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Can consumption habit spillovers be a source of equilibrium indeterminacy?

Been-Lon Chen · Yu-Shan Hsu · Kazuo Mino

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Abstract This paper investigates whether the external consumption habit can be a source of indeterminacy in a one-sector growth model when the labor supply is elastic. When there is a proper habit effect with a positive intertemporal elasticity of substitution, we find that the model exhibits indeterminacy if the coefficient of the habit formation is sufficiently large that allows for a substantial impact of current consumption on the habit. Indeterminacy arises even though the elasticity of the Frisch labor supply is positive and the elasticity of the labor demand in negative. In a calibrated version, we find that indeterminacy is empirically plausible when the habit effect is negative that features the "catching up with the Joneses" effect. Under given "catching up with the Joneses" effects, the external consumption habit can be a source of indeterminacy even if more than a half of the external consumption habit comes from past average consumption.

Keywords Habit · Catching up with the Joneses · Indeterminacy · Growth model

JEL Classification E21 · E32

B.-L. Chen (✉)
The Institute of Economics, Academia Sinica, 128 Academia Road Section 2, Taipei 11529, Taiwan
e-mail: bchen@econ.sinica.edu.tw

Y.-S. Hsu
Department of Economics, National Chung Cheng University, Chiayi City, Taiwan
e-mail: ecdysh@ccu.edu.tw

K. Mino
Kyoto Institute of Economic Research, Kyoto University, Kyoto, Japan
e-mail: mino@kier.kyoto-u.ac.jp

1 Introduction

This paper asks whether the external consumption habit can be a source of indeterminacy in a one-sector growth model with an elastic labor supply. Several authors have studied the multiplicity of the equilibrium path of the one-sector growth model with an elastic labor supply when production externalities are introduced. In particular, Benhabib and Farmer (1994) and Farmer and Guo (1994) showed that the equilibrium of the model with separable instantaneous utility may exhibit indeterminacy when production externalities are such that returns to labor are so large so that the labor demand is upward slopping and steeper than the labor supply curve. Those required increasing social returns were too high as compared with empirical estimates by Basu and Fernald (1997). Bennett and Farmer (2000) showed that if preferences were non-separable in consumption and leisure, indeterminacy arises even when increasing social returns are within estimates so that the elasticity of the labor demand is larger than the elasticity of the Frisch labor supply wherein the Frisch labor supply was downward sloping.[1] However, Hintermaier (2003) demonstrated that the conditions for indeterminacy in Bennett and Farmer (2000) implied a non-concave utility and indeterminacy cannot arise if the utility is concave.

In order to avoid the labor supply curve to cross the labor demand curve with abnormal slopes in one-sector growth models that exhibits indeterminacy, a number of papers have considered consumption externalities instead of production externalities. In one-sector growth models with endogenous time preference rates when the labor supply inelastic, Drugeon (1998) and Chen and Hsu (2007) discovered that positive consumption externalities can be a source of indeterminacy. However, due to an inelastic labor supply, Chen et al. (2010) found that the external consumption habit is not a source of indeterminacy when a negligible fraction of the habit is formed by past consumption.[2] In another paper, Alonso-Carrera et al. (2008) found that both positive and negative consumption externalities can be a source of indeterminacy in a one-sector growth model with an elastic labor supply. In particular, even though the utility in Alonso-Carrera et al. (2008) is concave, the model exhibits indeterminacy when the labor demand curve crosses the labor supply curve with normal slopes wherein the elasticity of the labor supply curve may be either positive, or negative but in this situation the labor supply is steeper than the labor demand.

In this paper, we extend the Alonso-Carrera et al. (2008) model and ask whether the introduction of external consumption habits can cause indeterminacy in a one-sector growth model when the labor supply is elastic. Different from the Alonso-Carrera et al. (2008) model, as the external consumption habit is formed partly by past external consumption, our model has an additional predetermined variable.

[1] The Frisch labor supply is the labor supply when the marginal utility of consumption is constant.

[2] Consumption externalities affect the time preference in Drugeon (1998) and the utility in Chen and Hsu (2007). Chen et al. (2010) extended these models and considered the external consumption habit.

The coefficient of the habit formation characterizes the strength of the influence that current external consumption affects the habit. When the coefficient of the habit formation is infinite, the external consumption habit is current external consumption and our model is the Alonso-Carrera et al. (2008) model that can exhibit indeterminacy. When the coefficient of the habit formation is finite so the external consumption habit is different from current external consumption in transitions, because of an elastic labor supply, we find that the equilibrium path is indeterminate if the coefficient of the habit formation is sufficiently large and if there is a proper habit effect with a positive *intertemporal elasticity of substitution* (henceforth, IES). A sufficiently large coefficient of the habit formation allows for current external consumption to exert a sizable impact on the external consumption habit. A positive IES allows for the tradeoff between consumption and savings. A proper habit effect assures a proper social complementarity in consumption which generates self-reinforcing mechanism connecting individual and joint choices so expectations-driven equilibrium paths may arise. In a calibrated version, we find that indeterminacy is empirically plausible when the habit effect is negative that features the "catching up with the Joneses" (henceforth, CUJ) effect. Given the CUJ effect, we find that the model exhibits indeterminacy even if more than a half of the external consumption habit comes from past external consumption.

The elastic labor supply plays an important role in order for the external consumption habit to be a source of indeterminacy. In our model, the elasticity of the Frisch labor supply depends on the coefficient of the habit formation. When the coefficient of the habit formation is zero, the external consumption habit is unchanged and our model is the standard one-sector growth model. In this case, the elasticity of the Frisch labor supply may be zero or positive and the steady state is a saddle and thus the equilibrium is determinate. When the coefficient of the habit formation is positive, the external consumption habit changes over time and the elasticity of the Frisch labor supply increases in the coefficient. When the coefficient of the habit formation increases to a finite threshold, a sizable fraction of the external consumption habit comes from current external consumption so the elasticity of the Frisch labor supply is sufficiently large. Then, as the amount of savings increases, the labor supply can increase substantially such that returns to savings are increasing in savings and thus the expectations-driven equilibrium can be self-fulfilling. Finally, if the coefficient of the habit formation increases further to the infinity, then the elasticity of the Frisch labor supply may be positive, or may be negative but the labor supply is steeper than the labor demand. This is the Alonso-Carrera et al. (2008) model that can exhibit indeterminacy. In contrast, when the coefficient of the habit persistence is zero, our model reduces to the standard real business cycle model. In this sense, our study includes the existing neoclassical growth model with or without consumption external effects as special cases.

Our model is the first one that finds the habit effect alone can create indeterminacy in a neoclassical growth model. In non-growth, dynamic models, if there is a cash-in-advance (henceforth, CIA) constraint, Auray et al. (2002) found that the external consumption habit brings about chaotic equilibrium paths and Auray et al. (2005) uncovered that the internal consumption habit causes indeterminate equilibrium paths. First, the two existing papers are not growth models. Moreover, our model has no distortion except for the external consumption habit. In contrast, these two existing papers relied on the interplay of the habit effect and the CIA constraint, and it has been shown that indeterminacy may arise under the CIA constraint with a given exogenous growth rate of money supply.[3]

Recent years have seen voluminous research in economics and finance that considered the external consumption habit.[4] A number of authors have analyzed the macroeconomic effects when people's utility is affected by external consumption habits. It was shown that the introduction of external consumption habits affects the process of economic growth (Carroll et al. 1997; Alvarez-Cuadrado et al. 2004; Doi and Mino 2008) and the efficiency of the competitive equilibrium (Alonso-Carrera et al. 2008). In finance, it was found that the introduction of external consumption habits in representative-agent models helps to resolve the equity premium puzzle (Abel 1990; Campbell and Cochrane 1999). Even with external consumption habits, these papers found that the equilibrium path toward a steady state is unique and thus determinate.[5]

As developed below, Sect. 2 sets up the model and analyzes the steady state. Section 3 investigates the conditions of endogenous investment fluctuations and offers quantitative analysis. Finally, concluding remarks are made in Sect. 4.

2 The model

Time is continuous. The basic model is an otherwise standard optimal growth model with leisure wherein we consider the consumption habit effect. The economy is populated by representative households with infinite lives and the population of households is fixed with a unit measure. The representative agent is endowed with one unit of time. At each point in time t the agent allocates to the market a fraction l_t as labor services and the remainder $1 - l_t$ as leisure. An agent obtains utility from his or her own consumption (c_t) and leisurely activities. Moreover, an agent's utility is affected by the

[3] Woodford (1994) found that in a monetary economy with a CIA constraint, the equilibrium path is indeterminate under a given exogenous growth rate of the money supply but is unique under a pegged nominal interest rate.

[4] The concept of the consumption habit may be traced to Hume (1748) who argued that preferences were influenced not simply by what a person did in the past, what his parents did, and what contemporary peers were doing but also by the behavior of past generations of peers. Similar contemporary ideas dated to Marshall (1898), Duesenberry (1949), Leibenstein (1950), and Hicks (1965). Subsequent research has identified two kinds of habit formation. One is referred to as external habit formation, expressed in terms of the past consumption of some outside reference group, usually the past consumption of the overall economy, and is the focus in the current study. The other is termed internal habit formation based upon an individual's own past consumption level.

[5] Recently, Wirl (2011) has analyzed general conditions for indeterminacy and multiple steady states in a model with an external stock in payoffs wherein the dynamical system includes a control and a stock. There are similarities between Wirl and our paper. First, both papers consider externalities. Moreover, externalities are both generated by stock variables. Finally, both papers derive the conditions that lead to indeterminacy. Our model has a negative externality and is a three-equation dynamical system and is different from a two-equation system in Wirl (2011) which considers a positive externality. Like our paper, the model by Antoci et al. (2009) has negative externalities and is a three-equation dynamical system. Unlike our paper, the negative externalities in the latter paper are generated by aggregate output which generates pollutions and reduces natural resources. Thus, their negative externalities affect the supply side, whereas our negative externalities affect the demand side. Moreover, Antoci et al. (2009) focus on global indeterminacy as opposed to local indeterminacy in our paper.

consumption habit in the society, H_t.[6] The lifetime utility is

$$\int_0^{\infty} e^{-\rho t} u(c_t, H_t, 1 - l_t) dt, \tag{1}$$

where u is the level of instantaneous utility and $\rho > 0$ is the instantaneous discount rate.

We assume that the instantaneous utility is twice continuously differentiable with the following properties: (i) $u_i > 0 > u_{ii}$, $i = 1, 3$, (ii) $u_{11} + u_{12} < 0$, (iii) $u_{11}u_{33} - (u_{13})^2 \geq 0$ and (iv) $\lim_{c \to 0} u_1 = \lim_{l \to 1} u_3 = \infty$, $\lim_{c \to \infty} u_1 = \lim_{l \to 0} u_3 = 0$. In (i), the utility displays positive and decreasing marginal utilities of consumption and leisure. The assumption (ii) guarantees a positive social IES. The assumption (iii) ensures a jointly concave utility in own consumption and leisure, while interior solutions are assured by the assumption (iv). We also assume that consumption and leisure are both normal goods. While we do not impose the sign of u_2 as the habit effect may be positive or negative, it is worth noting that if $u_{12} > 0$, then past habit in the society enhances an agent's marginal utility of consumption and there is thus the CUJ effect (Abel 1990).

The external consumption habit is a stock at time t. Following Ryder and Heal (1973), habit is accumulated from the distant past to the present and is a weighted average of past consumption flows in the economy, with weights declining exponentially in the distant past. Specifically, habit is

$$H_t = \beta e^{-\beta t} \int_{-\infty}^{t} e^{\beta \tau} C_\tau d\tau, \quad \beta \geq 0,$$

where C_t is average consumption in the society in t.[7] The above expression may be rewritten as follows.

$$\dot{H} = \beta(C_t - H_t), \quad \beta \geq 0, \quad \text{with } H_0 \text{ given.} \tag{2}$$

[6] Evidence of the external habit effect has been prevalent and was confirmed as early as the 1950s by Brown (1952) who estimated the habit effect by using the aggregate data in Canada. Recently, a growing body of empirical evidence concerning external habit persistence has emerged. Using time-series data in the US, Fuhrer (2000) strongly supported the hypothesis of consumption habit formation. More recently, using panel data in the US, Ravina (2005) and Korniotis (2010) both have provided strong evidence about external habit persistence in household consumption choices. Using data from other countries, supportive evidence of external habits has been offered by, among others, van de Stadt et al. (1985) who used longitudinal panel surveys of households in the Netherlands, Guariglia and Rossi (2002) who used the British Household Panel Survey, Case (1991) who used an Indonesian socio-economic survey, and Carrasco et al. (2005) who used household panel data from Spain.

[7] The formulation is different from that in Auray et al. (2002); Auray et al. (2005) which assumed $H_t = C_{t-1}$ and thus their habit is determined by the society's consumption last period. The Ryder and Heal's formulation is more general. Constantinides (1990) used the same habit formation regime as ours except his habit is internal.

This law of motion says that the society's future habit is increased by the difference between current average consumption and existing habit adjusted by a coefficient, β. The coefficient of habit formation characterizes the strength of the influence that current average consumption affects the habit. It is clear that the larger the value of β, the larger the influence of current average consumption in the formation of the habit. Two extreme cases are as follows. If $\beta = 0$, H_t is fixed and is given by H_0 for all t. In this case, our model is reduced to the standard one-sector optimal growth model with leisure. Conversely, if $\beta = \infty$, then the habit adjusts so fast such that the habit in the society is completely determined by current average consumption; namely, $H_t = C_t$. In this latter case, our model is a one-sector growth models with consumption externalities that is like those studied by Liu and Turnovsky (2005) and Alonso-Carrera et al. (2008).

The economy has a continuum of firms with a unit measure. A firm is endowed with a neoclassical production technology $f(k_t, l_t)$ where k_t is per capita capital stock and the marginal product of each input is positive and is decreasing in input. Firms are competitive and are thus price takers. Since the Cobb–Douglas technology is used in the indeterminacy literature following Benhabib and Farmer (1994) and Farmer and Guo (1994), we use it here given by

$$f(k_t, l_t) = Ak_t^{\alpha} l_t^{1-\alpha}, \quad 0 < \alpha < 1.$$

Notice that unlike Benhabib and Farmer (1994) and Farmer and Guo (1994), we do not assume the presence of production externalities.

The optimization problem in a decentralized economy is as follows. First, given wage rates, w_t, and rental rates of capital, r_t, a representative firm at each point in time $t$chooses optimal demands for capital and labor in order to maximize its profits. Denote δ as the depreciation rate of capital. The optimal conditions are as follows.

$$w_t = (1 - \alpha)Ak_t^{\alpha} l_t^{-\alpha}, \tag{3a}$$
$$r_t = \alpha Ak_t^{\alpha-1} l_t^{1-\alpha} - \delta. \tag{3b}$$

Next, taking wage rates and rental rates as given by the market and the habit as given by the society, the representative household's problem is to tradeoff between consumption and savings and tradeoff between working and leisure in order to maximize her lifetime utility (1), subject to the following budget constraint

$$\dot{k}_t = w_t l_t + r_t k_t - c_t. \tag{4}$$

The optimal conditions are

$$u_1(c_t, H_t, 1 - l_t) = \lambda_t, \tag{5a}$$
$$u_3(c_t, H_t, 1 - l_t) = w_t \lambda_t, \tag{5b}$$
$$\dot{\lambda}_t = (\rho - r_t)\lambda_t, \tag{5c}$$

along with $\lim_{t\to\infty} e^{-\rho t}\lambda_t k_t = 0$, which is the transversality condition. The variable λ_t is the co-state variable associated with capital and thus, the shadow price of capital. In these optimal conditions, (5a) and (5b) equates the marginal utility to the marginal cost for consumption and leisure, respectively, and (5c) is the Euler equation for capital.

For given k_0 and H_0, competitive equilibrium is a path $\{k, l, H, w, r, \lambda\}$ with $c = C$, and is determined by (2), (3a)–(3b), (4) and (5a)–(5c).

To determine the competitive equilibrium, first, we simplify equilibrium conditions by use of (3a), (5a) and (5b) and obtain

$$(1-\alpha)Ak_t^{\alpha}l_t^{-\alpha} = \frac{u_3(C_t, H_t, 1-l_t)}{u_1(C_t, H_t, 1-l_t)}, \tag{6a}$$

which equates the marginal product of labor to the marginal rate of substitution between consumption and leisure.

Next, (3b) and (5c) lead to

$$\dot{\lambda}_t = (\rho + \delta - \alpha A k_t^{\alpha-1} l_t^{1-\alpha})\lambda_t. \tag{6b}$$

Finally, we use (3a) and (3b) to rewrite the budget constraint (4) as a resource constraint.

$$\dot{k}_t = Ak_t^{\alpha}l_t^{1-\alpha} - \delta k_t - C_t. \tag{6c}$$

Therefore, equilibrium conditions are simplified to (2), (6a), (6b) and (6c). In a steady state, $\dot{k} = \dot{\lambda} = \dot{H} = 0$ and thus (2) indicates $H^* = C^*$. Then, (6a), (6b) and (6c) determine C^*, l^* and k^* in the same way as does in an otherwise standard growth model with leisure.

To analyze the existence of a steady state, (6b) gives

$$k^* = \left(\frac{\rho+\delta}{\alpha A}\right)^{\frac{-1}{1-\alpha}} l^*. \tag{7a}$$

From (7a), (6a) and (6c) respectively yield,

$$(1-\alpha)A\left(\frac{\rho+\delta}{\alpha A}\right)^{\frac{-\alpha}{1-\alpha}} = \frac{u_3(C^*, C^*, 1-l^*)}{u_1(C^*, C^*, 1-l^*)}, \tag{7b}$$

$$C^* = \left(\frac{\rho+\delta}{\alpha A}\right)^{\frac{-1}{1-\alpha}} \left[\frac{\rho}{\alpha} + \left(\frac{1}{\alpha} - 1\right)\delta\right] l^*. \tag{7c}$$

Thus, (7c) implies that C^* is linear in l^*, denoted by $C^* = C(l^*)$, where $C'(l^*) > 0$ and $C(0) = 0$. Substituting this relationship into (7b) gives

$$A(1-\alpha)\left(\frac{\rho+\delta}{\alpha A}\right)^{\frac{-\alpha}{1-\alpha}} u_1(C(l^*), C(l^*), 1-l^*) = u_3(C(l^*), C(l^*), 1-l^*). \tag{7d}$$

We can use (7d) to determine l^* in a steady state. For simplicity, we referred to the left-hand side and the right-hand side of (7d) as LHS and RHS, respectively. The result $C(0) = 0$ and Assumption (iv) $\lim_{c\to 0} u_1 = \infty$ and $\lim_{l\to 0} u_3 = 0$ indicate that LHS $= \infty >$ RHS $= 0$ as $l \to 0$. Moreover, $C(1)$ is a constant. Thus, Assumption (iv) $\lim_{l\to 1} u_3 = \infty$ suggests LHS<RHS $= \infty$ as $l \to 1$. Hence, LHS and RHS intersect at least once, in which case we can conclude that there exists at least one interior steady state $l^* \in (0, 1)$.

Proposition 1 *In a one-sector growth model with the external consumption habit, there exists a steady state.*

Although there exists at least a steady state, we cannot rule out the possibility of multiple steady states as LHS and RHS of (7d) are so non-linear that may intersect each other more than twice. The reason for multiple steady states has been well-known since Cooper and John (1988) in that spillovers (here, the external habit effect) create strategic complementarities in payoffs and coordination failures among agents which may give rise to multiple steady states.[8] When we use the constant elasticity of substitution (CES) utility in Sect. 3.3, we will find that the steady state may be unique or multiple depending on the magnitude of external effects relative to the elasticity of substitution.

3 Endogenous investment fluctuations

In this section, we study indeterminacy by investigating the stability property of the model.

3.1 The conditions of indeterminacy

In this subsection, we will show that, if the coefficient of the habit formation is sufficiently large and there is a proper habit effect with a positive IES, then equilibrium paths toward a steady state are indeterminate. Thus, there are endogenous investment fluctuations.[9]

We simplify the dynamic system into three variables. First, by the implicit function theorem, (6a) leads to the following relationship

$$l_t = l(C_t, k_t, H_t). \tag{8}$$

Differentiating (5a) with respect to time, with the use of (3b), (5c) and (8), yields the Keynes-Ramsey condition as follows.

[8] Readers are referred to the paper by Chen (2007) which found multiple balanced growth paths in a one-sector endogenous growth model with a negative consumption habit effect.

[9] When equilibrium is indeterminate, the equilibrium system cannot pin down the location of initial control variables (consumption) for given initial state variables (capital and habits). Agents' expectations about other's behavior affect the location of initial control variables. Since agents' expectations are like animal's spirits which can fluctuate a lot, the location of initial control variables fluctuates a lot. Thus, there are endogenous fluctuations in the economy. For a better account, see survey by Benhabib and Farmer (1999).

$$\dot{C}_t = \frac{C_t}{-\Omega}\left\{(\alpha+\varepsilon-\eta)\left(\alpha A k_t^{\alpha-1} l_t^{1-\alpha} - \delta - \rho\right) - \frac{\alpha\varepsilon}{k_t}\dot{k}_t - \frac{1}{H_t}[(\alpha+\varepsilon-\eta)\varsigma + \varepsilon\chi]\dot{H}_t\right\}, \tag{9}$$

where

$$\begin{aligned}
&\Omega \equiv (\eta-\alpha)\sigma + \varepsilon\phi < 0,\\
&\eta \equiv u_{33} l(c_t, k_t, H_t)/u_3 < 0,\\
&\sigma \equiv -u_{11} c_t/u_1 > 0,\\
&\varepsilon \equiv u_{13} l(c_t, k_t, H_t)/u_1,\\
&\phi \equiv u_{13} c_t/u_3,\\
&\varsigma \equiv -u_{12} H_t/u_1,\\
&\chi \equiv (u_{12}/u_1 - u_{23}/u_3)H_t.
\end{aligned}$$

As $u_{33} < 0$ and $u_{11} < 0$, it is obvious that $\eta < 0$ and $\sigma > 0$. Moreover, it is worth noting that if the preference exhibits the CUJ effect, then it is more likely $\chi > 0$. Moreover, under the joint concavity condition in c and l, $\varepsilon\phi/\sigma + \eta = [u_{11}u_{33} - (u_{13})^2]l/(u_{11}u_3) \leq 0$, and as $\sigma > 0$, thus $\eta\sigma + \varepsilon\phi \leq 0$,which indicates $\Omega < 0$. It is required that the IES of consumption is positive: $(\alpha+\varepsilon-\eta)/(-\Omega) > 0$. This implies $(\alpha + \varepsilon - \eta) > 0$. Moreover, the assumption of consumption and leisure both being normal goods gives

$$(\varepsilon - \eta)(\phi + \sigma) > 0. \tag{10a}$$

One crucial feature for understanding indeterminacy is the Frisch labor supply, which is obtained when the marginal utility of consumption is held constant. In our model, the elasticity of the Frisch labor supply is

$$\zeta(\beta) \equiv \frac{dw/w}{dl/l} = \left\{\frac{\varepsilon[\phi - (\varsigma+\chi)\frac{C}{H}x(\beta)]}{-\sigma - \varsigma\frac{C}{H}x(\beta)} - \eta\right\}, \tag{10b}$$

where $x(\beta)$ is the effect of the change in C_t on H_t in a small time interval $\Delta t > 0$. In Appendix we have shown that $x(\beta) = (1 - e^{\beta\Delta t})$ and thus $x(\beta) \in [0, 1]$ is increasing in β. First, as $\beta \to \infty$, $x(\beta) = 1$ and $\zeta(\beta) = \varepsilon[\phi - (\varsigma+\chi)]/(-\sigma-\varsigma) - \eta$. The elasticity of the Frisch labor supply may be positive or negative. Next, as β decreases from the infinity, the elasticity of the Frisch labor supply may be negative or positive when β is very large. Finally, when $\beta \to 0$, $x(\beta) = 0$ and $\zeta(\beta) = (\varepsilon\phi + \eta\sigma)/(-\sigma) \geq 0$, and then the elasticity of the Frisch labor supply is positive or zero. We will find that the equilibrium is indeterminate when there is a sufficiently large value of β. Thus, our model exhibits indeterminacy under $\zeta(\beta) > 0 > -\alpha$ when the elasticity of the Frisch labor supply is positive and the labor demand is unambiguously negative which is $-\alpha$.

The dynamic equilibrium system consists of (2), (6c) and (9) and determines the dynamic path of c_t, k_t and H_t. If we take Taylor's linear expansion of the dynamic equilibrium system in the neighborhood of the steady state, along with the use of (8), we obtain a Jacobean matrix, denoted as J. The characteristic polynomial of the Jacobean matrix is

$$G(\omega) = -\omega^3 + Tr(J)\omega^2 - Ds(J)\omega + Det(J) = 0, \tag{11}$$

where $Det(J)$ is the determinant, $Tr(J)$ is the trace, $Ds(J)$ is the sum of the determinant of the second-order principal minors, of the Jacobean matrix J,given, respectively, by

$$Det(J) = \beta N, \tag{12a}$$
$$Tr(J) = T + \beta\Gamma_1, \tag{12b}$$
$$Ds(J) = M + \beta\Gamma_2, \tag{12c}$$

where

$$N = \frac{(1-\alpha)(\rho+\delta)}{(-\Omega)}\frac{\rho+\delta(1-\alpha)}{\alpha}[(\phi+\sigma-\eta+\varepsilon)-\chi],$$
$$\Gamma_1 = \frac{1}{(-\Omega)}\{-(\varsigma+\sigma)(\alpha+\xi)-[1-x(\beta)](\varepsilon\varsigma+\varepsilon\chi-\eta\varsigma-\xi\varsigma)\},$$
$$\Gamma_2 = \rho\Gamma_1 + \frac{1}{(-\Omega)}\{(1-\alpha)(\rho+\delta)(\varsigma+\chi-\phi)+[\rho+\delta(1-\alpha)]\varepsilon\},$$
$$M = \frac{(1-\alpha)(\rho+\delta)[\rho+\delta(1-\alpha)]}{\Omega\alpha}(\phi+\sigma-\eta+\varepsilon) < 0,$$
$$T = \frac{1}{(-\Omega)}\{\alpha\rho(\phi+\sigma)-[\rho+\delta(1-\alpha)](\eta-\varepsilon)-\rho(\eta\sigma+\varepsilon\phi)\} > 0.$$

where M is the determinant in the standard growth model (in the case of $\beta = 0$) and thus $M < 0$. $M < 0$ and (10a) together indicate $(\phi + \sigma) > 0$ and $(\eta - \varepsilon) < 0$. Moreover, as $(\eta\sigma + \varepsilon\phi) \leq 0$, it follows that $T > 0$.

As the economic system includes two state variables with initial values determined at k_0 and H_0, a steady state is a sink if the number of eigenvalues with negative real parts is three. In this situation, there are multiple equilibrium paths towards the steady state and indeed there is a continuum of equilibrium paths towards the steady state. As we do not know the specific equilibrium path along which the economy is chosen to move toward the steady state, the equilibrium path is called indeterminate.

Examining the polynomial function $G(\omega)$, it is clear that $G(\omega) = -\infty$ when $\omega = \infty$ and $G(\omega) = \infty$ when $\omega = -\infty$. In view of the Routh-Hurwitz stability criterion, the necessary conditions for the presence of three stable roots are: (i) $G(0) = Det(J) < 0$, (ii) $G'(0) = -Ds(J) < 0$, (iii) $Tr(J) < 0$ and (iv) $-Ds(J) + Det(J)/Tr(J) < 0$.

Let $\beta_1 > \beta_2$ be two finite coefficients of the habit formation that satisfy $\beta_1\beta_2 = MT/(\Gamma_1\Gamma_2) > 0$ and $\beta_1 + \beta_2 = \{\Gamma_1 M + \Gamma_2 T - N\}/(-\Gamma_1\Gamma_2) > 0$. We can obtain the following result.

Theorem 1 *In a one-sector growth model with external consumption habits, the steady state is a sink if* (i) $\beta > \beta_1$ *and* (ii) $\chi > \phi + \sigma - \eta + \varepsilon, \Gamma_1 < 0$ *and* $\Gamma_2 > 0$.

Proof See Appendix.

It is worth noting that the condition (i) $\beta > \beta_1$ of a sufficiently large coefficient of the habit formation makes our model different from the standard optimal growth model which emerges under the case of $\beta = 0$. In the case of $\beta = 0$, there is no indeterminacy. A sufficiently large β indicates that a large fraction of the habit comes from current average consumption so current external consumption has a substantial impact on the external consumption habit. In the special case when β goes to ∞, the habit only comes from current average consumption and our model is the Alonso-Carrera et al. (2008) model in which case current average consumption can be a source of indeterminacy.

In the condition (ii), $\Gamma_1 < 0$ requires $(\alpha + \varepsilon - \eta)/(-\Omega) > 0$ which calls for a positive IES. Moreover, as the external habit effect appears in χ and ς, the conditions $\chi > \phi + \sigma - \eta + \varepsilon$ and $\Gamma_2 > 0$ require the external habit effect to be in a proper range.

Theorem 1 thus stipulates that with a positive IES and a proper habit effect, the steady state is a sink if there is a sufficiently large coefficient of the habit formation. A sufficiently large coefficient of the habit formation allows for current external consumption to exert a substantial impact on the external consumption habit. A positive IES allows for the tradeoff between consumption and savings. A proper degree of the external habit effect assures a social complementarity in consumption which generates self-reinforcing mechanism connecting individual and joint choices so expectations-driven equilibrium paths may arise.

3.2 Labor supply and indeterminacy

In Theorem 1, indeterminacy requires a sufficiently large coefficient of the habit formation. Moreover, the coefficient of the habit formation affects the elasticity of the Frisch labor supply. This sub-section analyzes the relationship between the labor supply and indeterminacy. Using (5a) and (5b), the Frisch labor supply is

$$w(l, \bar{u}_1) = \frac{u_3(C(l, \bar{u}_1), H(l, \bar{u}_1), 1 - l)}{\bar{u}_1(C(l, \bar{u}_1), H(l, \bar{u}_1), 1 - l)}.$$

If we differentiate the Frisch labor supply with respect to l and $\bar{u}$, we obtain the elasticity of the Frisch labor supply with respect to the marginal utility of private consumption.

$$\left(\frac{\partial l}{\partial \bar{u}_1}\right)\left(\frac{\bar{u}_1}{l}\right) = -\frac{[u_{31} + u_{32}x(\beta)]\frac{\partial C}{\partial \bar{u}_1} - \frac{u_3}{\bar{u}_1}}{[u_{31} + u_{32}x(\beta)]\frac{\partial C}{\partial l} - u_{33}}\left(\frac{\bar{u}_1}{l}\right), \tag{13a}$$

where, by using $u_1(C, H, 1 - l) = \bar{u}_1$, $\frac{\partial C}{\partial \bar{u}_1} = \frac{1}{u_{11}+u_{12}x(\beta)}$ and $\frac{\partial C}{\partial l} = \frac{u_{13}}{u_{11}+u_{12}x(\beta)}$.

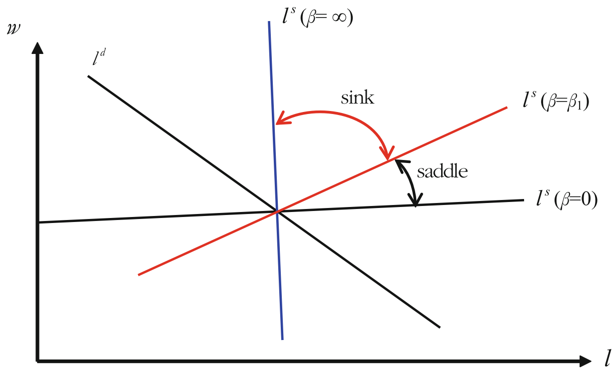

Fig. 1 Labor supply and the determinacy of the equilibrium path

Using the definitions of ϕ, σ, χ, ε, η and ς, (13a) is rewritten as

$$\left(\frac{\partial l}{\partial \bar{u}_1}\right)\left(\frac{\bar{u}_1}{l}\right) = -\frac{\phi + \sigma - \chi\frac{C}{H}x(\beta)}{\varepsilon\phi + \eta\sigma - (\varepsilon\varsigma + \varepsilon\chi - \eta\varsigma)\frac{C}{H}x(\beta)} = \frac{\phi + \sigma - \chi\frac{C}{H}x(\beta)}{\left[\sigma + \varsigma\frac{C}{H}x(\beta)\right]\zeta}. \tag{13b}$$

First, when $\beta = \infty$, $\kappa(\infty) = 1$ and $C = H$ for all t; thus the elasticity of the Frisch labor supply is $\zeta(\infty) = \frac{\varepsilon[\phi-(\varsigma+\chi)]}{-\sigma-\varsigma} - \eta$ which may be positive or negative. See $l^s(\beta = \infty)$ in Fig. 1. Then, (13b) is

$$\left(\frac{\partial l}{\partial \bar{u}_1}\right)\left(\frac{\bar{u}_1}{l}\right) = \frac{\phi + \sigma - \chi}{\sigma + \varsigma}\frac{1}{\zeta(\infty)}, \tag{14a}$$

whose sign is opposite to that of $\zeta(\infty)$ as $\frac{\phi+\sigma-\chi}{\sigma+\varsigma} < 0$.[10] This case is reduced to that of Alonso-Carrera et al. (2008) model and equilibrium indeterminacy emerges.

Next, if $\beta < \infty$, H is different from C in transitions and $x(\beta)$ is decreasing β. In this case, the elasticity of the Frisch labor supply decreases to $\zeta(\beta) = \frac{\varepsilon[\phi-(\varsigma+\chi)\frac{C}{H}x(\beta)]}{-\sigma-\varsigma\frac{C}{H}x(\beta)} - \eta$ and the labor supply curve rotates clockwise from $l^s(\beta = \infty)$ in Fig. 1. In this case, when β is very large, $\zeta(\beta)$ may be negative or positive. But, if β is smaller, $\zeta(\beta)$ is positive like $l^s(\beta = \beta_1)$ in Fig. 1. Then, (13b) is

$$\left(\frac{\partial l}{\partial \bar{u}_1}\right)\left(\frac{\bar{u}_1}{l}\right) = \frac{\phi + \sigma - \chi\frac{C}{H}x(\beta)}{\sigma + \varsigma\frac{C}{H}x(\beta)}\frac{1}{\zeta(\beta)}. \tag{14b}$$

Notice that the term $\frac{\phi+\sigma-\chi\frac{C}{H}x(\beta)}{\sigma+\varsigma\frac{C}{H}x(\beta)}$ in (14b) is still negative but its absolute value decreases as $x(\beta)$ decreases in β.

[10] While $\sigma + \varsigma > 0$, the condition (ii) in Theorem 1 is $\phi_+\sigma - \chi < \eta - \varepsilon < 0$, where the sign of $\eta - \varepsilon < 0$ follows from (10a).

In the case when the elasticity of the Frisch labor supply is negative ($\zeta(\beta) < 0$) which may emerge when β is very large, then the elasticity of the Frisch labor supply with respect to the marginal utility of private consumption in (14b) is positive. In this case, suppose that the agent who expects that other people increase savings now also increases savings. The resulting higher future consumption will lower the marginal utility of future private consumption. Given the positive elasticity of the Frisch labor supply with respect to the marginal utility of private consumption in (14b), the future labor supply curve shifts downward. As the elasticity of the Frisch labor supply is larger than the elasticity of the labor demand, a downward shift of the labor supply reduces the wage rate which in turn increases the employment, thereby increasing the marginal product of capital and thus returns to capital in equilibrium. As a result, the expectations of higher savings can be self-fulfilled and there is an expectations-driven equilibrium.

Alternatively, when the elasticity of the Frisch labor supply is positive, $\zeta(\beta) > 0$ and then the elasticity of the Frisch labor supply with respect to the marginal utility of private consumption in (14b) is negative. Under this condition, when the agent increases savings, higher future consumption decreases the marginal utility of future private consumption so the labor supply in the future increases. The marginal product of capital and thus returns to capital will be higher. As a result, higher savings brings about higher returns to capital and there is an expectations-driven equilibrium.

Finally, when β is decreased and is below the threshold β_1, then the elasticity of the Frisch labor supply is positive, $\zeta(\beta) > 0$ and the elasticity of the Frisch labor supply with respect to the marginal utility of private consumption in (14b) is also positive. To illustrate this case, let β decrease to $\beta = 0$. Then, $x(0) = 0$ and the elasticity of the Frisch labor supply is $\zeta(0) = (\varepsilon\phi + \eta\sigma)/(-\sigma) \geq 0$. See $l^s(\beta = 0)$ in Fig. 1. Thus, (13b) is

$$\left(\frac{\partial l}{\partial \bar{u}_1}\right)\left(\frac{\bar{u}_1}{l}\right) = \frac{\phi + \sigma}{\sigma}\frac{1}{\zeta(0)} \geq 0. \tag{14c}$$

In this case, the elasticity of the Frisch labor supply with respect to the marginal utility of private consumption is larger than or equal to zero. An expectations-driven increase in savings is not consistent with the equilibrium because the labor supply is either reduced or unchanged in response to lower marginal utility of consumption in the future.

3.3 Quantitative analysis

In this subsection, we employ a parametric version of our model and envisage whether indeterminacy is quantitatively plausible. The utility is assumed to take the following CES form,

$$u(c_t, H_t, 1 - l_t) = \left[(1-\mu)(c_t H_t^{\psi})^{1-\nu} + \mu(1-l_t)^{1-\nu}\right]^{\frac{1}{1-\nu}}. \tag{15}$$

In the utility, parameter $\mu > 0$ is the share of leisure relative to consumption. This utility function satisfies the joint concavity and is non-separable in c and $1 - l$.[11] The parameter $\nu \geq 0$ and $1/\nu$ measures the *elasticity of substitution* (henceforth, *ES*) between consumption and leisure. The utility is general and includes the following three special cases: (i) when $\nu = 0$, the *ES* between c_t and l_t is infinite and the utility is a linear form; (ii) when $\nu = 1$, the *ES* between c_t and l_t is one and the utility is a Cobb–Douglas form; (iii) when $\nu = \infty$, the *ES* between c_t and l_t is zero and the utility is a Leontief form. Moreover, as we will see below, $1/\nu$ also measures the degree of the IES as the IES is increasing in the value of $1/\nu$. Parameter ψ determines the degree of an external habit effect. Other things being equal, a larger stock of habit reduces an agent's utility if $\psi < 0$ while a larger stock of habit increases an agent's utility if $\psi > 0$. In particular, when $\psi(1 - \nu) > 0$, the habit exhibits the CUJ effect, which implies either (i) $\psi > 0$ and $1/\nu > 1$ or (ii) $\psi < 0$ and $1/\nu < 1$.

The utility (15) gives the following *marginal rate of substitution* (henceforth *MRS*) between agent‘s consumption and external consumption habit along the equilibrium path.

$$\frac{u_2(c_t, H_t, 1 - l_t)}{u_1(c_t, H_t, 1 - l_t)} = \frac{\psi C_t}{H_t}.$$

If the *MRS* is constant, the utility (15) satisfies the *restricted homotheticity* (henceforth *RH*) property with respect to C and H proposed by Alonso-Carrera et al. (2006). Although the marginal utilities u_1 and $u_2$are homogeneous of the same degree, the utility (15) is neither additively nor multiplicative separable between consumption and leisure. The utility satisfies the *RH* property if: (i) the economy is in a steady state wherein, $H^* = C^*$ or (ii) the coefficient of habit formation is infinite, $\beta = \infty$, $so H_t = C_t$ for all t. However, in the general case when $0 < \beta < \infty$ and thus $H_t \neq C_t$ in transitions, then the utility does not satisfy the *RH* property and thus the competitive equilibrium path may be inefficient.

Denote $\Delta = \alpha + \nu l/(1 - l) > 0$. Under the utility (15), the Keynes-Ramsey condition in (9) is

$$\dot{C}_t = \frac{C_t}{\alpha\sigma/\Delta}\left[\alpha A(k_t)^{\alpha-1}(l_t)^{1-\alpha} - (\rho + \delta) - \frac{\alpha\varepsilon}{\Delta k_t}\dot{k}_t - \frac{1}{H_t}\left(\varsigma + \frac{\varepsilon\psi(1-\nu)}{\Delta}\right)\dot{H}_t\right]. \tag{16}$$

The steady state is determined by (7b) and (7c) which, using the utility (15), together lead to

$$l^{*\frac{\psi(1-\nu)-\nu}{\nu}}(1 - l^*) = \left[\frac{\mu}{1-\mu}\frac{1}{A(1-\alpha)}\left(\frac{\rho+\delta}{\alpha A}\right)^{\frac{\alpha+\psi(1-\nu)-\nu}{1-\alpha}}\left(\frac{\alpha}{\rho+\delta(1-\alpha)}\right)^{\psi(1-\nu)-\nu}\right]^{\frac{1}{\nu}}. \tag{17}$$

[11] Carroll et al. (2000) and Chen (2007) used the utility function with $\psi < 0$ in a one-sector endogenous growth model with inelastic leisure.

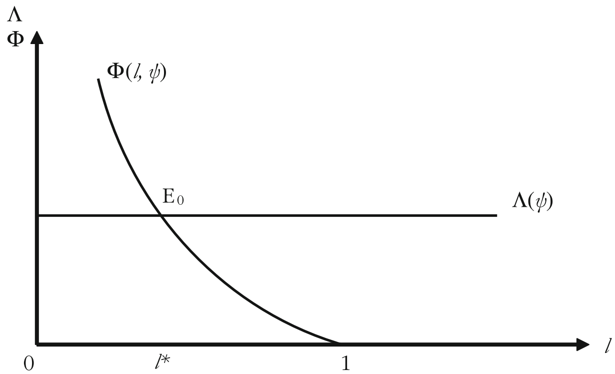

Fig. 2 Existence of steady state under $\psi(1-\nu) < \nu$

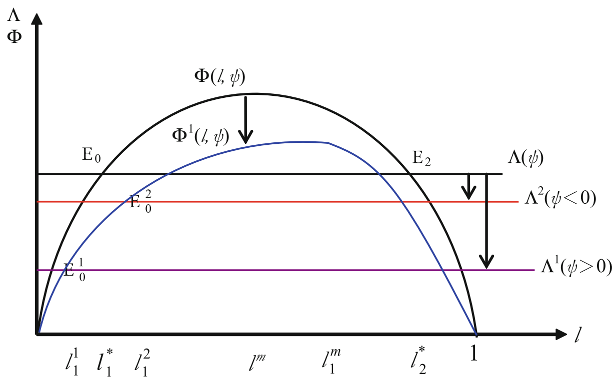

Fig. 3 Existence of steady state under $\psi(1-\nu) > \nu$ and the habit effect

Equation (17) determines the level of l^* in steady state. When l^* is obtained, we can solve k^*, C^* and H^* by $k^* = \left(\frac{\alpha A}{\rho+\delta}\right)^{\frac{1}{1-\alpha}} l^*$ and $H^* = C^* = \frac{\rho+\delta(1-\alpha)}{\alpha}\left(\frac{\alpha A}{\rho+\delta}\right)^{\frac{1}{1-\alpha}} l^*$.

To determine l^*, in a figure with l on the horizontal axis, the right-hand side of (17), denoted by $\Lambda(\psi)$, is independent of l and is thus a horizontal locus. The left-hand side of (17), denoted by $\Phi(l, \psi)$, have two types of shape depending on the value of $\psi(1-v) - v$.[12]

In Fig. 2, which is under the case of $\psi(1-v) < v$, the locus $\Phi(l, \psi)$ is monotonically decreasing in l for $l \le 1$ with the value of $\Phi(l, \psi)$ decreasing from infinite when $l = 0$ to zero when $l = 1$. In this case, there is a unique steady state, l^*.

In Fig. 3, which is under the case of $\psi(1-v) > v$, the locus $\Phi(l, \psi)$ is first increasing and then decreasing in l for $l \le 1$, with the value of $\Phi(l, \psi)$ increasing from zero when $l = 0$, reaching a top when $l^m = 1-\nu/[\psi(1-\nu)] < 1$ and finally returning to zero when $l = 1$. In this case, there are two steady states with the employment at l_1^* and l_2^*.

[12] The slope of $\Phi(l, \psi)$ is dictated by $\frac{d\Phi(l,\psi)}{dl} = \frac{1}{\nu} l^* \psi(\frac{1}{\nu} - 1) - 2\{[\psi(1-v) - v] - \psi(1-v)l\}$.

In the parameter region wherein a sink may arise, a negative determinant of the Jacobean matrix is equivalent to $\psi(1-\nu) > \nu/(1-l)$, which implies $\psi(1-\upsilon) > \upsilon$ and $l < l^m$. Thus, the steady state l^* in Fig. 2 is not a sink. In Fig. 3, although l_2^* is not a sink, the steady state l_1^* may be a sink and thus endogenous investment fluctuations may emerge.

It is worth noting that the condition $\psi(1-\upsilon) > \upsilon > 0$ stipulates that indeterminacy may arise only under either (i) $\psi > 0$ and $1/\nu > 1$, or (ii) $\psi < 0$ and $1/\nu < 1$. This indicates that if the utility is a Cobb–Douglas form (which arises under $\nu = 1$), the external consumption habit cannot be a source of indeterminacy in a one-sector growth model.

To see how the habit effect influences the steady state l_1^* in Fig. 3, the habit effect affects the employment level and thus, $l_1^* = l^*(\psi)$. First, under (i) $\psi > 0$ and $1/\nu > 1$, when ψ increases, the habit effect is larger and the value of $\psi(1-\nu)$ increases. Both loci $\Lambda(\psi)$ and $\Phi(l,\psi)$ then shift downwards with $\Lambda(\psi)$ shifting more than $\Phi(l,\psi)$ at the original employment level l_1^*. See $\Lambda^1(\psi > 0)$ and $\Phi^1(l,\psi)$ in Fig. 3. As a result, the steady state moves to $\mathrm{E}_0 1$ with a lower steady-state employment level $l_1^1 < l_1^*$.

Next, under (ii) $\psi < 0$ and $1/\nu < 1$, when ψ decreases, the habit effect is increased and the value of $\psi(1-\nu)$ is larger. Both loci $\Lambda(\psi)$ and $\Phi(l,\psi)$ also shift downwards, but $\Lambda(\psi)$ shifts downward less than $\Phi(l,\psi)$ at the original employment level l_1^*. Thus, under $\psi < 0$, if we assume that a decrease in ψ also shifts $\Phi(l,_{\psi})$ to $\Phi^1(l,\psi)$, then $\Lambda(\psi)$ is shifted downward to $\Lambda^2(\psi < 0)$ that is less than $\Lambda^1(\psi > 0)$. See $\Lambda^2(\psi > 0)$ and $\Phi^1(l,\psi)$ in Fig. 3. As a result, the steady state moves to E_0^2 with a higher steady-state employment level $l_1^2 > l_1^*$.

According to (16), the IES is given by

$$\frac{1}{\alpha\sigma/\Delta} = \frac{1}{\nu}\left(1+\frac{\nu}{\alpha}\frac{l^*(\psi)}{1-l^*(\psi)}\right) \times\left[1+\frac{\rho+\delta(1-\alpha)}{(1-\alpha)(\rho+\delta)}\left(\frac{\rho+\delta}{\alpha A}\right)^{\frac{(1+\psi)(1-\nu)}{1-\alpha}}\left(\frac{\alpha}{\rho+\delta(1-\alpha)}\right)^{(1+\psi)(1-\nu)}\frac{l^*(\psi)}{1-l^*(\psi)}\right] > 0, \tag{18}$$

Thus, the IES is increasing in $1/\nu$. In the case when $\psi > 0$, a higher habit effect reduces the IES indirectly through a lower l^*. On the other hand, when $\psi < 0$, a higher habit effect increases the IES indirectly through a higher l^*.

Denote

$$E \equiv \left\{\alpha\rho\upsilon\Gamma_1 + [\rho+\delta(1-\alpha)]\frac{\upsilon l}{(1-l)}\right\}(1-Q) - (1-\alpha)(\rho+\delta)\upsilon Q,$$

$$D \equiv \alpha(1+\psi)(1-Q) - \frac{\psi l^*}{1-l^*}Q,$$

$$Q \equiv \frac{(1-\mu)}{1-\mu+\mu(1-l^*)^{1-\nu}(C(l^*))^{-(1-\nu)(1+\psi)}} > 0.$$

To characterize the conditions of a sink under the utility (15), first, given a value of β, it is required that $\Gamma_1 < 0$. This condition demands $\psi/\nu > D/\alpha$ if $\psi > 0$ and $\psi/\nu < D/\alpha$ if $\psi < 0$.

Next, the condition $\chi > \phi + \sigma - \eta + \varepsilon$ is equivalent to $\psi(1-\nu) > \nu/(1-l) > 0$. This condition may be read as either $\psi < \psi_2 \equiv \nu/[(1-\nu)(1-l)]$ if $\psi < 0$ and $1/\nu < 1$, or $\psi > \psi_2$ if $\psi > 0$ and $1/\nu > 1$.

Finally, the condition $\Gamma_2 > 0$ requires $\psi < \psi_1 \equiv E/[(1-\alpha)(\rho+\delta)\nu Q]$. Thus, we obtain

Proposition 2 *With the CES utility (13), under a sufficiently large β, the conditions that the steady state l_1^* is a sink are*

(i) $-D/[\alpha(-\psi)] < 1/\nu < 1$ *and* $\psi < min\{\psi_1, \psi_2\}$ *if* $\psi < 0$,

(ii) $1/\nu > max\{D/(\alpha\psi), 1\}$ *and* $\psi_2 < \psi < \psi_1$ *if* $\psi > 0$.

Proposition 2 stipulates that both a positive habit effect and a negative habit effect can lead to indeterminacy. When the habit effect is negative ($\psi < 0$), a value of $1/v$ smaller than one can give rise to indeterminacy. Under a negative habit effect, a larger habit effect (a smaller ψ) increases the IES indirectly through a larger l^* in (18). Therefore, a smaller $1/\nu$ suffices to establish indeterminacy. Alternatively, when the habit effect is positive ($\psi > 0$), a value of $1/v$ larger than one is required in order to bring about indeterminacy. Under a positive habit effect, a larger habit effect (a larger ψ) lowers the IES indirectly through a smaller l. Thus, a larger $1/v$ is needed to exhibit indeterminacy.

While indeterminacy may emerge under both a positive habit effect and a negative habit effect in Proposition 2, as our calibration exercises below show, only a negative habit effect $\psi < 0$ is empirically plausible. Therefore, empirically plausible indeterminacy in our model requires only a small $1/v$ which implies a small IES.

Now, we quantitatively assess the plausibility of indeterminacy in our model. We choose $k^*/y^* = 4$ and $C^*/y^* = 0.8$, which are consistent with data in the US, and calibrate (6c) to obtain $\delta = 0.05$. Furthermore, we normalize $A = 1$ and set $\rho = 0.04$, and then use (6b) to calibrate and obtain $\alpha = 0.36$ and thus the elasticity of the labor supply demand is -0.36. We choose the coefficient of the habit formation at $\beta = 0.35$.[13] Although there is no empirical data about the values of μ, we can choose μ in order both to satisfy a positive IES in (18) and to ensure the existence of the steady state in (17). We set $\mu = 0.247$. Since the dynamic property of the steady state depends on the interaction between ψ and ν, we choose a combination of ψ and ν that gives rise to a sink. We choose the pair $\{\psi, \nu\} = \{-2.25, 2.5\}$ such that the calibrated value of l^* equal to 0.251 Prescott (2006).[14] Under this set of benchmark parameter values, we get following the steady state: $l^* = 0.251$, $k^* = 2.191$, $y^* = 0.547$ and $H^* = C^* = 0.438$.[15]

[13] While Constantinides (1990) employed $\beta = 0.6$, Carroll et al. (1997) and Alvarez-Cuadrado et al. (2004) used $\beta = 0.2$. Our value lies within these existing values used. As we will see from the quantitative results in Fig. 5 below, there is a tradeoff between the coefficient of the habit formation β and the CUJ effect $\psi(1-\nu)$. Thus, if we parameterize $\beta = 0.6$, the required CUJ effect that creates indeterminacy is smaller. Alternatively, if we parameterize $\beta = 0.2$, the required CUJ effect that creates indeterminacy is larger.

[14] Prescott (2006) pointed out that 25 % of productive time was allocated to market in the US.

[15] There is another steady state with $l_2^* = 0.2674$ which is a saddle.

Table 1 Numerical results of indeterminacy

Benchmark	$\psi = -2$	$\psi = -2.5$	$\psi > 0$
$1 < v < 3.301$	$1 < v < 2.927$	$1 < v < 4.619$	Not plausible
$(0.303 < 1/v < 1)$	$(0.342 < 1/v < 1)$	$(0.217 < 1/v < 1)$	

Benchmark parameter: $A = 1$, $\alpha = 0.36$, $\rho = 0.04$, $\delta = 0.05$, $\mu = 0.25$, $\beta = 0.35$, $\psi = -2.25$ and $v = 2.5$; benchmark steady state: $k^* = 2.181$, $l^* = 0.25$, $y^* = 0.545$ and $H^* = c^* = 0.436$

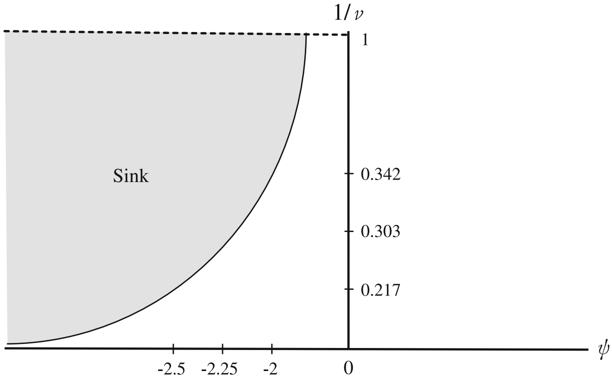

Fig. 4 Combination of $(\psi, 1/v))$ that exhibits indeterminacy. *Note* Other parameter values are the same as those of Table 1

We are ready to study the empirical plausibility that the steady state $l^* = 0.251$ is a sink. For a given degree of an external habit effect ψ and the coefficient of the habit formation β, we will find the range of $1/v$ under which the steady state is a sink. Later, for given ψ and $1/v$, we will find the threshold of the coefficient of the habit formation β under which the steady state is a sink. Given the benchmark parameter values, we find that the steady state is a sink if $1 < v < 3.301$. See Column 1 in Table 1. The range of $1 < v < 3.301$, or equivalently the range of $0.303 < 1/v < 1$, implies the value of the IES in the range of (0.384, 1.354) which seems to be reasonable. Under these benchmark parameter values, the implied elasticity of the Frisch labor supply ζ is 11.55 which is positive and larger than the elasticity of the labor demand, -0.36.

If we increase the value of ψ and thus lower the degree of an external habit effect, the range of $1/v$ decreases (Column 2); in contrast, if we decrease the value of ψ and thus increase the degree of an external habit effect, the range of $1/v$ increases (Column 3). However, if we increase the value of ψ so $\psi > 0$, we cannot find plausible values of v such that the steady state is a sink (Column 4). In Fig. 4, we draw the range of $(\psi, 1/v)$ that yields a sink (the shaded area). Thus, in order to have a proper habit effect that yields a sink, there is a tradeoff between the value of ψ and the value of $1/v$.

The results in Fig. 4 suggest that indeterminacy is empirically plausible only when the habit effect is negative ($\psi < 0$) that features the CUJ effect, ($\psi(1 - v) > 0$). In Theorem 1, both a proper habit effect (the CUJ effect) and the coefficient of habit

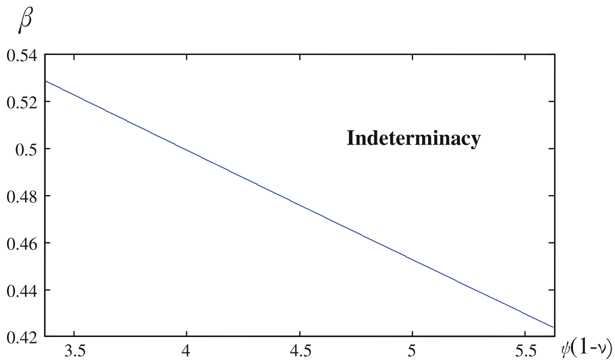

Fig. 5 Combination of $(\beta, \psi(1-\nu))$ that exhibits indeterminacy. *Note* Other parameter values are the same as those of Table 1

formation have roles in creating indeterminacy. Indeed, if one of the two effects is stronger, indeterminacy can still emerge if the other effect is weaker. In Fig. 5, we illustrate the tradeoff between these two effects for a range of the values of $\psi(1-v)$ that exhibits indeterminacy which, for a given value of $\psi(1-v)$, gives the threshold of the coefficient of the habit formation (β). Figure 5 suggests that if the CUJ effect is such that $\psi(1-v) = 5.5$, indeterminacy arises as long as the coefficient of the habit formation is as large as $\beta = 0.43$. In the case when $\Delta t = 1$, $\beta = 0.43$ indicates $\kappa(\beta) = 0.35$. This implies that the steady state is a sink if more than 35 % of the habit stock comes from current external consumption, or equivalently, if less than 65 % of the habit stock comes from past average consumption. Alternatively, if the CUJ effect is smaller such that $\psi(1-v) = 3.5$, indeterminacy emerges if the coefficient of the habit formation is as large as $\beta = 0.53$ which means less than 59 % of the habit stock coming from average consumption in the past. Thus, under the range of the CUJ effect in Fig. 5, the external consumption habit is a source of indeterminacy even if more than a half of the external consumption habit comes from average consumption in the past.

Finally, a negative effect of a rise in the stock of external consumption habits means that an individual household feels jealous of the other households' (past as well as current) consumption. In addition, the CUJ effect implies that conformism prevails in consumption activities. Jealousy and conformism have been frequently assumed by empirical oriented studies on the models with consumption externalities, because the households' Euler equations with these assumptions can be supported by the data more easily than the Euler equations with positive consumption externalities and anti-conformism. Therefore, our numerical experiments suggest that the economy with rapid formation of external habits may produce indeterminacy of equilibrium under empirically plausible conditions.

4 Concluding remarks

This paper investigates whether the introduction of the external consumption habit into an otherwise standard one-sector growth model can lead to indeterminacy when the labor supply is elastic. The external consumption habit stock is formed partly by current external consumption and partly by past external consumption habits, wherein the coefficient of the habit formation characterizes the strength of the influence that current external consumption affects the habit. We find that the equilibrium path is indeterminate if the coefficient of the habit formation is sufficiently large and there is a proper habit effect with a positive IES. A sufficiently large coefficient of the habit formation is to allow for a sufficiently large impact of current external consumption on the external consumption habit. A positive IES allows for the tradeoff between consumption and savings. A proper habit effect assures a proper social complementarity in consumption which generates self-reinforcing mechanism connecting individual and joint choices so expectations-driven equilibrium paths may arise.

Why do external consumption habits create indeterminacy? Due to a sunspot shock, agents anticipate a rise in the rate of return to capital in the future and they change the current consumption-saving decision. If these changes actually raise the future rate of return, then there may exist expectations-driven fluctuations generated by the presence of multiple equilibrium paths. If there are strong CUJ effects and the adjustment speed of habit formation is sufficiently rapid, a rise in the expected rate of return to capital would make the agents increase (not decrease) current consumption and leisure. This is because when all agents increase their consumption today, the social level of habit stock increases tomorrow. Thus if the CUJ effect is large and habit formation is fast, to catch up the social level of habits, the agents incase their consumption and leisure tomorrow as well, even though they increase today's consumption and leisure. If this is the case, tomorrow's capital stock decreases, which increases the rate of return to capital. In contrast, if there is no external habit or the habit formation is very slow, an increase in the today's consumption may not induce a larger consumption and leisure tomorrow. In this case tomorrow's labor supply increases, which may increase capital-labor ratio, so that the rate of return cannot rise and the equilibrium is determinate.

An elastic labor supply plays an important role in order for the external consumption habit to be a source of indeterminacy. The elasticity of the Frisch labor supply is decreasing with the coefficient of the habit formation. When the coefficient of the habit formation is above a threshold, as the amount of savings increases, the labor supply can increase sufficiently such that returns to savings are higher. Thus, an expectations-driven equilibrium can be self-fulfilled.

While a positive habit effect and a negative habit effect both may generate indeterminacy, using a calibrated version of the model we find that indeterminacy is empirically plausible only when the habit effect is negative that features the CUJ effect. Moreover, under given CUJ effects, we find that the external consumption habit can be a source of indeterminacy even if more than a half of the external consumption habit comes from average consumption in the past.

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Appendix 1: The elasticity of the Frisch labor supply

If we differentiate (5a) and keep λ fixed, we obtain

$$dc = \frac{u_{13}dl - u_{12}dH}{u_{11}}. \tag{19}$$

Under given λ, differentiating (5b) gives

$$\frac{dw}{w} = \frac{u_{31}dc + u_{32}dH - u_{31}dl}{u_3}. \tag{20}$$

Substituting (19) into (20) yields

$$\begin{aligned}\zeta(k,l,H) \equiv \frac{\frac{dw}{w}}{\frac{dl}{l}} &= \frac{l}{u_3}\left[\frac{u_{13}(u_{31} + u_{32}\frac{dH}{dc})}{u_{11} + u_{12}\frac{dH}{dc}} - u_{33}\right] \\ &= \left[\frac{\frac{lu_{13}}{u_1}(\frac{u_{31}}{u_3}c + \frac{u_{32}}{u_3}H\frac{c}{H}\frac{dH}{dc})}{\frac{u_{11}}{u_1}c + \frac{u_{12}}{u_1}H\frac{c}{H}\frac{dH}{dc}} - \frac{lu_{33}}{u_3}\right],\end{aligned}$$

and thus the elasticity of the Frisch labor supply

$$\zeta(k,l,H) = \left\{\frac{\varepsilon[\phi - (\varsigma + \chi)\frac{C}{H}\frac{dH}{dC}]}{-\sigma - \varsigma\frac{C}{H}\frac{dH}{dC}} - \eta\right\}. \tag{21}$$

In order to obtain dH/dC, we rewrite H_t as

$$H_t = \int_{-\infty}^{t} D(t,\tau)C_\tau d\tau, \tag{22}$$

where $D(t,\tau) = \beta e^{-\beta(\tau - t)}$ which satisfies

$$\int_{-\infty}^{t} D(t,\tau)d\tau = 1. \tag{23}$$

Hence, $D(t,\tau)$ measures the influence of C_τ on H_t at time $\tau \in (-\infty; t]$ and is thus a weight function defined on $\tau \in (-\infty; t]$ with the total weight equal 1 for all $\beta > 0$. $D(t,\tau)$ is a decreasing function of β for $\beta \geq \frac{1}{t-\tau}$ that approaches 0 as $\beta \to \infty$. Thus, as $\beta \to \infty$, $D(t,\tau)$ approaches to the function $\Upsilon_t(\tau)$ that satisfies

$$\Upsilon_t(\tau) = \begin{cases} 0, & \tau < t, \\ \infty, & \tau = t, \end{cases}$$

and $\int_{-\infty}^{t} \Upsilon_t(\tau)d\tau = 1$. This indicates that for $\beta \to \infty$, $H_t = C_t$.

To explore further how the change in C_t affects H_t, let us consider the case wherein C_τ is changed to $C_\tau + dC_\tau$ and H_t is changed to $H_t + dH_t$. Using (22), we obtain

$$dH_t = \int_{-\infty}^{t} D(t,\tau) dC_\tau d\tau. \tag{24}$$

We assume that for some $t_1 < t$,

$$dC_\tau = \begin{cases} 0, & \tau < t_1, \\ q, & t_1 \leq \tau \leq t. \end{cases}$$

Denote $\Delta t = t - t_1 > 0$. Then by (24), $dH_t = (1 - e^{-\beta\Delta t})q$ and the effect of the change in C_t on H_t becomes $x(\beta) \equiv \frac{dH_t}{dC_\tau} = \frac{dH_t}{q} = (1 - e^{-\beta\Delta t})$. One can see that $0 < x(\beta) < 1$ and $x(\beta)$ is increasing in β, with $x(\beta) \to 0$ as $\beta \to 0$ and $x(\beta) \to 1$ as $\beta \to \infty$.

Therefore, (21) can be rewritten as

$$\zeta(\beta) = \left\{ \frac{\varepsilon[\phi - (\varsigma + \chi)\frac{C}{H}x(\beta)]}{-\sigma - \varsigma\frac{C}{H}x(\beta)} - \eta \right\}.$$

Appendix 2: Proof of Theorem 1

In the Appendix, we prove the Theorem 1. Denote f_i, $i = l, k$ and lj, $j = C, k, H$, as partial derivatives with respect to i and j. If we take the linear Taylor's expansion of the dynamic equilibrium system (2), (6c) and (16) in the neighborhood of a steady state, along with the use of (8), we obtain

$$\begin{bmatrix} \dot{C}_t \\ \dot{k}_t \\ \dot{H}_t \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} & J_{13} \\ f_l l_c - 1 & f_k - \delta - f_l l_k & f_l l_H \\ \beta & 0 & -\beta \end{bmatrix} \begin{bmatrix} C_t - C^* \\ k_t - k^* \\ H_t - H^* \end{bmatrix}, \tag{25}$$

where

$$J_{11} = \frac{C}{\Omega}\left\{ [(\alpha + \varepsilon - \eta)\varsigma + \varepsilon\chi]\frac{\beta}{H} + (\eta - \varepsilon - \alpha) f_{kl} l_c + \frac{\alpha\varepsilon}{k}(f_l l_c - 1) \right\},$$

$$J_{12} = \frac{C}{\Omega}\left\{ (\eta - \varepsilon - \alpha)(f_{kl} l_k + f_{kk}) + \frac{\alpha\varepsilon}{k}(f_l l_k + f_k - \delta) \right\},$$

$$J_{13} = \frac{C}{\Omega}\left\{ -[(\alpha + \varepsilon - \eta)\varsigma + \varepsilon\chi]\frac{\beta}{H} + (\eta - \varepsilon - \alpha) f_{kl} l_H + \frac{\alpha\varepsilon}{k} f_l l_H \right\}.$$

Let J denote the Jacobean matrix in (25) and ω denote its corresponding eigenvalue. The characteristic polynomial is in (11), with $Det(J)$, $Tr(J)$ and $Ds(J)$ defined in (12a)–(12c).

It is clear from (11) that $G(\omega) = -\infty$ when $\omega = \infty$ and $G(\omega) = \infty$ when $\omega = -\infty$. A sink requires three stable roots. The necessary conditions for the presence of three stable roots are: (i) $G(0) = Det(J) < 0$ and (ii) $G'(0) = -Ds(J) < 0$. Moreover, according to the Routh-Hurwitz theorem, the requirement of no eigenvalues with positive real parts in the above characteristic polynomial suggests no variation in signs in the following series: $\{-1, Tr(J), \_ Ds(J) + Det(J)/Tr(J), Det(J)\}$.

This indicates the additional requirement of (iii) $Tr(J) < 0$ and (iv) $-Ds(J) + Det(J)/Tr(J) < 0$.

To investigate these conditions,

(i) $G(0) = Det(J) < 0$
Since $Det(J) = \beta(1-\alpha)(\rho+\delta)\,[\rho+\delta(1-\alpha)]\,[(\phi+\sigma+\varepsilon-\eta)-\chi]/(\text{-}\Omega\alpha)$ and $\Omega < 0$, it is obvious that this requires $\chi > \phi + \sigma - \eta + \varepsilon$.

(ii) $Tr(J) < 0$. As $T > 0$, $Tr(J) < 0$ requires both

$$\Gamma_1 < 0 \quad \text{and} \quad \beta > \beta_a \equiv T/(-\Gamma_1) > 0. \tag{26a}$$

(iii) $G'(0) = -Ds(J) < 0$
As $M < 0$, $Ds(J) > 0$ requires both

$$\Gamma_2 > 0 \quad \text{and} \quad \beta > \beta_b \equiv M/(-\Gamma_2) > 0. \tag{26b}$$

(iv) $-Ds(J) + Det(J)/Tr(J) < 0$
Under $Tr(J) < 0$ in (ii), condition (iv) is equivalent to $-Ds(J)Tr(J) + Det(J) > 0$. Using (12a)–(12c), this requires

$$L(\beta) = \beta^2 - \beta\left\{\frac{M}{-\Gamma_2} + \frac{T}{-\Gamma_1} + \frac{N}{-\Gamma_1\Gamma_2}\right\} + \frac{MT}{\Gamma_1\Gamma_2} > 0, \tag{27a}$$

where $N = \frac{(1-\alpha)(\rho+\delta)}{(-\Omega)}\frac{\rho+\delta(1-\alpha)}{\alpha}(\phi+\sigma-\eta+\varepsilon-\chi) < 0$, whose the negative sign comes from using (i).

When $L(\beta) = 0$ the polynomial has two roots β_1 and β_2, $\beta_1 \geq \beta_2$, as follows.

$$\frac{1}{2}\left\{\frac{\mathrm{N}}{\Gamma_1\Gamma_2} + \frac{\mathrm{M}}{-\Gamma_2} + \frac{\mathrm{T}}{-\Gamma_1} \pm \left[\left(\frac{\mathrm{N}}{\Gamma_1\Gamma_2} + \frac{\mathrm{M}}{-\Gamma_2} + \frac{\mathrm{T}}{-\Gamma_1}\right)^2 - 4\frac{\mathrm{MT}}{\Gamma_1\Gamma_2}\right]^{1/2}\right\}.$$

Under (i) $Det(J) < 0$, (ii) $Tr(J) < 0$ and (iii) $Ds(J) > 0$, both β_1 and β_2 are positive, as verified by

$$\beta_1\beta_2 = MT/(\Gamma_1\Gamma_2) > 0 \quad \text{and} \quad \beta_1 + \beta_2 = \{\Gamma_1 M + \Gamma_2 T - N\}/(-\Gamma_1\Gamma_2) > 0.$$

The inequality sign in (26a) is satisfied if any one of the following two cases holds: (a) $\beta > \beta_1 \geq \beta_2$ or (b) $\beta < \beta_2 \leq \beta_1$. However, case (b) is impossible as case (b) implies $\beta_2 < T/(-\Gamma_1) \equiv \beta_a$, which is against the requirement of $\beta > \beta_a$ for $Tr(J) < 0$ in (ii).

Therefore, (27a) and $-Ds(J)Tr(J) + Det(J) > 0$ both can be met only if

$$\beta > \beta_1. \tag{27b}$$

It is straightforward to show that $\beta_1 > \beta_a$ and $\beta_1 > \beta_b$. Thus, (26a), (26b) and (27b) indicate that the requirement of $\beta > \beta_1$.

Therefore, under $\beta > \beta_1$, the conditions of a sink are: $\chi > \phi + \sigma - \eta + \varepsilon$, $\Gamma_1 < 0$ and $\Gamma_2 > 0$.

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