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“Habits and Endogenous Investment Fluctuations”

Been-Lon Chen

Yu-Shan Hsu

Kazuo Mino

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# Habits and Endogenous Investment Fluctuations\*

Been-Lon Chen\*\*

Institute of Economics, Academia Sinica, Taiwan

Yu-Shan Hsu

Department of Economics, National Chung Cheng University, Taiwan

Kazuo Mino

Kyoto Institute of Economic Research, Kyoto University, Japan

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## Abstract

This paper envisages whether an external habit effect can produce indeterminate equilibrium paths thereby generating endogenous investment fluctuations. In an otherwise standard optimal growth model with leisure, we find that an external habit effect can cause endogenous investment fluctuations if there is a proper habit effect together with a proper intertemporal elasticity of substitution. In a calibrated version of the model, we find that endogenous investment fluctuations are plausible when the habit effect is negative with the “catching up with the Joneses” effect.

**Keywords:** catching up with the Joneses; habit; indeterminacy; one-sector growth model.

**JEL classification:** E21; E32

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\*\* Corresponding address: Been-Lon Chen, the Institute of Economics, Academia Sinica, 128 Academia Road Section 2, Taipei 11529, TAIWAN. Phone: (886-2)27822791 ext. 309; Fax: (886-2)2785-3946; Email: [bchen@econ.sinica.edu.tw](mailto:bchen@econ.sinica.edu.tw).

## 1. Introduction

Recent years have seen voluminous research in economics and finance that considered the habit effect.<sup>1</sup> In finance, for example, Constantinidis (1990), Abel (1990, 1999) and Campbell and Cochrane (1999) resolved the equity premium puzzle in the context of the representative-agent, consumption-based model with a habit added to the standard power utility function. In economics, the habit effect has been widely adopted in business cycle models to help account for aggregate fluctuations (Ljungqvist and Uhlig, 2000; Boldrin et al., 2001). The habit effect also assists in improving responses to monetary-policy actions (Fuhrer, 2000; Mansoorian and Michelis, 2005) and explaining the process of economic growth (Carroll et al., 1997; Alvarez-Cuadrado et al., 2004; Doi and Mino, 2008). Most of these existing papers considered an external habit effect. By allowing for adjacent complementarity in consumption, an external habit effect affects the intertemporal elasticity of substitution and thus helps to resolve the equity premium puzzle and to account for aggregate fluctuations and economic growth. Each of these papers found a determinate equilibrium path toward a steady state.

This paper studies an otherwise standard optimal growth model and investigates whether an external habit effect can create indeterminate equilibrium paths.<sup>2</sup> In otherwise standard optimal growth models, indeterminate equilibrium paths and thereafter equilibrium indeterminacy were first found by Benhabib and Farmer (1994) and Farmer and Guo (1994). Indeterminate equilibrium paths in these two papers required increasing social returns in

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<sup>1</sup> The concept of the habit effect may be traced to Hume (1748) who argued that preferences were influenced not simply by what a person did in the past, what his parents did, and what contemporary peers were doing but also by the behavior of past generations of peers. Similar contemporary ideas dated to Marshall (1898), Duesenberry (1949), Leibenstein (1950), and Hicks (1965). Subsequent research has identified two kinds of habit formation. One is referred to as external habit formation, expressed in terms of the past consumption of some outside reference group, usually the past consumption of the overall economy, and is the focus in the current study. The other is termed internal habit formation based upon an individual's own past consumption level.

<sup>2</sup> Evidence of the habit effect has been prevalent and was confirmed as early as the 1950s by Brown (1952) who estimated the habit effect by using the aggregate data in Canada. Recently, a growing body of empirical evidence concerning external habit persistence has emerged. Using time-series data in the U.S., Fuhrer (2000) strongly supported the hypothesis of consumption habit formation. More recently, using panel data in the U.S., Ravina (2005) and Korniotis (2010) both have provided strong evidence about external habit persistence in household consumption choices. Using data from other countries, supportive evidence of external habits has been offered by, among others, van de Stadt *et al.* (1985) who used longitudinal panel surveys of households in the Netherlands, Case (1991) who used an Indonesian socio-economic survey, and Carrasco *et al.* (2005) who used household panel data from Spain.

production that were higher than empirical estimates derived later by Basu and Fernald (1997) and others. This brought about research using more general specifications of production to revealing indeterminate equilibrium paths with much lower increasing social returns.<sup>3</sup> This approach also led to investigations employing more general specifications of preference further highlighting indeterminate equilibrium paths based on external current consumption flows.<sup>4</sup> However, with the exception of Auray *et al.* (2002, 2005), papers with more general specifications of preference did not analyze the relationship between the habit effect and indeterminate equilibrium paths.

Auray *et al.* (2002, 2005) found indeterminate equilibrium paths in dynamic models with labor as the only input and with a cash-in-advance (CIA) constraint and a habit effect. Although there was the interplay of the habit effect and the CIA constraint in these two models, indeterminate equilibrium paths resulted mainly from the CIA constraint with a given exogenous growth rate of money supply, a property that has been known since Woodford's (1994) contribution.<sup>5</sup> Indeed, even when the habit effect was internal as specified in Auray *et al.* (2005), indeterminate equilibrium paths spawned. An external habit effect alone did not generate indeterminate equilibrium paths in these two papers. In particular, Auray *et al.* (2002, 2005) did not consider capital. Thus, while the models by Auray *et al.* (2002, 2005) can successfully explain large endogenous fluctuations of labor hours in business cycles, they cannot explain large endogenous fluctuations of investment which is an important feature in the course of business cycles. By contrast, we analyze an otherwise standard optimal growth model save for the habit effect. Thus, indeterminate equilibrium paths in our model are resulted from the habit effect on its own. Moreover, our equilibrium manifests endogenous fluctuations in investment.

Specifically, we study a standard optimal growth model with leisure whose equilibrium path toward steady state is known to be determinate (e.g., Blanchard and Fischer, 1989). We add in an external habit effect.<sup>6</sup> We expect that when there is a proper external habit effect

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<sup>3</sup> See Wen (1998) in a one-sector model and Benhabib and Farmer (1996) and Benhabib and Nishimura (1998) in two-sector models.

<sup>4</sup> See Drugeon (1998), Chen and Hsu (2007) and Alonso-Carrera *et al.* (2008).

<sup>5</sup> Woodford (1994) found that in a monetary economy with a CIA constraint, equilibrium is indeterminate under a given exogenous growth rate of money supply but is unique under a pegged nominal interest rate.

<sup>6</sup> As in Auray *et al.* (2002, 2005), there is only a single aggregate good and habit arises from overall

together with a proper intertemporal elasticity of substitution (henceforth, IES), equilibrium paths are indeterminate featuring endogenous investment fluctuations. We posit that a proper external habit effect generates social complementarity in consumption that is ignored by the representative, rational agent. This creates a self-reinforcing mechanism linking private and collective choices so expectations-driven equilibrium paths may potentially arise. Through affecting the labor and leisure tradeoff, an external habit effect indirectly affects the social IES. Therefore, a proper IES is required in order to ease the tradeoff between consumption and investment/savings.

In the theoretical model, we find that a positive habit effect and a negative habit effect both can generate endogenous investment fluctuations. In a calibrated version of the model, we find that endogenous investment fluctuations are empirically plausible only when the habit effect is negative with the “catching up with the Joneses” effect.

As developed below, Section 2 sets up the model and analyzes the steady state. Section 3 investigates the conditions of endogenous investment fluctuations and offers quantitative analysis. Finally, concluding remarks are made in Section 4.

## 2. The Model

Time is continuous. The basic model is an otherwise standard optimal growth model with leisure wherein we consider the habit effect. The economy is populated by representative households with infinite lives and the population of households is fixed with a unit measure. The representative agent is endowed with one unit of time. At each point in time  $t$  the agent allocates to the market a fraction  $l_t$  as labor services and the remainder  $1-l_t$  as leisure. An agent obtains utility from his or her own consumption ( $c_t$ ) and leisurely activities. Moreover, an agent’s utility is affected by the consumption habit in the society,  $H_t$ . The lifetime utility is

$$\int_0^{\infty} e^{-\rho t} u(c_t, H_t, 1-l_t) dt, \quad (1)$$

where  $u$  is the level of instantaneous utility and  $\rho > 0$  is the instantaneous discount rate.

We assume that the instantaneous utility is twice continuously differentiable with the

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consumptive habits as demonstrated in our paper. This concept of consumptive habit is different from that of a “deep habit” studied in models with many consumption goods in which private agents form habits not only from their overall consumption levels but also from the consumption of different goods (Ravn *et al.*, 2006, 2010).

following properties: (i)  $u_i > 0 > u_{ii}$ ,  $i=1, 3$ , (ii)  $u_{11} + u_{12} < 0$  and (iii)  $u_{11}u_{33} - (u_{13})^2 \geq 0$ . In (i), the utility displays the standard positive and decreasing marginal utility of consumption and leisure. The assumption (ii) guarantees a positive IES, while the assumption (iii) ensures a jointly concave utility in own consumption and leisure. We also assume that consumption and leisure are both normal goods. While we do not impose the sign of  $u_2$  as the habit effect may be positive or negative, it is worth noting that if  $u_{12} > 0$ , then past habit in the society enhances an agent's marginal utility of consumption and there is thus the "catching up with the Joneses" effect (Abel, 1990).

The consumption habit is a stock in the society at time  $t$ . Following Ryder and Heal (1973, p.2), habit is accumulated from the distant past to the present and is a weighted average of past consumption flows in the economy, with weights declining exponentially in the distant past. Specifically, habit is

$$H_t = \beta e^{-\beta t} \int_{-\infty}^t e^{\beta \tau} C_\tau d\tau, \beta \geq 0,$$

where  $C_t$  is average consumption in the society in  $t$ .<sup>7</sup> The above expression may be rewritten as follows.

$$\dot{H} = \beta(C_t - H_t), \quad 0 < \beta < \infty, \quad \text{with } H_0 \text{ given.} \quad (2)$$

This law of motion says that the society's future habit is increased by the difference between current average consumption and existing habit adjusted by a non-negative, finite coefficient,  $\beta$ . The coefficient of habit formation characterizes the strength of the influence that current average consumption affects future habits. It is clear that the larger the value of  $\beta$ , the larger the influence of current average consumption in the formation of future habits. Two extreme cases are as follows. If  $\beta=0$ , then  $H_t$  is fixed and is given by  $H_0$  for all  $t$ . In this case, our model is reduced to the standard one-sector optimal growth model with leisure. Conversely, if  $\beta=\infty$ , then the habit adjusts so fast such that the habit in the society is completely determined by current average consumption; namely,  $H_t=C_t$ . In this case, our model is reduced to a one-sector growth model with current consumption externalities (cf. Liu and Turnovsky, 2005).

The economy has a continuum of firms with a unit measure. A firm is endowed with a

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<sup>7</sup> The formulation is different from that in Auray et al. (2002, 2005) which assumed  $H_t=C_{t-1}$  and thus their habit is determined by the society's consumption last period. The Ryder and Heal's formulation is more general. Constantanidis (1990) used the same habit formation regime as ours except his habit is internal.

neoclassical production technology  $f(k_t, l_t)$  where  $k_t$  is per capita capital stock and the marginal product of each input is positive and is decreasing in input. Firms are competitive and are thus price takers. Since the Cobb-Douglas technology is used in the indeterminacy literature following Benhabib and Farmer (1994) and Farmer and Guo (1994), we use it here given by

$$f(k_t, l_t) = Ak_t^\alpha l_t^{1-\alpha}, \quad 0 < \alpha < 1.$$

The optimization problem in a decentralized economy is as follows. First, given wage rates,  $w_t$ , and rental rates of capital,  $r_t$ , a representative firm at each point in time  $t$  chooses optimal demands for capital and labor in order to maximize its profits. Denote  $\delta$  as the depreciation rate of capital. The optimal conditions are as follows.

$$w_t = (1-\alpha)Ak_t^\alpha l_t^{-\alpha}, \quad (3a)$$

$$r_t = \alpha Ak_t^{\alpha-1} l_t^{1-\alpha} - \delta. \quad (3b)$$

Next, taking wage rates and rental rates as given by the market and the habit as given by the society, the representative household's problem is to tradeoff between consumption and savings and tradeoff between working and leisure in order to maximize her lifetime utility (1), subject to the following budget constraint

$$\dot{k}_t = w_t l_t + r_t k_t - c_t. \quad (4)$$

The optimal conditions are

$$u_1(c_t, H_t, 1-l_t) = \lambda_t, \quad (5a)$$

$$u_3(c_t, H_t, 1-l_t) = w_t \lambda_t, \quad (5b)$$

$$\dot{\lambda}_t = (\rho - r_t) \lambda_t, \quad (5c)$$

along with  $\lim_{t \rightarrow \infty} e^{-\rho t} \lambda_t k_t = 0$ , which is the transversality condition. The variable  $\lambda_t$  is the co-state variable associated with capital and thus, the shadow price of capital. In these optimal conditions, (5a) and (5b) equates the marginal utility to the marginal cost for consumption and leisure, respectively, and (5c) is the Euler equation for capital.

For given  $k_0$  and  $H_0$ , competitive equilibrium is a path  $\{k, l, H, w, r, \lambda\}$  with  $c=C$ , and is determined by (2), (3a)-(3b), (4) and (5a)-(5b).

To determine the competitive equilibrium, first, we simplify equilibrium conditions by use of (3a), (5a) and (5b) and obtain

$$(1-\alpha)Ak_t^\alpha l_t^{-\alpha} = \frac{u_3(C_t, H_t, 1-l_t)}{u_1(C_t, H_t, 1-l_t)}, \quad (6)$$

which equates the marginal product of labor to the marginal rate of substitution between consumption and leisure.

Next, we use (3a) and (3b) to rewrite the budget constraint (4) as a resource constraint.

$$\dot{k}_t = Ak_t^\alpha l_t^{1-\alpha} - \delta k_t - C_t. \quad (7)$$

Therefore, equilibrium conditions are simplified to (2), (5b), (6) and (7). In a steady state,  $\dot{k} = \dot{\lambda} = \dot{H} = 0$  and thus (2) indicates  $H^* = C^*$ . Then, (5b), (6) and (7) determine  $C^*$ ,  $l^*$  and  $k^*$  in the same way as does in an otherwise standard growth model with leisure.

### 3. Endogenous Investment Fluctuations

In this section, we study endogenous investment fluctuations by investigating the dynamic property of the model.

#### 3.1 The Conditions of Endogenous Investment Fluctuations

In this subsection, we will show that, under a finite, sufficiently large coefficient of habit formation, if there is a proper habit effect together with a proper IES, then equilibrium paths toward a steady state are indeterminate and thus there are endogenous investment fluctuations.

We simplify the dynamic system into three variables. First, by the implicit function theorem, (6) leads to the following relationship

$$l_t = l(C_t, k_t, H_t). \quad (8)$$

Differentiating (5a) with respect to time, with the use of (3b), (5c) and (8), yields the Keynes-Ramsey condition as follows.

$$\dot{C}_t = \frac{C_t}{\Omega} \{ (\eta - \alpha - \varepsilon)(f_k - \delta - \rho) + \frac{\alpha\varepsilon}{k_t} \dot{k}_t + \frac{1}{H_t} [(\alpha + \varepsilon - \eta)\zeta + \varepsilon\chi] \dot{H}_t \}, \quad (9)$$

where  $\Omega \equiv (\eta - \alpha)\sigma + \varepsilon\phi < 0$ ,

$$\eta \equiv u_{33}l(c_t, k_t, H_t) / u_3 < 0,$$

$$\sigma \equiv -u_{11}c_t / u_1 > 0,$$

$$\varepsilon \equiv u_{13}l(c_t, k_t, H_t) / u_1,$$

$$\phi \equiv u_{13}c_t / u_3,$$

$$\zeta \equiv -u_{12}H_t / u_1,$$

$$\chi \equiv (u_{12} / u_1 - u_{23} / u_3)H_t.$$



Note that if the preference exhibits the “catching up with the Joneses” effect, then it is more likely  $\chi > 0$ . Under the concavity condition,  $\varepsilon\phi/\sigma + \eta < 0$  and thus,  $\eta\sigma + \varepsilon\phi < 0$ , which indicates  $\Omega < 0$ . It is required that the IES of consumption is positive:  $(\eta - \alpha - \varepsilon)/\Omega > 0$ . This implies  $(\eta - \alpha - \varepsilon) < 0$ . Moreover, the assumption of consumption and leisure both being normal goods gives

$$(\varepsilon - \eta)(\phi + \sigma) > 0. \quad (10)$$

The dynamic equilibrium system consists of (2), (7) and (9) and determines the dynamic path of  $c_t$ ,  $k_t$  and  $H_t$ . If we take Taylor’s linear expansion of the dynamic equilibrium system in the neighborhood of the steady state, along with the use of (8), we obtain a Jacobean matrix, denoted as  $J$ . The characteristic polynomial of the Jacobean matrix is

$$G(\omega) = -\omega^3 + Tr(J)\omega^2 - Ds(J)\omega + Det(J) = 0, \quad (11)$$

where  $Det(J)$  is the determinant,  $Tr(J)$  is the trace,  $Ds(J)$  is the sum of the determinant of the second-order principal minors, of the Jacobean matrix  $J$ , given, respectively, by

$$Det(J) = (-\beta) \frac{(1-\alpha)(\rho+\delta)[\rho+\delta(1-\alpha)]}{\Omega\alpha} [(\phi + \sigma - \eta + \varepsilon) - \chi] \quad (12a)$$

$$Tr(J) = T + \beta \Gamma_1, \quad (12b)$$

$$Ds(J) = M + \beta \Gamma_2, \quad (12c)$$

where  $\Gamma_1 = \frac{1}{(-\Omega)} [(\eta - \alpha)(\zeta + \sigma) - \varepsilon(\zeta + \chi - \phi)]$ ,

$$\Gamma_2 = \rho\Gamma_1 + \frac{1}{(-\Omega)} \{ (1-\alpha)(\rho+\delta)(\zeta + \chi - \phi) + [\rho + \delta(1-\alpha)]\varepsilon \},$$

$$M = \frac{(1-\alpha)(\rho+\delta)[\rho+\delta(1-\alpha)]}{\Omega\alpha} (\phi + \sigma - \eta + \varepsilon) < 0, \quad ^8$$

$$T = \frac{1}{(-\Omega)} \{ \alpha\rho(\phi + \sigma) - [\rho + \delta(1-\alpha)](\eta - \varepsilon) - \rho(\eta\sigma + \varepsilon\phi) \} > 0. \quad ^9$$

As the economic system includes two state variables with initial values determined at  $k_0$  and  $H_0$ , a steady state is a sink and the equilibrium path toward the steady state is indeterminate if the number of eigenvalues with negative real parts is three. Examining the polynomial function  $G(\omega)$ , it is clear that  $G(\omega) = -\infty$  when  $\omega = \infty$  and  $G(\omega) = \infty$  when  $\omega = -\infty$ . In view of the Routh-Hurwitz stability criterion, the necessary conditions for the presence of three stable roots are: (i)  $G(0) = Det(J) < 0$ , (ii)  $G'(0) = -Ds(J) < 0$ , (iii)  $Tr(J) < 0$  and (iv) –

<sup>8</sup>  $M$  is the determinant in the standard growth model (in the case of  $\beta=0$ ) and thus  $M < 0$ .

<sup>9</sup>  $M < 0$  and (10) together indicate  $(\phi + \sigma) > 0$  and  $(\eta - \varepsilon) < 0$ . Moreover, the concavity condition implies  $(\eta\sigma + \varepsilon\phi) < 0$ . It follows that  $T > 0$ .

$$Ds(J)+Det(J)/Tr(J)<0.$$

We can obtain the following result.

**Theorem 1.** *Suppose that the coefficient of habit formation  $0<\beta<\infty$  is sufficiently large. Then under  $\chi>\phi+\sigma-\eta+\varepsilon$ ,  $\Gamma_1<0$  and  $\Gamma_2>0$ , the steady state is a sink.*

Proof: See Appendix.

It is worth noting that the restriction of  $\beta>0$  makes our model different from the standard optimal growth model which emerges under the case of  $\beta=0$ . On the other hand, the restriction of  $\beta<\infty$  makes our model different from a one-sector growth with current consumption externality studied by Liu and Turnovsky (2005) and Alonso-Carrera *et al.* (2008).<sup>10</sup> Finally, the restriction of a large  $\beta$  demands a large weight of current average consumption in the formation of future habits.

Examining the conditions in Theorem 1, the condition  $\Gamma_1<0$  requires  $(\eta-\alpha-\varepsilon)/\Omega>0$  which calls for a proper IES. Moreover, as the external habit effect appears in  $\chi$  and  $\varsigma$ , the conditions  $\chi>\phi+\sigma-\eta+\varepsilon$  and  $\Gamma_2>0$  require the external habit effect to be in a proper range.

Theorem 1 thus stipulates that under a sufficiently large coefficient of habit formation, the steady state is a sink and there are endogenous investment fluctuations if there is a proper habit effect together with a proper IES. A proper degree of the external habit effect assures a social complementarity in consumption which generates self-reinforcing mechanism connecting individual and joint choices so expectations-driven equilibrium paths may potentially arise. A proper IES makes easier the tradeoff between consumption and investment/savings so consumption can be increased or decreased more easily and thus, investment/savings can be decreased or increased more easily.

The intuition for generating endogenous investment fluctuations goes as follows. In order to exhibit endogenous investment fluctuations, an expectations-driven equilibrium requires higher investment to raise the marginal product of capital and thus the return to capital. Starting with an equilibrium path, a proper external habit effect can produce a self-reinforcing mechanism linking private and collective choices. If the representative agent expects that all other agents in the society will increase investment/savings and reduce

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<sup>10</sup> Both Liu and Turnovsky (2005) and Alonso-Carrera *et al.* (2008) studied an otherwise standard growth model except for including current consumption externalities. Liu and Turnovsky (2005) focused on the efficiency of the competitive equilibrium in the long run, while Alonso-Carrera *et al.* (2008) analyzed equilibrium indeterminacy.

consumption, the agent will also expect that all other agents will decrease leisure, as consumption and leisure are complements. When there is a proper IES, consumption and leisure both can be reduced sufficiently and the labor supply in the society is increased sufficiently. Through the Pareto complementarity, a large labor supply enhances the marginal product of capital sufficiently large that the direct negative effect of investment on the marginal product of capital is dominated by the indirect positive effect through a larger labor supply. Knowing these, the agent will decrease consumption and increase investment which earns a higher return. The expectations are thus self-fulfilling. Therefore, in equilibrium, depending on the expectations, all economic agents may choose to increase investment today or to reduce investment today. There are thus endogenous fluctuations in aggregate investment dictated by expectations.

The intuition shown above can be confirmed more clearly in a discrete-time setting. The discrete-time version of our model is to maximize the discounted sum of future utilities subject to a periodic budget constraint and the following habit accumulation equation<sup>11</sup>

$$H_{t+1} = \beta(C_t - H_t) + H_t, \quad \beta > 0.$$

Here, we assume  $u_{12} > 0$  ("catching up with the Joneses" effect) and  $u_{13} > 0$  (Edgeworth complementarity between consumption and leisure).

In the formulation the Euler equation for the optimal consumption is given by

$$\frac{u_1(C_{t+1}, H_{t+1}, 1-l_{t+1})}{u_1(C_t, H_t, 1-l_t)} = \frac{1+\rho}{1+r_{t+1}-\delta},$$

where  $r_{t+1} = \alpha A k_{t+1}^{\alpha-1} l_{t+1}^{1-\alpha}$  is the rate of return to capital in period  $t+1$ .

Now suppose that at the beginning of period  $t$  the economy stays in the steady state. We assume that a sunspot shock makes the households anticipate a rise in the rate of return to capital in period  $t+1$ . From the Euler equation an anticipated rise in  $r_{t+1}$  substitutes the current consumption  $C_t$  with the next period's consumption  $C_{t+1}$ , and thus  $C_t$  decreases and  $C_{t+1}$  will increase. If  $\beta=0$  so that the habit stock stays constant over time ( $H_t=H_{t+1}=H$ ), then due to the complementarity between consumption and leisure,  $l_t$  rises and  $l_{t+1}$  falls. As a result of a rise in capital stock due to a higher saving in period  $t$  and a reduction in labor supply in  $t+1$ , the rate of return to capital  $r_{t+1}$  will decline and, hence, the initial expectations are not

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<sup>11</sup> The discounted sum of utility is  $\sum_{t=0}^{\infty} (\frac{1}{1+\rho})^t u(c_t, H_t, 1-l_t)$ ,  $\rho > 0$  and the budget constraint is  $k_{t+1} = A k_t^\alpha l_t^{1-\alpha} + (1-\delta)k_t - c_t$ .

self-fulfilling.

When  $\beta > 0$ , the habit accumulation equation and the Euler equation yield

$$\frac{u_1(C_{t+1}, \beta C_t + (1-\beta)H_t, 1-l_{t+1})}{u_1(C_t, H_t, 1-l_t)} = \frac{1+\rho}{1+r_{t+1}-\delta}.$$

Note that a decrease in  $C_t$  lowers  $H_{t+1}$  through the habit accumulation equation. Other things being equal, a fall in  $H_{t+1}$  reduces the marginal utility in  $t+1$  because of our assumption  $u_{12} > 0$ . This effect would be large if the adjustment speed  $\beta$  is sufficiently high. Such an external effect in period  $t+1$  may not decrease but increase  $l_{t+1}$ . Therefore, a sufficiently large impact of the change in habit stock would produce a larger labor supply in period  $t+1$ , which may offset the decrease in the marginal product of capital in period  $t+1$  caused by a higher investment in period  $t$ . If this is the case, the initial expectations are self fulfilled and sunspot-driven investment fluctuations emerge.

### 3.2 Quantitative Analysis

In this subsection, we employ a parametric model and envisage whether endogenous investment fluctuations are quantitatively plausible.

The utility is assumed to take the following constant elasticity of substitution (CES) form,

$$u(c_t, H_t, 1-l_t) = [(1-\mu)(c_t H_t^\psi)^{1-\nu} + \mu(1-l_t)^{1-\nu}]^{\frac{1}{1-\nu}}. \quad (13)$$

In the utility, parameter  $\mu > 0$  is the share of leisure relative to consumption. This utility function satisfies the joint concavity and is non-separable in  $c$  and  $1-l$ . The parameter  $\nu \geq 0$  and  $1/\nu$  measures the elasticity of substitution (henceforth, ES) between consumption and leisure. The utility is general and includes the following three special cases: (i) when  $\nu=0$ , the ES between  $c_t$  and  $l_t$  is infinite and the utility is a linear form; (ii) when  $\nu=1$ , the ES between  $c_t$  and  $l_t$  is one and the utility is a Cobb-Douglas form; (iii) when  $\nu=\infty$ , the ES between  $c_t$  and  $l_t$  is zero and the utility is a Leontief form. Moreover, as we will see below,  $1/\nu$  also measures the degree of the IES as the IES is increasing in the value of  $1/\nu$ . Parameter  $\psi$  determines the degree of an external habit effect. Other things being equal, a larger stock of habit reduces an agent's utility if  $\psi < 0$  while a larger stock of habit increases an agent's utility if  $\psi > 0$ . In particular, when  $\psi(1-\nu) > 0$ , the habit exhibits the "catching up with the Joneses" effect, which implies either (i)  $\psi > 0$  and  $1/\nu > 1$  or (ii)  $\psi < 0$  and  $1/\nu < 1$ .

Under the CES utility in (13), if we denote  $\Delta = \alpha + \nu l / (1-l) > 0$ , the Keynes-Ramsey

condition in (9) is

$$\dot{C}_t = \frac{C_t}{\sigma - \varepsilon \nu / \Delta} [\alpha A(k_t)^{\alpha-1} (l_t)^{1-\alpha} - (\rho + \delta) - \frac{\alpha \varepsilon}{\Delta k_t} \dot{k}_t - \frac{1}{H_t} (\zeta + \frac{\varepsilon \psi (1-\nu)}{\Delta}) \dot{H}_t]. \quad (14)$$

The steady state is determined by (5b), (6) and (7). Using the CES utility in (13), these three equations together lead to

$$l^{*\frac{\psi(1-\nu)-\nu}{\nu}} (1-l^*) = \left[ \frac{\mu}{1-\mu} \frac{1}{A(1-\alpha)} \left( \frac{\rho+\delta}{\alpha A} \right)^{\frac{\alpha+\psi(1-\nu)-\nu}{1-\alpha}} \left( \frac{\alpha}{\rho+\delta(1-\alpha)} \right)^{\psi(1-\nu)-\nu} \right]^{\frac{1}{\nu}}. \quad (15)$$

Equation (15) determines the level of  $l^*$  in steady state. When  $l^*$  is obtained, we can solve  $k^*$ ,  $C^*$  and  $H^*$  by  $k^* = \left( \frac{\alpha A}{\rho+\delta} \right)^{\frac{1}{1-\alpha}} l^*$  and  $H^* = C^* = \frac{\rho+\delta(1-\alpha)}{\alpha} \left( \frac{\alpha A}{\rho+\delta} \right)^{\frac{1}{1-\alpha}} l^*$ .

To determine  $l^*$ , in a figure with  $l$  on the horizontal axis, the right-hand side of (15), denoted by  $\Lambda(\psi)$ , is independent of  $l$  and is thus a horizontal locus. The left-hand side of (15), denoted by  $\Phi(l, \psi)$ , have two types of shape depending on the value of  $\psi(1-\nu)-\nu$ .<sup>12</sup>

In Figure 1, which is under  $\psi(1-\nu) < \nu$ , the locus  $\Phi(l, \psi)$  is monotonically decreasing in  $l$  for  $l \leq 1$  with the value of  $\Phi(l, \psi)$  decreasing from infinite when  $l=0$  to zero when  $l=1$ . In this case, there is a unique steady state,  $l^*$ .

[Insert Figure 1 here]

In Figure 2, which is under  $\psi(1-\nu) > \nu$ , the locus  $\Phi(l, \psi)$  is first increasing and then decreasing in  $l$  for  $l \leq 1$ , with the value of  $\Phi(l, \psi)$  increasing from zero when  $l=0$ , reaching a top when  $l^m = 1 - \nu / [\psi(1-\nu)] < 1$  and finally returning to zero when  $l=1$ . In this case, there are two steady states with the employment at  $l_1^*$  and  $l_2^*$ .

[Insert Figure 2 here]

In the parameter region wherein a sink may arise, a negative determinant of the Jacobean matrix is equivalent to  $\psi(1-\nu) > \nu(1-l)$ , which implies  $\psi(1-\nu) > \nu$  and  $l < l^m$ . Thus, the steady state  $l^*$  in Figure 1 is not a sink. In Figure 2, although  $l_2^*$  is not a sink, the steady state  $l_1^*$  may be a sink and thus endogenous investment fluctuations may emerge.

It is worth noting that the condition  $\psi(1-\nu) > \nu > 0$  stipulates that endogenous investment fluctuations may arise only under either (i)  $\psi > 0$  and  $1/\nu > 1$ , or (ii)  $\psi < 0$  and  $1/\nu < 1$ . This indicates that if the utility is a Cobb-Douglas form (which arises under  $\nu=1$ ), the habit effect cannot bring about endogenous investment fluctuations in an otherwise standard optimal growth model.

<sup>12</sup> The slope of  $\Phi(l, \psi)$  is dictated by  $\frac{d\Phi(l, \psi)}{dl} = \frac{1}{\nu} l^{*\psi(\frac{1}{\nu}-1)-2} \{[\psi(1-\nu) - \nu] - \psi(1-\nu)l\}$ .

[Insert Figures 1 and 2 here]

Characterizing the steady state  $l_1^*$  in Figure 2, the habit effect affects the employment level and thus,  $l_1^* = l^*(\psi)$ . First, under (i)  $\psi > 0$  and  $1/\nu > 1$ , when  $\psi$  increases, the habit effect is larger and the value of  $\psi(1-\nu)$  increases. Both loci  $\Lambda(\psi)$  and  $\Phi(l, \psi)$  then shift downwards with  $\Lambda(\psi)$  shifting more than  $\Phi(l, \psi)$  at the original employment level  $l_1^*$ . See  $\Lambda^1(\psi > 0)$  and  $\Phi^1(l, \psi)$  in Figure 2. As a result, the steady state moves to  $E_0^1$  with a lower steady-state employment level  $l_1^1 < l_1^*$ .

Next, under (ii)  $\psi < 0$  and  $1/\nu < 1$ , when  $\psi$  decreases, the habit effect is increased and the value of  $\psi(1-\nu)$  is larger. Both loci  $\Lambda(\psi)$  and  $\Phi(l, \psi)$  also shift downwards, but  $\Lambda(\psi)$  shifts downward less than  $\Phi(l, \psi)$  at the original employment level  $l_1^*$ . Thus, under  $\psi < 0$ , if we assume that a decrease in  $\psi$  also shifts  $\Phi(l, \psi)$  to  $\Phi^1(l, \psi)$ , then  $\Lambda(\psi)$  is shifted downward to  $\Lambda^2(\psi < 0)$  that is less than  $\Lambda^1(\psi > 0)$ . See  $\Lambda^2(\psi < 0)$  and  $\Phi^1(l, \psi)$  in Figure 2. As a result, the steady state moves to  $E_0^2$  with a higher steady-state employment level  $l_1^2 > l_1^*$ .

According to (14), the IES is given by

$$\frac{1}{\sigma - \varepsilon \nu / \Delta} = \frac{1}{\nu} \left( 1 + \frac{\nu}{\alpha} \frac{l^*(\psi)}{1 - l^*(\psi)} \right) \left[ 1 + \frac{\rho + \delta(1 - \alpha)}{(1 - \alpha)(\rho + \delta)} \frac{l^*(\psi)}{1 - l^*(\psi)} \right] > 0, \quad (16)$$

which is increasing in  $1/\nu$ . In the case when  $\psi > 0$ , a higher habit effect reduces the IES indirectly through a lower  $l^*$ . On the other hand, when  $\psi < 0$ , a higher habit effect increases the IES indirectly through a higher  $l^*$ .

Denote

$$E \equiv \{\alpha \rho \nu \Gamma_1 + [\rho + \delta(1 - \alpha)] \frac{\nu l}{(1 - l)}\} (1 - Q) - (1 - \alpha)(\rho + \delta) \nu Q,$$

$$D \equiv \alpha(1 + \psi)(1 - Q) - \frac{\nu l^*}{1 - l^*} Q,$$

$$Q \equiv \frac{(1 - \mu)}{1 - \mu + \mu(1 - l^*)^{1 - \nu} (ml^*)^{-(1 - \nu)(1 + \psi)}} > 0.$$

To characterize the conditions of endogenous investment fluctuations under the CES utility in (13), first, under a given value of  $\beta$ , it is required that  $\Gamma_1 < 0$ . This condition demands  $\psi/\nu > D/\alpha$  if  $\psi > 0$  and  $\psi/\nu < D/\alpha$  if  $\psi < 0$ .

Next, the condition  $\chi > \phi + \sigma - \eta + \varepsilon$  is equivalent to  $\psi(1 - \nu) > \nu(1 - l) > 0$ . This condition may be read as either  $\psi < \psi_2 \equiv \nu / [(1 - \nu)(1 - l)]$  if  $\psi < 0$  and  $1/\nu < 1$ , or  $\psi > \psi_2$  if  $\psi > 0$  and  $1/\nu > 1$ .

Finally, the condition  $\Gamma_2 > 0$  requires  $\psi < \psi_1 \equiv E / [(1 - \alpha)(\rho + \delta) \nu Q]$ .

Thus, we obtain

Proposition 1. *Under the CES utility in (13) and a sufficiently large  $\beta$ , the conditions of endogenous investment fluctuations are*

- (i)  $-D[\alpha(-\psi)] < 1/\nu < 1$  and  $\psi < \min\{\psi_1, \psi_2\}$  if  $\psi < 0$ ,
- (ii)  $1/\nu > \max\{D/(\alpha\psi), 1\}$  and  $\psi_2 < \psi < \psi_1$  if  $\psi > 0$ .

Proposition 1 stipulates that both a positive habit effect and a negative habit effect can lead to endogenous investment fluctuations. When the habit effect is negative ( $\psi < 0$ ), a value of  $1/\nu$  smaller than one can give rise to endogenous investment fluctuations. However, when the habit effect is positive ( $\psi > 0$ ), a value of  $1/\nu$  larger than one is required in order to bring about endogenous investment fluctuations. The results come from the fact that a higher  $l^*$  increases the IES in (16). Under a negative habit effect, a larger habit effect (a smaller  $\psi$ ) increases the IES indirectly through a larger  $l^*$ . Therefore, a smaller  $1/\nu$  suffices to create endogenous investment fluctuations. Alternatively, under a positive habit effect, a larger habit effect (larger  $\psi$ ) lowers the IES indirectly through a smaller  $l^*$ . Thus, a larger  $1/\nu$  is needed in order to generate endogenous investment fluctuations.

In Proposition 1, while endogenous investment fluctuations may emerge under both a positive habit effect and a negative habit effect, as our calibration exercises below show, only a negative habit effect is empirically plausible.

Now, we quantitatively assess the plausibility of endogenous investment fluctuations in our model. We choose  $k^*/y^*=4$  and  $C^*/y^*=0.8$ , which are consistent with data in the US, and calibrate (14c) to obtain  $\delta=0.05$ . Furthermore, we normalize  $A=1$  and set  $\rho=0.04$ , and then use (14) to calibrate and obtain  $\alpha=0.36$ . We choose the coefficient of habit formation at  $\beta=0.35$ .<sup>13</sup> Although there is no empirical data about the values of  $\mu$ , we can choose  $\mu$  in order both to satisfy a positive IES in (16) and to ensure the existence of the steady state in (15). We set  $\mu=0.4$ . Since the dynamic property of the steady state depends on the interaction between  $\psi$  and  $\nu$ , we choose a combination of  $\psi$  and  $\nu$  that gives rise to endogenous investment fluctuations. We choose the pair  $\{\psi, \nu\}=\{-2.25, 2.5\}$  such that the calibrated value of  $l^*$  equal to 0.25 (Prescott, 2006).<sup>14</sup> Under this set of benchmark parameter

<sup>13</sup> While Constantinides (1990) employed  $\beta=0.6$ , Carroll *et al.* (1997) and Alvarez-Cuadrado *et al* (2004) used  $\beta=0.2$ . Our value lies within these existing values used.

<sup>14</sup> Prescott (2006) pointed out that the fraction of productive time allocated to market was 0.25 in the US.

values, the steady state is:  $k^*=2.181$ ,  $l^*=0.250$ ,  $y^*=0.545$  and  $H^*=C^*=0.436$ .

We are ready to study the empirical plausibility of endogenous investment fluctuations. For a given coefficient of habit formation  $\beta$  and a given degree of an external habit effect  $\psi$ , we will find the range of  $1/\nu$  under which endogenous investment fluctuations prevail. For the benchmark parameter values, we find that endogenous investment fluctuations arise if  $1 < \nu < 3.301$ , or equivalently if  $0.303 < 1/\nu < 1$ . See Table 1.

[Insert Table 1 here]

We conduct some robustness checks. First, if we increase the value of  $\beta$ , we find that the required range of  $1/\nu$  is the same. Next, if we increase the value of  $\psi$  and thus lower the degree of an external habit effect, the range the  $1/\nu$  decreases; if we decrease the value of  $\psi$  and thus increase the degree of an external habit effect, the range the  $1/\nu$  increases. Finally, if we increase the value of  $\psi$  so  $\psi > 0$ , we cannot find plausible values of  $\nu$  so the conditions of endogenous investment fluctuations are met. In Figure 3, we draw the range of  $(\psi, 1/\nu)$  that yields endogenous investment fluctuations. See the shaded area in Figure 3.

[Insert Figure 3 here]

Thus, our quantitative exercises indicate that endogenous investment fluctuations are empirically plausible only when the habit effect is negative,  $\psi < 0$ . Our results stipulate that endogenous investment fluctuations are empirically plausible only when the habit effect is negative that features the “catching up with the Joneses” effect,  $\psi(1-\nu) > 0$ . It is worth noting that a negative effect of a rise in the stock of external consumption habits means that an individual household feels jealous of the other households’ (past as well as current) consumption. In addition, the “catching up with the Joneses” effect implies that conformism prevails in consumption activities. Jealousy and conformism have been frequently assumed by empirical oriented studies on the models with consumption externalities, because the households’ Euler equations with these assumptions can be supported by the data more easily than the Euler equations with positive consumption externalities (i.e., admiration) and anti-conformism (i.e., the “falling-behind-the-Joneses” effect,  $u_{12} < 0$ ). Therefore, our numerical experiments suggest that the economy with rapid formation of external habits may produce indeterminacy of equilibrium under empirically plausible conditions.

#### 4. Concluding remarks

This paper investigates whether, in an otherwise standard optimal growth model, an



external habit effect can generate endogenous investment fluctuations. A standard optimal growth model with leisure yields a determinate investment path toward a steady state and thus fluctuations of investment are the result of exogenous shocks to technology, preferences and policies. By adding an external habit effect into an otherwise standard optimal growth model, we find that our model can endogenously generate investment fluctuations without resorting to exogenous shocks. The interplay of social complementarity in consumption and the intertemporal substitution between consumption and investment/savings is the key to generate endogenous investment fluctuations in our model.

We find that under a sufficiently large coefficient of habit formation, fluctuations in investment can emerge endogenously when there is a proper degree of an external habit effect together with a proper intertemporal elasticity of substitution. A proper degree of an external habit effect assures a social complementarity in consumption that is ignored by the representative, rational agent. This generates a self-reinforcing mechanism linking private and collective choices that may give rise to expectations-driven equilibrium. Through affecting the labor and leisure tradeoff, an external habit effect changes the social intertemporal elasticity of substitution. Thus, it is required a proper intertemporal elasticity of substitution in order to ease the tradeoff between consumption and investment/savings so consumption can be increased or decreased more easily and thus, investment can be decreased or increased more easily.

While a positive habit effect and a negative habit effect both may generate endogenous investment fluctuations, using a calibrated version of the model we find that only a negative habit effect can lead to endogenous investment fluctuations. The endogenous investment fluctuations are empirically plausible when the habit effect is negative that features the “catching up with the Joneses” effect.

## Appendix

In the Appendix, we prove the Theorem 1. If we take the linear Taylor’s expansion of the dynamic equilibrium system (2), (7) and (9) in the neighborhood of a steady state, along with the use of (8), we obtain

$$\begin{bmatrix} \dot{C}_t \\ \dot{k}_t \\ \dot{H}_t \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} & J_{13} \\ f_l l_c - 1 & f_k - \delta - f_l l_k & f_l l_H \\ \beta & 0 & -\beta \end{bmatrix} \begin{bmatrix} C_t - C^* \\ k_t - k^* \\ H_t - H^* \end{bmatrix}, \quad (\text{A1})$$

where  $J_{11} = \frac{c}{\Omega} \{[(\alpha + \varepsilon - \eta)\zeta + \varepsilon\chi] \frac{\beta}{H} + (\eta - \varepsilon - \alpha)f_{kl}l_c + \frac{\alpha\varepsilon}{k}(f_1l_c - 1)\}$ ,

$$J_{12} = \frac{c}{\Omega} \{(\eta - \varepsilon - \alpha)(f_{kl}l_k + f_{kk}) + \frac{\alpha\varepsilon}{k}(f_1l_k + f_k - \delta)\},$$

$$J_{13} = \frac{c}{\Omega} \{ -[(\alpha + \varepsilon - \eta)\zeta + \varepsilon\chi] \frac{\beta}{H} + (\eta - \varepsilon - \alpha)f_{kl}l_H + \frac{\alpha\varepsilon}{k}f_1l_H \}.$$

Let  $J$  denote the Jacobean matrix in (A1) and  $\omega$  denote its corresponding eigenvalue. The characteristic polynomial is in (11), with  $Det(J)$ ,  $Tr(J)$  and  $Ds(J)$  defined in (12a)-(12c).

It is clear from (11) that  $G(\omega)=-\infty$  when  $\omega=\infty$  and  $G(\omega)=\infty$  when  $\omega=-\infty$ . A sink requires three stable roots. The necessary conditions for the presence of three stable roots are: (i)  $G(0)=Det(J)<0$  and (ii)  $G'(0)=-Ds(J)<0$ . Moreover, according to the Routh-Hurwitz theorem, the requirement of no eigenvalues with positive real parts in the above characteristic polynomial suggests no variation in signs in the following series:  $\{-1, Tr(J), -Ds(J)+Det(J)/Tr(J), Det(J)\}$ . This indicates the additional requirement of (iii)  $Tr(J)<0$  and (iv)  $-Ds(J)+Det(J)/Tr(J)<0$ .

To investigate these conditions,

(i)  $G(0)=Det(J)<0$

Since  $Det(J)=\beta(1-\alpha)(\rho+\delta)[\rho+\delta(1-\alpha)][(\phi+\sigma+\varepsilon-\eta)-\chi]/(-\Omega\alpha)$  and  $\Omega<0$ , it is obvious that this requires  $\chi>\phi+\sigma-\eta+\varepsilon$ .

(ii)  $Tr(J)<0$ .

As  $T>0$ ,  $Tr(J)<0$  requires both

$$\Gamma_1<0 \text{ and } \beta>\beta_a \equiv T/(-\Gamma_1)>0. \quad (A2a)$$

(iii)  $G'(0)=-Ds(J)<0$

As  $M<0$ ,  $Ds(J)>0$  requires both

$$\Gamma_2>0 \text{ and } \beta>\beta_b \equiv M/(-\Gamma_2)>0. \quad (A2b)$$

(iv)  $-Ds(J)+Det(J)/Tr(J)<0$

Under  $Tr(J)<0$  in (ii), condition (iv) is equivalent to  $-Ds(J)Tr(J)+Det(J)>0$ . Using (12a)-(12c), this requires

$$L(\beta) = \beta^2 - \beta \left\{ \frac{M}{-\Gamma_2} + \frac{T}{-\Gamma_1} + \frac{N}{-\Gamma_1\Gamma_2} \right\} + \frac{MT}{\Gamma_1\Gamma_2} > 0, \quad (A3a)$$

where  $N \equiv \frac{(1-\alpha)(\rho+\delta)}{\Omega} \frac{\rho+\delta(1-\alpha)}{\alpha} (\phi+\sigma-\eta+\varepsilon-\chi) > 0$ , whose positive sign comes from using

(i).

When  $L(\beta)=0$ , the polynomial has two roots  $\beta_1$  and  $\beta_2$ ,  $\beta_1 \geq \beta_2$ , as follows.

$$\frac{1}{2} \left\{ \frac{N}{-\Gamma_1\Gamma_2} + \frac{M}{-\Gamma_2} + \frac{T}{-\Gamma_1} \pm \left[ \left( \frac{N}{-\Gamma_1\Gamma_2} + \frac{M}{-\Gamma_2} + \frac{T}{-\Gamma_1} \right)^2 - 4 \frac{MT}{\Gamma_1\Gamma_2} \right]^{1/2} \right\}.$$

Under (i)  $Det(J)<0$ , (ii)  $Tr(J)<0$  and (iii)  $Ds(J)>0$ , both  $\beta_1$  and  $\beta_2$  are positive, as verified by

$$\beta_1\beta_2=MT/(\Gamma_1\Gamma_2)>0 \text{ and } \beta_1+\beta_2=\{N+\Gamma_1M+\Gamma_2T\}/(-\Gamma_1\Gamma_2)>0.$$

The inequality sign in (A3a) is satisfied if any one of the following two cases holds: (a)  $\beta>\beta_1\geq\beta_2$  or (b)  $\beta<\beta_2\leq\beta_1$ . However, case (b) is impossible as case (b) implies  $\beta_2<T/(-\Gamma_1)\equiv\beta_a$ , which is against the requirement of  $\beta>\beta_a$  for  $Tr(J)<0$  in (ii).

Therefore, (A3a) and  $-Ds(J)Tr(J)+Det(J)>0$  both can be met only if

$$\beta > \beta_1. \tag{A3b}$$

It is straightforward to show that  $\beta_1>\beta_a$  and  $\beta_1>\beta_b$ . Thus, (A2a), (A2b) and (A3b) indicate that the requirement of  $\beta>\beta_1$ .

Therefore, under  $\beta>\beta_1$ , the conditions of a sink are:  $\chi>\phi+\sigma-\eta+\varepsilon$ ,  $\Gamma_1<0$  and  $\Gamma_2>0$ . ■

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Figure 1: Existence of steady state under  $\psi(1-\nu)<\nu$ .

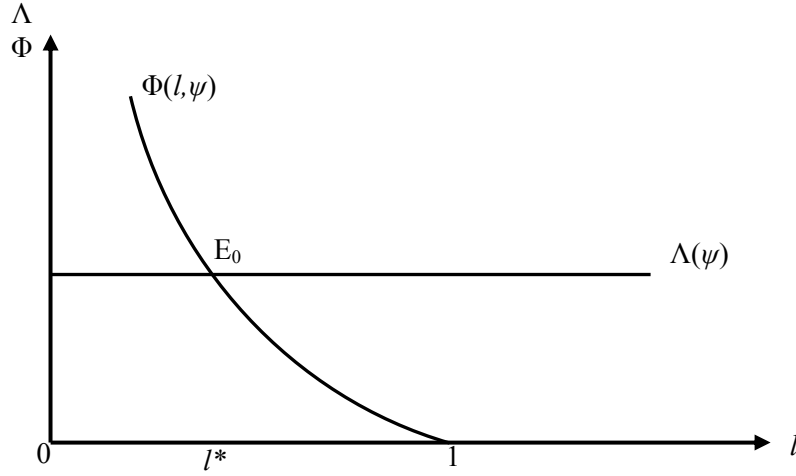


Figure 2: Existence of steady state under  $\psi(1-\nu)>\nu$  and an increase in the habit effect.

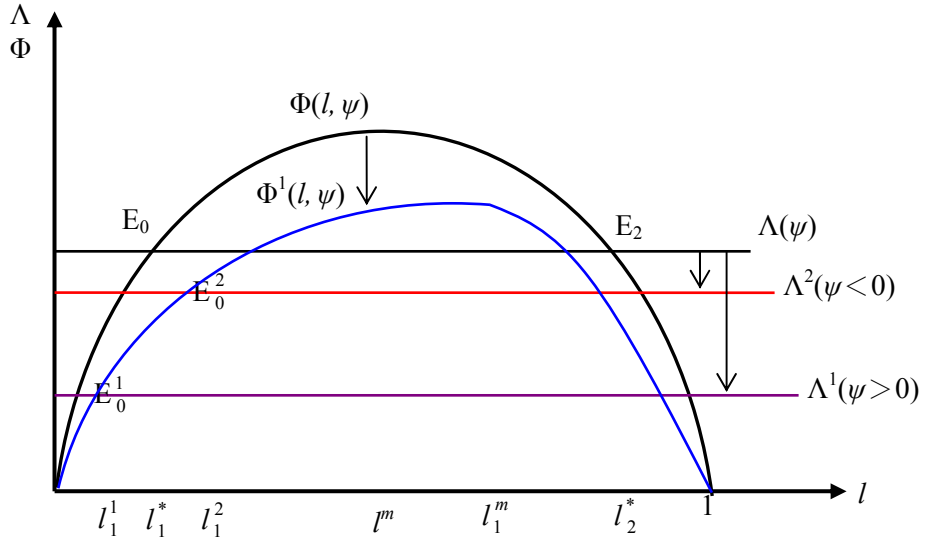


Figure 3: Different combination of  $(\psi, 1/\nu)$  that exhibits endogenous investment fluctuations.

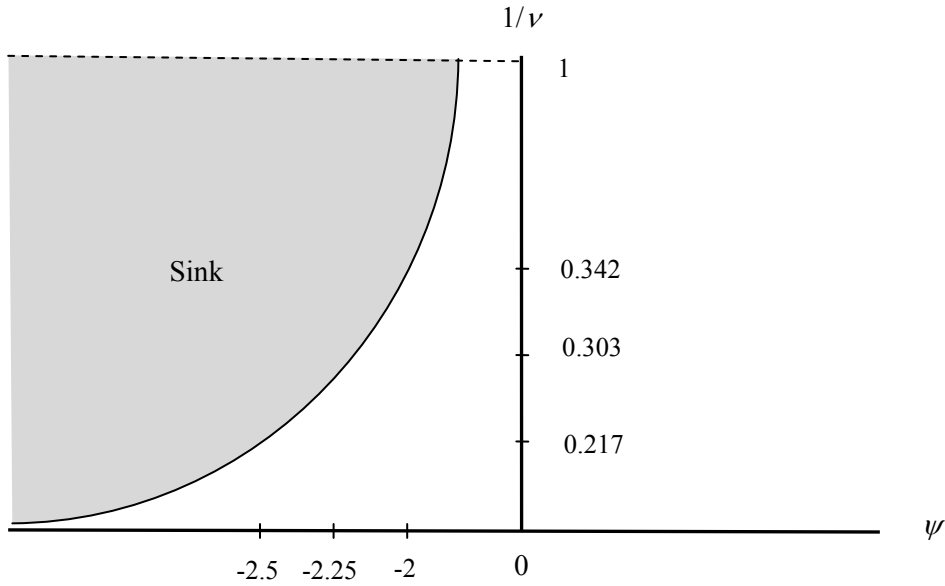


Table 1: Numerical results of local indeterminacy

Benchmark	$\beta$ increases	$\psi=-2$	$\psi=-2.5$	$\psi>0$
$1 < \nu < 3.301$	$1 < \nu < 3.301$	$1 < \nu < 2.927$	$1 < \nu < 4.619$	Not plausible
$(0.303 < 1/\nu < 1)$	$(0.303 < 1/\nu < 1)$	$(0.342 < 1/\nu < 1)$	$(0.217 < 1/\nu < 1)$	

Note: benchmark parameter:  $A=1$ ,  $\alpha=0.36$ ,  $\rho=0.04$ ,  $\delta=0.05$ ,  $\mu=0.4$ ,  $\beta=0.36$ ,  $\psi=-2.25$  and  $\nu=2.5$ ; benchmark steady state:  $k^*=2.181$ ,  $l^*=0.250$ ,  $y^*=0.545$  and  $H^*=c^*=0.436$ .