Journal of Macroeconomics 33 (2011) 247–258

Contents lists available at ScienceDirect

Journal of Macroeconomics

journal homepage: www.elsevier.com/locate/jmacro

The dynamic welfare cost of seignorage tax and consumption tax in a neoclassical growth model with a cash-in-advance constraint

Chia-Hui Lu [a], Been-Lon Chen [b,*], Mei Hsu [a]

[a] Department of Economics, National Taipei University, 151, University Road, San Shia, Taipei 237, Taiwan
[b] Institute of Economics, Academia Sinica, 128 Academia Road Section 2, Taipei 11529, Taiwan

ARTICLE INFO

Article history:
Received 15 September 2009
Accepted 13 November 2010
Available online 26 November 2010

JEL classification:
E13
E52
O23

Keywords:
Growth model
Cash-in-advance
Seignorage tax
Consumption tax
Welfare cost

ABSTRACT

Using a public finance approach, this study investigates welfare costs between seignorage and consumption taxes in a standard growth model. One of these two taxes is used to finance exogenous public spending to balance the government budget. The steady-state welfare cost of consumption taxes is lower if the consumption effect dominates the leisure effect. This paper compares equilibrium along transitional dynamic and steady-state paths and finds that because of lower consumption and leisure and thus higher welfare costs of consumption taxes during early periods, the welfare cost of consumption taxes is larger than the welfare cost of seignorage taxes.

1. Introduction

Monetary economists have devoted considerable effort to measuring the welfare cost of seignorage. The traditional approach treats real balances as a consumption good and seignorage as an inflation tax on real balances. Although some studies have found a large welfare cost of inflation and others uncovered a small welfare cost of inflation, the findings generally established a negative relationship between inflation and welfare.[1] In this body of research, the optimal inflation tax is such that the nominal rate of interest calls for deflation at the rate of time preference, or a zero nominal interest rate, known as the Friedman rule (Friedman, 1969).

Alternatively, there is a line of research that studies monetary policy following a public finance approach. This approach was started by Phelps (1973). In an environment where a lump-sum tax is not available, Phelps (1973) argued against the Friedman rule and showed that all goods, including real balances, would be taxed in a Ramsey-like fashion. In dynamic general equilibrium models, the issue was re-examined by Chari et al. (1991) and Braun (1994) in terms of the welfare cost between a seignorage tax and an income tax. These two sets of authors employed a Lucas and Stokey (1983) model without capital and with cash goods and credit goods. Under a class of preference, the results in Chari et al. (1991) were supportive

* Corresponding author. Tel.: +886 2 27822791x309; fax: +886 2 2785 3946.
E-mail addresses: chlu@mail.ntpu.edu.tu (C.-H. Lu), bchen@econ.sinica.edu.tw (B.-L. Chen), mhsu@mail.ntpu.edu.tu (M. Hsu).

[1] Examples include Fischer (1981), Cooley and Hansen (1989), Kimbrough (1986), Prescott (1987), Cole and Stockman (1992), Gillman (1993), Gomme (1993), Mino and Shibata (1995), Dotsey and Ireland (1996), Ireland (1994, 2007), Aiyagari et al. (1998), Wu and Zhang (2000), Erosa and Ventura (2002), Itaya and Mino (2003) and Chen et al. (2008). See Lucas (2000) for a survey.

0164-0704/$ - see front matter © 2010 Elsevier Inc. All rights reserved.
doi:10.1016/j.jmacro.2010.11.004

of the Friedman rule of zero nominal interest rate, while under a more general specification of preference, the conclusion of Braun (1994) was in favor of a seignorage tax instead of the Friedman rule.[2]

Later, Palivos and Yip (1995) followed the public finance approach used by Chari et al. (1991) and Braun (1994). Palivos and Yip (1995) investigated endogenous growth models with a cash-in-advance constraint. In their work, consumption and a fraction of investment are cash constrained. They compared the welfare cost between a seignorage tax and an income tax as alternative ways of financing exogenous public spending along a balance growth path, as do Chari et al. (1991) and Braun (1994). They found that in the long run, the welfare cost of a seignorage tax is lower than that of an income tax on the condition that a sufficiently large fraction of investment is with a cash constraint. Their results are in supportive of Braun (1994) but not Chari et al. (1991) and suggested an optimal inflation tax higher than the Friedman rule. All of the papers above compared a seignorage tax with an income tax, a tax that is known to be dynamically inefficient in the long run since Judd (1985) and Chamley (1986).[3]

In a recent paper Ho et al. (2007) offered a comparison of the welfare cost between a seignorage tax and a consumption tax in the public finance approach in a model with real balances and leisure in utility. They found that without a production externality, a seignorage tax always had a higher welfare cost than a consumption tax in the long run. With a production externality, a seignorage tax not only had a smaller welfare cost than a consumption tax but may have a welfare gain.

In this paper, we revisit the comparison of the welfare cost between a seignorage tax and a consumption tax in the public finance approach. In particular, we compare the overall welfare cost along both the transitional dynamic path and the steady state. Existing literature studied welfare comparisons in steady state but not along transitions. We envisage the issue in a model with a cash-in-advance (henceforth, CIA) constraint. We choose a CIA approach because, except for Ho et al. which used a money-in-utility (henceforth, MIU) approach, all of the above-mentioned dynamic growth models are analyzed in a CIA approach. We study an otherwise standard growth with a general a CIA constraint in which the government spending is financed by consumption and seignorage taxes.

We find the following results. First of all, when investment is constrained by cash, switching from a consumption tax to a seignorage tax lowers consumption but has an ambiguous effect on leisure in the long run. As a result, the welfare cost of a consumption tax is lower than the welfare cost of a seignorage tax only if the harmful effect through lower consumption dominates. Second, when we consider the transition effect, a consumption tax reduces more consumption and has a higher welfare cost during early periods than a seignorage tax. Thus, the overall welfare ranking between the two taxes is theoretically ambiguous. Finally, our quantitative results indicate that under plausible rates of time preferences, the overall welfare cost of a seignorage tax is lower than that of a consumption tax.

In our first result, the welfare cost of a seignorage tax is lower than that of a consumption tax only if there is a cash constraint on investment and the harmful effect via consumption dominates. Without a cash constraint on investment, a seignorage tax is like a consumption tax and both taxes affect equilibrium allocation in a symmetric way. This result cannot arise in Ho et al. even if the degree of consumption is the same as the degree of real balances in utility since real balances directly affect the utility so the marginal utility of real balances affects the marginal cost of holding capital and thus the tradeoff between consumption and savings. Moreover, when investment is constrained by cash, leisure may increase in response to both taxes. In particular, a seignorage tax may result in a higher leisure than a consumption tax, which case materializes if the positive effect on leisure due to the complement of capital and labor in production (and thus the substitute between capital and leisure) dominates the negative effect on leisure due to the complement between consumption and leisure in utility. As a result, the welfare cost of a seignorage tax may or may not be higher than the welfare cost of a consumption tax in the long run. The welfare cost of a seignorage tax is unambiguously higher than the welfare cost of a consumption tax in the long run only when the consumption effect dominates.

The second result is the main innovation in our study. Our second and third results together stipulate that the overall welfare cost of a seignorage tax is smaller than a consumption tax. This result is similar to that in Ho et al., but is based on a different time horizon. While the result in Ho et al. is held only in the long run, our result is obtained in the transition and the steady state as a whole.

Since our second result is the key innovation in this study, we explain its reasons as follows. Under a CIA constraint on investment, the constraint affects the agent's tradeoff between consumption in t and savings/investment in t. Investment in t (which is related to consumption in $t + 1$) needs real balances and is affected by the after-tax shadow price of real balances in $t + 1$. As a result, if a period t consumption tax is imposed, it affects the intertemporal tradeoff between consumption in period $t - 1$ and t. However, if a period t seignorage tax is imposed, it affects the intertemporal tradeoff between consumption in period t and $t + 1$, but not the tradeoff between consumption in period $t - 1$ and t. Thus, while a consumption tax in t exerts effects on the tradeoff between consumption in $t - 1$and t, a seignorage tax in t affects the tradeoff between consumption in t and $t + 1$. Because of this one-period earlier effect in a consumption tax, when the tax code is changed in period t, consumption is reduced more under period t consumption tax than under period t seignorage tax. This effect persists over time so the path of consumption is lower under a consumption tax. As leisure is a complement to consumption, leisure is also lower in a

[2] While Chari et al. (1991) required the preference to be homogenous in the two consumption goods and weakly separable in leisure activities, Braun (1994) assumed a more general preference that includes the one used in Chari et al. (1991) as a special case.

[3] In an endogenous growth model with a loan constraint in a Diamond and Dybvig (1983) framework, Espinosa-Vega and Yip (2002) also compared the welfare cost of a seignorage tax with that of an income tax. They failed to find support in an optimal inflation tax higher than the Friedman rule as they could not meet the condition that the binding loan constraint for welfare-maximization be smaller than the binding loan constraint for inflation-minimization according to their proposition 7.

consumption tax. Since a seignorage tax has an extra distortion on capital, in finite periods this effect will dominate the former transition effect. After then, the level of consumption in a seignorage tax is lower than in a consumption tax. Thus, a seignorage tax exerts a lower welfare cost in early periods of tax changes and a higher welfare cost in the long run. As a consequence, the overall welfare comparison tradeoffs the short run and the long run. Our calibration results indicate that, under plausible rates of the time preference, the overall welfare cost of a seignorage tax is lower than that of a consumption tax.

The remainder of this paper is organized as follows. Section 2 sets up a growth model with a CIA constraint and studies the optimization and equilibrium conditions, while Section 3 studies the welfare costs of alternative ways of finance. Finally, we offer concluding remarks in Section 4.

2. The basic model

2.1. Environment

The model is based on Stockman (1981) and Wang and Yip (1992) and is conducted in discrete time extended to include leisure.[4] At time 0, the representative agent's lifetime utility is

$$U = \sum_{t=0}^{\infty} \frac{1}{(1+\rho)^t} u(c_t, l_t), \tag{1}$$

where c_t is consumption, l_t is leisure and $\rho > 0$ is the time preference rate. The felicity $u(c_t, l_t)$ has standard properties and is increasing and concave in c_t and l_t: $u_i(c_t, l_t) > 0 > u_{ii}(c_t, l_t)$, $i = 1, 2$. Moreover, we assume that consumption and leisure are complements: $u_{ij}(c_t, l_t) > 0$ and $u_{ji}(c_t, l_t) > 0$, $i, j = 1, 2$, $i \neq j$.

The representative agent's budget constraint is

$$[k_{t+1} - (1-\delta)k_t] + [(1+\pi_t)m_{t+1} - m_t] = f(k_t, L - l_t) - (1+\tau_{c_t})c_t, \tag{2}$$

where $f(k_t, L - l_t)$ is the output per capita in t, k_t is capital per capita in t, and $L - l_t$ is labor hours per capita in t in which L is the agent's time endowment. Moreover, m_t is real money holdings per capita in t, τ_{ct} is the consumption tax rate in t, π_t is the inflation rate in t and δ is the depreciation rate of capital.[5] Eq. (2) stipulates that output is used either as consumption or savings and savings may be in terms of investment or real balances.

The production technology $f(k_t, L - l_t)$ is a standard neoclassical technology with a positive and decreasing marginal product in each input: $f_i(k_t, L - l_t) > 0 > f_{ii}(k_t, L - l_t)$, $i = 1, 2$. It is well-known that a standard neoclassical technology with two inputs also implies that the two inputs must be complements: $f_{ij}(k_t, L - l_t) > 0$, $i, j = 1, 2$, $i \neq j$.

Both consumption and investment may be subject to CIA constraints. We consider

$$m_t \geqslant (1+\tau_{c_t})c_t + \varphi[k_{t+1} - (1-\delta)k_t], 0 \leqslant \varphi \leqslant 1. \tag{3}$$

Because taxes are paid in cash, consumption taxes $\tau_{ct}c_t$ are a part of the CIA constraint. The above CIA constraint is the one utilized in Wang and Yip (1992) and Palivos and Yip (1995) and is general that includes the following special cases. If $\varphi = 1$, consumption and investment are equally cash constrained. Such a constraint is employed by Stockman (1981) and Abel (1985). If $\varphi = 0$, only consumption is constrained by liquidity as assumed in Clower (1967) and Lucas (1980).[6]

We assume that a fraction of investment is constrained by cash. Previous studies in economics and finance have identified many motives for firms to hold cash. Moreover, evidence indicates that firms have consistently held much more cash after 1980 with the average ratio of cash to total assets more than doubling in the past 20 years (Bates et al., 2006). These results suggest that at least a fraction of firm's investment may require cash.[7]

To complete the model, we specify government behavior. We assume that in each period the government may finance an exogenously given expenditure sequence $\{G_t\}$ that is not valued by private agents. As in the line of research in public finance approach, we abstract the effects of government expenditures and assume that government expenditures affect neither the production nor preferences. This assumption isolates the distortions generated by government expenditures. Money supply and consumption taxes are thus the two sources of distortions.

Following existing studies, we assume that the government spending is an exogenously given fraction of total output: $G_t = \beta y_t$, where $0 \leqslant \beta < 1$.[8] To finance its expenditures, the government relies on two sources of revenue: a seignorage tax (or the additional printing of money) and a consumption tax. The money supply is initially predetermined. Let μ_t be the growth rate of nominal money. The government budget constraint is given by

$$\mu_t m_t + \tau_{ct}c_t = G_t = \beta f(k_t, L - l_t). \tag{4}$$

[4] A version of the model with inelastic leisure is available upon request.

[5] To save space we do not consider income taxes, because the income tax was found to be more costly than the seignorage tax (Braun, 1994; Palivos and Yip, 1995).

[6] In this case consumption is a cash good and investment is a credit good. As investment is produced from the same sector as the consumption good but is not used for consumption in our model, the nature of our credit good is different from that in Lucas and Stokey (1983) and Chari et al. (1991).

[7] These are transaction motives (Mulligan, 1997), precautionary motives (Opler et al., 1999), cost reduction motives (Finnerty, 1980), agency motives (Jensen, 1986) and tax motives (Foley et al., 2006).

[8] For example, see Cooley and Hansen (1991), Palivos and Yip (1995), Lucas (2000) and Ho et al. (2007).

2.2. Optimization conditions

The representative agent's problem is to maximize (1), subject to (2), (3), taking as given the monetary growth rate, the tax rate, initial capital and initial nominal money holdings. Let $\lambda_t > 0$ be the co-state variable associated with the budget constraint, and $\xi_t > 0$ be the Lagrange multiplier of the CIA constraint. The necessary conditions are

$$u_1(c_t, l_t) = (1 + \tau_{ct})(\lambda_t + \xi_t), \tag{5a}$$

$$u_2(c_t, l_t) = \lambda_t f_2(k_t, L - l_t), \tag{5b}$$

$$\frac{1}{1+\rho}\{\lambda_{t+1}[f_1(k_{t+1}, L - l_{t+1}) + 1 - \delta] + \xi_{t+1}\varphi(1-\delta)\} = [\lambda_t + \varphi\xi_t], \tag{5c}$$

$$\frac{1}{1+\rho}(\lambda_{t+1} + \xi_{t+1}) = (1 + \pi_t)\lambda_t, \tag{5d}$$

and the transversality conditions: $\lim_{t\to\infty} \frac{1}{(1+\rho)^t}\lambda_t k_{t+1} = 0$ and $\lim_{t\to\infty} \frac{1}{(1+\rho)^t}\lambda_t m_{t+1} = 0$.

In these conditions, (5a) equalizes the marginal utility of consumption to the marginal cost of consumption. The marginal cost of consumption includes not only the shadow price of real balances but also the shadow price of the CIA constraint on consumption. Next, condition (5b) is the optimal condition for the labor and leisure tradeoff and stipulates that in optimum, the marginal utility of leisure must equal the marginal product of labor. Moreover, conditions (5c) and (5d) are the intertemporal no-arbitrage conditions for capital and real balances, respectively. Finally, the two transversality conditions are the usual "no Ponzi game" conditions on the two assets.

2.3. Equilibrium conditions

In equilibrium, the money and the goods markets must clear. The goods market clearing condition is $y_t = c_t + (k_{t+1} - k_t) + \delta k_t + G_t$ which, using (4), is

$$k_{t+1} - k_t = (1-\beta)f(k_t, L - l_t) - \delta k_t - c_t. \tag{6a}$$

The money market clearing condition is

$$(1 + \pi_t)m_{t+1} = (1 + \mu_t)m_t. \tag{6b}$$

Perfect-foresight equilibrium is a time path $\{c_t, l_t, m_t, k_t, \lambda_t, \xi_t, \pi_t\}$ and an endogenous policy variable of either μ_t or τ_{ct} that satisfy the agent's optimization, (5a), (5b), (5c), (5d), the money and the goods market clearance, (6a), (6b), the government budget constraint (4), and the binding CIA constraint (3).[9]

To determine the equilibrium, first, we substitute (5a) and (5b) into (5c) to obtain

$$\begin{aligned}&\frac{1}{(1+\rho)}\left\{\frac{[f_1(k_{t+1}, L - l_{t+1}) + (1-\delta)]u_2(c_{t+1}, l_{t+1})}{f_2(k_{t+1}, L - l_{t+1})} + \varphi(1-\delta)\left[\frac{u_1(c_{t+1}, l_{t+1})}{1+\tau_{ct+1}} - \frac{u_2(c_{t+1}, l_{t+1})}{f_2(k_{t+1}, L - l_{t+1})}\right]\right\}\\ &\quad = \frac{u_2(c_t, l_t)}{f_2(k_t, L - l_t)} + \varphi\left[\frac{u_1(c_t, l_t)}{1+\tau_{ct}} - \frac{u_2(c_t, l_t)}{f_2(k_t, L - l_t)}\right].\end{aligned} \tag{7a}$$

Condition (7a) is an intertemporal no-arbitrage condition for capital accumulation. It is worth noting that this condition is not directly affected by a seignorage tax. Moreover, the condition is not influenced by a consumption tax if $\varphi = 0$. However, if $\varphi > 0$ and thus investment is constrained by cash, this condition is directly affected by consumption taxes in periods t and $t + 1$. This is because now investment needs to use real balances and thus the after-tax shadow price of real balances influences the tradeoff between consumption and savings.

Next, substituting (5a) and (5b) into (5d) gives

$$\frac{1}{(1+\rho)}\frac{u_1(c_{t+1}, l_{t+1})}{(1+\tau_{ct+1})} = \frac{u_2(c_t, l_t)}{f_2(k_t, L - l_t)}(1 + \pi_t), \tag{7b}$$

where $1 + \pi_t = (1 + \mu_t)\frac{(1+\tau_{c_t})c_t + \varphi[(1-\beta)f(k_t, L-l_t) - c_t]}{(1+\tau_{c_{t+1}})c_{t+1} + \varphi[(1-\beta)f(k_{t+1}, L-l_{t+1}) - c_{t+1}]}$.[10]

Condition (7b) comes from an intertemporal no-arbitrage condition for real balances which governs the tradeoff between consumption and leisure. In this condition, the discounted, after-tax marginal utility of consumption in period $t + 1$ is equal

[9] Following Lucas (1980) and Wang and Yip (1992), we restrict our attention to an analysis of the case when the CIA constraint is binding in equilibrium. This requires that the monetary growth rate be greater than or equal to the growth rate of the discounted marginal rate of substitution for consumption in two consecutive periods.

[10] We use (6a) and the CIA constraint in (3) to attain $m_t = (1 + \tau_{ct} - \varphi)c_t + \varphi(1-\beta)f(k_t, L - l_t)$. The expression for $1 + \pi_t$ is obtained by combining this relationship with (6b) yields.

to the inflation-adjusted marginal utility of leisure relative to the marginal product of labor in periods t. If $\varphi = 0$, this condition becomes $\frac{1}{(1+\rho)}\frac{u_1(c_{t+1},l_{t+1})}{(1+\mu_t)(1+\tau_{c_t})} = \frac{u_2(c_t,l_t)}{f_2(k_t,L-l_t)}\frac{c_t}{c_{t+1}}$ and thus only the seignorage tax in period t and the consumption tax in period t affect the tradeoff between consumption in the next period and leisure in this period. However, if $\varphi > 0$, the consumption tax in period $t+1$ also influences the tradeoff between consumption in period $t+1$ and leisure in period t. Since investment needs real balances, the after-tax shadow price of real balances affects the tradeoff between consumption in period $t+1$ and leisure in period t.

It is worth noting that in (7a) and (7b) when $\varphi > 0$, a seignorage tax in period t affects the tradeoff about consumption and savings in periods t and $t+1$, but a consumption tax in period t affects the tradeoff about consumption and savings in periods $t-1$ and t. Thus, when there is a consumption tax change in period t, the agent will start to react to the change one-period earlier. As will be seen in the next section, this feature generates different transitional effects on consumption in response to the two tax changes.

Finally, (6a), (7a), and (7b) are the dynamical system and determine the equilibrium paths of c_t, k_t and l_t. The equilibrium paths of λ_t, ξ_t, π_t, m_t and μ_t or τ_{ct} are in turn determined from other equations.

In steady state, $c_t = c_{t+1} = c^*$, $k_t = k_{t+1} = k^*$, $l_t = l_{t+1} = l^*$ and $m_t = m_{t+1} = m^*$. Thus, (6b) leads to $\pi^* = \mu$.[11] The steady-state values of (k^*, c^*, l^*) are determined by[12]

$$f_1(k^*, L-l^*) = (\rho+\delta)\{1+\varphi[(1+\rho)(1+\mu)-1]\}, \tag{8a}$$

$$c^* = (1-\beta)f(k^*, L-l^*) - \delta k^*, \tag{8b}$$

$$\frac{u_2(c^*,l^*)}{u_1(c^*,l^*)} = \frac{1}{(1+\tau_c)(1+\mu)}\frac{f_2(k^*,L-l^*)}{(1+\rho)}. \tag{8c}$$

Finally, along the equilibrium path in steady state, the level of welfare is

$$U^* = \frac{1+\rho}{\rho}u(c^*,l^*). \tag{8d}$$

Clearly, the long-run welfare depends on the level of consumption and leisure in the long run.

3. Welfare costs of alternative ways of finance

We are ready to study the welfare effect of alternative ways of public financing: an exogenous increase in the share of public expenditure, $d\beta > 0$, financed either by an increase in a consumption tax rate, $d\tau_c > 0$, or an increase in the rate of monetary growth, $d\mu > 0$, in order to balance the government budget. To simplify the analysis, in the following we assume that initially there is no government expenditure and thus, both the tax rate and the monetary growth rate are zero; i.e., $\beta = \tau_c = \mu = 0$.

3.1. Steady-state effect

We now characterize the steady-state effect. The balance of the government budget, evaluated at $\beta = \mu = \tau_c = 0$, implies

$$d\beta = \frac{1}{f(k^*, L-l^*)}(m^* d\mu + c^* d\tau_c).$$

If the increase in government expenditure is financed by a consumption tax only, the effects on consumption and leisure in the long run, respectively, are

$$\frac{dc}{d\tau_c} = \frac{1}{\Delta}[(-J_{22})c^* + J_{12}] < 0, \tag{9a}$$

$$\frac{dl}{d\tau_c} = \frac{1}{\Delta}(J_{21}c^* - 1), \tag{9b}$$

where $\Delta \equiv J_{22} - J_{12}J_{21} < 0$, $J_{22} \equiv \left[\frac{u_{22}}{u_2} - \frac{u_{12}}{u_1} + \frac{1}{f_2}\left(f_{22} - \frac{f_{21}f_{12}}{f_{11}}\right)\right] < 0$, $J_{12} \equiv \left[\frac{(f_1-\delta)f_{12}}{-f_{11}} + f_2\right] > 0$ and $J_{21} \equiv \left(\frac{u_{21}}{u_2} - \frac{u_{11}}{u_1}\right) > 0$.

By contrast, if the increase in government expenditure is financed by a seignorage tax only, the effects on consumption and leisure in the long run, respectively, are

$$\frac{dc}{d\mu} = \frac{1}{\Delta}\left\{[(-J_{22})c^* + J_{12}] + \varphi\left[(-J_{22})\chi - J_{12}\frac{f_{21}}{f_2}\frac{\rho+\delta}{f_{11}}(1+\rho)\right]\right\} < 0, \tag{10a}$$

[11] An asterisk is used to denote a value in steady state.

[12] While (8b) comes from (6a) and (8c) from (7b), (8a) is from the combination of (7a) and (7b).

$$\frac{dl}{d\mu} = \frac{1}{\Delta}\left\{[J_{21}c^* - 1] + \varphi\left[J_{21}\chi + \frac{f_{21}}{f_2}\frac{\rho+\delta}{f_{11}}(1+\rho)\right]\right\}, \tag{10b}$$

where $\chi \equiv \frac{(\rho+\delta)(1+\rho)(f_1-\delta)}{-f_{11}} + \delta k^* > 0$.

In comparing (9a), (9b) with (10a), (10b), it is obvious that if $\varphi = 0$ and thus investment is not constrained by cash, the effects on consumption and leisure in the long run are the same in both types of taxes. Since welfare in the long run depends on the levels of consumption and leisure, both ways of the public finance have the same welfare cost. Note that while both taxes reduce consumption unambiguously, the effect on leisure is ambiguous. The reason is that these taxes generate a crowding-out effect away from leisure since the government takes resources away (through the term $J_{21}c^*/\Delta$). However, these two taxes create a substitution effect toward leisure because the price of consumption is higher relative to leisure (through the term $-1/\Delta$). The net effect depends on which effect dominates.

If $\varphi > 0$ and investment is constrained by cash, a seignorage tax exerts an extra distortion on capital that leads to a larger crowding-out effect on consumption (through the extra term $\varphi[\frac{(-J_{22})\chi}{\Delta} - \frac{J_{12}}{\Delta}\frac{f_{21}}{f_2}\frac{\rho+\delta}{f_{11}}(1+\rho)] < 0$ in (10a)). However, a seignorage tax may not lower leisure as compared to a consumption tax. Leisure is reduced due to complements of consumption and leisure in utility (through the extra term $J_{21}\varphi\chi/\Delta$ in (10b)), while there is a positive effect on leisure because of complements of capital and labor and thus substitutes of capital and leisure in the production (through the extra term $\varphi\frac{1}{\Delta}\frac{f_{21}}{f_2}\frac{\rho+\delta}{f_{11}}(1+\rho) > 0$ in (10b)). As a result, we obtain the following result.

Proposition 1. *In an optimal growth model when investment is constrained by cash, a switch from a consumption tax to a seignorage tax reduces consumption but has an ambiguous effect on leisure in the long run.*

The reasons for the above results are well-known in existing studies with inelastic labor. Intuitively, when investment is not constrained by cash, $(1 + \mu)m^* = (1 + \tau_c)c^*$ and a seignorage tax is just a tax on consumption. The efficient tax structure in the sense of Ramsey (1927) indicates that a seignorage tax should be the same as a consumption tax and thus they have the same welfare cost. When investment is constrained by cash, $(1 + \mu)m^* = (1 + \tau_c)c^* + \delta k^*$ in the long run and a seignorage tax generates a further distortion on capital. As capital is reduced in the long run, consumption is reduced more in a seignorage tax. A seignorage tax thus has a higher welfare cost than a consumption tax in the long run.

When there is a labor-leisure tradeoff, the same reason goes on except that now leisure is affected. However, we are not sure whether leisure is higher or lower than the initial level in response to the two taxes because of a crowding-out effect away from and a substitution effect toward leisure. In particular, when investment is constrained by cash, because of offsetting effects generated by a complement of consumption and leisure in utility and a complement of capital and labor in production, a seignorage tax may lead to a higher leisure than a consumption tax in the long run. As a result of an ambiguous effect on leisure, the welfare cost of a switch from a consumption tax to a seignorage tax is ambiguous. Only in the case when the harmful effect from consumption dominates, a seignorage tax has a higher welfare cost than a consumption tax.

Our results may be compared with Ho et al. (2007) wherein they found that under no production externalities, the welfare cost of a seignorage tax is always higher than the welfare cost of a consumption tax. Our result is different when only consumption is constrained by cash, the welfare cost of a seignorage tax is not higher than, but the same as, that of a consumption tax. This effect cannot emerge in Ho et al. (2007) under a MIU approach, even when the degree of real balances in utility is assumed to be the same as the degree of consumption, because real balances directly affect the utility. Thus, their marginal utility of real balances affects the marginal cost of holding capital and the tradeoff between consumption and savings.

Moreover, when investment is constrained by cash, we find that leisure may be increasing in both taxes. In particular, a seignorage may lead to more leisure than a consumption tax. This result is also different from that in Ho et al. which obtains a lower leisure in a seignorage tax. As a result, the welfare cost of a seignorage tax may be higher or lower than that of a consumption tax in our model. The welfare cost of a seignorage tax is higher than that of a consumption tax in our model only when the detrimental effect through consumption dominates.[13]

3.2. Dynamic effects

We have shown that when investment is constrained by cash, a seignorage tax leads to a lower consumption level than a consumption tax in the long run. In this sub-section, we will show that in early periods of tax increases, the level of consumption in a consumption tax is lower than that in a seignorage tax.

To analyze this effect, we substitute the tradeoff condition between consumption in period $t + 1$ and leisure in period t in (7b) to the tradeoff condition between consumption and savings in period t in (7a) and obtain

[13] In our numerical analysis below, the steady-state value of leisure under a seignorage tax is higher than that under a consumption tax. As the effect via lower consumption dominates, the welfare cost under a seignorage tax is quantitatively higher than that under a consumption tax in a steady state.

$$\left\{\frac{[f_1(k_{t+1},L-l_{t+1})+(1-\delta)]u_2(c_{t+1},l_{t+1})}{f_2(k_{t+1},L-l_{t+1})}+\varphi(1-\delta)\Lambda(t+1)\right\}$$
$$=\frac{u_1(c_{t+1},l_{t+1})c_{t+1}}{(1+\mu_t)(1+\tau_{c_{t+1}})c_t}\Omega(t+1,t;\varphi)+\varphi\left[\frac{(1+\rho)u_1(c_t,l_t)}{1+\tau_{ct}}-\frac{u_1(c_{t+1},l_{t+1})c_{t+1}}{(1+\mu_t)(1+\tau_{c_{t+1}})c_t}\Omega(t+1,t;\varphi)\right], \tag{11a}$$

where $\Lambda(t+1)\equiv\left[\frac{u_1(c_{t+1},l_{t+1})}{1+\tau_{ct+1}}-\frac{u_2(c_{t+1},l_{t+1})}{f_2(k_{t+1},L-l_{t+1})}\right]$ and $\Omega(t+1,t;\varphi)\equiv\frac{(1+\tau_{c_{t+1}})+\varphi[(1-\beta)f(k_{t+1},L-l_{t+1})-c_{t+1}]/c_{t+1}}{(1+\tau_{c_t})+\varphi[(1-\beta)f(k_t,L-l_t)-c_t]/c_t}$.

The right-hand side in (11a) is the after-tax, inflation-adjusted marginal utility of consumption in the next period, while the left-hand side is the marginal product of capital in the next period.[14] The above relationship is thus an intertemporal tradeoff between consumption in t and $t+1$.

In the case when φ = 0, (11a) is

$$\frac{[f_1(k_{t+1},L-l_{t+1})+(1-\delta)]u_2(c_{t+1},l_{t+1})}{f_2(k_{t+1},L-l_{t+1})}=\frac{u_1(c_{t+1},l_{t+1})c_{t+1}}{(1+\mu_t)(1+\tau_{c_t})c_t}. \tag{11b}$$

Now, because of no cash constraints on investment, the shadow price of real balances does not affect the intertemporal tradeoff for consumption. It is clear to see from (11b) that the intertemporal tradeoff condition is adjusted by a seignorage tax in t and a consumption tax in t. As only consumption requires real balances, it follows that a seignorage tax is just a consumption tax. Thus, both taxes have symmetric effects on the intertemporal tradeoff for consumption in (11b). As a result, their welfare costs are the same along a transitional path.

By contrast, when $\varphi > 0$, (11a) stipulates that the intertemporal tradeoff condition is also adjusted by a consumption tax $t+1$. This feature creates different dynamic effects under both taxes. If only a consumption tax is used, (11a) is

$$\left\{\frac{[f_1(k_{t+1},L-l_{t+1})+(1-\delta)]u_2(c_{t+1},l_{t+1})}{f_2(k_{t+1},L-l_{t+1})}+\varphi(1-\delta)\Lambda(t+1)\right\}$$
$$=\frac{u_1(c_{t+1},l_{t+1})c_{t+1}}{(1+\tau_{c_{t+1}})c_t}\Omega(t+1,t;\varphi)+\varphi[\frac{(1+\rho)u_1(c_t,l_t)}{1+\tau_{ct}}-\frac{u_1(c_{t+1},l_{t+1})c_{t+1}}{(1+\tau_{c_{t+1}})c_t}\Omega(t+1,t;\varphi)], \tag{12a}$$

while if only a seignorage tax is employed, (11a) becomes

$$\left\{\frac{[f_1(k_{t+1},L-l_{t+1})+(1-\delta)]u_2(c_{t+1},l_{t+1})}{f_2(k_{t+1},L-l_{t+1})}+\varphi(1-\delta)\Lambda^s(t+1)\right\}$$
$$=\frac{u_1(c_{t+1},l_{t+1})c_{t+1}}{(1+\mu_t)c_t}\Omega^s(t+1,t;\varphi)+\varphi\left[(1+\rho)u_1(c_t,l_t)-\frac{u_1(c_{t+1},l_{t+1})c_{t+1}}{(1+\mu_t)c_t}\Omega^s(t+1,t;\varphi)\right], \tag{12b}$$

where $\Lambda^s(t+1)\equiv[u_1(c_{t+1},l_{t+1})-\frac{u_2(c_{t+!},l_{t+1})}{f_2(k_{t+1},L-l_{t+1})}]$ and $\Omega^s(t+1,t;\varphi)\equiv\frac{1+\varphi[(1-\beta)f(k_{t+1},L-l_{t+1})-c_{t+1}]/c_{t+1}}{1+\varphi[(1-\beta)f(k_t,L-l_t)-c_t]/c_t}$.

Notice that τ_{ct+1} affects the tradeoff between c_t and c_{t+1} while μ_{t+1} affects the tradeoff between c_{t+1} and c_{t+2}. Thus, a consumption tax affects the consumption tradeoff one-period earlier.

To see the difference in the dynamic effect, suppose that the government increases the government expenditure permanently starting from period T onward. Suppose further that the government finances the expenditure by a consumption tax from T onward: $\tau_{ct}=0$ if $t<T$ and $\tau_{ct}>0$ if $t\geqslant T$. In period $T-1$ (12a) is

$$\left\{\frac{[f_1(k_T,L-l_T)+(1-\delta)]u_2(c_T,l_T)}{f_2(k_T,L-l_T)}+\varphi(1-\delta)\Lambda(T)\right\}=\frac{u_1(c_T,l_T)}{(1+\tau_T)}\frac{c_T}{c^*}\Omega(T,T-1;\varphi)$$
$$+\varphi\left[(1+\rho)u_1(c^*,l^*)-\frac{u_1(c_T,l_T)}{(1+\tau_{c_T})}\frac{c_T}{c^*}\Omega(T,T-1;\varphi)\right], \tag{13a}$$

where $\Lambda(T)\equiv\left[\frac{u_1(c_T,l_T)}{1+\tau_{cT}}-\frac{u_2(c_T,l_T)}{f_2(k_T,L-l_T)}\right]$ and $\Omega(T,T-1;\varphi)\equiv\frac{(1+\tau_{c_T})+\varphi[(1-\beta)f(k_T,L-l_T)-c_T]/c_T}{1+\varphi[(1-\beta)f(k^*,L-l^*)-c^*]/c^*}$.[15]

By contrast, suppose that the government finances the government expenditure by a seignorage tax; thus, $\mu_t=0$ if $t<T$ and $\mu_t>0$ if $t\geqslant T$. Then, in period $T-1$ (12b) is

$$\left\{\frac{[f_1(k_T,L-l_T)+(1-\delta)]u_2(c_T,l_T)}{f_2(k_T,L-l_T)}+\varphi(1-\delta)\Lambda^s(T)\right\}=u_1(c_T,l_T)\frac{c_T}{c^*}\Omega^s(T,T-1;\varphi)+\varphi[(1+\rho)u_1(c^*,l^*)$$
$$-u_1(c_T,l_T)\frac{c_T}{c^*}\Omega^s(T,T-1;\varphi)], \tag{13b}$$

where $\Lambda^s(T)\equiv[u_1(c_T,l_T)-\frac{u_2(c_T,l_T)}{f_2(k_T,L-l_T)}]$ and $\Omega^s(T,T-1;\varphi)\equiv\frac{1+\varphi[(1-\beta)f(k_T,L-l_T)-c_T]/c_T}{1+\varphi[(1-\beta)f(k^*,L-l^*)-c^*]/c^*}$.

[14] Recall from (5b) that $u_2/f_2=\lambda$ and is the shadow price of capital.

[15] Endogenous variables in periods before T are at initial steady-state levels.

It is clear from (13a) that the consumption tax τ_{cT} affects the intertemporal tradeoff between c_{T-1} and c_T in which $c_{T-1} = c^*$. By contrast, in (13b) the seignorage tax μ_T does not affect the intertemporal tradeoff between c_{T-1} and c_T. Thus, the consumption tax imposed in period T, τ_{cT}, exerts the intertemporal tradeoff effect one-period earlier than the seignorage tax imposed in period T, μ_{T}.

In the period $T - 1$, the intertemporal condition in (13a), the consumption tax τ_{cT} reduces after-tax marginal utility of c_T that makes c_T more expensive than c^*; as a result, c_T is lower in order to increase the marginal utility. A lower c_T indicates a required higher consumption tax rate to balance the government budget in period $T + 1$. Through the intertemporal tradeoff between c_T and c_{T+1} in (12a), c_{T+1} is even lower. The same reasoning indicates that c_t is decreasing over time until a new steady state that ends at an even lower level.

The effect of a seignorage tax μ_T starts from the period T intertemporal condition, a tradeoff between c_T and c_{T+1}. In the period T intertemporal condition which is obtained by evaluating (12b) at $t = T$, the marginal utility of c_{T+1} is decreased by $(1 + \mu_T)$ as compared to the marginal utility of c_T. This reduces the price of c_T relative to c_{T+1} and there is a switch of consumption away from c_{T+1} toward c_T. Thus, although c_T is reduced by μ_T, it is decreased by a smaller amount as compared to a consumption tax. In the intertemporal condition in period $T + 1$, the marginal utility of c_{T+2} is decreased by $(1 + \mu_{T+1})$ as compared to the marginal utility of c_{T+1}. Thus, c_{T+2} is smaller than c_{T+1}. The same reasoning indicates that c_t is decreasing over time until a new steady state that ends is at a lower level. Leisure is higher in a seignorage tax than in a consumption tax in the short run due to the complement between leisure and consumption and the substitute between leisure and capital.

To summarize our results,

Proposition 2. *In an optimal growth model with a cash constraint on investment, a switch from a consumption tax to a seignorage tax leads to higher consumption and higher leisure and the welfare cost is smaller in the short run.*

Since a seignorage tax also distorts capital accumulation, the level of consumption under a seignorage tax will be smaller than that under a consumption tax in finite periods. As a consequence, while the welfare cost of a seignorage tax is lower than the welfare cost of a consumption tax in the short run, the welfare cost of a seignorage tax is larger than the welfare cost of a consumption tax in the long run when the consumption effect dominates. The net effect thus depends on whether the short-run effect or the long-run effect dominates.

3.3. Calibration analysis

Now, we conduct a calibration analysis to quantify the relative welfare cost along both the transitional path and the steady state.

To quantify the model, we take Taylor's linear expansion of the equilibrium system (6a), (7a), and (7b) in the neighborhood of the unique steady state and obtain a Jacobean matrix. The values of c_t, k_t, l_t along the unique equilibrium path are then each represented by the sum of their own new steady state c^*, k^*, l^* and a product of three components: a coefficient, a stable root to the power of the time, and the corresponding eigenvector of the stable root. The coefficient is determined by boundary conditions.[16]

We now calibrate our model. We follow Keller (1976) and use the *constant elasticity of substitutability* (henceforth, CES) utility function: $u(c_t, l_t) = [ac_t^{\varepsilon} + (1-a)l_t^{\varepsilon}]^{\frac{1}{\varepsilon}}$, where $\varepsilon \equiv (\sigma - 1)/\sigma$ and $\sigma > 0$ is the *elasticity of substitutability* (henceforth, ES) and a is the intensity of consumption in utility relative to leisure. A felicity with a constant ES is consistent with a balanced growth path. We use the Cobb-Douglas production technology: $f(k) = Ak^{\alpha}(L - l)^{1-\alpha}$, where $0 < \alpha < 1$ is the share of capital and $A > 0$ is the coefficient of productivity. We calibrate the model in steady state to reproduce some key features representative of the US economy.

Following Cooley (1995), in our benchmark model we chose the annual rate of the capital depreciation at $\delta = 5\%$, the annual rate of the time preference at $\rho = 4\%$, and the value of the ES in consumption at $\sigma = 2.5$; thus, $\varepsilon = 0.6$. There is no data about the fraction of investment constrained by the liquidity. To be consistent with the observation that firms began to hold more cash after 1980 (Bates et al., 2006), we chose 5% of investment constrained by the liquidity and thus, $\varphi = 0.05$.[17]

In the theoretical model we assume the initial size of government expenditure is zero and to be consistent, we set $\beta = \mu = \tau_c = 0$ in calibration.[18] The time endowment is assumed to be $L = 10$ units. The leisure time is set at $l^* = 7.5$ in consistence with the fraction of time allocated to market at around 25%, as pointed out by Prescott (2006). Finally, we normalize $A = 1$, and then use (8a) to calibrate the share of capital in production in consonance with the annual capital-output ratio at 3.32 (Cooley, 1995, p.21). We obtain $\alpha = 0.3$. Then we combine (8b) and (8c) and calibrate the intensity of consumption in utility relative to leisure; we obtain $a = 0.3952$. Under the set of benchmark parameter values, we find a positive eigenvalue whose absolute value is smaller than 1 (stable root) and two complex eigenvalues whose absolute values are larger than 1 (unstable root). This quantitative result guarantees a unique equilibrium path toward the steady state. The unique steady state is $k^* = 13.9210$, $c^* = 3.4886$, $y^* = 4.1846$, $l^* = 7.5$ and $U^* = 150.0341$.

Now, suppose that the government increases the share of public expenditure from 0% to 10% of total output once and for all. When the spending is financed by a seignorage tax, a higher monetary growth rate is required in order to balance the

[16] Detailed methods concerning the determination of the equilibrium path of the endogenous variables under a tax change are available upon request.

[17] Our results hold if other values are used for the fraction of investment that is constrained by cash.

[18] When we change β in quantitative analysis later, both μ and τ_c are endogenously determined in a way to balance the government budget.

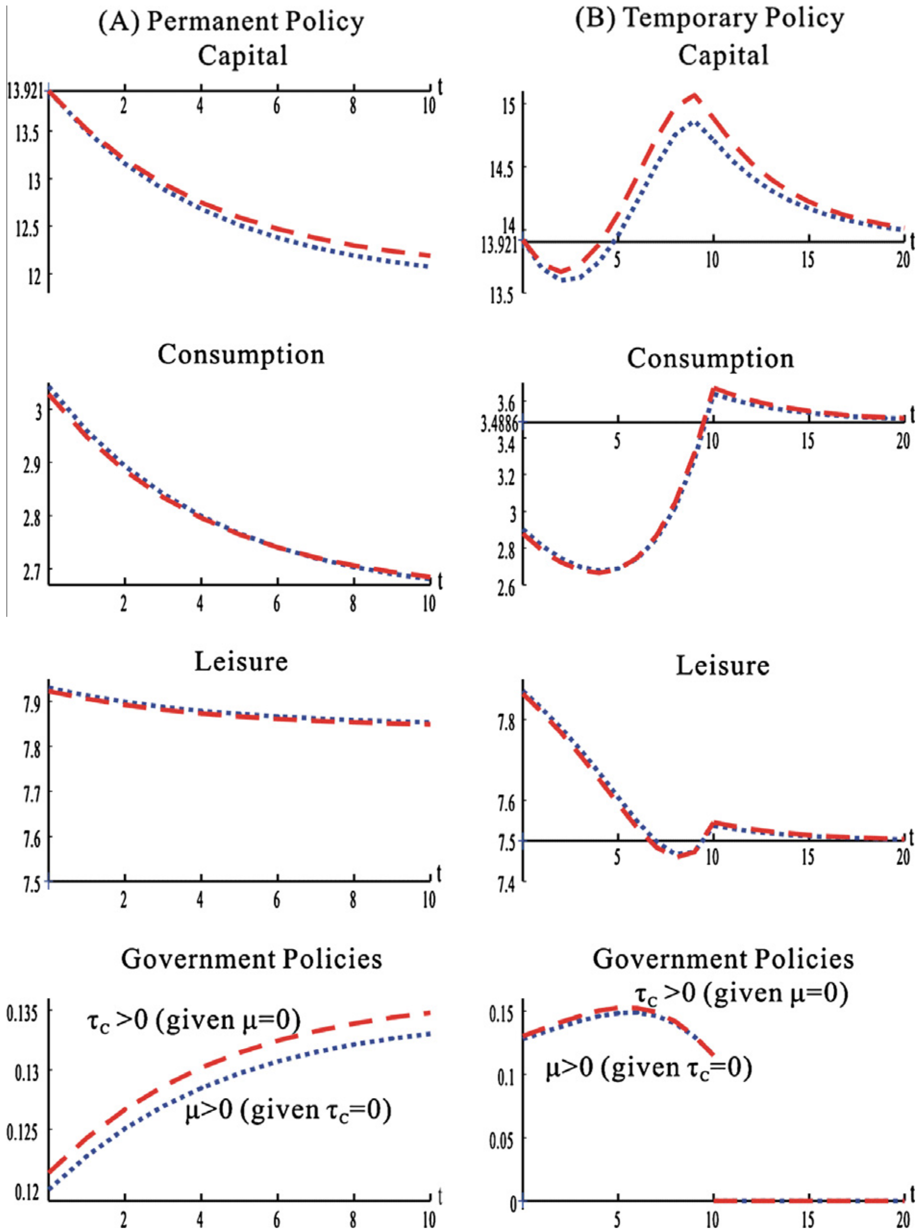

Fig. 1. Effects of alternative permanent ways of public financing in optimal growth model. *Notes*: (1) • • •: β = 10%, $\mu > 0$, and $\tau_c = 0$; – – –: β = 10%, $\tau_c > 0$ and $\mu = 0$. Other parameters are $\varepsilon = 0.6$, $\rho = 0.04$, $\varphi = 0.05$, $A = 1$, $\alpha = 0.3$, $a = 0.3952$, $\delta = 0.05$ and $L = 10$. (2) Initial steady state, under $\beta = \tau_c = \mu = 0$, is represented by the intersection of the horizontal and the vertical axes, except for consumption (whose initial steady state is $c^* = 3.4886$) and government policies (whose initial steady state is $\tau_c = \mu = 0$) in the panel A.

government budget in each period. Alternatively, in a consumption tax, it is necessary to levy a consumption tax rate in order to balance the government budget.[19] The corresponding monetary growth rate and the consumption tax rate and the effects on the equilibrium paths of capital stock, consumption and leisure are reported in panel A of Fig. 1.

It is clear from Fig. 1 that due to the crowding-out effect, both taxes have an adverse effect on capital, with a stronger adverse effect under a seignorage tax because of an additional distortion due to cash constraints on investment. Both types of taxes generate harmful effects on consumption, with a stronger adverse effect under a seignorage tax during later periods near a steady state; however, the negative effect under a consumption tax is stronger during earlier periods. Both types of

[19] When we increase β to 10%, our calculation finds a similar set of eigenvalues with a smaller absolute value of the stable eigenvalue under ($\mu > 0$, $\tau_c = 0$) and an even smaller absolute value of the stable eigenvalue under ($\tau_c > 0$, $\mu = 0$). We have checked the eigenvalues in every robustness analysis. In all cases of robustness analysis reported below, we find only one stable root.

Table 1
Welfare comparison between consumption tax and seignorage tax[a].

β	ε	φ	ρ	Δ_U (consumption tax)[b]	Δc (consumption tax)[c]	Δ_U (seignorage tax)[b]	Δc (seignorage tax)[c]
A. Permanent changes							
0.1	0.6	0.05	0.10	−3.7241	13.7619	−3.7200	13.7591
0.1	0.6	0.05	0.04	−3.5086	14.4640	−3.5006	14.4446
0.1	0.6	0.05	0.01	−3.4568	16.1827	−3.4468	16.1357
0.1	0.6	0.05	0.001	−3.4691	17.4525	−3.4577	17.3862
0.2	0.6	0.05	0.04	−6.7199	34.1890	−6.6845	34.0190
0.05	0.6	0.05	0.04	−1.7883	6.6977	−1.7868	6.6961
0.1	0.9	0.05	0.04	−2.8642	20.4422	−2.8574	20.4117
0.1	0.2	0.05	0.04	−3.6389	13.7166	−3.6309	13.6853
0.1	0.6	0.001	0.04	−3.5070	14.4621	−3.5069	14.4606
0.1	0.6	0	0.04	−3.5070	14.4620	−3.5070	14.4620
B. Temporary changes							
0.1	0.6	0.05	0.10	−2.2991	7.8296	−2.2919	7.8111
0.1	0.6	0.05	0.04	−1.1891	4.2524	−1.1758	4.2245
0.1	0.6	0.05	0.01	−0.3573	1.3682	−0.3524	1.3506
0.1	0.6	0.05	0.001	−0.0391	0.1536	−0.0384	0.1509
0.2	0.6	0.05	0.04	−2.4755	9.3324	−2.4089	9.0751
0.05	0.6	0.05	0.04	−0.5826	2.0481	−0.5813	2.0440
0.1	0.9	0.05	0.04	−1.1303	4.0986	−1.1167	4.0667
0.1	0.2	0.05	0.04	−1.2093	4.2721	−1.2028	4.2498
0.1	0.6	0	0.04	−1.1938	4.2661	−1.1938	4.2661

Notes:
[a] Baseline parameters are β = 0.1, ε = 0.6, φ = 0.05 and ρ = 0.04. Other common parameters are A = 1, α = 0.3, a = 0.3952, δ = 0.05 and L = 10.
[b] Δ_U is a percentage change in welfare from the benchmark, defined as $(U^* - U_0^*)/U_0^*$, where $U^* = \sum_{t=0}^{\infty}\frac{1}{(1+\rho)^t}u(c_t, l_t)$ and $U_0^* = \sum_{t=0}^{\infty}\frac{1}{(1+\rho)^t}u(c_0, l_0)$ are the welfare under a tax change and the welfare in the benchmark case, respectively.
[c] Δ_C is the consumption equivalence, calculated as $\sum_{t=0}^{\infty}\frac{1}{(1+\rho)^t}u((1+\Delta_C)c_t, l_t) = U_0^*$, where Δ_C is constrained to be the same along the dynamic equilibrium path.

taxes cause positive effects on leisure, with a stronger effect under a seignorage tax during early periods. The net effect on the welfare depends on whether the effect during early periods or the effect during later periods dominates.

We calculate two kinds of welfare costs. One kind is a percentage welfare change, denoted as Δ_U, that is calculated by the difference in the level of discounted utility under a tax change from its benchmark level divided by the benchmark level. We also calculate the welfare cost in terms of consumption equivalence, denoted as Δ_C. We find that under our calibrated discount rate of ρ = 0.04, the discounted utility under a consumption tax is lower than the discounted utility under a seignorage tax (see panel A in Table 1). In terms of consumption equivalence, the overall welfare cost of a 10% permanent increase in the government expenditure financed by a seignorage tax is 14.44%. This welfare cost is much higher than those of a 10% seignorage tax in Cooley and Hansen (1989) and Lucas (2000) due to two differences. First and foremost, a fraction of investment is constrained by cash in our model. Second, a 10% increase in the government expenditure requires more than 10% seignorage taxes.

As the discounted lifetime utility depends on the discount rate, one may wonder if other discounted rates may lead to a reverse welfare ranking order. We have experimented with the effects under many rates of time preferences from ρ = 0.001 to ρ = 0.1. With each ρ we recalculate the model under $\beta = \tau_c = \mu = 0$ in order to attain the equilibrium. We then increase β by 10% and calculate the welfare change under each way of finance. As our results show, in these plausible values of the discount rate, the welfare cost of a consumption tax is higher than the welfare cost of a seignorage tax.

We also study the sensitivity of the size of public spending and the curvature of utility. When we change the size of public spending to 20% and 5%, we find the same welfare ranking (Panel A, Table 1). When the curvature of utility is changed to one that is more risk averse, the welfare ranking is the same. For example, see Table 1 for ε = 0.2 and 0.9.

Since a cash constraint on investment is crucial to the welfare ranking, we reduce the value of φ and see if the welfare ranking changes. In our numerical exercises, when φ is reduced, the welfare cost difference is smaller but the welfare cost of a consumption tax is higher than that of a seignorage tax even if the value of φ is as small as 0.001. Only in the special case when investment is not constrained by cash (φ = 0), a permanent seignorage tax is like a permanent consumption tax (see Panel A in Table 1).[20]

We have also conducted quantitative exercises of temporary tax changes. Since a seignorage tax has a less harmful effect on capital than its consumption counterpart in the short run, it is expected that a temporary consumption tax has a higher

[20] A referee pointed out that the welfare cost of inflation tax in our model is larger than those in existing literature. We have done many other quantitative exercises and found that the welfare cost of inflation tax is lower only if β is reduced so the required money supply is smaller. We also found that even along a steady-state path the welfare cost of inflation tax is large in our model, although in this situation the welfare cot of inflation tax is larger, instead of smaller, than the welfare cost of consumption tax. These results are available upon request.

Table 2
Welfare comparison between temporary consumption tax and permanent seignorage tax[a].

Period (consumption tax)	Δ_U (consumption tax)[b]	Period (seignorage tax)	Δ_U (seignorage tax)[b]
10	−1.1891	∞	−3.5006
20	−1.9423	∞	−3.5006
30	−2.4488	∞	−3.5006
40	−2.7954	∞	−3.5006
50	−3.0287	∞	−3.5006
60	−3.1820	∞	−3.5006
70	−3.2887	∞	−3.5006
80	−3.3620	∞	−3.5006
90	−3.4086	∞	−3.5006
100	−3.4420	∞	−3.5006
110	−3.4619	∞	−3.5006
120	−3.4753	∞	−3.5006
130	−3.4886	∞	−3.5006
140	−3.4953	∞	−3.5006
150	−3.4990	∞	−3.5006
155	−3.5007	∞	−3.5006
160	−3.5021	∞	−3.5006

Notes:
[a] Parameter values are $\beta = 0.1$, $\varepsilon = 0.6$, $\varphi = 0.05$, $\rho = 0.04$, $A = 1$, $\alpha = 0.3$, $a = 0.3952$, $\delta = 0.05$ and $L = 10$.
[b] See the calculation method in Table 1.

welfare cost than a temporary seignorage tax. When we experiment with a 10-period increase in the share of public expenditure from 0% to 10% of total output, our results confirm this point (Panels B, Table 1 and Fig. 1).

Finally, one interesting case is the comparison between a temporary consumption tax and a permanent inflation tax. In Table 1, it is clear that under the benchmark parameter values, a 10-period consumption tax (see Panel B) has a lower welfare cost than a permanent seignorage tax (Panel A). We expect that if the number of period is increased until a threshold period, a temporary consumption tax has a lower welfare cost than that of a permanent seignorage tax. In Table 2, under the benchmark parameter values, we calculate the welfare cost of a temporary consumption tax in different periods of consumption tax changes. We find that if the number of periods of a consumption tax change is less than 155 periods, a temporary consumption tax has a welfare cost lower than that in a permanent seignorage tax.

4. Concluding remarks

Monetary economists have devoted considerable effort to measuring the welfare cost of seignorage taxes. Many studies have investigated monetary policy in the public finance approach. Except one study, this body of research has compared the welfare between a seignorage tax and an income tax as alternative ways of public financing. Moreover, they only compared their welfare costs in the long run. This study contributes to existing literature with two lines of thought. First, we compare the welfare of a seignorage tax with a consumption tax. Second, we compare the overall welfare by taking into account equilibrium paths along transitional dynamics.

We study the standard growth model with leisure and with a CIA constraint in which an exogenous stream of public spending is financed by either a seignorage tax or a consumption tax. We find that when investment is constrained by cash, a switch from a consumption tax to a seignorage tax lowers consumption but has an ambiguous effect on leisure in the long run. As a result, a seignorage tax has a higher welfare cost than a consumption tax only when investment is constrained by cash and the consumption effect dominates. Moreover, in the short run, a switch from a consumption tax to a seignorage tax leads to higher consumption and leisure and thus the welfare cost is smaller. Thus, when we take into account the transition effect and the steady-state effect, the overall welfare ranking between the two taxes are ambiguous. A consumption tax has a higher welfare cost during early periods while a seignorage tax has a higher welfare cost during later periods. Finally, our calibration exercises indicate that if investment is constrained by cash, under plausible rates of the time preference, the overall welfare cost of a consumption tax is higher than that of a seignorage tax.

As with all findings, our paper will certainly be improved upon over time and with the prospect of offering avenues for further research. We offer two possibilities of further research. First, as in most existing lines of research, our monetary model is a reduced-form approach that imposes a CIA constraint. There is no micro-foundation for the demand for money in our model. Lagos and Wright (2005) have proposed a new framework for monetary policy analysis based on a micro-founded search-theoretical model of money. With the money search, a buyer's share is less than 100% in the bargaining problem which fails to meet the Hosios (1990) condition for efficiency. The agent who holds money thus faces the holdup problem which leads to a higher welfare cost of inflation in the long run. It is interesting to extend our model by introducing the money search *a la* Largo and Wright (2005) and compare the overall welfare cost between a seignorage tax and a consumption tax along the transitional dynamics and steady state. As now the long-run cost of inflation is higher, we conjecture that the required rates of the time preference will be larger so the overall welfare cost of a seignorage tax is smaller than that of a consumption tax.

Second, our model has a representative agent, but a seignorage tax may have redistribution effects. Recently, Chiu and Molico (2010) have extended the model of Lagos and Wright (2005) to a model with heterogeneous agents. In their model, an agent needs to pay a random fixed cost in order to participate in the trade for the central market goods. An agent with a high realized fixed cost does not participate in such a trade and consumes zero central market goods. In this environment, mildly expansionary monetary policy can relax the liquidity constraint of some agents that works against a liquidity risk. As a result of the redistribution effect, the welfare cost of inflation is not only lower than that of Lagos and Wright (2005) but also lower than those using a reduced-form approach (e.g., Lucas, 2000). It is interesting to extend our model to the heterogeneous agent framework following Chiu and Molico (2010) and compare the overall welfare cost between a seignorage tax and a consumption tax along the transitional dynamics and steady state. As now the long-run cost of inflation is much lower, we conjecture that our results would continue to hold.

Acknowledgments

We'd like to thank two anonymous referees, Gary Hansen, Selo Imrohoroglu and participants at the GRIPS (Tokyo) Meetings, National Taiwan University and Osaka University for comments and suggestions. Part of the research was conducted when the authors visited the Department of Economics, Washington University in St. Louis. They thank the hospitality offered by the Department, especially Ping Wang, the Department Chair.

References

Abel, A.B., 1985. Dynamic behavior of capital accumulation in a cash-in-advance model. Journal of Monetary Economics 16, 55–71.
Aiyagari, S.R., Braun, R.A., Eckstein, Z., 1998. Transaction services, inflation, and welfare. Journal of Political Economy 106, 1274–1301.
Bates, T.W., Kahle, K.M., Stulz, R.M., 2006, Why do U.S. firms hold so much more cash than they used to? NBER Working Paper No. 12534.
Braun, R.A., 1994. How large is the optimal inflation tax. Journal of Monetary Economics 34, 201–214.
Chamley, C., 1986. Optimal taxation of capital income in general equilibrium with infinite lives. Econometrica 54, 607–622.
Chari, V.V., Christiano, L.J., Kehoe, P.J., 1991. Optimal fiscal and monetary policy: some recent results. Journal of Money, Credit, and Banking 23, 519–539.
Chen, B.-L., Hsu, M., Lu, C.-H., 2008. Inflation and growth: impatience and a qualitative equivalence. Journal of Money, Credit and Banking 40, 1309–1323.
Chiu, J., Molico, M., 2010. Liquidity, redistribution, and the welfare cost of inflation. Journal of Monetary Economics 57, 428–438.
Clower, R.W., 1967. A reconsideration of the micro foundations of monetary theory. Western Economic Journal 6, 1–9.
Cole, H.L., Stockman, A.C., 1992. Specialization, transaction technologies, and money growth. International Economic Review 33, 283–298.
Cooley, T.F., 1995. Frontiers of Business Cycle Research. Princeton University Press, Princeton.
Cooley, T.F., Hansen, G.D., 1989. The inflation tax in a real business cycle model. American Economic Review 79, 733–748.
Cooley, T.F., Hansen, G.D., 1991. The welfare costs of moderate inflation. Journal of Money, Credit and Banking 23, 483–503.
Diamond, D.W., Dybvig, P.H., 1983. Bank runs, deposit insurance, and liquidity. Journal of Political Economy 91, 401–419.
Dotsey, M., Ireland, P., 1996. The welfare cost of inflation in general equilibrium. Journal of Monetary Economics 37, 29–47.
Erosa, A., Ventura, G., 2002. On inflation as a regressive consumption tax. Journal of Monetary Economics 49, 761–795.
Espinosa-Vega, M.A., Yip, C.K., 2002. Government financing in an endogenous growth model with financial market restrictions. Economic Theory 20, 237–257.
Finnerty, J.D., 1980. Real money balances and the firm's production function. Journal of Money, Credit and Banking 12, 666–671.
Fischer, S., 1981. Towards an understanding of the costs of inflation: II. Camegie-Rochester Conference Series on Public Policy 15, 5–42.
Foley, C.F., Hartzell, J.C., Titman, C., Twite, G.J., 2006. Why do firms hold so much cash? A tax based explanation. NBER Working Paper No. 12649.
Friedman, M., 1969, The optimum quantity of money. In: The Optimum Quantity of Money, and Other Essays. Aldine Pub. Co., Chicago, pp. 1–50.
Gillman, M., 1993. The welfare cost of inflation in a cash-in-advance economy with costly credit. Journal of Monetary Economics 31, 97–115.
Gomme, P., 1993. Money and growth revisited: measuring the costs of inflation in an endogenous growth model. Journal of Monetary Economics 32, 51–77.
Ho, W.-M., Zeng, J., Zhang, J., 2007. Inflation taxation and welfare with externalities and leisure. Journal of Money, Credit and Banking 39, 105–131.
Hosios, A.J., 1990. On the efficiency of matching and related models of search and unemployment. Review of Economic Studies 57, 279–298.
Ireland, P.N., 1994. Money and growth: an alternative approach. American Economic Review 84, 47–65.
Ireland, P.N., 2007. Changes in the Federal Reserve's inflation target: causes and consequences. Journal of Money, Credit and Banking 39, 1851–1882.
Itaya, J.-I., Mino, K., 2003. Inflation, transaction costs and indeterminacy in monetary economies with endogenous growth. Economica 70, 451–470.
Jensen, M., 1986. Agency costs of free cash flow, corporate finance and takeovers. American Economic Review 76, 323–329.
Judd, K.L., 1985. Redistributive taxation in a simple perfect foresight model. Journal of Public Economics 28, 59–83.
Keller, W.J., 1976. A nested CES-type utility function and its demand and price-index functions. European Economic Review 7, 175–186.
Kimbrough, K.P., 1986. The optimum quantity of money rule in the theory of public finance. Journal of Monetary Economics 18, 227–284.
Largo, R., Wright, R., 2005. A unified framework for monetary theory and policy analysis. Journal of Political Economy 113, 463–484.
Lucas Jr., R.E., 1980. Equilibrium in a pure currency economy. Economic Inquiry 18, 203–220.
Lucas Jr., R.E., 2000. Inflation and welfare. Econometrica 68, 247–274.
Lucas Jr., R.E., Stokey, N.L., 1983. Optimal fiscal and monetary policy in an economy without capital. Journal of Monetary Economics 12, 55–93.
Mino, K., Shibata, A., 1995. Monetary policy, overlapping generations, and patterns of growth. Economica 62, 179–194.
Mulligan, C.B., 1997. Scale economies, the value of time, and the demand for cash: longitudinal evidence from firms. Journal of Political Economy 105, 1061–1079.
Opler, T., Pinkowitz, L., Sulz, R., Williamson, R., 1999. The determinants and implications of corporate cash holdings. Journal of Financial Economics 52, 3–46.
Palivos, T., Yip, C.K., 1995. Government expenditure financing in an endogenous growth model: a comparison. Journal of Money, Credit and Banking 27, 1159–1178.
Phelps, E.S., 1973. Inflation in the theory of public finance. Swedish Journal of Economics 75, 67–82.
Prescott, E.C., 1987. A multiple means-of-payment model. In: Barnett, W.A., Singleton, K.J. (Eds.), New Approaches to Monetary Economics. New York, Cambridge University Press, pp. 42–51.
Prescott, E.C., 2006. Nobel lecture: the transformation of macroeconomic policy and research. Journal of Political Economy 114, 203–235.
Ramsey, F.P., 1927. A contribution to the theory of taxation. Economic Journal 37, 47–61.
Stockman, A.C., 1981. Anticipated inflation and the capital stock in a cash-in-advance economy. Journal of Monetary Economics 8, 387–393.
Wang, P., Yip, C.K., 1992. Alternative approaches to money and growth. Journal of Money, Credit and Banking 23, 553–562.
Wu, Y., Zhang, J., 2000. Monopolistic competition, increasing returns to scale, and the welfare costs of inflation. Journal of Monetary Economics 46, 417–440.