

# Public Finance Review

<http://pfr.sagepub.com/>

---

## General Fund Financing, Earmarking, Economic Stabilization, and Welfare

Been-Lon Chen and Shun-Fa Lee

*Public Finance Review* published online 24 February 2009

DOI: 10.1177/1091142109331634

The online version of this article can be found at:

<http://pfr.sagepub.com/content/early/2009/02/24/1091142109331634>

A more recent version of this article was published on - Aug 31, 2009

---

Published by:



<http://www.sagepublications.com>

Additional services and information for *Public Finance Review* can be found at:

**Email Alerts:** <http://pfr.sagepub.com/cgi/alerts>

**Subscriptions:** <http://pfr.sagepub.com/subscriptions>

**Reprints:** <http://www.sagepub.com/journalsReprints.nav>

**Permissions:** <http://www.sagepub.com/journalsPermissions.nav>

[Version of Record - Aug 31, 2009](#)

>> [OnlineFirst Version of Record - Feb 24, 2009](#)

[What is This?](#)

# General Fund Financing, Earmarking, Economic Stabilization, and Welfare

Been-Lon Chen

*Institute of Economics, Academia Sinica*

Shun-Fa Lee

*Tamkang University*

Discussion has been made concerning the pros and cons of financing public projects via either earmarking or a general fund. The article studies the desirability of earmarked and general fund financing based on economic stabilization in a two-sector growth model. Regardless of the nature of public goods, earmarked taxes contribute to aggregate stabilization, while general fund financing may be destabilizing and cause fluctuations. The underlying mechanism in favor of earmarked taxes against general fund financing is that general fund financing creates intersectoral externalities and strategic complementarities that are sufficiently large to exert endogenously persistent and recurring fluctuations in aggregate activities in the absence of shocks to fundamentals. Earmarked taxing generates only sector-specific externalities that are too small to exert local indeterminacy. In a calibrated version, we compute the level of long-run welfare, and the results reflect favorably on the use of earmarked taxing.

**Keywords:** *earmarked tax; general fund finance; indeterminacy; welfare*

## I. Introduction

This article studies the desirability of earmarked taxing against general fund financing based on the implications for economic stabilization. Under general fund financing, the tax revenues from different sources are placed in a general fund, from which different government programs are financed. For example, in the case of national defense, federal taxes from many

---

**Authors' Note:** The authors gratefully acknowledge the discussions with C. C. Yang, J. Alm, and F.-S. Hung. Please address correspondence to Been-Lon Chen, The Institute of Economics, Academia Sinica, 128 Academia Rd, Section 2, Taipei, Taiwan 11529; e-mail: [bchen@econ.sinica.edu.tw](mailto:bchen@econ.sinica.edu.tw).

sources are collected and placed within the treasury, and then Congress spends the amount it deems appropriate for national defense without regard to where the revenues were raised. In earmarked taxes, the revenues are designated to particular spending activities, thus providing a direct link between tax revenues from one source and earmarked expenditure for a particular task. For instance, the federal gasoline taxes and motor vehicle fees, airport and air ticket taxes, television licensing fees, tolls, and hotel room and tourist taxes in some countries are earmarked for the activities in the transportation and tourist sectors, including the building and maintenance of transportation services; other examples are cigarette taxes, carbon taxes, and air pollution fees that are earmarked for the activities in the health service and environmental sectors.<sup>1</sup> Coffee taxes in Colombia are also a famous example of earmarked taxes. According to Teja and Bracewell-Milnes (1991), a number of taxes are levied on coffee exports. A considerable percentage of the proceeds are earmarked for its price stabilization activities and subsidizing domestic coffee consumption. Thus, earmarked spending is not only in infrastructure but also in other aspects. In all these sectors, the tax revenues from a particular source/sector go toward a relevant spending destination/sector.<sup>2</sup>

Conventional wisdom provides extensive discussion concerning the pros and cons of earmarking and general fund. Earmarking allows for the median voter to choose which quantities of the public goods should be provided at which tax prices, and calls for a simultaneous choice in a level of taxation and expenditure on an item-by-item basis. A traditional argument for earmarked taxes is that earmarking protects high-priority programs from shifting majorities, inefficiency, and corruption and entails a very process of budgetary choice, which links directly between revenues of a particular tax and expenditures for a particular public task. Moreover, earmarked taxation works as a commitment solution solving time-inconsistency problems in tax policy. Finally, the earmarked tax is better able to reflect personal preferences in collective choices and is a method of channeling the incentives of politicians in socially useful directions. The argument in favor of earmarked taxing began with the classic article based on the public finance approach by Buchanan (1963), followed by Goetz (1968), Browning (1975), and Marsilliani and Renstrom (2000), among others.

Alternatively, general fund financing separates choices of taxation from choices of spending and conceptually creates a two-step decision process, with the aggregate taxation level chosen first and the distribution of those revenues among expenditure programs decided thereafter. Some of the opponents of earmarked taxes in support of general fund financing maintain

that earmarking introduces inflexibility and leads to a misallocation of resources (McMahon and Sprenkle 1972), while others argue the erosion of budgetary efficiency (Teja 1988), the inefficiency under political uncertainty (Brett and Keen 2000), and low internal rates of returns and high distortions (Butler 2000). Recently, Bös (2000) considered the situation where taxing and spending are performed by two separate agents in a principal-agent setting and found that earmarking is not optimal and hence no longer as desirable as advocated by Buchanan (1963) and others.

This article argues for the attractiveness of earmarked taxes based on the implications affecting economic stabilization, as a method different from a traditional public finance approach. The stability of a policy means that the underlying policy guarantees a determinate equilibrium path, whereas destabilization indicates that the equilibrium path is locally indeterminate and there are endogenous, welfare-reducing fluctuations unrelated to economic fundamentals.<sup>3</sup> To our knowledge, this study is the first attempt to evaluate the two tax regimes based on stability properties.

Specifically, we set up a standard endogenous growth model with two sectors, where one of the sectors producing pure consumption goods and the other sector producing either pure investment goods or composite goods that may be consumed or invested. There is output taxation on both sectors, with provisions of public expenditure in accordance with one of the two fiscal regimes. Public goods under provision may be partly productive and partly consumptive in nature.<sup>4</sup> In this article, public goods are formulated either as production- or as utility-enhanced.

There are various ways of implementing public spending of tax revenues from earmarking and general fund tax regimes. With tax revenues, public spending may be put into service in terms of productive public goods, consumptive public goods, transfers, or other kinds such as public wastes. We do not choose transfers and public wastes as the ways of public spending. Our study covers only provisions of public goods among ways of public spending for the following reasons. Much of public spending is in terms of provisions of public goods. Provisions of public goods, either in terms productive or consumptive public goods, are often seen in existing two-sector growth models. Moreover, our choice may serve as a benchmark for further analysis in other types of public spending.

Different destinations of useful public services are distinguished by the activities in different sectors in our model. The general fund financing regime is represented here by output taxation from two sectors for public services toward these two sectors, and the earmarked taxing regime corresponds to output taxation in one sector with public spending toward that

sector. Although earmarking is a special case of general fund financing, such a feature is not characterized in our analysis. The main difference between these two regimes is that the allocation of resources for earmarking is intrasector and that for general fund financing is intersector. Although the configuration may capture only one aspect of the differences about the two regimes in practice, such a differentiation is a trade-off we make in a simple general equilibrium model.

A brief account of the results is as follows: while earmarked taxing contributes to aggregate stabilization, general fund financing may generate endogenously persistent and recurring fluctuations unrelated to economic fundamentals. The argument against general fund financing is the presence of intersectoral resource reallocation resulting from the provisions of public goods to one sector using resources from the other sector. This intersectoral externality generates strategic complementarities between sectors that are so large that self-fulfilling prophecies emerge driving the economy either to experience rapid capital accumulation and high growth or to suffer from slow capital accumulation and low growth. In both the cases of production- and utility-enhanced public goods, local indeterminacy is established if the degree of intersectoral externality exceeds a threshold. Earmarked taxing creates a sector-specific externality, but the externality here is not large enough to exert local indeterminacy.

Dynamic stability has been one of the popular research topics in a general equilibrium, two-sector model. The contribution of our study to this type of a two-sector setup lies in comparisons of the dynamic stability properties in two popular tax regimes. In particular, our study finds that an intersectoral externality may arise easily from a particular fiscal regime that is emerged not only in production-related but also in consumption-related public good provisions.<sup>5</sup> The existing literature has paid no attention to the source of dynamic instabilities in association with particular types of tax regimes. Moreover, in a calibrated version of the model, we also compute the long-run welfare in these two tax regimes. We find the level of welfare in an earmarked tax is higher than the level of welfare in a general fund tax. Therefore, this article provides support in favor of earmarked taxing against general fund financing based on both aggregate stabilization in transitional dynamics and the level of welfare in the long run.

Section 2 below studies the model with production-enhanced public goods, and section 3 investigates the model with utility-enhanced public goods. Section 4 calibrates the model and compares the level of welfare in the long run. Section 5 concludes the article.

## 2. Production-Enhancing Public Goods

The economy consists of households, firms, and the government, with two sectors that are competitive and for convenience, are called sectors  $X$  and  $Y$ . Capital is the only private input in both sectors, but it may be thought of as a composite of physical and human capital (Rebelo 1991). There is a continuum of infinitely lived, representative agent whose measure is normalized to unity. There is a representative firm, endowed with an AK-type technology, the simplest technology in consistency with perpetual growth.

The production in both sectors is externally enhanced by public services in the fashion in Barro (1990) and others. Following the two-sector setup in Boldrin and Rusticini (1994) and Drugeon, Poulsen, and Venditti (2003), sector  $X$  is pure consumption goods and sector  $Y$  is pure investment goods.<sup>6</sup> The technologies in both sectors are as follows:

$$X = (sK)^{1-\alpha} G_X^\alpha, 0 < \alpha < 1, \quad (1a)$$

$$Y = a[(1-s)K]^{1-\beta} G_Y^\beta, 0 < \beta < 1, \quad (1b)$$

where  $a$  is the productivity coefficient in sector  $Y$ ,  $s$  is the fraction of capital  $K$  allocated to sector  $X$ , and  $G_X$  and  $G_Y$  are the provisions of public goods toward sectors  $X$  and  $Y$ , with  $\alpha$  and  $\beta$  their contribution to the production, respectively.

The government levies output taxes in both sectors with a tax rate  $\tau$ .<sup>7</sup> Let  $T_X$  and  $T_Y$  be the tax revenues in sectors  $X$  and  $Y$ . Then,

$$T_X = \tau X, T_Y = \tau Y \quad (2)$$

The household budget constraints are

$$C = (1 - \tau)X, \quad (3a)$$

$$\dot{K} = (1 - \tau)Y - \delta K, K(0) \text{ given}, \quad (3b)$$

where  $C$  is the consumption and  $\delta$  is the depreciation rate of capital.

The representative agent is assumed to possess a discounted lifetime utility, with a felicity exhibiting a constant, intertemporal elasticity of substitution as follows:

$$\int_0^\infty e^{-\rho t} \frac{C^{1-\sigma} - 1}{1-\sigma} dt, \sigma \geq 0, \rho > 0, \quad (4)$$

where  $\sigma$  is the reciprocal of the intertemporal elasticity of substitution and  $\rho$  is the discount rate.

The fiscal system is classified into general fund financing and earmarking. Different activities in our model are distinguished by sectors. The general fund financing is formalized by output taxation from two sectors for spending toward these two sectors. The earmarked tax is formalized by output taxation in one sector for expenditure on public services in that sector.

We assume that the government divides the taxes from each sector's resource into two parts as follows:

$$T_X = uT_X + (1 - u)T_X \quad (5a)$$

$$T_Y = vT_Y + (1 - v)T_Y \quad (5b)$$

where  $u \in (0, 1)$  and  $v \in (0, 1)$ , determined by the government, are the fraction of tax on sectors  $X$  and  $Y$ , respectively, allocated to the provision of one kind of public service.

The taxes are then used to provide public services. To distinguish the two regimes, we need a general technology of the public goods provision. Moreover, to be consistent with sustainable growth, the technology must be of constant returns. In line with the Cobb-Douglas form in (1a) and (1b), we formalize the technologies of public good provision as follows:

$$G_X = G_{X1}^\theta G_{X2}^{1-\theta} \quad (6a)$$

$$G_Y = G_{Y1}^\eta G_{Y2}^{1-\eta} \quad (6b)$$

where  $G_{X1}$  and  $G_{X2}$  (respectively  $G_{Y1}$  and  $G_{Y2}$ ) stand for the two inputs used by the government to provide public services toward sector  $X$  ( $Y$ ), and  $\theta$  and  $1-\theta$  ( $\eta$  and  $1-\eta$ ) represent the contribution of inputs toward the provision of public services for sector  $X$  ( $Y$ ). We remark that in equation (6) it is impossible to directly add up the two inputs when providing public services. The reasons are that if the two inputs come from different sectors, they have different shadow prices.

Let us remark on the use of private capital in the provision of public services. The consideration of the use of private capital in the production of public inputs will not change the main insights, but the algebra becomes much more complicated. To see why, we may denote  $\chi$  as the fraction of private capital that goes to the public sector and thus  $1-\chi$  as the remaining fraction to the private sector. For the former fraction  $\chi$ , if fraction  $\omega$  is used in the production of  $G_X$ , then  $1-\omega$  is in the production of  $G_Y$ . Then,  $\chi$  and  $\omega$  are two new variables and are control variables. While  $\chi$  is determined by

equalizing the marginal products between private goods and public goods,  $\omega$  is determined by equalizing the marginal products between two public goods. In equilibrium conditions, we may express  $\chi$  and  $\omega$  as functions of  $s$ :  $\chi = \chi(s)$  and  $\omega = \omega(s)$ . As will be seen below, the dynamic system is summarized by the single variable  $s$ . If private capital is used in the provisions of public services, the dynamic system in  $s$  is more complicated as it involves  $\chi = \chi(s)$  and  $\omega = \omega(s)$ . Other than making the dynamic equation more complicated, this will not change the results as it is the degree of intersectoral externality that is produced by a general fund tax and drives aggregate destabilization.

Since the taxes are divided into four parts and there are two kinds of public good provisions, there are six ways in the combination of resources into inputs. Among the six types of combination, only one type can be used to capture the earmarking regime: tax sources from sector  $X$  ( $Y$ ) are used to provide public services toward sector  $X$  ( $Y$ ). Specifically, earmarking is represented by the following<sup>8</sup>:

$$G_{X1} = uT_X, \quad G_{X2} = (1 - u)T_X; \quad (7a)$$

$$G_{Y1} = vT_Y; \quad G_{Y2} = (1 - v)T_Y. \quad (7b)$$

However, general fund financing is described by the other five combinations. Without loss of generality, we only examine the following benchmark case:

$$G_{X1} = uT_X, \quad G_{X2} = vT_Y; \quad (8a)$$

$$G_{Y1} = (1 - u)T_X, \quad G_{Y2} = (1 - v)T_Y. \quad (8b)$$

The main difference between these two regimes is that the allocation of resources for earmarking is intrasector but that for general fund financing is intersector.

It is easy to see adding up conditions for public inputs. In earmarked taxes, equations (5a) and (7a) yields  $G_{X1} + G_{X2} = T_X = \tau X$ , and equations (5b) and (7b) lead to  $G_{Y1} + G_{Y2} = T_Y = \tau Y$ . In a general fund regime, equations (5a) and (8a) generate  $G_{X1} + G_{Y1} = T_X = \tau X$ , and equations (5b) and (8b) give rise to  $G_{X2} + G_{Y2} = T_Y = \tau Y$ .

Given  $K(0)$ ,  $\tau$ ,  $u$ ,  $v$ , and public goods, the representative agent's problem is to choose  $C$ ,  $s$ , and  $K$  to maximize its discounted utility in equation (4), subject to the constraints in equations (3a,b) and (1a,b). If we let  $\lambda(t)$  be the co-state variable associated with  $K(t)$ , then the necessary conditions are as follows:

$$(1 - \alpha)C^{-\sigma} \frac{X}{s} = \lambda(1 - \beta) \frac{Y}{1 - s}, \quad (9a)$$

$$(1 - \alpha)C^{-\sigma} \frac{(1 - \tau)X}{K} + \lambda[(1 - \beta) \frac{(1 - \tau)Y}{K} - \delta] = \lambda\rho - \dot{\lambda}, \quad (9b)$$

and the transversality condition  $\lim_{t \rightarrow \infty} e^{-\rho t} \lambda K = 0$ .

While equation (9a) equates the marginal product of capital between the consumption and the investment sector, equation (9b) is the Euler equation for capital. We are ready to evaluate the stabilization properties in each tax regime. We start with an earmarked taxing regime, followed by a general fund financing regime.

## Earmarking Taxes

The equilibrium conditions include equations (1a,b), (2), (3a,b), (6a,b), (7a,b), and (9a,b) that determine  $T_X$ ,  $T_Y$ ,  $X$ ,  $Y$ ,  $G_X$ ,  $G_Y$ ,  $G_{X1}$ ,  $G_{X2}$ ,  $G_{Y1}$ ,  $G_{Y2}$ ,  $C$ ,  $K$ ,  $s$ , and  $\lambda$ . To analyze the equilibrium, we transform nonstationary into stationary variables and eventually summarize the dynamic system in one equation.

First, let  $p$  denote the shadow price of the investment good in terms of the consumption good. If we use equation (9a), together equations (1a,b), (2), (6a,b), and (7a,b), we obtain

$$p \equiv \lambda C^\sigma = \frac{1 - \alpha}{(1 - \beta)} a^{-\frac{1}{1-\beta}} [u^0(1 - u)^{1-0}]^{\frac{\alpha}{1-\beta}} [v^0(1 - v)^{1-0}]^{-\frac{\beta}{1-\beta}} \tau^{\frac{\alpha}{1-\alpha} - \frac{\beta}{1-\beta}}. \quad (10)$$

Then, using equations (9a,b), with equations (1b), (2), (6a,b), and (7a,b), yields a relationship for  $\frac{\dot{\lambda}}{\lambda}$ . Moreover, we use equations (1b) and (3b) to obtain an expression for  $\frac{\dot{K}}{K}$ . Finally, if we substitute equations (1a) and (3a) into (10) and then differentiate, with the use of relationships  $\frac{\dot{\lambda}}{\lambda}$  and  $\frac{\dot{K}}{K}$ , we obtain

$$\dot{s} = \frac{s}{\sigma} \{(\sigma - 1)\delta - \rho + (1 - \tau)\phi_1[(1 - \beta) - \sigma(1 - s)]\} \equiv \Theta(s), \quad (11)$$

where  $\phi_1 \equiv \{a[\tau v^\eta(1 - v)^{(1-\eta)\beta}]\}^{\frac{1}{1-\beta}}$ .

Equation (11) is a first-order differential equation in the fraction of capital allocated to the consumption good sector and summarizes the dynamics of the economy in equilibrium. Once the equation determines  $s$ , other variables are determined by substituting  $s$  into the other equations.

A steady-state equilibrium is a balanced growth path (BGP) when  $\dot{s} = 0$ . It is evident that there exists a unique BGP at  $s^* = 1 - \frac{1-\beta}{\sigma} - \frac{(\sigma-1)\delta-\rho}{\sigma\phi_1(1-\tau)}$ . The BGP is interior when  $0 < s^* < 1$ , which is so if  $0 < (1-\beta)(1-\tau)\phi_1 - \delta - \rho < \sigma[\phi_1(1-\tau) - \delta]$ . The condition is standard in an endogenous growth model: while the first inequality requires high productivity for positive growth in a BGP, the second inequality demands a bound on productivity to meet the transversality condition.

As  $s$  is a control variable that adjusts instantaneously, a BGP is a saddle if the eigenvalue has a positive real part, and thus the equilibrium trajectory in the neighborhood of the BGP diverges from the BGP. The BGP is a sink if the eigenvalue has a negative real part, and thus the equilibrium trajectory in the neighborhood of the BGP converges to the BGP. As  $\dot{s} = \Theta(s)$  is increasing in  $s$ , the eigenvalue has a positive real part. Therefore, the BGP is a saddle, and thus the equilibrium path toward the BGP is unique.

## 2.2. General Fund Financing Taxes

In this regime, the only difference is the composition of public services that is summarized in equations (8a,b). Denote  $\alpha_1 \equiv \alpha\theta, \alpha_2 \equiv \alpha(1-\theta), \beta_1 \equiv \beta\eta,$  and  $\beta_2 \equiv \beta(1-\eta)$ , where  $\alpha_2$  and  $\beta_1$  represent the intersectoral externality in sectors  $X$  and  $Y$ , respectively.<sup>9</sup>

To analyze the equilibrium, we use equation (9a), with equations (2), (6a,b), (8a,b), and (1a,b), to obtain

$$p \equiv \lambda C^\sigma = \frac{(1-\alpha)\Phi_1}{(1-\beta)\Phi_2} \left( \frac{s}{1-s} \right)^{\frac{-[\alpha_2(1-\beta)+\beta_1(1-\alpha)]}{\phi_2}}, \tag{12}$$

$$\text{where } \Phi_1 = \{ \tau^{\alpha_1(1-\beta_2)+\alpha_2(1+\beta_1)} (u^{\alpha_1} v^{\alpha_2})^{1-\beta_2} [(1-u)^{\beta_1} (1-v)^{\beta_2}]^{\alpha_2} \}^{\frac{1}{\phi_2}},$$

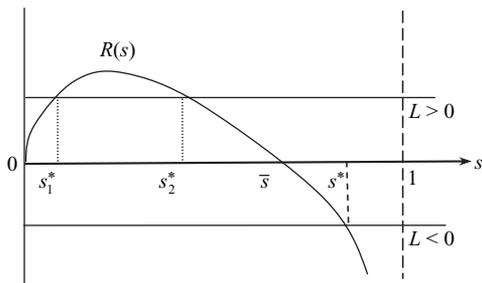
$$\Phi_2 = \{ \alpha^{(1-\alpha_1)} \tau^{\beta_1(1+\alpha_2)+\beta_2(1-\alpha_1)} (u^{\alpha_1} v^{\alpha_2})^{\beta_1} [(1-u)^{\beta_1} (1-v)^{\beta_2}]^{1-\alpha_1} \}^{\frac{1}{\phi_2}},$$

$$\phi_2 = (1-\beta_2)(1-\alpha_1) - \alpha_2\beta_1 > 0.$$

If we use the relationship  $\frac{\dot{\lambda}}{\lambda}$ , derived based on equations (1a,b), (2), (6a,b), (8a,b), and (9a,b), and the relationship  $\frac{\dot{K}}{K}$ , resulted from equations (1b), (2), (3b), (6a,b), and (8a,b), then the substitution of equation (3a) into equation (12) and differentiation leads to the following equation summarizing the dynamics in equilibrium.

$$\dot{s} = M(s)\Delta(s) = \Lambda(s), \tag{13}$$

**Figure 1**  
**Existence of Balanced Growth Paths in General Fund Financing Taxes**



where  $M(s) \equiv \frac{\phi_2 s(1-s)}{\sigma\phi_2(1-s) + \beta_1(1-\alpha) - (\sigma-1)\alpha_2(1-\beta)}$ ,

$$\Delta(s) \equiv (\sigma - 1)\delta - \rho - (1 - \tau)\Phi_2 \left(\frac{s}{1-s}\right)^{\frac{\beta_1(1-\alpha)}{\phi_2}} [\sigma(1-s) - (1-\beta)].$$

The feasibility demands  $M(s) < \infty$ , which implies a threshold  $\hat{s} \equiv 1 - \frac{(\sigma-1)\alpha_2(1-\beta) - \beta_1(1-\alpha)}{\sigma\phi_2}$  such that  $s \neq \hat{s}$ . Note that  $M(0) = M(1) = 0$ , and  $M(s) > (resp. <)$  0 if  $s < (resp. >)$   $\hat{s}$ . Once  $s$  is determined from equation (13), we can determine other variables.

The BGP is determined when  $\dot{s} = 0$ , which is

$$(\sigma - 1)\delta - \rho = (1 - \tau)\Phi_2 \left(\frac{s}{1-s}\right)^{\frac{\beta_1(1-\alpha)}{\phi_2}} [\sigma(1-s) - (1-\beta)]. \quad (14)$$

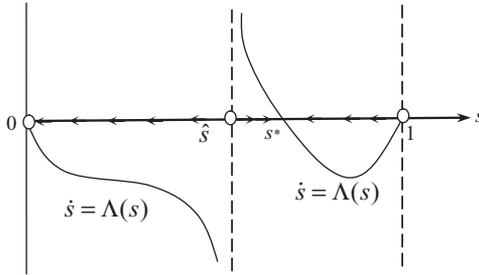
It is obvious that the left-hand side of equation (14), denoted as  $L$ , is a constant. The right-hand side of equation (14), denoted as  $R(s)$ , is zero at both  $s = 0$  and  $s = \bar{s} \equiv 1 - (1 - \beta)/\sigma$ , with  $0 < \bar{s} < 1$  if  $\sigma > 1 - \beta$ . Thus,  $R(s)$  has a humped shape with a positive (negative) value for all  $s < (>)$   $\bar{s}$  and approaching to negative infinity as  $s$  is close to 1 (figure 1).<sup>10</sup>

It is easy to see that a unique BGP emerges if  $L \equiv (\sigma - 1)\delta - \rho < 0$ , while there are two BGPs if  $L \equiv (\sigma - 1)\delta - \rho > 0$  and the following condition is met:<sup>11</sup>

$$\text{Condition A : } (\sigma - 1)[(1 - \tau)\Phi_2 \beta^{\frac{\beta_1(1-\alpha)}{\phi_1}} (1 - \beta)^{1 - \frac{\beta_1(1-\alpha)}{\phi_1}} - \delta] > \rho.$$

The proof of dynamic stability is in appendix A. In the case with a unique BGP, dynamic stability is as demonstrated in figure 2. We have shown in

**Figure 2**  
**Unique Sink Balanced Growth Path:  $\Sigma \in \Sigma_{12}$ .**



appendix A that the unique BGP  $s^*$  is a sink, and thus the equilibrium path toward it is indeterminate if the following condition is met:

$$\alpha_2 > \max\{\tilde{\alpha}, \hat{\alpha}\}$$

where  $\tilde{\alpha} \equiv \frac{\beta_1(1-\alpha) + \sigma(1-\beta_2)(1-\alpha_1)[1-s^*(\beta_1)]}{(\sigma-1)(1-\beta) + \sigma\beta_1[1-s^*(\beta_1)]}$  and  $\hat{\alpha} \equiv \frac{\beta_1(1-\alpha_1)}{(\sigma-1)(1-\beta) + \beta_1}$ .

This condition says that given  $\beta_1$ , that is, the degree of intersectoral externalities in sector  $Y$ , the unique BGP is a sink if the degree of intersectoral externalities in sector  $X$  is above a minimal level.

In the case with two BGPs, dynamic stability is as illustrated in figures 3–5, where the two BGPs being  $s_1^*$  and  $s_2^*$ , with  $s_1^* < s_2^*$ . Figure 3 is when  $s_2^* < \hat{s}$ , and in this circumstance it is evident that  $\dot{s} = \Lambda(s)$  is negatively (respectively positively) sloping at  $s_1^*$  (respectively  $s_2^*$ ). Therefore, while BGP  $s_2^*$  is a saddle, BGP  $s_1^*$  is a sink.

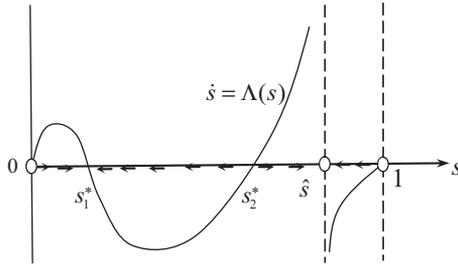
Alternatively, for  $s_2^* > \hat{s}$ , two situations emerge depending on if  $s_1^*$  is larger or smaller than  $\hat{s}$ . In figures 4 and 5, BGP  $s_2^*$  is always a sink, as  $\dot{s} = \Lambda(s)$  is always negatively sloping around  $s_2^*$ . BGP  $s_1^*$  is a saddle in figure 5 as  $\dot{s} = \Lambda(s)$  is positively sloping at  $s_1^*$ ; however,  $s_1^*$  is a sink in figure 4 as  $\dot{s} = \Lambda(s)$  is negatively sloping at  $s_1^*$ , like that in figure 3.

To summarize the above results, we have:

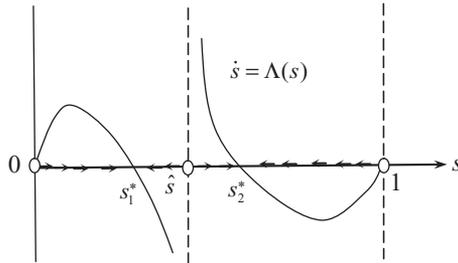
Proposition 1: Suppose the public goods are productive. Then,

1. when the tax is earmarked, the equilibrium path toward the BGP is unique and determinate;
2. when the fiscal regime is governed by general fund financing, the equilibrium path toward at least one BGP is destabilized if, given the degree of

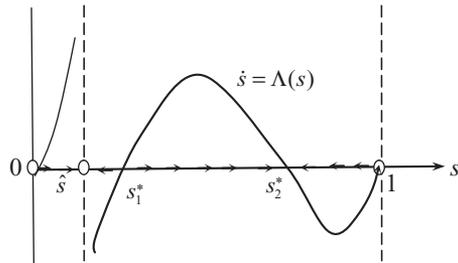
**Figure 3**  
Sink Balanced Growth Path  $s_1^*$ :  $\Sigma \in \Sigma_{21}$



**Figure 4**  
Sink Balanced Growth Path  $s_1^*$  and  $s_2^*$ :  $\Sigma \in \Sigma_{22}$



**Figure 5**  
Sink Balanced Growth Path  $s_2^*$ :  $\Sigma \in \Sigma_{22}$



intersectoral externalities in the investment good sector, the degree of intersectoral externalities in the consumption good sector is above a level.

Note that earmarked spending may exert intersectoral externalities if  $G_X = G_Y = G$ . We argue that this situation may not emerge for two reasons. First, it is difficult, if not impossible, to obtain the condition  $G_X = G_Y$ . The reason is that, even with an equal share of tax revenues from each sector of  $X$  and  $Y$  allocated to  $G_{JK}$ ,  $J = X, Y$ ,  $K = 1, 2$ ,  $X = Y$  is required to obtain  $G_X = G_Y$ . Generically, it is rarely for two different sectors to produce the same amount of output. Moreover, even if  $G_X = G_Y = G$  is possible, there is insufficient intersectoral externalities to trigger equilibrium indeterminacy as earmarked spending in one sector may only have a minor effect on the other sector. This is so as earmarked public spending is also in aspects other than infrastructure and in the case of infrastructure; it may not be shared with other sectors. In the example of earmarked taxes with respect to coffee taxes in Colombia (Teja and Bracewell-Milnes 1991), earmarked taxes are spent mostly in the area of coffee price stabilization activities and subsidizing domestic coffee consumption. Even in infrastructure in the coffee-producing area, the infrastructure is mainly used by the coffee industry. In this case, there are only minor intersectoral externalities, if any, whose magnitude is below the threshold and thus cannot trigger equilibrium indeterminacy.

### 3. Utility-Enhancing Public Goods

Suppose now that public goods are consumptive and enhance the agent's utility. We need two kinds of consumption goods for public services to enhance consumption in both sectors. Thus, while the commodity in sector  $X$  is still a pure consumption good, the commodity in sector  $Y$  is now modified as a composite good that may be either consumed or accumulated as capital stock. As there are no productive public goods now, the technologies in equations (1a,b) are modified as  $X = sK$  and  $Y = a(1 - s)K$ , respectively. Denote  $C_X$  and  $C_Y$  the consumption of goods  $X$  and  $Y$ , respectively. Now the budget constraints in equations (3a,b) become

$$C_X = (1 - \tau)X, \quad (15a)$$

$$\dot{K} = (1 - \tau)Y - \delta K - C_Y, \quad K(0) \text{ given}, \quad (15b)$$

while the utility in equation (4) is modified as follows:

$$\int_0^{\infty} e^{-\rho t} \left\{ \frac{1}{1-\sigma_1} \left[ (C_X G_X^{\gamma_1})^{1-\sigma_1} - 1 \right] + \frac{1}{1-\sigma_2} \left[ (C_Y G_Y^{\gamma_2})^{1-\sigma_2} - 1 \right] \right\} dt, \quad (16)$$

$\rho > 0, \sigma_i \geq 0, \gamma_1 > 0$  and  $\gamma_2 > 0,$

where the government budget constraint equation (5) or equation (6) is satisfied.

To be consistent with a BGP, it is required that  $\sigma_1 + (\sigma_1 - 1)\gamma_1 = \sigma_2 + (\sigma_2 - 1)\gamma_2 \equiv \phi_3 > 0$  and  $\sigma_1 = \sigma_2$ , denoted as  $\sigma$  hereafter. The requirement implies  $\gamma_1 = \gamma_2$ , henceforth denoted as  $\gamma$ . The condition for  $\phi_3 > 0$  is  $\sigma > \frac{\gamma}{1+\gamma}$ , which is easily met if  $\sigma \geq 1$ . Thus, we impose

$$\text{Condition B: } \gamma_1 = \gamma_2 \equiv \gamma \text{ and } \sigma_1 = \sigma_2 \equiv \sigma \geq 1.$$

Given  $K(0)$ ,  $\tau$  and public goods, the representative agent's optimization problem is to choose  $C_X$ ,  $C_Y$ ,  $s$ , and  $K$  to maximize equation (16), subject to constraints equations (15a,b). The necessary conditions are

$$\frac{1}{a} C_X^{-\sigma} G_X^{\gamma(1-\sigma)} = \lambda, \quad (17a)$$

$$C_Y^{-\sigma} G_Y^{\gamma(1-\sigma)} = \lambda, \quad (17b)$$

$$C_X^{-\sigma} G_X^{\gamma(1-\sigma)} (1-\tau)s + \lambda[a(1-\tau)(1-s) - \delta] = \lambda\rho - \dot{\lambda}, \quad (17c)$$

and the transversality condition  $\lim_{t \rightarrow \infty} e^{-\rho t} \lambda K = 0$ .

While the conditions in equations (17a,b) equalize the marginal utilities of the consumption of goods  $X$  and  $Y$  with the shadow price of capital, (17c) is the Euler equation for capital. Note that the relative price of  $Y$  in terms of  $X$  is  $p \equiv \lambda C_X^{\sigma} G_X^{\gamma(1-\sigma)}$ .

### 3.1. Earmarking Taxes

Substituting equations (6a,b), (7a,b), and (15a) into (17a) and differentiating, together the relationships  $\frac{\dot{\lambda}}{\lambda}$  from equations (6a,b), (7a,b), (17a), and (17c) and  $\frac{\dot{k}}{k}$  from equations (6a,b), (7a,b), (15b), and (17a,b), yields the following equation summarizing the dynamics in equilibrium.

$$s = s \left\{ \frac{1}{\phi_3} [a(1-\tau) - \delta - \rho] + \delta - (1-\tau) \left[ a(1-s) - s\phi_4 \left( \frac{s}{1-s} \right)^{\frac{\gamma(\sigma-1)}{\sigma}} \right] \right\} \equiv \Psi(s), \quad (18)$$

where  $\phi_4 = \{a[u^{\theta}(1-u)^{1-\theta}v^{-\eta}(1-v)^{1-\eta}]^{\gamma(\sigma-1)}\}^{\frac{1}{\sigma}}$ .

In a BGP,  $\dot{s} = 0$ , and thus

$$\frac{1}{\phi_3} [a(1 - \tau) - \delta - \rho] + \delta = (1 - \tau) \left[ a(1 - s) - s\phi_4 \left( \frac{s}{1 - s} \right)^{\frac{\gamma(\sigma-1)}{\sigma}} \right]. \quad (19)$$

The left-hand side of the above relationship is positive under condition B. The right-hand side, denoted as  $RH(s)$ , is  $RH(0) = a(1 - \tau) > 0$  and  $RH(1) \leq 0$ , and is decreasing in  $s$  for all  $s \in (0, 1)$ .<sup>12</sup> Thus, there exists a unique interior BGP  $s^*$  if  $(1 - \tau) > [a(1 - \tau) - \delta - \rho] / \phi_3 + \delta$ , or equivalently if  $a(1 - \tau) - (\delta + \rho) > -\rho\phi_3 / (\phi_3 - 1)$ . In particular, the economic growth rate in a BGP is positive if  $a(1 - \tau) - (\delta + \rho) > 0$ .

Finally, to investigate local dynamics,  $\dot{s} = \Psi(s)$  is increasing in  $s$  under condition B. As a result, the BGP is always a saddle and thus the equilibrium path toward the BGP is unique.

### 3.2. General Fund Financing Taxes

For analytical simplicity, we assume  $\theta + \eta = 1$  in equation (6) so that the total contribution to the provisions of public services toward both sectors of a tax source from any one of the two sectors is summed to unity. While this assumption is not necessary; it makes  $\gamma_1\theta = \gamma_2(1 - \eta) \equiv \varepsilon < \gamma$  and  $\gamma_1(1 - \theta) = \gamma_2\eta \equiv \psi < \gamma$  under condition B, so that  $\varepsilon$  now representing the sector-specific externality on consumption is simplified to be the same in both sectors, and  $\psi$  representing the *intersectoral* externality on consumption is also the same in both sectors.

Substituting equations (6a,b), (8a,b), and (16a) into (17a) and differentiating, using the relationships  $\frac{\lambda}{\lambda}$  from (6a,b), (8a,b), (17a), and (17c) and  $\frac{\kappa}{\kappa}$  from (6a,b), (8a,b), (15b), and (17a,b), leads to the following equation summarizing the dynamics in equilibrium.

$$\dot{s} = W(s)\Gamma(s) \equiv \Pi(s), \quad (20)$$

where  $W(s) = \frac{s(1-s)}{\sigma(1-s) - (\sigma-1)[(\varepsilon+\psi)s - \varepsilon]}$ ,

$$\Gamma(s) \equiv a(1 - \tau) - \delta - \rho - \phi_3 \left\{ (1 - \tau) \left[ a(1 - s) - \phi_5 s \left( \frac{s}{1 - s} \right)^{\frac{(\sigma-1)(\varepsilon-\psi)}{\sigma}} \right] - \delta \right\},$$

$$\phi_5 \equiv a^{\frac{1}{\sigma}} \left( \frac{u^\varepsilon v^\psi}{(1 - v)^\varepsilon (1 - u)^\psi} \right)^{\frac{\sigma-1}{\sigma}} > 0.$$

Feasibility requires  $W(s) < \infty$ , which implies a threshold  $\tilde{s} \equiv \frac{\sigma + (\sigma - 1)\varepsilon}{\sigma + (\sigma - 1)(\varepsilon + \psi)} < 1$  such that  $s \neq \tilde{s}$ . Note that  $\tilde{s} < 1$  under condition B. It follows that  $W(s) > 0$  (respectively  $< 0$ ) if  $s < (\text{resp. } >) \tilde{s}$ .

In a BGP,  $\dot{s} = 0$  so equation (20) leads to

$$\frac{1}{\phi_3} [a(1 - \tau) - \delta - \rho] + \delta = (1 - \tau) \left[ a(1 - s) - \phi_3 S \left( \frac{s}{1 - s} \right)^{\frac{(\sigma - 1)(\varepsilon - \psi)}{\sigma}} \right].$$

The left-hand side of the above relationship is positive under condition B. The right-hand side, denoted as  $RHS(s)$ , is  $RHS(0) = a(1 - \tau) > 0$  and  $RHS(1) \leq 0$  and is decreasing in  $s$  for all  $s \in (0, 1)$ . Thus, there exists a unique interior BGP  $s^* = s^*(\tau, \delta, \rho; \psi)$  if  $a(1 - \tau) - (\delta + \rho) > -\rho\phi_3 / (\phi_3 - 1)$ , with  $\frac{\partial s^*}{\partial \psi} > 0$  if  $\frac{s^*}{1 - s^*} \geq \frac{v}{1 - u}$ , and the economic growth rate is positive if  $a(1 - \tau) - (\delta + \rho) > 0$ .

Finally, to analyze the stabilization we note that  $\Gamma(\mathbf{s})$  is increasing in  $s$ . As a result, if  $W(s) < 0$ , then  $\dot{s} = \Pi(s)$  is decreasing in  $s$  and thus the BGP is a sink.

The condition for  $W(s) < 0$  is  $s^* > \tilde{s}$ , that implies the prerequisite of  $\psi > \psi^* \equiv \frac{1 - s^*}{s^*} \left[ \frac{\sigma}{\sigma - 1} + \varepsilon \right]$ . The condition thus requires the level of intersectoral externalities above a threshold to exhibit local indeterminacy and to make the economy destabilized.

To summarize the results in this section, we obtain

Proposition 2: Suppose that a public good is utility-enhancing and condition B is met. Then,

1. in the earmarked tax regime, the equilibrium path to BGP is always unique and determinate;
2. in the general fund financing regime, the equilibrium path toward BGP is indeterminate if the level of intersectoral externalities is above a threshold.

## 4. Welfare Analysis

This section compares the welfare in both earmarked taxes and general fund taxes. As shown in sections 2 and 3, if the level of intersectoral externalities is above a threshold, at least one of the BGP is a sink in general fund taxes. Then, there is a continuum of equilibrium paths toward the BGP that is a sink. It is impossible to compute the welfare along an infinite number of equilibrium paths in general fund taxes. Under the circumstances, we may compute the welfare along the BGP.

Specifically, under given initial states, we can calculate the discounted lifetime utility along the BGP. We calibrate the model economy in consistency with the US economy and then quantify the level of discounted lifetime utility. We start with the economy with production-enhancing public goods, followed by the economy with utility-enhancing public goods. As will be seen, in both economies the level of welfare in earmarked taxes is higher than that in general fund taxes.

### 4.1. Production Enhancing Public Goods

Along the BGP, the discounted lifetime utility in equation (4) is written as follows:

$$U_1 = \frac{C_0^{1-\sigma}}{(1-\sigma)[\rho + (\sigma-1)g]} - \frac{1}{(1-\sigma)\rho}, \tag{21}$$

where  $C_0$  is the initial consumption and  $g$  denotes the economic growth rate along the BGP. Values of  $C_0$  and  $g$  are determined by the fraction of capital  $K$  allocated to the consumption good sector, a predetermined initial level of capital stock and the tax regime. Initial capital stock  $K(0)$  is the same in both tax regimes, whereas the initial consumption and the economic growth rate along a BGP both respond to tax regimes and thus may be different in a different tax regime.

Under an earmarking tax regime, we have

$$C_0 = (1-\tau) \left[ \tau u^0 (1-u)^{1-\theta} \right]^{\frac{1}{1-\sigma}} s^* K(0);$$

$$g = \frac{1}{\sigma} \{ (1-\tau)(1-\beta)\phi_1 - \delta - \rho \}.$$

Alternately, under a general fund tax regime we have

$$C_0 = (1-\tau)\Phi_1 s^{*\frac{[1-\beta(1-\eta)](1-\alpha)}{\phi_2}} (1-s^*)^{\frac{\alpha(1-\theta)(1-\beta)}{\phi_2}} K(0);$$

$$g = \frac{1}{\sigma} \left\{ (1-\tau)(1-\beta)\Phi_2 \left( \frac{s^*}{1-s^*} \right)^{\frac{(1-\alpha)\eta\beta}{\phi_2}} - \delta - \rho \right\}.$$

To compare the level of welfare between these two regimes, we quantitatively assess the discounted lifetime utility as follows: First, we calibrate the model based on the following parameter values representative of the economy in the United States and consistent with a 2 percent long-run real

economic growth rate. The total tax revenues in the United States, on average, account for 20 percent of its GDP after 1980, and hence  $\tau = 0.2$  is chosen. Following Turnovsky (2000), we choose the degree of the externality of public goods at  $\alpha = \beta = 0.08$ . For the time preference rate, we set  $\rho = 0.025$  in accordance with Benhabib and Perli (1994). Following Mulligan and Sala-i-Martin (1993), we choose the inverse of intertemporal elasticity of substitution at  $\sigma = 2$  and the depreciation rate at  $\delta = 0.05$ . An equal share of inputs in the public good production is chosen ( $\theta = \eta = 0.5$ ) for simplicity. Similarly, an equal share in the allocation of tax revenues is set ( $u = v = 0.5$ ).

Using the above parameter values, we calibrate the productivity coefficient in sector  $Y$  and obtain  $a = 0.2179$  under an earmarking regime and  $a = 0.2064$  under a general fund regime. Note that the resulting calibrated value for  $a$  is insensitive to different values of  $\theta$ ,  $\eta$ ,  $u$ , and  $v$ .<sup>13</sup> We normalize the value of predetermined initial capital at  $K(0) = 1$ . Under the set of benchmark parameter values, the unique BGP in earmarking taxes is  $\{s^* = 0.32, C_0 = 0.2095, g = 0.02\}$  and the level of welfare along the BGP is  $U_1 = -66$ . There are two BGPs in general fund taxes:  $\{s_1^* = 0.01, C_0 = 0.0078, g = 0.0108\}$  and  $\{s_2^* = 0.40, C_0 = 0.2662, g = 0.0196\}$ . While the level of welfare at the former BGP is  $U_1 = -3508$ , the level of welfare at the latter BGP is  $U_1 = -44$ . The former BGP is a sink and the latter is a saddle. Thus, in the long-run equilibrium paths almost surely converge to the former BGP. The level of welfare along the BGP in general fund taxes is  $U_1 = -3508$  with the probability at 1. As a result, the level of welfare along the BGP in a general fund tax regime is lower than that in an earmarked tax regime.

To see how robust our above results are, we also simulate the model and compare the welfare between the two regimes in different sets of parameter values. We find that our above results hold true in the following parameter space:  $\rho \in [0.02, 0.05]$ ,  $\delta \in [0.05, 0.20]$ ,  $\sigma \in [1.5, 7]$ ,  $\theta \in [0.01, 0.99]$ ,  $\eta \in [0.01, 0.99]$ ,  $\alpha \in [0.01, 0.20]$ , and  $\beta \in [0.01, 0.20]$ . The range of parameter space is sufficiently large, and parameter values are plausible. This indicates that our results are robust.

## 4.2. Utility-Enhancing Public Goods

Along the BGP, the discounted lifetime utility in equation (16) is written as follows:

$$U_2 = \frac{\Omega_1 + \Omega_2}{(1 - \sigma)[\rho + (\sigma - 1)(1 + \gamma)g]} - \frac{2}{(1 - \sigma)\rho}, \quad (22)$$

where  $\Omega_1 \equiv (C_{X0}G_{X0}\gamma)^{(1-\sigma)}$  and  $\Omega_1 \equiv (C_{Y0}G_{Y0}\gamma)^{(1-\sigma)}$  are composites of initial consumption of private and public goods.

Under an earmarking regime,

$$\Omega_1 = \left\{ (1-\tau)s^*K(0) \left[ u^0(1-u)^{1-\theta} \tau s^*K(0) \right]^\gamma \right\}^{(1-\sigma)};$$

$$\begin{aligned} \Omega_2 &= \left\{ (1-\tau)s^*K(0) \left[ a \left( u^0(1-u)^{1-\theta} v^{-\eta} (1-v)^{-(1-\eta)} \frac{s^*}{1-s^*} \right) \gamma (\sigma-1) \right]^{\frac{1}{\sigma}} \right\}^{(1-\sigma)} \\ &\quad \left[ v^\eta (1-v)^{1-\eta} \tau a (1-s^*)K(0) \right]^\gamma (1-\sigma); \\ g &= \frac{1}{\sigma + \gamma(\sigma-1)} \left[ a(1-\tau) - \delta - \rho \right]. \end{aligned}$$

Alternately, under a general fund regime,

$$\Omega_1 = \left\{ (1-\tau)s^*K(0) \left[ (s^*u)^0 [a(1-s^*)v]^{1-\theta} \tau K(0) \right]^\gamma \right\}^{(1-\sigma)};$$

$$\begin{aligned} \Omega_2 &= \left\{ (1-\tau)s^*K(0) \left[ a \left[ u^0 v^{1-\theta} (1-u)^{-\eta} (1-v)^{-(1-\eta)} \right. \right. \right. \\ &\quad \left. \left. \left. \left( \frac{s^*}{1-s^*} \right)^{(\theta-\eta)} \right]^\gamma (\sigma-1) \right]^{\frac{1}{\sigma}} \right\}^{(1-\sigma)} \times \left\{ [s^*(1-u)]^\eta [a(1-s^*)(1-v)] \right\}^{1-\eta} \\ &\quad \tau K(0) \right\}^\gamma (1-\sigma); \end{aligned}$$

$$g = \frac{1}{\sigma + \gamma(\sigma-1)} \left[ a(1-\tau) - \delta - \rho \right].$$

We continue to use the benchmark parameter values in section 4.1 except for the factor share of public goods in production,  $\alpha$  and  $\beta$ . Following Turnovsky (2000), we choose the intensity of public goods consumption on the preference relative to private consumption at  $\gamma = 0.3$ . We then calibrate the productivity coefficient in sector  $Y$  and obtain  $a = 0.1512$  in both tax regimes.<sup>14</sup> Under the set of benchmark parameter values and the normalized value of predetermined initial capital at  $K(0) = 1$ , the unique BGP in earmarking taxes is  $\{s^* = 0.24, \Omega_1 = 15.94, \Omega_2 = 60.78, g = 0.02\}$  and the level of welfare along the BGP is  $U_2 = -1424$ . Alternatively, in general fund taxes, the unique BGP is  $\{s^* = 0.10, \Omega_1 = 47.51, \Omega_2 = 122.17, g = 0.02\}$  and the level of welfare along the BGP  $U_2 = -3247$ . Thus, the level of welfare along the BGP in a general fund tax regime is lower than that in an earmarked tax regime.

Finally, we also simulate the model and compare the welfare in the two tax regimes in different sets of parameter values to examine the robustness of the above results. We find that our results hold true in a wide range of parameter space as follows:  $\rho \in [0.02, 0.05]$ ,  $\delta \in [0.05, 0.20]$ ,  $\sigma \in [1.5, 7]$ ,  $\theta \in [0.01, 0.99]$ ,  $\eta \in [0.01, 0.99]$ , and  $\gamma \in [0.01, 1.5]$ . This indicates that the level of welfare in a general fund tax regime is lower than the level of welfare in an earmarked tax regime.

## 5. Concluding Remarks

Extensive discussion has been concerning the pros and cons of earmarking and general fund in conventional wisdom. This article evaluates the implications of desirability of earmarked tax and general fund financed tax based on aggregate economic stabilization. We use a simple growth model with two sectors to illustrate the implications. We find that regardless of the nature of public goods, earmarked taxes contribute to aggregate stabilization, while general fund financing may be destabilizing and thus causing economic fluctuations unrelated to economic fundamentals. An earmarked tax generates sector-specific externalities that are too small to exert destabilizing forces. General fund financing taxes, however, create intersectoral externalities and strategic complementarities that may be large enough so that it is prone to indeterminacy of equilibrium, thereby generating persistent and recurring fluctuations in aggregate activities in the absence of shocks to fundamentals. We also quantify the model and compare the level of welfare along a BGP in these two tax regimes. Our results reveal that under plausible parameter values, the level of welfare along a BGP in an earmarked tax regime is higher than in a general fund tax regime. Thus, our results are robust to the nature of public goods. Our results support the use of earmarked taxing against general fund financing based on aggregate stabilization in transitional dynamics and level of welfare in the long run.

Let us point out three possible limitations found in our article. First, the earmarked tax is like a user fee for the public input or service.<sup>15</sup> When firms in each sector pay taxes and receive public input or service, the external effect of public goods is internalized. As a result, the BGP in an earmarked tax is always determinate. In a similar fashion in a general fund financing regime, if the government is able to charge firms in different sectors the marginal cost for the public service, intersectoral externalities as produced by public services are fully internalized. Then, we expect the BGP to be determinate in the general fund financing regime. If, however, the government

is not able to charge the marginal cost for public services, which is the case in practice, the BGP is indeterminate in a general fund financing regime.

Second, other than the two-sector model of pure consumption and pure investment goods used in our article, another popular two-sector, endogenous growth model is the Lucas (1988) model. This type of model has been extended by Benhabib and Perli (1994), Bond, Wang, and Yip (1996), Mino (2001), Benhabib, Meng, and Nishimura (2000), Ben-Gad (2003), and Chen and Lee (2007). In this type of two-sector model, there is not only physical capital but also human capital. The human capital sector uses a production technology to produce pure investment goods. In appendix B, we extend our model to take into account of human capital accumulation. We find that equilibrium paths are determinate in both kinds of taxing regimes. Such a result is consistent with the finding in Chen and Lee (2007). Chen and Lee (2007) have shown that in a two-sector Lucas model when productive public capital is introduced, equilibrium paths are always determinate unless there is a congestion effect in the use of public services. Thus, our conclusion in favor of earmarked taxes in terms of economic stability is applicable to the environment where labor cannot be used to form human capital.

Finally, we have assumed a given tax rate, and thus public goods are not provided in an optimal fashion. We abstract from an optimal provision of public goods for analytical tractability. In appendix C, we consider a benevolent government that chooses its first-best policy in the provision of public goods, given the production technology and resource constraints. We have shown that in earmarked taxes, the optimal public good provision in each sector is a fixed proportion of the output produced in each sector. As a result, the dynamic stability properties of the BGP in earmarked taxes are the same as those analyzed in the text and equilibrium paths are determinate. In general fund financing taxes, however, optimal provisions of the two types of public goods are very nonlinear functions of consumption and the shadow price of capital. There are thus four more conditions. The dynamic stability conditions are thus a  $5 \times 5$  system, including the dynamic evolution of the fraction of capital allocated to the consumption sector analyzed in the text and these four additional conditions. It is impossible to analyze the dynamic stability in this  $5 \times 5$  system unless the model is further simplified. As a result, we cannot be sure if a general fund financing regime is more destabilized when public goods are optimally provided. Nevertheless, this may provide an interesting avenue for further research.

## Appendix A

### Proof of Indeterminacy in General Fund Financing Taxes in Section 2.2

In analyzing transitional dynamics in general fund financing taxes in the model of production enhancing public goods, denote  $\Sigma$  as the parameter space. Using  $\sigma = 1 + \rho/\delta$  as a threshold, we split  $\Sigma$  into the following two subsets:

$$\Sigma_1 \equiv \left\{ \Sigma \in \Sigma \mid 1 - \beta < \sigma < 1 + \frac{\rho}{\delta} \right\},$$

$$\Sigma_2 \equiv \left\{ \Sigma \in \Sigma \mid 1 + \frac{\rho}{\delta} \leq \sigma \text{ and } \sigma \geq (\sigma - 1)[1 - \tau)\Phi_2 \beta^{\frac{\beta_1(1-\alpha)}{\phi_2}} (1 - \beta)^{1 - \frac{\beta_1(1-\alpha)}{\phi_2}} - \delta] > \rho \right\}.$$

The dynamic stability of the system  $\dot{s} = \Lambda(s)$  in equation (13) depends on the signs of  $M(s)$  and  $\Delta(s)$  and is investigated as follows.

#### Case 1. Unique Balanced Growth Path (BGP)

This case arises for all parameters in  $\Sigma_1$  and results in a unique BGP,  $s^*$ .

As  $M(s) >$  (respectively  $<$ ) 0 if  $s <$  (*resp.*  $>$ )  $\hat{s} \equiv 1 - \frac{(\sigma-1)\alpha_2(1-\beta)-\beta_1(1-\alpha)}{\sigma\phi_1}$ , we may divide set  $\Sigma_1$  into two mutually exclusive subsets as follows:

$$\Sigma_{11} \equiv \left\{ \Sigma \in \Sigma \mid 1 - \beta < \sigma < 1 + \frac{\rho}{\delta} \text{ and } s^* < \hat{s} \right\},$$

$$\Sigma_{12} \equiv \left\{ \Sigma \in \Sigma \mid 1 - \beta < \sigma < 1 + \frac{\rho}{\delta} \text{ and } s^* > \hat{s} \right\}.$$

When the parameter subspace is  $\Sigma_{11}$ , then the slope of  $\dot{s} = \Lambda(s)$  is positive at  $s^*$ . The trajectory in the neighborhood of the unique BGP  $s^*$  diverges from  $s^*$ , and thus  $s^*$  is a saddle.

Alternatively, when the relevant subspace is  $\Sigma_{12}$ , the slope of  $\dot{s} = \Lambda(s)$  is negative at  $s^*$ , and thus the BGP is a sink.

Thus, the feasibility of local indeterminacy under  $\Sigma_1$  requires  $s^* > \hat{s}$  and  $\hat{s} < 1$ .

First, condition  $s^* > \hat{s}$  requires  $s^*(\beta_1) > \hat{s} \equiv 1 - \frac{(\sigma-1)\alpha_2(1-\beta)-\beta_1(1-\alpha)}{\sigma\phi_2}$ , where  $\frac{\partial s^*(\beta_1)}{\partial \beta_1} >$  or  $<$  0. Under a given degree of intersectoral externalities in sector Y,  $\beta_1$ , the requirement is a minimal degree of intersectoral externalities in sector X,

$$\alpha_2 > \tilde{\alpha} \equiv \frac{\beta_1(1 - \alpha) + \sigma(1 - \beta_2)(1 - \alpha_1)[1 - s^*(\beta_1)]}{(\sigma - 1)(1 - \beta) + \sigma\beta_1[1 - s^*(\beta_1)]}$$

Next, condition  $\hat{s} < 1$  is met when  $\sigma > 1 + \frac{\beta_1(1-\alpha)}{\alpha_2(1-\beta)}$ . If we combine with the conditions in  $\Sigma_{12}$ , the requirement is again a minimal degree of intersectoral externalities in sector  $X$ .

$$\alpha_2 > \hat{a} \equiv \frac{\beta_1(1 - \alpha_1)}{(\sigma - 1)(1 - \beta) + \beta_1}$$

Combining the requirements for  $s^* > \hat{s}$  and  $\hat{s} < 1$ , under a given  $\beta_1$ , the unique BGP is a sink if  $\alpha_2 > \max\{\tilde{\alpha}, \hat{a}\}$ .

## Case 2. Two BGPs

This case arises for all parameters in  $\Sigma_2$ . Denote the two BGPs as  $s_1^*$  and  $s_2^*$ , with  $s_1^* < s_2^*$ .

Then,  $\Delta(s) > 0$  for  $s < s_1^*$  and  $s > s_2^*$ , and  $\Delta(s) < 0$  for  $s$  in  $(s_1^*, s_2^*)$ . As the sign of  $M(s)$  depends on whether  $s_2^*$  is larger or smaller than  $\hat{s}$ , we may separate  $\Sigma_2$  into two mutually exclusive subsets as follows:

$$\Sigma_{21} \equiv \left\{ \Sigma \in \Sigma \mid 1 + \frac{\rho}{\delta} \leq \sigma, (\sigma - 1) \left[ (1 - \tau)\Phi_2\beta^{\frac{\beta_1(1-\alpha)}{\phi_1}} (1 - \beta)^{\frac{\beta_1(1-\alpha)}{\phi_1}} - \delta \right] > \rho \text{ and } s_2^* < \hat{s} \right\},$$

$$\Sigma_{22} \equiv \left\{ \Sigma \in \Sigma \mid 1 + \frac{\rho}{\delta} \leq \sigma, (\sigma - 1) \left[ (1 - \tau)\Phi_2\beta^{\frac{\beta_1(1-\alpha)}{\phi_1}} (1 - \beta)^{1 - \frac{\beta_1(1-\alpha)}{\phi_1}} - \delta \right] > \rho \text{ and } s_2^* < \hat{s} \right\}.$$

In the case where  $s_2^* > \hat{s}$ , the relevant parameter subspace is  $\Sigma_{21}$ , and the local dynamics are illustrated in figure 3. As  $M(s) >$  (respectively  $>$ )  $0$  for all  $s <$  (*resp.*  $>$ )  $\hat{s}$ , it is evident that  $\hat{s} = \Lambda(s)$  is negatively (respectively positively) sloping at  $s_1^*$  (respectively  $s_2^*$ ). While BGP  $s_2^*$  is a saddle, BGP  $s_1^*$  is a sink.

Alternatively, for  $s_2^* < \hat{s}$ , the relevant parameter subspace is  $\Sigma_{22}$ . Two situations emerge depending if  $s_1^*$  is larger than  $\hat{s}$  (figures 4 and 5). It is apparent that BGP  $s_1^*$  is a saddle in figure 5 as  $\hat{s} = \Lambda(s)$  is positively sloping at  $s_1^*$ ; however,  $s_1^*$  is a sink in figure 4 as  $\hat{s} = \Lambda(s)$  is negatively sloping at

$s_1^*$ , like that in figure 3. BGP  $s_2^*$  is always a sink in both figures, as  $\dot{s} = \Lambda(s)$  is always negatively sloping around  $s_2^*$ .

## Appendix B

### An Extension to Include Human Capital

If we consider human capital as another private input, our model may be modified as follows: The technologies in both sectors are

$$X = (sK)^{1-\alpha-\zeta}(zH)^\zeta G_X^\alpha, \quad (\text{B1a})$$

$$Y = [(1-s)K]^{1-\beta-\xi}[(1-z)H]^\xi G_Y^\beta, \quad (\text{B1b})$$

where  $s$  and  $z$  are the fraction of physical capital,  $K$ , and human capital,  $H$ , respectively, allocated to sector  $X$ , and  $\zeta$  and  $\xi$  are, respectively, the contribution to the production in each sector. While sector  $X$  produces consumable investment goods, sector  $Y$  produces human capital goods.  $G_X$  and  $G_Y$  are the provisions of public goods toward sectors  $X$  and  $Y$ , with  $\alpha$  and  $\beta$  being their contribution to the production, respectively.

For simplicity, we assume there is no depreciation for the stock of physical and human capital. The motions of the two kinds of capital stock for the representative agent are given by

$$\dot{K} = (1-\tau)X - c, \quad (\text{B2a})$$

$$\dot{H} = (1-\tau)Y. \quad (\text{B2b})$$

Given  $K(0)$ ,  $H(0)$ ,  $\tau$ ,  $u$ ,  $v$ , and public goods, the representative agent's problem is to choose  $C$ ,  $s$ ,  $z$ ,  $K$ , and  $H$  to maximize its discounted utility in equation (4), subject to the constraints in (B1a, 2b). If we let  $\lambda(t)$  and  $\mu(t)$  be the co-state variable associated with  $K(t)$  and  $H(t)$ , respectively, then the necessary conditions are

$$C^{-\sigma} = \lambda, \quad (\text{B3a})$$

$$\lambda(1-\alpha-\zeta)\frac{X}{s} = \mu(1-\beta-\xi)\frac{Y}{1-s}, \quad (\text{B3b})$$

$$\lambda \zeta \frac{X}{z} = \mu \xi \frac{Y}{1-z}, \tag{B3c}$$

$$\lambda(1-\alpha-\zeta) \frac{(1-\tau)X}{K} + \mu(1-\beta-\xi) \frac{(1-\tau)Y}{K} = \lambda \rho - \dot{\lambda}, \tag{B3d}$$

$$\lambda \zeta \frac{(1-\tau)X}{H} + \mu \xi \frac{(1-\tau)Y}{H} = \mu \rho - \dot{\mu}, \tag{B3e}$$

together transversality conditions  $\lim_{t \rightarrow \infty} e^{-\rho t} \lambda K = 0$ , and  $\lim_{t \rightarrow \infty} e^{-\rho t} \mu H = 0$ .

Condition (B3a) equates the marginal utility of consumption to the marginal cost, the shadow price of physical capital, while (B3b) and (B3c) equate the marginal product of physical capital and human capital between the goods and the education sector. Finally, (B3d) and (B3e) are the Euler equations governing the optimal accumulation for physical and human capital, respectively.

Finally, the government’s behavior is the same in the text.

Since the model is similar to Chen and Lee (2007), we may follow their dynamic analysis. We transform the economic system into the structure with variables  $\{p, n, m\}$ , where  $p \equiv \mu/\lambda$ ,  $n \equiv C/H$ , and  $m \equiv K/H$ . First, from the Pareto complements in physical and human capital in the technology, we obtain

$$z = \frac{(1-\alpha-\zeta)\xi s}{(1-\alpha-\zeta)\zeta s + (1-\beta-\xi)\zeta(1-s)}, \tag{B4}$$

with  $z'(s) = \frac{s(1-s)}{z(1-z)} > 0$ .

If we use equations (B1a, 1b), (B4), and (6), we rewrite (B3b) as

$$p = \frac{\zeta}{\xi} \frac{X/(sK)}{Y/[(1-z)H]} \frac{sK}{zH}. \tag{B5a}$$

Under an earmarking regime, (B5a) can be rewritten as

$$\frac{\zeta \Delta_1}{\zeta \Delta_2} \left( \frac{s(z)}{z} \right)^{\frac{\zeta(1-\alpha)-\zeta(1-\beta)}{(1-\alpha)(1-\beta)}} = pm \frac{\zeta(1-\beta)-\zeta(1-\alpha)}{(1-\alpha)(1-\beta)}, \tag{B5b}$$

where  $\Delta_1 = [\tau u^\theta (1-u)^{1-\theta}]^{\frac{\alpha}{1-\alpha}}$ , and  $\Delta_2 = [\tau v^\eta (1-v)^{1-\eta}]^{\frac{\beta}{1-\beta}}$ . Note that the above equation is similar to equation (9a) in Chen and Lee (2007).

Based on equation (B5b), it is easy to obtain  $z$  as a function of  $p$  and  $m$  and

$$\frac{\partial z}{\partial p} = \frac{-\xi(1-\alpha-\zeta)z}{p\Gamma},$$

$$\frac{\partial z}{\partial m} = \frac{-\xi(1-\alpha-\zeta)z[\xi(1-\alpha)-\zeta(1-\beta)]}{(1-\alpha)(1-\beta)m\Gamma},$$

where  $\Gamma = [\xi(1-\alpha) - \zeta(1-\beta)]^2 > 0$ .

Based on the proposition 3 in Chen and Lee (2007, 2497), it is easy to show that the condition for indeterminacy is  $\Gamma = < 0$ . However,  $\Gamma = > 0$  under an earmarking regime, and thereby the equilibrium path toward the BGP is determinate.

Alternately, under general fund regime, equation (B5a) can be rewritten as

$$\frac{\zeta\Delta_3}{\xi\Delta_4} \left( \frac{s(z)}{1-s(z)} \right)^{\frac{\alpha(1-\beta)(1-\theta)}{\Phi_2}} \left( \frac{z}{1-z} \right)^{\frac{-(1-\alpha)\eta\beta}{\Phi_2}} \left( \frac{s(z)}{1-z} \right)^{\frac{\xi(1-\alpha)-\zeta(1-\beta)}{\Phi_2}} = pm^{\frac{\zeta(1-\beta)-\xi(1-\alpha)}{\Phi_2}}, \quad (B6)$$

where

$$\Delta_3 = \left\{ \tau^{\alpha\beta(1-\theta)}(1-u)^{\alpha\beta\eta(1-\theta)}(1-v)^{\alpha\beta\eta(1-\theta)(1-\eta)}[\tau(u^\theta v^{1-\theta})]^{\alpha[(1-\beta)(1-\eta)]} \left( \frac{\zeta(1-\beta-\xi)}{\xi(1-\alpha-\xi)} \right) - \alpha\xi(1-\theta) \right\}^{\frac{1}{\Phi_2}}; \Delta_4 = \left\{ [\tau(1-u)^\eta(1-v)^{(1-\eta)}]^\beta [1-\alpha\theta] [\tau(u^\theta v^{1-\theta})]^{\alpha\beta\eta} \left( \frac{\zeta(1-\beta-\xi)}{\xi(1-\alpha-\xi)} \right)^{(1-\beta-\xi)(1-\alpha\theta)} \right\}^{\frac{1}{\Phi_2}}.$$

Equation (B6) also yields  $z$  as a function of  $p$  and  $m$  with

$$\frac{\partial z}{\partial p} = \frac{-\Phi_2\xi(1-\alpha-\zeta)z(1-z)}{p\Gamma},$$

$$\frac{\partial z}{\partial m} = \frac{\xi(1-\alpha-\zeta)z(1-z)[\xi(1-\alpha)-\zeta(1-\beta)]}{m\Gamma},$$

where  $\Gamma = [\xi(1-\alpha) - \zeta(1-\beta)]^2 s(1-z) + \alpha(1-\beta)(1-\theta)\xi(1-\alpha-\zeta) + \beta(1-\alpha)\eta\xi(1-\alpha-\zeta) > 0$ .

Thus, the equilibrium path toward the BGP is determinate.

## Appendix C

### An Extension When the Government Chooses Public Goods Optimally

---

We assume the government is benevolent and choose its first-best policy in the provision of public goods, given production technology and resource constraints.

#### 1. Earmarking Financing

---

In an earmarking regime, using equations (5), (6), and (7) yields the following government budget constraints:

$$\tau X = u^{-\alpha\theta}(1-u)^{-\alpha(1-\theta)} G_X, \quad (C1)$$

$$\tau Y = v^{-\beta\eta}(1-v)^{-\beta(1-\eta)} G_Y. \quad (C2)$$

If we substitute equations (C1) and (C2) into equations (3a) and (3b), we obtain economy's resource constraints.

$$X = C + u^{-\alpha\theta}(1-u)^{-\alpha(1-\theta)} G_X, \quad (C3)$$

$$Y = \dot{K} + \delta K + v^{-\beta\eta}(1-v)^{-\beta(1-\eta)} G_Y. \quad (C4)$$

To determine the first-best policy, we need to solve the social planner's problem. The social planner's problem is to choose  $C$ ,  $s$ ,  $K$ ,  $G_X$ , and  $G_Y$  to maximize the representative agent's utility in equation (4), subject to production technologies equations (1a) and (1b) and the resource constraints in equations (B3) and (B4), taking  $K(0)$ ,  $u$ , and  $v$  as given. If we let  $\mu(t)$  be the co-state variable associated with  $K(t)$ , then the necessary conditions are combined into four equations and are as follows:

$$(1-\alpha)C^{-\sigma}\frac{X}{s} = \mu(1-\beta)\frac{Y}{1-s}, \quad (C5)$$

$$(1-\alpha)C^{-\sigma}\frac{X}{K} + \mu\left[(1-\beta)\frac{Y}{K} - \delta\right] = \rho\mu - \dot{\mu}, \quad (C6)$$

$$\frac{G_X}{X} = \alpha u^{\alpha\theta} (1-u)^{\alpha(1-\theta)} \equiv \omega_1, \quad (C7)$$

$$\frac{G_Y}{Y} = \beta v^{\eta\beta} (1-v)^{\beta(1-\eta)} \equiv \omega_2. \quad (C8)$$

The conditions for  $G_X$  and  $G_Y$  are in equations (C7) and (C8), which indicate that public spending in each sector accounts for a fixed fraction of output produced in each sector. If we substitute equations (C7) and (C8) into equations (1a) and (1b), the production functions are rewritten as follows:

$$X = \omega_2^{\frac{\alpha}{1-\alpha}} (sK), \quad (C9)$$

$$Y = \omega_2^{\frac{\beta}{1-\beta}} [a(1-s)K]. \quad (C10)$$

It is obvious that the production functions are of an AK form with the production coefficient in proportion to a fixed fraction of optimal public good provisions in each sector. The dynamic stability in this economy remains characterized by equation (11). As a result, under an optimal public provision, the equilibrium path toward BGP is determinate in earmarked financing.

## 2. General Fund Financing

---

For a general fund regime, using equations (5), (6), and (7) yields the following government budget constraints:

$$\tau X = \Delta_1 [G_X^{-(1-\eta)} G_Y^{(1-\theta)}]^{\frac{1}{\eta-\theta}}, \quad (C9)$$

$$\tau Y = \Delta_2 [G_X^\eta G_Y^{-\theta}]^{\frac{1}{\eta-\theta}}, \quad (C10)$$

where  $\Delta_1 = \left[ u^{\theta(1-\eta)} (1-u)^{-\eta(1-\theta)} \left( \frac{v}{1-v} \right)^{(1-\theta)(1-\eta)} \right]^{\frac{1}{\eta-\theta}}$ ;

$$\Delta_2 = \left[ v^{-\eta(1-\theta)} (1-v)^{\theta(1-\eta)} \left( \frac{u}{1-u} \right)^{-\theta\eta} \right]^{\frac{1}{\eta-\theta}}.$$

If we substitute equations (C9) and (C10) into equations (3a) and (3b), the economy's resource constraints are:

$$X = C + \Delta_1 \left[ G_X^{-(1-\eta)} G_Y^{(1-\theta)} \right]^{\frac{1}{\eta-\theta}}, \tag{C11}$$

$$Y = \dot{K} + \delta K + \Delta_2 \left[ G_X^\eta G_Y^{-\theta} \right]^{\frac{1}{\eta-\theta}}, \tag{C12}$$

The social planner’s problem is to choose  $C$ ,  $s$ ,  $K$ ,  $G_X$ , and  $G_Y$  to maximize the representative agent’s utility in equation (4), subject to technology equations (1a) and (1b), and resource constraints in equations (C11) and (C12), taking as given  $K(0)$ ,  $u$ , and  $v$ . If we let  $\mu(t)$  be the co-state variable associated with  $K(t)$ , then the necessary conditions are combined into four equations and are as follows:

$$(1 - \alpha)C^{-\sigma} \frac{X}{s} = \mu(1 - \beta) \frac{Y}{1 - s}, \tag{C13}$$

$$(1 - \alpha)C^{-\sigma} \frac{X}{K} + \mu \left[ 1 - \beta \right] \frac{Y}{K} - \delta = \rho\mu - \dot{\mu}, \tag{C14}$$

$$C^{-\sigma} \left[ \alpha X + \frac{(1 - \eta)\Delta_1}{\eta - \theta} \left( G_X^{-(1-\eta)} G_Y^{(1-\theta)} \right)^{\frac{1}{\eta-\theta}} \right] = \mu \frac{\eta\Delta_2}{\eta - \theta} \left[ G_X^\eta G_Y^{-\theta} \right]^{\frac{1}{\eta-\theta}}, \tag{C15}$$

$$C^{-\sigma} \frac{(1 - \theta)\Delta_1}{\eta - \theta} \left[ G_X^{-(1-\eta)} G_Y^{(1-\theta)} \right]^{\frac{1}{\eta-\theta}} = \mu \left[ \beta Y + \frac{\theta\Delta_2}{\eta - \theta} \left( G_X^\eta G_Y^{-\theta} \right)^{\frac{1}{\eta-\theta}} \right], \tag{C16}$$

with the transversality condition  $\lim_{t \rightarrow \infty} e^{-\rho t} \mu K = 0$ .

Equations (C15) and (C16) are, respectively, the optimal conditions for  $G_X$  and  $G_Y$ . These two conditions determine  $G_X$  and  $G_Y$  but unlike in earmarked taxes, there are no closed-form expressions. Both  $G_X$  and  $G_Y$  are each a nonlinear function of  $C$  and  $\mu$ . As the dynamics of  $\mu$  is governed by equation (B14), through which the dynamics of  $C$  are influenced. Given these conditions, the dynamic stability in the general fund financing taxes is determined by a  $5 \times 5$  system including equations (13) and (C13,16). Finally, it is impossible to analyze the dynamic stability in this  $5 \times 5$  system.

## Notes

1. See Wagner (1991, 110) for gasoline taxes and vehicle fees, Stiglitz (1988, 177) for airport and airline ticket taxes and tolling, and Bailey (1995, 216) for road fuel duties, road fund license fees, television licensing fees, hotel bed taxes, cigarette taxes, carbon taxes, and air pollution fees.

2. In the United States, for example, the fraction of earmarked tax in the state tax dropped from 51.4 percent in 1954 to 41.1 percent in 1964, to 23 percent in 1979 (Wyrick and Arnold 1989), and to a stable 24 percent in 2000 (Novarro 2002). In Canada, the fraction of earmarked tax was 36 percent at the federal level and 25 percent at the provincial level (Hickling Corporation 1991). Finally, in Japan, only 20.6 percent of indirect tax was earmarked for specific appropriations of government-provided services in the 1990 fiscal year (Ishi 2001).

3. There is growing literature that analyzes aggregate economic stability of government policy rules. See, for example, the study of balanced-budget fiscal policy rules by Schmitt-Grohé and Uribe (1997) and of the inflation-forecast targeting rule by Benhabib, Schmitt-Grohé, and Uribe (2001).

4. Recent studies concerning public goods such as Barro (1990), Glomm and Ravikumar (1994), and Chen (2003) have adopted the production specification, while other studies such as Cazzavillan (1996), Bianconi and Turnovsky (1997), and Devereux and Wen (1998) have used the utility strategy. Chen (2006) used both types of specification.

5. In addition to externality, local indeterminacy may arise from distortionary factor taxation with two sectors (Bond, Wang, and Yip 1996), from the presence of increasing returns with one sector (Farmer and Guo 1994) and trade (Nishimura and Shimomura 2002). See Benhabib and Farmer (1999) for a survey of the literature.

6. See also Benhabib and Farmer (1996), Benhabib and Nishimura (1998), Harrison (2001), Weder (2001), Benhabib, Nishimura, and Venditti (2002), and Nishimura and Venditti (2002), among others.

7. Although a fraction of the government's revenue may be from consumption, the assumption of output tax is innocuous. To allow for a consumption tax in our model, two consumption goods are necessary. In the environment with two consumption goods in section 3 below, if the output taxation is replaced by a consumption tax, the underlying dynamic structure is the same and the results remain unchanged. We thus maintain the taxation of output throughout the article. Literature concerning the output taxation in an endogenous growth model starts from Barro (1990) and Rebelo (1991).

8. In this presentation, earmarking regime is reduced to  $G_X = T_X$  and  $G_Y = T_Y$  when  $\theta = \eta = 1/2$ .

9. Together equations (6a,b) and (8a,b), the production functions (1a) and (1b) are  $X = (sK)^{1-\alpha}(uT_X)^{\alpha\theta}(vT_Y)^{\alpha(1-\theta)}$ , and  $Y = [(1-s)K]^{1-\beta}[(1-u)T_X]^{\beta\eta}[(1-v)T_Y]^{\beta(1-\eta)}$ .

10. Alternatively, if  $\sigma < 1 - \beta$ , the  $R(s)$  is zero only at  $s = 0$ , with the value being negative and decreasing in  $s$  and approaching to negative infinity as  $s$  is close to 1. In general  $\sigma \geq 1$ , this case is less interesting.

11. Let  $\bar{s} \equiv \left\{ [\phi_2 + \beta_1(1-\alpha)] - \sqrt{[\phi_2 + \beta_1(1-\alpha)]^2 - 4\phi_2\beta_1(1-\alpha)(1-(1-\beta)/\sigma)} \right\} / 2\phi_2$ . Then,  $R(\bar{s})$  is the maximal value. It is obvious that  $R(\bar{s}) > R(\beta) = (1-\tau)\phi_2(\sigma-1)\beta^{\frac{\beta_1(1-\alpha)}{\theta_1}}(1-\beta)^{1-\frac{\beta_1(1-\alpha)}{\theta_1}}$ . Two BGP's are obtained if  $L \leq R(\bar{s})$ . It suffices to require  $L \leq R(\beta)$ , from which condition A is obtained.

12.  $\frac{\partial RH(s)}{\partial s} = -(1-\tau) \left\{ \alpha + \phi_4 \left( \frac{s}{1-s} \right)^{\frac{\gamma(\sigma-1)}{\sigma}} + \phi_4 \frac{\gamma(\sigma-1)}{\sigma} \frac{1}{1-s} \left( \frac{s}{1-s} \right)^{\frac{\gamma(\sigma-1)}{\sigma}} \right\} = \frac{-(1-\tau)}{\sigma s(1-s)} \{ \sigma s(1-s) + [\sigma(1-s) + \gamma(\sigma-1)]N(s) \}$ , where  $N(s) \equiv s \left( \frac{s}{1-s} \right)^{\frac{\gamma(\sigma-1)}{\sigma}}$ . If  $\sigma(1-s) + \gamma(\sigma-1) \geq 0$ , then  $\frac{\partial RH(s)}{\partial s} < 0$ . Alternatively, in a less plausible case where  $\sigma(1-s) + \gamma(\sigma-1) < 0$ , we may use  $\bar{s} = 0$  to rewrite  $N(s) = (1-s) - \frac{1}{1-\tau} \left[ \frac{1}{\phi_2} (1-\tau-\delta-\rho) + \delta \right]$ . Then, under condition B,  $N(s) < (1-s)$ . That means  $\frac{\partial RH(s)}{\partial s} < \frac{-(1-\tau)}{\sigma s(1-s)} \{ \sigma s(1-s) + [\sigma(1-s) + \gamma(\sigma-1)](1-s) \} = \frac{-(1-\tau)\phi_2}{\sigma s} < 0$  under condition B. As a consequence,  $RH(s)$  is decreasing in  $s$  under condition B.

13. To summarize the benchmark parameter values, economic growth rate = 0.02,  $\tau = 0.2$ ,  $\delta = 0.05$ ,  $\rho = 0.025$ ,  $\sigma = 2$ ,  $\theta = \eta = 0.5$ ,  $u = v = 0.5$ ,  $\alpha = \beta = 0.8$ ,  $a$  (earmarking) = 0.2179 and  $a$  (general fund) = 0.2064.

14. To summarize the benchmark parameter values, economic growth rate = 0.02,  $\tau = 0.2$ ,  $\delta = 0.05$ ,  $\rho = 0.025$ ,  $\sigma = 2$ ,  $\theta = \eta = 0.5$ ,  $u = v = 0.5$ ,  $\gamma = 0.3$ , and  $a = 0.1512$ .

15. We thank an anonymous referee for bringing this point to our attentions.

## References

- Bailey, S. J. 1995. *Public sector economies: Theory, policy and practice*, Hampshire: Macmillan.
- Barro, R. J. 1990. Government spending in a simple model of endogenous growth. *Journal of Political Economy* 98:S103–25.
- Ben-Gad, Michael. 2003. Fiscal policy and indeterminacy in models of endogenous growth. *Journal of Economic Theory* 108:322–44.
- Benhabib, J., R. Farmer. 1996. Indeterminacy and sector specific externalities. *Journal of Monetary Economics* 37:397–419.
- . 1999. Indeterminacy and sunspots in macroeconomics. In *Handbook of macroeconomics*, ed. J. B. Taylor and M. Woodford, 387–448. Amsterdam: Elsevier Science.
- Benhabib, J., Q. Meng, and K. Nishimura. 2000. Indeterminacy under constant returns to scale in multisector economics. *Econometrica* 68:1541–8.
- Benhabib, J., K. Nishimura. 1998. Indeterminacy and sunspots with constant returns. *Journal of Economic Theory* 81:58–96.
- Benhabib, J., K. Nishimura, and A. Venditti. 2002. Indeterminacy and cycles in two-sector discrete-time model. *Economic Theory* 20:217–35.
- Benhabib, J., R. Perli. 1994. Uniqueness and indeterminacy: On the dynamics of endogenous growth. *Journal of Economic Theory* 63:113–42.
- Benhabib, J., S. Schmitt-Grohe, and M. Uribe. 2001. Monetary policy and multiple equilibrium. *American Economic Review* 91:167–86.
- Bianconi, M., S. Turnovsky 1997. International effects of government expenditure in international economies. *Canadian Journal of Economics* 30:57–84.
- Boldrin, M., A. Rusticini 1994. Indeterminacy of equilibria in models with infinitely-lived agents and external effects. *Econometrica* 62:3223–42.
- Bond, E. W., P. Wang, and C. Yip. 1996. A general two-sector model of endogenous growth with human and physical capital: Balanced growth and transitional dynamics. *Journal of Economic Theory* 68:149–73.
- Bös, D. 2000. Earmarked taxation: Welfare versus political support. *Journal of Public Economics* 75:439–62.
- Brett, C., M. Keen 2000. Political uncertainty and the earmarking of environmental taxes. *Journal of Public Economics* 75:315–40.
- Browning, E. K. 1975. Collective choice and general fund financing. *Journal of Political Economy* 83:377–90.
- Buchanan, J. M. 1963. The economics of earmarked taxes. *Journal of Political Economy* 71:457–69.
- Butler, M. 2000. The political feasibility of pension reform options: The case of Switzerland. *Journal of Public Economy* 75:389–416.

- Cazzavillan, G. 1996. Public spending, endogenous growth, and endogenous fluctuations. *Journal of Economic Theory* 71:394–415.
- Chen, B.-L. 2003. Tax evasion in a model of endogenous growth. *Review of Economic Dynamics* 6:381–403.
- . 2006. Economic growth with optimal public spending composition. *Oxford Economic Papers* 58:123–36.
- Chen, B.-L., and S.-F. Lee. 2007. Congestible public goods and indeterminacy: A two-sector endogenous growth model. *Journal of Economic Dynamics and Control* 31:2486–518.
- Devereux, M., J. F. Wen. 1998. Political instability, capital taxation and growth. *European Economic Review* 42:1635–52.
- Druegeon, J. P., O. Poulsen, and A. Venditti. 2003. On intersectoral allocations, factor substitutability and multiple long-run growth paths. *Economic Theory* 21:175–83.
- Farmer, R., and J.-T. Guo. 1994. Real business cycles and the animal spirits hypothesis. *Journal of Economic Theory* 63:42–72.
- Glomm, G., B. Ravikumar. 1994. Public investment in infrastructure in a simple growth model. *Journal of Economic Dynamics and Control* 18:1173–88.
- Goetz, C.-J. 1968. Earmarked taxes and majority rule budgetary processes. *American Economic Review* 58:128–36.
- Harrison, S. G. 2001. Indeterminacy in a model with sector-specific externalities. *Journal of Economic Dynamics and Control* 25:747–64.
- Hickling Corporation. 1991. Earmarking of revenues and program management. Prepared for Royal Commission on Passenger Transport, Ottawa.
- Ishi, H. 2001. *The Japanese tax system*. 3rd ed. Oxford: Oxford University Press.
- Marsilliani, L., and T. Renstrom. 2000. Time inconsistency in environmental policy: Tax earmarking as a commitment solution. *Economic Journal* 110:C123–38.
- McMahon, W. W., C. M. Sprenkle. 1972. Earmarking and the theory of public expenditure. *National Tax Journal* XXV:229–30.
- Mino, K. 2001. Indeterminacy and endogenous growth with social constant returns. *Journal of Economic Theory* 97:203–22.
- Mulligan, C., and X. Sala-i-Martin. 1993. Transitional dynamics in two-sector models of endogenous growth. *Quarterly Journal of Economics* 108:737–73.
- Nishimura, K., K. Shimomura. 2002. Trade and indeterminacy in a dynamic general equilibrium model. *Journal of Economic Theory* 105:244–60.
- Nishimura, K., A. Venditti. 2002. Intersectoral externalities and indeterminacy. *Journal of Economic Theory* 105:140–57.
- Novarro, N. K. 2002. Does earmarking matter? The case of state lottery profits and educational spending, SIEPR discussion paper No. 02-19.
- Rebelo, S. 1991. Long-run policy analysis and long-run growth. *Journal of Political Economy* 99:500–21.
- Schmitt-Grohe, S., and M. Uribe. 1997. Balanced-budget rules, distortionary taxes, and aggregate instability. *Journal of Political Economy* 105:976–1000.
- Stiglitz, J. E. 1988. *Economics of the public sector*. New York: W. W. Norton.
- Teja, R. S. 1988. The case for earmarked taxes. *IMF Staff Papers* 35:523–33.
- Teja, R. S., and B. Bracewell-Milnes. 1991. *The case for earmarked taxes: Government spending and public choice*. Research Monograph. London: Institute of Economic Affairs.

- Turnovsky, S. J. 2000. Fiscal policy, elastic labor supply, and endogenous growth. *Journal of Monetary Economics* 45:185–210.
- Wagner, R. E. 1991. *Charging for government: User charges and earmarked taxes in principle and practice*. London: Routledge.
- Weder, M. 2001. Indeterminacy in a small open economy Ramsey growth model. *Journal of Economic Theory* 98:339–56.
- Wyrick, T. L., R. A. Arnold. 1989. Earmarking as a deterrent to rent-seeking. *Public Choice* 60:283–91.

**Been-Lon Chen** is research fellow in the Institute of Economics, Academia Sinica and adjunct professor in the Department of Economics, National Taiwan University. He holds a PhD degree in Economics, University of California at Los Angeles. His area of research is economic growth and macroeconomics.

**Shun-Fa Lee** is assistant professor in the Department of Industrial Economics, Tamkang University. He received his doctorate in economics from National Taiwan University. Much of his research has examined equilibrium indeterminacy in an endogenous growth model and trade policies in a two-county growth model.