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Admiration is a source of indeterminacy

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Abstract

Are consumption externalities sources of indeterminacy? The answer is negative in conventional wisdom. This paper shows consumption externalities establish local indeterminacy if the degree of decreasing impatience is sufficiently large. Dynamic models with indeterminacy are valuable as they tend to explain economic fluctuations without relying on exogenous shocks and question interpretation of simple estimation obtained by pooling data, among others.

Keywords: Consumption externalities; Neoclassical growth model; Indeterminacy

JEL classification: E62; O40

1. Introduction

Recently, consumption externalities have been introduced in dynamic general equilibrium models in the areas of asset pricing (e.g., Abel, 1990; Constantinides, 1990) and business cycles (e.g., Ljungqvist and Uhlig, 2000). Our current understanding holds that neoclassical growth models with consumption externalities generate *intertemporal* inefficiency, but the steady-state equilibrium is efficient and the dynamic equilibrium is always unique (e.g., Fisher and Hof, 2000; Alonso-Carrera et al., 2004, 2005a). If labor-leisure choices are allowed for, *intratemporal* substitution is changed and the steady-state equilibrium

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is inefficient (Dupor and Liu, 2003), but the dynamic equilibrium is still determinate (Liu and Turnovsky, 2005).[1] Thus, consumption externalities are not a source of indeterminacy in conventional wisdom.

In this paper, we show that it is possible for a one-sector, neoclassical growth model with consumption externalities to exhibit local indeterminacy if the rate of time preference is decreasing in individual consumption.[2] The reason is that consumption externalities are a source of intertemporal inefficiency while decreasing impatience generates intratemporal substitution. Individual consumption affects optimal capital demand when individuals respond to expectations about others' consumption. If the degree of decreasing impatience is sufficiently large, expectations about others' consumption is self-fulfilling in equilibrium.

The endogenous rate of time preference has been recognized by Fisher (1930), who showed that a person's impatience decreases as the economy develops. In particular, in a class of model with recursive utility Koopmans (1960) made an argument in favor of decreasing impatience.[3] In an empirical work using post-war time-series data in Japan, Ogawa (1993) found that the rate of time preference is decreasing in consumption in the early stage of development.

Local indeterminacy is established in existing one-sector, neoclassical growth models based on the assumption of increasing returns or externalities in production, pioneered by Benhabib and Farmer (1994) and Farmer and Guo (1994)[4], but the assumption is challenged by empirical studies (e.g., Burnside, 1996; Basu and Fernald, 1997). Dynamic models with indeterminacy are valuable as they tend to explain economic fluctuations without relying on exogenous shocks, and question interpretation of simple estimation obtained by pooling data, among others.[5] We show that endogenous fluctuations can easily arise in relation to consumption externalities independent of the challenged arguments of increasing returns or externalities in production.

As developed below, Section 2 sets up the basic model and analyzes the existence of a unique steady state. Section 3 is the main body that examines the issue of local indeterminacy. Finally, some concluding remarks are presented in Section 4.

2. The model

A representative household supplies labor inelastically. He owns the shares of firms and decides consumption and savings at each instant of time t, the latter of which accumulates capital for production at time $t+\mathrm{d}t$. The lifetime preference of the representative agent is

$$U = \int_0^\infty u(c(t), C(t))X(t)\mathrm{d}t, \tag{1}$$

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[1] Recently, in a paper assuming non-constant marginal rate of substitution between private and social consumption, Alonso-Carrera et al. (2005b) showed that by allowing for labor and leisure choices as in Ljungqvist and Uhlig (2000), a one-sector neoclassical growth model with consumption externalities generates local indeterminacy if the price elasticity of the Frisch labor supply is large enough.

[2] The rate of time preference is also called "*perspective undervaluation of future*" by Böhm-Bawerk (1889) and "*human impatience*" by Fisher (1930).

[3] Uzawa (1965) represents another class of argument in favor of increasing impatience.

[4] Benhabib and Farmer (1994) show that the simple one-sector model with increasing returns is identical, in the sense of giving rise to the same reduced form, to a model with monopolistic competition and constant markups. See the survey by Benhabib and Farmer (1999).

[5] In a recent paper Cazzavillan (1996) finds local indeterminacy arising from government externalities in utility in an unbounded growth model, but he assumes the public good externalities in production.

where u is the felicity function, c is individual consumption, C is consumption per capita in the economy that is consumption externalities, and X is the discount.

There are two concepts concerning the ways through which the consumption externalities influence an individual's utility. First, for given an individual consumption, average consumption directly affects an individual's utility. Following Dupor and Liu (2003), it is said that the individual feels either *jealous* if $u_2(c,C)<0$, or *admiring* if $u_2(c,C)>0$. Second, average consumption may have an external effect on an individual's marginal utility of his own consumption. It is referred to as *keeping up with the Joneses* if $u_{12}(c,C)>0$ (e.g., Gali, 1994) or as *running away from the Joneses* if $u_{12}(c,C)<0$ (e.g., Dupor and Liu, 2003).

The felicity is assumed to be twice differentiable with

Assumption 1. (*i*) $u_1(c,C)>0>u_{11}(c,C)$ and $u_2(c,C)>0$ for any c and C;
(*ii*) $u_{11}(c,C)+u_{12}(c,C)<0$ and $u_{11}(c,C)u_{22}(c,C)-2u_{12}(c,C)>0$ for any c and C.

While the first part of Assumption 1(*i*) is standard, the second part asserts that the consumption externality augments an individual's utility directly; i.e., individuals feel admiration. Assumption 1(*ii*) assures that the utility is concave in consumption at the social level. Both keeping up with and running away from the Joneses are included.

The discount is defined as $X(t)\equiv \int_0^t \exp[-\rho(c(t))]d\tau$ or equivalently,

$$\dot{X} = -\rho(c(t))X(t), \text{ with } X(0) \text{ given}, \tag{2}$$

where $\rho(c(t))$ is the instantaneous discount rate in t.

We assume that the instantaneous rate of discount is twice differentiable with

Assumption 2. $\rho'(c)<0$ and $-\frac{\rho''}{\rho}<-\frac{u_{11}+u_{12}}{u}$ for any c and C.

While in the former decreasing impatience is assumed as in Fisher (1930) and Koopmans (1960), the latter demands the curvature of the utility be larger than that of the discount.

The representative firm is endowed with a technology $Y=F(K,L)$ where Y is output, L is labor input and K is capital input that for simplicity is assumed not to depreciate. The technology is assumed to be twice differentiable and satisfies the standard properties in a neoclassical production function. In a per capita form, the technology is rewritten as $y=f(k)$, where y is output per capita and k is capital stock per capita with $k(0)$ given initially. The technology satisfies

Assumption 3. $f'(k)>0>f''(k)$, $f''(k)<\rho'(c)f'(k)$, $f(0)=0$ and $f'(0)>\rho(0)$ for any k and c.

In addition to the neoclassical properties, the remaining assumptions in Assumption 3 are made in order to satisfy the second order conditions for an interior equilibrium.

The representative household allocates income to consumption and savings. Thus,

$$\dot{k} = f(k(t))-c(t). \tag{3}$$

The representative household's problem is to maximize Eq. (1), subject to Eqs. (2) and (3), taking as given the consumption externalities. The Hamiltonian associated to the program is

$$H(c,k,X,\theta,\lambda_X) = X\{u(c,C) + \theta[f(k)-c]-\lambda[\rho(c)]\},$$

where $\theta > 0$ is the co-state variable associated with capital and $-\lambda > 0$ is the co-state variable associated with the discount. The necessary conditions are

$$\theta = u_1(c, C) - \lambda\rho'(c), \tag{4a}$$

$$\dot{\lambda} = -u(c, C) + \lambda\rho(c), \tag{4b}$$

$$\dot{\theta} = \theta[-f'(k) + \rho(c)], \tag{4c}$$

and the transversality conditions, $\lim_{t\to\infty} \lambda(t)X(t) = 0$ and $\lim_{t\to\infty} \theta(t)X(t)k(t) = 0$.

In the optimal conditions, the first condition equates the marginal cost and the discounted marginal utility, while the second and the third conditions are the Euler equations for discount and capital, respectively. Finally, consumption externalities generate intertemporal inefficiency in Eq. (4a), while consumption affects intratemporal substitution via decreasing impatience in Eq. (4c).

Definition 1. Under Assumptions 1–3, for $k(0)$ and $X(0)$, a symmetric market equilibrium is a path $\{C(t), k(t), \theta(t), \lambda(t)\}$, $c(t) = C(t)$, which solves Eqs. (3), (4a) (4b) (4c) and the transversality conditions.

Definition 2. A steady-state market equilibrium is a symmetric market equilibrium with a constant C^* and $\dot{k} = \dot{\theta} = \dot{\lambda} = 0$.

The following conditions characterize the steady-state market equilibrium.

$$\theta^* = u_1(C^*, C^*) - \lambda^*\rho'(C^*), \tag{5a}$$

$$u(C^*, C^*) = \lambda^*\rho(C^*), \tag{5b}$$

$$f'(k^*) = \rho(C^*), \tag{5c}$$

$$f(k^*) = C^*. \tag{5d}$$

It is obvious that under 1–3, there exists a unique steady state in Eqs. (5a) (5b) (5c) (5d). If we substitute Eq. (5d) into Eq. (5c), then we obtain a unique k^* according to Assumption 3. Substituting k^* into Eq. (5d) yields C^*, and C^* into Eq. (5b) yields λ^*. Finally, substituting λ^* into Eq. (5a) yields θ^*. Thus, there exists a unique steady state. Indeed, Eqs. (4b) and (5b) indicate that the shadow price of the discount at t is the discounted sum of utility of future consumption streams,

$$\lambda^*(t) = \int_t^\infty u(C^*(\tau), C^*(\tau))e^{\int_0^\tau -\rho(C(s)^*)\mathrm{d}s}\mathrm{d}\tau = \frac{u(C^*, C^*)}{\rho(C^*)}.$$

The efficiency of the allocation is obtained by studying the central planner's problem. As the central planner internalizes the consumption externalities, the necessary conditions are

$$\theta^* = u_1(C^*, C^*) + u_2(C^*, C^*) - \lambda^*\rho'(C^*), \tag{6}$$

along with Eqs. (4b)–(4c). Therefore, the first-best steady state is determined by Eqs. (6) and (5b) (5c) (5d).

The allocation of capital and consumption is dictated solely by the optimal condition for capital and the market clearing condition in Eqs. (5c) and (5d). It is clear that consumption externalities affect neither Eqs. (5c) nor (5b) and thus do not affect the steady-state allocation. Therefore, even though there is decreasing impatience, consumption externalities do not cause market failures in terms of allocation in the steady state. We thus obtain

Proposition 1. *Under Assumptions 1–3, there exists a unique interior steady state. Consumption externalities do not exert any effect on the steady-state allocation.*

The consumption externalities affect the shadow price of capital in two ways (cf. Eq. (4a)), directly through the effect of either keeping up with or running away from the Joneses, and indirectly via changing the shadow price of discount whose effect disappears if the rate of time preference is exogenous. They thus affect the felicity not only directly but also indirectly through the shadow prices of discount and capital (cf. Eq. (4b)).

As a result, although the consumption externalities do not affect the allocations in the steady state, they exert effects on the allocations in transitional dynamics through their influences on the shadow prices of discount and capital. If the interactions between decreasing impatience and consumption externalities are large enough, the dynamic equilibrium is locally indeterminate.

3. Macroeconomic dynamics

We now analyze the dynamic equilibrium. If we differentiate Eq. (4a) and use Eq. (4c), we obtain

$$\dot{C} = \frac{1}{\Omega}\left(f' - \rho - \frac{\rho'}{u_1 - \lambda\rho'}\dot{\lambda}\right), \tag{7}$$

where $\Omega \equiv -\frac{u_{11}+u_{12}-\lambda\rho''}{u_1-\lambda\rho'} > 0$ is the reciprocal of the intertemporal elasticity of substitution, which is positive under Assumption 2.

The dynamic equilibrium is summarized by $\dot{k}$, $\dot{\lambda}$ and $\dot{C}$ in Eqs. (3), (4b) and (7). If we take a linear Taylor's expansion around the steady state $\{C^*, k^*, \lambda^*\}$, we obtain

$$\begin{bmatrix} \dot{C} \\ \dot{k} \\ \dot{\lambda} \end{bmatrix} = \begin{bmatrix} \frac{1}{\Omega}\frac{u_2\rho'}{u_1-\lambda\rho'} & \frac{1}{\Omega}f'' & \frac{1}{\Omega}\frac{-\rho\rho'}{u_1-\lambda\rho'} \\ -1 & f' & 0 \\ -(u_1+u_2-\lambda\rho') & 0 & \rho \end{bmatrix} \begin{bmatrix} C-C^* \\ k-k^* \\ \lambda-\lambda^* \end{bmatrix}. \tag{8}$$

The economic system in Eq. (8) involves one state variable, *k*, whose initial value is predetermined, and two control variables, *λ* and *C*, that can adjust instantaneously. As a result, if there exists one eigenvalue with negative real parts, the equilibrium path in the neighborhood of the unique steady state is determinate; on the other hand, if there exist two or more eigenvalues with negative real parts, the dynamic equilibrium path is indeterminate.

Denote $\boldsymbol{J}$ as the Jacobean matrix in Eq. (8) and ω as its eigenvalues. If we subtract $\boldsymbol{J}$ from matrix $\omega\boldsymbol{I}$, where $\boldsymbol{I}$ is a 3×3 identity matrix, the eigenvalues are determined by $|\boldsymbol{J}-\omega\boldsymbol{I}|=0$; i.e.,

$$\Psi(\omega) = -\omega^3 + A\omega^2 + B\omega + D = 0, \tag{9}$$

where

$$A\equiv\left(f' + \rho + \frac{1}{\Omega}\frac{u_2\rho'}{u_1-\lambda\rho'}\right),$$
$$B\equiv\left[\frac{f''}{\Omega} - f'\rho - \frac{1}{\Omega}\frac{u_2\rho'}{u_1-\lambda\rho'}\rho + \frac{1}{\Omega}\rho\rho'\right],$$
$$D\equiv\frac{\rho}{\Omega}(f''-\rho'f')<0.$$ [6]

It is obvious $\Psi(-\infty)>0$ and $\Psi(\infty)<0$. If we differentiate Eq. (8), we obtain

$$\Psi'(\omega) = -3\omega^2 + 2A\omega + B. \tag{9}$$

If we let $\Psi'(\omega)=0$, we obtain two extreme values, denoted by ω_1 and ω_2, as

$$\omega_i = \frac{A\pm\sqrt{A^2+3B}}{3}, i = 1, 2. \tag{10}$$

As $D<0$, there exist three roots with negative real parts if the locus $\Psi(\omega)$ intersects the ω-axis at $\omega<0$ three times. This situation occurs if the extreme values such that $\Psi'(\omega)=0$ are both negative. The required conditions are $A<0$ and $B<0$. There are four cases.

Case 1. Suppose there are no consumption externalities and no endogenous rate of time preference; i.e., $u_2=u_{12}=\rho'=\rho''=0$. Then $A\equiv f'+\rho>0$, and $B\equiv\frac{f''}{\Omega}-f'\rho<0$, which violate the required condition $A<0$.

Case 2. Suppose the rate of time preference is exogenous; i.e., $\rho'(c)=\rho''(c)=0$. Then, $A\equiv f'+\rho>0$ and $B\equiv\frac{f''}{\Omega}-f'\rho<0$, that are again not satisfied with condition $A<0$.

Case 3. Suppose there are no consumption externalities; i.e., $u_2=u_{12}=0$. Then, $A\equiv f'+\rho>0$ and $B\equiv\frac{f''}{\Omega}-f'\rho+\frac{1}{\Omega}\rho\rho'<0$. Condition $A<0$ is not met.

Case 4. Suppose there are consumption externalities and decreasing impatience as in Assumptions 1 and 2. Then, $A<0$ if

$$-\rho'\left(\frac{u_2}{u_1-\lambda\rho'}\right)>\Omega(f'+\rho)>0, \tag{11a}$$

and $B<0$ if

$$-\rho'\left(-1+\frac{u_2}{u_1-\lambda\rho'}\right)>f''-\Omega f'\rho. \tag{11b}$$

[6] In deriving B and D, Eq. (5c) is used.

Conditions (11a) and (11b) indicate that in order for consumption externalities to create local indeterminacy, it is necessary to exhibit a large enough degree of decreasing impatience.

The specific reason goes as follows. Suppose the representative household expects that other households increase consumption. Suppose further the representative household responds by increasing consumption. Such expectations and responses are consistent with the optimality and equilibrium conditions only if the degree of decreasing impatience is large.

Under decreasing impatience, the commodity market clearing condition in Eq. (5d) requires higher capital in order to satisfy higher consumption. The resulting decreasing marginal product of capital in the optimal capital condition in Eq. (5c) is satisfied because of intratemporal substitution due to decreasing impatience. Moreover, Eq. (5b) indicates a higher shadow price of the discount, but the shadow price of capital in Eq. (5a) is ambiguous. However, if there is exogenous or increasing impatience, the optimal capital condition in Eq. (5c) fails to satisfy and individuals do not increase their consumption in response to the expectations of other's higher consumption.

Intuitively, decreasing impatience causes intratemporal substitution so that intertemporal inefficiency produced by consumption externalities results in re-optimization in capital demand. Thus, individual consumption affects optimal capital demand via an individual's best response to expectations about consumption externalities. If the degree of decreasing impatience is sufficiently large, expectations about consumption are self-fulfilling in equilibrium.

To summarize the local dynamic properties, we obtain

Proposition 2. *Under Assumptions 1–3, the dynamic equilibrium in neoclassical growth models with consumption externality is indeterminate if there is a large degree of decreasing impatience.*

4. Concluding remarks

Previous analyses have shown that consumption externalities generate intertemporal inefficiency, but they cannot be a source of indeterminacy. Our analysis shows that it is possible for consumption externalities to establish local indeterminacy in a one-sector neoclassical growth model, if there is a large enough degree of decreasing impatience.

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