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On R&D spillovers, multiple equilibria and indeterminacy

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Abstract Empirical studies often find significant and positive R&D spillovers across firms. In this note, we incorporate this spillover effect into a scale-invariant quality-ladder model. We find that the modified model features multiple steady states (1) a high-R&D steady state, (2) a low-R&D steady state and (3) a zero-R&D steady state. As for dynamics, when R&D spillovers are small, only the zero-R&D steady state is stable, and it emerges as a no-growth trap. In this case, the economy is subject to sunspot fluctuations around this trap (i.e., local indeterminacy). When R&D spillovers are large, both the zero-R&D and high-R&D steady states are stable and locally indeterminate. In this case, increasing patent breadth may cause the high-R&D steady state to become unstable and the economy to converge to the no-growth trap. Therefore, strengthening patent protection may stifle innovation through the occurrence of a bifurcation.

Keywords Endogenous-growth model · R&D spillovers · Indeterminacy · Multiple equilibria · Bifurcation

JEL Classification O31 · O41 · E32

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1 Introduction

Empirical studies, such as Jaffe (1986), Bernstein and Nadiri (1988, 1989) and Los and Verspagen (2000), often find significant and positive R&D spillovers across firms. For example, Jaffe (1986) estimates the elasticity of patents with respect to other firms' R&D to be about 1.1 for an average R&D firm. Given the empirical importance of R&D spillovers, we incorporate this spillover effect into an otherwise standard R&D-based growth model. We find that the modified model features multiple steady states (1) a high-R&D steady state, (2) a low-R&D steady state and (3) a zero-R&D steady state. As for dynamics, when R&D spillovers are small, only the zero-R&D steady state is stable, and it emerges as a no-growth trap. In this case, the economy is subject to sunspot fluctuations around this trap (i.e., local indeterminacy). When R&D spillovers are large, both the zero-R&D and high-R&D steady states are stable and locally indeterminate. In this case, increasing patent breadth may cause the high-R&D steady state to become unstable and the economy to converge to the no-growth trap. As a result, strengthening patent protection may stifle innovation and economic growth through the occurrence of a bifurcation. It is worth noting that the *intratemporal* R&D spillover effect that we consider in this study is different from the *intertemporal* knowledge spillover effect (i.e., the effect of previous innovations on current R&D productivity) that is commonly discussed in the R&D-based growth literature.

Since the seminal studies by Benhabib and Farmer (1994, 1996), Benhabib and Perli (1994) and Boldrin and Rustichini (1994), many studies have analyzed indeterminacy in endogenous-growth models.[1] While most studies in this literature focus on AK growth models, Benhabib et al. (1994) consider an R&D-based growth model *a la* Romer (1990) and find that complementarity between intermediate inputs and a greater-than-unity elasticity of intertemporal substitution may generate indeterminacy.[2] A number of recent studies, such as Haruyama and Itaya (2006) and Arnold and Kornprobst (2008), also show that indeterminacy may arise in R&D-based growth models when the elasticity of intertemporal substitution is greater than one. Arnold and Bauer (2009) consider a version of the Romer model with competitive sectors and find that a no-growth trap may emerge. This note complements these interesting studies by showing that R&D spillovers can be an alternative source of indeterminacy and a no-growth trap in R&D-based growth models.

Our study also relates to the literature on patent policy and economic growth. Jaffe and Lerner (2004) and Bessen and Meurer (2008) provide detailed case studies to show an important and surprising finding that strengthening patent protection may stifle innovation. Many growth-theoretic studies, such as O'Donoghue and Zweimuller (2004), Furukawa (2007), Horii and Iwaisako (2007), Chu (2009) and Chu et al. (2009), have analyzed the ambiguous effects of patent protection on innovation and growth via different channels. Our study relates to this literature by proposing a novel

[1] For some recent studies in this literature, see Drugeon and Venditti (2001), Garnier et al. (2007), Lloyd-Braga et al. (2006, 2007), Nishimura and Venditti (2002, 2004a,b, 2005, 2007) and Nourry and Venditti (2001).

[2] For a complete characterization of the stability properties of the original Romer model, see Arnold (2000a,b).

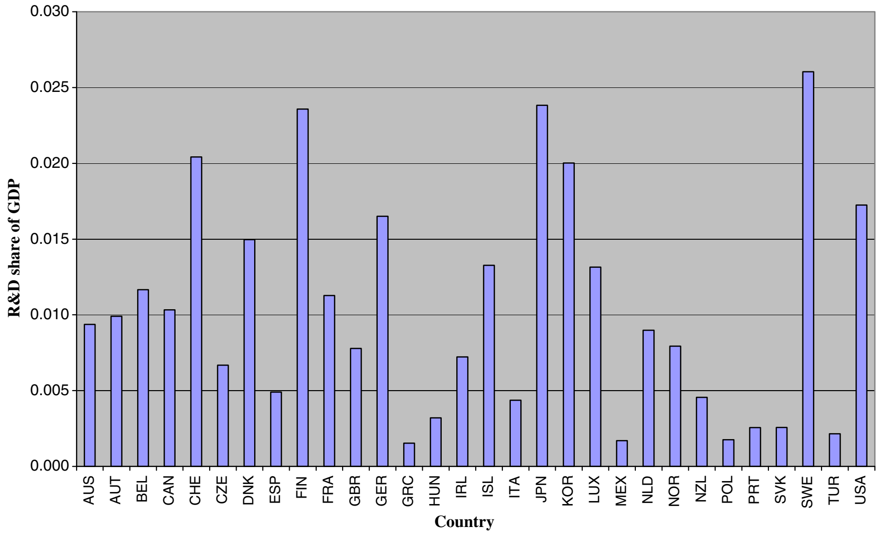

Fig. 1 Business R&D shares of GDP across OECD countries. We obtain the data on R&D financed by business enterprises from UNESCO Institute for Statistics and compute the average business R&D share of GDP between 2001 and 2005 for each OECD country

channel through which patent protection can have a negative effect on innovation and growth through the occurrence of a bifurcation. Finally, our finding also sheds some light on a stylized fact that the business R&D share of GDP varies significantly even across OECD countries (see Fig. 1). We show that in the presence of R&D spillovers, there are multiple equilibrium levels of R&D spending, and these equilibria can be indeterminate.

The rest of this note is organized as follows. Section 2 presents the model. Section 3 analyzes the steady states while Sect. 4 characterizes the dynamics of the model. The final section concludes with policy implications.

2 A scale-invariant quality-ladder model with R&D spillovers

In this section, we modify the quality-ladder model from Grossman and Helpman (1991) by adding three features (1) R&D spillovers, (2) variable patent breadth as in Li (2001) and (3) an exogenous process of variety expansion for removing scale effects as in Howitt (2000).[3] Given that quality-ladder models have been well-studied, the model's familiar components will be briefly described below to conserve space while the new features will be described in more details.

[3] See, for example, Jones (1999) for a discussion on scale effects in R&D-based growth models.

2.1 Households

There is a unit continuum of identical households, who have a standard iso-elastic utility function.

$$U = \int_0^\infty e^{-(\rho - g_L)t} \left(\frac{c_t^{1-\sigma} - 1}{1-\sigma} \right) dt, \tag{1}$$

where $\sigma \geq 1$ is the inverse of the intertemporal elasticity of substitution.[4] The parameters ρ and g_L are respectively the subjective discount rate and the population growth rate. Each household has $L_t = L_0 e^{g_L t}$ members at time t, and c_t denotes consumption per capita. To ensure that lifetime utility is bounded, we impose the following lower bound on the discount rate.

Condition D (Discount rate): $\rho > \sigma g_L$.

Households maximize (1) subject to

$$\dot{a}_t = (r_t - g_L) a_t + w_t - c_t, \tag{2}$$

where a_t is the value of assets owned by each member of a household, and r_t is the real rate of return. Each household member supplies one unit of labor at each instant of time to earn a real wage income w_t. The familiar Euler equation derived from the household's intertemporal optimization is

$$\frac{\dot{c}_t}{c_t} = \frac{1}{\sigma}(r_t - \rho). \tag{3}$$

2.2 Final goods

The final-goods sector is characterized by perfect competition, and the producers take both the output and input prices as given. The production function for final goods y_t is a standard Cobb–Douglas aggregator over a continuum of differentiated intermediate goods $x_t(j)$ for $j \in [0, n_t]$ given by

$$y_t = \exp\left(\frac{1}{n_t} \int_0^{n_t} \ln x_t(j) dj \right), \tag{4}$$

[4] We focus on the empirically relevant case of a less-than-unity elasticity of intertemporal substitution as evidenced in existing empirical studies (e.g., Guvenen 2006). As for the case of a greater-than-unity elasticity of intertemporal substitution, Benhabib et al. (1994), Haruyama and Itaya (2006) and Arnold and Kornprobst (2008) have shown that indeterminacy may arise. However, given that our focus is on R&D spillovers, we do not explore this channel here.

where n_t is the number of existing varieties.[5] This formulation of normalizing the production function by the number of varieties follows from Aghion and Howitt (1998, chapter 12), Aghion and Howitt (2005) and Howitt (2000).[6] From profit-maximization, the conditional demand function is $p_t(j)x_t(j) = y_t/n_t$, where $p_t(j)$ is the price of $x_t(j)$.

We follow Howitt (2000) to consider a law of motion for n_t given by $\dot{n}_t = \xi L_t$, where L_t is the population size at time t. Therefore, the number of varieties grows as a result of unintended imitation, and each person's propensity to imitate is given by the parameter $\xi > g_L$. The monopolistic profit made in this variety-expanding process is transferred to the households. If we define a state variable $s_t \equiv L_t/n_t$, then the law of motion for s_t would be

$$\dot{s}_t/s_t = g_L - \xi s_t. \tag{5}$$

Equation (5) implies that this independent dynamic system is globally stable, and s_t must converge to its steady state $s = g_L/\xi$ given any initial s_0. For the rest of the analysis, we assume that s_t has reached its steady state. In this case, the number of varieties increases at the rate of population growth such that the amount of R&D labor per variety is constant over time. As a result, a larger population does not lead to a higher quality growth rate (i.e., the absence of scale effects).

2.3 Intermediate goods

There is a continuum of industries producing the differentiated quality-enhancing intermediate goods. In each industry, there is a monopolistic leader, who holds a patent on the latest innovation and dominates the market until the next innovation occurs. The production function of the current leader in industry j is

$$x_t(j) = z^{q_t(j)} l_{x,t}(j). \tag{6}$$

$l_{x,t}(j)$ is the labor input for producing goods j. $z^{q_t(j)}$ is industry j's highest level of technology at time t. $z > 1$ is the step size of each technological improvement. $q_t(j)$ is the number of improvements that has occurred as of time t. The lowest marginal cost of production in industry j is $mc_t(j) = w_t/z^{q_t(j)}$.

As commonly assumed in the literature, the current and previous industry leaders engage in Bertrand price competition, and the profit-maximizing price for the current leader is a constant markup over her marginal cost of production given by

$$p_t(j) = \mu(z, b)mc_t(j), \tag{7}$$

[5] Our result is robust to the case of $n_t = 1$ for all t. However, in this case, the size of population must be constant in order to have a balanced-growth path.

[6] In this study, we suppress the growth effect of variety expansion because it is driven by an exogenous process.

where $\mu(z, b) = z^b$, and $b \in (0, 1]$ denotes the level of patent breadth.[7] In Grossman and Helpman (1991), there is complete patent protection against imitation such that $b = 1$. Li (2001) generalizes the policy environment to allow for incomplete patent protection against imitation such that $b \in (0, 1)$. Because of incomplete patent breadth, the current leader's quality improvement enables the previous leader to imitate and increase her productivity by a factor of z^{1-b} without infringing the current leader's patent. Therefore, the limit-pricing markup for the current leader is given by z^b. From now on, we denote patent protection as $\mu \equiv \mu(z, b)$ for convenience and consider changes in μ coming from changes in b only. From (6) and (7), the amount of monopolistic profit in industry j is

$$\pi_{x,t}(j) = (\mu - 1)w_t l_{x,t}(j) = \left(\frac{\mu - 1}{\mu}\right)\frac{y_t}{n_t}. \tag{8}$$

Equation (8) shows that the amount of profit is the same across industries.

Finally, for a new variety, we make the standard assumption that the quality of each new variety is randomly drawn from the quality distribution of existing varieties. Furthermore, we assume that the patentholder of a new variety can only protect a fraction b of a quality increment. Given that the previous quality increments are unpatented, Bertrand competition drives the markup down to $\mu = z^b$ as well.

2.4 R&D

Denote the value of an invention in industry j as $v_t(j)$. Equal profits and equal arrival rates of innovation across industries together imply $v_t(j) = v_t$ for $j \in [0, n_t]$.[8] The familiar no-arbitrage condition is

$$r_t v_t = \pi_{x,t} + \dot{v}_t - \lambda_t v_t. \tag{9}$$

Equation (9) equates the interest rate to the expected return per unit of asset. The right-hand side of (9) is the sum of (i) the monopolistic profit $\pi_{x,t}$ generated by this asset, (ii) the potential capital gain $\dot{v}_t$, and (iii) the expected capital loss $\lambda_t v_t$, where λ_t is the Poisson arrival rate of innovation.

There is a unit continuum of R&D entrepreneurs $h \in [0, 1]$, who employ R&D labor to create quality-improving inventions in each industry j. The expected profit for entrepreneur h is

$$\pi_{r,t}(h) = v_t \lambda_t(h) - w_t l_{r,t}(h). \tag{10}$$

7 The degree of patent breadth b can be interpreted as the fraction of an invention that is protected by its patent. In reality, when an inventor applies for a patent to protect her invention, she makes a number of claims about this invention to be reviewed by a patent examiner. Patent breadth determines the broadness of these claims, and specific (i.e., narrow) claims are unlikely to be infringed upon (i.e., ineffective patent protection).

8 We follow the standard approach in the literature to focus on the symmetric equilibrium. See Cozzi et al. (2007) for a theoretical justification for the symmetric equilibrium in quality-ladder growth models.

The Poisson arrival rate of innovation for entrepreneur h is $\lambda_t(h) = \bar{\varphi}_t l_{r,t}(h)$, where $\bar{\varphi}_t$ captures the productivity of R&D at time t. Free entry leads to zero expected profit in the R&D sector such that

$$v_t \bar{\varphi}_t = w_t. \tag{11}$$

This condition determines the allocation of labor between production and R&D. When $v_t \bar{\varphi}_t < w_t$, the R&D sector shuts down, and the economy is in a no-growth trap.

To introduce intra-industry R&D spillovers, we assume that

$$\bar{\varphi}_t = \varphi l_{r,t}^{\phi}. \tag{12}$$

The parameter $\phi \in (-1, 1)$ captures the degree of spillovers from R&D spending $l_{r,t}$ within an industry. On one hand, the R&D-based growth literature usually assumes a negative intratemporal R&D externality (i.e., $\phi \in (-1, 0)$) across firms to capture the patent-race and duplication effects; see, for example, Kortum (1993) and Jones (1999) for a discussion. On the other hand, motivated by the empirical evidence cited in the introduction, theoretical studies on R&D cooperation have long considered positive R&D spillovers (i.e., $\phi \in (0, 1)$) in partial-equilibrium models. Seminal studies in this literature include D'Aspremont and Jacquemin (1988) and Kamien et al. (1992). While these studies assume that each firm benefits from other firms' R&D in an additively-separable form, more recent studies, such as Bensaid et al. (1994) and Anbarci et al. (2002), emphasize complementarity in firms' R&D that are also present in the empirical framework in Jaffe (1986) and Los and Verspagen (2000).

2.5 Aggregation

Define $Z_t \equiv \exp(\frac{1}{n_t} \int_0^{n_t} q_t(j) dj \ln z)$ as aggregate technology and $l_{x,t} = l_{x,t}(j)$ as labor input per variety. Substituting (6) into (4) yields the aggregate production function of final goods given by

$$y_t = Z_t l_{x,t}. \tag{13}$$

The market-clearing condition for final goods is $y_t = c_t L_t$. The factor payments are $y_t = n_t(w_t l_{x,t} + \pi_{x,t})$, where $n_t \pi_{x,t}$ is the total amount of monopolistic profit and is equal to

$$n_t \pi_{x,t} = y_t(\mu - 1)/\mu. \tag{14}$$

The factor payment for production labor is

$$n_t w_t l_{x,t} = y_t/\mu. \tag{15}$$

The level of aggregate technology can be rewritten as

$$Z_t \equiv \exp\left(\frac{1}{n_t}\int_0^{n_t} q_t(j)dj \ln z\right) = \exp\left(\int_0^t \lambda_\tau d\tau \ln z\right), \tag{16}$$

where the last equality uses the law of large numbers. Differentiating (16) with respect to time yields

$$g_{Z,t} \equiv \dot{Z}_t/Z_t = \lambda_t \ln z, \tag{17}$$

where the arrival rate of innovation for each variety is $\lambda_t = \bar{\varphi}_t l_{r,t} = \varphi l_{r,t}^{1+\phi}$. Later on, we will show that $l_{r,t}$ is independent of L_t, so that scale effects are absent in this model.

3 Steady states

To conserve space in the main text, a formal definition of the equilibrium is relegated to Appendix B. In this section, we firstly derive the conditions that characterize the steady-state allocation. Imposing the balanced-growth condition on (9) yields $v_t = \pi_{x,t}/(\lambda + r - g_c) = \pi_{x,t}/(\lambda + \rho + (\sigma - 1)g_c)$,[9] where the second equality is obtained from (3) and $\dot{\pi}_{x,t}/\pi_{x,t} = \dot{c}_t/c_t \equiv g_c = g_Z - g_L$ on the balanced-growth path. Substituting the above expression of v_t into (11) yields $\bar{\varphi}_t \pi_{x,t}/(\lambda + \rho + (\sigma - 1)g_c) = w_t$. Then, we substitute (14), (15) and (17) into this condition to derive

$$\bar{\varphi}_t l_x(\mu - 1) = [\lambda + \rho + (\sigma - 1)(\lambda \ln z - g_L)], \tag{18}$$

where $\lambda = \bar{\varphi} l_r = \varphi\ l_r^{1+\phi}$. To close the model, we use the labor-market clearing condition given by

$$n_t(l_x + l_r) = L_t \Leftrightarrow l_x + l_r = s = g_L/\xi. \tag{19}$$

Lemma 1 *The equilibrium condition that characterizes the steady-state value(s) of l_r is given by*

$$F(l_r) \equiv \left(\frac{\rho + (1-\sigma)g_L}{\varphi(\mu - 1)}\right)\frac{1}{l_r^{\phi}} + \left(\frac{(\sigma - 1)\ln z}{\mu - 1} + \frac{\mu}{\mu - 1}\right) l_r - \frac{g_L}{\xi} = 0. \tag{*}$$

Proof Solve (18) and (19). Also, note that Condition D implies $\rho + (1 - \sigma)g_L > 0$. □

[9] It is useful to note that the expected return on R&D is $\lambda v_t = \lambda \pi_{x,t}/(\lambda + r - g_c)$, where the λ in the numerator is the business-stealing effect while the λ in the denominator is the creative–destruction effect. Given R&D spillovers, the arrival rate of innovation is $\lambda = \varphi l_r^{1+\phi}$. Therefore, R&D spillovers affect the arrival rate of innovation which in turn affects the expected return on R&D.

To ensure that equilibrium R&D is non-negative, we impose a lower bound on R&D productivity.

Condition R (R&D productivity):

$$\varphi > (\rho + (1-\sigma)g_L)\left(\frac{(\sigma-1)\ln z + \mu}{\phi}\right)^{\phi}\left(\frac{1+\phi}{\mu-1}\right)^{1+\phi}\left(\frac{\xi}{g_L}\right)^{1+\phi}.$$

3.1 No R&D spillovers

We firstly consider the case of no R&D spillovers. In this case, (*) shows that there is a unique steady state, and the steady-state R&D labor per variety is given by

$$l_r(\phi = 0) = \frac{1}{(\sigma-1)\ln z + \mu}\left((\mu-1)\frac{g_L}{\xi} - \frac{\rho + (1-\sigma)g_L}{\varphi}\right). \tag{20}$$

At $\phi = 0$, Condition R simplifies to $\varphi > \left(\frac{\rho+(1-\sigma)g_L}{\mu-1}\right)\frac{\xi}{g_L}$, which together with $\sigma \geq 1$ implies that $l_r(\phi = 0) > 0$. Taking a simple differentiation of (20) shows that $\partial l_r(\phi = 0)/\partial\mu > 0$. In other words, a larger patent breath increases equilibrium R&D in the absence of R&D spillovers.

3.2 Negative R&D spillovers

As for the case of negative R&D spillovers $\phi \in (-1, 0)$, there is also a unique steady state. First, $F(l_r)$ is an increasing function in l_r. Second, $F(l_r)$ must cross the zero-axis exactly once because $F(0) < 0$ and $F(s) > 0$. Therefore, there exists a unique steady-state allocation $l_r(\phi < 0)$ that is implicitly defined by (*). Finally, because $F(l_r)$ is decreasing in μ for a given l_r, we once again have the following result that $\partial l_r(\phi < 0)/\partial\mu > 0$. Therefore, a larger patent breath also increases equilibrium R&D in the presence of negative R&D spillovers.

3.3 Positive R&D spillovers

In the case of positive R&D spillovers, there are three steady states: (i) a zero-R&D steady state, (ii) a low-R&D steady state and (iii) a high-R&D steady state. The zero-R&D steady state exists because when entrepreneurs expect aggregate R&D $l_{r,t}$ to be zero, they also expect R&D productivity $\bar{\varphi}_t = \varphi l_{r,t}^{\phi}$ to be zero given $\phi > 0$. As a result, they have no incentive to invest in R&D, and this outcome is consistent with profit maximization under positive R&D spillovers. As for the two interior steady states, they are characterized by (*), and their existence is guaranteed by Condition R.

Figure 2 plots (*) along with the interior steady-state values of l_r under positive R&D spillovers. l_r^h denotes the steady state in which there are a high level of equilibrium R&D and a high growth rate $g_Z^h = (l_r^h)^{1+\phi}\varphi \ln z$. l_r^l denotes the steady state in which there are a low level of equilibrium R&D and a low growth rate $g_Z^l = (l_r^l)^{1+\phi}\varphi \ln z$.

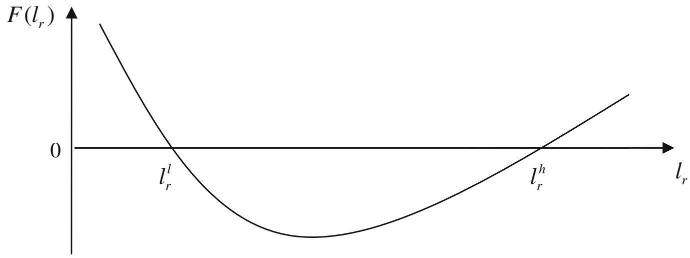

Fig. 2 Interior steady states under positive R&D spillovers

Intuitively, in the presence of positive R&D spillovers, when R&D entrepreneurs expect a high (low) level of R&D spending in their industry, they have more (less) incentives to invest in R&D and their expectations are self-fulfilling. The next lemma considers the comparative statics of l_r^h and l_r^l with respect to the level of patent breadth under positive R&D spillovers.

Lemma 2 *Under positive R&D spillovers, l_r^h and l_r^l are respectively increasing and decreasing in the level of patent breadth.*

Proof An increase in μ decreases the left-hand side of (*) and shifts down the curve in Fig. 2. As a result, l_r^h goes up while l_r^l goes down. □

The effect of patent breadth on l_r^h is standard. A higher level of patent breadth improves the incentives for R&D and increases growth. As for the counterintuitive effect on l_r^l, it is due to the instability of the low-R&D steady state (to be characterized below). Given Samuelson (1983) seminal correspondence principle, it is perhaps not surprising that the comparative statics of an equilibrium may depend on its stability property. Therefore, the intriguing question is on the determinacy of each steady state. To answer this question, we characterize the dynamics of the model in the following section.

4 Dynamics of the model

To characterize the dynamics of the model, there are four cases to consider (i) no R&D spillovers $\phi = 0$, (ii) negative R&D spillovers $\phi \in (-1, 0)$, (iii) small positive R&D spillovers $s\phi/(\sigma + \phi) < l_r^h$,[10] and (iv) large positive R&D spillovers $s\phi/(\sigma + \phi) > l_r^h$.[11] In Lemma 3, we derive the differential equation (**) that characterizes the model's dynamics.

[10] It is useful to note that $s\phi/(\sigma + \phi) < l_r^l$ is an empty parameter space. Thus, $s\phi/(\sigma + \phi) < l_r^h$ is in fact referring to $l_r^l < s\phi/(\sigma + \phi) < l_r^h$.

[11] The value of ϕ, at which the condition $s\phi/(\sigma + \phi) = l_r^h$ holds, is unique because l_r^h is decreasing in ϕ. To see this, differentiating $F(l_r)$ in (*) with respect to ϕ yields $-(\rho + (1 - \sigma)g_L) \ln l_r/[\varphi(\mu - 1)l_r^{\phi}] > 0$ given $l_r \in (0, s)$ and $\ln l_r < 0$.

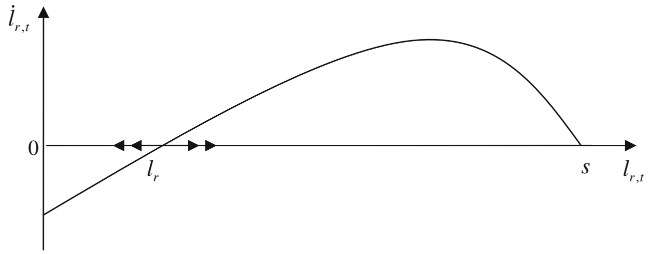

Fig. 3 No R&D spillovers

Lemma 3 *The dynamics of the model can be summarized by the following differential equation.*

$$\dot{l}_{r,t} = \underbrace{\left(\frac{l_{r,t}(s - l_{r,t})}{s\phi - (\sigma + \phi)l_{r,t}}\right)}_{first\ term} \times \underbrace{\left(s(\mu - 1)\varphi l_{r,t}^{\phi} - (\mu + (\sigma - 1)\ln z)\varphi l_{r,t}^{1+\phi} - (\rho + (1 - \sigma)g_L)\right)}_{second\ term}, \qquad (**)$$

where $s = g_L/\xi$.

Proof See Appendix A. □

4.1 No R&D spillovers

For the case of no R&D spillovers (i.e., $\phi = 0$), the model is a standard scale-invariant quality-ladder growth model. In this case, the dynamic system is characterized by saddle-point stability. The phase diagram is plotted in Fig. 3, and $l_{r,t}$ must jump to the unique steady state. Otherwise, $l_{r,t} \to s$ implies zero consumption and violates the utility maximization of households. Similarly, $l_{r,t} \to 0$ violates the profit maximization of R&D entrepreneurs when $\phi = 0$.

4.2 Negative R&D spillovers

As for the case of negative R&D spillovers (i.e., $\phi \in (-1, 0)$), the dynamic system is also characterized by saddle-point stability. The phase diagram is similar to Fig. 3 except that $\dot{l}_{r,t} = 0$ as $l_{r,t} \to 0$. Again, $l_{r,t}$ must also jump to the unique steady state. Otherwise, the economy converges to either (i) $l_{r,t} \to s$ implying zero consumption and violating the utility maximization of households or (ii) $l_{r,t} \to 0$ violating the profit maximization of R&D entrepreneurs when $\phi < 0$.

4.3 Small positive R&D spillovers

For the case of small positive R&D spillovers (i.e., $s\phi/(\sigma + \phi) < l_r^h$), the model features three steady states $\{0, l_r^l, l_r^h\}$. At $l_{r,t} = 0$, the first term of (**) is zero. When

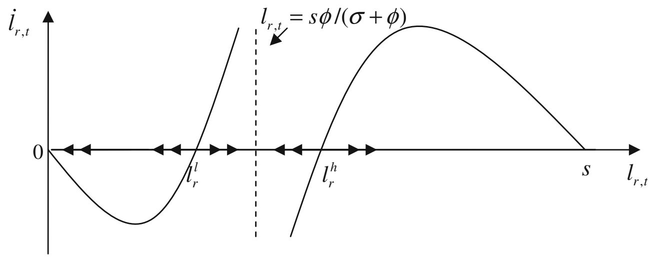

Fig. 4 Small positive R&D spillovers

$0 < l_{r,t} < l_r^l$, the first term becomes positive while the second term of (**) is negative. When $l_r^l < l_{r,t} < s\phi/(\sigma + \phi)$, the second term becomes positive. As $l_{r,t}$ approaches $s\phi/(\sigma + \phi)$ from below (above), $\dot{l}_{r,t}$ approaches positive (negative) infinity. When $s\phi/(\sigma + \phi) < l_{r,t} < l_r^h$, the first term of (**) is negative while the second term is positive. When $l_{r,t} > l_r^h$, the second term of (**) becomes negative. The phase diagram is plotted in Fig. 4.

We follow Palivos et al. (2003) to consider global indeterminacy and local indeterminacy in a one-dimensional dynamic system of a jump variable. Palivos et al. (2003) define global indeterminacy as the situation in which there exist at least two initial points $l_{r,0}$ such that each corresponding solution $l_{r,t}$ converges to a steady-state equilibrium point. According to this definition, the multiplicity of steady-state equilibrium points is a sufficient condition for global indeterminacy because $l_{r,t}$ is a jump variable. As for local indeterminacy, a steady-state equilibrium point l_r^* is said to be locally indeterminate if there exists a non-degenerate interval Ω such that for every initial point $l_{r,0} \in \Omega$, the solution $l_{r,t}$ converges to l_r^*. Figure 4 shows that the two interior steady states are unstable while the zero-R&D steady state is stable and becomes a no-growth trap. Therefore, the economy is subject to sunspot fluctuations around the zero-R&D steady state (i.e., local indeterminacy) while the two interior steady states are locally determinate.[12]

Proposition 1 *When R&D spillovers are positive and small, the two interior steady states are unstable and locally determinate while the zero-R&D steady state is stable and locally indeterminate.*

Proof See Fig. 4. □

4.4 Large positive R&D spillovers

For the case of large R&D spillovers (i.e., $s\phi/(\sigma + \phi) > l_r^h$), the model also features three steady states $\{0, l_r^l, l_r^h\}$. For the dynamics around l_r^l, it is the same as before. When $l_r^l < l_{r,t} < l_r^h$, the second term of (**) is positive. When $l_r^h < l_{r,t} < \phi/(\sigma + \phi)$, the second term becomes negative. The phase diagram is plotted in Fig. 5.

[12] Note that when $l_{r,t}$ starts from a point within the interval (l_r^l, l_r^h), the economy converges to $s\phi/(\sigma + \phi)$ at which point the dynamic system (**) becomes undefined.

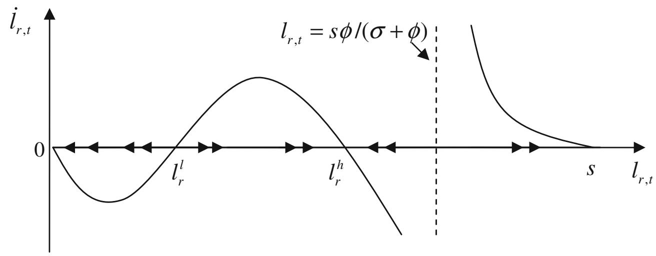

Fig. 5 Large positive R&D spillovers

As in the previous section, the presence of multiple steady-state equilibrium points implies global indeterminacy because $l_{r,t}$ is a jump variable. As for locally indeterminacy, both the zero-R&D and high-R&D steady states are locally stable under large positive R&D spillovers. In other words, there are now two locally indeterminate steady states in the economy. Finally, the low-R&D steady state continues to be unstable and locally determinate.

Proposition 2 *When R&D spillovers are positive and large, both the zero-R&D and high-R&D steady states are stable and locally indeterminate while the low-R&D steady state is unstable and locally determinate.*

Proof See Fig. 5. □

What happens when policymakers increase the level of patent breadth? In this case, as the next proposition states, it is possible that strengthening patent protection leads to a surprisingly negative effect on innovation and growth through the occurrence of a bifurcation.

Proposition 3 *When R&D spillovers are positive and large, a larger patent breadth leads to an increase in l_r^h, which may cause the condition $s\phi/(\sigma+\phi) > l_r^h$ to be reversed. When this reversal occurs, the high-R&D steady state becomes unstable, and the economy may fall into the no-growth trap.*

Proof Apply Lemma 2. Also, compare Figs. 4 and 5. □

4.5 Discussion of the zero-growth trap

In the model, we have considered a particular functional form of $\bar{\varphi}_t = \varphi l_{r,t}^{\phi}$ for R&D spillovers. Given this specification, if aggregate R&D is zero, then R&D productivity would also be zero under positive R&D spillovers. Suppose we consider an alternative specification of $\bar{\varphi}_t = \alpha + \varphi l_{r,t}^{\phi}$. In this case, the zero-growth trap disappears because R&D productivity remains to be positive even when aggregate R&D is zero. In other words, the emergence of a zero-growth trap is the result of a functional-form assumption. However, we believe that the theoretical analysis based on the particular functional form that we use is still interesting because the specification is consistent with the literature of infinite horizon models on indeterminacy initiated by

Benhabib and Farmer (1994) in the context of bounded growth and Benhabib and Perli (1994) in the context of unbounded growth.[13] In other words, $\bar{\varphi}_t = \varphi l_{r,t}^{\phi}$ closely resembles the common specification in an important branch of related literature.

5 Conclusion

In this note, we have analyzed the effects of R&D spillovers in a scale-invariant quality-ladder model. Our findings can be summarized as follows. First, positive R&D spillovers generate multiple steady states that feature different levels of R&D and economic growth. Second, when R&D spillovers are small, the zero-R&D steady state emerges as a no-growth trap and is locally indeterminate. Third, when R&D spillovers are large, both the zero-R&D and high-R&D steady states are stable and locally indeterminate. All these results are obtained in a standard R&D-based growth model with the addition of positive R&D spillovers. Given the empirical evidence for R&D spillovers and their drastic impact on the equilibrium properties of R&D-based growth models, it is important to take into account these spillover effects when deriving policy implications from these models. For example, strengthening patent protection may have a surprisingly negative effect on innovation in the presence of R&D spillovers, and this theoretical finding is potentially consistent with the recent concerns about the patent system stifling the innovation process. Therefore, for achieving the socially optimal level of R&D, it may be more effective to employ other innovation policies, such as R&D subsidies, prize for innovation, and protection of trade secrets, in conjunction with patent policy.

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Appendix A: Proof of Lemma 1

Using (12) and (15), we can rewrite (11) as

$$\varphi l_{r,t}^{\phi} = \frac{1}{\mu v_t}\left(\frac{y_t}{l_{x,t} n_t}\right) = \frac{1}{\mu v_t}\left(\frac{Z_t}{n_t}\right), \tag{A1}$$

where the second equality follows from (13). Taking the log of (A1) and then differentiating the resulting expression with respect to time yield

$$\phi \frac{\dot{l}_{r,t}}{l_{r,t}} = \frac{\dot{Z}_t}{Z_t} - \frac{\dot{v}_t}{v_t} - \frac{\dot{n}_t}{n_t}, \tag{A2}$$

[13] For example, in Sect. 3 of Benhabib and Perli (1994), labor externality in the knowledge production function is also introduced in such a way that if aggregate labor supply to knowledge production is zero, then productivity at the individual level would also become zero.

where $\dot{n}_t/n_t = g_L$. Rearranging (9) yields

$$\frac{\dot{v}_t}{v_t} = \lambda_t + r_t - \frac{\pi_{x,t}}{v_t}. \tag{A3}$$

Combining (14) and the first equality of (A1) yields

$$\frac{\pi_{x,t}}{v_t} = (\mu - 1)\varphi l_{r,t}^{\phi} l_{x,t}. \tag{A4}$$

Taking the log of $c_t L_t = Z_t l_{x,t}$ and differentiating the resulting expression with respect to time yield

$$\frac{\dot{c}_t}{c_t} = \frac{\dot{Z}_t}{Z_t} + \frac{\dot{l}_{x,t}}{l_{x,t}} - g_L. \tag{A5}$$

Substituting (17), (A3) and (A4) into (A2) yields

$$\phi\frac{\dot{l}_{r,t}}{l_{r,t}} = \lambda_t \ln z - \lambda_t - r_t + (\mu - 1)\varphi l_{r,t}^{\phi} l_{x,t} - g_L, \tag{A6}$$

where $\lambda_t = \varphi l_{r,t}^{1+\phi}$ and $r_t = \rho + \sigma(\dot{c}_t/c_t) = \rho + \sigma(\lambda_t \ln z + \dot{l}_{x,t}/l_{x,t} - g_L)$, which uses (3), (17) and (A5). Differentiating $l_{x,t} + l_{r,t} = s$ with respect to time yields

$$\frac{\dot{l}_{x,t}}{l_{x,t}} = -\frac{\dot{l}_{r,t}}{s - l_{r,t}}. \tag{A7}$$

Finally, substituting (A7) into (A6) and applying a few steps of mathematical manipulation yield (**). □

Appendix B: Equilibrium definition

The decentralized equilibrium is a time path of allocations $\{c_t, y_t, x_t(j), l_{x,t}(j), l_{r,t}(h)\}_{t=0}^{\infty}$ and a time path of prices $\{w_t, r_t, p_t(j), v_t\}_{t=0}^{\infty}$. Also, at each instant of time,

(a) the representative household chooses $\{c_t\}$ to maximize utility taking $\{r_t, w_t\}$ as given;
(b) the competitive final-goods firms choose $\{x_t(j)\}$ to maximize profit taking $\{p_t(j)\}$ as given;
(c) the monopolistic leader in industry j chooses $\{p_t(j), l_{x,t}(j)\}$ to maximize profit subject to Bertrand price competition and taking $\{w_t\}$ as given;
(d) R&D entrepreneur h chooses $\{l_{r,t}(h)\}$ to maximize expected profit taking $\{v_t, w_t\}$ as given;
(e) the market for final goods clears such that $y_t = c_t L_t$; and
(f) the labor market clears such that $n_t(l_{x,t} + l_{r,t}) = L_t \Leftrightarrow l_{x,t} + l_{r,t} = s_t = g_L/\xi$ for all t.

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