

A backward-bending labor supply curve without an income effect

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This paper proposes an explanation of the backward-bending labor supply curve that is not based on the premise that the income effect dominates the substitution effect. Unlike the classical labor supply theory that treats working hours and work effort as being synonymous, this paper treats them as distinct variables in an efficiency wage model. A wage rate increase is shown to give rise to two direct substitution effects that motivate the worker to provide more effort and hours. When a greater effort exerts a cross substitution effect that reduces hours, the hour supply curve may bend backward in the absence of an income effect.

1. Introduction

The empirical evidence concerning labor supply indicates that a higher wage may result in a smaller number of working hours.¹ This fact reveals that the labor supply curve may slope downward or bend backward. The standard explanation can be found in textbooks as being that where the income effect dominates the substitution effect. Such an explanation is apparently a straightforward application of the Hicksian income-substitution effect apparatus.²

The backward-bending labor supply curve is not only empirically significant and theoretically interesting, but it also has important policy implications. It is well known that supply-side economics advocates the implementation of a strategy whereby taxes are reduced so as to increase people's incentive to work. The income tax reduction policy of Ronald Reagan's administration is the most famous example. A cut in income taxes is equivalent to an increase in the wage rate and will give rise to both a substitution effect and an income effect. The extent to which the belief of supply-siders regarding the effectiveness of an income tax rate cut is theoretically sound depends on whether the labor supply curve is backward-bending or not. The other well-known example is the shortage of registered nurses,

¹ For a comprehensive review, see Killingsworth (1983). For a brief review, see Ehrenberg and Smith (1991).

² A formal mathematical derivation can be found in Gilbert and Pfouts (1958). Hanoch (1965) finds some particular preference maps that generate a backward-bending curve. Barzel and McDonald (1973) show that the assets held by the individual play an important role in determining the shape of the labor supply curve.

whereby the common proposal is to increase the flow of services from the existing stock of trained registered nurses by increasing the wage rate. The effectiveness of this proposal crucially depends on the belief that the hour supply curve of registered nurses is upward sloping. If it is not, then the effect of such a proposal will turn out to be just the opposite of what was originally intended.³

According to the classical static textbook model of the labor-leisure choice, the labor supply curve in terms of on-the-job hours is upward sloping in the absence of an income effect. However, a number of empirical studies yield contrasting results and suggest that the hour labor supply curve without an income effect may slope downward.⁴ This finding gives rise to a need to find an explanation beyond the classical model.

In the classical labor supply model, the supply of hours and the supply of effort are treated as being synonymous. This synonymous setting is reasonable where work effort is controlled by technology, but in practice, equal-time jobs may have very different levels of effort requirement. Some jobs are hard, but others may be easy. A notable difference between working hours and work effort is that the former is easy to define, measure, and monitor, while the latter is not. In the past, at least until the development of principal-agent models, economists were not good at dealing with the imperfectly observable effort. The apparent distinction between work effort and working hours has thus long been ignored.

This paper proposes a possible explanation of a backward-bending labor supply curve that is not based on the premise that the income effect dominates the substitution effect. In contrast to the synonymous setting, this paper treats working hours and work effort as two distinct variables. By using a modified shirking model of Shapiro and Stiglitz (1984), it is shown that the labor supply curve in terms of working hours may slope downward or bend backward, even if the income effect is absent.

The idea of allowing workers a dual choice of hours and effort to generate a backward-bending hour supply curve without an income effect is not totally new. Dickinson (1999) yielded this result in a piece-rate model where workers are paid based on their effort, and he found support for this in laboratory experiments. An implicit assumption in his model is that work effort is perfectly observable, but this may not fit certain labor environments where effort is not perfectly observable and workers cannot be compensated according to their effort. To help fill this gap, the present research extends Dickinson's (1999) piece-rate model to a shirking model of efficiency wages and focuses on the scenario where effort is incompletely observable and compensation is based on workers' on-the-job hours.

The rest of this paper is organized as follows. The following section presents and analyzes an efficiency wage model. The final section concludes.

³ In fact, the estimates of Link and Settle (1981) suggest that several cohorts are either currently on, or approaching, the backward-bending portions of their respective supply curves.

⁴ See the references cited in Dickinson (1999).

2. The model

This model is closely related to Rasmussen's (1998) efficiency wage model, which is a continuous effort version of Shapiro and Stiglitz's (1984) model. This paper extends the Rasmussen model from an income-effort setting to an income-effort-hour one. Workers are assumed to enjoy income (consumption) and leisure, but dislike putting forth effort. A typical worker's instantaneous utility is

$$U(c, l, e); U_c > 0, U_l < 0, U_e < 0, U_{ll} < 0, U_{ee} < 0 \quad (1)$$

where c is consumption or income, l is the number of paid-for or on-the-job hours, and e is effort. Effort e ($0 \leq e \leq 1$) is defined as the ratio of the number of hours that the worker actually works to the number of on-the-job hours. Accordingly, el is the actual or effective working time during the paid-for hours. Two properties of this setting are worth mentioning. First, this utility function is different from that of the classical labor supply theory, in the sense that this paper treats l and e as two different components, rather than as being synonymous. Second, in contrast to the canonical shirking model of Shapiro and Stiglitz (1984) where effort is a binary variable (zero and one), following Pisauro (1991) and Rasmussen (1998), and others, effort is a continuous variable in this paper.

There is an instantaneous probability ρ that a worker will be caught shirking. In general, $\rho = \rho(e)$ is a decreasing function of the worker's effort (i.e. $\rho' < 0$). This means that the greater the effort is that one provides, the lower the probability will be that one is caught shirking. Without loss of generality, we assume as in Pisauro (1991) that $\rho(e) = 1 - e$, so that $\rho' < 0$ and $\rho'' = 0$. Workers who are caught shirking will be fired and enter the unemployed pool. Before being rehired by other firms, they will receive unemployment benefit (b) from the government. Workers who are not caught shirking will be paid $c = wl$ (w is the hourly wage), that is, the workers' remuneration is based on the number of on-the-job hours. The probability that an unemployed worker will obtain a new job is denoted by ψ .

There are two states of the world that a typical worker may face at any moment: he or she may either be employed or unemployed. Let subscripts E and U represent the status of being employed and unemployed, and let the expected life-time utility of an employed and unemployed worker be denoted by V^E and V^U , respectively. Following Rasmussen (1998), we can obtain the following two asset equations

$$rV^E = U(wl, l, e) + \rho(e)(V^U - V^E) \quad (2)$$

$$rV^U = U(b, 0, 0) + \psi(V^E - V^U) \quad (3)$$

Each of these equations states that the interest rate times the asset value equals flow benefits (dividends) plus expected capital gains (or losses). Since the unemployment benefit is not the focus of our analysis, we assume that $b = 0$ and let $U(0, 0, 0) = 0$. From (2)–(3), we obtain

$$V^E = \frac{(r + \psi)U(wl, l, e)}{r(r + \psi + \rho(e))} \quad (4)$$

A worker chooses l and e so as to maximize (4), and the first-order conditions are

$$wU_c(wl, l, e) + U_l(wl, l, e) = 0 \quad (5)$$

$$(r + \psi + \rho(e))U_e(wl, l, e) - U(wl, l, e)\rho'(e) = 0 \quad (6)$$

The second-order conditions for an interior maximum require that

$$w^2U_{cc} + 2wU_{lc} + U_{ll} < 0 \quad (7)$$

$$D \equiv \phi^2\{(w^2U_{cc} + 2wU_{lc} + U_{ll})U_{ee} - (wU_{ec} + U_{el})^2\} > 0 \quad (8)$$

where $\phi = (r + \psi)/[r(r + \psi + \rho)] > 0$.

For ease of exposition, it is useful to first express the total effect of a wage rate increase on the worker's optimal hours of work (namely l_w) in a Slutsky equation. By letting \bar{U} be the initial utility before an increase in w , the equation is

$$l_w = \frac{\partial l}{\partial w} \Big|_{U=\bar{U}} + \frac{\partial l}{\partial e} \frac{\partial e}{\partial w} \Big|_{U=\bar{U}} + \frac{\partial l}{\partial c} \frac{\partial c}{\partial w} \quad (9)$$

This is equivalent to saying that

Total effect

= direct substitution effect + cross substitution effect + income effect

The total effect of a higher wage rate on the number of paid-for hours can be divided into three effects in this income-effort-hour setting. The direct substitution effect that arises from a higher wage rate motivates the workers to directly substitute on-the-job hours for off-the-job leisure. The cross substitution effect is due to a higher wage rate indirectly affecting the supply of on-the-job hours by influencing effort. The income effect refers to the indirect impact of a higher wage rate on on-the-job hours by raising the income. It is well known that the direct substitution effect is positive and that the sign of the income effect depends upon whether or not off-the-job leisure is an inferior good. The sign of the cross effect is dependent upon the sign of $(\partial l/\partial e)(\partial e/\partial w|_{U=\bar{U}})$, where the direct substitution effect of a higher wage rate on work effort $(\partial e/\partial w|_{U=\bar{U}})$ is positive, but the effect of a greater effort on paid-for hours $(\partial l/\partial e)$ may be negative. Accordingly, regardless of whether the income effect is taken into account, the slope of the hour supply curve (the total effect) may be negative as long as the cross substitution effect (or $\partial l/\partial e$) is negative.

The main focus of this paper is to show that the hour supply curve may be negatively sloping in the absence of an income effect. A straightforward approach to our focus is therefore to exclude the income effect by assuming that the typical worker's preference is characterized by a utility function with a constant marginal utility of income, such as $U(c, l, e) = c + u(l, e)$. This leads to $U_{cc} = U_{cl} = U_{ce} = 0$.

From (5)–(6), the effects of a wage rate increase on the worker's optimal hours and effort (namely, l_w and e_w) are⁵

$$l_w = \frac{-U_{ee}}{U_{ll}U_{ee} - U_{le}U_{el}} + \frac{lU_{el}}{(r + \psi + \rho)(U_{ll}U_{ee} - U_{le}U_{el})} \geq 0 \quad (10)$$

$$e_w = \frac{-lU_{ll}}{(r + \psi + \rho)(U_{ll}U_{ee} - U_{le}U_{el})} + \frac{U_{el}}{U_{ll}U_{ee} - U_{le}U_{el}} \geq 0 \quad (11)$$

By substituting the first term (the direct effect on effort) in (11) into the second term in (10), eq. (10) can be expressed in terms of the Slutsky equation in (9) as

$$l_w = \frac{\partial l}{\partial w} \Big|_{U=\bar{U}} - \frac{U_{le}}{U_{ll}} \frac{\partial e}{\partial w} \Big|_{U=\bar{U}} \quad (9a)$$

where $\partial l / \partial e = -U_{le} / U_{ll}$. Since both direct substitution effects ($\partial l / \partial w|_{U=\bar{U}}$) and ($\partial e / \partial w|_{U=\bar{U}}$) are positive, and $U_{ll} < 0$, the term U_{le} (i.e. the effect of a greater effort on the marginal utility of on-the-job hours) plays a crucial role in determining the sign of l_w .

To highlight the above point, let us investigate a special case $u(l, e)$ is an additively separate function in l and e (i.e. $U_{le} = 0$). That is, a change in e has no impact on the marginal utility of l and vice versa. In this situation, the sign of l_w is positive since it is only generated by the direct or pure substitution effect, such as

$$l_w = \frac{-1}{U_{ll}} > 0 \quad (10a)$$

The result $l_w > 0$ states that a higher wage rate induces a worker to supply more on-the-job hours by raising the opportunity cost of off-the-job leisure. In the absence of the income effect, this result—the higher the wage, the greater the hours of work—is the standard result of the textbook labor supply theory. In other words, when $U_{le} = 0$, the cross substitution effect between effort and hours is zero, and so the slope of the hour supply curve will be positive in the absence of an income effect. This confirms the argument that U_{le} plays a key role in determining whether $l_w < 0$ will hold.⁶

From (10), the necessary and sufficient condition for $l_w < 0$ is

$$U_{el} < \frac{(r + \psi + \rho)U_{ee}}{l} < 0 \quad (10b)$$

That is, if the income effect is ignored, then the case for $l_w < 0$ appears only when $U_{le} < 0$. That is to say, without an income effect, $l_w < 0$ may occur if a greater

⁵Note that $U_{ll}U_{ee} - U_{le}U_{el} > 0$ in (10)–(11). This is because when the utility function is $U(c, l, e) = c + u(l, e)$ (i.e. $U_{cc} = U_{cl} = U_{ce} = 0$), the second-order condition in (8) degenerates to $D \equiv \phi^2[U_{ll}U_{ee} - (U_{el})^2] > 0$. A detailed derivation of the main results may be obtained from the author upon request.

⁶Similarly, from (11), we obtain $e_w = -l / (r + \psi + \rho) U_{ee} > 0$ under $U_{le} = 0$. This indicates that a higher wage rate induces a greater effort by increasing the expected cost of shirking. This result—the higher the wage, the greater the effort—is the basic tenet of efficiency wage models.

effort increases the marginal disutility of spending time at the workplace (on-the-job hours).⁷

Intuitively, when the income effect is absent, a higher wage rate increases the relative attractiveness of spending time at the workplace (paid-for or on-the-job hours) to off-the-job leisure as well as of spending time working (non-shirking) to on-the-job leisure (shirking). This generates two direct substitution effects that motivate the worker to supply more hours and provide greater effort. These are the first terms in (10) and (11), respectively. By increasing the marginal disutility of spending time at the workplace ($U_{le} < 0$), a greater effort in turn generates a negative cross substitution effect and induces the worker to reduce his/her on-the-job hours. This is the second term in (10) or (9a). As a result, when the cross effect is negative and large enough (in terms of absolute value) to dominate the positive direct effect on on-the-job hours, the labor supply curve in terms of on-the-job hours may slope downward, even though the income effect is absent.

It is quite reasonable to argue that the greater the effort supplied is during a given number of paid-for hours, the greater the extent (or probability) is to which a rise in effort will increase the marginal disutility of paid-for hours. Since the direct substitution effect of a higher wage rate on effort is positive ($(\partial e / \partial w)|_{U=\bar{U}} > 0$), the extent (or possibility) to which a rise in the wage rate raises the marginal disutility of paid-for hours will increase with the wage rate. This implies that the higher the wage rate is, the greater will be the negative cross substitution effect in terms of absolute value (the second term in (10) or (9a)), and so the greater the extent (or possibility) to which the negative cross substitution effect may dominate the positive direct substitution effect (the first term in (10) or (9a)). The hour supply curve may therefore tend to slope upward at a lower wage rate, but slope downward as the wage rate increases beyond a critical value, resulting in a backward-bending curve.⁸

After regarding hours of work and work effort as different variables, what would be of interest for policy purposes would be the effect of a wage rate increase on actual or effective working time (el), and not just on the supply of paid-for hours (l). The two policy examples given in the Introduction (the income tax reduction of the Reagan Administration and the proposal to increase the wage rate of registered nurses) have unintended consequences if a downward slope characterizes the

⁷ One example for the case where $U_{le} < 0$ is that the worker's utility is characterized by $U(c, e, l) = c - e^2 P$. Moreover, this condition is the same as that in Dickinson (1999). In his piece-rate model, Dickinson (1999) found that a condition for a negatively-sloping hour supply curve without an income effect is that an increase in on-the-job leisure decreases the marginal utility of off-the-job leisure. This condition is supported by his experimental results.

⁸ Although the foregoing analysis focuses on the possibility that the hour supply curve may slope downward without an income effect, it is also worth noting that the effort supply curve may also slope downward ($e_w < 0$) as well. The economic intuition behind this possibility is similar to that behind the backward-sloping hour supply curve. The necessary and sufficient condition for $e_w < 0$ is $U_{el} < IU_{ll}/(r + \psi + \rho) < 0$.

effective labor supply curve. In fact, it is possible that a wage rate increase may, under certain conditions, cause the worker's optimal paid-for hours to decrease while the optimal effort increases, and then the total output for a given work day may still rise even though less hours are spent at the workplace. Therefore, in considering the policy implications, it would be most interesting here to investigate whether the response of the effective labor supply to a wage rate increase is negative or not.

By using (10)–(11), under $h \equiv el$, the slope of the effective labor supply curve ($h_w \equiv e_w l + e l_w$) is

$$h_w = -\frac{(r + \psi + \rho)eU_{ee} + l^2 U_{ll}}{(r + \psi + \rho)(U_{ll}U_{ee} - U_{le}U_{el})} + \frac{(e + r + \psi + \rho)lU_{el}}{(r + \psi + \rho)(U_{ll}U_{ee} - U_{le}U_{el})} \stackrel{?}{\leq} 0 \quad (12)$$

The first term (total direct effect) is positive, while the sign of the second term (total cross effect) is uncertain. Hence, if the total cross substitution effect is negative and dominates the positive total direct substitution effect, then the slope of the effective labor supply curve may be negative. Moreover, the necessary and sufficient condition for $h_w < 0$ is

$$U_{el} < \frac{(r + \psi + \rho)e}{(e + r + \psi + \rho)l} U_{ee} + \frac{l}{(e + r + \psi + \rho)} U_{ll} < 0 \quad (12a)$$

As a consequence, when a greater effort increases the marginal disutility of spending time at the workplace ($U_{le} < 0$), the effective labor supply curve may slope downward.

3. Concluding remarks

The backward-bending labor supply curve is not only empirically significant and theoretically interesting, but it also has important policy implications. The widely-accepted explanation is that the income effect dominates the substitution effect. However, a number of empirical studies give rise to contrasting results and suggest that the hour supply curve without an income effect may actually slope downward. This paper proposes an efficiency wage model where working hours and work effort are treated as distinct variables in order to provide an explanation.

The model in this paper is admittedly rudimentary. In particular, it does not take into account the firm's behavior. Whether or not this shirking model can be supported by empirical evidence or laboratory experimental results is also ignored. Nevertheless, it is hoped that the model presented here may well serve as a useful alternative in which the reason for the backward-bending labor supply curve could be potentially explained.

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