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Price competition and quality differentiation with multiproduct firms

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Abstract This paper examines a two-stage competition where firms simultaneously choose the number of products and qualities in the first stage, and then compete in prices. It is shown that a monopolist must sell a single product. In addition, in any equilibrium of multiproduct duopoly, there are segmented patterns of quality differentiation. Entangled configurations never emerge because each firm has an incentive to reduce the number of products facing direct competition with its rival. This result contrasts sharply with the equilibrium of non-segmented quality differentiation when firms compete in quantities. Furthermore, we find that the high-quality firm never offers more products than the low-quality firm, and quality differentiation between firms is greater than that within a firm.

Keywords Multiproduct firms · Vertical product differentiation · Quality differentiation · Quality competition · Bertrand competition

JEL Classification D43 · L11 · L13

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1 Introduction

Multiproduct firms abound in reality. Firms in various industries provide multiple quality-differentiated goods. However, the literature on vertical product differentiation often assumes that each firm can produce only a single quality of products due to analytical convenience. Although there are some notable exceptions, such as Katz (1984) and Gilbert and Matutes (1993), in their models, boosting product lines (products with distinct qualities) is assumed to be costless. As long as one firm gains from selling some products, it is profitable for the other firm to produce and sell it at a slightly lower price. Accordingly, each firm produces all the qualities in the domain and obtains zero profits. Since each firm offers a full product line, there is no quality differentiation between their products.[1]

However, a casual survey of the evidence suggests that any two multiproduct oligopolies usually differentiate the qualities of their products. In fact, the patterns of their quality differentiation can be classified into the *segmented* type and the *non-segmented* type. The former type indicates that the products of one firm always have higher quality and more highly-priced than those of the other firm. For example, in the market for cellular baseband processors, Qualcomm is producing chipsets for high-end smartphones and 3G mobile phones offering the Snapdragon series that includes models QSD 8250, 8650, MSM 7230, 8260, 8660, etc.; MediaTek is producing chipsets for low-end feature phones offering the MT series that includes models MT 6225, 6253, 6573, etc. All Qualcomm models are more highly-priced than those of MediaTeK. On the other hand, the latter type denotes completely different patterns where the qualities as well as prices of the products provided by two firms are entangled, such as Mercedes-Benz's C, E and S classes of sedans versus BMW's 3, 5 and 7 series.[2]

Cheng and Peng (2012) have examined the emergence of non-segmented differentiation under a quality/variety-then-quantity competition. They document that multiproduct duopolists always have incentive to avoid the strong cannibalization effect in segmented patterns, and result in an entangled configurations. However, while the non-segmented type of differentiation has been justified, the related literature has not shed enough light on how the segmented patterns arises. The literature dealing with the segmented differentiation usually exogenously assumes a segmented type of quality differentiation without justification. In Champsaur and Rochet (1989, 1990) and Cheng et al. (2011), the qualities provided by one firm are all assumed to be higher than those of the other firm. Champsaur and Rochet (1989) directly assume that each firm offers a consecutive interval of qualities, and they show that there must be a gap between the two intervals in order to avoid the keen price competition. Cheng et al. (2011) examines a variety/quality-then-price competition in a multiproduct duopoly,[3]

[1] The analysis of Gilbert and Matutes (1993) is based on both a one-stage game and a three-stage game (the two firms choose product quality sequentially) respectively, and they show that firms may specialize only product provision if they can make a commitment to restrict the product offerings.

[2] These prices are from their official websites; they denote the suggested retail price of each class with basic equipment.

[3] The term "variety/quality" denotes the case where firms may choose both the number of products and the associated quality levels of their products.

where the qualities provided by one firm are assumed to be always higher than those of the other firm.[4]

This paper thus intends to examine the equilibrium types of quality differentiation for the multiproduct firms engaging in a variety/quality-then-price competition when firms are allowed to provide products with entangled qualities. *The main purpose of this study is to justify the emergence of the segmented differentiation, and to characterize the endogenously determined patterns of quality differentiation between a high-quality firm and a low-quality firm*. Therefore, we generalize a standard model of endogenous vertical differentiation like Tirole (1988) and Motta (1993) to multiproduct offerings while maintaining the remaining structure of the original model on the setting of the determination of market size as well as the cost with the quality improvement, and make a comparison of our findings with those in the related literature.[5] A two-stage competition is specified, where firms simultaneously choose the number of products and the associated qualities of their products in the first stage, and then compete in prices in the second stage.[6]

Bonnisseau and Lahmandi-Ayed (2006) assume linear costs of quality improvement and show that the incumbent who is allowed to provide multiple qualities (with a maximum of two) always offers a single product. Nevertheless, Cheng et al. (2011) find that firms have incentives to provide multiple qualities when the unit costs of quality are convex and quadratic, while there must be single-product firms when the unit cots are concave (including linear) in quality. Because our focus is on the multiproduct offerings, we consider that the unit costs of quality improvement are convex and specify a quadratic form for tractability as in Motta (1993), Lambertini and Tedeschi (2007) and Cheng et al. (2011), among others.[7] The convex costs of quality improvement is, in fact, reasonable since increasing quality is often increasingly expensive due to the decreasing returns to scale of human technologies.

In reality, boosting the number of product lines often involves additional setup costs due to the requirements of the distinct or advanced facilities, and thus a firm normally restricts itself to produce fewer varieties than a full product line due to the limitation

[4] In contrast to the finding of single-quality outcome in Champsaur and Rochet (1989, Proposition 3; 1990, Proposition 2) where the market is assumed to be fully covered, Cheng et al. (2011) show that each firm always has an incentive to provide multiple qualities in a partially covered market (i.e., the market size is endogenously determined) when the unit costs of quality are increasing and quadratic.

[5] The standard model on endogeneous vertical differentiation like the paper by Tirole (1988) and Motta (1993), they examine a single-product duopoly where consumers' valuations for quality are different and distributed uniformly, and shows that firms differentiate their product qualities in the market. In contrast, Herweg and Relaxing (2012) examines two-part tariff competition in a single-product duopoly where consumers have *identical* quality tastes and each of them may purchase more than one product. He shows that firms will differentiate their qualities if and only if two-part tariffs are feasible.

[6] In contrast to the literature on endogeneous choice of vertical quality differentiation where quality is a long-run variable and thus the firms usually compete sequentially on quality and price, Chioveanu (2012) examines the one-stage competition of sing-product firms with simultaneous quality-and-price choices in a model of two-type consumers and two qualities where quality differentiation is exogenously given. Her analysis is suitable for the markets where quality choices do not require long-term investments, such as professional services.

[7] In Lambertini and Tedeschi (2007), the quadratic costs of quality improvement fall upon the fixed costs, and firms choose their quality sequentially rather than simultaneously.

of its resources. Thus, we involve a fixed setup cost for each kind of product; that is, a fixed cost will be incurred each time a product is introduced. Moreover, since it is not our purpose to discuss product proliferation, we follow Cheng and Peng (2012) to involve a restriction on the maximum number of products where each firm will not provide more than two products as long as there is a positive fixed setup cost.

Unlike Katz (1984) and Gilbert and Matutes (1993), this paper shows that multi-product firms will differentiate the qualities of their products rather than providing identical qualities. The asymmetric equilibrium patterns provide an explanation for the segmented type of differentiation, and reveal the properties of quality differentiation in a multiproduct duopoly. The set of equilibrium outcomes in this study is much richer compared to the symmetric outcome where each firm provides a full product line, and it also contrasts with the literature on vertical product differentiation where each firm can produce only single quality.

There is a brief preview of the examination results. First, compared with the multi-product monopoly in the previous literature, we show that a natural monopolist always sells a single product. Second, in contrast to the single-product results in Champsaur and Rochet (1989, 1990) and Bonnisseau and Lahmandi-Ayed (2006), we find that the configuration with multiproduct duopoly emerges in the equilibrium. Moreover, in any equilibrium of multiproduct duopoly, it reveals the existence of segmented patterns with quality differentiation because each firm intends to reduce the number of products facing direct competition with the rival to avoid keen price competition. This finding contrasts sharply with the non-segmented equilibrium outcome in Cheng and Peng (2012) where firms compete on quantity choice in the second stage. Third, it is shown that the high-quality firm never offers more products than the low-quality firm, and the quality differentiation among neighboring products between firms is greater than that within a firm.

The remainder of this paper is organized as follows. Section 2 outlines the general framework of our model. Section 3 characterizes the SPNE under quadratic costs of quality improvement. Section 4 concludes.

2 General framework

There are two firms, $R = A, B$, with identical production cost of quality improvement. Firm R produces n_R quality-differentiated products $rs = r1, r2 \dots, rn_R$ associated with qualities $q_{rs} = q_{r1}, q_{r2} \dots, q_{rn_R}$ in the vertical quality spectrum, where $q_{rs} > q_{r,s+1}$ for $s = 1, 2 \dots, n_R - 1$, and these products of different quality are sold at the prices $p_{rs} = p_{r1}, p_{r2} \dots, p_{rn_R}$.

Each consumer purchases one unit at most. In other words, she buys one unit of the products from firm A or firm B, or she does not buy at all. Consumers are assumed to distribute uniformly over the taste interval $[\underline{\theta}, \bar{\theta}]$ with the density normalized to 1. In the following analysis, we focus on the equilibrium that the two firms do *not* cover the whole market.[8] $\underline{\theta} = 0$ is assumed so that there must be some consumers who will

[8] Some literature examines an uncovered market, such as Katz (1984), Moorthy (1988), Choi and Shin (1992) and Motta (1993), the others assume a covered market, such as Crampes and Hollander (1995) and

never buy the goods, and thus we would not need to make other explicit conditions that the market is not fully covered. The consumer indexed by a taste parameter $\theta \in [0, \bar{\theta}]$ maximizes the following utility function:

$$U_\theta(q_{rs}) = \begin{cases} \theta q_{rs} - p_{rs} & \text{if she/he buys the product with quality } q_{rs} \text{ at price } p_{rs} \text{ from firm } R. \\ 0 & \text{otherwise.} \end{cases}$$

This utility function implies that all consumers prefer higher quality at a given price, and the consumer indexed by a higher θ is willing to pay more for higher quality.

Consider the demand for each product. Suppose that the total number of products in the market is N, and that these products can be ordered based on their associated qualities such that $O^1(q) > O^2(q) > \cdots > O^N(q) > 0$, where $q = (q_{a1}, q_{a2}, \ldots, q_{an_A}, q_{b1}, q_{b2}, \ldots, q_{bn_B})$ and $O^i(\cdot)$ is an operator which selects the i-th highest quality level from $(\cdot)$. In order to simplify the notation, let us introduce $\widetilde{q}_i \equiv O^i(q)$, and the price of the product with quality $\widetilde{q}_i$ is denoted by $\widetilde{p}_i$.

The marginal consumer with index θ_i for which $\theta_i \widetilde{q}_i - \widetilde{p}_i = \theta_i \widetilde{q}_{i+1} - \widetilde{p}_{i+1}$, where $i = 1, \ldots, N-1$, will be indifferent between buying a product with quality $\widetilde{q}_i$ and a product with quality $\widetilde{q}_{i+1}$. Any consumer with an index greater than θ_i will prefer to buy the product with quality $\widetilde{q}_i$ than to buy the product with quality $\widetilde{q}_{i+1}$. Moreover, the marginal consumer with an index θ_N for which $\theta_N \widetilde{q}_N - \widetilde{p}_N = 0$ is indifferent between buying the product with the lowest quality $\widetilde{q}_N$ to not buying at all. Any consumer with an index greater than θ_N will prefer to buy the product of quality $\widetilde{q}_N$ than not to buy at all. Accordingly, the demand for all these products can be expressed as follows. Let $\widetilde{p} = (\widetilde{p}_1, \widetilde{p}_2, \ldots, \widetilde{p}_N)$ and $\widetilde{q} = (\widetilde{q}_1, \widetilde{q}_2, \ldots, \widetilde{q}_N)$.

$$\widetilde{x}_i(\widetilde{p}, \widetilde{q}) = \begin{cases} \bar{\theta} - \theta_i = \bar{\theta} - \frac{\widetilde{p}_1 - \widetilde{p}_2}{\widetilde{q}_1 - \widetilde{q}_2} & \text{if } i = 1. \\ \theta_{i-1} - \theta_i = \frac{\widetilde{p}_{i-1} - \widetilde{p}_i}{\widetilde{q}_{i-1} - \widetilde{q}_i} - \frac{\widetilde{p}_i - \widetilde{p}_{i+1}}{\widetilde{q}_i - \widetilde{q}_{i+1}} & \text{if } i = 2, 3 \ldots, N-1. \\ \theta_{N-1} - \theta_N = \frac{\widetilde{p}_{N-1} - \widetilde{p}_N}{\widetilde{q}_{N-1} - \widetilde{q}_N} - \frac{\widetilde{p}_N}{\widetilde{q}_N} & \text{if } i = N. \end{cases} \quad (1)$$

where $\widetilde{x}_i(\widetilde{p}, \widetilde{q})$ is the demand for the product with quality $\widetilde{q}_i$, namely, the demand for the i-th highest quality.[9]

We assume that the costs of quality improvement fall upon variable costs, which means that fixed costs are independent of quality. In addition, the production activities are fully additive so that the unit cost of each product is independent of its quantities because our purpose is not to discuss the relationships between qualities based on the cost interactions.[10] The unit cost of quality improvement is denoted by $c(q_{rs})$. Besides, there is a fixed production cost, denoted by $f > 0$, for a variety/product line. Thus, the profit of firm R, denoted by π_R, is given by

Footnote 8 continued
Champsaur and Rochet (1989). Meanwhile, the market coverage is endogenously determined in some studies such as Wauthy (1996).

[9] Note that if $N = 1$, the demand is reduced to $\widetilde{x}_1(p, q) = \bar{\theta} - \theta_1 = \bar{\theta} - \frac{\widetilde{p}_1}{\widetilde{q}_1}$.

[10] In avoid to mix the effect on the scale production for each product. This assumption is quite common in the related literature such as Tirole (1988), Choi and Shin (1992) and Motta (1993).

$$\pi_R = \sum_{s=1}^{n_R} [p_{rs} - c(q_{rs})]x_{rs} - n_R f \tag{2}$$

where n_R is the number of products offered by firm R and x_{rs} is the demand for the product with quality q_{rs} which can be derived from (1) after ordering all the products by their qualities.

Game structure

Two firms, A and B, play a two-stage game. In the first stage, they simultaneously choose the number of products n_R and associated qualities (q_{r1}, q_{r1}) or (q_{r1}, q_{r2}). Moreover, a firm enters the market only if its profit is strictly positive. In the second stage, they simultaneously choose the prices of their products, having observed the number of products and qualities.

In equilibrium, eighteen configurations possibly arise. Excluding axisymmetric cases, we therefore consider the following nine possible alternatives of vertically spatial configurations:

$$(a),\ (aa),\ (ab),\ (aab),\ (abb),\ (aba),\ (aabb),\ (abba),\ (abab) \tag{3}$$

where (a) is the monopoly with a single product, (aa) is the monopoly with two products, (ab) is the duopoly with a single product, (aab), (abb), $(aabb)$ are all referred to the segmentation configuration since the products provided by a single firm have neighboring qualities, and (aba), $(abba)$, $(abab)$ are respectively referred to as sandwich, enclosure and interlacing.

For example, if firm A offers two products of different qualities (q_{a1}, q_{a2}) and firm B offers a single product of quality q_{b1} such that $q_{a1} > q_{a2} > q_{b1}$, we denote this configuration by (aab) and its profit by $\pi_A(aab)$. According to Eq. (2), the profits are

$$\pi_A(aab) = [p_{a1} - c(q_{a1})]x_{a1} + [p_{a2} - c(q_{a2})]x_{a2} - 2f$$
$$\pi_B(aab) = [p_{b1} - c(q_{b1})]x_{b1} - f$$

where $x_{a1}(p, q)$, $x_{a2}(p, q)$ and $x_{b1}(p, q)$ can be derived from (1) according to their order $q_{a1} > q_{a2} > q_{b1}$, i.e., $(\widetilde{q}_1, \widetilde{q}_2, \widetilde{q}_3) = (q_{a1}, q_{a2}, q_{b1})$. Thus, $(\widetilde{x}_1, \widetilde{x}_2, \widetilde{x}_3) = (x_{a1}, x_{a2}, x_{b1})$ so that the demands for each product are

$$x_{a1}(x, q) = \left(\bar{\theta} - \frac{p_{a1}-p_{a2}}{q_{a1}-q_{a2}}\right)$$
$$x_{a2}(x, q) = \left(\frac{p_{a1}-p_{a2}}{q_{a1}-q_{a2}} - \frac{p_{a2}-p_{b1}}{q_{a2}-q_{b1}}\right)$$
$$x_{b1}(x, q) = \left(\frac{p_{a2}-p_{b1}}{q_{a2}-q_{b1}} - \frac{p_{b1}}{q_{b1}}\right).$$

Similarly, $(abab)$ indicates that both firm A and firm B produce two products with qualities such that $q_{a1} > q_{b1} > q_{a2} > q_{b2}$. Thus, as we mentioned above, according to Eqs. (1) and (2), the profits of firm A and firm B, respectively, can be written as

$$\pi_A(abab) = [p_{a1} - c(q_{a1})]\left(\bar{\theta} - \frac{p_{a1} - p_{b1}}{q_{a1} - q_{b1}}\right) + [p_{a2} - c(q_{a2})]\left(\frac{p_{b1} - p_{a2}}{q_{b1} - q_{a2}} - \frac{p_{a2} - p_{b2}}{q_{a2} - q_{b2}}\right) - 2f$$
$$\pi_B(abab) = [p_{b1} - c(q_{b1})]\left(\frac{p_{a1} - p_{b1}}{q_{a1} - q_{b1}} - \frac{p_{b1} - p_{a2}}{q_{b1} - q_{a2}}\right) + [p_{b2} - c(q_{b2})]\left(\frac{p_{a2} - p_{b2}}{q_{a2} - q_{b2}} - \frac{p_{b2}}{q_{b2}}\right) - 2f.$$

The profit functions of the other configurations in (3) can be expressed in a similar way. For any configuration where N products exist, each product should have a strictly positive demand.

We ignore all configurations in which the two firms produce identical qualities, since competition between firms will drive the prices of identical products to their marginal costs, and in turn, offering identical products brings no profit, and moreover it divvies the demand for the established products. Thus, firms will not provide identical qualities in equilibrium.

3 Equilibrium characterization

In this section, we develop the model where the unit cost of quality improvement is assumed to be quadratic for tractability as in Champsaur and Rochet (1990) and Motta (1993), among others. The unit cost, $c(q_{rs})$, is given by αq_{rs}^2, where $\alpha > 0$. As stated in the previous section, we first solve for the optimal qualities of products and associated prices within each configuration of (3). Let us start with the monopolistic ones.

Monopoly with a single product (a). By solving the first-order conditions, the optimal quality and price are, respectively,[11]

$$q_{a1} = \frac{\bar{\theta}}{3\alpha} \quad and \quad p_{a1} = \frac{2\bar{\theta}^2}{9\alpha}$$

By substituting this into the profit, we have

$$\pi_A(a) = \frac{\bar{\theta}^3}{27\alpha} - f$$

Monopoly with two products (aa). Solving the first-order conditions, the optimal qualities and prices are, respectively,[12]

$$q_{a1} = \frac{2\bar{\theta}}{5\alpha} \quad and \quad p_{a1} = \frac{7\bar{\theta}^2}{25\alpha}$$
$$q_{a2} = \frac{\bar{\theta}}{5\alpha} \quad and \quad p_{a2} = \frac{3\bar{\theta}^2}{25\alpha}$$

By substituting this into the profit function (2), we have

$$\pi_A(aa) = \frac{\bar{\theta}^3}{25\alpha} - 2f$$

[11] The other solution, $q_{a1} = \frac{\bar{\theta}}{\alpha}$ and $p_{a1} = \frac{\bar{\theta}^2}{\alpha}$, does not satisfy the second-order conditions.

[12] The other solution, $q_{a1} = \frac{2\bar{\theta}}{3\alpha}$, $q_{a2} = \frac{\bar{\theta}}{3\alpha}$, $p_{a1} = \frac{5\bar{\theta}^2}{9\alpha}$ and $p_{a2} = \frac{2\bar{\theta}^2}{9\alpha}$, does not satisfy the second-order conditions.

Table 1 The candidates for the SPNEs

	(a)	(aa)	(ab)	(aab)	(abb)	(aba)	$(aabb)$
π_A^*	$0.0370\frac{\overline{\theta}^3}{\alpha}-f$	$0.04\frac{\overline{\theta}^3}{\alpha}-2f$	$0.0164\frac{\overline{\theta}^3}{\alpha}-f$	$0.0183\frac{\overline{\theta}^3}{\alpha}-2f$	$0.0146\frac{\overline{\theta}^3}{\alpha}-f$	$0.01526\frac{\overline{\theta}^3}{\alpha}-2f$	$0.0163\frac{\overline{\theta}^3}{\alpha}-2f$
π_B^*			$0.0121\frac{\overline{\theta}^3}{\alpha}-f$	$0.0107\frac{\overline{\theta}^3}{\alpha}-f$	$0.0132\frac{\overline{\theta}^3}{\alpha}-2f$	$0.01118\frac{\overline{\theta}^3}{\alpha}-f$	$0.0118\frac{\overline{\theta}^3}{\alpha}-2f$
$\widetilde{q}_1^*$	$0.333\frac{\overline{\theta}}{\alpha}$	$0.4\frac{\overline{\theta}}{\alpha}$	$0.410\frac{\overline{\theta}}{\alpha}$	$0.457\frac{\overline{\theta}}{\alpha}$	$0.4208\frac{\overline{\theta}}{\alpha}$	$0.4207\frac{\overline{\theta}}{\alpha}$	$0.463\frac{\overline{\theta}}{\alpha}$
$\widetilde{q}_2^*$		$0.2\frac{\overline{\theta}}{\alpha}$	$0.199\frac{\overline{\theta}}{\alpha}$	$0.371\frac{\overline{\theta}}{\alpha}$	$0.220\frac{\overline{\theta}}{\alpha}$	$0.214\frac{\overline{\theta}}{\alpha}$	$0.389\frac{\overline{\theta}}{\alpha}$
$\widetilde{q}_3^*$				$0.183\frac{\overline{\theta}}{\alpha}$	$0.110\frac{\overline{\theta}}{\alpha}$	$0.033\frac{\overline{\theta}}{\alpha}$	$0.204\frac{\overline{\theta}}{\alpha}$
$\widetilde{q}_4^*$							$0.102\frac{\overline{\theta}}{\alpha}$
$\widetilde{p}_1^*$	$0.222\frac{\overline{\theta}^2}{\alpha}$	$0.556\frac{\overline{\theta}^2}{\alpha}$	$0.227\frac{\overline{\theta}^2}{\alpha}$	$0.274\frac{\overline{\theta}^2}{\alpha}$	$0.2313\frac{\overline{\theta}^2}{\alpha}$	$0.2311\frac{\overline{\theta}^2}{\alpha}$	$0.274\frac{\overline{\theta}^2}{\alpha}$
$\widetilde{p}_2^*$		$0.222\frac{\overline{\theta}^2}{\alpha}$	$0.075\frac{\overline{\theta}^2}{\alpha}$	$0.195\frac{\overline{\theta}^2}{\alpha}$	$0.085\frac{\overline{\theta}^2}{\alpha}$	$0.079\frac{\overline{\theta}^2}{\alpha}$	$0.206\frac{\overline{\theta}^2}{\alpha}$
$\widetilde{p}_3^*$				$0.065\frac{\overline{\theta}^2}{\alpha}$	$0.036\frac{\overline{\theta}^2}{\alpha}$	$0.007\frac{\overline{\theta}^2}{\alpha}$	$0.075\frac{\overline{\theta}^2}{\alpha}$
$\widetilde{p}_4^*$							$0.032\frac{\overline{\theta}^2}{\alpha}$
$\widetilde{x}_1^*$	$0.333\overline{\theta}$	$0.2\overline{\theta}$	$0.279\overline{\theta}$	$0.086\overline{\theta}$	$0.270\overline{\theta}$	$0.262\overline{\theta}$	$0.074\overline{\theta}$
$\widetilde{x}_2^*$		$0.2\overline{\theta}$	$0.345\overline{\theta}$	$0.221\overline{\theta}$	$0.290\overline{\theta}$	$0.340\overline{\theta}$	$0.219\overline{\theta}$
$\widetilde{x}_3^*$				$0.339\overline{\theta}$	$0.110\overline{\theta}$	$0.198\overline{\theta}$	$0.290\overline{\theta}$
$\widetilde{x}_4^*$							$0.102\overline{\theta}$
SPNE	✓		✓		✓		✓
$\widetilde{\pi}_1^*$	$0.0370\frac{\overline{\theta}^3}{\alpha}$		$0.0164\frac{\overline{\theta}^3}{\alpha}$		$0.0146\frac{\overline{\theta}^3}{\alpha}$		$0.0044\frac{\overline{\theta}^3}{\alpha}$
$\widetilde{\pi}_2^*$			$0.0121\frac{\overline{\theta}^3}{\alpha}$		$0.0105\frac{\overline{\theta}^3}{\alpha}$		$0.0119\frac{\overline{\theta}^3}{\alpha}$
$\widetilde{\pi}_3^*$					$0.00026\frac{\overline{\theta}^3}{\alpha}$		$0.0096\frac{\overline{\theta}^3}{\alpha}$
$\widetilde{\pi}_4^*$							$0.0022\frac{\overline{\theta}^3}{\alpha}$

Note that π_R^* is firm R's gross profit, and $\widetilde{\pi}_i^* = (\widetilde{p}_i^* - \alpha\widetilde{q}_i^{*2})\widetilde{x}_i^*$ is the gross profit from product i

Table 2 Product differentiation and average qualities

	(a)	(ab)	(abb)	$(aabb)$
$\triangle\widetilde{q}_1$		$0.211\,\frac{\overline{\theta}}{\alpha}$	$0.201\frac{\overline{\theta}}{\alpha}$	$0.074\frac{\overline{\theta}}{\alpha}$
$\triangle\widetilde{q}_2$			$0.110\frac{\overline{\theta}}{\alpha}$	$0.185\frac{\overline{\theta}}{\alpha}$
$\triangle\widetilde{q}_3$				$0.102\frac{\overline{\theta}}{\alpha}$
$AvgQ$	$0.333\frac{\overline{\theta}}{\alpha}$	$0.2933\,\frac{\overline{\theta}}{\alpha}$	$0.2829\frac{\overline{\theta}}{\alpha}$	$0.2369\frac{\overline{\theta}}{\alpha}$

The notation $\triangle\widetilde{q}_i = \widetilde{q}_i^* - \widetilde{q}_{i+1}^*$ refers to the product differentiation of neighboring products, and $AvgQ = \sum_i q_i^* x_i^* / \sum_i x_i^*$ denotes the weighted average quality provided in the market

As for the duopolistic configurations, due to the complexity of the reaction functions (first-order conditions), solving for the actual equilibrium qualities is tedious. For simplicity, we omit the detailed derivations,[13] and summarize the results of the equilibrium qualities, quantities, prices and profits of the duopolistic configurations in Table 1 as long as the equilibrium emerges. It has been proved that all the outcomes in Table 1 satisfy the associated second-order conditions and are consistent with the assumption that the provisions of both firms do not cover the whole market. Those equilibrium configurations in Table 1 are the candidates for the SPNEs. Note that non-segmented configurations $(abba)$ and $(abab)$ are not shown in Table 1 because there is no admissible interior solution satisfying the quality ordering of its own configuration in these two configurations.[14] Besides, we present the solutions to the monopolistic configurations as decimal figures for comparison with the duopolistic outcomes (Table 2).

In Table 1, the equilibrium qualities are proportional to $\frac{\overline{\theta}}{\alpha}$. It implies that the equilibrium quality decreases with the coefficient for the costs of quality improvement (α) and increases with $\overline{\theta}$, which is a constant proportion of the average willingness to pay for quality ($\frac{\overline{\theta}}{2}$). In addition, it is revealed that the equilibrium prices are proportional to $\frac{\overline{\theta}^2}{\alpha}$. The prices decrease with the coefficient for the costs of quality (α) due to the decline in the equilibrium qualities. Furthermore, the equilibrium quantity is in proportion to $\overline{\theta}$ which measures the total market demand of the product. In other words, the equilibrium quantity of outputs is greater while the market demand for the product is larger.

We now examine which of these candidates in Table 1 are indeed SPNEs. In the following analysis, we assume that any quality deviation is in proportion to $\frac{\overline{\theta}}{\alpha}$. It is demonstrated that the candidates (aa), (aab) and (aba) can not be selected as the SPNEs. Only the candidates (a), (ab), (abb) and $(aabb)$ may emerge in equilibrium for a specific range of the fixed cost, f.

[13] The detailed derivation is available from the authors.

[14] The equilibrium qualities in $(abba)$ and $(abab)$ are too complex to solve analytically, so we treat 0.1 as an interval to make all combinations of $\{r, s, t\}$ initial values, and employ Newton's method to search for the solution to $\{r, s, t\}$, where $r = \frac{\widetilde{q}_2}{\widetilde{q}_1}$, $s = \frac{\widetilde{q}_3}{\widetilde{q}_1}$ and $t = \frac{\widetilde{q}_4}{\widetilde{q}_1}$. It is shown that there is no admissible interior solution. The derivation will be provided upon request.

We first examine the SPNE candidate (aa) in Table 1. We find that if the fixed cost f is small enough for firm A to offer two products, then firm B must have incentive to enter the market. Thus, (aa) must not be an SPNE configuration. Accordingly, we conclude with the following proposition:

Proposition 1 *A monopolist always produces a single product/quality when there is a potential entrant along with price competition.*

Proof See Appendix 1. □

Proposition 1 documents that a natural monopolist never sells multiple products. It derives from the proof that candidate (aa) cannot be an SPNE, it implies that if it is sufficiently profitable for a firm to offer two products, it must be profitable for the other firm to enter the market. This is because the profit from the first product is greater than that from the second one.

Similarly, while examining the candidate (aab) in Table 1, we find that if the fixed cost f is small enough such that firm A has an incentive to provide two products, it must be profitable for firm B to offer two products. Moreover, by examining the candidate (aba) in Table 1, we also find that firm A always has an incentive to deviate from (aba) to (aab) by selling two products with qualities (q_{a1}, q_{a2}) such that $q_{a1} > q_{a2} > 0.214\frac{\overline{\theta}}{\alpha}$. This is because firm A faces two-side price competition in both high and low quality market in the (aba) configuration, however, it only faces one-side price competition in the low quality market in the configuration of (aab). Thus, (aba) cannot be selected as an SPNE. So far, we have shown that (aa), (aab), (aba), $(abba)$ and $(abab)$ cannot be selected as SPNEs. Two preliminary findings are thus established as follows. We thus have the following remark:

Remark 1 In a two-stage game where firms compete on numbers of products, qualities and then prices,

(i) Sandwich, enclosure and interlacing configurations never emerge in equilibrium.
(ii) The high-quality firm never offers more products than the low-quality firm.

Proof See Appendix 2. □

In Remark 1, (i) shows that none of the non-segmented configurations of multiproduct firms can be selected as an equilibrium. This is because, in the non-segmented patterns, there are more products facing direct competition from the rival's products compared to that of the segmented configurations. *Each multiproduct firm thus has an incentive to reduce the number of its products facing direct competition from its rival, in order to avoid the keen price competition in the next stage.* This finding sharply contrasts with the result of the Cournot competition in Cheng and Peng (2012) where only the entangled patterns of differentiation emerge in the multiproduct equilibrium because firms are eager to avoid the strong cannibalization effect in a segmented pattern. The intuition for (ii) is that the segmentation of the low-quality market has an additional gain in that it acquires new consumers compared to the segmentation of the high-quality market. Thus, as long as the high-quality firm has an incentive to segment the market by selling two neighboring qualities, the low-quality firm must find it profitable to do that as well. Accordingly, the high-quality firm never offers more products than the low-quality firm.

On the comparison between the Bertrand and Cournot competition, some of the literature assumes that the degrees of product substitutability is exogenous, and it is shown that the two types of competition yields different results depending on the products being complements or substitutes. For example, Vives (2008) examines the effects of different competition modes on process innovation, and indicates that, for the complements, the Bertrand and Cournot competition yields different effects on R&D incentives while this is not the case for the highly substitutable products. By contrast, the literature on vertical product differentiation considers that the degree of product substitutability is endogenously determined by the first-stage decisions of quality differentiation. For a single-product duopoly, Motta (1993) shows that the Bertrand competition leads to more differentiation than Cournot one, which suggests the products are less substitutable under the Bertrand competition than under the Cournot one. Moreover, our study examines the patterns of quality differentiation between multiproduct firms, and further reveals the products substitutability between multiproduct firms. The segmented patterns of qualities under the Bertrand competition presents that one multiproduct firm focuses on high-quality products and the other focuses on low-quality ones. This implies that there is less product substitutability between multiproduct firms under the Bertrand than under the Cournot competition, in comparison with the entangled patterns of the Cournot result in Cheng and Peng (2012).

To ensure that the other candidates in Table 1, i.e., (a), (ab), (abb) and $(aabb)$, can be selected as SPNEs, we have to further ensure that it is not profitable for any firm to deviate to any other configuration and that each firm enters the market only if its profit is strictly positive. Formally, the three conditions to ensure that (ab) is an SPNE can be stated as follows:

Duopoly with a single product (ab).

$$\pi_A^*(ab) \geq \max\{\pi_A(ab), \pi_A(ba)\} \text{ and } \pi_B^*(ab) \geq \max\{\pi_B(ab), \pi_B(ba)\}.$$

$$\pi_A^*(ab) \geq \max\{\pi_A(aab), \pi_A(aba), \pi_A(baa)\} \text{ and } \pi_B^*(ab) \geq \max\{\pi_B(abb), \pi_B(bab), \pi_B(bba)\}.$$

$$\pi_A^*(ab) > 0 \text{ and } \pi_B^*(ab) > 0.$$

The first condition implies that both firms have no profitable deviation by altering the ordering of the qualities. The former inequality means that, given $q_{b1} = 0.199\frac{\overline{\theta}}{\alpha}$, firm A has no incentive to alter the ordering of qualities; meanwhile the latter inequality is that, given $q_{a1} = 0.410\frac{\overline{\theta}}{\alpha}$, firm B has no incentive to alter the ordering of qualities, either. The second condition means that neither firm has a profitable deviation by altering the number of products as well as their associated qualities. The last condition ensures that both firms profitably enter the market.[15]

Based on all of these no deviation conditions in the Nash concept and the entering conditions, the following remark is thus obtained:

Remark 2 In a two-stage game where firms *compete on numbers of products, qualities and then prices,* (i)–(v) *exhibit an SPNE for this two-stage game.*

[15] The conditions for configurations (a), (abb) and $(aabb)$ to be an SPNE can be found in Appendix 3.

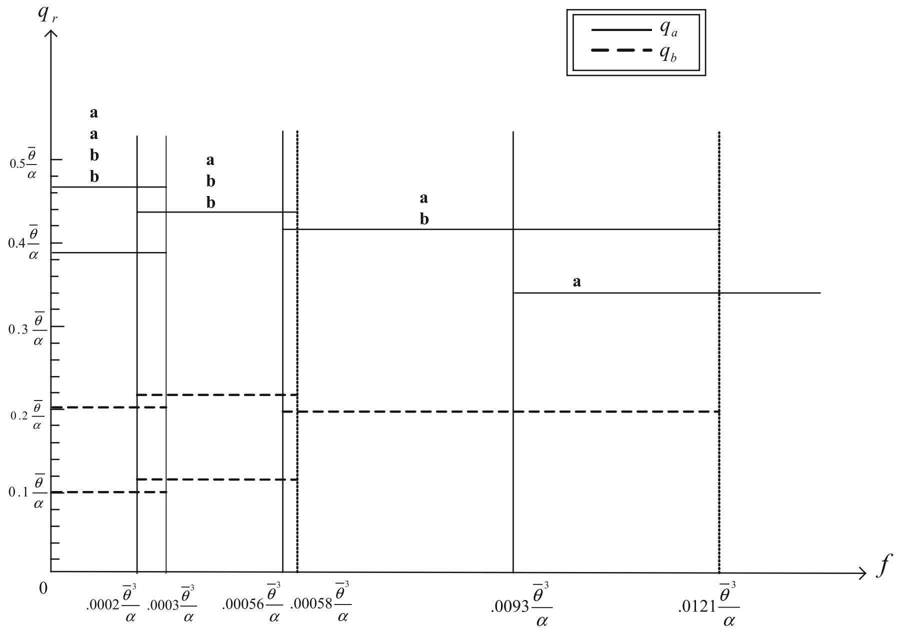

Fig. 1 Equilibrium qualtites

(i) *When* $0 < f \leq 0.0003\frac{\overline{\theta}^3}{\alpha}$, *the high-quality firm offers two products with qualities* $(0.463\frac{\overline{\theta}}{\alpha}, 0.389\frac{\overline{\theta}}{\alpha})$. *The low-quality firm offers two products with qualities* $(0.204\frac{\overline{\theta}}{\alpha}, 0.102\frac{\overline{\theta}}{\alpha})$. */Segmentation* $(aabb)$.

(ii) *When* $0.0002\frac{\overline{\theta}^3}{\alpha} \leq f \leq 0.00058\frac{\overline{\theta}^3}{\alpha}$, *the high-quality firm offers a single product with quality* $0.4208\frac{\overline{\theta}}{\alpha}$. *The low-quality firm offers two distinct products with quality* $(0.220\frac{\overline{\theta}}{\alpha}, 0.110\frac{\overline{\theta}}{\alpha})$. */Segmentation* (abb).

(iii) *When* $0.00056\frac{\overline{\theta}^3}{\alpha} \leq f < 0.0121\frac{\overline{\theta}^3}{\alpha}$, *both firms offer a single product. The associated qualities are* $0.410\frac{\overline{\theta}}{\alpha}$ *and* $0.199\frac{\overline{\theta}}{\alpha}$. */Duopoly with a single product* (ab).

(iv) *When* $0.0093\frac{\overline{\theta}^3}{\alpha} \leq f < 0.0370\frac{\overline{\theta}^3}{\alpha}$, *a single firm monopolizes the market by offering a product with quality* $0.333\frac{\overline{\theta}}{\alpha}$. */Monopoly with a single product* (a).

(v) *When* $f \geq 0.037\frac{\overline{\theta}^3}{\alpha}$, *no firm enters the market.*

Remark 2 is illustrated in Fig. 1. It reveals that the equilibrium configuration with multiproduct firms may emerge, and both of the segmentation (*abb*) and (*aabb*) may possibly be the alternative of the multiproduct equilibrium.

In contrast to the single-quality results outlined in Champsaur and Rochet (1989, Proposition 3; 1990, Proposition 2) where the market is assumed to be fully covered, we find that each firm may produce multiple qualities when the market is partially covered. Specifically, introducing a new product yields two effects on profits: a positive effect of segmentation and a negative effect of cannibalization. The former implies that firms may extract consumer surplus by introducing a product to segment the market while the latter means that the newly introduced product will divvy the market share of its own original products. When the market is not given to be fixed, introducing a product to segment the market will enlarge the total market size because more consumers will make a purchase. Because of the market expansion, the segmentation profit increases and the negative effect of cannibalization weakens. Accordingly, it is profitable for firms to produce multiple qualities. This result supports the equilibrium of multiproduct firms in Cheng et al. (2011).

In addition, while Bonnisseau and Lahmandi-Ayed (2006) assume linear costs of quality improvement and show that a firm always produces a single quality in equilibrium, Remark 2 reveals that multiproduct firms may emerge in the equilibrium when the costs of quality improvement is quadratic. This is because the utility is specified by $u = \theta q_i$, which implies the willingness to pay for a unit quality is constant. Hence, when the marginal cost of quality is concave (including linear), the higher the quality is, the lower the marginal cost of improving a unit quality will be. Therefore, firms have incentives to sell qualities at as high a price as possible; this implies that they have no reason to offer goods of lower qualities and segment the market, as shown in Cheng et al. (2011).

Some other points are worth to mention. First, the equilibrium in the literature on single product firms is only a possible outcome of our equilibrium results under a specific range of fixed costs. Second, the equilibrium outcome depends on the fixed cost f, and there are multiple equilibria for some ranges of f. Moreover, it is shown that firms never offer greater numbers of products as the fixed cost is higher. Third, in the equilibrium of a multiproduct duopoly, only the equilibrium configuration with market segmentation (abb) and ($aabb$) may emerge. As we have mentioned, sandwich, enclosure and interlacing configurations do not emerge in equilibrium because firms have the incentive to reduce the number of products facing direct keen price competition.

Furthermore, in the multiproduct equilibrium, there are strategic effects on equilibrium qualities and market shares of firms based on the nature of the strategic interaction from multi-product offerings. Consider that both firms produce a single product, and then the low-quality firm (firm B) introduces a new product with lower quality than the existing ones. For the given quality of the pre-existing products, the new product brings pressure on prices for both existing products. In order to relax the competition from the new product, firm B has an incentive to raise the quality of its existing product. For firm B, this leads to a negative effect on increasing the competition from firm A, but there is also a positive effect on lowering the competition from its new product, together with a demand expansion. For firm A, instead, there is only a negative effect forcing this firm to raise its quality and shrinking its demand. A symmetric argument holds for the high-quality firm to introduce a new product with higher quality than the existing one. These effects on qualities and market shares are implied by the outcomes listed in Table 1.[16]

[16] Compared the outcome (ab) with (abb), the market share of firm A decreases from $0.279\overline{\theta}$ to $0.270\overline{\theta}$ while the market share of firm B increases from $0.345\overline{\theta}$ to $0.4\overline{\theta}$.

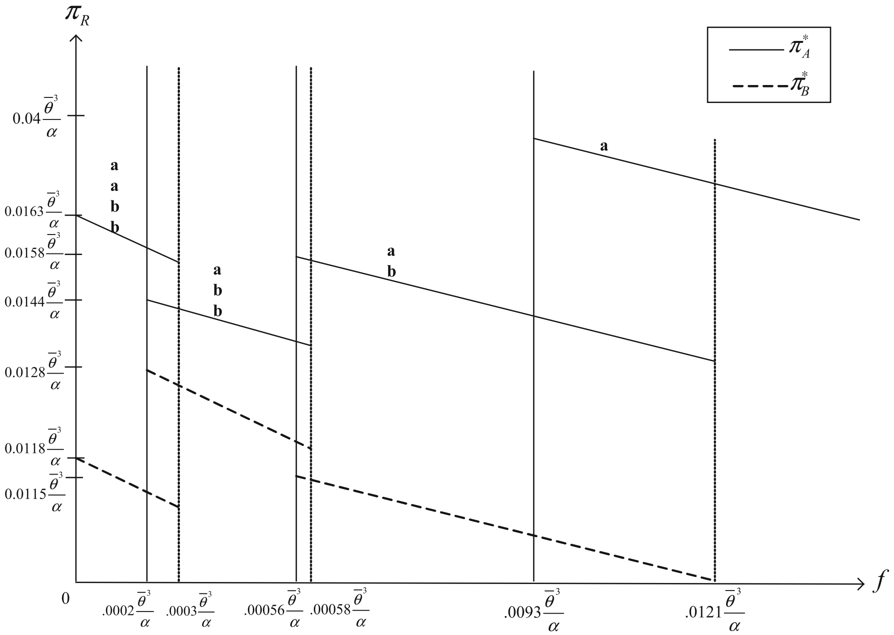

Fig. 2 Equilibrium profits

According to the possible multiproduct equilibrium, we further conclude the following property of quality differentiation among multiproduct firms:

Proposition 2 *In the segmentation equilibrium,*

(i) *The quality differentiation of two neighboring products between firms is greater than that within a firm.*
(ii) *Quality differentiation in the low-quality market is greater than that in the high-quality market.*

This result is intuitively reasonable since the competition between neighboring products offered by two distinct firms is more serious than the competition between those products offered by a single firm. The equilibrium profits of multiproduct duopolists are illustrated in Fig. 2. We notice that the profits of firms are not monotonic with regard to the fixed cost. As the fixed cost increases, the profit of a high-quality firm drops at first and then experiences a jump. However, the profit of a low-quality firm goes in the reverse direction.

4 Conclusion

In this paper, we have analyzed the variety/quality-then-price competition in which firms compete in terms of the numbers of products, quality and then prices. The equilibrium results are in sharp contrast to those in the literature on the vertically spatial competition of product differentiation, and provide an explanation for the segmented patterns of market structure in the real world.

We find that, when considering a competitor, a natural monopolist always sells a single quality of products. However, a multiproduct duopoly may emerge. In addition, in any equilibrium of a multiproduct duopoly, there are segmented patterns of product differentiation because firms have the incentive to reduce the number of products facing direct competition to avoid keen price competition. This result is in sharp contrast to the entangled patterns of differentiation in a literature with Cournot model. Furthermore, it is shown that the high-quality firm never offers more products than the low-quality firm, and the quality differentiation among neighboring products between firms is greater than that within a firm.

Our results were obtained from a specific model with several assumptions regarding the preferences of consumers, production technology and the setting of market size which are common in the related literature. There are also some possible extensions following this study. One possibility is that firms enter the market sequentially rather than simultaneously. In addition, a different assumption regarding the distribution of consumers' preferences may result in a different equilibrium outcome since it can be imagined that a non-uniform distribution of preference may be helpful to offset the disadvantage of aggravating the competition of the market sandwich, enclosure and interlacing configurations.

Appendix 1: Proof of Proposition 1

Proof To ensure that it is not profitable for firm A to deviate to configuration (a), we must have $f \leq 0.003\frac{\overline{\theta}^3}{\alpha}$. However, given $(q_{a1}^*, q_{a2}^*) = (0.4\frac{\overline{\theta}}{\alpha}, 0.2\frac{\overline{\theta}}{\alpha})$, we solve the first-order condition whereby firm B maximizes its profit $\pi_B(aab)$ and then obtain $q_{b1}^{**} = 0.105\frac{\overline{\theta}}{\alpha}$ and $\pi_B(0.4\frac{\overline{\theta}}{\alpha}, 0.2\frac{\overline{\theta}}{\alpha}, 0.105\frac{\overline{\theta}}{\alpha}) = 0.005\frac{\overline{\theta}^3}{\alpha} - f$. To ensure that firm B has no incentive to enter the market, we require that $f \geq 0.005\frac{\overline{\theta}^3}{\alpha}$. Thus, configuration (aa) cannot be an SPNE. □

Appendix 2: Proof of Remark 1

(i) The following proof shows that the candidate (aba) in Table 1, where $(q_{a1}^*, q_{a2}^*, q_{b1}^*) = (0.421\frac{\overline{\theta}}{\alpha}, 0.033\frac{\overline{\theta}}{\alpha}, 0.214\frac{\overline{\theta}}{\alpha})$, cannot be selected as an SPNE.

Proof Given $q_{b1} = 0.214\frac{\overline{\theta}}{\alpha}$, by solving the first-order conditions whereby firm A maximizes its profit $\pi_A(aab)$ by selling two products with qualities (q_{a1}, q_{a2}) such that $q_{a1} > q_{a2} > 0.214\frac{\overline{\theta}}{\alpha}$, we derive $q_{a1} = 0.466\frac{\overline{\theta}}{\alpha}$ and $q_{a2} = 0.398\frac{\overline{\theta}}{\alpha}$. Substituting them into the profit of firm A , we obtain $\pi_A(0.466\frac{\overline{\theta}}{\alpha}, 0.398\frac{\overline{\theta}}{\alpha}, 0.214\frac{\overline{\theta}}{\alpha}) = 0.01533\frac{\overline{\theta}^3}{\alpha} - 2f$, which is always higher than $\pi_A(0.421\frac{\overline{\theta}}{\alpha}, 0.033\frac{\overline{\theta}}{\alpha}, 0.214\frac{\overline{\theta}}{\alpha}) = 0.01526\frac{\overline{\theta}^3}{\alpha} - 2f$. □

(ii) The following proves that the candidate (aab) in Table 1, where $(q_{a1}^*, q_{a2}^*, q_{b1}^*) = (0.457\frac{\overline{\theta}}{\alpha}, 0.371\frac{\overline{\theta}}{\alpha}, 0.183\frac{\overline{\theta}}{\alpha})$, cannot be selected as an SPNE.

Proof Given that firm B sells a single product with quality $0.183\frac{\overline{\theta}}{\alpha}$, we solve the first-order condition whereby firm A maximizes its profit $\pi_A(ab)$ and derive $q_{a1} = 0.401\frac{\overline{\theta}}{\alpha}$ and $\pi_A(0.401\frac{\overline{\theta}}{\alpha}, 0.183\frac{\overline{\theta}}{\alpha}) = 0.0178773\frac{\overline{\theta}^3}{\alpha} - f$. Accordingly, to ensure that firm A will offer two products rather than one, it must be the case that $0.0182949\frac{\overline{\theta}^3}{\alpha} - 2f \geq 0.0178773\frac{\overline{\theta}^3}{\alpha} - f$, i.e., $f \leq 0.0004176\frac{\overline{\theta}^3}{\alpha}$. Moreover, we solve the first-order condition that firm B maximizes its profit $\pi_B(aabb)$ as firm A sells two products with qualities $(0.457\frac{\overline{\theta}}{\alpha}, 0.371\frac{\overline{\theta}}{\alpha})$, and obtain $q_{b1} = 0.195\frac{\overline{\theta}}{\alpha}$, $q_{b1} = 0.097\frac{\overline{\theta}}{\alpha}$ and $\pi_B(0.457\frac{\overline{\theta}}{\alpha}, 0.371\frac{\overline{\theta}}{\alpha}, 0.195\frac{\overline{\theta}}{\alpha}, 0.097\frac{\overline{\theta}}{\alpha}) = 0.011071\frac{\overline{\theta}^3}{\alpha} - 2f$. Therefore, to ensure that firm B will not deviate to selling two products, we require that $0.0106517\frac{\overline{\theta}^3}{\alpha} - f \geq 0.0110711\frac{\overline{\theta}^3}{\alpha} - 2f$, i.e., $f \geq 0.0004194\frac{\overline{\theta}^3}{\alpha}$. Thus, the candidate (aab) cannot be an SPNE since, if it is profitable for firm A to sell two products, firm B must have an incentive to deviate to selling two products. □

Appendix 3: No deviation conditions of configurations (*a*), (*abb*) and (*aabb*)

Monopoly with a single product (a).

$\pi_A^*(a) \geq \max\{\pi_A(aa)\}$ and

$0 \geq \max\{\pi_B(ab), \pi_B(ba)\}$; $0 \geq \max\{\pi_B(abb), \pi_B(bab), \pi_B(bba)\}$.

$\pi_A^*(a) > 0$.

Sandwich (abb).

$\pi_A^*(abb) \geq \max\{\pi_A(bab), \pi_A(bba)\}$ and $\pi_B^*(abb) \geq \max\{\pi_B(bab), \pi_B(bba)\}$.

$\pi_A^*(abb) \geq \max\{\pi_A(ab), \pi_A(ba)\}$ and

$\pi_A^*(abb) \geq \max\{\pi_A(aabb), \pi_A(abba), \pi_A(abab), \pi_A(bbaa), \pi_A(baab), \pi_A(baba)\}$.

$\pi_A^*(abb) > 0$ and $\pi_B^*(abb) > 0$.

Enclosure (aabb).

$\pi_A^*(aabb) \geq \max\{\pi_A(abba), \pi_A(abab), \pi_A(bbaa), \pi_A(baab), \pi_A(baba)\}$ and

$\pi_B^*(aabb) \geq \max\{\pi_B(abba), \pi_B(abab), \pi_B(bbaa), \pi_B(baab), \pi_B(baba)\}$.

$\pi_A^*(aabb) \geq \max\{\pi_A(abb), \pi_A(bab), \pi_A(bba)\}$ and

$\pi_B^*(aabb) \geq \max\{\pi_B(aab), \pi_B(aba), \pi_B(aab)\}$.

$\pi_A^*(aabb) > 0$ and $\pi_B^*(aabb) > 0$.

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