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A ONE-SECTOR GROWTH MODEL WITH CONSUMPTION STANDARD: INDETERMINATE OR DETERMINATE?*

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In a representative agent, one-sector growth model in which the discounting is decreasing in the consumption standard measured as the current average consumption flow, Drugeon (1998) establishes local indeterminacy. This paper extends Drugeon's setup in the discount rate. In our setup, the consumption standard is a habit stock that a weighted average of the whole history of average consumption flows in the past. Local indeterminacy emerges only when the speed of habit formation tends to infinity; otherwise, local indeterminacy cannot appear, no matter how large the habit affects the discount rate.
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1. Introduction

This paper studies a standard representative-agent, one-sector growth model with endogenous discounting. It re-examines the determinacy and indeterminacy of the equilibrium path around a steady state in relation to the role of the external effects of consumption standards. In this paper, the consumption standard in the society is the whole history of average consumption flows. The whole history of average consumption evolves in a manner just like those in the literature of habit formation, as pioneered by Ryder and Heal (1973).[1]

In a recent paper, Drugeon (1998) studies the role of consumption in a standard representative-agent, one-sector growth model with endogenous discounting. He formulates subjective discounting which increases in private consumption as in Uzawa (1968) and decreases in the current consumption standard in the society as measured by the average level of current consumption and finds that local indeterminacy arises in the neighbourhood of a steady state. Local indeterminacy is established in an existing one-sector, bounded growth framework based on the assumption of increasing returns in production in a competitive model, as pioneered by Benhabib and Farmer (1994) and Farmer and Guo (1994),[2] but the assumption of increasing returns in production is challenged by empirical

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[1] According to Ryder and Heal (1973, p. 2), the habit "is thus a weighted average of past consumption levels, with the weights declining exponentially into the past."

[2] In a two-sector model with bounded and unbounded growth, local indeterminacy is pioneered by Boldrin and Rusticini (1994) and extended by Venditti (1998) and Benhabib and Nishimura (1998). See survey by Benhabib and Farmer (1999). The model with increasing returns is identical, in the sense of giving rise to the same reduced form, to a model with monopolistic competition and constant markups, according to Benhabib and Farmer (1994). Recent studies have shown that indeterminacy under constant returns to scale is possible in two-sector unbounded growth models (e.g. Mino, 2001).

studies (e.g. Burnside, 1996; Basu and Fernald, 1997). Drugeon's work is important as it establishes local indeterminacy on the basis of the assumption of externalities in preference without relying upon the assumption of increasing returns to production.

Although Drugeon's work is valuable, it assumes the consumption standard in a society is equal to current average consumption flows.[3] Alternatively, the consumption standard in a society may be formed by average consumption in the past. While the setting of current consumption in Drugeon (1998) represents a flow consumption standard in the discount rate, the weighted average consumption in the past is a stock in the discount rate. The purpose of this paper is to consider the stock of the consumption standard in the discount rate.

Evidence that past consumption behaviour affects current consumption and thus consumption habits are formed in the past has been confirmed by, among others, Fuhrer (2000) in the use of time series data in the USA and by Carrasco *et al.* (2005) in the employment of panel data in Spain. Thus, a society's consumption standard cannot be formed by current consumption only but is formed from the past. Indeed, this idea of habit persistence from the past is hardly new, and dates to Marshall (1898) and Dusenbery (1949). Recent years have seen applications in macroeconomics. Literature on asset pricing under habit persistence in preferences has been developing; see Abel (1990), Constantinides (1990) and Campbell and Cochrane (1999). The feature of habit persistence in preference has been adopted in models, in order to explain stylized features in business cycles (Boldrin *et al.*, 2001); to show how procyclical tax policy affects the economy counter-cyclically (Ljungqvist and Uhlig, 2000); and to explain different economic growth rates (Chen, 2007; Doi and Mino, 2008).

Specifically, in this paper the consumption habit is formed by the whole history of consumption with the speed of adjustment in the way as formulated in Ryder and Heal (1973). Using the different formulation of the discount rate, the steady-state allocation is the same as in Drugeon. However, local dynamics are totally different. Local indeterminacy is possible only when the speed of the habit formation tends to infinity; otherwise, it is impossible. The reasons are as follows.

When the adjustment speed in the habit formation tends to infinity, the habit at each point in time t is equal to average current consumption flows in time t and thus the change in the habit stock is equal to zero for all time and the habit is a flow variable. In this environment, the dynamic equilibrium conditions are a 3×3 system with two control variables, which are consumption and the shadow price of impatience, and a state variable, which is capital with a predetermined initial value. When the negative effect of the consumption habit on the discount rate is sufficiently large in an absolute value, there are two stable roots and thus local indeterminacy emerges.

However, when the adjustment speed in habit formation is finite, the habit stock changes gradually and evolves in transitions. Thus, the change in the habit stock is equal to zero only in steady state and the habit is a stock variable. In this situation, the dynamic equilibrium conditions are a 4×4 system with an additional state variable, which is the habit stock with a predetermined initial value. As a result, the condition with a sufficiently large absolute value of the negative effect of the consumption habit on the discount rate that generates two stable roots, can now exactly pin down the unique equilibrium path and thus local indeterminacy is not possible.

[3] A similar local indeterminacy result was obtained in Chen and Hsu (2007) and Alonso-Carrera *et al.* (2008) wherein current average consumption enters in the felicity rather than the discount.

The intuition for local indeterminacy and determinacy goes as follows.[4] When the adjustment speed in the habit formation is infinite, the level of external habits is the level of current external consumption. In the household's optimization of consumption choices, optimal consumption is positively related to the shadow price of capital. Then, expectations of a higher rate of capital will raise the shadow price of capital, which in turns increases the demand for consumption, which declines capital accumulation. Thus, the rate of return to capital will rise and the sunspot-driven expectations change is self-fulfilled. Alternatively, when the adjustment speed in the habit formation is finite, however, the level of external habits is different from the level of current external consumption in transitions. In this circumstance, the household's optimization of consumption choices implies a negative relation between optimal consumption and the shadow price of capital. Moreover, the relation between optimal consumption and external habits is positive as external habits negatively affect the discount rate. Then, a sunspot-driven shock that leads to anticipations of a higher return to capital will raise the shadow price of capital. This reduces the demand for consumption as optimal consumption is negatively related to the shadow price of capital. A fall in consumption makes the level of consumption lower than the level of habits and thus, habits start declining. As optimal consumption is positively related to external habits, consumption declines further. Thus, current investment increases and the rate of return to capital will fall. As a consequence, the initial expectations are not self-fulfilled.

As developed below, Section 2 sets up the model and studies the steady state. Section 3 is the main body that examines the issue of local indeterminacy and determinacy. Finally, some concluding remarks are made in Section 4.

2. The model

Time is continuous. There is a continuum of a representative household with an infinite life, and a representative firm producing a single commodity serving as consumption and capital. Households own shares of firms and decide how much to consume and to save at each instant of time t. The savings accumulate capital that augments the stock of capital input in production at time $t + dt$.

The lifetime preference of the representative agent is represented as:

$$U = \int_0^\infty u(c(t))\,X(t)\,dt, \tag{1}$$

where u is the felicity function that depends additively on consumption c at the various dates, and X is the discount factor in t.

The discount is $X(t) \equiv \exp\left\{-\int_0^t \rho(c(\tau), H(\tau)d\tau)\right\}$, and is thus evolved as

$$\dot{X} = -\rho(c(t), H(t))\,X(t), \text{with } X(0) \text{ given}, \tag{2}$$

where $\rho(c, H)$ is the instantaneous discount rate in t. Notation $H(t)$ is the consumption standard in the society in t. Drugeon (1998) explains the reason why consumption standards

[4] We thank Kazuo Mino, the coeditor, for providing the intuition below.

affect the discount rate. Once a society has sufficient consumption and living standards, its inhabitants benefit from an increasing length of their planning horizon and increase their valuation of future outcomes.

We postulate the consumption standard as the stock of consumption habits in the society. The stock of consumption habits in the society in t is formed as a weighted average of past average consumption levels in the society, with the weights declining exponentially in the past, in the same way as in Ryder and Heal (1973, p. 2):

$$H(t) \equiv \theta \exp(-\theta t) \int_{-\infty}^{t} \exp(\theta \tau) C(\tau) d\tau, \theta \geq 0$$

where $C(t)$ is the average amount of consumption in the society in t. Thus, the stock of consumption habits is evolved as

$$\dot{H} = \theta(C(t) - H(t)), \text{with } H(0) \text{ given.} \tag{3}$$

Equations 2 and 3 indicate that the discount is endogenous, depending upon the individual's own consumption. Moreover, it also depends upon the consumption standard, which is the whole past weighted average consumption flow in the society.

Parameter θ is the speed of adjustment that the stock of consumption habits is formed from past average consumption flows. The larger θ is, the larger is the influence of the current average consumption in the determination of new consumption habits. Two extreme cases are as follows. First, if $\theta \to 0$, then no matter how the average consumption flow changes, the stock of consumption habits is never affected. As a result, $H(t)$ is fixed for all t and given by $H(0)$. Under the situation, the discounting function in Uzawa (1968) is obtained as $\rho = \rho(c(t), H(0))$. Second, if $\theta \to \infty$, consumption habits adjust at an infinite speed. Then, the consumption standard must adjust instantaneously; so fast that $H(t) = C(t)$ for all t.[5] In this case, the discount in Drugeon (1998) is obtained.

The following assumptions are imposed on the preference structure.

Assumption A:

1. *$u(c)$ is twice differentiable with $u' > 0 > u''$ for any $c > 0$.*

2. *$\rho(c, H)$ is twice differentiable with $\rho_1 > 0 > \rho_{11}$, $\rho_2 < 0$ and $\rho_1 + \rho_2 > 0$ for any $c > 0$ and $H > 0$, and $\rho_0 \equiv \rho(0, 0) \in (0, \infty)$.*

3. *$-\rho_{11}/\rho_1 < -u_{cc}/u_c$ for any $c > 0$.*

While Assumption A.1 is common in existing literature in order to assure an increasing and concave utility of individual consumption, the assumption $\rho_1 > 0 > \rho_{11}$ in A.2 has been a standard assumption in the literature of endogenous discounting since Uzawa (1968). Assumption A.2 assumes that an agent is more patient as the consumption standard in the society increases, an assumption that was conceived by Fisher (1987, p. 336) regarding wealth and the rate of preference for the present, which Drugeon (1998) thought to echo. Assumption A.2 assumes a decreasing social impatience as average consumption habit reduces the discount.[6] Moreover, we assume $\rho_1 + \rho_2 > 0$, so the

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[5] Because if not, the economy is unstable as shown by $H = \infty$ (*resp.* $-\infty$) if $H(t) >$ (*resp.* $<$) $C(t)$.

[6] In Drugeon (1998), the example $\rho(c, X) = (1 - \varepsilon)^{-1} c^{1-\varepsilon} X^{\upsilon-1}$, $\varepsilon, \upsilon \in (0, 1)$ satisfies Assumption A.2.

discount rate is increasing in private and social consumption as a whole. There is evidence that supports an increasing time preference as consumption increases; e.g. Ogawa (1993).[7] Finally, Assumption A.3 is a technical assumption that requires the curvature of the felicity function with respect to individual consumption be larger than the curvature of the discounting function with respect to individual consumption.

Another school of research is to consider average consumption or past habits in the felicity. Two methods have been used in the inclusion of average consumption in the felicity. One is to utilize average current consumption, like that used in Drugeon (1998). Studies in this fashion include Gali (1994), Palivos *et al.* (1997), Campbell and Cochrane (1999), Ljungqvist and Uhlig (2000) and Dupor and Liu (2003). Another method is to use past consumption habits, such as those used in our paper. Articles in the latter method comprise Abel (1990), Constantinides (1990), Boldrin *et al.* (2001), Alvarez-Cuadrado *et al.* (2004) and Chen (2007). Our formulation of habits in the discount is equivalent to placing habits in the felicity as seen in the following.

If we follow the solution in Uzawa (1968, pp. 486–494) and take $\Delta \equiv \int_0^t \rho(c(\tau), H(\tau))\,d\tau$ as the independent variable instead of t in the maximization, we obtain $U \equiv \int_0^t W(c, H)\,d\Delta$. The instantaneous felicity is now $W(c, H) = u(c)/\rho(c, H)$, where $W_2(c, H) > 0$ as $\rho_2 < 0$. Thus, the formulation of consumption externalities in a discount is equivalent to a formulation of consumption externalities in a felicity.

Finally, the representative firm is endowed with a neoclassical technology $Y = f(k)$, in which k is capital stock per capita with $k(0)$ given. For simplicity we assume k has no depreciation. The following standard neoclassical assumptions are made.

Assumption B: $f(0) = 0$, *and* $f' > 0 > f''$ *for any* k, *and* $\lim_{k\to 0} f'(k) = \infty$ *and* $\lim_{k\to\infty} f'(k) = 0$.

As the household is assumed to own the share of firms, income not spent on consumption now becomes savings and accumulates capital as follows.

$$\dot{k} = f(k(t)) - c(t). \tag{4}$$

The representative household problem is to maximize the lifetime utility in (1), given the constraints in (2) and (4), taking as given the consumption habit in (3). The Hamiltonian associated to the household's program is as follows.[8]

$$\mathcal{H}(c, k, X, \xi, \lambda) = X\{u(c) + \xi[f(k) - c] - \lambda\rho(c, H)\},$$

where ξ is the co-state variable associated with capital and λ is the co-state variable associated with the discounting.

Applying the Pontryagin maximum principle, we derive the necessary optimal conditions.

[7] Using the post-war annual time-series data, Ogawa (1993) found time preferences increased after the 1970s in Japan and Taiwan as the two economies grew, indicating an increasing time preference.

[8] To save space, the time index is omitted in the analysis below.

$$u' - \lambda\rho_1 = \xi, \tag{5a}$$

$$\dot{\xi} = \xi(-f' + \rho) \tag{5b}$$

$$\dot{\lambda} = -u + \lambda\rho, \tag{5c}$$

$$\lim_{t\to\infty} \xi k(t) = 0, \tag{5d}$$

$$\lim_{t\to\infty} \lambda X(t) = 0. \tag{5e}$$

In the optimal conditions, (5a) equates the marginal costs and the net discounted marginal utility, which is the discounted marginal utility adjusted for the resulting increase in discounting. Equations (5b) and (5c) are the Euler equations for capital and discounting. Finally, (5d) and (5e) are the transversality conditions assuring that the value of capital and discounting is not exploded.

Definition 1: *Under Assumptions A and B, for* $k(0)$ *and* $H(0)$, *a symmetric equilibrium is a tuple* $\{C, k, \xi, \lambda, H\}$ *with* $c = C$, *which solves* (3), (4) *and* (5a) – (5c).

Definition 2: *Under Assumptions A and B, for* $k(0)$ *and* $H(0)$, *a steady-state equilibrium is a tuple* $\{C^*, k^*, \xi^*, \lambda^*, H^*\}$ *with a constant C and* $\dot{\xi} = \dot{k} = \dot{\lambda} = \dot{H} = 0$.

Thus, the steady state is determined by

$$\xi^* = u'(C^*) - \lambda^*\rho_1(C^*, H^*), \tag{6a}$$

$$\rho(C^*, H^*) = f'(k^*), \tag{6b}$$

$$\lambda^* = \frac{u(C^*)}{\rho(C^*, H^*)}, \tag{6c}$$

$$f(k^*) = C^*, \tag{6d}$$

$$C^* = H^*. \tag{6e}$$

We obtain

Proposition 1. *Under Assumptions A and B, there exists a unique interior steady state* (C^*, k^*, H^*, λ^*, ξ^*).

Proof: First, we substitute $C^* = H^*$ in (6e) into (6a) – (6d). Then, (6d) results in $k^* = k(H^*)$ with $k(H^*) = 0$ at $H^* = 0$, according to Assumption B, and [Reinsert] while (6b) becomes

$$\rho(H^*, H^*) = f'(k(H^*)) \tag{7}$$

While the left-hand side of (7) is strictly increasing in H^* as $\rho_1 + \rho_2 > 0$ according to Assumption A.2 the right-hand side is strictly decreasing in H^* as [Reinsert]. Moreover, at $H^* = 0$, $f'(0) = \infty$, according to Assumption B, and $\rho(0, 0) = \rho_0 < \infty$, according to Assumption A.2, and thus $\rho(0, 0) < f'(0)$. As a result, there exists a unique interior H^* in steady state such that (7) is satisfied. A unique H^* in turn determines a unique interior C^*, a unique interior k^* in (6d), a unique interior λ^* in (6c), and finally, a unique interior ξ^* in (6a). ■

We note that if instead $\rho_1 + \rho_2 < 0$, the left-hand side of (7) is negatively sloping. Moreover, if $\rho_1 + \rho_2$ is not monotonically decreasing in H, then there may be multiple steady states. In Assumption A.2 we rule out this case for two reasons. First, evidence supports an increasing time preference as consumption increases (e.g. Ogawa, 1993). Second, attention is paid to the existence of a unique steady state in order to study local indeterminacy around the unique steady state. With multiple steady states, local indeterminacy arises more easily in equilibrium and, moreover, there may be global indeterminacy even if each steady state is locally a saddle (e.g. Chen, 2007).

We must point out from (6e) that in a steady state, consumption habits equal current equilibrium consumption flows. Substituting $C^* = H^*$ into other equations in the system, we find the same unique equilibrium conditions in a steady state as in Drugeon (1998). However, as consumption habits evolve in transitions as long as $\theta < \infty$ and thus the speed of habit formation is finite, the consumption standard differs from the amount of average current consumption flows in transitions. Therefore, as we analyze below, transitional dynamics are totally different if θ is less than infinity.

3. Indeterminacy and determinacy of dynamic path

This section investigates the dynamic properties in the neighbourhood of the unique steady state. Before we analyze the stability properties in our general model, let us consider the Drugeon (1998) model, which is a special case in our model that emerges only when the speed of habit formation tends to infinity, $\theta \to \infty$.

3.1 The speed of habit formation tends to infinity

When the speed of habit formation tends to infinity, it is necessary to have $H_t = C_t$ at all times; otherwise, there is no steady state. Thus, habits are formed instantaneously and [Reinsert] at any point in time t. Therefore, the expression for [Reinsert] in (3) is not a part of the dynamic equilibrium conditions. The dynamic equilibrium conditions include (4), (5a), (5b) and (5c) with $\rho = \rho(C, C)$ that determine the path of C, λ, ξ and k.

If we take the linearization of the dynamic equilibrium system, evaluated at the unique steady state, we obtain:[9]

$$\begin{bmatrix} 0 \\ \dot{\xi} \\ \dot{\lambda} \\ \dot{k} \end{bmatrix} = \begin{bmatrix} \chi & -1 & -\rho_1 & 0 \\ \xi(\rho_1+\rho_2) & 0 & 0 & -\xi f'' \\ -u' + \lambda(\rho_1+\rho_2) & 0 & \rho & 0 \\ -1 & 0 & 0 & f' \end{bmatrix} \begin{bmatrix} C - C^* \\ \xi - \xi^* \\ \lambda - \lambda^* \\ k - k^* \end{bmatrix}, \tag{8a}$$

where $\chi \equiv u'' - (\rho_{11} + \rho_{12})\lambda = u'' - (\rho_{11} + \rho_{12})(u/\rho)$, is "the *equilibrium* marginal value of wealth as a function of consumption" in Drugeon (1998, p. 355).

[9] Following the method used in Chen *et al.* (2008), in the stability analysis below, (5a) and (5b) are not combined into the law of motion of consumption. In an earlier version available upon request, we provided the stability analysis when (5a) and (5b) were combined to obtain the law of motion of consumption. Both methods lead to the same roots.

Let ω be the corresponding eigenvalue for the Jacobean matrix in (8a). The characteristic roots are determined when the value of the following determinant is zero,

$$\begin{vmatrix} \chi & -1 & -\rho_1 & 0 \\ \xi(\rho_1+\rho_2) & 0-\omega & 0 & -\xi f'' \\ -u'+\lambda(\rho_1+\rho_2) & 0 & \rho-\omega & 0 \\ -1 & 0 & 0 & f'-\omega \end{vmatrix} = 0, \tag{8b}$$

which leads to the following characteristic polynomial

$$\Lambda(\omega) = (\rho-\omega)\Phi(\omega) = 0,$$

where $\Phi(\omega) \equiv a_0\omega^2 + a_1\omega + a_2$,

$$a_0 \equiv \chi,$$
$$a_1 \equiv [u' - \lambda(\rho_1+\rho_2)]\rho_1 - \rho\chi - \xi(\rho_1+\rho_2) = -\rho\chi - \rho_2 u'$$ [10]
$$a_1 \equiv -\xi f'' + \xi(\rho_1+\rho_2) > 0.$$

Along the Epstein (1987) class of result, it is readily known that $\omega = \rho > 0$ is a trivial root. We need to investigate the other two roots such that $\Phi(\omega) = 0$. As the economic system involves a state-like variable whose initial value is predetermined, $k(0)$, the equilibrium path in the neighbourhood of the unique steady state is determinate if there is a negative root (i.e. stable root) and indeterminate if there are two roots with negative real parts.

Drugeon (1998, Corollary 1) considers the joint holding of $\chi > 0$ and $\rho_2 < 0$, which we follow below. As $a_0 = \chi > 0$, the signs of the two roots in $\Phi(\omega) = 0$, denoted as ω_1 and ω_2, satisfy the following two conditions.

$$\text{Sign}\{\omega_1\omega_2\} = \text{Sign}\{a_2\} > 0,$$

$$\text{Sign}\{\omega_1 + \omega_2\} = -\text{Sign}\{a_1\}.$$

It is clear that $\omega_1\omega_2 > 0$ and thus, the roots are both either negative or positive. Examination of the sign of $\omega_1 + \omega_2$ indicates that both roots are negative if $a_1 > 0$, or

$$\rho + \rho_2 u'/\chi < 0. \tag{9}$$

The condition (9) is exactly identical to the condition in the part (i), Corollary 1 in Drugeon (1998, pp. 356). This condition establishes local indeterminacy near a steady state in an economy without an external effect in production. The condition (9) may be rewritten as

$$-\rho_2 > \rho\chi/u'.$$

The above expression says that if there is a sufficiently negative effect of the consumption standard on the discount, then there are two stable roots. Thus, in Drugeon (1998), a sufficient negative effect of average consumption flows on the discount leads to local indeterminacy.

The intuitive reason for Drugeon's result can be easily understood if we use (5a) and obtain the relationship of optimal consumption $c = c(\xi)$, with $c'(\xi) = \partial c/\partial\xi = 1/\chi$. Under

[10] The relationship (5a) is used in the derivation of the second equality of a_1.

the condition $\chi > 0$, $c'(\xi) > 0$ means that a rise in the shadow price of capital lowers optimal current consumption. Suppose that the economy initially stays at a steady state. Suppose further that a sunspot-driven shock hits the economy and that the households anticipate that the rate of return to capital increases. This raises the shadow price of capital. A rise in the shadow price of capital increases the optimal demand for consumption, which declines capital accumulation. Thus, the rate of return to capital will rise and the sunspot-driven expectations change is self-fulfilled.

3.2 The speed of habit formation is finite

We now turn to the model when the speed of habit formation is finite. Now, H adjusts gradually and has transitional dynamics and thus $\dot{H} \neq 0$ unless a steady state is reached. Thus, the expression for $\dot{H}$ in (3) is a part of the dynamic equilibrium conditions. The dynamic equilibrium system is therefore composed of (3), (4), (5a), (5b) and (5c).

If we take the linearization of the dynamic equilibrium system, evaluated at the unique steady state, we obtain

$$\begin{bmatrix} 0 \\ \dot{\xi} \\ \dot{\lambda} \\ \dot{k} \\ \dot{H} \end{bmatrix} = \begin{bmatrix} u'' - \lambda\rho_{11} & -1 & -\rho_1 & 0 & -\lambda\rho_{12} \\ \xi\rho_1 & 0 & 0 & -\xi f'' & \xi\rho_2 \\ -\xi & 0 & \rho & 0 & \lambda\rho_2 \\ -1 & 0 & 0 & f' & 0 \\ \theta & 0 & 0 & 0 & -\theta \end{bmatrix} \begin{bmatrix} C - C^* \\ \xi - \xi^* \\ \lambda - \lambda^* \\ k - k^* \\ H - H^* \end{bmatrix}, \tag{10a}$$

Denote as μ the corresponding eigenvalue of the Jacobean matrix in (10a). The characteristic roots are determined when the value of the following determinant is zero,

$$\begin{vmatrix} u'' - \lambda\rho_{11} & -1 & -\rho_1 & 0 & -\lambda\rho_{12} \\ \xi\rho_1 & 0-\mu & 0 & -\xi f'' & \xi\rho_2 \\ -\xi & 0 & \rho-\mu & 0 & \lambda\rho_2 \\ -1 & 0 & 0 & f'-\mu & 0 \\ \theta & 0 & 0 & 0 & -\theta-\mu \end{vmatrix} = 0, \tag{10b}$$

which yields the following characteristic polynomial

$$\Gamma(\mu) = \xi\sigma(\mu - \rho)\Omega(\mu) = 0, \tag{11}$$

where $\Omega(\mu) \equiv a_0\mu^3 + a_1\mu^2 + a_2\mu + a_3$,

$$a_0 = -1 < 0,$$

$$a_1 = -(\theta - \rho) - \frac{\theta}{\sigma}\frac{\lambda}{u' - \lambda\rho_1}\rho_{12},$$

$$a_2 = \theta\rho + \frac{\rho\rho_1}{\sigma} - \frac{1}{\sigma}\left(\frac{\theta u'}{u' - \lambda\rho_1}\rho_2 + f''\right) + \frac{\theta}{\sigma}\frac{u}{u' - \lambda\rho_1}\rho_{12},$$

$$a_3 = \frac{\theta}{\sigma}[\rho(\rho_1 + \rho_2) - f''] > 0.$$

$$\sigma \equiv \frac{-(u'' - \lambda\rho_{11})}{u' - \lambda\rho_1} > 0.$$

Obviously, a trivial eigenvalue is $\mu = \rho > 0$. We now investigate $\Omega(\mu) = 0$ for the remaining three roots. According to the Routh–Hurwitz theorem, the number of eigenvalues with positive real parts of the polynomial $\Omega(\mu) = 0$ is equal to the number of variations in signs of the following series,

$$\left\{a_0, a_1, \frac{\Delta_2}{a_1}, a_3\right\}, \tag{12}$$

where $\Delta_2 = a_1a_2 - a_0a_3$.

As the economic system involves two state-like variables whose initial values are predetermined at $k(0)$ and $H(0)$, the equilibrium path in the neighbourhood of the unique steady state is indeterminate only if the number of stable roots is three, which requires no positive root and thus no variation in signs in (12).

To analyze the number of stable roots, we first note in a_3 that $f'' < 0$ and $\rho_1 + \rho_2 > 0$. Under Assumption A3 that the curvature of the felicity with respect to individual own consumption is larger than the curvature of the discount function with respect to individual own consumption, σ is positive. Indeed, σ is the reciprocal of the intertemporal elasticity of substitution and is in general positive. As a result, $a_3 > 0$.

In order to obtain local indeterminacy of the equilibrium path near the unique steady state, it must be $a_0 > 0$, $a_1 > 0$ and $\Delta_2 > 0$, so the number of variations in signs of the series in (12) is zero. This is impossible as $a_0 < 0$. As a result, local indeterminacy will not emerge in this model.

The reasons for local determinacy in our model are as follows. The speed of the habit formation is finite and the habit stock then evolves gradually in transitions. The habit stock is thus a state variable rather than a control variable. Therefore, the equilibrium system now has two state variables, k and H. In this situation, the condition with a sufficiently large absolute value of the negative effect of the consumption habit on the discount rate that generates two stable roots, now can exactly pin down the unique equilibrium path. It is thus impossible for local indeterminacy to emerge.

Intuitively, the results may be understood if we use (5a) and obtain the relationship of optimal consumption $c = c(\xi, H)$, with $c_1(\xi, H) = \partial c/\partial\xi = 1/(u'' - \lambda\rho_{11}) < 0$ according to Assumption A3 and $c_2(\xi, H) = \partial c/\partial H = (\lambda\rho_2)/(u'' - \lambda\rho_{11}) > 0$. Suppose that the economy initially stays at a steady state. Suppose further that a sunspot-driven shock hits the economy and that the households anticipate that the rate of return to capital increases. This raises the shadow price of capital and because of $c_1(\xi, H) < 0$, the demand for consumption is reduced. In addition, a fall in consumption makes the level of consumption lower than the level of habits, so that habits start declining. As $c_2(\xi, H) > 0$, a fall in habits yields a further decline in consumption. Thus, current investment increases and the rate of return to capital will fall. As a consequence, the initial expectations are not self-fulfilled.

Summarizing the results in this subsection, we now obtain:

Proposition 2: *Under Assumptions A and B and a finite speed of habit formation, the equilibrium path in the neighbourhood of a steady state is never locally indeterminate.*

4. Concluding remarks

This paper re-examines the determinacy and indeterminacy of the steady state in relation to the role of the consumption standard in a standard representative-agent, one-sector growth model with the endogenous discount dependent on the consumption standard. A key departure of this paper from existing literature is the way the consumption standard is formed. The consumption standard is a habit stock in our paper and is formed by the whole past average consumption flows. The feature suggests the equilibrium is self-fulfilling under a sufficiently large negative effect of the habit stock on the discount only if the speed of habit formation is infinite so that the habit stock is always equal to current average consumption and thus is intrinsically a control-like variable. If the adjustment speed of habit formation is less than infinite, the habit stock is different from current average consumption. Then, the habit stock can only adjust gradually and is thus a stock variable. Therefore, the dynamic equilibrium system then has an additional state variable. Local indeterminacy cannot emerge under two stable roots in the environment of a sufficiently large negative effect of the habit stock on the discount that would otherwise generate local indeterminacy in a model when consumption standards are a flow variable.

Recently, in a one-sector growth model with leisure choices and fixed discount rates, Alonso-Carrera *et al.* (2008) have obtained local indeterminacy if the instantaneous utility is not homothetic with respect to the private and social levels of consumption. As argued in our paper and documented in empirical research, consumption habits in a society are formed from the past and thus, the speed of habit formation is finite. In such an environment, we expect that their indeterminacy result may be eliminated. This thus points to a direction of further research.

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