



# Consumption externality, efficiency and optimal taxation in one-sector growth model

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## ABSTRACT

In an economy with consumption externalities, existing studies find that a competitive equilibrium is efficient in the long run and remains efficient in transitions if preferences are homothetic. This paper revisits the efficiency issue in an otherwise standard one-sector growth model where consumption externalities affect a utility via their effects on the time preference. We find that even if preferences are homothetic, the externality changes the marginal rate of substitution between now and future and leads to a disparity in the intertemporal elasticity of substitution between the centrally planned economy and a decentralized economy. As a result, a competitive equilibrium is inefficient in transition dynamics. We characterize an optimal tax/subsidy structure that enables the allocation in a decentralized economy to replicate the social optimum.

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## 1. Introduction

Recently, it has been shown that a competitive equilibrium is efficient in the long run, even if there are economy-wide current consumption externalities (e.g., Alonso-Carrera et al., 2004; Alvarez-Cuadrado et al., 2004).<sup>1</sup> Moreover, a competitive equilibrium remains efficient during the transition if preferences are homothetic with respect to individual consumption and consumption externalities along the equilibrium path (e.g., Fisher and Hof, 2000; Alonso-Carrera et al., 2006).

These existing studies represented preferences by a function in which a felicity is discounted by a constant time preference. The specification is analytically tractable and can easily describe how tastes and opportunities interact to determine the paths of consumption and capital. However, their preferences imply that the *marginal rate of substitution* (henceforth, MRS) between time  $t_1$  and  $t_2$  is independent of consumption at any time  $t \neq t_1, t_2$ .

A popular class of felicity is the constant elasticity of substitution function (hereafter, CES). This class of felicity has been widely used in macroeconomics. In particular, in endogenous growth models it is

necessary to use a CES felicity in order to assure the existence of a balanced growth path. It is known that a CES felicity with individual and average consumption is homothetic along the equilibrium path. The resulting competitive equilibrium is always efficient if time preferences are constant. However, when an agent's time preferences are affected by consumption externalities and thus endogenous, then competitive equilibrium is no longer efficient even if the felicity is of the homothetic CES form. As a result, the conclusions and explanatory powers, especially concerning welfare, may be incomplete and limited in models of CES felicities with a constant discount.

This paper investigates the efficiency issue concerning current consumption externalities. We consider a generalized class of preferences in which time preferences are influenced by current average consumption. In this class of preferences, consumption at any time  $t \geq t_1$  affects the MRS between time  $t_1$  and  $t_2 > t_1$ . As a consequence, even if a felicity is homothetic, there is a divergence between the MRS in a decentralized and in a centralized economy. This leads to a disparity in the *intertemporal elasticity of substitution* (henceforth, IES) between a decentralized and a centralized economy. The competitive equilibrium path in a decentralized economy is thus suboptimal.

In this paper, we study an otherwise standard optimal growth model except for the effect of the consumption externality in the discount. The influence of social forces on individual time preferences was stressed by earlier economists. Notably, Rae (1834, p. 198) saw culture as a critical determinant of differences in time preferences across various economies. Beginning with Fisher (1930), it has long been argued theoretically and empirically that time preferences are related to his income, wealth or consumption level. In this paper we

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<sup>1</sup> This conclusion holds as long as there is no labor–leisure tradeoff as found in Dupor and Liu (2003) and Liu and Turnovsky (2005).

represent the external effect in the discount by average consumption.<sup>2</sup> In this formulation, a negative consumption externality is represented by average consumption that externally increases an individual's discount rate, while a positive consumption externality is average consumption that externally decreases an individual's discount rate. Through the discount rate, the consumption externality exerts effects on not only the consumption decision at current point in time but also the consumption decision afterward. A consumption externality brings different growth rates of the capital price between a centralized economy and a decentralized economy. As a consequence, this competitive equilibrium path is inefficient in transition dynamics.

We find that the growth rate of consumption in the market equilibrium may be lower or higher than the efficient growth rate depending on positive or negative externalities. We characterize a tax structure that enables a competitive equilibrium path to replicate the social optimum path. Optimal taxes or subsidies bring change the IES in a decentralized economy in order to correct the effect of average consumption on discounting that is ignored by an agent. Whether it is optimal to tax or to subsidize depends on initial capital relative to its steady-state level, because the marginal product of capital and endogenous impatience are both at work. The condition for a tax when initial capital is smaller than its steady state is the condition for a subsidy when initial capital is larger than its steady state.

As developed below, Section 2 studies a decentralized economy. Section 3 is a centrally planned economy. Section 4 analyzes the growth rate of consumption and optimal taxation. Finally, concluding remarks are offered in Section 5.

## 2. The model

We consider an otherwise standard optimal growth model except for the effect of average consumption on the discount. The economy consists of a large number of identical agents, normalized to unity. The representative agent supplies labor inelastically and trades off consumption over time in order to maximize the discounted lifetime utility. The lifetime utility is

$$U = \int_0^\infty [u(c(t))X(t)]dt, \tag{1}$$

where  $u$  is the felicity function and  $c$  is individual consumption.  $X(t)$  is the discount at time  $t$  and is  $X(t) \equiv e^{-\int_0^t \rho(C(\tau))d\tau}$ , which changes according to

$$\dot{X} = -\rho(C(t))X(t), \text{ with } X(0) \text{ given,} \tag{2}$$

where  $\rho(C(t))$  is the discount-rate function and  $C$  is average consumption in the economy at time  $t$ . Thus, the discount rate is assumed to be influenced by average consumption.<sup>3</sup>

The influence of social forces on individual discount has been emphasized by Rae (1834) and Fisher (1930). It was argued that culture is a central determinant of differences in time preferences across different societies. A simple and tractable formulation of such a cultural influence is to employ relevant average variables in a society. Our paper follows the line of research by Drugeon (1998), Shi (1999), Schmidt-Grohé and Uribe (2003), Bian and Meng (2004) and Meng (2006) and places average consumption in the discount has recently been used.<sup>4</sup> Drugeon (1998) set up a model with discount rates dependent on individual and average consumption, and

also Shi (1999) postulated that individual discount relies on average consumption habits. Schmidt-Grohé and Uribe (2003) employed average felicity to influence discounts, and Bian and Meng (2004) and Meng (2006) modeled average consumption and income in the discount.<sup>5</sup>

We assume that the felicity and the discount-rate functions are well-behaved and are twice continuously differentiable. In addition, the following assumptions are made.

### Assumption 1.

- (i)  $u'(C) > 0 > u''(C)$ ;
- (ii)  $\rho > 0, \rho'(C) > 0$  or  $\rho'(C) < 0$ ;
- (iii)  $\frac{u'(C)}{u(C)} N \frac{\rho'(C)}{\rho(C)}; -\frac{u''(C)}{u(C)} \geq -\frac{\rho''(C)}{\rho(C)}$ . (iii)

Assumption 1 (i) is standard. Assumption 1 (ii) asserts that impatience is positive and may be increasing or decreasing in average consumption. Assumption 1 (iii) are the technical conditions in order to assure a positive IES. The conditions require that the slope of the felicity be larger than and decreasing in a speed faster than the slope of the discount.

Let us comment on an increasing or a decreasing impatience in Assumption 1 (ii). There is extensive dispute over whether or not impatience should increase or decrease as consumption rises. Koopmans (1960) made an argument in favor of decreasing impatience. Authors like Blanchard and Fischer (1989) and Barro and Sala-i-Martin (1995) found it counterintuitive that an agent would be more impatient as his level of consumption rises. On the other hand, Epstein (1987) offered counter-arguments and argued that the proper interpretation of a discount rate is that individuals who know that they will have a large level of consumption in the future will evaluate current consumption more highly. Lucas and Stokey (1984) pointed out that increasing impatience, a sort of diminishing returns to savings, is often needed to produce unique, stable and non-degenerate steady-state wealth distributions in a deterministic infinite-horizon setting with a fixed set of agents. Authors like Obstfeld (1990) and Palivos, et al. (1997) followed the assumption of increasing impatience in order to assure the stability. We allow for the possibilities of either increasing or decreasing impatience as average consumption rises.<sup>6</sup>

Unlike Alonso-Carrera et al. (2004, 2006), Chen and Hsu (2007) and Liu and Turnovsky (2005), in our paper we place consumption externalities in the discount and do not place consumption externalities directly in a felicity. Our modeling approach captures the effect of consumption externalities on the lifetime utility and simplifies the model through the effect on the discount.<sup>7</sup> To see this, we follow Uzawa (1968, pp. 486–494) and take  $\Delta \equiv \int_0^t \rho(C(\tau))d\tau$  as the independent variable instead of  $t$  in the maximization. The utility is then  $U = \int_0^t W(c, C)e^{-\Delta}d\Delta$  and the instantaneous felicity is  $W(c, C) = u(c)/\rho(C)$ , where  $W_2(c, C) > 0$  if  $\rho'(C) < 0$  and  $W_2(c, C) < 0$  if  $\rho'(C) > 0$ . Thus, our formulation of consumption externalities in a discount is similar to a formulation of

<sup>5</sup> Alternatively, there is a body of research that models the effect of individual's own consumption on the discount. This is an internal effect on the discount. The influence of an agent's own consumption on his discount was first postulated by Uzawa (1968), and has been extended in subsequent research (e.g., Lucas and Stokey, 1984; Epstein, 1987; Obstfeld, 1990; Palivos et al., 1997). In earlier versions, we have considered an internal discount and the results are the same. To maintain the same message with a simple story, in this paper we do not consider an internal discount.

<sup>6</sup> Empirical evidence is mixed regarding increasing or decreasing time preferences. Using the Panel Study of Income Dynamics, Lawrence (1991) found time preferences of the poor are three to five percentage points higher than those of the rich, indicating a decreasing time preference. Using the post-war annual time-series data, Ogawa (1993) found time preferences in Japan and Taiwan increase as the two economies grow, indicating an increasing time preference.

<sup>7</sup> In an earlier version that is available upon request, we considered consumption externalities in felicities and discounts and the results are the same.

<sup>2</sup> Placing average consumption in the discount has recently been used in Drugeon (1998), Schmidt-Grohé and Uribe (2003) and Meng (2006).

<sup>3</sup> If the lifetime utility is formulated as  $U = \int_0^\infty [u(c(t))e^{-M(t)}]dt$ , then  $\dot{M} = -\rho(C(t))$ , with  $M(0)$  given.

<sup>4</sup> See also Shi (1999) who postulated that individual discount relies on average consumption habits.

consumption externalities in a felicity except that we abstract from private consumption in the discount rate.<sup>8</sup> When consumption in a society externally increases (or decreases) the discount rate, an agent's lifetime utility is decreased (or increased). This is called a *jealousy* (or *admiration*) effect (e.g., Dupor and Liu, 2003).

The single good is produced by  $y=f(k)$ , where  $y$  is output per capita and  $k$  is capital stock per capita with  $k(0)$  given initially; capital does not depreciate. The technology exhibits standard properties in a neoclassical production function and the Inada conditions as follows.

**Assumption 2.**  $f'(k) > 0 > f''(k)$ ,  $\lim_{k \rightarrow 0} f'(k) = \infty$  and  $\lim_{k \rightarrow \infty} f'(k) = 0$ .

The budget constraint of the representative household is

$$\dot{k} = f(k(t)) - c(t). \tag{3}$$

It we let  $\lambda_k > 0$  be the co-state variable associated with capital, taking average consumption as given by the society, the necessary conditions of the household's optimization are

$$\lambda_k = u'(c), \tag{4a}$$

$$\dot{\lambda}_k = \lambda_k [-f'(k) + \rho(C)], \tag{4b}$$

along with the transversality condition  $\lim_{t \rightarrow \infty} X(t)\lambda_k(t)k(t) = 0$ .

In these conditions, Eq. (4a) equates the marginal cost with the marginal utility, while Eq. (4b) is the Euler equation for capital.

**Definition 1.** Under Assumptions 1–2, given  $k(0)$ , a symmetric equilibrium path in the decentralized economy is  $\{C^*(t), k^*(t), \lambda_k^*(t)\}$  with  $\dot{c}^*(t) = C^*(t)$  which solves Eqs. (3) and (4a–b).

A steady state is a symmetric equilibrium path  $\{C^*, k^*, \lambda_k^*\}$  with a constant  $C^*$  and  $\dot{k}^* = \dot{\lambda}_k^* = 0$  in Eqs. (3) and (4b). In Appendix 1 we have established the following results.

**Proposition 1.** Under Assumptions 1–2 and  $\rho'(C) > \frac{f''(k)}{f'(k)}$ , there exists a unique interior steady state which is a saddle.

The steady state is determined by the market clearing condition (Eq. (3)) and the modified golden rule (Eq.(4b)) at  $\dot{\lambda}_k = 0$ . The two conditions are illustrated by a positively sloping locus for the market clearing condition and a negatively sloping locus for the golden rule in Fig. 1 with the steady state at  $E$ . Different from models with consumption externalities and exogenous discount, average consumption now exerts an effect on the steady-state equilibrium allocation via its effect on the impatience in our model. As a result, depending on the direction of the external consumption effect on impatience, a shock has different steady-state effects.

To illustrate this point, suppose that the productivity is increased. The shock shifts the market clearing condition locus leftward in Fig. 1. The golden-rule locus is shifted rightward under negative externalities ( $\rho'(C) \geq 0$ ) and leftward under positive externalities ( $\rho'(C) < 0$ ). In particular, while an improved technology (represented by  $A$  in Fig. 1) increases consumption in a steady state, the level of consumption depends on externalities. The level of consumption is the highest under positive externalities, followed by zero externalities ( $\rho' = 0$ ) and finally, negative externalities. The level of capital per capita is also increased the highest under positive externalities, followed by zero externalities. However, under negative, capital per capita may decrease in a steady state even though the technology is improved. This result emerges if the negative externalities are so strong that  $\rho' > f'/f > 0$ . The

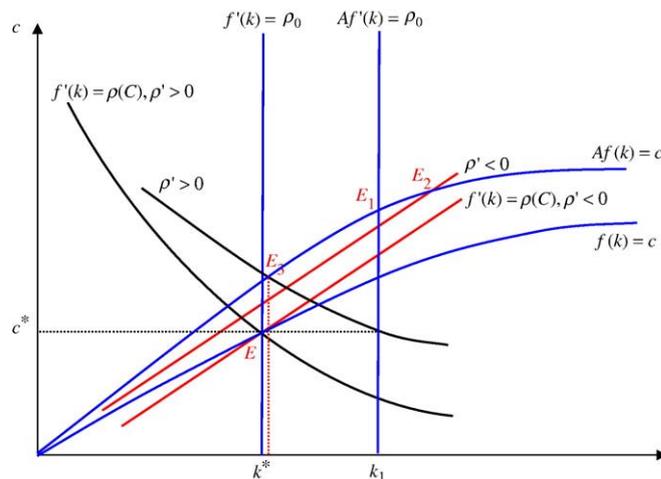


Fig. 1. Steady state and effects of technology shock.

reason is that under an externally higher discount, future consumption is substituted toward current consumption in equilibrium. When this effect is sufficiently strong, the effect of an improved technology is predominated. As a result, capital is reduced in the long run. These results are different from those not only in a standard Ramsey–Cass optimal growth model but also in an optimal growth model with consumption externality in a felicity (e.g., Alonso-Carrera et al., 2004; Alvarez-Cuadrado et al., 2004).

We must point out that in an otherwise standard growth model with consumption externalities, equilibrium paths toward a steady state may be indeterminate when the discount rate is affected by consumption (e.g., Drugeon, 1998; Chen and Hsu, 2007). In this paper, our attention is paid to a unique saddle path in order to focus on comparisons between a competitive equilibrium and a centrally planned optimum. Indeed, it is easy to show that the condition necessary to guarantee the existence of the steady state in our paper is also the condition to assure a saddle path: the negative dependence of the growth rate of consumption with respect to consumption. This is easily reasoned if we compute implicitly the derivative  $dk/dc$  from the market clearing condition (Eq. (3)) and take a derivative of the growth rate of consumption (Eq. (6a) below) evaluated at a steady state.

### 3. Centrally planned economy

Different from a decentralized economy where the representative agent neglects the consumption externality, the planner in a centrally planned economy internalizes external effects. Denote  $\tilde{\lambda}_X > 0$  the co-state variable associated with the discount in the planner's problem. If we use a tilde to denote a variable at the social optimum, the necessary conditions for the central planner's problem are

$$\tilde{\lambda}_k = u'(\tilde{c}) - \tilde{\lambda}_X \rho'(\tilde{c}), \tag{5a}$$

$$\dot{\tilde{\lambda}}_k = \tilde{\lambda}_k [-f'(\tilde{k}) + \rho(\tilde{c})], \tag{5b}$$

$$\dot{\tilde{\lambda}}_X = -u(\tilde{c}) + \tilde{\lambda}_X \rho(\tilde{c}), \tag{5c}$$

with the transversality conditions  $\lim_{t \rightarrow \infty} \tilde{X}(t)\tilde{\lambda}_k(t)\tilde{k}(t) = 0$  and  $\lim_{t \rightarrow \infty} \tilde{X}(t)\tilde{\lambda}_X(t) = 0$ .

As the social planner internalizes the effect of consumption externalities on the discount, the marginal utility in Eq. (5a) is reduced (resp. increased) by a negative (resp. positive) external effect via a higher (or lower) discount rate. The internalization of the discount gives rise to an evolution of the shadow price of discounting in Eq. (5c).

<sup>8</sup> In earlier versions, in addition to external consumption, we placed individual's own consumption in the discount and thus  $\rho = \rho(c, C)$ . There is no extra insight but complicated algebra, and thus the discount rate is now abstracted from the effect of individual's own consumption.

**Definition 2.** Under Assumptions 1–2, given  $k(0)$  and  $X(0)$ , an efficient allocation is a path  $\{\tilde{C}, \tilde{k}, \tilde{\lambda}_k, \tilde{\lambda}_x\}$  which solves Eq. (3), with  $\tilde{c} = \tilde{C}$ , and Eqs. (5a–c).

3.1. Steady state

Comparing a decentralized economy with a centrally planned economy, allocation is the same in a steady state. Allocation in a centrally planned economy is determined by the modified golden rule (Eq. (5b)) at  $\dot{\lambda}_k = 0$  and the market clearing condition (Eq. (3)) at  $k = 0$ . These two conditions determine the efficient allocation of  $C$  and  $k$  in the steady state in exactly the same way as Eq. (3) and Eq. (4b) at  $\dot{\lambda}_k = 0$  determine the competitive market equilibrium values of  $C$  and  $k$  in the steady state. As a result, even with endogenous discounts, the competitive market equilibrium is efficient in a steady state. The result corroborates with models with exogenous discounts in Alonso-Carrera et al. (2004) and Liu and Turnovsky (2005) in which current consumption externalities do not create inefficiency in a steady state.

Once the efficient allocation of  $C$  and  $k$  in a steady state is determined, the shadow price of the discount in a steady state is determined by (5c). It is easy to obtain the shadow price of the discount in the steady state as  $\tilde{\lambda}_x = u(\tilde{C})/\rho(\tilde{C})$  which is just the discounted value of the utility along the steady-state consumption stream.

3.2. Dynamic equilibrium

Although the market competitive equilibrium is efficient in a steady state, we will show that the market equilibrium path is inefficient in dynamic transitions.

We compare the dynamic paths between decentralized and centrally planned economies.<sup>9</sup> The dynamic path is best represented by the growth rate of consumption. As conditions (Eq. (4a)) and (Eq. (5a)) are different if  $\rho' \neq 0$ , there are different growth rates of consumption. Differentiating Eq. (4a), with the use of Eq. (4b), yields the consumption path in a decentralized economy

$$\dot{C} = \sigma(t)[f'(k(t)) - \rho(C(t))], \tag{6a}$$

where  $\sigma(t) \equiv \frac{u'(t)}{-u''(t)} > 0$  If we divide both side of Eq.(6a) by  $C(t)$ , then  $\frac{\sigma(t)}{\tilde{C}(t)}$  is the IES and Eq. (6a) growth rate of consumption in a decentralized economy.

Differentiating Eq. (5a), with the use of Eq. (5b), yields the efficient consumption path in transitions in a centrally planned economy

$$\dot{\tilde{C}} = \tilde{\sigma}(t)[f'(\tilde{k}(t)) - \tilde{\rho}(t) + \tilde{B}(t)], \tag{6b}$$

where  $\tilde{\rho}(t) \equiv \rho(\tilde{C}(t)); \tilde{B}(t) \equiv \tilde{\rho}'(t) \frac{\tilde{u}(t) - \tilde{\lambda}_x(t)\tilde{\rho}(t)}{\tilde{u}'(t) - \tilde{\lambda}_x(t)\tilde{\rho}'(t)}; \tilde{\sigma}(t) \equiv -\frac{\tilde{u}'(t) - \tilde{\lambda}_x(t)\tilde{\rho}'(t)}{[\tilde{u}''(t) - \tilde{\lambda}_x(t)\tilde{\rho}''(t)]} > 0$ . If

we divide both sides of Eq. (6b) by  $\tilde{C}(t)$ , then  $\frac{\tilde{\sigma}(t)}{\tilde{C}(t)}$  is the IES and Eq. (6b) is the growth rate of consumption in a centrally planned economy. Under Assumption 1 (iii), the IES is positive in a steady state when  $\lambda_x = u/\rho$ . The sign of  $\tilde{B}(t)$  is determined by the sign of  $[\tilde{u}(t) - \tilde{\lambda}_x(t)\tilde{\rho}(t)]$  and  $\tilde{\rho}'(t)$ . In the case when the initial capital is smaller than its steady-state level and thus,  $f' - \rho > 0$ , consumption increases in transitions and thus the shadow price of the discount is decreasing. This indicates  $\tilde{\lambda}_x = \tilde{u}(t) - \tilde{\lambda}_x(t)\tilde{\rho}(t) > 0$  and the sign of  $\tilde{B}(t)$  dictates the sign of  $\tilde{\rho}'(t)$  Alternatively, when the initial capital is larger than its steady-state level, the sign of  $\tilde{B}(t)$  is opposite to the sign of  $\tilde{\rho}'(t)$ .

<sup>9</sup> To compare the efficient allocation with the market equilibrium allocation, it is necessary to assure that there is a unique efficient equilibrium path and a unique market equilibrium path. Recall that in Appendix A, under a mild condition  $\rho'(C) > f'(k)/f''(k)$ , the steady-state equilibrium allocation is a saddle. In Appendix B, we have shown that the steady-state efficient allocation is a saddle under the same condition as in the market equilibrium. Thus, efficient equilibrium path and market equilibrium path are both unigue.

Obviously, if there is not a consumption externality and thus  $\rho' = \rho'' = 0$ , then  $\tilde{B} = 0$  and  $\tilde{\sigma}(t) = \sigma(t)$ . As a result, consumption in the market equilibrium changes in the same as the efficient consumption path. However, if there is a consumption externality and thus  $\rho' \neq 0$ ,  $\tilde{\sigma}(t) \neq \sigma(t)$  and  $\tilde{B} \neq 0$ . Therefore, Eq. (6a)  $\neq$  Eq. (6b). Thus, the consumption path in transitions in a market economy is different from the consumption path in a centrally planned economy. To summarize the results,

**Proposition 2.** Under Assumptions 1–2,  $\rho'(C) > \frac{f''(k)}{f'(k)}$ , and  $\rho'(C) \neq 0$ , dynamic paths in a market economy is not efficient.

A popular class of felicity is the CES function which has been widely used in economics. In particular, a CES form is required in order to assure the existence of a balanced growth path in endogenous growth models. It is known that in a model with exogenous discount rates, a CES felicity with consumption and consumption externalities is homothetic along the equilibrium path and as a result, the dynamic competitive equilibrium path is always efficient. As we have just shown, a model with a CES felicity and consumption externalities in discount may have inefficient growth rate of consumption. Two examples are in order. In these two examples, we suppose that initial capital is smaller than its steady-state level. Moreover, we consider the standard CES felicity form that is homothetic as follows.

$$u = u(c) = \frac{1}{1-\gamma} c^{1-\gamma}, \gamma \geq 1,$$

where  $\gamma$  is the degree of risk aversions and  $1/\gamma$  is the intertemporal elasticity of substitution in the consumption.

**Example 1.** Let  $\rho(C) = \alpha_0 - e^{-\eta C} > 0$ , where  $\eta < 1$  may be positive ( $\eta < 0$ ) or negative ( $\eta > 0$ ).

The consumption path in the market equilibrium is  $\dot{C} = \frac{C(t)}{\gamma} [f'(k) - \rho(C)]$  and the consumption path in the optimal allocation are  $\dot{\tilde{C}} = \tilde{\sigma}(t)[f'(\tilde{k}) - \rho(\tilde{C}) + \tilde{B}(t)]$ , where  $\tilde{B}(t) \equiv \eta e^{-\eta \tilde{C}} \frac{\tilde{C}^{1-\gamma} / (1-\gamma) - \tilde{\lambda}_x \rho'(\tilde{C})}{\tilde{C}^{-\gamma} - \eta \tilde{\lambda}_x e^{-\eta \tilde{C}}} > 0$  and  $\tilde{\sigma}(t) \equiv \frac{\tilde{C}^{-\gamma} - \eta \tilde{\lambda}_x e^{-\eta \tilde{C}}}{\gamma \tilde{C}^{-(\gamma+1)} - \eta^2 \tilde{\lambda}_x e^{-\eta \tilde{C}}} > 0$ . Then,  $\dot{\tilde{C}} \leq \dot{C}$  if  $\eta \geq 0$ .

**Example 2.** Let  $\rho(C) = \alpha_0 + \alpha_1 C > 0$ . With a linear discount, the utility has a larger curvature.

The consumption paths have the same forms as those in Example 1, except for  $\tilde{B}(t) \equiv \alpha_1 \frac{\tilde{C}^{1-\gamma} / (1-\gamma) - \tilde{\lambda}_x \rho'(\tilde{C})}{\tilde{C}^{-\gamma} - \alpha_1 \tilde{\lambda}_x} > 0$  and  $\tilde{\sigma}(t) \equiv \frac{\tilde{C}^{-\gamma} - \alpha_1 \tilde{\lambda}_x}{\gamma \tilde{C}^{-(\gamma+1)}} > 0$ . Then,  $\dot{\tilde{C}} \leq \dot{C}$  if  $\alpha_1 \geq 0$ .

4. Optimal tax policy

We have thus found that an equilibrium path in a decentralized economy is inefficient when consumption externalities affect time preferences. This is so as the externality distorts the growth rate of the shadow price of capital in a decentralized economy. We now characterize a tax structure that enables market equilibrium to replicate the first-best optimum. There are two forces at work: the marginal product of capital and endogenous time preferences via consumption externalities. We thus consider capital and consumption taxes.

Let  $\tau_k(t)$  and  $\tau_c(t)$  be the time  $t$  tax rate on capital income and consumption, respectively. In the replicated equilibrium,  $C^{**}(t) = \tilde{C}(t)$ ,  $k^{**}(t) = \tilde{k}(t)$ ,  $\lambda_k^{**}(t) = \tilde{\lambda}_k(t)$  for all  $t$ , in which two asterisks are used to denote the equilibrium with optimal taxes. It is easy to show that the optimal tax rate at time  $t$  is (see Appendix C)

$$\tau_c(t) = -1 + \exp\left\{\int_0^t \frac{\Lambda(s)}{\sigma^{**}(s)} ds\right\}, \text{ or } \tau_k(t) = \frac{\Lambda(t)}{\sigma^{**}(t)f'(k^*(t))}, \tag{8}$$

where  $\Lambda(t) \equiv \sigma^{**}(t)[f'(k^{**}) - \rho(C^{**})] - \tilde{\sigma}(t)[f'(\tilde{k}) - \rho(\tilde{C})] + \tilde{B}$  is the difference between changes in equilibrium consumption under optimal taxes, evaluated at zero optimal tax rates, and changes in consumption in the tax-free competitive equilibrium, and  $\sigma^{**}(t)/C^{**} > 0$  is the IES under optimal taxes.

Thus, if changes in equilibrium consumption evaluated at zero optimal tax rates are larger than changes in consumption in the tax-free competitive equilibrium, an optimal consumption tax is necessary in order for the competitive equilibrium to be efficient. Alternatively, if changes in consumption evaluated at zero optimal tax rates are smaller than changes in consumption in the tax-free competitive equilibrium, an optimal consumption subsidy is necessary in order for the competitive equilibrium to be efficient.

In addition to a consumption tax/subsidy, a capital income tax/subsidy is optimal. There is equivalence between taxation on capital income and taxation on consumption in a model of relative consumption by Fisher and Hof (2000). In the model with exogenous time preferences in Fisher and Hof (2000), the MRS between time  $t_1$  and  $t_2$  is independent of average consumption in a society at any time  $t \neq t_1, t_2$ . If the felicity is of a CES form, the equilibrium path is the same in both a decentralized economy and a centrally planned economy and then, no optimal taxes are required. In contrast, in the present model average consumption in a society affect time preferences and thus in a centrally planned economy, the MRS between time  $t_1$  and  $t_2$  is influenced by average consumption at any time  $t$  in  $[t_1, t_2]$ . As a result, even if the felicity is of a CES form, consumption externalities change the IES in a centrally planned economy through the effect on time preferences that are otherwise ignored by individuals. Taxation is made on the market allocation in an effort to adjust the ratio of IESs and the time preference rates between a decentralized economy and a centrally planned economy. Intuitively, through the effect on the discount, the consumption externality twists the growth rates of the shadow price of capital in the market equilibrium. This distortion affects capital accumulation and it is optimal to tax capital income to correct such a distortion. Both tax policies affect the relative price between today and future and could achieve efficiency.

To see how consumption externalities in discounting determines whether a tax or a subsidy is optimal, we consider the case when the initial level of capital is lower than the steady-state level, i.e.,  $k(0) < \tilde{k} = k^{**}$ . When initial capital is smaller than its steady-state level,  $f'(k^{**}(0)) - \rho^{**}(0) > 0$  and the level of consumption increases over time toward a steady state.

1.  $\rho'(C) > 0$ .

Suppose that  $\rho'(C) > 0$  and thus, the consumption externality is negative. Therefore, before a steady state is reached, the increase in consumption in the competitive equilibrium is smaller than the increase in consumption in the environment with optimal taxes, evaluated at zero optimal tax rates, i.e.,  $\Lambda(t) > 0$ . Then, a tax on either consumption or capital income is optimal.

The reason is easily understood. Under  $\rho'(C) > 0$  and thus a negative consumption externality, the agent discounts the future less than he would have done when the externality had been internalized. A tax is optimal as it would reduce individual consumption which will reduce the effect of negative externalities.

2.  $\rho'(C) < 0$ .

Alternatively, suppose that  $\rho'(C) < 0$  and thus, the consumption externality is positive. In this case, before a steady state is reached, the increase in consumption under the environment of optimal taxes, evaluated at zero optimal tax rates, is smaller than the increase in consumption in the competitive equilibrium, i.e.,  $\Lambda(t) < 0$ . Thus, it is optimal to subsidize either consumption or capital.

Intuitively, the agent discounts more than he would have done because he does not internalize the effect of an increase in average

consumption. A subsidy is optimal as it would increase individual consumption which will increase the effect of positive externalities.

To summarize optimal taxation,

**Proposition 3.** Under Assumptions 1–2 and  $\rho'(C) > \frac{f''(k)}{f'(k)}$ , suppose that initial capital is smaller than the steady-state level. Then, it is optimal to tax (resp. subsidize) consumption or capital income if the consumption externality is negative (resp. positive).

Suppose instead that the initial capital is larger than the steady-state level. The level of consumption is decreasing over time toward a steady state. It follows that a subsidy is optimal if  $\rho'(C) > 0$  and a tax is optimal if  $\rho'(C) < 0$ .

The above results have the following implications. We use as an example of a negative consumption externality  $\rho'(C) > 0$ . The result of an example of a positive consumption externality is just the opposite.

First, suppose initially the steady state is at  $k_1(0)$  in Fig. 2. Suppose further that there is a productivity shock that increases capital and consumption in a new steady state as represented by  $k^{**}$  in Fig. 2. In the equilibrium path where consumption is increasing from  $k_1(0)$  toward the new steady state, the level of consumption, when there is no taxes, is higher than the efficient level and as a result, it is optimal to tax either consumption or capital.

Alternatively, suppose initially the steady state is represented by  $k_2(0)$  in Fig. 2. Suppose further that there is now an adverse productivity shock. Then consumption and capital are reduced in a new steady state, represented also by  $k^{**}$  in Fig. 2. In the equilibrium path when consumption is decreasing from  $k_2(0)$  toward the new steady state, the level of consumption with zero tax and subsidy is smaller than the efficient level and as a result, it is optimal to subsidize either consumption or capital.

Therefore, other things being equal, when taxation is optimal in the event of a shock that increases capital and consumption toward a new steady state, a subsidy is optimal in the face of an adverse shock that decreases capital and consumption toward a new steady state.

Finally, we should make note of the time inconsistency issue as we depart from the typical formulation with a constant discount rate. When a discount is affected by variables other than time distance, the MRS for consumption between time  $t_1$  and  $t_2$  as of planning date  $\tau_1$  may be different from that as of planning date  $\tau_2 > \tau_1$ . The optimal program may be time inconsistent. The discount-rate function in our model depends on average consumption in a modified Uzawa type. As Blanchard and Fischer (1989, p. 72) clearly pointed out, such formulation leads to a rate of time preferences that changes through time but still implies time consistency of the optimal program.

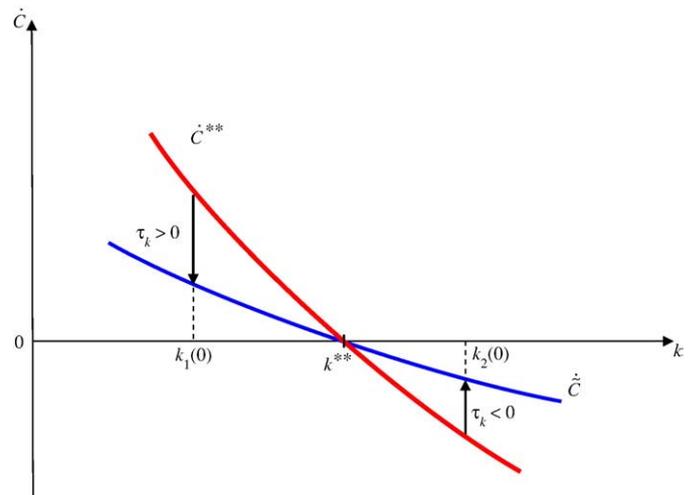


Fig. 2. Growth rate of consumption under case  $\rho'(C) > 0$ .

**5. Concluding remarks**

This paper studies the consumption externality and the efficiency in a standard optimal growth model with current consumption externalities in time preferences. A negative consumption externality reduces the utility through a higher time preference rate, while a positive consumption externality increases the utility through a lower time preference rate. In the present framework, as current consumption externalities affect instantaneous utilities through the effect on the discount rate, the effect then works not only at a current point in time and but also afterward. Thus, even if the felicity is homothetic along the equilibrium path, the discounted MRS between consecutive periods in the centrally planned economy is influenced by consumption externalities and is no longer constant. It follows that the IES for consumption differs between a centrally planned economy and a decentralized economy. The resulting equilibrium in a decentralized economy is thus suboptimal.

We find that the consumption path in a decentralized economy may be smaller or larger than the efficient path depending on negative or positive external consumption effects. We characterize a tax structure that enables the equilibrium in a decentralized economy to replicate the social optimum. In the replication, optimal taxation brings change the IES in a decentralized economy in order to correct the external effect of consumption on discounting that is ignored by an agent. Because endogenous time preferences and the marginal product of capital are both at work, the condition for optimal taxation depends not only on a positive or a negative consumption externality, but also on initial capital relative to its steady-state level. When initial capital is smaller than its steady state, a negative consumption externality requires a tax while a positive consumption externality requires a subsidy. When initial capital is larger than its steady state, a negative consumption externality requires a subsidy while a positive consumption externality requires a tax.

Finally, we offer comments about other forms of consumption in the discount. It is possible that an agent's past consumption may form an internal habit stock and relate to the discount, an idea that has been studied by Shi and Epstein (1993), followed by Chen (2007). The internal habit effect does not cause inefficiency. Alternatively, average consumption in a society may form a habit stock and externally affect an agent through his discount, thus exemplifying an effect of catching-up with or running-away from the Joneses. The effect has been placed in the felicity in Alonso-Carrera, et al. (2004), Alvarez-Cuadrado et al. (2004). When there is an external habit effect in the discount, we expect the steady-state allocation in a decentralized economy is efficient. The reason is the stock of habit equals the level of consumption in a steady state in a neoclassical growth model. With external habit formulation, there is an additional stock variable (habit stock) initially predetermined and one more shadow price for the variable. The transitional dynamics is more complicated. The dynamic path of allocation in a decentralized economy is expected to be inefficient. The reason is that external consumption from the very past affects habits and influences the MRS between now and the future. Yet, as there is more than one state variable, it would be more difficult to characterize a set of tax structure to replicate the equilibrium. Yet, this may offer yet another avenue for further research as, for example, not only current consumption pollution may have a negative effect on an individual, but also past consumption pollution may exert a negative effect an individual in a society.

**Appendix A. Proof of proposition**

**Proof.** The steady state in the market equilibrium is determined by

$$f'(k^*) = \rho(C^*), \tag{A1}$$

$$f(k^*) = C^*. \tag{A2}$$

The locus (Eq. (A2)) is upward sloping in the (k, C) plane according to Assumption 2. If  $\rho' > 0$ , the locus (Eq. (A1)) is downward sloping. Then, there exists a unique interior  $C^* > 0$  and  $k^* > 0$  (see E in Fig. 1). Alternatively, if  $\rho' < 0$ , the locus (Eq. (A1)) is upward sloping. As we will show below that a saddle stability of a steady state requires  $\rho' > f''/f'$ , which demands the locus (Eq. (A1)) be steeper than Eq. (A2). Then, there exists a unique steady state if  $\rho' < 0$ .

To show the saddle stability, differentiating Eq. (4a) with respect to time, together Eq. (4b), yields

$$\dot{C}(t) = \sigma(C)[f'(k(t)) - \rho(C)], \tag{A3}$$

where

$$\sigma(C) = -\frac{u'}{u''} \equiv \sigma > 0.$$

The stability of the market equilibrium is analyzed by the Taylor expansion of Eqs. (3) and (A3) around the steady state  $\{k^*, C^*\}$

$$\begin{bmatrix} \dot{k} \\ \dot{C} \end{bmatrix} = \begin{bmatrix} f'(k^*) & -1 \\ \sigma f''(k^*) & -\sigma\rho' \end{bmatrix} \begin{bmatrix} k - k^* \\ C - C^* \end{bmatrix}. \tag{A4}$$

Denote the Jacobean matrix in Eq. (A4) as  $J$  and the corresponding two characteristic roots of  $J$  as  $\mu_1$  and  $\mu_2$ . To characterize the root of  $J$ , note that the economic system (Eq. (A4)) involves one state variable whose initial value is predetermined and one control variable that can adjust instantaneously. As a result, the dynamic equilibrium near the unique steady state is a saddle path if there is one root with a negative real part, and is indeterminate if there are two roots with negative real parts.

Thus, a saddle requires

$$\mu_1 + \mu_2 = \sigma(-f'\rho' + f'') < 0. \tag{A5}$$

If  $\rho' > 0$ , then Eq. (A5) is met. If  $\rho' < 0$ , Eq. (A5) is met if the following condition is met

$$\rho' > f''/f'. \tag{A6}$$

Under this condition, the locus Eq. (A1) is upward sloping and steeper than the locus Eq. (A2).

**Appendix B. Stability of efficient allocation**

The stability of efficient allocation is analyzed by the linear Taylor expansion of Eqs. (3), (5b) and (6b) around the steady-state efficient allocation,  $\{\tilde{k}, \tilde{\lambda}_x, \tilde{C}\}$ ,

$$\begin{bmatrix} \dot{k} \\ \dot{\lambda}_x \\ \dot{C} \end{bmatrix} = \begin{bmatrix} f'(\tilde{k}) & 0 & -1 \\ 0 & \tilde{\rho} & -[u' - \tilde{\lambda}_x \tilde{\rho}'] \\ \tilde{\sigma} f''(\tilde{k}) & \tilde{\sigma} \frac{-\tilde{\rho}' \tilde{\rho}}{\tilde{u}'(t) - \tilde{\lambda}_x(t) \tilde{\rho}'} & 0 \end{bmatrix} \begin{bmatrix} k - k^* \\ \lambda_x - \lambda_x^* \\ C - C^* \end{bmatrix}. \tag{B1}$$

The characteristic polynomial of the Jacobean matrix  $J$  is

$$Y(w) = -w^3 + Zw^2 + Fw + G, \tag{B2}$$

where

$$Z \equiv f' + \tilde{\rho} > 0,$$

$$F \equiv -\tilde{\sigma} f'' - f' \tilde{\rho} + \tilde{\rho}' \tilde{\rho},$$

$$G \equiv \tilde{\sigma} \tilde{\rho}' [f'' - f' \tilde{\rho}'] < 0.$$

The Jacobean matrix in Eq. (B1) has three characteristic roots. The dynamic equilibrium near the unique steady state is a saddle path if there is only one root with a negative real part, and is indeterminate if there are two or more than two roots with negative real parts.

1.  $\rho'(C) > 0$ . Then  $\Upsilon(0) = G < 0$ . There are either three roots or one root with negative real parts. Obviously, we obtain  $\Upsilon(-\infty) > 0$  and  $\Upsilon(\infty) < 0$ . If we differentiate Eq. (B2), we obtain  $\Upsilon'(w) = -3w^2 + 2Zw + F$ . If we let  $\Upsilon'(w) = 0$ , we obtain two extreme values, denoted by  $w_1$  and  $w_2$ , as

$$w_i = \frac{Z \pm \sqrt{Z^2 + 3F}}{3}, i = 1, 2.$$

If at least one of the two extreme values,  $\mu_1$  and  $\mu_2$ , is positive, then there exists only one root with negative real parts. Note that  $Z > 0$  and thus, at least one extreme value is positive.

2.  $\rho'(C) < 0$ . Under  $\tilde{\rho}' > f'/f$  in Eq. (A6), then  $G < 0$  and the same reasoning in the above applies. Therefore, there exists only one root with negative real parts.

**Appendix C. Optimal taxation**

Let  $\tau_k(t)$  and  $\tau_c(t)$  be the time  $t$  tax rate on capital income and consumption, respectively, and  $S(t)$  be the transfer to a representative agent at time  $t$ . The representative agent's budget constraint is

$$\dot{k} = (1 - \tau_k)f(k(t)) - (1 + \tau_c)c(t) + S(t). \tag{C1}$$

The government maintains a balanced budget as follows.

$$\tau_k f(k(t)) + \tau_c c(t) = S(t). \tag{C2}$$

The necessary conditions of the agent's problem, together  $c^{**} = C^{**}$ , are

$$(1 + \tau_c)\lambda_k^{**} = u'(C^{**}), \tag{C3}$$

$$\dot{\lambda}_k^{**} = \lambda_k^{**} \left[ -(1 - \tau_k)f'(k^{**}) + \rho(C^{**}) \right], \tag{C4}$$

with

$$\lim_{t \rightarrow \infty} X^{**}(t)\lambda_k^{**}(t)k^{**}(t) = 0.$$

Allocation in a decentralized equilibrium is determined by Eqs. (3) and (C3)–(C4). Replication of the equilibrium involves setting these tax rates so that  $k^{**}(t) = \tilde{k}(t)$ ,  $C^{**}(t) = \tilde{C}(t)$ , and  $\lambda_k^{**}(t) = \tilde{\lambda}_k(t)$ . Differentiating Eq. (C3), with the use of Eq. (C4), yields

$$\dot{C}^{**} = \sigma^{**}(t) \left[ (1 - \tau_k)f'(k^{**}(t)) - \frac{\dot{\tau}_c}{1 + \tau_c} - \rho(C^{**}(t)) \right], \tag{C5}$$

where  $\sigma^{**}(t) > 0$  is as defined in Eq. (6a). The growth rate of consumption in a centrally planned economy is in Eq. (6b). In the

replicated equilibrium,  $C^{**}(t) = \tilde{C}(t)$ ,  $k^{**}(t) = \tilde{k}(t)$ ,  $\lambda_k^{**}(t) = \tilde{\lambda}_k(t)$  for all  $t$ . This implies  $\dot{C}^{**}(t) = \dot{\tilde{C}}(t)$  in Eq. (C5) and (6b), which leads to

$$\sigma^{**}(t) \left[ \tau_k(t)f'(k^{**}(t)) + \frac{\dot{\tau}_c}{1 + \tau_c} \right] = \Lambda(t) \equiv \sigma^{**}(t) \left[ f'(k^{**}) - \rho(C^{**}) \right] - \tilde{\sigma}(t) \left[ f'(\tilde{k}) - \rho(\tilde{C}) \right] + \tilde{B}. \tag{C6}$$

The optimal tax rate at time  $t$  is thus the form in Eq. (8), rewritten as follows.

$$\tau_c(t) = -1 + \exp \left\{ \int_0^t \frac{\Lambda(s)}{\sigma^{**}(s)} ds \right\}, \text{ or } \tau_k(t) = \frac{\Lambda(t)}{\sigma^{**}(t)f'(k^*(t))}.$$

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