, Sweden, Switzerland, and United Kingdom.

# D Proof of Proposition 2: Equivalence of Equilibrium with Collateral Constraints at the Bank or Household Level

Consider processes for $l_t$, $r_t$, $d_t$, $i_t^l$, $i_t^d$, $c_t^T$, $p_t$, $\lambda_t$, and $\mu_t$ that constitute a competitive equilibrium of the economy with the collateral constraint at the household level, that is, processes that satisfy Definition 1. We wish to show that this allocation, with the exception of $i_t^l$, also satisfies Definition 3. Since (6), (7), (11), (12), (13), (17), and (20), belong to both Definition 1 and Definition 3, and do not feature $i_t^l$, what needs to be shown is that equilibrium conditions (26)-(29) hold.

Let $i_t^{l,B}$ denote the equilibrium lending rate in the economy with the collateral constraint at the bank level. Suppose in a given date and state the collateral constraint is slack, $l_t < \kappa(y_t^T + p_t y_t^N)$, so that $\mu_t = 0$. Set $\mu_t^B = \mu_t = 0$ and $i_t^{l,B} = i_t^l$. Then (26)-(29) are satisfied. Next, consider the case that the collateral constraint binds in the baseline allocation, $l_t = \kappa(y_t^T + p_t y_t^N)$. Note that in this case $l_t > 0$. Set

$$\mu_t^B = \frac{\mu_t}{1-\mu_t}\frac{(1+i_t^l)}{(1+i_t^d)},$$

which implies that $\mu_t^B$ is strictly positive. The facts that $\mu_t^B$ is positive and that the collateral constraint holds with equality ensure that (27)-(28) hold. It remains to show that (26) and (29) are satisfied. Pick $i_t^{l,B}$ so that it satisfies the left-hand side expression of (26) with equality. This then implies that the middle and the right-hand side expression of (26) also hold and yields

$$1 + i_t^{l,B} = (1+i_t^d)(1+\Gamma_l(l_t, r_t)) + \mu_t \frac{\lambda_t}{\beta E_t \lambda_{t+1}}.$$

Now use the left expression of (5) holding with equality to replace $(1+i_t^d)(1+\Gamma_l(l_t, r_t))$ with $1 + i_t^l$, and replace $(1+i_t^l)$ in turn with (14), which yields $(1+i_t^l) = (1-\mu_t)\frac{\lambda_t}{\beta E_t \lambda_{t+1}}$. Substituting these expressions in the above displayed equation, we obtain

$$(1+i_t^{l,B}) = \frac{\lambda_t}{\beta E_t \lambda_{t+1}}.$$

It follows that (29) is satisfied, which is what we set out to show. Finally, to see that $i_t^{l,B} \geq i_t^l$ use the facts that $\frac{\lambda_t}{\beta E_t \lambda_{t+1}} = (1+i_t^l)/(1-\mu_t)$ and that $0 \leq \mu_t < 1$.■

# E Proof of Claim 1: Non-Equivalence of Interest on Reserves and Reserve Requirements

Consider the processes $c_t^T$, $r_t$, $l_t$, and $d_t$ that solve the optimization problem in Definition 2. By construction, these processes satisfy the bank's balance sheet (7) and the economy's sequential resource constraint (20). Set $\lambda_t$ and $p_t$ to satisfy equilibrium conditions (12) and (13). The collateral constraint (11) is then satisfied by construction. Consider a date in which $l_t > 0$ and the collateral constraint is slack.[5] Then, $\mu_t = 0$, which implies that equilibrium conditions (15) and (16) are satisfied. The interest rate on loans, $i_t^l$, is then determined residually by the Euler equation (14). It remains to check whether (17) and (31)-(33) also hold. Because $l_t > 0$, the left expression of (32) holds with equality. If $\eta_t = 0$, then this expression and (33) form a system of two equations in one unknown, $i_t^d$, which is in

[5]One can show that $l_t > 0$ at all times under the relatively weak assumption $\Gamma_l(0, r_t) = 0$, which is satisfied by the functional forms used in the quantitative analysis (see Section 3.1). The quantitative analysis further shows that under the assumed calibration in the constrained optimal allocation the collateral constraint does not bind at all times.

general inconsistent. If, on the other hand, $\eta_t$ is different from zero, then the left expression in (31) holds with equality. This expression, the left expression in (32) holding with equality, and (33) represent a system of three equations in three unknowns, $\delta_t$, $i_t^d$, and $\eta_t$. There are no guarantees, however, that the solution to this system will yield a non-negative value for the Lagrange multiplier on the reserve requirement, $\eta_t$.[6]

# F Optimality of Positive Loans in an Economy Without a Collateral Constraint

This appendix presents the proof of the following claim used in the proof of Proposition 4:

**Claim F1.** *In the absence of a collateral constraint, the constrained optimal allocation given in Definition 6 features positive loans,* $l_t > 0$.

To establish this claim, note from Definition 6 that in the absence of the collateral constraint, given $d_t$, the social planner chooses the variables $l_t$ and $r_t$ to minimize the sum of the private resource cost of bank intermediation and the public resource cost of reserve management, subject to the balance sheet constraint. Formally, given $d_t$, the planner solves

$$\min_{\{l_t, r_t\}} C(l_t, r_t) \equiv \Gamma(l_t, r_t) + \Gamma^r(r_t)$$

subject to (7). Given our focus on equilibria in which $d_t > 0$ for all $t$, it is clear from (7) that either $l_t$ or $r_t$ must be positive. Differentiating the objective function and the constraint, we get

$$dC(l_t, r_t) = \Gamma_l(l_t, r_t)dl_t + \Gamma_r(l_t, r_t)dr_t + \Gamma^{r'}(r_t)dr_t$$

and

$$dl_t + dr_t + \Gamma_l(l_t, r_t)dl_t + \Gamma_r(l_t, r_t)dr_t = 0.$$

Suppose, contrary to the claim, that $l_t = 0$, so that $r_t > 0$. In this case, by Assumption 1, $\Gamma_l(l_t, r_t) = \Gamma_r(l_t, r_t) = 0$. Then, using the differentiated constraint to get rid of $dr_t$ in the differentiated cost, we get

$$dC(0, r_t) = -\Gamma^{r'}(r_t)dl_t,$$

which is negative for $dl_t > 0$. This shows that $l_t = 0$ cannot be optimal, as the planner could reduce the resource cost by increasing loans.

[6]For example, the real allocation in the calibrated economy of Section 3.2 implies values of $\eta_t$ ranging from -0.03 to 0.05.

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