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Inflation and Growth: Impatience and a Qualitative Equivalence

This paper studies the role of an endogenous time preference on the relationship between inflation and growth in the long run in both the money-in-utility-function (MIUF) and transactions-costs (TC) models. We establish a qualitative equivalence between the two models in a setup without a labor–leisure tradeoff. When the time preference is decreasing (or increasing) in consumption and real balances, both the MIUF and TC models are qualitatively equivalent in terms of predicting a negative (or positive) relationship between inflation and growth in a steady state. Both a decreasing and an increasing time preference in consumption are consistent with the arguments found within the literature. While a decreasing time preference in real balances corroborates with empirical evidence, there is no evidence in support of an increasing time preference in real balances.

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THE RELATION BETWEEN inflation and growth has been one of the central issues in macroeconomics since the work first performed by Tobin (1965) and Sidrauski (1967). Different approaches of introducing money yield incompatible predictions concerning the effect of anticipated inflation on capital accumulation. Competing approaches include the money-in-utility-function (MIUF) model,

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the transactions-costs (TC) approach, and the cash-in-advance (CIA) approach.[1] The MIUF and TC approaches were first compared by Dornbush and Frenkel (1973), and their equivalence was first indicated by an example in Brock (1974).[2] Feenstra (1986) established a *functional* equivalence between the MIUF and TC approaches.[3] Since then, numerous developments in comparisons of these competing approaches have come about.

Among these developments, there are two successful advancements that established a *qualitative* equivalence between alternative approaches. First, Wang and Yip (1992) departed from Feenstra and used the utility employed in Brock (1974) with direct utility interactions between leisure and real balances. A qualitative equivalence was created when higher monetary growth leads to lower capital and higher leisure in a steady state. The required conditions were weaker than those necessary for a functional equivalence. Zhang (2000) also obtained a *qualitative* equivalence between inflation and growth, but unlike Wang and Yip (1992), the equivalence was among different cases in the TC approach. With labor–leisure trade-offs, Zhang (2000) considered a general TC function and assumed that money holdings reduce the transaction costs of consumption, production, or investment. He obtained comparative-static results similar to Wang and Yip (1992) when assuming a dominant consumption effect.

These existing studies represented preferences with labor–leisure trade-offs and a functional in which a utility function is discounted by constant time preferences. The specification is attractive because it is analytically tractable and easily describes how tastes and opportunities interact to determine an economy's paths of consumption and capital accumulation. However, its rigid structure severely limits the conclusions and explanatory power of the corresponding models. Their preference implies that the marginal rate of substitution between time t_1 and t_2 is independent of consumption at any time $t \neq t_1, t_2$. As a result, when a labor–leisure trade-off is not possible, this type of model implies the neutrality of money in the long run. This paper considers a generalized class of preferences that has an attractive feature in which the rate of time preference is endogenous. This class of preferences allows the demands of consumption and real balances at any time $t \geq t_1$ to affect the marginal rate of substitution between time t_1 and $t_2 > t_1$. Thus, if there is a shock affecting consumption and real balances, the marginal rate of substitution between now and future is influenced. As a result, even if a labor–leisure trade-off is not possible, capital accumulation and growth may be changed. In this paper, we study the economic implications on the

1. While Tobin (1965) used the TC approach in a nonoptimized model and obtained a positive relationship between inflation and growth as a result of portfolio shift from non-interest-bearing real balances to capital, Sidrauski (1967) employed the MIUF approach with individuals' rational choices in an infinite horizon model and found growth independent of money growth or inflation, known as the super-neutrality of money.

2. Dornbush and Frenkel (1973) used a nonoptimized model with inelastic labor, whereas Brock (1974) employed an individual optimized, infinite-horizon model with leisure and thus, elastic labor.

3. In a model without capital and labor–leisure choice, Feenstra showed a duality between the two approaches by redefining choice variables using then unpopular utility specifications.

relationship between inflation and growth in the long run when a class of preference with endogenous time preferences is considered.

Specifically, in the present model, we study the non-superneutrality of money in an otherwise standard optimal growth model. The departure here is to take account of an endogenous time preference. Labor supply is perfectly inelastic as is usually assumed in optimal growth models. We choose the setup of inelastic labor not because it is more realistic. Rather, existing studies by Brock (1974) and Wang and Yip (1992) have relied on elastic labor in order to establish non-superneutrality. Our setup highlights the significance of an endogenous time preference in establishing the non-superneutrality result without relying on leisure–labor trade-offs.

An endogenous time preference has been stressed at least as early as Böhm-Bawerk (1989). Later, Fisher (1930, p. 61–94) observed the changes in the rate of time preference over time as consumption, income, risks, and personal factors change. Koompmans (1960) elaborated endogenous impatience in a model class with a recursive utility. In a neoclassical growth framework, endogenous impatience was first formalized by Uzawa (1968), followed by Wan (1970). Endogenous impatience has since then been extensively used in optimal growth models (e.g., Lucas and Stokey 1984, Epstein 1987, Obstfeld 1990).

In a standard optimal growth model, steady-state capital is determined by the commodity clearance condition and the Keynes–Ramsey rule. In both the MIUF and TC models, a higher money supply reduces real balances because of higher inflation and thus, a higher opportunity cost of holding money. In our study, the effect of anticipated inflation on real balances is transmitted to capital via endogenous impatience. Money is not superneutral in a steady state because of an endogenous response of a time preference. We establish a qualitative equivalence between the MIUF and TC approaches, under not only a positive but also a negative relationship between inflation and growth. A positive relationship, a Tobin effect, emerges when the time preference is *increasing* in both consumption and money, while a negative relationship, a reverse Tobin effect, arises when the time preference is *decreasing* in both consumption and money. As a result, the long-run relationship between inflation and growth depends crucially on the response of a time preference to consumption and money. There is evidence consistent with a time preference both decreasing and increasing in consumption. However, there is only evidence in support of a time preference decreasing in real balances, as was partly evidenced by Becker and Mulligan (1997). Under plausible decreasing impatience, we thus establish a qualitative equivalence between the MIUF and TC approaches in line with that in Wang and Yip (1992) and Zhang (2000). Different from these two existing studies, our equivalence result relies on neither a labor–leisure trade-off nor a dominant consumption effect.

Finally, this paper is not the first attempt to analyze the implication of an endogenous time preference on the non-superneutrality. Earlier, Epstein and Hynes (1983, section V) argued that in an MIUF model because of the substitutability between real balances and consumption, inflation increases steady-state capital under endogenous time preferences. Thus, only the Tobin effect is observed in Epstein and Hynes. Our model renders the result in Epstein and Hynes as a special case when real balances increase impatience. We predict a negative relationship when real balances decrease impatience. Moreover, we establish a qualitative equivalence between the MIUF and TC models, an unaddressed issue in Epstein and Hynes (1983).

The remainder of this paper is organized as follows. Section 1 analyzes the MIUF model. Section 2 studies the TC model and then establishes a qualitative equivalence between the MIUF and TC models. Finally, some concluding remarks are offered in Section 3.

1. AN MIUF MODEL

We consider an extension of the standard one-sector optimal growth model with inelastic labor supply. The agent seeks to maximize the following discounted sum of lifetime felicities:

$$U = \int_0^\infty u(c(t), m(t))X(t)\,dt, \tag{1}$$

in which u is the felicity function, c and m are individual consumption and real balances, and X is the cumulative discount at time t. Following Sidrauski (1967), money directly enters the felicity, on the argument that this represents a reduced form equation in a world of transaction costs. By facilitating transactions, money yields a direct utility to the agent that is not associated with other assets such as capital, which then further yields an indirect utility through the income they generate and the consumption goods they enable the agent to purchase.

Denote $\rho(\cdot,\cdot)$, the discounting function. The cumulative discount changes as follows:

$$\dot{X} = -\rho(c(t), m(t))X(t), \text{ with } X(0) \text{ given.} \tag{2}$$

ASSUMPTION 1:

(i) $u_c(c, m) > 0 > u_{cc}(c, m)$, $u_m(c, m) > 0 > u_{mm}(c, m)$, *for any* $c > 0$ *and* $m > 0$;

(ii) $u_{cc}/u - \rho_{cc}/\rho < 0$, *for any* $c > 0$ and $m > 0$.

Assumption 1(i) postulates a positive and decreasing marginal utility of consumption; the same assumption is made for real balances following existing studies. Assumption 1(ii) requires the curvature of felicity larger than that of discount rate with respect to consumption. This assumption is necessary for a positive intertemporal elasticity of substitution.

A correlation between consumption and time preferences is well postulated, yet there is a considerable disagreement over whether impatience should increase or decrease as consumption rises. Fisher (1930) proposed that a person's impatience decreases as the economy develops. Koompmans (1960) made arguments in favor of decreasing impatience. Authors like Blanchard and Fischer (1989) found it counterintuitive that people would be more impatient as their level of consumption rises.

Alternatively, Epstein (1987), who surveys the debates on this issue, offers several counterarguments and argues that the proper interpretation of a discount rate is that individuals who know they will have a large level of consumption in the future evaluate current consumption more highly. Lucas and Stokey (1984) point out that increasing impatience is often needed to produce unique, stable and nondegenerate steady-state wealth distributions. Authors like Obstfeld (1990) follow the assumption of increasing impatience in order to assure stability. In this paper, we allow for the possibilities of both $\rho_c > 0$ and $\rho_c < 0$. Empirical evidence is also mixed in support of either one of the two relationships.[4]

The correlation between real cash holdings and time preference is justified as follows. In the formulation with consumption and money in utility, the rate of time preference at time t is the Volterra derivative of the present value (in utility terms) due to an upward perturbation of the consumption path at time equal and larger than t, according to Epstein and Hynes (1983) and Epstein (1987). Applying this method, Obstfeld (1990) showed that the rate of time preference depends on consumption and the shadow price of the cumulated discount. The shadow price at time t represents the discounted present value of the future felicities at and after time t. When both consumption and money appear in a felicity, both arguments are in relation to the rate of time preference through the shadow price. As a result, money is in relation to time preferences. While there is little evidence about a relation between money and time preferences, Becker and Mulligan (1997) have used data in the Panel Study of Income Dynamics and uncovered a positive relationship. As money accounts for a fraction of wealth, their result may be viewed as supportive of the positive relationship between money and time preferences, namely, $\rho_m < 0$. Although such a relation is what will be focused on, in the analysis that follows we do not rule out the possibility of a positive relationship between money and impatience, exemplified by $\rho_m > 0$.

The production technology is $y = f(k)$, where y is output per capita and k is capital per capita, with $k(0)$ given initially, and, for simplicity, is assumed not to depreciate.

Assumption 2:

(i) $f_k(k) > 0 > f_{kk}(k)$, $f(0) = 0$, $\lim_{k\to 0} f_k(k) = \infty$, and $\lim_{k\to\infty} f_k(k) = 0$;
(ii) $f_{kk}/f_k < \rho_c$, *for all k and c.*

While a concave technology in Assumption 2(i) is standard, 2(ii) is a technical condition that is necessary in order to satisfy the Correspondence Principle (Samuelson 1948).[5]

4. Using household cross-section data based upon the Panel Study of Income Dynamics, Lawrence (1991) uncovered the time preference rate of the poor is 3%–5% points higher than those of the rich. Using the post-war annual time series data from Japan and Taiwan, Ogawa (1993) found the time preference rates were declining up to a certain point and then increasing as the two economies grew. See also evidence cited in Becker and Mulligan (1997) concerning the hypothesis that (i) patience varies across individuals and (ii) wealth causes patience. However, using household-level panel data from India, Ogaki and Atkeson (1997) found constant time preference rates when the intertemporal elasticity of substitution was relaxed to vary.

5. This latter condition is thus a variant of the Brock–Gale condition that requires the increase in the discount rate to dominate the increase in the marginal product of capital in the steady-state equilibrium.

Nominal money supply is assumed to grow at a constant rate μ with the amount of nominal money supply given initially. The real transfer from the government, ν, is financed by seigniorage, so $\nu = \mu m$. The budget constraint of the representative agent is $\dot{a} = f(k) - \pi m + \nu - c$, where $a = k + m$ is the agent's total wealth and π is the rate of inflation.

In equilibrium, the money and goods markets are clear: $\dot{m} = (\mu - \pi)m$ and $\dot{k} = f(k) - c$. We have derived the representative agent's optimality conditions and the dynamic equilibrium system.[6] In a steady state, $\dot{c} = \dot{m} = \dot{k} = 0$ and thus $\pi^* = \mu$.[7] The conditions in a steady state are

$$f_k(k^*) = \rho(c^*, m^*), \tag{3a}$$

$$f(k^*) = c^*, \tag{3b}$$

$$f_k(k^*) + \mu = \left(u_m^*/u^* - \rho_m^*/\rho^*\right)/\left(u_c^*/u^* - \rho_c^*/\rho^*\right). \tag{3c}$$

To investigate the relationship between money and growth in the long run, we start with the special case when real balances do not affect the degree of impatience, followed by the general case when real balances affect the degree of impatience.

Consider the special case when real balances do not affect the degree of impatience, $\rho_m = 0$. The Keynes–Ramsey rule equation (3a), referred to as the KR rule, and the goods market clearance condition (3b), referred as the CC condition, simultaneously determine the unique level of capital and consumption. Specifically, while the CC condition is positively sloping in the (k, c) plan, the KR rule may be negative or positive sloping depending on $\rho_c \geq 0$ and $\rho_c \leq 0$ (Figures 1 and 2). Under $\rho_c \leq 0$, the KR rule must be steeper than the CC condition in order to satisfy the Samuelson Correspondence Principle.[8] The relative slopes of the two loci imply the requirement of $f_{kk} - \rho_c\, \rho < 0$ in Assumption 2(ii). Thus, under $f_{kk} - \rho_c \rho < 0$, there exists a unique steady state.

In both conditions of $\rho_c \geq 0$ and $\rho_c \leq 0$, consumption and capital are completely determined by (3a) and (3b) independent real balances (E_0 in Figures 1 and 2). Then, the growth rate of nominal money, μ, exerts effects on real balances completely determined by equation (3c). In this economy, nominal money is thus superneutral in the long run even though real balances interact in a utility term with consumption and consumption affects time preferences. Indeed, as we have shown, this special superneutrality feature is shared with a CIA constraint on consumption.

6. The algebra is available at the first author's website.

7. An asterisk is used to denote a steady-state value.

8. Under $\rho_c \leq 0$, should the KR rule be flatter than the CC condition, then a lower productivity shock would have led to an increase, rather than a decrease, in both capital and consumption in a steady state.

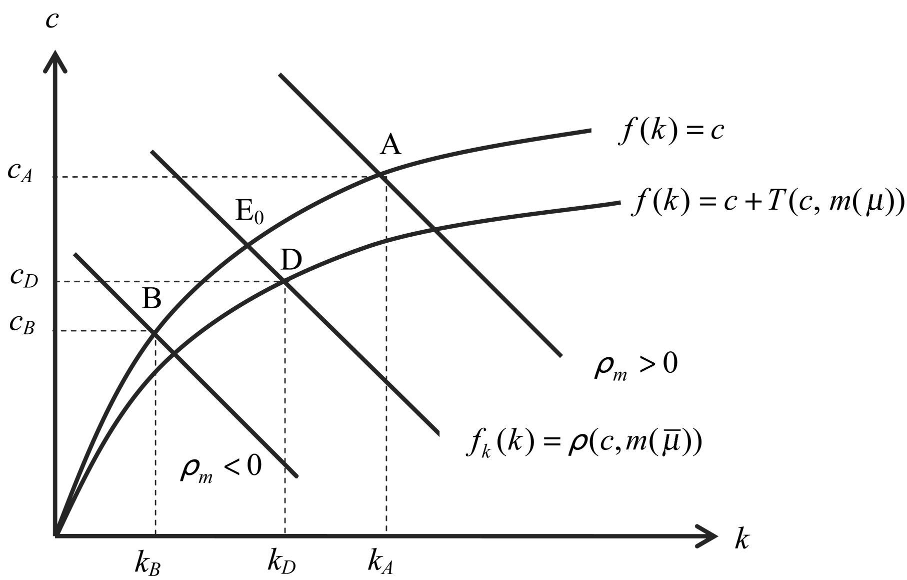

FIG. 1. A Higher μ and Thus $dm < 0$ in the MIUF and TC Models: Case $\rho_c \geq 0$.

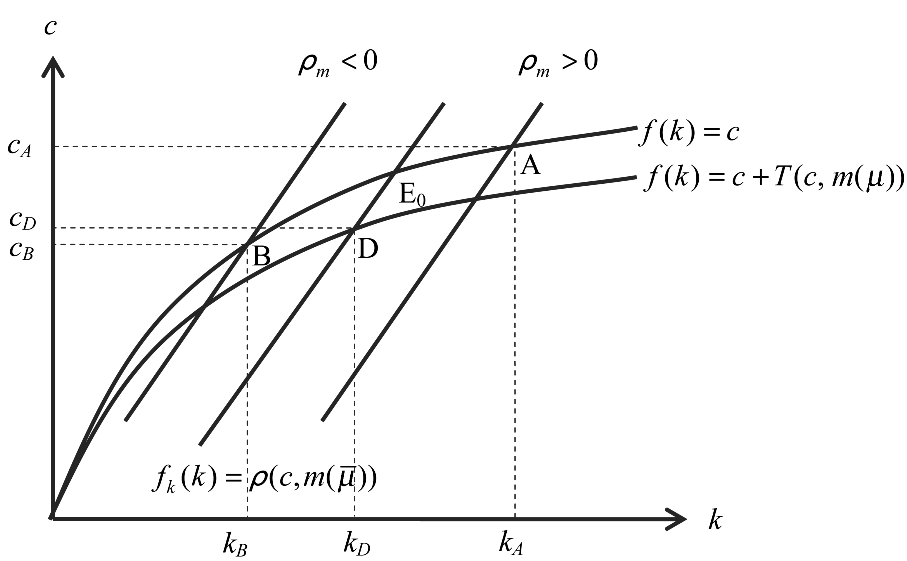

FIG. 2. A Higher μ and Thus $dm < 0$ in the MIUF and TC Models: Case $\rho_c \leq 0$.

Next, consider the general case when real balances affect the degree of impatience, i.e., $\rho_m \neq 0$. In this case, it may be $\rho_m > 0$ and $\rho_m < 0$. If we differentiate equations (3a) and (3b), we obtain

$$(f_{kk} - \rho_c \rho)\, dk = \rho_m \, dm, \tag{4a}$$

where $f_{kk} - \rho_c \rho < 0$ according to Assumption 2.

Next, differentiating equation (3c), together (3b), yields

$$\begin{aligned} &\rho\{f_{kk}\theta/\rho + J_{21}/m + J_{24}\theta/\rho m\}\, dk \\ &\quad + \{J_{22}/m + \theta(\rho+\mu)J_{24}/\rho m\}\, dm = -d\mu, \end{aligned} \tag{4b}$$

where$J_{21} = -[u_{cm} - \lambda\rho_{cm} - (\rho+\mu)(u_{cc} - \lambda\rho_{cc})]m^*/(u_c - \lambda\rho_c)$, $J_{22} = -[u_{mm} - \lambda\rho_{mm} - (\rho+\mu)(u_{cm} - \lambda\rho_{cm})]m^*/(u_c - \lambda\rho_c)$, $J_{24} = -[-\rho_m + (\rho+\mu)\rho_c]m^*/(u_c - \lambda\rho_c)$.

Together equations (4a) and (4b), we obtain

$$dm/d\mu = -(f_{kk} - \rho_c\rho)/\Lambda < 0, \tag{5a}$$

where $\Lambda = -\Delta(u_{cc} - \lambda\rho_{cc})/\theta\rho m < 0$ and $\Delta < 0$ when the steady state is a saddle.

It follows from equation (5a) that an increase in the growth rate of money unambiguously reduces the holdings of real balances. A lower level of real balances then affects the holding of capital and consumption in a steady state through equations (4a) and (4b).

Substituting equations (5a) into (4a) yields

$$dk/d\mu = \rho_m/(f_{kk} - \rho_c\rho) * dm/d\mu = -\rho_m/\Lambda \gtreqless 0 \quad \text{if} \quad \rho_m \gtreqless 0. \tag{5b}$$

The effect on capital depends on how real balances affect impatience. When the degree of impatience is increasing in real balances, capital holdings are higher in a steady state (point A in Figures 1 and 2). This is the Tobin effect, a result predicted by Epstein and Hynes (1983) in the case with an endogenous time preference. Alternatively, if the degree of impatience is decreasing in real balances, capital holdings are lower in a steady state (point B in Figures 1 and 2).

Similarly, if we substitute equation (5b) into equation (4b), a higher growth rate of money affects the steady-state consumption in the same direction as that of capital:

$$dc/d\mu = -\rho_m f_k(k)/\Lambda = sign\{dk/d\mu\}. \tag{5c}$$

Intuitively, a higher growth rate of money leads to lower real balances. If real balances increase the degree of impatience, an individual is more patient as he has less real balances. The agent tends to consume less and saves more. It follows that capital stock is higher and output and consumption are higher in a steady state. Alternatively, if real balances decrease impatience, the agent becomes less patient as real balances decrease. He then consumes more and saves less. Consequently, capital stock, output, and consumption are lower in a steady state.

Using equations (5a) and (5b), we have shown that the wealth is increasing (decreasing) in the long run if $\rho_m \geq (\leq) - (f_{kk} - \rho_c \rho) > 0$. If lower real balances increase impatience, real balances and capital both are lower in a steady state. As a result, wealth decreases in a steady state. However, if $\rho_m > 0$ and is sufficiently large, lower real balances increase patience and the agent saves so much that the increase in capital dominates the decrease in real balances. Wealth increases in a steady state.

Finally, the utility in a steady state is $U = u(c^*, m^*)/\rho(c^*, m^*)$. Lower real balances reduce the felicity but may decrease or increase the discount rate, thus an ambiguous welfare effect. Moreover, consumption may be higher or lower, making the welfare effect even more ambiguous. The net effect is positive (or negative) if $\rho_m \geq (\leq) (1 + \mu/\rho)(\rho_c \rho - f_{kk}) > 0$. If $\rho_m < 0$, the level of welfare must be lower because of a lower consumption level and a higher discount rate resulting from lower real balances. Alternatively, if $\rho_m > 0$, the effect on the level of welfare is ambiguous. In the situation with a sufficiently large ρ_m, consumption increases so much that the welfare is higher.

2. TRANSACTION COSTS MODEL

In our examination of the TC model, following Saving (1971) and Wang and Yip (1992), we assume that only consumption transactions are costly and money holdings facilitate transactions. Specifically, the transaction costs are assumed to take the form: $T(t) = T(c(t), m(t))$, where $T_c > 0, T_{cc} > 0, T_m < 0 < T_{mm}, T(0, m) = 0$ and $T_{cm} \leq 0$. Thus, the transaction costs are positive if there is positive consumption. Moreover, the transaction costs function is convex in consumption and decreasing in real balances in a diminishing way. Finally, assumption $T_{cm} \leq 0$ indicates that higher real balances tend to lower the marginal transaction costs of consumption.

With transaction costs, the representative agent's budget constraint in equation (3) now becomes

$$\dot{a} = f(k) - c - \pi m + \nu - T(c, m). \tag{6}$$

The discounted lifetime utility is given by equation (1), but the felicity $u(c, m)$ in equation (1) is replaced by $u(c)$ and the discount rate $\rho(c, m)$ in equation (2) by $\rho(c)$. The Volterra derivative indicates that ρ is a function of c and the shadow price of the cumulative discount. The shadow price of the cumulative discount at time t in turn is the present value of the future discounted felicity at and after time t, which is a function of $c(t)$. As a result, we replace $\rho(c(t), m(t))$ in equation (2) by $\rho(c(t))$.

In equilibrium, the money market condition is unchanged, but the goods market condition is now $\dot{k} = f(k) - c - T(c, m)$. We have derived the optimization problem and simplified the equilibrium dynamic system. In a steady state, $\dot{c} = \dot{m} = \dot{k} = 0$. The steady-state conditions are

$$f_k(k^*) = \rho(c^*), \tag{7a}$$

$$f(k^*) = c^* + T(c^*, m^*), \tag{7b}$$

$$f_k(k^*) + \mu = -T_m. \tag{7c}$$

To analyze the relationship between inflation and growth, the KR rule (equation (7a)) and the CC condition (equation (7b)) in the (k, c) plan are illustrated in Figures 1 and 2 with the steady state at E_0. When $\rho_c \leq 0$, the Correspondence Principle requires $f_{kk}(1 + T_c) - \rho\rho_c < 0$, which is assured under Assumption 2 and $T_c > 0$. Both loci together yield $\{f_{kk} - [\rho_c\rho/(1 + T_c)]\}\, dk = -[\rho_c T_m/(1 + T_c)]\, dm$. Similar to our MIUF model in Section 1, changes in capital depend on changes in real balances, but the effect is now via reducing transaction costs $T_m < 0$. If the time preference is independent of consumption, $\rho_c = 0$, then even if real balances reduce transaction costs, they only affect consumption in equation (7b) without spreading out the effect to equation (7a), thereby exerting no effect on capital. Capital is solely determined by equation (7a) in this special case. As a result, money is superneutral in a steady state. This is the result shared in the TC models in Wang and Yip (1992) and Zhang (2000) when their labor supply is inelastic. However, if consumption affects time preferences, then even if labor supply is inelastic, real balances remain exerting effects on capital.

The negative effect of a higher growth rate of money on real balances is easily obtained by differentiating equation (7c). Real balances are decreasing even if $\rho_c = 0$. As real balances decrease, the CC condition shifts downward (Figures 1 and 2). Obviously, even if $\rho_c = 0$, consumption unambiguously decreases in a steady state because of higher transaction costs. However, the effect on capital depends upon how consumption affects the degree of impatience. In Figure 1 where $\rho_c \geq 0$, capital increases (see point D), in contrast, in Figure 2 where $\rho_c \leq 0$, capital decreases.

Intuitively, lower real balances increase the transaction costs and thus discourage consumption. In an economy where an agent is *more patient* as he consumes more, $\rho_c < 0$, lower consumption makes him more patient so he saves more. As a result, capital is higher in a steady state. Alternatively, when an individual is *less patient* as he consumes more, $\rho_c > 0$, lower consumption makes him less patient. Thus, savings and capital are lower in a steady state.

The wealth in a steady state is increasing (or decreasing) if $\rho_c T_m \leq (\geq) f_{kk}(1 + T_c) - \rho_c\rho < 0$. In the case where $\rho_c \leq 0$, as then $\rho_c T_m > 0$, real balances and capital both decrease and thus the wealth decreases (in Figure 2). Alternatively, when $\rho_c \geq 0$, as $\rho_c T_m < 0$, the effect on wealth is ambiguous because real balances decrease while capital increases (Figure 1). It is possible that wealth increases. This situation emerges when $\rho_c > f_{kk}(1 + T_c)/(\rho + T_m) > 0$. Under such a condition, the agent becomes sufficiently patient so an increase in capital is more than a decrease in real balances. Finally, the lifetime utility in a steady state is always lower as consumption is lower.

TABLE 1

COMPARATIVE-STATIC RESULTS OF A HIGHER GROWTH RATE OF MONEY

		m	K	a	C	U
MIUF	$\rho_m > 0$	−	+	+[a]	+	+[b]
	$\rho_m < 0$	−	−	−	−	−
TC	$\rho_c > 0$	−	+	+[c]	−	−
	$\rho_c < 0$	−	−	−	−	−

[a] $\rho_m > \rho\rho_c - f_{kk} > 0$.
[b] $\rho_m > (1 + \mu/\rho)(\rho\rho_c - f_{kk}) > 0$.
[c] $\rho_c > f_{kk}(1 + T_c)/(\rho + T_m) > 0$.

We now briefly compare the MIUF and TC models. Table 1 conveniently summarizes the comparative-static results for the two models with an endogenous time preference.

If a time preference is affected by real balances in the MIUF model, money is not superneutral even without labor–leisure trade-offs. The non-superneutrality result is in line with Brock (1974) and his followers in models that rely on labor–leisure trade-offs. The result differs from the superneutrality in Sidrauski (1967) in models with an exogenous time preference. Our results reveal that a higher growth rate of money, and thus higher inflation, reduces capital, wealth, consumption, and welfare in the long run in the case where $\rho_m < 0$, but increases capital and wealth in the case where $\rho_m > 0$. The positive relationship between inflation and capital in the latter case features a Tobin effect. This positive relationship is what has been argued and emphasized by Epstein and Hynes (1983) in the context of an endogenous time preference. The result in Epstein and Hynes, however, is one of the cases here that emerges only if real balances increase impatience. If real balances decrease impatience, the relationship between inflation and capital is negative in a steady state. Existing empirical evidence is in support of this case (Becker and Mulligan 1997), but Epstein and Hynes (1983) neglected this result.

In the special case where $\rho_m = 0$, real balances do not affect a time preference. In this case, we obtain the money superneutrality even when consumption affects a time preference. In a MIUF model with an endogenous time preference, Asako (1983) and Hayakawa (1995) obtained the superneutrality only under a utility that is separable in real balances and consumption. In contrast, our MIUF model obtains the superneutrality under a general utility function that allows for substitutability between real balances and consumption.

For the TC model with an endogenous time preference, results in Table 1 indicate that a higher growth rate of money, and thus higher inflation, unambiguously decreases real balances and consumption, and thus wealth, because of the transaction costs of consumption. This result is in line with the prediction in existing works with an exogenous time preference by Wang and Yip (1992) and Zhang (2000). Like these existing studies, money supply is not superneutral. However, depending on the response of time preferences to consumption, the relationship between inflation and capital here may be negative or positive in a steady state. The relationship is negative only when the degree of impatience is decreasing in consumption, $\rho_c < 0$. When $\rho_c > 0$ and the degree of impatience is sufficiently increasing in consumption, such a relationship is positive. Therefore, different from existing studies with an exogenous time preference, there is possibly a Tobin effect in the TC model with an endogenous time preference.

Following Wang and Yip (1992) and Zhang (2000), we now establish a qualitative equivalence between the MIUF and TC models. It is impossible for higher money supplies to increase all capital, wealth, consumption, and welfare. Alternatively, if we only focus on the effect on the reallocation of assets, it is possible to identify a set of conditions so the Tobin effect emerges in both models. Indeed, higher money supplies increase capital and wealth in both the MIUF and TC models when $\rho_m > 0$ and $\rho_c > 0$ hold and their magnitudes are sufficiently large. In Wang and Yip (1992, table 1, p. 555) and Zhang (2000, table 2, p. 10), there are no unified parameter restrictions so the Tobin effect emerges in their MIUF and TC models.

There is a qualitative equivalence in terms of a reversed Tobin effect between the MIUF and TC models. When both conditions $\rho_m < 0$ and $\rho_c < 0$ hold, higher money supplies decrease capital, wealth, consumption, and welfare in both the MIUF and TC models. In Wang and Yip (1992, table 1), a dominant consumption effect over a real balance effect is required in order to establish a qualitative equivalence between the MIUF and TC models. A similar condition is also required to create a qualitative equivalence between the different TC models in Zhang (2000, table 2). Moreover, labor–leisure trade-offs are required in these two existing studies. In our model, there is the requirement of neither a dominant consumption effect nor labor–leisure trade-off. Our equivalence result is established under endogenous time preferences.

Finally, we should mention the plausibility of the conditions for a qualitative equivalence in terms of the relationship between inflation and growth where $\rho_c > 0$ and $\rho_m > 0$ for a positive relationship and $\rho_c < 0$ and $\rho_m < 0$ for a negative relationship. In the former case, the required condition $\rho_c > 0$ is consistent with Uzawa (1968), Lucas and Stokey (1984), and Obstfeld (1990). However, we find no existing evidence in support of the condition $\rho_m > 0$. Thus, a Tobin effect is less plausible. In the latter case, condition $\rho_c < 0$ is consistent with that proposed by Fisher (1930), Koompmans (1960), Blanchard and Fischer (1989). Moreover, condition $\rho_m < 0$ is consistent with the evidence in Becker and Mulligan (1997). In view of a support in favor of $\rho_m < 0$, it is more plausible that both the MIUF and TC models are qualitatively equivalent in terms of a negative relationship between inflation and growth in a steady state.

3. CONCLUDING REMARKS

This paper revisits the issue of the relationship between inflation and growth in the long run. Differing from the work by Brock (1974), Wang and Yip (1992), and Zhang (2000), we do not rely on a labor–leisure trade-off in order to establish the non-superneutrality of money. We focus on the role of an endogenous time preference upon the demand for assets between real balances and capital in an optimal growth model between the MIUF and TC approaches. Consideration of an endogenous time preference influences the marginal rate of substitution between consumption now and in the future and thus changes capital accumulation. As a result, we find that a qualitative equivalence between the MIUF and TC models is easy to establish without relying on labor–leisure trade-offs and a dominant consumption effect. Our results are in sharp contrast to those obtained in existing models with an exogenous time preference. In particular, even in the absence of elastic labor, an endogenous time preference easily spreads the effect of real balances over to the optimal demand for capital and thus exerts an effect on capital in a steady state.

In these two models, a higher inflation always leads to lower real balances. However, the effect on capital and real assets depends on the degree of impatience in response to consumption and real balances. Under increasing impatience in consumption and in real balances, we find higher capital and wealth in association with a higher inflation in both the MIUF and TC models. As a result, the relationship between inflation and growth is positive in a steady state, as was argued in Tobin (1965). However, there is no evidence pointing to increasing impatience in real balances. Alternatively, under decreasing impatience in both consumption and real balances, a higher inflation reduces capital and wealth in both the MIUF and TC models, thus a reverse Tobin effect. Decreasing impatience in consumption is in line with Fisher (1930) and his followers, and decreasing impatience in real balances is also consistent with the evidence in Becker and Mulligan (1997). Therefore, in this plausible case the MIUF and TC models are qualitatively equivalent in terms of a negative relationship between inflation and growth in a steady state.

Finally, we only consider a time preference affected by individual consumption and individual real balances. Alternatively, time preferences may be affected by average consumption and real balances in an economy; thus, there are admiration and zealousy effects (e.g., Meng 2006). Moreover, the discount rate may be affected by either own past consumption (individual habits, e.g., Chen 2007) or average past consumption (social habits, e.g., Alvarez-Cuadrado et al. 2004); thus, there are the effects of catching up with the Joneses. It may be interesting to see how consideration of different ways of formulating endogenous time preferences may affect the agent's saving behavior and thus capital formation, an avenue for future research.

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