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Multiple BGPs in a Growth Model with Habit Persistence

This paper establishes multiple balanced growth paths in an otherwise standard, competitive growth model without externality or distortions and with households' preference dependent upon how his/her consumption compares to a habit stock formed by his/her own past consumption. The key feature in our model is that consumption forms habits in combination with existing habits. This model establishes multiple equilibria because habit persistence induces an internal, intertemporal complementarity effect among consumption flows, with current consumption reinforcing future consumption. As a result, there exist two balanced-growth paths, with one path exhibiting low consumption and habits and high economic growth, and the other path exhibiting high consumption and habits and low growth that is not necessarily a development trap. Both steady states are saddles, but global indeterminacy arises where a high balanced growth path co-exists with a low balanced growth path where the two equilibrium paths cannot be pareto ranked and history need not matter for the selection of the equilibrium path.

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Recently there has been resurgent interest in economic growth models with multiple steady states. This strand is motivated by several empirical studies that document the existence of multiple development clubs (e.g., Baumol and Wolff 1988, Quah 1996, Durlauf and Quah 1999). In accordance with the facts are models in economic development that display poverty traps, where economies with low initial capital stocks or incomes converge to a steady state with low per capita

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income, while economies with high initial capital stocks converge to a different steady state that corresponds to high per capita income.

The existing literature of economic development with multiple steady states may be broadly classified into five lines. One line concerns models of club convergency where countries differ in their initial level of human capital around different steady-state equilibria in the presence of social increasing returns to scale from human capital formation (e.g., Azariadis and Drazen 1990) and capital market imperfection (e.g., Galor and Zeira 1993). A second line is the introduction of heterogenous agents so that through income distribution, otherwise similar countries except for their initial distribution of income may cluster around different steady-state equilibria (e.g., Galor and Zeira 1993, Benabou 1996). A third line examines models of fertility choice and a tradeoff between the time allocated to bearing and rearing children and the time devoted to educating children, with the result that households find a large number of children in their best interests, leading to an equilibrium with low per capita income (e.g., Becker, Murphy, and Tamura 1990, Galor and Weil 1996). A fourth line is sectoral complementarities that may generate multiple steady-state equilibria due to aggregate demand spillovers (e.g., Krugman 1991, Chen, Mo, and Wang 2002).[1] Finally, a fifth line investigates costly intermediation in which a participation externality, where the cost of financial intermediation depends negatively on the mass of consumers, generates multiple steady states (e.g., Cooper and Ejarque 1995, Becsi, Wang, and Wynne 1999). Except for few works,[2] all these studies incorporate some forms of externality, which generate multiple long-run equilibria through an *external* complementarity effect among households/investors.

This paper develops a model to establish multiple steady states with global indeterminacy in a standard one-sector unbounded growth model with habit consumption and without relying on externality. The introduction of habit formation into the growth model is motivated as follows. Empirical supports of the influences of habits on consumption have been made since Houthakker and Taylor (1970). The idea of habits in preferences dates to Marshall (1898) and Duesenberry (1949). According to Duesenberry (1949), the source of variations in habit formation is a learning process which begins in childhood, and a permanent shock will not change consumption behavior too much in the short run, but over time an individual responds to it and reaches a new consumption pattern that becomes habitual in the same way as the original pattern.[3] Ryder and Heal (1973) and Boyer (1975) have investigated the implication of habit

1. A static version of the line of research was pioneered by Rosenstein-Rodan (1943) and first formalized by Murphy, Shleifer, and Vishny (1989).

2. These include Galor and Ryder (1989), Galor (1992), Galor and Weil (1996), and Shimomura (2004). In a one-sector model by Galor and Ryder (1989) and a two-sector model by Galor (1992), they obtain multiple steady states in overlapping-generations growth models, a framework where market participation is limited to those consumers alive as noted in Cass and Shell (1973). While the steady states in Galor and Weil (1996) are either locally a saddle or a sink but there is no global indeterminacy, all of the steady states in our model are locally a saddle but global indeterminacy arises. Finally, Shimomura (2004) obtained multiple steady states in a two-country trade model based upon negative income effects so the durable good is a Giffen good for a range of its shadow price.

3. The source of variations in habit formation "is difficult to describe in a short place because it is a genetic process which begins in childhood. . . . Suppose a man suffers a 50 per cent reduction in his income

formation in the neoclassical optimal growth model, showing that the models lead to richer dynamic behavior of consumption and other main variables around the steady state.[4]

Understanding consumption, and thus savings, has important bearings on aggregate activities and economic growth. Apparent data indicate excess smoothness of aggregate consumption (Campbell and Deaton 1989, Fuhrer 2000). Moreover, data also indicate that increases in economic growth cause subsequent increases in aggregate savings using both cross-country (Carroll and Weil 1994, Attainasio, Picci, and Scorcu 2000, Rodrik 2000) and case studies (Paxson 1996, Deaton and Paxon 2000, Rodrik 2000). The data are mainly confined to East Asian countries. These evidences favor the use of a utility with habits that exhibits consumption dynamics consistent with the above facts. Recently there have been several empirical studies that compared the model with habits in preference with the model without habits and rejected the latter.[5] These evidences motivate the use of a model with consumption habits in order to analyze the implications upon the long-run economic growth.

Specifically, our model generalizes the endogenous growth model of habits by Carroll, Overland, and Weil (2000) and of recursive preferences by Palivos, Wang, and Zhang (1997) by allowing both current consumption flows and the existing habitual stock to contribute to the increment of future habits. The generalization of habit formation is important because it enables rich dynamics. This is an issue that has not been thoroughly studied; while Carroll, Overland, and Weil focus on how habit persistence implies that higher growth can induce more savings, Palivos, Wang, and Zhang concentrate on recursive preferences of the form of Uzawa (1968).

Our general form of habit formation leads to a threshold ratio of consumption to habit resulting in two balanced growth paths (hereafter BGP). This model creates multiple BGPs for the following reasons. Existing habit enhances habit formation and affects preferences. This feature induces an *internal*, *intertemporal complementarity effect*, with current consumption reinforcing future consumption. High/low future consumption is expected to be associated with high/low current consumption, in order to attain a given level of utility for a household. Thus, for a given initial state, a small disturbance may only lead the economy to shift *locally* to a new growth path around the original BGP. However, it is also possible that a small disturbance may also change the threshold significantly so that the economy converges *globally* to a

and expects this reduction to be permanent. Immediately after the change he will tend to act in the same way as before . . . but he eventually will learn . . . and responds by buying cheap substitutes. . . . Eventually he will reach a new consumption pattern . . . is likely to become habitual in the same way as the original pattern" (Duesenberry 1949, p.24).

4. Using the optimal growth models Avearez-Cuadrado, Goncalo Monteiro, and Turnovsky (2004) studied economic dynamics while Alonso-Carrera, Caballe, and Raurich (2004) analyzed economic dynamics and optimal tax policies. The framework has been extended to an endogenous growth framework recently (Carroll, Overland, and Weil 2000, Alonso-Carrera, Caballe, and Raurich 2001). Recent years have seen applications in finance (Abel 1990, Constantinides 1990, Campbell and Cochrane 1999) and business cycles (e.g., Mansoorian 1996, Ljungqvist and Uhlig 2000, Boldrin, Christiano, and Fisher 2001).

5. See, for example, the studies using time-series data in G-7 countries by Further and Klein (1998), time-series data in Japan by Pagano (2004), and the household panel survey in Britain by Guariglia and Ross (2002).

new growth path around the other BGP. As one of the two BGPs has a higher long-run economic growth rate, such a small shock may lead to a large difference in the economic growth rate in the long run.

While local dynamics are determinate (saddle) in our model, global indeterminacy arises where there is a high balanced growth path co-existing with a low balanced growth path that is not necessarily a development trap. Here the two equilibrium paths cannot be pareto ranked and history need not matter for the selection of the path. These features differentiate the results of our model from the aforementioned studies with regard to two steady states. In some of these works, the steady states are both saddle points and there exists a threshold level such that an initial condition history determines the steady state to which an economy converges (e.g., Kurz 1968, Palivos 1995, Galor 1996, Galor and Weil 1996). In the remaining works, one of the steady states is a sink and the other is a saddle point, and *globally*, for most given initial conditions an economy converges to the steady state that is a sink (e.g., Krugman 1991, Chen and Shimomura 1998).

In this paper, the theoretical model will be presented in the next section, while Section 2 studies balanced-growth paths and transitional dynamics, and Section 3 investigates global indeterminacy. Concluding remarks are made in Section 4.

1. BASIC MODEL

Our basic model draws on Romer (1986), Palivos, Wang, and Zhang (1997), and Carroll, Overland, and Weil (2000). Consider an economy populated by households and firms. There exists a continuum of infinite-lived, identical households, with no population growth. There also exists a continuum of representative firms, with the shares owned by households. It follows that the economy is a world of representative household-producers.

1.1 Environment

The representative household is assumed to possess the following discounted, life-time utility

$$U = \int_0^\infty e^{-\rho t} u(c(t)/S^\gamma(t))\, dt, \quad \rho > 0, 0 < \gamma < 1, \tag{1a}$$

where u is the felicity function in t, $c(t)$ is the instantaneous private consumption flow in t, and $S(t)$ is the habitual stock. Parameter γ indexes the importance of habits. If $\gamma = 0$ then only the absolute level of consumption is important; while if $\gamma = 1$, then consumption relative to the habitual stock is what matters. For values of γ between 0 and 1, both the absolute and relative levels are important. For this study, we assume $0 < \gamma < 1$ so that absolute consumption level is not a consumer's concern. To facilitate the analysis, we have adopted a parametric felicity by assuming the following CES functional form

$$u(c(t)/S^{\gamma}(t)) = 1/(1-\sigma)[(c(t)/S^{\gamma}(t))^{1-\sigma} - 1], \quad \sigma > 1. \tag{1b}$$

Assumption $\sigma > 1$ is crucial for the results of multiple interior steady states. The presumption rules out the logarithmic felicity form, requiring one with curvature steeper than a logarithmic form. Restriction $\sigma > 1$ is consistent with most empirical findings that intertemporal elasticity of substitution is smaller than one.

The stock of habits is assumed to evolve according to

$$\dot{S} = Bc^{\mu}(t)S^{1-\mu}(t) - \delta_s S(t), \quad 1/2 < \mu \leq 1, S(0) > 0 \textit{ given}, \tag{2}$$

in which $B > 0$ represents a technology coefficient that forms past consumption flows into habits, with $\delta_s \geq 0$ describing how existing habitual stock depreciates. Thus, a current consumption flow generates a long-lasting effect in a manner summarized by the stock of habits.

equation (2) is more general than the specification in Carroll, Overland, and Weil (2000). Here we follow other authors, such as Campbell and Cochrane (1999) and Lettace and Uhlig (2000) to assume that consumption accumulates future habits in relation to existing habitual stocks. More specifically, we assume that current consumption flows contribute toward future habit formation, possibly in association with existing habits; therefore, restriction $\mu \leq 1$ is made,[6] and to be consistent with a perpetual growth framework the formation technology is assumed to be of constant returns with respect to existing habits and current consumption. It should be remarked that when $B = \delta_s = 0$, past consumption does not form the habitual stock. Then, the model is reduced to a conventional one-sector, endogenous growth model (e.g., Romer 1986).

The representative firm is assumed to own the following production technology

$$y(t) = Ak(t), \quad k(0) > 0 \textit{ given}, \tag{3}$$

where $y(t)$ is output, $k(t)$ is capital stock, and $A > 0$ is a parameter. The technology is abstracted from labor so capital should be interpreted broadly to include physical as well as human capital (e.g., Rebelo 1991).

Finally, as households own firms' shares, the representative household's budget constraint is

$$\dot{k} = y(t) - c(t) - \delta_k k(t), \tag{4}$$

where $\delta_k \geq 0$ is capital's depreciation rate. This equation says that disposable income, not consumed currently, becomes savings, which augments capital.

1.2 Optimization

The representative household's problem is to choose consumption, in order to maximize its discounted; lifetime utility (equations (1a and 1b)), subject to habit-

6. While habitual stock is $S(t) = \{[S(0)e^{-\delta_s t}]^{\mu} + B\mu\int_0^t [e^{-\delta_s(t-\tau)}c(\tau)]^{\mu}\, d\tau\}^{1/\mu}$ when $\mu \leq 1$, it is $S(t) = S(0)e^{-\delta_s t} + B\int_0^t e^{-\delta_s(t-\tau)}c(\tau)\, d\tau$ when $\mu = 1$.

ual stock formation (equation (2)); production technology (equation (3)); and budget constraints (equation (4)), taking existing capital stock $k(t)$ and habits $S(t)$ as predetermined. To solve the dynamic optimization problem, we define the following current-value Hamiltonian

$$H(c, k, S, \lambda_k \lambda_s) = 1/(1-\sigma)[(c/S^\gamma)^{1-\sigma} - 1] + \lambda_k[Ak - c - \delta_k k]$$
$$- \lambda_s[Bc^\mu S^{1-\mu} - \delta_s S],$$

where λ_k and λ_s denote the costate variables associated with equations (2) and (4), respectively. We should note that the shadow price of habit stock, $-\lambda_s$, is negative. The negative shadow price on habit stocks reflects the fact that a greater initial habit stock yields a lower utility flow for equation (1). Without abuse of terminology, in what follows we call the absolute value of the shadow price of habit stocks, λ_s, as the shadow habit price. Although under $\sigma > 1$ the felicity is strictly concave in $c(t)$, for $0 < \gamma \leq 1$ it is not concave in $S(t)$ as a higher existing habit stock lowers utility. Since felicity function u is not concave in c and S, the Mangasarian sufficient theorem cannot be used. Instead, we need to apply the Arrow sufficient theorem to guarantee the concavity of the Hamiltonain (see Arrow and Kurz 1970). Denote

$$\hat{H}(k, S, \lambda_k, \lambda_s) = \underset{\{c \in R^+\}}{\text{Max}} H(c, k, S, \lambda_k, \lambda_S).$$

We have shown in the Appendix that when $\mu = 1$, under a mild condition $\hat{H}(k, S, \lambda_k(t), \lambda_s(t))$ is concave in k and S for fixed values of λ_k, $\lambda_k t$.[7] For Case $\mu < 1$, as long as μ is large enough, by continuity a similar condition assures $\hat{H}(k, S, \lambda_k(t), \lambda_s(t))$ to be concave in k and S for fixed values of λ_k, λ_k, and t. Therefore, $k^*(t)$, $S^*(t)$, and $c^*(t)$ solve Problems (1)–(4). As $\hat{H}(k, S, \lambda_k(t), \lambda_s(t))$ is not strictly concave in k and S, therefore $k^*(t)$, $S^*(t)$, and $c^*(t)$ are not necessarily unique.

Denote ρ as the time-preference rate. Applying the Pontryagin maximum principle, we obtain the following first-order conditions

$$(c/S^\gamma)^{1-\sigma} 1/c = \lambda_k - \lambda_s(B\mu S^{1-\mu}/c^{1-\mu}), \tag{5a}$$

$$A - \delta_k = \rho - \dot{\lambda}_k/\lambda_k, \tag{5b}$$

$$-(c/S^\gamma)^{1-\sigma}[\gamma/(\lambda_s S)] + \delta_s - B(1-\mu)(c^\mu/S^\mu) = \dot{\lambda}_s/\lambda_s - \rho. \tag{5c}$$

together with equations (2) and (4), and the transversality condition[8]

$$\lim_{t\to\infty} e^{-\rho t} H(t) = 0. \tag{5d}$$

equation (5a) equates the marginal utility of current consumption to the marginal costs of foregone savings, net of the effect via habit formation. Conditions (5b)

7. The condition is $1/(\lambda\delta_S) + B/\delta_S > 1(1 - 1/\sigma)[(1-\gamma) + \gamma/\sigma]/[(1+\gamma)(\sigma-1)^2 + \gamma] + 1$.

8. On the transversality condition in an infinite horizon, optimal problems, see Michel (1982).

and (5c) are two Euler equations that equate the net marginal productivity of capital stock, as well as the marginal utility of habitual stock, respectively, net of depreciation, to the time-preference rate, net of their respective capital gains (or losses).

2. BALANCED GROWTH PATHS AND TRANSITIONAL DYNAMICS

We are now ready to analyze the competitive market equilibrium.

DEFINITION. A perfect-foresight equilibrium is a tuple $\{\dot{S}/S(t), y(t)/S(t), \dot{k}/k(t), c(t)/S(t), k(t)/S(t), \lambda_s(t)/\lambda_k(t)\}$ that satisfies

(i) habitual stock formation; i.e., equation (2);
(ii) production technology; i.e., equation (3);
(iii) households' budgets; i.e., equation (4);
(iv) representative household-firm optimization; i.e., equations (5a)–(5d).

To analyze the market equilibrium, we transform the economic system into a 3×3 system in three variables $\{x, \lambda, z\}$, where $x \equiv c/S$, $\lambda \equiv \lambda_s/\lambda_k$, and $z \equiv k/S$. By keeping the relative shadow price in the system, our 3×3 system does not involve the second-order time derivatives of consumption and is more simplified than the method utilized in Carroll, Overland, and Weil (2000). This solution method is in line with Bond, Wang, and Yip (1996). In order to derive the three-variable system, first we divide equation (5a) by equation (5c) to obtain

$$\dot{\lambda}_s/\lambda_s = \delta_s + \rho - \gamma(x/\lambda) - B[1 - \mu + \mu\gamma]x^{\mu}. \tag{6a}$$

Next, differentiating equation (5a) with respect to time yields

$$[\dot{\lambda}_k/\lambda_k + \gamma(1-\sigma)\dot{S}/S + \sigma\dot{c}/c] + B\mu(\lambda/x^{1-\mu})\{\dot{\lambda}_s/\lambda_s + [\gamma(1-\sigma) + (1-\mu)]\dot{S}/S - (1-\sigma-\mu)\dot{c}/c\} = 0,$$

which, together with equations (2), (5b), and (6a), can be rewritten as

$$\dot{c}/c = \{(A - \delta_k - \rho) + [\gamma(1 + (\sigma-1)/\mu)x/\lambda - (\rho + \mu\delta_s + \gamma(\sigma-1)\delta_s) + \gamma\mu B x^{\mu}]B\mu(\lambda/x^{1-\mu}) + \gamma(\sigma-1)(B^2\mu(\lambda/x^{1-2\mu}) - \delta_s)\}/\{\sigma + (\sigma+\mu-1)B\mu(\lambda/x^{1-\mu})\}, \tag{6b}$$

Notice that since $(A - \delta_k)$ is the marginal product of capital, expression $\dot{c}/c$ reduces to the formulation in existing one-sector, endogenous growth models when $B = \mu = 0$, with the long-run, intertemporal elasticity of substitution equal $1/\sigma$. With $B > 0$ and $\mu > 0$ for habit formation, equation (6b) suggests that in the long run, intertemporal elasticity of substitution is $[\sigma + (\sigma+\mu-1)B\mu(\lambda/x^{1-\mu})]^{-1}$, which for

$\sigma + \mu > 1$ is smaller than $1/\sigma$, even for the case $\mu = 1$.[9] Intuitively, current consumption forms habit stock that complements future consumption, thus lowering intertemporal elasticity of substitution for consumption. Therefore, we obtain

PROPOSITION 1. *An economy with habit formation with $B > 0$ and $\mu > 0$ has a lower, long-run, intertemporal elasticity of substitution for consumption than one without habit formation.*

Finally, the economic system cannot be analyzed without transforming the growing variables into great ratios. We take differences between equations (6b) and (2), equations (6a) and (5b), and equations (4) and (2) to obtain the following 3×3 economic system

$$\begin{aligned}\dot{x}/x = \big\{(A - \delta_k - \rho) + \big[\gamma(1 + (\sigma - 1)/\mu)x/\lambda \\ - (\rho + \mu\delta_s + \gamma(\sigma - 1)\delta_s) + \gamma\mu B x^{\mu}\big] B\mu(\lambda/x^{1-\mu}) \\ + \gamma(\sigma - 1)(B^2\mu(\lambda/x^{1-2\mu}) - \delta_s)\big\}/ \\ \{\sigma + (\sigma + \mu - 1)B\mu\lambda/x^{1-\mu}\} - Bx^{\mu} + \delta_s,\end{aligned} \tag{7a}$$

$$\dot{\lambda}/\lambda \equiv \dot{\lambda}_s/\lambda_s - \dot{\lambda}_k/\lambda_k = \delta_s - \gamma(x/\lambda) - B[1 - (1 - \gamma)\mu]x^{\mu} + A - \delta_k, \tag{7b}$$

$$\dot{z}/z \equiv \dot{k}/k - \dot{S}/S = A - (x/z) - \delta_k - Bx^{\mu} + \delta_s. \tag{7c}$$

Typically, a three-variable system is difficult to analyze. This is not the case here due to the block-recursive nature of the system. Capital to habit ratio, z, enters the system only through expression (7c); the other two equations form a separate subsystem in x and λ. Thus, while consumption and thus x affects physical capital accumulation and thereby the ratio of capital to habits, the ratio of capital to habits affects lifetime utility and consumption only by determining the initial optimal choices of these variables. Once these initial choices are made, λ and x evolve accordingly. More specifically, while expressions (7a) and (7b) jointly determine $\{\lambda(t), x(t)\}$, expression (7c) determines $z(t)$. When the three-variable system is solved, all other endogenous variables can be subsequently determined: $\dot{S}/S(t)$ is determined by equation (2), $y(t)/S(t)$ is determined by equation (3), $\dot{k}/k(t)$ is determined by equation (4), and finally, the consumption growth rate, $\dot{c}/c(t)$, is determined by equation (6b). As a result, the market equilibrium is completely determined.

2.1 Balanced Growth Paths

We now determine the balanced-growth path (BGP). A BGP is a steady-state, competitive market equilibrium, in which all variables grow at a constant rate over time.

9. This result is in contrast to the finding of the long-run, intertemporal elasticity being larger than $1/\sigma$ in an economy with habit formation in Carroll, Overland, and Weil (2000), which resorts to the second-order time derivatives of c in order to derive the transitional dynamics of their economic system.

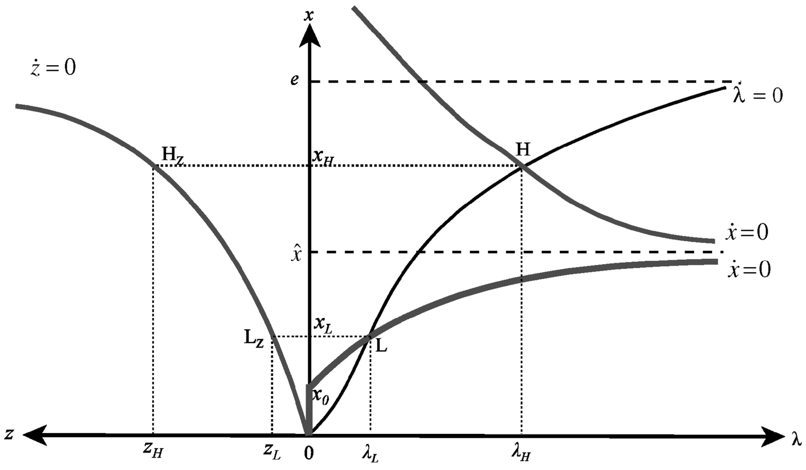

FIG. 1. Balanced Growth Paths

Therefore, $\dot{x} = \dot{\lambda} = \dot{z} = 0$ in the steady state. We determine the BGP by investigating the shape of loci summarized by expressions (7a)–(7c) in two planes.

First, we start with Locus $\dot{\lambda} = 0$ in a (λ, x) plane. Differentiating Expression (7b) with respect to x and λ, we find Locus $\dot{\lambda} = 0$ is upward sloping. Moreover, we find that Locus $\dot{\lambda} = 0$ starts from the origin and approaches horizontal line $x = e \equiv \{(A + \delta_s - \delta_k)/[B(1-(1-\gamma)\,\mu)]\}^{1/\mu} > 0$, when λ approaches infinity (see Figure 1).

Next, in order to analyze the shape of Locus $\dot{x} = 0$, we consider

Condition R: $(\sigma - 1)(1-\gamma)\delta_s > \rho$.

Condition R is easy to meet as ρ is a small value at less than 4% while σ is a large number, more than 1 in calibration. Note that for $\gamma < 1$ Condition R implies $(\sigma - 1)(1 - \gamma\ \delta_s) > \rho$. Under Condition R there is a positive threshold separating the behavior for Locus $\dot{x} = 0$. Differentiating Locus $\dot{x} = 0$ yields

$$dx/d\lambda|_{\dot{x}=0} = -a^*_{12}/a^*_{11} \lesseqgtr 0, \quad if\ x \lesseqgtr \hat{x}, \tag{8}$$

where

$$a^*_{11} \equiv -[B\mu(1-\gamma+(\sigma-1)/\mu - \gamma(\sigma-1)/(B\mu x^{1-\mu})](2\mu-1)(\lambda/x^{1-\mu}) - [(\sigma-1)(1-\gamma\delta_s)-\rho]\lambda/x - [\sigma(1-\gamma)+\gamma(1-\mu)] < 0,$$

$$a^*_{12} \equiv -\big[\rho-(\sigma-1)(1-\gamma)\delta_s + B(\sigma-(1-\mu))(1-\gamma)x^{\mu})\big] \lesseqgtr 0,$$

$$\text{if}\, x \lesseqgtr \hat{x} \equiv \{[(\sigma-1)(1-\gamma)\delta_s - \rho]/[B(\sigma+\mu-1)(1-\gamma)]\}^{\frac{1}{\mu}} > 0,$$

We examine some special cases as follows.

First, when $\mu \to 0$ and $\gamma \to 0$, we obtain $a_{11}^* = -(\sigma - 1 - \rho)(\lambda/x) - \sigma < 0$ and $a_{12}^* \equiv -\rho(\sigma - 1)(B - \delta_s) < 0$.

Then, Locus $\dot{x} = 0$ is positively sloping if the locus exists. The problem with $\mu \to 0$ is that equation (7a) becomes $\dot{x}/x = (A - \delta_k - \rho)/\sigma - [1 - \gamma(\sigma - 1)/\sigma](B - \delta_s)$, so $\dot{x}$ is usually not zero and Locus $\dot{x} = 0$ does not exist. The economic system must be transformed in a different way. The model is then reduced to Rebelo (1991).

Second, restricting $\mu > 0$, when $\gamma \to 0$, we find

$$a_{12}^* \equiv -[\rho - (\sigma - 1)\delta_s + B(\sigma + \mu - 1)x^{\mu}] < (>)\, 0 \text{ if } x > (<)\, \hat{x}, \text{ where}$$

$$\hat{x} \equiv \{[(\sigma - 1)\delta_s - \rho]/[B(\sigma + \mu - 1)]\}^{1/\mu} > 0 \text{ if } \sigma > 1, \text{ and}$$

$$a_{11}^* \equiv \lambda/x\{-[B(\sigma + \mu - 1)(2\mu - 1)]x^{\mu} - [(\sigma - 1) - \rho]\} - \sigma.$$

When μ is large enough, it is easy to show that $a_{11}^* < -\sigma < 0$.[10]

Finally, allowing for both $\mu > 0$ and $\gamma > 0$, the sign of a_{12}^* remains dependent on whether x is larger or smaller than the threshold $\hat{x}$, and the sign of a_{11}^*, by continuity, is negative if μ is large enough.

Thus, under Condition R if $\mu > 0$ is large enough, Locus $\dot{x} = 0$ slopes negatively when $x \geq \hat{x}$, and positively when $x \leq \hat{x}$ and is thus not a monotonic locus as illustrated in Figure 1. Moreover, for $x \geq \hat{x}$, we have shown that Locus $\dot{x} = 0$ has an infinite slope and has the x axis as the asymptote when x approaches infinity, and Line $x = \hat{x}$ as the asymptote when λ approaches infinity. Alternatively, for $x \leq \hat{x}$, Locus $\dot{x} = 0$ intersects the x axis at the region $[0, x_0]$, where $x_0 \equiv \{[A - (\delta_k + \rho) - (\sigma + \gamma(\sigma - 1))\delta_s]/\{B[\sigma(1 - \gamma) + \gamma(1 - \mu)]\}\}^{1/\mu} > 0$, with a positive slope at Point $(0, x_0)$,[11] and approaches Line $x = \hat{x}$ when λ approaches infinity.

Given the shapes of Loci $\dot{\lambda} = 0$ and $\dot{x} = 0$ in Figure 1, as $e > \hat{x}$,[12] these two loci have two interior intersections.[13] Therefore, there exist two interior BGPs, as indicated by points H and L in the figure, with BGP H having a higher ratio of consumption to habits and a higher ratio of habit price to capital price, while BGP L having a lower ratio of consumption to habits and a lower ratio of habit price to capital price.

As is clear, the equilibrium with two steady states emerges from the existence of the threshold. The threshold in the model makes Locus $\dot{x} = 0$ opposite in the slope when the locus is located above or below the threshold. The threshold arises from two internal, intertemporal complement effects. One is the habit in preference, represented by γ, and the other is the complement of current habits in forming new habits, represented by $1 - \mu$, so that consumption accumulates future habits in relation to existing habitual stocks. When the habit stock is small relative to consumption so the ratio of consumption to habits is above the threshold, the two complement effects are weak so that $a_{12}^* < 0$, i.e., the growth rate of the ratio of consumption to habits

10. Using the result $x > x_0$, it requires $\mu > \equiv \{-(\sigma - 3/2) + \sqrt{\sigma(\sigma - 1) + 1/4 - 2[2\sigma - (1 + \rho)]\sigma/[A - \delta_k + \rho - \sigma]}\}/2$, in order to guarantee $a_{11}^* < -\sigma$.

11. The slope is $dx/d\lambda|_{\dot{x}=0\,\lambda_0=0,x=x_0} = -[\rho - (\sigma - 1)(1 - \gamma)\delta_s + B(\sigma + \mu - 1)(1 - \gamma)x_0]/[\sigma(1 - \gamma) + \gamma(1 - \mu)] > 0$, as $x_0 < \hat{x}$.

12. The condition for $e > \hat{x}$ is $(1 - \gamma)[(\sigma + \mu - 1)(\mathrm{A} - \delta_k) + \mu\delta_s] + \rho + (1 + \gamma)\,\mu[(\sigma - 1)(1 - \gamma)\,\delta - \rho] > 0$, which is met under Condition R.

13. A third intersection is a degenerated steady state with zero consumption and shadow price of habits.

decreases in the ratio of the shadow price of habits to the shadow price of capital. Alternatively, when the ratio of consumption to habits is below the threshold, the two complement effects are strong so that $a_{12}^* > 0$.

Finally, in order to compare the economic growth rate for the two BGPs, we analyze Locus $\dot{z} = 0$.

We find that the slope of Locus $\dot{z} = 0$ is positively sloping and, moreover, is concave in a (x, z) plane. Therefore, BGP H is associated with a higher ratio of capital to habit stock, while BGP L is associated with a lower ratio of capital to habit.

The economic growth rate can be rewritten using equation (4), along with equation (3), to obtain

$$\dot{k}/k = \dot{y}/y = A - \delta_k - x^*/z^*. \tag{9}$$

Therefore, $\dot{k}/k$, and thus $\dot{y}/y$, is decreasing in x/z. Notice that the ratio of consumption to habits relative to the ratio of capital to habits is just the ratio of consumption to capital, c/k. Since the long-run x/z in BGP H is x_H/z_H, which is smaller than x_L/z_L in BGP L, the long-run, economic growth rate in BGP H is thereby larger than in BGP L. Since relationship $x_H/z_H > x_L/z_L$ implies $c_H/k_H < c_L/k_L$, the ratio of consumption to capital is lower for BGP H. As lower consumption in BGP H leads to slower habit accumulation, the price of habit is thus higher in BGP H, resulting in $\lambda_H > \lambda_L$ in Figure 1.

The reasons for two possible equilibrium paths for an initial state are as follows. With the consumption habit in preferences, a household's higher current consumption is expected to lead to higher future consumption, in order for a ratio of consumption to habit to attain a given level of utility. This effect induces an interaction among consumption flows, with current consumption reinforcing future consumption. When an agent expects to gain a certain utility level in the future, s/he may choose optimally to consume more now. As high current consumption forms more habits, s/he has to consume more in the future to have a proper ratio of consumption to habit in order to obtain the desired utility level. Alternatively, s/he could optimally choose to consume little now and in the future, in order to obtain the same utility level. As a result, there are two possible long-run equilibrium paths for consumption, with high/low future consumption expectations leading to high/low current consumption choices, and all the equilibrium paths are consistent with expectations in equilibrium. Consequently, habits are formed faster in the equilibrium path associated with high consumption than in the equilibrium path associated with low consumption, while capital stock is accumulated slowly in the former equilibrium path, and faster in the latter equilibrium path. As an effect of capital accumulation, the economic growth rate is low in the steady state of the former equilibrium path, and high in the latter equilibrium path.

2.2 Transitional Dynamics

We now proceed to analyze the dynamic properties of the three-variable system by examining its transitional dynamics. The transitional dynamics of the economic system can be analyzed if we linearize the dynamic system of Expressions (7a)–(7c), evaluated at a BGP $\{x^*, \lambda^*, z^*\}$, to yield

$$\begin{pmatrix} \dot{x} \\ \dot{\lambda} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & 0 & a_{33} \end{pmatrix} \begin{pmatrix} x - x^* \\ \lambda - \lambda^* \\ z - z^* \end{pmatrix} \tag{10}$$

where

$$a_{11} \equiv B\mu(x^{\mu}/\Psi)a_{11}^*,$$

$$a_{12} \equiv B\mu(x^{\mu}/\Psi)a_{12}^*,$$

$$a_{21} \equiv -\gamma - [1-(1-\gamma)\mu]B\mu\lambda^*/x^{*^{1-\mu}} < 0, \quad \text{and}$$
$$= -\gamma - \gamma B\mu\lambda^* < 0, \quad \text{if } \mu = 1,$$

$$a_{22} \equiv \gamma x^*/\lambda^* > 0,$$

$$a_{31} \equiv -\left(1 + B\mu x^{*^{\mu-1}} z^*\right) < 0, \quad \text{and} \quad = -(1 + Bz) < 0, \quad \text{if } \mu = 1,$$

$$a_{33} \equiv x^*/z^* > 0,$$

$$\Psi \equiv \sigma + (\sigma + \mu - 1)B\mu\lambda/x^{1-\mu} > 0, \quad \text{and} \quad = \sigma + \sigma B\lambda > 0, \quad \text{if } \mu = 1.$$

As the dynamic system of Expressions (7a)–(7c) involves one state variable, z, and two control variables, x and λ, there exists a unique equilibrium saddle path toward a BGP if the number of negative eigenvalues near the BGP is one, and there exists a continuum of equilibrium paths toward a BGP if the number of negative eigenvalues near the BGP is larger than one.

Denote $\mathbf{J}$ as the Jacobean matrix in equation (10) and θ as its eigenvalues. When we subtract matrix $\mathbf{J}$ from matrix $\theta\mathbf{I}$, where $\mathbf{I}$ is an identity matrix of order 3, then the eigenvalues are determined by equating determinant $|\mathbf{J} - \theta\mathbf{I}|$ to zero. If we expand $|\mathbf{J} - \theta\mathbf{I}| = 0$, we obtain the following characteristic function:

$$[\theta^2 - (a_{11} + a_{22})\theta + (a_{11}a_{22} - a_{21}a_{22})](\theta - a_{33}) = 0.$$

Solving the above polynomial function yields the following three eigenvalues:

$$\theta_1 = [(a_{11} + a_{22}) - \sqrt{(a_{11} + a_{22})^2 - 4(a_{11}a_{22} - a_{12}a_{21})}]/2 < 0,$$

$$\theta_2 = [(a_{11} + a_{22}) + \sqrt{(a_{11} + a_{22})^2 - 4(a_{11}a_{22} - a_{12}a_{21})}]/2 > 0,$$

$$\theta_3 = x^*/z^* > 0.$$

Given $a_{11} < 0$, $a_{21} < 0$, and $a_{22} > 0$, and in the neighborhood of BGP H, $a_{12} < 0$, we obtain $a_{11}a_{22} - a_{12}a_{21} < 0$. Then, $(a_{11} + a_{22}) < \sqrt{(a_{11} + a_{22})^2 - 4(a_{11}a_{22} - a_{12}a_{21})} > 0$, and thus, $\theta_1 < 0 < \theta_2$ for BGP H. On

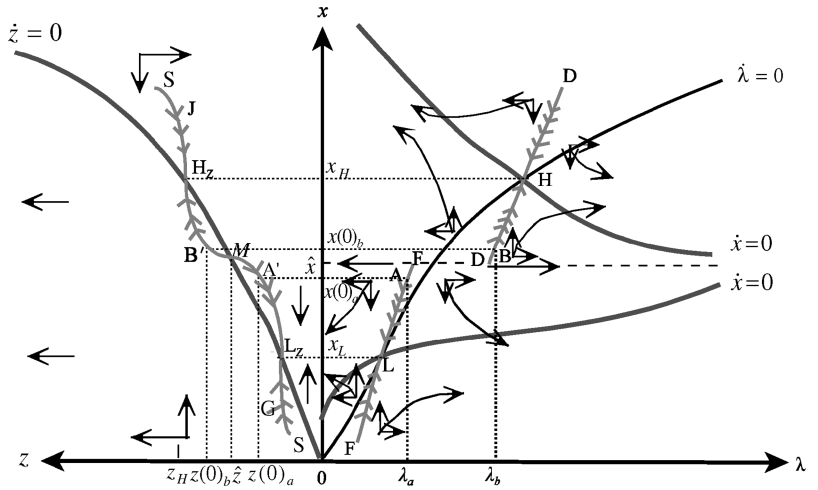

Fig. 2. Transitional Dynamics: Saddles

the other hand, $a_{12} > 0$ in the neighborhood of BGP L. As the slopes of both Loci $\dot{x} = 0$ and $\dot{\lambda} = 0$ are positive and the latter, $-a_{22}/a_{21} > 0$, is steeper than the former, $-a_{12}/a_{11} > 0$, we obtain $a_{11}a_{22} - a_{12}a_{21} < 0$, and thus $\theta_1 < 0 < \theta_2$ for BGP L. Therefore, there is only one negative root around each BGP, and both BGPs are saddle points. Consequently, there is a unique dynamic growth path toward any one of the two BGPs. In Figure 2, Path DD is the unique saddle path toward BGP H, while Path FF is the unique saddle path toward BGP L.

In the (z, x) plane, if we draw a horizontal auxiliary line from $\hat{x}$, it intersects the $\dot{z} = 0$ locus. We call the intersection point M and the corresponding value of $z = \hat{z}$. Path SS is the unique saddle path, and it is discontinuous at the threshold $\dot{z}$. For any initial $z(0) < \hat{z}$, say at $z(0)_a$ in Figure 2, x must jump to the Saddle SS and converge to x_L in the long run; for any $z(0) > \hat{z}$, say at $z(0)_b$ in Figure 2, x must jump to the Saddle SS and converge to x_H in the long run.

Summarizing the BGP and transitional dynamics, we obtain

Proposition 2. *In an economy with habit formation, under Condition R, if* $\mu > 0$ *is large enough there exist two BGPs that are saddles, with one having higher economic growth rates and the other, lower economic growth rates.*

Previous papers with path-dependent utility may have two nondegenerate steady-state equilibria (under the exogenous growth framework), but one stationary point is a saddle and the other is a sink (e.g., Krugman 1991, Palivos 1995, Galor and Weil 1996). If both steady states are saddles, history, or predetermined state variables govern the steady state to which the equilibrium moves. However, although both steady states are saddles in our model, history need not select the equilibrium path.

As is clear in the next section, global indeterminacy arises in that starting from an initial state in the neighborhood of *one* BGP the economy may move toward the neighborhood of *the other* BGP. As a result, although the low- and the high-growth equilibria are saddles, the low-growth equilibrium is not necessarily a development trap.

3. GLOBAL INDETERMINACY

We analyze the global indeterminacy of small disturbances. As an illustrative example, we investigate two disturbances, with one in relation to habits and the other in relation to productivity. Specifically, we consider faster habit formation (a higher B) and larger productivity (a higher A) and envisage the equilibrium dynamics. Then, we analyze the saving rates in the short run and in the long run. Finally, we extend the model and consider endogenous rates of time preference so that global indeterminacy is eliminated.

3.1 When More Habits Are Formed by Consumption

Suppose that the economy is originally at high-growth equilibrium, with a BGP at H_z and thus the initial state $z_H(0)$ larger than the threshold, $\dot{z}$ (Figure 3). When a given level of consumption forms more habits, Loci $\dot{x} = 0$, $\dot{\lambda} = 0$, and $\dot{z} = 0$ all shift downward. As a result, an agent reduces consumption instantaneously (to A) so the ratio of habit price to capital price increases instantaneously (to H_1). As lower

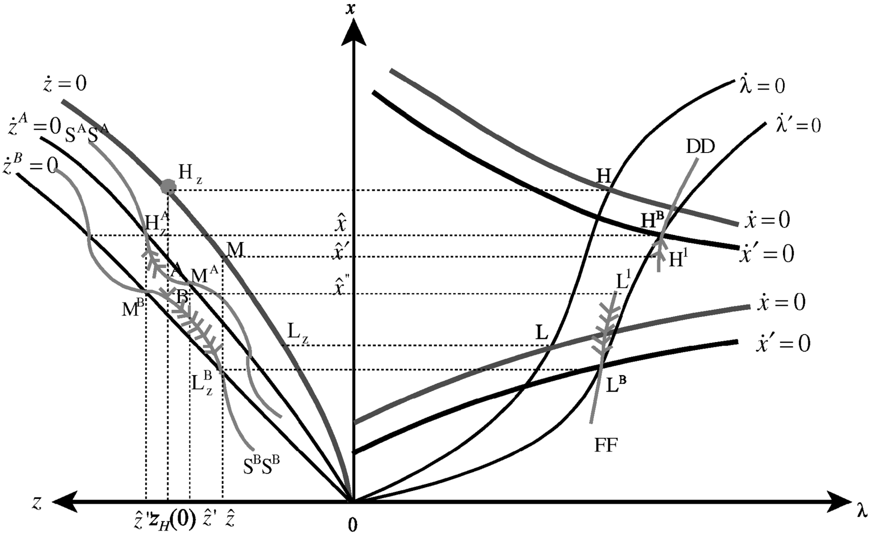

Fig. 3. A Faster Habit Formation (Higher B): Global Dynamics

consumption in the short run leads to more capital accumulation, the ratio of capital to habit increases along Saddle S^AS^A over time (to H_z^B). Moreover, the ratios of consumption to habit and of habit price to capital price move along path DD (to H^B). Thus, the ratios of capital to habit and of consumption to habit are larger in the long run.

However, when the habit formation coefficient is increased so Locus $\dot{z} = 0$ shifts downward significantly, the threshold, $\hat{x}$ and thus $\hat{z}$, may be reduced to $\hat{z}''$. Then, the initial state $z_H(0)$ is smaller than the threshold, $\hat{z}'$. As a result, the agent reduces consumption a lot instantaneously (to B) in order to slow down the formation of habitual stock. Over time, as habit is formed faster than capital formation, the ratio of capital to habit is reduced to L_z^B (along Saddle S^BS^B), and the ratio of habit price to capital price decreases (along path FF). Consequently, all three ratios are smaller, rather than larger, in the long run.

3.2 Increased Productivity: Global Dynamics

Next, we consider a large productivity. Suppose that the economy is originally at low-growth equilibrium, with a BGP at L_z and the initial state $z_L(0)$ smaller than the threshold $\hat{z}$ (Figure 4). When productivity is increased, Loci $\dot{x} = 0$, $\dot{\lambda} = 0$, and $\dot{z} = 0$ all shift upward. Anticipation of higher income increases consumption instantaneously resulting in a higher ratio of consumption to habit (A), and the consequent accumulation of habits reduces the ratio of habit price to capital price (to L^1). Over time, the ratio of capital to habit is decreased (resp. increased) along Saddle S^AS^A (to L_z^A) if Locus $\dot{x} = 0$ shifts leftward less (resp. more) than Locus $\dot{\lambda} = 0$. The ratios of habit price to capital price and of consumption to habits move along path FF (to

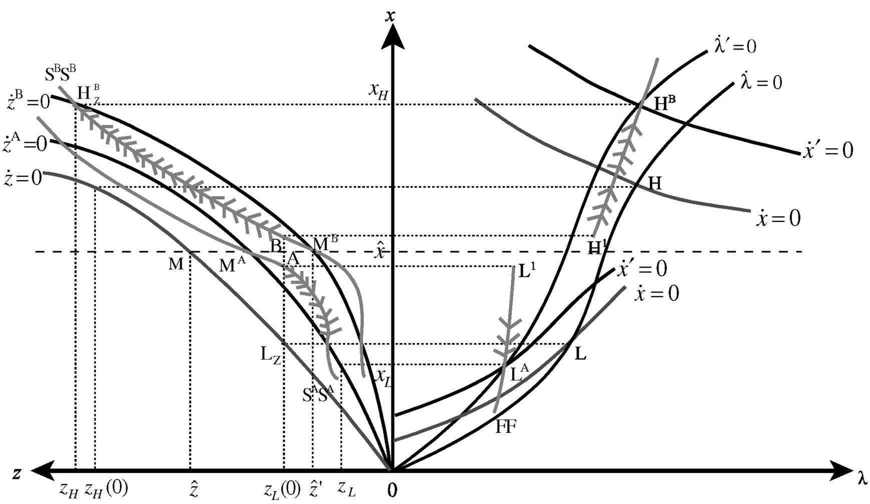

FIG. 4. A Larger Productivity (Higher A)

A^A). In the new BGP, the ratio of capital to habit is decreasing in the long run, while the ratios of consumption to habit and of habit price to capital price may be larger or smaller than that of the original BGP. The effect on the economic growth rate in the new BGP is in general positive as there is a positive direct effect that generally dominates the ambiguous indirect effect through changing the ratio of consumption to capital.

However, when productivity is increased so Locus $\dot{z} = 0$ shifts upward significantly, the threshold $\hat{z}$ is reduced to $\hat{z}'$ that is smaller than the initial state, $z_L(0)$. As a result, consumption increases instantaneously so the ratio of consumption to habits is at B in equilibrium. As income is higher, capital is accumulated faster than habits so that the ratio of capital to habits increases along Saddle $S^B S^B$ over time (to H_z^B), and as a result, the ratio of habit price to capital price increases along path DD over time (to H^B). In the new BGP, all three ratios are increasing in the long run. The economic growth rate is higher through a positive direct effect and a positive indirect effect via a lower ratio of consumption to capital.

3.3 Short-run and Long-run Savings Rates

Carroll, Overland, and Weil (2000) derived the condition under which the derivative of the gross savings rate with respect to the growth rate of output in steady state is positive. They found that habit formation increases the range of parameter values for the relationship to be positive. It seems natural to characterize the short-run and long-run savings rate around the high and low equilibrium.

According to equation (4), the savings rate in our model is

$$\chi(t) \equiv [y(t) - c(t)]/y(t) = [\dot{k} - \delta_k k(t)]/Ak(t) = (1/A)[\dot{k}/k(t) - \delta_k],$$

where, from equation (9), $\dot{k}/k(t)$ is inversely related to the ratio of consumption to capital.

The savings rate depends upon the location of initial state. Suppose the economy is initially either at $z(0)_a$ or at $z(0)_b$ in Figure 2. Then, the initial equilibrium is J and G, respectively. As the ratio of consumption to capital at G is smaller than the ratio of consumption to capital at J, the savings rate at G is larger than the savings rate at B′ in the short run.

In the long run, the economy initially at $z(0)_a$ moves from A′ along Saddle SS to L_z, the low-growth BGP, while the economy initially at $z(0)_b$ moves from B′ to H_z, the high-growth BGP in Figure 2. As the ratio of consumption to capital at H_z is smaller than that at L_z, the savings rate of the economy initially at $z(0)_b$ is higher than the economy initially at $z(0)_a$ in the long run. As a result, there is a possibility that an economy with a higher savings rate in the short run has a lower savings rate in the long run.

Moreover, there is global indeterminacy. A small disturbance may change the economic structure so that the threshold changes significantly, according to Figures 3 and 4. Thus, the economy initially at $z(0)_a$ in Figure 2 may move to a BGP in the neighborhood of H_z while the economy initially at $z(0)_b$ may move to a BGP in the neighborhood of L_z. As a consequence, the savings rate in the economy initially at $z(0)_b$ is lower, rather than higher, than the economy initially at $z(0)_a$ in the long run.

To summarize global indeterminacy, we obtain

PROPOSITION 3. *In an economy with habit formation, under Condition R and if $\mu > 0$ is large enough, a small disturbance may change the economic structure so global indeterminacy arises. As a result, the economic growth rate and the saving rate are globally indeterminate.*

A remark must be made for the low long-run growth BGP in our model. The low-growth BGP is not interpreted as a development trap. Existing works interpreted a long-run low-growth equilibrium as a trap because it is locally a sink and thus, once the economy is in the neighborhood of the long-run equilibrium it is difficult to move toward another long-run equilibrium (e.g., Krugman 1991, Galor and Weil 1996). The low long-run BGP in our model is locally a saddle, and globally it may move to the high long-run BGP and is thus not a development trap.

3.4 Extension: Endogenous Rates of Time Preferences

Global indeterminacy arises in this model because of the following two features: the internal, intertemporal complementarity effect and the existence of two steady states. If the threshold is not feasible, we rule out the existence of two steady states. Then, the equilibrium with two steady states exhibiting global indeterminacy is reduced to the equilibrium with a unique steady state exhibiting global determinacy. The existence of two steady states is ruled out if Condition R no longer holds.

Suppose that the rate of time preference is not constant but is endogenous in the sense of Uzawa (1968). Specifically, following Palivos, Wang, and Zhang (1997), we assume $\rho = \rho(u(xS^{1-\gamma}))$, with $\rho'(u) > 0$ and $\rho(u)$ reaching asymptotically an upper bound $\bar{\rho} > (\sigma - 1)(1 - \gamma)\delta_s$. Then, $\rho = \rho(u)$ may increase over time when there is a positive, even small, economic growth rate so that S increases over time. Eventually, $\rho(u)$ increases to a level that Condition R fails to meet. As a result, eventually the threshold $\hat{x} < 0$ in Figures 1 and 2 and the steady state L are thus not feasible. Therefore, the equilibrium with two steady states disappears, while the equilibrium with a unique steady state emerges. The unique steady state is both locally and globally determinate. Global indeterminacy is eliminated and history determines the fates of economies.

4. CONCLUDING REMARKS

In this paper, we have analyzed a simple competitive, one-sector, endogenous growth model that has been extended to allow for a utility function exhibiting habit persistence in preferences. The key feature in our model is that consumption forms consumption habits in combination with existing habits. We use a standard solution procedure for a three-variable dynamic system without involving second-order time derivatives for consumption in the system. Habit persistence in preferences brings forth a non-linear economic system resulting in two interior balanced growth paths, with one exhibiting low consumption and habit formation and high economic growth, and the other displaying high consumption, high habit formation, and low economic growth.

The two steady states are saddle points locally, but for an initial ratio of capital and habits global indeterminacy arises with a high balanced growth path co-existing with a low balanced growth path such that history need not matter for the selection of the equilibrium path. Moreover, the two equilibrium paths cannot be pareto ranked and the low growth BGP is not a trap. We have investigated the global indeterminacy under two disturbances, one pertaining to habit formation and the other to productivity. In addition, we have analyzed the resulting implications on short-run and long-run savings rate and an extension to endogenous rates of time preference in order to eliminate global indeterminacy.

Finally, there are restrictions in the model and for simplicity we point out just one. In the model both BGPs have long-run positive economic growth rates. The model can only be used to highlight the epoch of sustained economic growth after the industrialization. Such a model is thus a non-unified growth theory in the language of Galor (2005). In order to interpret the growth process over the entire history of the human species from the state of Malthusian stagnation to the post-Malthusian regime and then the sustained regime, the model needs modification. To do that, following Galor and Weil (2000), Galor and Moav (2002) and Galor (2005), we may consider the fertility choice, endogenous labor supply, education choice, and technical progress, along with other building blocks like the Malthusian elements and the Darwin elements. We may then analyze how the habit in preference affects the growth dynamics and process over the entire economic growth history. This points to a direction for further research.

APPENDIX
PROOF OF THE HAMILTONIAN SATISFYING THE ARROW SUFFICIENT THEOREM

Maximizing the Hamiltonian with respect to c, i.e.,

$$\underset{\{c\}}{\text{Max}}\, H(c, k, S, \lambda_k \lambda_S) = 1/(1-\sigma)[(c/S^{\gamma})^{1-\sigma} - 1] + \lambda_k[Ak - c - \delta_k k] - \lambda_s[Bc^{\mu}S^{1-\mu} - \delta_s S],$$

leads to the following necessary and sufficient condition

$$H_c = c^{-\sigma}/S^{\gamma(1-\sigma)} - \lambda_k - \lambda_s\left[B\mu S^{1-\mu}/c^{1-\mu} - \delta_s\right] = 0, \tag{A1a}$$

$$H_{cc} = -\sigma c^{-(\sigma+1)}/S^{\gamma(1-\sigma)} + \lambda_s B\mu(1-\mu)S^{1-\mu}/c^{-\mu} < 0. \tag{A1b}$$

It is obvious equation (A1a) implies

$$\lambda_k + \lambda_s B\mu S^{1-\mu}/c^{1-\mu} = c^{-\sigma}/S^{\gamma(1-\sigma)} + \lambda_s\delta_s, \tag{A1c}$$

and leads to Relationship $c = c(S)$, that satisfies

$$\begin{aligned} c'(S) &= \gamma(\sigma-1)/\sigma c/S > 0, \quad \text{for } \mu = 1, \\ &= [\gamma(\sigma-1)c/S - \Omega]/\Lambda > 0, \quad \text{if } \exists\underline{\mu} > 0, \ni\mu < \underline{\mu}, \quad \text{for } \mu < 1, \end{aligned} \tag{A2a}$$

$$c''(S) = [\gamma(\sigma-1)/\sigma][\gamma(\sigma-1)/\sigma - 1]c/S^2 > 0, \quad \text{for } \mu = 1, \tag{A2b}$$

$$\begin{aligned} c''(S) = \big\{&[\gamma(\sigma-1)/\sigma(c'(S)/S - c/S^2)] - \lambda_s B\mu(1-\mu)/ \\ &\sigma[(\sigma+\mu)c^{\sigma+\mu-1}/S^{\gamma(\sigma-1)+\mu}c'(S) - [\gamma(\sigma-1)+\mu]c^{\sigma+\mu}/ \\ &S^{\gamma(\sigma-1)+\mu-1}] - \gamma(\sigma-1)/\sigma c'(S)/ \\ &S[(\sigma+\mu-1)c^{\sigma+\mu-2}/S^{\gamma(\sigma-1)+\mu-1}c'(S) \\ &-(\gamma(\sigma-1)+\mu-1)c^{\sigma+\mu-1}/S^{\gamma(\sigma-1)+\mu-2}]/\Lambda\big\}/\Lambda, \\ &\text{if } 0 < \mu < 1, \end{aligned} \tag{A2c}$$

$$\begin{aligned} &= \gamma(\sigma-1)/\sigma[c'(S) - c/S]/(\Lambda S) - \Omega/ \\ &\Lambda[(\sigma+\mu)c'(S) - [\gamma(\sigma-1)+\mu]c/S]/c - (1-\Lambda)/\Lambda c'(S)/ \\ &c[(\sigma-1+\mu)c'(S) - [\gamma(\sigma-1)-1+\mu]c/S] < 0, \\ &\text{if } \exists\mu_0 > 0,\ \mu < \mu_0, \end{aligned} \tag{A3a}$$

where $\Omega = [\lambda_s B\mu(1-\mu)/\sigma][c^{\sigma+\mu}/S^{\gamma(\sigma-1)+\mu}] > 0$ and $0 < \Lambda = 1 - [\lambda_s B\mu(1-\mu)/\sigma][c^{\sigma-1+\mu}/S^{\gamma(\sigma-1)-1+\mu}] < 1$.

For the terms in the three large brackets in equation (A2c), we have

$$\begin{aligned} &c'(S) - c/S = [\gamma(\sigma-1)-\sigma]/\sigma c/S < 0, \\ &(\sigma+\mu)c'(S) - [\gamma(\sigma-1)+\mu]c/S \\ &\quad = \mu[\gamma(\sigma-1)-\sigma]c/(\sigma\Lambda S)(1 + \lambda_s B(1-\mu)c^{\sigma+\mu-1}/S^{\gamma(\sigma-1)+\mu-1}) < 0, \\ &(\sigma-1+\mu)c'(S) - (\gamma(\sigma-1)-1+\mu)c/S \\ &\quad = (1-\mu)[\gamma(\sigma-1)-\sigma]c/\sigma\Lambda S(-1 + \lambda_s B\mu c^{\sigma+\mu-1}/S^{\gamma(\sigma-1)+\mu-1}) \lesseqgtr 0. \end{aligned}$$

As in equation (A2a), by continuity there exists a $\mu_0 > 0$ large enough that the first term in equation (A2c), which is negative, dominates the positive second term and the ambiguous third term, and therefore $c''(S)$ in equation (A2c) is negative.

Substituting Relationship $c = c(S)$ into the Hamiltonian, we obtain a new Hamiltonian as follows:

$$\underset{\{S,k\}}{\text{Max}}\ \hat{H}(k, S, \lambda_k \lambda_S) = 1/(1-\sigma)[(c(S)/S^{\gamma})^{1-\sigma} - 1] + \lambda_k[Ak - c(S) - \delta_k k]$$
$$- \lambda_s\big[Bc(S)^{\mu} S^{1-\mu} - \delta_s S\big].$$

Then, we can derive the following conditions

$$\hat{H}_k = \lambda_k(A - \delta_k), \tag{A3a}$$

$$\hat{H}_{kk} = 0, \tag{A3b}$$

$$\hat{H}_{kS} = 0, \tag{A3c}$$

$$\hat{H}_S = c^{-\sigma}(S)c'(S)/S^{\gamma(1-\sigma)} - \gamma c^{1-\sigma}(S)/S^{\gamma(1-\sigma)+1} - \lambda_k c'(S)$$
$$- \lambda_s B[\mu c^{\mu-1}(S)c'(S)S^{1-\mu} + (1-\mu)c^{\mu}(S)S^{-\mu}] + \lambda_s \delta_S. \tag{A3d}$$

The condition for $\hat{H}_{SS}$ is more complicated, and we analyze it for (1) Case $\mu = 1$, and (2) Case $\mu < 1$.

A1 For Case $\mu = 1$

$$\hat{H}_{SS} = -\sigma c^{-(\sigma+1)}(S)/S^{\gamma(1-\sigma)}[c'(S)]^2 + c^{-\sigma}(S)/$$
$$S^{\gamma(1-\sigma)}[c''(S) - \gamma(\sigma-1)c(S)/S] - \gamma c^{-\sigma}(S)/$$
$$S^{\gamma(1-\sigma)+1}\{(1-\sigma)c'(S) - [\gamma(1-\sigma)+1]c/S\} - (\lambda_k + \lambda_s B)c''(S). \tag{A4a}$$

If we substitute into equations (A2a)–(A2b) under $\mu = 1$, equation (A4a) becomes

$$\hat{H}_{SS} = -\gamma/\sigma c^{-\sigma+1}/S^{\gamma(1-\sigma)+2}[(1+\gamma)(\sigma-1)^2 + \gamma\sigma]$$
$$+ [c^{-\sigma}/S^{\gamma(1-\sigma)} - (\lambda_k + \lambda_s B)c''(S), \tag{A4b}$$

which using equation (A1c) can be rewritten as

$$\hat{H}_{SS} = -\gamma/\sigma c^{-\sigma+1}/S^{\gamma(1-\sigma)+2}[(1+\gamma)(\sigma-1)^2 + \gamma\sigma]$$
$$- \lambda_s\delta_s[\gamma(\sigma-1)/\sigma][\gamma(\sigma-1)/\sigma - 1]c/S^2$$
$$= -\gamma/\sigma c/S^2 \lambda_s \delta_s \big\{c^{-\sigma}\big/\big(S^{\gamma(1-\sigma)}\lambda_s\delta_s\big)[(1+\gamma)(\sigma-1)^2 + \gamma\sigma]$$
$$-(\sigma-1)[1 - [\gamma(\sigma-1)]/\sigma]\big\}. \tag{A4c}$$

As $c^{-\sigma}/(S^{\gamma(1-\sigma)}\ \lambda_s\ \delta_s) = (\lambda_k + \lambda_s\ B)/(\lambda_s\ \delta_s) - 1 \equiv [(1 + \lambda B)/\lambda\ \delta_s\ ] - 1$ according equation (A1c), where $\lambda \equiv \lambda_s/\lambda_k$ as defined in the text, then $\hat{H}_{SS} < 0$ in equation (A4c), if the following condition is satisfied.

Condition S: $1/\lambda\delta_S + B/\delta_S > (1 - 1/\sigma)[(1 - \gamma) + \gamma/\sigma]/[(1 + \gamma)(\sigma - 1)^2 + \gamma] + 1$.

Condition S is met if δ_S and λ are small, and B is large.
Then, the Hessian is

$$\begin{vmatrix} \hat{H}_{SS} & \hat{H}_{Sk} \\ \hat{H}_{kS} & \hat{H}_{kk} \end{vmatrix} = 0. \tag{A5}$$

Therefore, when $\mu = 1$, the Hamiltonian is concave and satisfies Arrow's sufficient theorem.

A2 For Case 0 < μ <1

$$\begin{aligned}\hat{H}_{SS} = &-\sigma c^{-(\sigma+1)}(S)/S^{\gamma(1-\sigma)}[c'(S)]^2 + c^{-\sigma}(S)/\\ &S^{\gamma(1-\sigma)}[c''(S) - \gamma(\sigma - 1)c(S)/S] - \gamma c^{-\sigma}(S)/\\ &S^{\gamma(1-\sigma)+1}\{(1-\sigma)c'(S) - [\gamma(1-\sigma) + 1]c/S\}\\ &- \left(\lambda_k + \lambda_S B\mu c^{\mu-1}S^{1-\mu}\right)c''(S)\\ &- \lambda_S B\mu(1-\mu)c^{\mu}S^{-\mu}(1/S - 1/c)(S/cc'(S) - 1),\end{aligned} \tag{A6}$$

whose first three terms are the same as in equation (A4a), the fourth term is also similar to equation (A4a) if μ is close to 1, and the last term is an extra term that is ambiguous and is small if μ is close to 1.

Given the similarity between form equation (A6) and form equation (A4a), by continuity there exists μ_1 large enough such that for all $\mu < \mu_1$, a condition similar to Condition S can guarantee equation (A6) to be negative, and thus the Hamiltonian to be concave in (S, k).

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