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The Dynamic Relationship between Inflation and Output Growth in a Cash-Constrained Economy*

Chia-Hui Lu, Been-Lon Chen, and Mei Hsu

Abstract

This paper studies the dynamic relationship between inflation and output growth in neoclassical growth models with endogenous cash constraints. We show this dynamic relationship is negative if the degree of cash constraints on investment is smaller than the degree of cash constraints on consumption but is positive if otherwise.

KEYWORDS: capital accumulation, cash constraint, dynamics

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1 Introduction

This paper investigates the effects of monetary growth on the dynamic equilibrium path of output in a neoclassical growth model with a cash-in-advance (hereafter CIA) constraint. The CIA model advocated by Clower (1967) and followed by Lucas (1980) has been one of the most popular methods to best introduce money into optimizing models. In an influencial paper, Stockman (1981) assumed that the CIA constraint applies equally to consumption and investment. His model predicted a negative long-run relationship between output and inflation, as in Friedman (1977), as opposed to long-run neutrality in the Clower-Lucas model where the CIA constraints apply to consumption only. Using the model in Stockman, Abel (1985) found that money affects the speed of adjustment along the transition path toward the steady state if the CIA constraint applies to gross investment as well as consumption.

The CIA constraint, as used in Stockman (1981) and Abel (1985), implicitly assumes that the degree of CIA constraints on investment relative consumption is 1.[1] The relative degree of CIA constraints on investment and consumption is more general than what were indicated by Stockman (1981) and Abel (1985). Recently, some researchers have considered the degree of CIA constraints on investment relative to that on consumption smaller than 1; e.g., Wang and Yip (1992), Palivos, et al (1993) and Chang and Tsai (2003).[2] However, there is no reason a priori to rule out the prospect that the degree of CIA constraints on investment relative to consumption is greater than the concept of one. Such a possibility is higher in an economy where firms' motives to hold cash are high and consumers use more credit. Indeed, economics and finance literature have identified at least five motives for firms to hold cash and moreover, evidence seems to indicate that firms hold more cash after 1980 with an increasing average ratio of cash to total assets in the past 20 years.[3] Furthermore, consumer credit has doubled during the postwar period, with particularly sharp

[1] Other papers employing an equal CIA constraint on consumption and investment include Crettez, et al (1999), Dotsey and Sarte (2000), and Gong and Zou (2001), among others.

[2] While Wang and Yip (1992) studied a qualitative equivalence between alternative monetary approaches in a steady state, Palivos, et al (1993) analyzed the velocity of money in a steady state and Chang and Tsai (2003) examined how the fraction of CIA constraints on investment relative to consumption affects the relationship between inflation and capital in a steady state.

[3] Motives include a transaction motive (Baumol, 1952; Mulligan, 1997), precautionary motive (Opler, et al, 1999), cost reduction motive (Finnerty, 1980), agency motive (Jensen, 1986) and/or tax motive (Foley, et al, 2006). Using all Compustat firm-year observations from 1980 to 2003 in the U.S., Bates, et al (2006, Table 1) found that the average cash ratio, measured as the ratio of cash and marketable securities to the book value of total assets, had an increasing trend from 0.1038 in 1980 to 0.2304 in 2003.

and sustained increases occurring over the past 20 years.[4] The evidence suggests that firms hold more cash and consumers use more credit than what one might be expected to assume. Even if there is no sufficient evidence to suggest that the degree of CIA constraints on investment is higher than that on consumption, the recent evidence does show such a possibility, at least during some periods of time.

The purpose of this paper is to study the dynamic behavior between inflation and output in an otherwise standard neoclassical growth model with general cash constraints. Our model has two important features. First, a general cash constraint is considered. We allow for the fraction of cash constraints on investment to that on consumption to be higher than one as opposed to those only smaller than or equal to one in existing setups.[5] Second, we allow the agent to spend resources in supplying credit for consumption.[6] The credit supply leads to an endogenous degree of cash constraints on consumption and thus a variable consumption velocity of money. As a result, the relative CIA constraint on investment and consumption is endogenous. We show that the dynamic relationship between inflation and output depends crucially on whether the degree of CIA constraints on investment is larger or smaller than the initial degree of CIA constraints on consumption. The reason may be understood using an example of a permanent increase in the growth rate of money.

With a permanent higher money growth rate and thus permanent higher inflation, there is an ambiguous effect on the labor supply as inflation directly increases leisure but lowers consumption which indirectly reduces leisure. However, when the intertemporal elasticity of substitution for consumption is smaller than one as in the data, inflation increases the labor supply. Moreover, the agent increases resources to supply credit and relax the degree of cash constraints on consumption. This renders more real balances available for investment which increases capital stock. For given initial cash constraints on consumption, if the degree of cash constraints on investment is large (*resp*. small), the agent has more (*resp*. less) incentives to supply credit for consumption in order to release more (*resp*. less) real balances for investment. Thus, capital increases more (*resp*. less) if the degree of cash constraints on investment is large (*resp*. small). As capital and labor are complements in production, the labor supply in the goods sector increases more if the degree of cash constraints on investment is large. Yet, the labor supply in the goods sector increases less or

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[4] Using consumer installment credit collected by the Board of Governors of the Federal Reserve, Ludvigson (1999) found the ratio of consumer credit to personal income in the U.S. increased from 7% in 1953 to 10% in 1980 and to nearly 13% in 1993.

[5] In a one-sector endogenous growth model with a CIA constraint, Chen and Guo (2008) allow for the possibility of the degree of CIA constraints on investment to be larger than the degree on consumption. They studied the conditions under which local indeterminacy arises.

[6] We follow Gillman and Kejak (2005, 2009) in setting the credit sector.

does not increase at all if the degree of cash constraints on investment is small because the use of capital in the credit supply may reduce capital employed in the goods production. Thus, there are three conflicting effects on output production. First, the positive capital stock effect is beneficial to output production. Next, due to a large fraction of capital allocated to credit production, the negative capital reallocation effect is harmful to output production. Finally, the ambiguous labor supply in the goods sector has an ambiguous effect on output production. A larger degree of cash constraints on investment enhances the positive capital stock effect, turns the labor supply from a negative or a small positive effect to a large positive effect, but also strengthens the negative capital reallocation effect. When the degree of cash constraints on investment is smaller than the initial degree of cash constraints on consumption, the positive capital stock effect and the labor supply effect are dominated by the negative capital reallocation effect. Thus, output is lower. However, when the degree of cash constraints on investment is larger than the initial degree of cash constraints on consumption, the positive capital stock effect is larger and there is a large positive labor supply effect. Although the negative capital reallocation effect is also larger, it is dominated by the two positive effects. As a result, output increases.

Our dynamic system involves three equations and is complex. In particular, as the cash constraint on consumption is endogenous, in an analytical solution it is difficult to see whether along the transition the degree of cash constraints on consumption is larger or smaller than the degree of cash constraints on investment. Quantitative analysis provides an alternative tool which allows us to control the environment under which the degree of cash constraints on consumption is larger or smaller than the degree of cash constraints on investment. We solve a parameterized version of our model and characterize the model in a steady state. We calibrate the model to be consistent with the U.S. using annual data and then quantitatively analyze the effects of expansionary monetary changes. Two kinds of changes are carried out: a permanent higher money growth rate and a temporary higher money growth rate.

Here, we offer two main findings. First, in a permanent higher money growth rate, the relationship between inflation and output is positive if the degree of cash constraints on investment is larger than the initial degree of cash constraints on consumption but is negative if otherwise. Second, in a temporary higher money growth rate, the relationship between inflation and output is always negative in the period when the policy change is in effect, but after the policy change is ended, the relationship turns positive if the degree of cash constraints on investment is larger than the initial degree of cash constraints on consumption but is still negative if otherwise.

Our model sheds light on an ambiguous dynamic relationship between inflation and output/growth in existing studies. For example, using international panel data (Gillman et al. 2004) and international G7 time series (Fountas et al. 2006), these authors found that inflation caused a negative long-run effect on output. However, using a 5-year average or annual data across countries, Bruno and Easterly (1998) found no robust relationship between inflation and growth except for high inflation countries. Earlier, Mankiw (1989) even stipulated that the inflation-growth correlation is positive at cyclical frequencies in developed economies. Boyd and Smith (1998) argued that developing countries with less developed financial systems appeared to suffer more severely from the effects of inflation than do developed countries. These later studies suggest that the inflation-output correlation is ambiguous in the short run and depends on financial systems. At different development stages, financial systems may offer different types of credits and apply different credit limits to firms and consumers. This leads to different degrees of liquid constraints on firms relative to consumers, and as a result, the relationship is ambiguous between inflation and growth.

This paper is organized as follows. Section 2 sets up the model and studies the optimization and the steady state. Section 3 investigates the patterns of equilibrium paths under two types of monetary policies. Finally, some concluding remarks are offered in Section 4.

2 The Model

The model is based on Stockman (1981), Abel (1985), Wang and Yip (1992) and Gillman and Kejak (2005, 2009). A representative agent is endowed with one unit of time. The agent allocates the fraction l of the time endowment to the goods sector, and the fraction n to the credit sector, with the remaining fraction x=1-l-n to leisure. The representative individual's lifetime utility is

$$U = \int_0^\infty u(c,x)\, e^{-\rho t} dt. \tag{1}$$

where c is individual's consumption and ρ>0 is the time preference rate. The felicity is strictly increasing and strictly concave in consumption and leisure; i.e., $u_i(c,x)>0>u_{ii}(c,x)$, $i=c,x$. To simplify the analysis, we assume that the felicity is separable in consumption and leisure and is the following form: [7] $u(c,x)=\frac{c^{1-\sigma}-1}{1-\sigma}+\chi\frac{x^{1-\varepsilon}-1}{1-\varepsilon}$. This utility function allows for differences between the intertemporal elasticity of substitution (IES) for consumption and the IES for leisure.

[7] This utility has been widely used; see Benhabib and Perli (1994) and Ladrón-de-Guevara et al. (1999), among others.

The goods production function is $y=f(k_y, l)$, where k_y is capital per capita used in the goods production. We assume that the goods production function is strictly increasing and strictly concave in capital and labor. To simplify the analysis, we use the following Cobb-Douglas function $f(k_y)=A(k_y)^{\alpha}(l)^{1-\alpha}$, where $A>0$ and $0<\alpha<1$.

The representative agent's budget constraint is thus

$$\dot{k}+\dot{m}=f(k_y,l)-c-\pi m-\delta k+v, \tag{2}$$

where m is real money holdings per capita, v is real transfers per capita from the government, π is the inflation rate, and δ is the depreciation rate of capital. Nominal money supply is initially given and assumed to grow at a constant rate μ.

Denote I the gross investment per capita. The gross investment net of the depreciation forms new capital in the way as follows.

$$\dot{k}=I-\delta k. \tag{3}$$

The representative agent faces the following CIA constraint

$$\varphi_c c+\varphi_I I\le m,\ 0\le\varphi_I\le 1. \tag{4a}$$

It is interesting to note that if $\varphi_c=\varphi_I$, consumption and investment are both equally cash constrained. This is the constraint employed by Stockman (1981) and Abel (1985). If $\varphi_c\geq\varphi_I$, consumption is more cash constrained than investment. This is the constraint as utilized in Wang and Yip (1992). Our setting is more general than all these existing cases in that we allow for both cases of $\varphi_c\geq\varphi_I$ and $\varphi_c\leq\varphi_I$. Moreover, we consider a flexible φ_c, which indicates a variable consumption velocity of money, $1/\varphi_c$.

To allow for flexible degrees of cash constraints, we assume that the agent can allocate resource to the production of credit, which is used as an alternative to money. We assume that, given φ_I, the agent chooses φ_c by determining how much resource to spend supplying credit. Denote the total real credit by d. Then, the total real credit is equal to the fraction of consumption not bought by cash, i.e. $d\equiv(1-\varphi_c)c$. Following Gillman and Kejak (2005, 2009), the production function of credit is given by

$$d=cB(\tfrac{k_d}{c})^{\gamma_1}(\tfrac{n}{c})^{\gamma_2},\ ,\ B>0,\ 0<\gamma_1,\gamma_2<1,\ \gamma_1+\gamma_2<1, \tag{4b}$$

where k_d is capital per capita used in the credit production.

This credit production function exhibits constant returns to scale in its three factors, capital, labor and consumption. Given $\gamma_1+\gamma_2<1$, the function brings about an upward sloping marginal cost of the credit supply per unit of consumption. With d, we obtain $\varphi_c = 1-\frac{d}{c} = 1- B(\frac{k_d}{c})^{\gamma_1}(\frac{n}{c})^{\gamma_2}$. Thus, the fraction of cash constraints on consumption is increasing in consumption and decreasing in capital and labor.

In any period, capital stock may be allocated to the goods production and the credit production. Thus, capital usages face a resource constraint: $k=k_y+k_d$. Let s be the fraction of total capital allocated to the credit production, $k_d=sk$. Then, $1-s=k_y/k$ is the fraction of total capital allocated to the goods production.

2.1 Optimization conditions

The representative agent's problem is to maximize (1), subject to (2)-(4). Let $\lambda_k>0$ and $\lambda_m>0$ be the co-state variables associated with capital and real money holdings, respectively, and $\xi>0$ be the Lagrange multiplier of the CIA constraint. The necessary conditions are

$$u_c(c,x) = \lambda_m + \xi[1-(1-\gamma_1-\gamma_2)B(sk)^{\gamma_1} n^{\gamma_2} c^{-\gamma_1-\gamma_2}], \tag{5a}$$

$$\lambda_k = \lambda_m + \xi\varphi_I, \tag{5b}$$

$$\lambda_m \alpha A[(1-s)k]^{\alpha-1} l^{1-\alpha} = \xi\gamma_1 B(sk)^{\gamma_1-1} n^{\gamma_2} c^{1-\gamma_1-\gamma_2}, \tag{5c}$$

$$u_x(c,x) = \lambda_m (1-\alpha) A[(1-s)k]^{\alpha} l^{-\alpha}, \tag{5d}$$

$$u_x(c,x) = \xi\gamma_2 B(sk)^{\gamma_1} n^{\gamma_2-1} c^{1-\gamma_1-\gamma_2}, \tag{5e}$$

$$\dot{\lambda}_k = (\rho+\delta)\lambda_k - \lambda_m \alpha A[(1-s)k]^{\alpha-1} l^{1-\alpha}, \tag{5f}$$

$$\dot{\lambda}_m = \rho\lambda_m - [-\lambda_m \pi + \xi], \tag{5g}$$

and the transversality conditions $\lim_{t\to\infty} e^{-\rho t}\lambda_{kt}k_t = 0$ and $\lim_{t\to\infty} e^{-\rho t}\lambda_{mt}m_t = 0$.

In these conditions, (5a) equalizes the marginal utility of consumption to the marginal cost of consumption, the latter being the sum of the shadow price of real balances and the shadow price of the CIA constraint on consumption. Next, in (5b) optimal investment requires no arbitrage between capital and real balances. Thus, the shadow price of capital must equal the shadow price of real balances and the shadow price of the CIA constraint on investment. Moreover, in (5c) optimal allocation of capital is determined by equalizing the marginal products between the two sectors. Furthermore, (5d) and (5e) are optimal conditions that tradeoff between leisure and working in the goods sector and tradeoff between leisure and working in the credit sector, respectively. Finally, conditions (5f) and

(5g) are the intertemporal no-arbitrage conditions which govern how each of the two Hamiltonian shadow prices changes over time.

2.2 Equilibrium and Steady State

In equilibrium, real transfers from the government are financed by the increase in the money supply; thus, $v=\mu m$. The money and the goods markets are both clear,

$$\dot{m} = (\mu - \pi)m, \tag{6a}$$

$$\dot{k} = A\big((1-s)k\big)^{\alpha} l^{1-\alpha} - c - \delta k. \tag{6b}$$

Using (4a) and (4b), the binding cash-in-advance constraint is[8]

$$m = (1-\varphi_I)c + \varphi_I A\big((1-s)k\big)^{\alpha} l^{1-\alpha} - B(sk)^{\gamma_1} n^{\gamma_2} c^{1-\gamma_1-\gamma_2}. \tag{6c}$$

A perfect foresight equilibrium is a time path $\{c, m, k, s, l, n, \lambda_k, \lambda_m, \xi, \pi\}$. The time path satisfies agent's optimization, (5a)-(5g), the money and the goods market equilibrium conditions, (6a)-(6b), and the binding CIA constraint (6c).

We will simplify the equilibrium conditions into a three-dimensional dynamical system with state vector (c, k, l).

First, combining (5c)-(5e) yields

$$n = \tfrac{\alpha}{1-\alpha}\tfrac{\gamma_2}{\gamma_1} l \tfrac{s}{1-s} \equiv n(s,l), \tag{7a}$$

which indicates that the labor in the credit sector is positively correlated to the labor in the goods sector. This result emerges because the two kinds of labor need to tradeoff with leisure and more the marginal product of labor between the two sectors needs to be equal.

With the use of (7a), (5d) leads to

$$\lambda_m = \tfrac{u_x(c,x)}{(1-\alpha)A}[(1-s)k]^{-\alpha} l^{\alpha} \equiv \lambda_m(c,k,l,s). \tag{7b}$$

Then, substituting ξ from (5b) into (5c), along with (7a), gives

[8] Following Lucas (1980) and Wang and Yip (1992), we assume the CIA constraint is binding in equilibrium. In our continuous-time framework, this requires that the monetary growth rate be greater than or equal to the discounted marginal rate of substitution between consumption in two consecutive points in time.

$$\lambda_k = \lambda_m \{1 + \tfrac{\varphi_I \alpha A}{\gamma_1 B}(1-s)^{\alpha-1} s^{1-\gamma_1} k^{\alpha-\gamma_1} l^{1-\alpha} n^{-\gamma_2} c^{\gamma_1-\gamma_2-1}\} \equiv \lambda_k(c,k,l,s). \tag{7c}$$

Moreover, if we substitute ξ from (5b) to (5a), along with (5c) and (5d), we obtain

$$\frac{u_c(c,x)(1-\alpha)A[(1-s)k]^{\alpha}}{u_x(c,x)l^{\alpha}} = 1 + \frac{\alpha A s^{1-\gamma_1} k^{\alpha-\gamma_1} l^{1-\alpha}}{\gamma_1 B(1-s)^{1-\alpha} n^{\gamma_2} c^{1-\gamma_1-\gamma_2}} - \frac{(1-\gamma_1-\gamma_2)\alpha A s k^{\alpha} l^{1-\alpha}}{\gamma_1 c(1-s)^{1-\alpha}}. \tag{7d}$$

This equation equalizes the marginal rate of substitution (MRS) between consumption and leisure and the marginal rate of technical substitution (MRTS) of the capital allocation between the goods production and the credit production. With the use (7a), (7d) stipulates the relationship s=$s(c, k, l)$.

Next, if we differentiate (7b), (7c) and (7d), along with (7a) and after manipulation, we obtain

$$\frac{\dot{l}}{l} = \frac{-1}{\Theta_1}\left\{-\left(\frac{u_{cc}c-\Lambda}{1-\gamma_1-\gamma_2}\frac{1}{\lambda_k-\lambda_m}+\frac{1}{\varphi_I}\right)\dot{\lambda}_k + \frac{\left[\frac{u_{cc}c-\Lambda}{1-\gamma_1-\gamma_2}\frac{\lambda_k}{\lambda_k-\lambda_m}-u_c+\frac{\lambda_k}{\varphi_I}-\frac{\Xi}{\Omega_1}\right]\dot{\lambda}_m}{\lambda_m} + \frac{\left[\frac{u_{cc}c-\Lambda}{1-\gamma_1-\gamma_2}(\alpha-\gamma_1)+\Lambda\alpha-\frac{\alpha\Xi}{\Omega_1}\right]\dot{k}}{k}\right\}, \tag{(8a)}$$

$$\frac{\dot{c}}{c} = \frac{1}{1-\gamma_1-\gamma_2}\left\{\frac{-\dot{\lambda}_k}{\lambda_k-\lambda_m}+\frac{\lambda_k}{\lambda_k-\lambda_m}\frac{\dot{\lambda}_m}{\lambda_m}+\frac{\left[\frac{(1-\alpha)s}{1-s}+1-\gamma_1-\frac{\gamma_2}{1-s}\right]}{\frac{u_{xx}}{u_x}\frac{n}{1-s}-\frac{\alpha s}{1-s}}\left[-\frac{\dot{\lambda}_m}{\lambda_m}-\alpha\frac{\dot{k}}{k}+\left(\alpha-\frac{u_{xx}(l+n)}{u_x}\right)\frac{\dot{l}}{l}\right]+\frac{(\alpha-\gamma_1)\dot{k}}{k}+\frac{(1-\alpha-\gamma_2)\dot{l}}{l}\right\}, \tag{8b}$$

where $\Omega_1 = \frac{u_{xx}}{u_x}\frac{n}{1-s}-\frac{\alpha s}{1-s}$,

$\Lambda = (1-\frac{1}{\varphi_I})\lambda_m + \frac{1}{\varphi_I}\lambda_k - u_c$,

$\Xi = \frac{u_{cc}c-\Lambda}{1-\gamma_1-\gamma_2}\left[\frac{(1-\alpha)s}{1-s}+1-\gamma_1-\frac{\gamma_2}{1-s}\right]+\Lambda(1-\frac{(\alpha-1)s}{1-s})$,

$\Theta_1 = \frac{u_{cc}c-\Lambda}{1-\gamma_1-\gamma_2}(1-\alpha-\gamma_2)+\Lambda(1-\alpha)+\frac{\Xi}{\Omega_1}\left[\alpha-\frac{u_{xx}}{u_x}(\ell+n)\right]$.

Similarly, totally differentiating (6c), along with (6a), (7a)-(7d) and (8a)-(8b), yields

$$\pi = \frac{1}{m+\tilde{H}}\left[\mu m-\tilde{G}\dot{\lambda}_k-\tilde{H}\left(\rho+\frac{1}{\varphi_I}-\frac{1}{\varphi_I}\frac{\lambda_k}{\lambda_m}\right)-\tilde{F}\frac{\dot{k}}{k}\right] \equiv \pi(c,k,l), \tag{9a}$$

where $\tilde{G} = \frac{\left[(1-\varphi_I)c-B(sk)^{\gamma_1}n^{\gamma_2}c^{1-\gamma_1-\gamma_2}(1-\gamma_1-\gamma_2)\right]}{1-\gamma_1-\gamma_2}\frac{-1}{\lambda_k-\lambda_m}+\left[\frac{\Gamma}{\Theta_1\Omega_1}\left[\alpha-\frac{u_{xx}}{u_x}(\ell+n)\right]+\frac{\tilde{Z}}{\Theta_1}\right](\frac{u_{cc}c-\Lambda}{1-\gamma_1-\gamma_2}\frac{1}{\lambda_k-\lambda_m}+\frac{1}{\varphi_I})$,

$\tilde{H} = \frac{\left[(1-\varphi_I)c-B(sk)^{\gamma_1}n^{\gamma_2}c^{1-\gamma_1-\gamma_2}(1-\gamma_1-\gamma_2)\right]}{1-\gamma_1-\gamma_2}\frac{\lambda_k}{\lambda_k-\lambda_m}-\frac{\Gamma}{\Omega_1}-\left[\frac{\Gamma}{\Theta_1\Omega_1}\left[\alpha-\frac{u_{xx}}{u_x}(\ell+n)\right]+\frac{\tilde{Z}}{\Theta_1}\right](\frac{u_{cc}c-\Lambda}{1-\gamma_1-\gamma_2}\frac{\lambda_k}{\lambda_k-\lambda_m}-u_c+\frac{\lambda_k}{\varphi_I}-\frac{\Xi}{\varphi_I})$,

$$\tilde{F} = \frac{\left[(1-\varphi_I)c - B(sk)^{\gamma_1} n^{\gamma_2} c^{1-\gamma_1-\gamma_2}(1-\gamma_1-\gamma_2)\right](\alpha-\gamma_1)}{1-\gamma_1-\gamma_2} + \alpha\varphi_I A(1-s)^{\alpha} k^{\alpha} \ell^{1-\alpha} - \gamma_1 B(sk)^{\gamma_1} n^{\gamma_2} c^{1-\gamma_1-\gamma_2} - \frac{\alpha\Gamma}{\Omega_1}$$
$$-\left[\frac{\Gamma}{\Theta_1\Omega_1}\left[\alpha - \frac{u_{xx}}{u_x}(\ell+n)\right] + \frac{\tilde{Z}}{\Theta_1}\right]\left[\frac{u_{cc}c-\Lambda}{1-\gamma_1-\gamma_2}(\alpha-\gamma_1) + \Lambda\alpha - \frac{\alpha\Xi}{\Omega_1}\right],$$
$$\Gamma = \frac{\left[(1-\varphi_I)c - B(sk)^{\gamma_1} n^{\gamma_2} c^{1-\gamma_1-\gamma_2}(1-\gamma_1-\gamma_2)\right]\left[\frac{(1-\alpha)s}{1-s} + 1 - \gamma_1 - \frac{\gamma_2}{1-s}\right]}{1-\gamma_1-\gamma_2} - \alpha s\varphi_I A(1-s)^{\alpha-1} k^{\alpha} \ell^{1-\alpha} - B(sk)^{\gamma_1} n^{\gamma_2} c^{1-\gamma_1-\gamma_2}\left(\gamma_1 + \frac{\gamma_2}{1-s}\right),$$
$$\tilde{Z} = \frac{\left[(1-\varphi_I)c - B(sk)^{\gamma_1} n^{\gamma_2} c^{1-\gamma_1-\gamma_2}(1-\gamma_1-\gamma_2)\right](1-\alpha-\gamma_2)}{1-\gamma_1-\gamma_2} + (1-\alpha)\varphi_I A(1-s)^{\alpha} k^{\alpha} \ell^{1-\alpha} - \gamma_2 B(sk)^{\gamma_1} n^{\gamma_2} c^{1-\gamma_1-\gamma_2}.$$

Furthermore, substituting ξ from (5b) into (5g), together with (7b), (7c) and (9a), yields

$$\dot{\lambda}_m = \lambda_m(c,k,l)\left[\rho + \frac{1}{\varphi_I} + \pi(c,k,l)\right] - \frac{\lambda_k(c,k,l)}{\varphi_I}. \tag{9b}$$

Finally, by using (5f), (7a)-(7d) and (9a)-(9b), the equilibrium system is simplified to the three dynamical equations in (6b), (8a) and (8b). These dynamic equations determine the equilibrium paths of c, k and l. Then, equilibrium paths of n, λ_m, λ_k, s, ξ, π and m are in turn determined by other equations.

In analyzing the equilibrium characterized by (6b), (8a) and (8b), we note that $\dot{c} = \dot{k} = \dot{l} = 0$ in a steady state. In a steady state, these three equations and (7d) lead to, respectively,

$$c^* = A[(1-s^*)k^*]^{\alpha} l^{*1-\alpha} - \delta k^*, \tag{10a}$$
$$\alpha A[(1-s^*)k^*]^{\alpha-1} l^{*1-\alpha} = [1+\varphi_I(\rho+\mu)](\rho+\delta), \tag{10b}$$
$$\alpha A[(1-s^*)k^*]^{\alpha-1} l^{*1-\alpha} = (\rho+\mu)\gamma_1 B\left(\frac{sk}{c}\right)^{\gamma_1-1}\left(\frac{n}{c}\right)^{\gamma_2}, \tag{10c}$$
$$\frac{u_c(c,x)(1-\alpha)A[(1-s)k]^{\alpha}}{u_x(c,x)l^{\alpha}} = 1+\rho+\mu - \frac{(1-\gamma_1-\gamma_2)\alpha Ask^{\alpha}l^{1-\alpha}}{\gamma_1 c(1-s)^{1-\alpha}}. \tag{10d}$$

The inflation rate is $\pi^*=\mu$ in a steady state according to (6a).

These equations uniquely determined the steady state. The steady-state value of s^* is

$$s^* = \frac{\frac{\rho+\delta}{\alpha} - \frac{\delta}{\varphi_I(\rho+\mu)+1}}{\frac{\rho+\delta}{(\rho+\mu)B\gamma_1}\left(\frac{\alpha}{1-\alpha}\frac{\gamma_1}{\gamma_2}\right)^{-\gamma_2}\left\{\frac{\rho+\delta}{\alpha A}[\varphi_I(\rho+\mu)+1]\right\}^{\frac{-\gamma_2}{1-\alpha}} + \frac{\rho+\delta}{\alpha}}, \quad \frac{\partial s^*}{\partial\mu} > 0. \tag{11a}$$

Two features are in order. First, a higher monetary growth rate (higher μ) increases the fraction of capital allocated to the credit production even if the investment is not cash constrained (φ_I=0). Intuitively, a permanently higher money growth rate increases the inflation rate. Individuals have incentives to increase the fraction of consumption bought by credit so as to lower the cash constraint on consumption. This requires a larger fraction of capital allocated to the credit sector. Next, a larger degree of cash constraints on investment results in an increase in capital in the credit production. Intuitively, when the degree of cash constraints on investment is larger, the agent will invest even more in supplying credit for consumption so as to release more real balances for investment.

The steady-state values of k^* and c^* are functions of the labor supply in the goods production as follows.

$$k^* = \left\{ \tfrac{\rho+\delta}{\alpha A}[\varphi_I(\rho+\mu)+1] \right\}^{\frac{-1}{1-\alpha}} \tfrac{l^*}{1-s^*} = k^*(l^*), \;\; \tfrac{\partial k^*}{\partial l^*} > 0, \tag{11b}$$

$$c^* = A\left\{ \tfrac{\rho+\delta}{\alpha A}[\varphi_I(\rho+\mu)+1] \right\}^{\frac{-\alpha}{1-\alpha}} l^* - \delta \left\{ \tfrac{\rho+\delta}{\alpha A}[\varphi_I(\rho+\mu)+1] \right\}^{\frac{-1}{1-\alpha}} \tfrac{l^*}{1-s^*} = c^*(l^*), \;\; \tfrac{\partial c^*}{\partial l^*} > 0. \tag{11c}$$

The results in (11b) and (11c) suggest that capital stock and the labor supply in the goods production are positively correlated, reason being that capital and labor are complement in production. Moreover, consumption is positively correlated with the labor supply in the goods production because output available for consumption is increasing in the labor supply.

Finally, the steady-state value of the labor supply is determined by

$$\tfrac{A(1-\alpha)}{c^*(l^*)^\sigma} \left\{ \tfrac{\rho+\delta}{\alpha A}[\varphi_I(\rho+\mu)+1] \right\}^{\frac{-\alpha}{1-\alpha}} = \left\{ 1+\rho+\mu - \frac{1-\gamma_1-\gamma_2}{\gamma_1\left[\frac{(\rho+\delta)[\varphi_I(\rho+\mu)+1]}{\alpha} - \frac{\delta}{1-s^*}\right]} \frac{s^*(\rho+\mu)[\varphi_I(\rho+\mu)+1]}{1-s^*} \right\} \frac{\chi}{[1-l^*(1+\frac{\alpha}{1-\alpha}\frac{\gamma_2}{\gamma_1}\frac{s^*}{1-s^*})]^\varepsilon}. \tag{11d}$$

To envisage whether there exists a unique steady-state labor supply in the goods production, it is clear that the left-hand side of (11d) is decreasing in l and the right-hand side of (11d) is increasing in l. See the $LHS(l)$ locus and the $RHS(l)$ locus, respectively, in Figure 1. To intuitively explain the slope, note that the left-hand side is the MRS between consumption and leisure and the right-hand side is the MRTS of the capital allocation between the goods production and the credit provision. A larger labor supply to the goods sector increases consumption and reduces leisure which decreases the marginal utility of consumption and increases the marginal utility of leisure, thereby decreasing the MRS. Thus, the $LHS(l)$ locus is decreasing in l. Moreover, because capital and labor are complement in production, a larger labor supply to the goods sector increases the marginal product of capital in the goods sector, thereby increasing the MRTS. Thus, the $RHS(l)$ locus is increasing in l. Further, the value of $LHS(l)$ approaches the infinity at l=0 and equal L>0 at l=1. The value of $LHS(l)$ is equal to R>0 at l=0 and approaches the infinity as l approaches

$\bar{l} = (1+\frac{\alpha}{1-\alpha}\frac{\gamma_2}{\gamma_1}\frac{s^*}{1-s^*})^{-1} < 1.$[9] As a result, there exists a unique steady-state labor supply l^* as illustrated by E_0 in the Figure 1. With the unique l^* in a steady state, we can solve for other variables in a steady state.

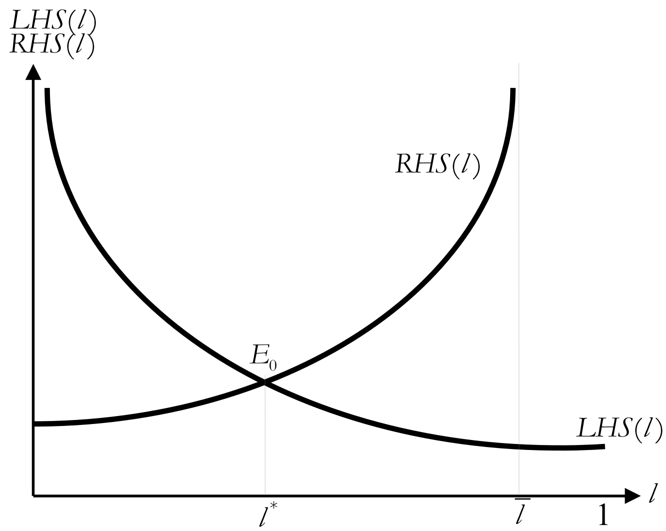

Figure 1: The existence of the steady state in the economy with endogenous labor

Now, we characterize the effect of an increase in the monetary growth rate in a steady state. First, we envisage the effect on the labor supply in the goods production and start with the effect on the *LHS*(*l*) locus. There is a direct negative effect of inflation which causes the *LHS*(*l*) locus to shift downward. But, there is also an indirect positive effect through lower consumption and thus a higher marginal utility of consumption which causes the *LHS*(*l*) locus to shift upward. To the extent the IES for consumption is smaller than one, as it is in empirical evidence, then σ>1 and the indirect positive effect dominates the direct negative effect. In this situation, the *LHS*(*l*) locus shifts upward. For the effect on the *RHS*(*l*) locus, there is a direct negative effect of inflation which shifts the *RHS*(*l*) locus downward. But, there is also a positive effect due to the capital reallocation away from the goods production toward the credit production which increases the MRTS of the capital allocation between the two sector and causes the *RHS*(*l*) locus to shift upward. In the case when the degree of cash constraints on investment is sufficiently large, the direct negative effect dominates the

9 $L=\frac{(1-s^*)^{\sigma}A(1-\alpha)}{\left\{(1-s^*)(\rho+\delta)[\varphi_1(\rho+\mu)+1]/\alpha-\delta\right\}^{\sigma}}\left\{\frac{(\rho+\delta)[\varphi_1(\rho+\mu)+1]}{\alpha A}\right\}^{\frac{\sigma-\alpha}{1-\alpha}}>0$ and $R=\left\{1+\rho+\mu-\frac{1-\gamma_1-\gamma_2}{\gamma_1\left[(\rho+\delta)[\varphi_1(\rho+\mu)+1]/\alpha-\delta/(1-s^*)\right]}\frac{s^*(\rho+\mu)[\varphi_1(\rho+\mu)+1]}{1-s^*}\right\}\chi>0.$

indirect positive effect and the *RHS*(*l*) locus shifts downward. Thus, when the IES for consumption is smaller than one and the degree of cash constraints on investment is sufficiently large, an increase in the monetary growth rate increases the labor supply in the goods production.

Next, we examine the effect on capital in the goods production in a steady state. We rewrite (11b) as

$$k^*_y = (1-s^*)k^* = \left\{\tfrac{\rho+\delta}{\alpha A}[\varphi_I(\rho+\mu)+1]\right\}^{\frac{-1}{1-\alpha}} l^*. \tag{12}$$

It is clear that there is a direct negative effect of inflation on capital in the goods production due to the cash constraint on investment. There is also an indirect negative effect because of more capital used in the credit production. However, there is a positive effect due to a larger labor supply in the goods production and the complement of capital and labor. Finally, there is yet another positive effect due to the availability of real balances. The credit supplied for consumption relaxes cash constraints on consumption and leaves more real balances available for investment. The net effect on capital in the goods production is thus ambiguous depending on which effects predominate. In the situation when the degree of cash constraints on investment is large, the positive effects are stronger. In this situation, more capital is employed in the goods production.

Finally, we study the effect on consumption. Examining (11c) we find one direct negative effect from inflation and two indirect effects from the capital reallocation and the labor supply. The indirect effect due to more capital in the credit production is negative while the indirect effect due to more labor supply in the goods production is positive. While the net effect is ambiguous, in general the two negative effects will dominate the positive effect, thereby leading to a lower consumption. In particular, when the degree of cash constraints on investment is large, the negative effects are stronger which lowers consumption.

3 The Dynamic Effects of Expansionary Monetary Policies

This section investigates the dynamic effects of different expansionary monetary policies under different CIA constraints on investment relative to consumption. As the cash constraint on consumption is endogenous, in an analytical solution it is difficult to see whether or not during the transition the degree of cash constraints on consumption is larger or smaller than the degree of cash constraints on investment. Quantitative analysis provides an alternative tool as it allows us to control the environment under which the cash constraint on consumption is larger or smaller than the cash constraint on investment.

To envisage the dynamic effects, we study a permanently higher money growth rate and a temporarily higher money growth rate. The dynamic effects of each policy are analyzed under the following two environments of the cash constraint: (1) consumption is more cash-constrained than investment and (2) consumption is less cash-constrained than investment.

To investigate the dynamic effects of expansionary monetary policies, we start by taking a linear Taylor's expansion of the dynamical system (6b), (8a) and (8b) in the neighborhood of the unique steady state and obtain a Jacobean matrix. We must find a negative and two positive eigenvalues associated with the Jacobean matrix in order to guarantee a unique equilibrium path toward the steady state. The values of $c(t)$, $k(t)$, $l(t)$ in the unique equilibrium path are then each represented by the sum of their own new steady state c^*, k^*, l^* and a product of three components: (i) a coefficient, (ii) an exponential to the power of the negative eigenvalue multiplied by time t, and (iii) the corresponding eigenvector of the negative eigenvalue. The coefficient is determined by boundary conditions depending upon the types of policy as will be explained later.[10]

We calibrate the model in a steady state to reproduce some key features representative of the U.S. economy at an annual frequency. We consider an annual monetary growth rate at μ=5% in our benchmark model. The annual capital-output ratio is around 3.32, according to Cooley (1995).

We normalize the technology coefficient in output production so A=1. Following Andolfatto (1996) and Hansen and Imrohoroglu (2008), we choose the capital share in output at α=0.36. The annual rate of time preference is set at 4% as used by Kydland and Prescott (1991). We start with the degree of cash constraints on investment at φ_I=0.1, and we will relax the value later. An agreement is held that the IES of consumption is less than one, so we choose the coefficient of risk aversion at σ=2 so the IES 0.5.[11]

As pointed out by Prescott (2006), the fraction of time allocated to market is around 25 percent. We choose l^*=0.25. The empirical literature has not come to an agreement concerning the value of the IES of leisure, $1/\varepsilon$. Yet, Imai and Keane (2004) used a model that allows for the IES of consumption to be different from the IES of labor/leisure, and their estimation result may shed light on the value of ε. Prescott (2006) pointed out Imai and Keane's (2004) labor supply elasticity is equivalent to an IES for leisure of 1.2 if the fraction of

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[10] See detailed methods from a linear Taylor's expansion to calculation of the dynamic paths in the Appendix.

[11] In an endogenous growth model, Espinosa-Vega and Yip (1999) found it crucial for the possibility of local indeterminacy whether the coefficient of relative risk aversion is above or below 1. Our primary results are unchanged when the coefficient of relative risk aversion is smaller than one.

productive time is 0.25, as it is for the United States. Following this line of reasoning, we set ε=0.83.

With these data, we use the capital-output ratio and (10b) to calibrate and obtain s^*=0.0230 and k^*=1.6089, which also indicate y^*=0.4846 and I^*=0.1126. Using (10a) we compute and obtain c^*=0.3720. Moreover, we set φ_c=0.5 and γ_2=0.1, and using the binding CIA constraint and (10c) we calibrate γ_I=0.2457 and B=1.5457. Then, we also obtain the fraction of labor allocated to the credit sector at n^*=0.0013 and to leisure at x^*= 0.7487. Finally, we use (11d) to calibrate χ=6.6481. Using these above data and parameter values, we obtain m^*=0.1973 and π^*=0.05.

Table 1 Eigenvalues in all sets of parameter changes

μ (%)	φ_I	φ_c	Eigenvalues		
5	0.1	0.5	0.1370+ 0.0700i	0.1370- 0.0700i	-0.0883
5.5	0.1	0.4856	0.1378+ 0.0724i	0.1378- 0.0724i	-0.0905
5	0.2	0.5	0.1284+ 0.0624i	0.1284- 0.0624i	-0.0865
5.5	0.2	0.4785	0.1286+ 0.0616i	0.1286- 0.0616i	-0.0909
5	0.4	0.5	-0.1224	0.0805	0.1517
5.5	0.4	0.4529	-0.1346	0.0655	0.1674
5	0.5	0.5	-0.1456	0.0462	0.1799
5.5	0.5	0.4257	-0.1590	0.0349	0.1946
5	0.6	0.5	-0.1578	0.0255	0.1980
5.5	0.6	0.3655	-0.1657	0.0159	0.2161
5	0.7	0.5	-0.1515	0.0108	0.2116
5.5	0.7	0.1208	-0.1483	0.0028	0.2458

We are now ready to analyze the dynamic effects of expansionary monetary policies. In each exercise below, we distinguish the case $\varphi_c \geq \varphi_I$ from the case $\varphi_I \geq \varphi_c$. In the case under $\varphi_c \geq \varphi_I$, we set φ_I=0.1, 0.2 and 0.4 so it is smaller than the initial φ_c=0.5 over time. In the case under $\varphi_I \geq \varphi_c$, we set φ_I=0.5, 0.6 and 0.7 so it is larger than or equal to φ_c=0.5.[12] Thus, in each of the two policy changes, in the baseline parameter values we first increase only the growth rate of money and analyze how the degree of cash constraints on consumption is endogenously changed and how other variables like output, labor and consumption change. Then, together with the increased growth rate of money, we also change the degree of cash constraints on investment and analyze the same set of variables. In all our comparative-dynamic exercises, we compute the eigenvalues in steady state and make sure that there is only one negative

[12] We cannot use the parameter value $\varphi_I \geq 0.8$ as the resulting value of $\gamma_1 + \gamma_2$ would be lager than 1 which violates our assumptions of the credit production function in (4b).

eigenvalue in order to guarantee a unique dynamic equilibrium path toward the steady state. The corresponding eigenvalues in the baseline steady state and all our comparative-static exercises are illustrated in Table 1. As is clearly seen, there is only one negative eigenvalue in each set of exogenous changes with regard to parameter values.

3.1 A Permanent Increase in Monetary Growth

First, we investigate the effect of a permanent increase in the growth rate of money. This is the money policy investigated in Abel (1985) and others.[13] Specifically, we assume that the money growth rate is increased once and for all from 5% to 5.5% at time 0. In this type of monetary policy, only one coefficient needs to be determined in the equilibrium path. Capital per capita is not affected at the timing when the money growth rate increases. This condition is used to determine the coefficient. The results are in Figure 2, with the results under $\varphi_c \geq \varphi_I$ illustrated in panel A and the results under $\varphi_I \geq \varphi_c$ displayed in panel B.[14]

Our quantitative results in both panels A and B of Figure 2 indicate that a permanently higher monetary growth rate and thus permanent higher inflation reduce consumption. Moreover, the higher the degree of cash constraints on investment, the lower is the level of consumption. The results confirm that the direct negative effect of inflation and the indirect negative effect of more capital allocation in the credit production are large and become stronger when the degree of cash constraints on investment is larger.

Given the degree of cash constraints on investment, in order to increase credit for consumption, the agent increases the fraction of capital in the credit production. The higher the degree of cash constraints on investment is, the higher is the increase in capital to the credit production. Thus, a higher degree of cash constraints on investment reduces the fraction of capital allocated to the goods sector (1-*s*). The production of credit for consumption relaxes cash constraints on consumption and thus lowers φ_c. This leaves more real balances available for investment. This indirect positive effect on investment dominates the direct negative effect of cash constraints on investment. Thus, net investment increases which accumulates capital over time until the steady state. Moreover, if the degree of cash constraints on investment is larger, more credit is supplied for consumption and more real balances are available for investment. Then, capital stock is higher.

[13] See also Mino and Shibata (1995) who analyzed the effect of money on dynamic paths of economic growth in an infinite-lived overlapping-generations model with money in utility.

[14] While the initial steady states are normalized to the initial steady state of φ_I=0.1 and φ_c=0.5 in Panel A, the initial steady states are normalized to the initial steady state of φ_I=0.7 and φ_c=0.1 in Panel B.

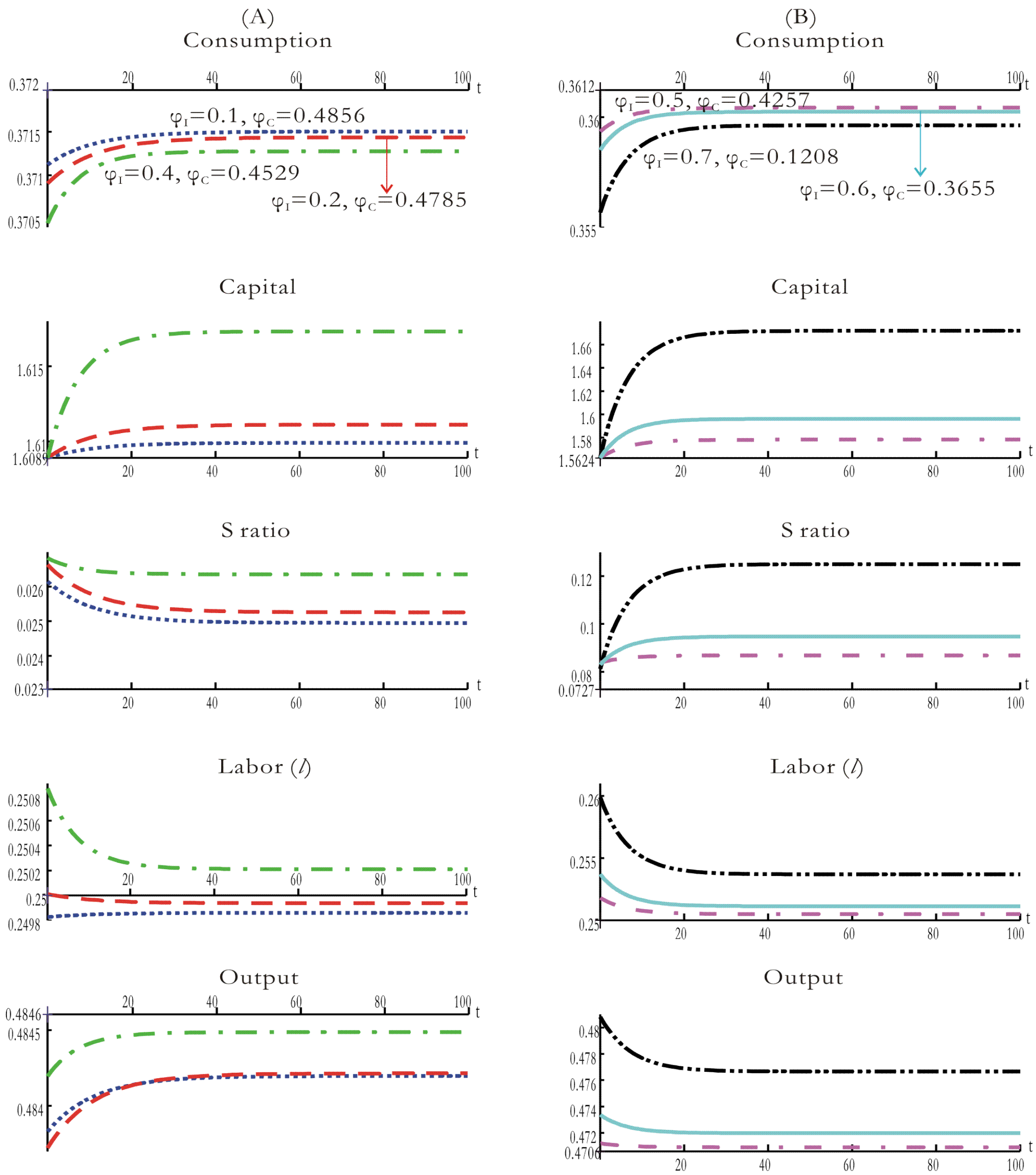

Figure 2: The Dynamic Effects of Permanent Monetary Policies in the economy with endogenous labor

Note: The intersection of the horizontal and the vertical axis in the panel (A) is the initial steady-state value normalized to that of the case that φ_I=0.1 and φ_c=0.5; and the intersection of the horizontal and the vertical axis in the panel (B) is the initial steady-state value normalized to that of the case that φ_I=0.7 and φ_c=0.5.

In response to permanent monetary expansion, the effect on the labor supply in the goods sector is negative when the degree of cash constraints on investment is as small as φ_I=0.2. But, when the degree of cash constraints on investment is larger, the labor supply in the goods sector increases.

Thus, a permanent higher money growth rate has three conflicting effects on output production. First, capital stock is higher which is beneficial to output production. Next, a smaller fraction of capital is allocated to the goods sector which is harmful to output production. Finally, the labor supply in the goods sector may be lower or higher which has an ambiguous effect on output production. A larger degree of cash constraints on investment enhances the positive capital stock effect, turns the labor supply effect from negative to positive, but also strengthens the negative capital reallocation effect. Our quantitative results indicate that as the degree of cash constraints on investment is larger, the positive capital stock effect and the labor supply effect increase more than the negative capital reallocation effect. Thus, the output production is increasing in the degree of cash constraints on investment. However, as is clear from panel A, when the initial degree of cash constraints on investment is smaller than the initial degree of cash constraints on consumption, output is always lower than the initial level.

On the other hand, when the initial degree of cash constraints on investment is larger than or equal to the initial degree of cash constraints on consumption, the results are different. In Panel B of Figure 2, the labor supply in the goods production is increased by a large amount in response to a permanently higher money growth rate. While capital allocated to the credit sector is increased by a large fraction, this effect creates large credit to consumption and thus releases a large quantity of real balances available for investment. As a result, investment is increased by a large amount which causes capital stock to increase by a large amount. The positive capital stock and the positive labor supply effect are thus both large. Although the negative capital reallocation effect is also large, it is quantitatively dominated by the other two positive effects. Therefore, the output production is higher than its initial level. In particular, when the initial degree of cash constraints on investment is higher, the two strong positive effects make output to increase even more.

To sum up the effects of a permanent higher money growth rate, we find that the relationship between money and output is positive if the degree of cash constraints on investment is larger than the initial degree of cash constraints on consumption but is negative if otherwise.

3.2 A Temporary Increase in Monetary Growth

Next, we investigate the dynamic behavior of a temporary higher money supply. Specifically, we assume that the growth rate of money is increased to 5.5% temporarily in the next 10 periods and returns to 5% from the 10^{th} period onward. With this monetary policy, four coefficients need to be determined in the unique equilibrium path. Before period 10, there are three coefficients associated with the three eigenvalues in the 3x3 equilibrium system. After period 10, there is one coefficient associated with the negative eigenvalue. To determine the four coefficients, note that capital is not affected when the money growth rate is increased. Immediately before and after quarter 10, equilibrium paths for $c(t)$, $k(t)$ and $l(t)$ must be equal. These four conditions determine the four coefficients. The quantitative dynamic behavior is illustrated in Figure 3.

It is clear that because of a short-term policy change, equilibrium paths of consumption, total capital, the capital reallocation and output are different from those in the long-term policy change in Figure 2. As is clear from Figure 3, no matter whether the degree of cash constraints on investment is smaller or larger than the initial degree of cash constraints on consumption, before period 10 when short-term policy change is in effect, the fraction of capital allocated to the credit production increases, but the level is lower if the degree of cash constraints on investment is larger. This effect is in sharp contrast to a long-term policy change wherein the fraction of capital allocated to the credit production is higher if the degree of cash constraints on investment is larger. Intuitively, in a temporary increase in the money growth rate, temporary inflation makes the agent to shy away from current investment to future investment. Therefore, the agent invests less on the credit provision because she does not need to release many real balances for investment. Thus, capital is always decreasing in the first ten periods, which is different from that in a permanent change. As a result, the level of capital stock is lower when the degree of cash constraints on investment is larger.

On the other hand, the labor supply in the goods production is increasing in the degree of cash constraints on investment when $\varphi_I<\varphi_c$ but is decreasing in the degree of cash constraints on investment when $\varphi_I>\varphi_c$. Nevertheless, the labor supply in the goods production always decreases before period 10. Together with lower capital stock and a smaller fraction of capital allocated to the goods production, output is unambiguously lower before period 10.

However, the effects do not end in period 10. After period 10, when the inflation rate is returned to its initial level, the labor supply in the goods sector is increased to a level that is larger than its initial level when $\varphi_I>0.2$. This results because the agent substitutes the labor supply away from the period with higher inflation to the period with lower inflation. Moreover, investment increases which accumulates capital toward the initial level. In particular, while the fraction of capital allocated to the credit production is still higher than the initial level after period 10 when $\varphi_I<\varphi_c$, the fraction of capital allocated to the credit production is lower than the initial level after period 10 under $\varphi_I>\varphi_c$.

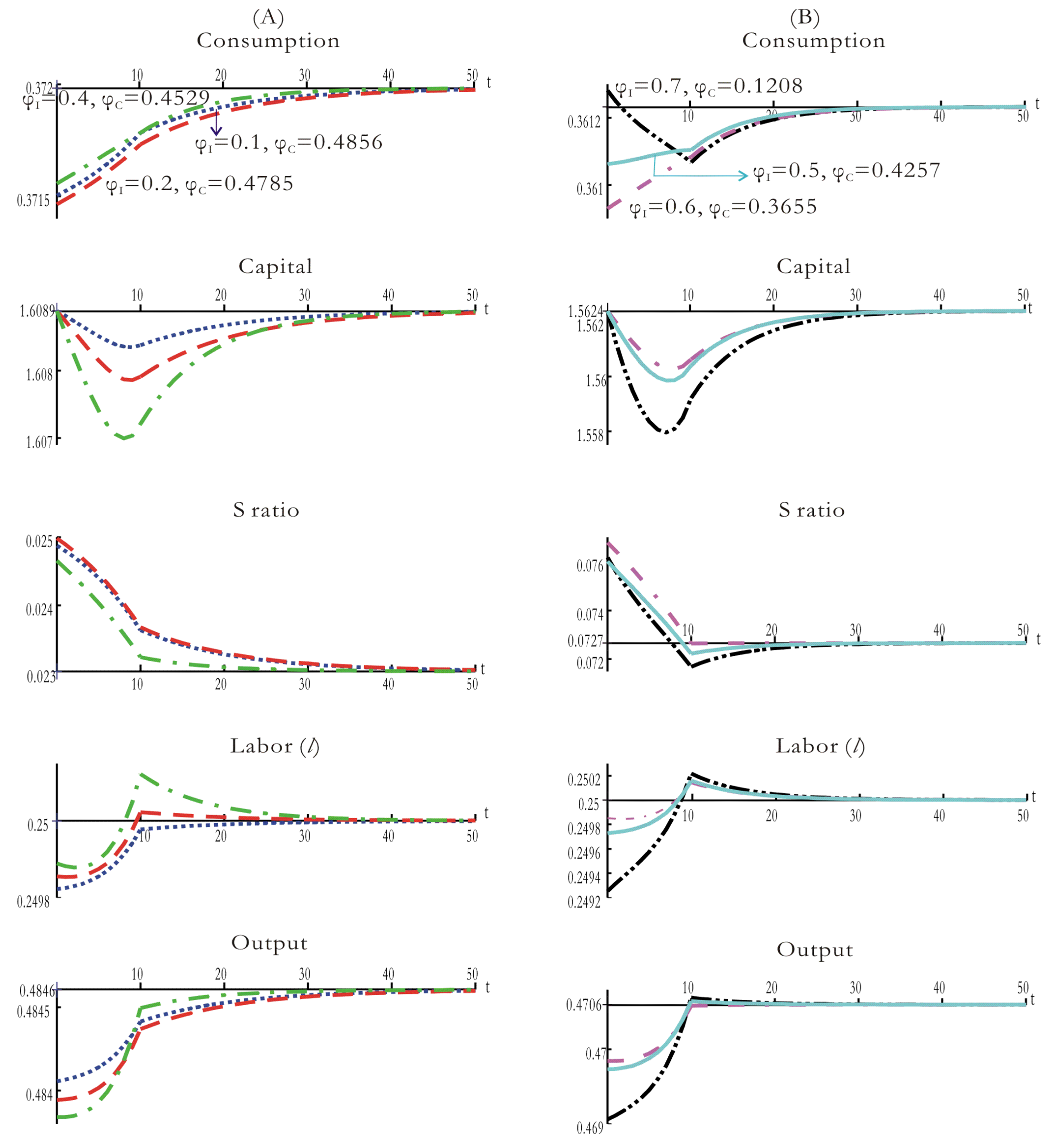

Figure 3: The Dynamic Effects of Temporary Monetary Policies in the economy with endogenous labor

Note: The intersection of the horizontal and the vertical axis in the panel (A) is the initial steady-state value normalized to that of the case that φ_I=0.1 and φ_c=0.5; and the intersection of the horizontal and the vertical axis in the panel (B) is the initial steady-state value normalized to that of the case that φ_I=0.7 and φ_c=0.5.

Thus, there is a larger fraction of capital in the goods sector after period 10 when $\varphi_I > \varphi_c$. Although the capital stock effect is still negative when $\varphi_I > \varphi_c$, the positive labor supply effect and the positive capital reallocation effect dominate the negative capital stock effect. As a consequence, output is higher than its initial level for a while after period 10 when $\varphi_I > \varphi_c$.

To sum up the effects of a temporary higher money growth rate, the relationship between money and output is always negative during the period when the expansionary monetary policy change is undergoing. However, after the period when the policy change is ended, the relationship between money and output becomes positive if the degree of cash constraints on investment is larger than the initial degree of cash constraints on consumption but is still negative if otherwise.

4 Concluding Remarks

In a neoclassical growth model, Stockman (1981) and Abel (1985) assumed the degree of CIA constraints on investment relative to that on consumption is 1 while others assumed the relative degree is smaller than or equal to 1. This paper departs from this line of reasoning in two directions. First, we allow for a general cash constraint with the degree of cash constraints on investment relative to that on consumption to be smaller as well as larger than 1. Second, we allow the agent to spend resources to supply credit for consumption and thus the relative cash constraint on investment and consumption is endogenous. We analyze the dynamic relationship between inflation and output in this framework.

Thus, the dynamic relationship depends crucially on whether the degree of cash constraints on investment is larger or smaller than the initial degree of cash constraints on consumption. In a permanent higher monetary growth rate, the relationship between inflation and output is positive if degree of cash constraints on investment is larger than the initial degree of cash constraints on consumption but is negative if otherwise. In a temporary higher money growth rate, the relationship between inflation and output is negative during the policy change period, but after the policy change is over, the relationship between inflation and output becomes positive if degree of cash constraints on investment is larger than the initial degree of cash constraints on consumption but is still negative if otherwise.

Appendix

Appendix 1 Transition Dynamic Path of a Permanent Increase in the Money Growth

This appendix presents technical details regarding the transitional dynamics of a permanent increase in the money growth. If we take Taylor's expansion of system (6b), (8a) and (8b) in the neighborhood of the unique steady state, we obtain

$$\begin{bmatrix} \dot{k} \\ \dot{c} \\ \dot{l} \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{bmatrix} \begin{bmatrix} k-k^* \\ c-c^* \\ l-l^* \end{bmatrix}, \tag{A1}$$

where $J_{11} = \alpha A(1-s)^{\alpha} k^{\alpha-1} \ell^{1-\alpha} - \delta - \alpha A(1-s)^{\alpha-1} k^{\alpha} \ell^{1-\alpha} \frac{\partial s}{\partial k}$,

$J_{12} = -1 - \alpha A(1-s)^{\alpha-1} k^{\alpha} \ell^{1-\alpha} \frac{\partial s}{\partial c}$,

$J_{13} = (1-\alpha) A(1-s)^{\alpha} k^{\alpha} \ell^{-\alpha} - \alpha A(1-s)^{\alpha-1} k^{\alpha} \ell^{1-\alpha} \frac{\partial s}{\partial l}$,

$$J_{31} = -\frac{\ell}{\Theta_1}\{-(\frac{u_{cc}c-\Lambda}{1-\gamma_1-\gamma_2}\frac{1}{\lambda_k-\lambda_m}+\frac{1}{\varphi_I})[(\rho+\delta)\frac{\partial\lambda_k}{\partial k}-\frac{A\alpha\ell^{1-\alpha}}{(1-s)^{1-\alpha}k^{1-\alpha}}\frac{\partial\lambda_m}{\partial k}-\frac{\lambda_m A\alpha\ell^{1-\alpha}(\alpha-1)}{(1-s)^{1-\alpha}k^{2-\alpha}}+\frac{(\alpha-1)\lambda_m A\alpha\ell^{1-\alpha}}{(1-s)^{2-\alpha}k^{1-\alpha}}\frac{\partial s}{\partial k}]$$
$$+[\frac{u_{cc}c-\Lambda}{1-\gamma_1-\gamma_2}\frac{\lambda_k}{\lambda_k-\lambda_m}-u_c+\frac{\lambda_k}{\varphi_I}-\frac{\Xi}{\Omega_1}][\frac{\partial\pi}{\partial k}-\frac{1}{\varphi_I}\frac{1}{\lambda_m}\frac{\partial\lambda_k}{\partial k}+\frac{1}{\varphi_I}\frac{\lambda_k}{\lambda_m^2}\frac{\partial\lambda_m}{\partial k}]+[\frac{(u_{cc}c-\Lambda)(\alpha-\gamma_1)}{1-\gamma_1-\gamma_2}+\Lambda\alpha-\frac{\alpha\Xi}{\Omega_1}]\frac{J_{11}}{k}\},$$

$$J_{32} = -\frac{\ell}{\Theta_1}\{-(\frac{u_{cc}c-\Lambda}{1-\gamma_1-\gamma_2}\frac{1}{\lambda_k-\lambda_m}+\frac{1}{\varphi_I})[(\rho+\delta)\frac{\partial\lambda_k}{\partial c}-\frac{A\alpha\ell^{1-\alpha}}{(1-s)^{1-\alpha}k^{1-\alpha}}\frac{\partial\lambda_m}{\partial c}+\frac{(\alpha-1)\lambda_m A\alpha\ell^{1-\alpha}}{(1-s)^{2-\alpha}k^{1-\alpha}}\frac{\partial s}{\partial c}]$$
$$+[\frac{u_{cc}c-\Lambda}{1-\gamma_1-\gamma_2}\frac{\lambda_k}{\lambda_k-\lambda_m}-u_c+\frac{\lambda_k}{\varphi_I}-\frac{\Xi}{\Omega_1}][\frac{\partial\pi}{\partial c}-\frac{1}{\varphi_I}\frac{1}{\lambda_m}\frac{\partial\lambda_k}{\partial c}+\frac{1}{\varphi_I}\frac{\lambda_k}{\lambda_m^2}\frac{\partial\lambda_m}{\partial c}]+[\frac{u_{cc}c-\Lambda}{1-\gamma_1-\gamma_2}(\alpha-\gamma_1)+\Lambda\alpha-\frac{\alpha\Xi}{\Omega_1}]\frac{J_{12}}{k}\},$$

$$J_{33} = -\frac{\ell}{\Theta_1}\{-(\frac{u_{cc}c-\Lambda}{1-\gamma_1-\gamma_2}\frac{1}{\lambda_k-\lambda_m}+\frac{1}{\varphi_I})[(\rho+\delta)\frac{\partial\lambda_k}{\partial\ell}-\frac{A\alpha\ell^{1-\alpha}}{(1-s)^{1-\alpha}k^{1-\alpha}}\frac{\partial\lambda_m}{\partial\ell}-\frac{\lambda_m(1-\alpha)A\alpha}{(1-s)^{1-\alpha}k^{1-\alpha}\ell^{\alpha}}+\frac{(\alpha-1)\lambda_m A\alpha\ell^{1-\alpha}}{(1-s)^{2-\alpha}k^{1-\alpha}}\frac{\partial s}{\partial l}]$$
$$+[\frac{u_{cc}c-\Lambda}{1-\gamma_1-\gamma_2}\frac{\lambda_k}{\lambda_k-\lambda_m}-u_c+\frac{\lambda_k}{\varphi_I}-\frac{\Xi}{\Omega_1}][\frac{\partial\pi}{\partial\ell}-\frac{1}{\varphi_I}\frac{1}{\lambda_m}\frac{\partial\lambda_k}{\partial\ell}+\frac{1}{\varphi_I}\frac{\lambda_k}{\lambda_m^2}\frac{\partial\lambda_m}{\partial\ell}]+[\frac{u_{cc}c-\Lambda}{1-\gamma_1-\gamma_2}(\alpha-\gamma_1)+\Lambda\alpha-\frac{\alpha\Xi}{\Omega_1}]\frac{J_{13}}{k}\},$$

$$J_{21} = \frac{c}{1-\gamma_1-\gamma_2}\{\frac{-1}{\lambda_k-\lambda_m}[(\rho+\delta)\frac{\partial\lambda_k}{\partial k}-\frac{A\alpha\ell^{1-\alpha}}{(1-s)^{1-\alpha}k^{1-\alpha}}\frac{\partial\lambda_m}{\partial k}-\frac{\lambda_m(\alpha-1)A\alpha\ell^{1-\alpha}}{(1-s)^{1-\alpha}k^{2-\alpha}}+\frac{(\alpha-1)\lambda_m A\alpha\ell^{1-\alpha}}{(1-s)^{2-\alpha}k^{1-\alpha}}\frac{\partial s}{\partial k}]$$
$$+\frac{\lambda_k}{\lambda_k-\lambda_m}[\frac{\partial\pi}{\partial k}-\frac{1}{\varphi_I}\frac{1}{\lambda_m}\frac{\partial\lambda_k}{\partial k}+\frac{1}{\varphi_I}\frac{\lambda_k}{\lambda_m^2}\frac{\partial\lambda_m}{\partial k}]+\frac{[-\frac{(\alpha-1)s}{1-s}+1-\gamma_1-\frac{\gamma_2}{1-s}]}{\Omega_1}[-\frac{\partial\pi}{\partial k}+\frac{1}{\varphi_I}\frac{1}{\lambda_m}\frac{\partial\lambda_k}{\partial k}-\frac{1}{\varphi_I}\frac{\lambda_k}{\lambda_m^2}\frac{\partial\lambda_m}{\partial k}-\frac{\alpha J_{11}}{k}+(\alpha-\frac{u_{xx}(\ell+n)}{u_x})\frac{J_{31}}{\ell}]$$
$$+\frac{\alpha-\gamma_1}{k}J_{11}+(1-\alpha-\gamma_2)\frac{J_{31}}{\ell}\},$$

$$J_{22} = \frac{c}{1-\gamma_1-\gamma_2}\{\frac{-1}{\lambda_k-\lambda_m}[(\rho+\delta)\frac{\partial\lambda_k}{\partial c}-\frac{A\alpha\ell^{1-\alpha}}{(1-s)^{1-\alpha}k^{1-\alpha}}\frac{\partial\lambda_m}{\partial c}+\frac{(\alpha-1)\lambda_m A\alpha\ell^{1-\alpha}}{(1-s)^{2-\alpha}k^{1-\alpha}}\frac{\partial s}{\partial c}]+\frac{\lambda_k}{\lambda_k-\lambda_m}[\frac{\partial\pi}{\partial c}-\frac{1}{\varphi_I}\frac{1}{\lambda_m}\frac{\partial\lambda_k}{\partial c}+\frac{1}{\varphi_I}\frac{\lambda_k}{\lambda_m^2}\frac{\partial\lambda_m}{\partial c}]$$
$$+\frac{[-\frac{(\alpha-1)s}{1-s}+1-\gamma_1-\frac{\gamma_2}{1-s}]}{\Omega_1}\{-\frac{\partial\pi}{\partial c}+\frac{1}{\varphi_I}\frac{1}{\lambda_m}\frac{\partial\lambda_k}{\partial c}-\frac{1}{\varphi_I}\frac{\lambda_k}{\lambda_m^2}\frac{\partial\lambda_m}{\partial c}-\frac{\alpha J_{12}}{k}+(\alpha-\frac{u_{xx}(\ell+n)}{u_x})\frac{J_{32}}{\ell}\}+\frac{\alpha-\gamma_1}{k}J_{12}+(1-\alpha-\gamma_2)\frac{J_{32}}{\ell}\},$$

$$J_{23} = \frac{c}{1-\gamma_1-\gamma_2}\{\frac{-1}{\lambda_k-\lambda_m}[(\rho+\delta)\frac{\partial\lambda_k}{\partial\ell} - \frac{A\alpha\ell^{1-\alpha}}{(1-s)^{1-\alpha}k^{1-\alpha}}\frac{\partial\lambda_m}{\partial\ell} - \frac{\lambda_m(1-\alpha)A\alpha\ell^{-\alpha}}{(1-s)^{1-\alpha}k^{1-\alpha}} + \frac{(\alpha-1)\lambda_m A\alpha\ell^{1-\alpha}}{(1-s)^{2-\alpha}k^{1-\alpha}}\frac{\partial s}{\partial l}]$$
$$+\frac{\lambda_k}{\lambda_k-\lambda_m}[\frac{\partial\pi}{\partial\ell} - \frac{1}{\varphi_I}\frac{1}{\lambda_m}\frac{\partial\lambda_k}{\partial\ell} + \frac{1}{\varphi_I}\frac{\lambda_k}{\lambda_m^2}\frac{\partial\lambda_m}{\partial\ell}] + \frac{(\alpha-\gamma_1)J_{13}}{k} + \frac{(1-\alpha-\gamma_2)J_{33}}{\ell}$$
$$+\frac{[-\frac{(\alpha-1)s}{1-s}+1-\gamma_1-\frac{\gamma_2}{1-s}]}{\Omega_1}[-\frac{\partial\pi}{\partial\ell} + \frac{1}{\varphi_I}\frac{1}{\lambda_m}\frac{\partial\lambda_k}{\partial\ell} - \frac{1}{\varphi_I}\frac{\lambda_k}{\lambda_m^2}\frac{\partial\lambda_m}{\partial\ell} - \frac{\alpha J_{13}}{k} + (\alpha - \frac{u_{xx}(\ell+n)}{u_x})\frac{J_{33}}{\ell}]\},$$

$$s_c = \frac{u_{cc}c}{u_c} - \frac{u_{xc}c}{u_x} - \frac{\lambda_m}{u_c}\left[-\frac{\alpha A(1-\gamma_1-\gamma_2)s^{1-\gamma_1}k^{\alpha-\gamma_1}\ell^{1-\alpha}}{B\gamma_1(1-s)^{1-\alpha}n^{\gamma_2}c^{1-\gamma_1-\gamma_2}} + \frac{(1-\gamma_1-\gamma_2)\alpha As}{\gamma_1 c}(1-s)^{\alpha-1}k^{\alpha}\ell^{1-\alpha}\right],$$

$$s_\ell = -\frac{u_{cx}}{u_c}(\ell+n) + \frac{u_{xx}}{u_x}(\ell+n) - \alpha - \frac{\lambda_m}{u_c}[\frac{\alpha A(1-\alpha-\gamma_2)s^{1-\gamma_1}k^{\alpha-\gamma_1}\ell^{1-\alpha}}{B\gamma_1(1-s)^{1-\alpha}n^{\gamma_2}c^{1-\gamma_1-\gamma_2}} - \frac{(1-\gamma_1-\gamma_2)s\alpha Ak^{\alpha}\ell^{1-\alpha}(1-\alpha)}{\gamma_1 c(1-s)^{1-\alpha}}],$$

$$s_k = \alpha - \frac{\lambda_m}{u_c}[\frac{\alpha A(\alpha-\gamma_1)s^{1-\gamma_1}k^{\alpha-\gamma_1}\ell^{1-\alpha}}{B\gamma_1(1-s)^{1-\alpha}n^{\gamma_2}c^{1-\gamma_1-\gamma_2}} - \frac{(1-\gamma_1-\gamma_2)s\alpha^2 Ak^{\alpha}\ell^{1-\alpha}}{\gamma_1 c(1-s)^{1-\alpha}}],$$

$$s_s = \frac{\alpha s}{1-s} + (\frac{u_{cx}}{u_c} - \frac{u_{xx}}{u_x})\frac{n}{1-s} + \frac{\lambda_m}{u_c}[\frac{\alpha As^{1-\gamma_1}k^{\alpha-\gamma_1}\ell^{1-\alpha}\left(\frac{(1-\alpha)s}{1-s}+1-\gamma_1-\frac{\gamma_2}{1-s}\right)}{B\gamma_1(1-s)^{1-\alpha}n^{\gamma_2}c^{1-\gamma_1-\gamma_2}} - \frac{(1-\gamma_1-\gamma_2)s\alpha Ak^{\alpha}\ell^{1-\alpha}(1-\frac{(\alpha-1)s}{1-s})}{\gamma_1 c(1-s)^{1-\alpha}}],$$

$$\frac{\partial s}{\partial k} = \frac{s_k}{s_s}\frac{s}{k},\ \frac{\partial s}{\partial c} = \frac{s_c}{s_s}\frac{s}{c},\ \frac{\partial s}{\partial l} = \frac{s_l}{s_s}\frac{s}{l},\ \frac{\partial\lambda_m}{\partial k} = -\frac{\alpha\lambda_m}{k} - \frac{\lambda_m}{s}\left(\frac{u_{xx}n}{u_x(1-s)} - \frac{\alpha s}{1-s}\right)\frac{\partial s}{\partial k},$$

$$\frac{\partial\lambda_m}{\partial c} = \frac{u_{xc}\lambda_m}{u_c} - \frac{\lambda_m}{s}\left(\frac{u_{xx}n}{u_x(1-s)} - \frac{\alpha s}{1-s}\right)\frac{\partial s}{\partial c},$$

$$\frac{\partial\lambda_m}{\partial l} = \frac{\lambda_m}{l}\left(\alpha - \frac{u_{xx}(n+l)}{u_x}\right) - \frac{\lambda_m}{s}\left(\frac{u_{xx}n}{u_x(1-s)} - \frac{\alpha s}{1-s}\right)\frac{\partial s}{\partial l},$$

$$\frac{\partial\lambda_k}{\partial k} = \frac{\lambda_k}{\lambda_m}\frac{\partial\lambda_m}{\partial k} + \frac{(\alpha-\gamma_1)(\lambda_k-\lambda_m)}{k} + \frac{(\lambda_k-\lambda_m)}{s}\left(\frac{(1-\alpha)s}{1-s} + 1 - \gamma_1 - \frac{\gamma_2}{1-s}\right)\frac{\partial s}{\partial k},$$

$$\frac{\partial\lambda_k}{\partial c} = \frac{\lambda_k}{\lambda_m}\frac{\partial\lambda_m}{\partial c} - \frac{(1-\gamma_1-\gamma_2)(\lambda_k-\lambda_m)}{c} + \frac{(\lambda_k-\lambda_m)}{s}\left(\frac{(1-\alpha)s}{1-s} + 1 - \gamma_1 - \frac{\gamma_2}{1-s}\right)\frac{\partial s}{\partial c},$$

$$\frac{\partial\lambda_k}{\partial l} = \frac{\lambda_k}{\lambda_m}\frac{\partial\lambda_m}{\partial l} + \frac{(1-\alpha-\gamma_2)(\lambda_k-\lambda_m)}{l} + \frac{(\lambda_k-\lambda_m)}{s}\left(\frac{(1-\alpha)s}{1-s} + 1 - \gamma_1 - \frac{\gamma_2}{1-s}\right)\frac{\partial s}{\partial l},$$

$$\frac{\partial\pi}{\partial k} = -\frac{1}{m+\tilde{H}}\{\tilde{G}[(\rho+\delta)\frac{\partial\lambda_k}{\partial k} - \frac{A\alpha\ell^{1-\alpha}}{(1-s)^{1-\alpha}k^{1-\alpha}}\frac{\partial\lambda_m}{\partial k} - \frac{\lambda_m A\alpha\ell^{1-\alpha}(\alpha-1)}{(1-s)^{1-\alpha}k^{2-\alpha}} + \frac{(\alpha-1)\lambda_m A\alpha\ell^{1-\alpha}}{(1-s)^{2-\alpha}k^{1-\alpha}}\frac{\partial s}{\partial k}]$$
$$+\tilde{H}[-\frac{1}{\varphi_I}\frac{1}{\lambda_m}\frac{\partial\lambda_k}{\partial k} + \frac{1}{\varphi_I}\frac{\lambda_k}{\lambda_m^2}\frac{\partial\lambda_m}{\partial k}] + \frac{\tilde{F}}{k}J_{11}\},$$

$$\frac{\partial\pi}{\partial c} = -\frac{1}{m+\tilde{H}}\{\tilde{G}[(\rho+\delta)\frac{\partial\lambda_k}{\partial c} - \frac{A\alpha\ell^{1-\alpha}}{(1-s)^{1-\alpha}k^{1-\alpha}}\frac{\partial\lambda_m}{\partial c} + \frac{(\alpha-1)\lambda_m A\alpha\ell^{1-\alpha}}{(1-s)^{2-\alpha}k^{1-\alpha}}\frac{\partial s}{\partial c}] + \frac{\tilde{H}}{\varphi_I}\frac{1}{\lambda_m}[-\frac{\partial\lambda_k}{\partial c} + \frac{\lambda_k}{\lambda_m}\frac{\partial\lambda_m}{\partial c}] + \frac{\tilde{F}J_{12}}{k}\},$$

$$\frac{\partial\pi}{\partial\ell} = -\frac{1}{m+\tilde{H}}\{\tilde{G}[(\rho+\delta)\frac{\partial\lambda_k}{\partial\ell} - \frac{A\alpha\ell^{1-\alpha}}{(1-s)^{1-\alpha}k^{1-\alpha}}\frac{\partial\lambda_m}{\partial\ell} - \frac{\lambda_m(1-\alpha)A\alpha}{(1-s)^{1-\alpha}k^{1-\alpha}\ell^{\alpha}} + \frac{(\alpha-1)\lambda_m A\alpha\ell^{1-\alpha}}{(1-s)^{2-\alpha}k^{1-\alpha}}\frac{\partial s}{\partial l}]$$
$$+\frac{\tilde{H}}{\varphi_I\lambda_m}[-\frac{\partial\lambda_k}{\partial\ell} + \frac{\lambda_k}{\lambda_m}\frac{\partial\lambda_m}{\partial\ell}] + \frac{\tilde{F}}{k}J_{13}\}.$$

The equilibrium dynamic system involves one variable whose initial value is predetermined and two control variables which may adjust instantaneously. The dynamic equilibrium path toward a steady state is unique if the characteristic function in association with the Jacobean matrix in (A1) has only one negative eigenvalue.

Let θ_i, i=1, 2, 3, be the eigenvalue of the Jacobean matrix in (A1) and θ_1 be the negative eigenvalue. Then, equilibrium time paths for the consumption, capital stock, and the ratio of capital allocated to the credit sector are as follows.

$$k_t = k^* + v_{11}e^{\theta_1 t}b_1 + v_{12}e^{\theta_2 t}b_2 + v_{13}e^{\theta_3 t}b_3,$$
$$c_t = c^* + v_{21}e^{\theta_1 t}b_1 + v_{22}e^{\theta_2 t}b_2 + v_{23}e^{\theta_3 t}b_3,$$
$$l_t = \lambda_m^* + v_{31}e^{\theta_1 t}b_1 + v_{32}e^{\theta_2 t}b_2 + v_{33}e^{\theta_3 t}b_3,$$

where v_{ji}, j=1, 2, 3, is the eigenvector corresponding to θ_i, i=1, 2, 3, and coefficient b_i, i=1, 2, 3, is determined by boundary conditions.

To determine b_i, note that all the three variables must converge to their steady-state value when $t \to \infty$. As θ_2, and θ_3 are positive, this is possible only if $b_2 = b_3 = 0$. As a result, the equilibrium time paths become

$$k_t = k^* + v_{11}e^{\theta_1 t}b_1, \tag{A2a}$$
$$c_t = c^* + v_{21}e^{\theta_1 t}b_1, \tag{A2b}$$
$$l_t = l^* + v_{31}e^{\theta_1 t}b_1. \tag{A2c}$$

Let k_0 (k^*) be the steady state before (after) a change in the money growth rate. As capital is not affected at t=0, thus (A2a) must satisfy the following condition

$$k_0 = k^* + v_{11}b_1. \tag{A2d}$$

The above relationship determines coefficient b_1. If we substitute b_1 into (A2b) and (A2c) for time 0, we obtain c_0 and s_0 which are adjusted instantaneously to the saddle arm. Using b_1 and (A2a)-(A2c), we obtain the equilibrium time paths of all variables at any point of time.

Appendix 2 Transition Dynamic Path of a Temporary Increase in the Money Growth

When the growth rate of money is increased for time 0 to 9, using the same method as in Appendix 1 the equilibrium time paths of key variables before time 10 are

$$\begin{cases} k_t = k^{1*} + v_{11}e^{\theta_1 t}b_1 + v_{12}e^{\theta_2 t}b_2 + v_{13}e^{\theta_3 t}b_3 \\ c_t = c^{1*} + v_{21}e^{\theta_1 t}b_1 + v_{22}e^{\theta_2 t}b_2 + v_{23}e^{\theta_3 t}b_3 \\ l_t = l^{1*} + v_{31}e^{\theta_1 t}b_1 + v_{32}e^{\theta_2 t}b_2 + v_{33}e^{\theta_3 t}b_3 \end{cases}, \text{when } t = 0^+ \sim 10^-, \tag{A3a}$$

where 0^+ indicates the moment when the growth rate of money is increased to 5.5% and 10^- indicates the moment before the growth rate of money is returned to 5%.

After the moment when the growth rate of money is returned to the normal level, the equilibrium time paths of key variables are

$$\begin{cases} k_t = k^* + v_{11}' e^{\theta_1' t} b_1' \\ c_t = c^* + v_{21}' e^{\theta_1' t} b_1' \\ l_t = l^* + v_{31}' e^{\theta_1' t} b_1' \end{cases} \text{ when } t \geq 10^+. \tag{A3b}$$

where time 10^+ indicates the exact moment when the growth rate of money is returned to 5%.

In the notations, a variable with a superscript 1* denotes a steady-state value under μ=5.5%, while that with * stands for a steady-state value under μ=5%; θ_i is the eigenvalue of the Jacobean matrix in (A1) associated with μ=5.5% while θ_i' is the eigenvalue of the Jacobean matrix (A1) associated with μ=5%. The eigenvector for θ_i is v_{ji} and the eigenvector for θ_i' is v_{ji}'. The four coefficients, b_1, b_2, b_3 and b_1', are determined as follows.

First, at t=0 when the growth rate of money is increased, the value of capital is not affected. Thus, we obtain the following condition

$$k_0 = k^{1*} + v_{11} b_1 + v_{12} b_2 + v_{13} b_3 . \tag{A4}$$

Next, because the representative agent's expectations are perfect foresighted, the continuity of all key variables suggests that the equilibrium time paths of key variables are equal right before and after the timing when the growth rate of money is returned to μ=5%.

$$k^{1*} + v_{11} e^{10\theta_1} b_1 + v_{12} e^{10\theta_2} b_2 + v_{13} e^{10\theta_3} b_3 = k^* + v_{11}' e^{10\theta_1'} b_1', \tag{A5a}$$

$$c^{1*} + v_{21} e^{10\theta_1} b_1 + v_{22} e^{10\theta_2} b_2 + v_{23} e^{10\theta_3} b_3 = c^* + v_{21}' e^{10\theta_1'} b_1', \tag{A5b}$$

$$l^{1*} + v_{31} e^{10\theta_1} b_1 + v_{32} e^{10\theta_2} b_2 + v_{33} e^{10\theta_3} b_3 = l^* + v_{31}' e^{10\theta_1'} b_1'. \tag{A5c}$$

Conditions (A4) and (A5a)-(A5c) determine the values for coefficients b_1, b_2, b_3, and b_1'. With these values of coefficients, we then use (A3a) and (A3b) to obtain the time paths of endogenous variables at any point of time.

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