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Progressive taxation and macroeconomic stability in two-sector models with social constant returns

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Abstract It has been shown that, in the two-sector Benhabib–Farmer–Guo model with technologies of *social increasing returns* that exhibits indeterminacy, progressive income taxes de-stabilize the economy. This paper revisits the robustness of the tax implication in the two-sector Benhabib–Nishimura model with technologies of *social constant returns* that exhibits indeterminacy. We show that a progressive income tax stabilizes the economy against sunspot fluctuations, and thus the tax implication based on the two-sector Benhabib–Farmer–Guo model is not robust.

Keywords Two-sector model · Progressive income taxes · Indeterminacy · No-income-effect utility · Social constant returns

JEL Classification E32 · O41

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1 Introduction

Equilibrium indeterminacy is a well-established result in infinite-horizon models with production externalities. The seminal work put forth by Benhabib and Farmer (1994) and Farmer and Guo (1994) analyzed a Cobb–Douglas economy with endogenous labor supply and proved the existence of an indeterminate steady state that can be exploited to generate business-cycle fluctuations driven by "animal spirits" of agents.[1] Their utility function is characterized by the presence of positive income effects on the demand for leisure.

Early criticism of the Benhabib–Farmer–Guo model questioned the empirical plausibility of its indeterminacy result, since it required a level of social increasing returns in the production function that was at odds with the existing estimates. Following work within this area has resulted in examples of model economies that are characterized by indeterminacy with lower levels of *social increasing* returns; e.g., Benhabib and Farmer (1996), Wen (1998), Weder (2000), Harrison (2001), and Chen et al. (2015).

There is another strand of two-sector models put forth by Benhabib and Nishimura (1998) and Benhabib et al. (2002) wherein indeterminacy can arise with technologies of *social constant* returns. This line of research uses a preference which is linear in consumption, which is different from the Benhabib–Farmer–Guo model that uses a preference that is concave in consumption.[2]

One common feature in all these models is that "animal spirits" of agents can be an independent impulse to endogenous business-cycle fluctuations. It follows that the policy implications are in line with the conventional view that policy rules, which operate like an automatic stabilizer, are designed to insulate the economy from belief-driven fluctuations.

Recently, economists have studied whether progressive income taxes help stabilize the economy. In one-sector models, Guo and Lansing (1998) and Christiano and Harrison (1999) found that progressive income tax policy can help stabilize the economy against belief-driven fluctuations. Their findings are consistent with the traditional perspectives that the progressive federal income tax has a significant role as an automatic stabilizer (e.g., Auerbach and Feenberg 2000). However, in a more recent two-sector model, Guo and Harrison (2001) uncovered the result that progressive income tax policy destabilizes, rather than stabilizes, the economy. In a series of subsequent two-sector models, Guo and Harrison (2015) and Chen and Guo (2013a, b, 2014, 2016, 2017) also confirmed the result. All these two-sector models employ technologies of social increasing returns, but we wonder whether or not the de-stabilization result of progressive income tax policy is robust when the technologies are of social constant returns. The purpose of our paper is to envisage the robustness of such a tax implication

[1] We use the terms "animal spirits", "sunspots" and "self-fulfilling beliefs" interchangeably. All refer to any randomness in the economy that is not related to uncertainties about economic fundamentals such as technology, preferences and endowments. For a review of this literature, see Benhabib and Farmer (1999).

[2] Following work with technologies of *social constant* returns (e.g., Nishimura and Venditti 2007; Garnier et al. 2013) has allowed a preference with curvature in consumption. Yet, the curvature is still very small, so the utility is intrinsically linear in consumption.

in two-sector models with social constant returns by Benhabib and Nishimura (1998) and Benhabib et al. (2002). We found that the tax implication is not robust.

Specifically, our paper allows for progressive income taxes in the two-sector model with two technologies of social constant returns and a preference that is linear in consumption set forth by Benhabib and Nishimura (1998) and Benhabib et al. (2002). In the absence of income taxes, the model exhibits equilibrium indeterminacy, when the consumption good is capital intensive from the private perspective and sector-specific externalities render the consumption good to be labor intensive from the social perspective. We show that a sufficiently progressive income tax policy can stabilize the economy against belief-driven fluctuations.

To understand the reasons behind these results, first we discuss the case without income taxes. Given that the consumption good is capital intensive from the private perspective, if the agents anticipate a higher return to capital tomorrow, they will increase the demand for investment goods today. This raises the relative price of investment goods today and accumulates the capital stock tomorrow. Because consumption goods are capital intensive from the private perspective, the larger capital stock tomorrow, via the Rybczynski effect, increases the output of consumption goods and decreases investment goods, which in turn raises the price of investment goods tomorrow. As there are sector-specific externalities, investment goods are capital intensive from the social perspective, so the duality between the Rybczynski and the Stolper–Samuelson effects is destroyed. Thus, via the Stolper–Samuelson effect and investment goods being capital intensive from the social perspective, a higher price of investment goods lowers the return to labor and raises the return to capital tomorrow. As a result, initial beliefs of a higher return to capital are self-fulfilling.

However, in the presence of progressive income taxes, the duality between the Rybczynski and the Stolper–Samuelson effects can be restored from the sense of post-tax factor prices. Now, the increase in the capital stock tomorrow and the resulting higher investment price tomorrow also raise the marginal tax rate tomorrow, which may offset the initial increase on the gross return to capital. We show that, when the tax rate is sufficiently progressive, the tax effect dominates the initial increase on the gross return to capital, so the post-tax return to capital is decreased. As a result, the economy is stabilized.

In addition to the above stabilization result under sufficiently progressive income taxes, we obtain a new result when the degree of the tax progressivity is below the threshold. In this situation, the duality is not restored by progressive income tax rates. Then, equilibrium indeterminacy still arises. In this case, we find that weaker labor supply elasticity makes the possibility of indeterminacy easier. The reason is that a lower labor supply elasticity has the same effect as a smaller degree of the tax progressivity in dampening the negative effect of capital on labor supply, which makes it easier for indeterminacy to arise.

This result is in sharp contrast to the existing models with positive-income-effect preferences and social increasing-return technologies by Benhabib and Farmer (1994, 1996) and Harrison (2001) wherein greater labor supply elasticity makes it easier to establish indeterminacy. Our result is also different from the two-sector models with social constant-return technologies by Benhabib and Nishimura (1998) which found that the possibility of indeterminacy is independent of the labor supply elasticity. We note that this result is similar to that in two-sector models that use preferences without an income effect by Guo and Harrison (2010) and Dufourt et al. (2015). These two papers consider the Greenwood et al. (1988) specification for individual preferences, with Guo and Harrison (2010) using the specification case of a logarithmic specification while Dufourt et al. (2015) employ a general case with different intertemporal elasticity of substitution in consumption. Yet, the mechanism in our model is based on a progressive income tax system, which is different from social increasing returns in Guo and Harrison (2010) and Dufourt et al. (2015).[3]

A roadmap outlines our discussion. In Sect. 2, we set up a two-sector model with and without income taxes and analyze the equilibrium conditions. In Sect. 3, we examine the stability properties. Finally, concluding remarks are offered in Sect. 4.

2 The model

The model is a discrete-time version of the two-sector model put forth by Benhabib and Nishimura (1998) and Benhabib et al. (2002). We extend their model to consider progressive income taxes.

2.1 The basic structure

The model has a representative firm, a representative household and a government. The firm produces consumption goods (Y_C) and investment goods (Y_I). The household consumes and supplies labor. The government levies income taxes and makes transfers.

In each sector, output is produced by the following technology with sector-specific externalities:

$$Y_{jt} = K_{jt}^{a_j} L_{jt}^{b_j} \bar{K}_{jt}^{\alpha_j} \bar{L}_{jt}^{\beta_j}, \quad j = C, I, \tag{1}$$

where K_{jt} and L_{jt} are capital and labor employed in sector $j = C, I$, with $a_j > 0$ and $b_j > 0$ being their share, respectively. $\bar{K}_{jt}$ and $\bar{L}_{jt}$ are economy-wide average capital and labor used in sector $j = C, I$, with $\alpha_j \geq 0$ and $\beta_j \geq 0$ measuring the extent to which the sector-specific externalities affect the production in sector $j = C, I$. As in standard two-sector models, capital and labor are assumed to be perfectly mobile across the sectors. Thus, firms in both sectors face the same equilibrium factors prices. In a symmetric equilibrium, all firms in a sector make the same decisions, so $\bar{K}_{jt} = K_{jt}$ and $\bar{L}_{jt} = L_{jt}$, $j = C, I$, for all t.

We assume that the technology in both sectors exhibits social constant returns; namely, $a_j + b_j + \alpha_j + \beta_j = 1$, $j = C, I$. In the case of $\alpha_j > 0$ or $\beta_j > 0$, the technology features decreasing returns to scale from the private perspective; namely, $a_j + b_j < 1$. We denote $\Delta \equiv (a_c b_I - a_I b_c)$, which measures the factor intensity ranking from the private perspective. If $\Delta > 0$, the consumption good is capital intensive from the private viewpoint; if $\Delta < 0$, the consumption good is labor intensive from the private viewpoint. Moreover, we denote $\theta_c \equiv (a_c + \alpha_c)$ and $\theta_I \equiv (a_I + \alpha_I)$,

[3] Recently, Chen et al. (2018) have studied a two-sector model with social increasing-return technologies and individual preferences that have varying degrees of income effects on the labor supply.

and thus $(\theta_c - \theta_I)$ measures the factor intensity ranking from the social perspective. When $(\theta_c - \theta_I) > 0$, the consumption good is capital intensive from the social perspective, and when $(\theta_c - \theta_I) < 0$, the consumption good is labor intensive from the social perspective.

As in Benhabib and Nishimura (1998) and Benhabib et al. (2002), the representative household lives forever and has a lifetime utility function linear in consumption given by:

$$\sum_{t=0}^{\infty} \rho^t \left(C_t - \frac{L_t^{1+\chi}}{1+\chi} \right), \quad \chi > 0, \tag{2}$$

where C_t denotes consumption and L_t is labor supply. Parameter $0 < \rho < 1$ is the discount factor and $\chi > 0$ is the reciprocal of the (Frisch) labor supply elasticity.[4] Keeping otherwise the same environment as in Benhabib and Nishimura (1998) and Benhabib et al. (2002) except for income taxes, we will show that it is progressive income taxes, rather than regressive income taxes, that can stabilize the economy.

Households receive a wage income in exchange for labor, and an interest income for physical capital, and since there are decreasing returns to scale at the private level, they also receive dividends. The budget constraint faced by the representative household is:

$$C_t + p_t I_t = (1 - \tau_t)(r_t K_t + w_t L_t + \Pi_t) + TR_t, \tag{3}$$

where p_t is the price of investment goods in terms of consumption goods, r_t is the rental rate of capital, w_t is the wage rate, K_t is the household's capital stock, I_t is gross investment, τ_t is the income tax rate, TR_t is the lump-sum transfer and Π_t is profits.

The law of motion of the capital stock is:

$$K_{t+1} = I_t + (1 - \delta) K_t, \tag{4}$$

where $\delta < 1$ is the depreciation rate.

The government determines the income tax policy to balance its budget in each period. In order to focus on the effects of the tax policy on macroeconomic stability, we assume that the government transfers all its tax revenues to households in a lump sum, $G_t = TR_t$.[5] The government's periodic budget constraint is:

$$G_t = \tau_t Y_t. \tag{5a}$$

We remark that because of decreasing private returns, the total factor income $r_t K_t + w_t L_t$ is not equal to aggregate income $Y_t \equiv Y_{ct} + p_t Y_{It}$, and the difference is profits.

[4] In an extension to this paper later, Benhabib et al. (2002. Footnote 4) showed that the model with preferences linear in consumption and technologies of *social constant* returns to scale is compatible with the model with a nonlinear single-period utility function and technologies with *private constant* returns to scale. Note that, in a two-sector *endogenous growth* model with technologies of social constant returns, a utility nonlinear in consumption can be allowed for. See Benhabib et al. (2002) and Mino (2001).

[5] We do not allow for the government expenditure in a household's utility or a firm's production, in order to isolate the effect of progressive income taxes from that of government expenditure.

Thus, profits are given by:

$$\Pi_t \equiv Y_{ct} + p_t Y_{It} - (r_t K_t + w_t L_t). \tag{5b}$$

Combining (3), (5a) and (5b) leads to the following aggregate resource constraint for the economy: $C_t + p_t I_t = Y_t$.

To introduce the income taxes, following Guo and Lansing (1998) and Guo and Harrison (2001), we postulate an income tax rate that takes the form given by:

$$\tau_t = 1 - \eta \left(\frac{\bar{Y}}{Y_t} \right)^{\varphi}, \quad \eta \in (0, 1), \varphi \geq 0, \tag{6}$$

where $\bar{Y}$ denotes the steady-state income level which is taken as given by the household. We remark that households internalize the progressivity of the tax rate by taking into account the impact of their decisions on the rate of income taxes. Note that if households ignore the impact of their decisions on the tax rate, sufficiently progressive income taxes will not work like a stabilizer.

The parameters η and φ govern the level and the slope of the tax schedule, respectively. The marginal tax rate is the change in taxes paid by the household when the taxable income changes, and thus $\tau_t^m = \tau_t + \eta\varphi(\bar{Y}/Y_t)^{\varphi}$. Comparing the marginal tax rate τ_t^m with the average tax rate τ_t, if φ is positive, the marginal tax rate is larger than the average tax rate, and thus the income tax schedule is progressive. By contrast, if φ is negative, the marginal tax rate is smaller than the average tax rate, and thus, the tax schedule is regressive. Moreover, if φ is 0, the marginal and the average income tax rates coincide, and thus the tax schedule is flat and equal to $1 - \eta$. According to Chen and Guo (2013a), the US federal household income tax is characterized by several tax brackets and branches of income that are taxed at progressively higher rates. The specification of $\varphi > 0$ will be the focus of our model.

Note that if $\tau_t = 0$, then $TR_t = 0$. Hence, the government has no role. In this case, our model degenerates to the one studied by Benhabib and Nishimura (1998).

2.2 The optimization problem

First, the representative firm's problem is standard. It chooses labor and capital in order to maximize profits. The first-order conditions, with the use of symmetric equilibrium conditions, give:

$$r_t = a_C \frac{Y_{Ct}}{K_{Ct}} = p_t a_I \frac{Y_{It}}{K_{It}}, \tag{7a}$$

$$w_t = b_C \frac{Y_{Ct}}{L_{Ct}} = p_t b_I \frac{Y_{It}}{L_{It}}. \tag{7b}$$

Next, households receive a wage income in exchange for labor, an interest income for physical capital, and since there are decreasing returns to scale at the private level, they also receive dividends. Households solve their maximization problem by taking all these payments as given. Thus, the household's optimization problem is to choose consumption, labor supply and capital in order to maximize (2) subject to (3), (4) and (6). The first-order conditions give:

$$L_t^{\chi} = (1 - \tau_t^m) w_t, \tag{8a}$$

$$1 = \rho \frac{\left(1 - \tau_{t+1}^m\right) r_{t+1} + (1 - \delta) p_{t+1}}{p_t}, \tag{8b}$$

$$\lim_{t \to \infty} \rho^t K_{t+1} = 0. \tag{8c}$$

In these conditions, (8a) equates the marginal disutility of labor to the after-tax wage rate, (8b) is the consumption Euler equation, and (8c) is the transversality condition. Notice that as the household takes into account the effect of its income upon the tax rate, the marginal tax rate enters conditions (8a) and (8b).

2.3 Market equilibrium

Now, we determine the equilibrium. A competitive equilibrium consists of sequences of perfectly anticipated prices and profits $\{w_t, r_t, p_t, \Pi_t\}_{t \geq 0}$ and allocations $\{K_{It}, K_{Ct}, L_{It}, L_{Ct}, Y_{It}, Y_{Ct}, K_t, L_t, C_t, I_t, TR_t\}_{t \geq 0}$ that satisfy the following conditions.

(i) Given prices and profits $\{w_t, r_t, p_t, \Pi_t\}_{t \geq 0}$, the sequence of quantities $\{K_{It}, K_{Ct}, L_{It}, L_{Ct}, Y_{It}, Y_{Ct}\}_{t \geq 0}$ solves the firm's problem with the technology in (1) and the necessary conditions in (7a) and (7b).
(ii) Given prices and profits $\{w_t, r_t, p_t, \Pi_t\}_{t \geq 0}$, the sequence of quantities $\{K_t, L_t, C_t, I_t\}_{t \geq 0}$ solves the household's problem that maximizes (2) subject to (3), (4) and (6) with the necessary conditions in (8a)–(8c).
(iii) The transfer $\{TR_t\}_{t \geq 0}$ solves for the government budget constraint (5).
(iv) Profits $\{\Pi_t\}_{t \geq 0}$ are given by (5b).
(v) All markets clear: $C_t = Y_{ct}$, $I_t = Y_{It}$, and

$$K_t = K_{ct} + K_{It}, \tag{9a}$$

$$L_t = L_{ct} + L_{It}, \tag{9b}$$

where the first two equations are markets for consumption goods and investment goods, respectively, and the last two equations are markets for capital and labor, respectively.

To determine the equilibrium, these conditions are simplified as follows. First, by using the factor allocation conditions between sectors in (7a) and (7b), the factor market clearing conditions (9a) and (9b) give the following sector output.

$$Y_{ct} = \frac{b_I r_t K_t - a_I w_t L_t}{\Delta}, \tag{10a}$$

$$Y_{It} = \frac{a_c w_t L_t - b_c r_t K_t}{p_t \Delta}. \tag{10b}$$

For given prices, if we take differentiation of (10b) with respect to capital, the Rybczynski effect is:

$$\frac{dY_{It}}{dK_t} = -\frac{b_c}{\Delta}\frac{r_t}{p_t} \mathop{\gtrless}^{<} 0 \quad \text{if } \Delta \mathop{\lessgtr}^{>} 0, \tag{11a}$$

$$\frac{dY_{It}}{dL_t} = \frac{a_c}{\Delta}\frac{w_t}{p_t} \mathop{\lessgtr}^{>} 0 \quad \text{if } \Delta \mathop{\lessgtr}^{>} 0. \tag{11b}$$

Next, substituting (7a) and (7b) into (1), we obtain the following factor prices that depend only on the relative price of goods.

$$r_t = \left(p_t a_I^{\theta_I} b_I^{(1-\theta_I)} a_c^{-\frac{\theta_c(1-\theta_I)}{1-\theta_c}} b_c^{-\frac{(1-\theta_c)(1-\theta_I)}{1-\theta_c}} \right)^{\frac{1-\theta_c}{\theta_I-\theta_c}} \equiv r_t(p_t), \tag{12a}$$

$$w_t = \left(r_t(p_t) a_c^{-1} b_c^{-\frac{1-\theta_c}{\theta_c}} \right)^{-\frac{\theta_c}{1-\theta_c}} \equiv w_t(p_t). \tag{12b}$$

From (12a) and (12b), we can derive the Stolper–Samuelson effect:

$$R_P \equiv \frac{dr_t}{dp_t}\frac{p_t}{r_t} = \frac{1-\theta_c}{\theta_I-\theta_c} \mathop{\lessgtr}^{>} 0 \quad \text{if } \theta_c \mathop{\gtrless}^{<} \theta_I, \tag{13a}$$

$$W_P \equiv \frac{dw_t}{dp_t}\frac{p_t}{w_t} = \frac{-\theta_c}{\theta_I-\theta_c} \mathop{\gtrless}^{<} 0 \quad \text{if } \theta_c \mathop{\gtrless}^{<} \theta_I. \tag{13b}$$

It is interesting to note that the signs of the Rybczynski effect in (11a) and (11b) depend on the factor intensity ranking from the private perspective, Δ. By contrast, the signs of the Stolper–Samuelson effect in (13a) and (13b) depend on the factor intensity ranking from the social perspective, $(\theta_c - \theta_I)$.

The marginal tax rate is a function of the household's income: $Y_t = r_t(p_t)K_t + w_t(p_t)L_t + \Pi_t$, where $r_t(p_t)$ is in (12a) and $w_t(p_t)$ is in (12b). Using these forms, we can write the fraction of income that is disposable as follows:

$$1 - \tau_t^m = \left[\eta(1-\varphi) \left(\frac{\bar{Y}}{r_t(p_t)K_t + w_t(p_t)L_t + \Pi_t} \right)^{\varphi} \right]. \tag{14a}$$

Thus, the capital stock, the labor supply and the investment price all affect the disposable fraction of income. Differentiating (14a) with respect to capital, labor and prices, respectively, their effects are as follows:

$$\frac{\partial(1-\tau_t^m)}{\partial K_t} = \left(-\varphi \frac{r_t K_t}{Y_t} \right) \frac{1-\tau_t^m}{K_t} < 0; \tag{14b}$$

$$\frac{\partial(1-\tau_t^m)}{\partial L_t} = \left(-\varphi \frac{w_t L_t}{Y_t} \right) \frac{1-\tau_t^m}{L_t} < 0; \tag{14c}$$

$$\frac{\partial(1-\tau_t^m)}{\partial p_t} = -\frac{\varphi}{\theta_I-\theta_c} \left((1-\theta_c)\frac{r_t K_t}{Y_t} - \theta_c \frac{w_t L_t}{Y_t} \right) \frac{1-\tau_t^m}{p_t}. \tag{14d}$$

The results indicate that the disposable fraction of income is affected by capital, labor and investment prices only if $\varphi > 0$. In this case, as rises in the capital stock and the labor supply raise income and increase the tax rate, they both decrease the disposable fraction of income. Yet, the effect of an increase in the investment price on the disposable fraction of income is ambiguous.

Finally, we use (8a) to derive the labor supply L_t as a function of K_t and p_t. With (14a), we rewrite (8a) as

$$L_t^{\chi} = \left[\eta(1-\varphi)\left(\frac{\bar{Y}}{r_t(p_t)K_t + w_t(p_t)L_t + \Pi_t}\right)^{\varphi}\right] w_t(p_t), \tag{15}$$

which implies the labor supply as a function of capital and the investment price:

$$L_t = L(K_t, p_t),$$

with

$$L_K \equiv \frac{\partial L_t}{\partial K_t}\frac{K_t}{L_t} = -\frac{\varphi\frac{r_t K_t}{Y_t}}{\chi + \varphi\frac{w_t L_t}{Y_t}} \leq 0, \tag{16a}$$

$$L_P \equiv \frac{\partial L_t}{\partial p_t}\frac{p_t}{L_t} = \frac{W_P}{\chi + \varphi\frac{w_t L_t}{Y_t}} - \frac{\varphi\left(\frac{r_t K_t}{Y_t}R_P + \frac{w_t L_t}{Y_t}W_P\right)}{\chi + \varphi\frac{w_t L_t}{Y_t}}, \tag{16b}$$

where (14b)–(14d) are used in (16a) and (16b).

It is worth noting that, in the absence of progressive taxes ($\varphi = 0$), the labor supply has only the substitution effect, via the effect of changes in the investment price on the wage, with the effect depending on the factor intensity from the social perspective. In the presence of progressive taxes ($\varphi > 0$), there is an income tax effect due to changes in income. Then, the labor supply is affected not only by the investment price but also by capital. In particular, there is a negative effect of capital on the labor supply ($L_K < 0$). Intuitively, the higher capital stock increases the marginal tax rate, which decreases the after-tax wage, and thus exerts a negative effect on the labor supply. In particular, because of a larger marginal tax rate, the larger is φ, the stronger is the negative effect. Moreover, it is clear to see that a smaller χ has the same effect as a larger φ in strengthening the negative effect of capital on the labor supply.

The equations of motion for the system are given by (4) and (8b). Using (12a), (12b), (14a) and the goods market clearance conditions, these two equations of motion are written in terms of capital and the price of investment goods as follows:

$$K_{t+1} = \frac{a_c w_t(p_t)L_t(K_t, p_t) - b_c r_t(p_t)K_t}{\Delta p_t} + (1-\delta)K_t, \tag{17a}$$

$$1 = \rho\frac{\left[1 - \tau_{t+1}^m(K_{t+1}, p_{t+1})\right] r_{t+1}(p_{t+1}) + (1-\delta)p_{t+1}}{p_t}. \tag{17b}$$

These two equations solve for the equilibrium path $\{K_t, p_t\}_{t=0}^{t=\infty}$. The equilibrium is a steady state if $K_{t+1} = K_t = K$ and $p_{t+1} = p_t = p$ for all t. It follows that in the steady state, $C_{t+1} = C_t = C$, $r_{t+1} = r_t = r$ and $w_{t+1} = w_t = w$ for all t. From (17a) and (17b), it is straightforward to show that, in the steady state, the real interest rate and the ratio of income that accrue to labor and capital are unique as follows[6]:

$$\frac{r}{p} = \frac{[1-\rho(1-\delta)]}{\rho\eta(1-\varphi)}, \tag{18a}$$

$$\Psi \equiv \frac{wL}{rK} = \frac{b_c[1-\rho(1-\delta)] + \delta\Delta\rho\eta(1-\varphi)}{a_c[1-\rho(1-\delta)]}. \tag{18b}$$

Using market clearing conditions ($C = Y_c$ and $I = Y_I$) and substituting (10a)–(10b) and (18b) into the aggregate resource constraint ($C + pI = Y$), the labor share in income in the steady state is

$$\Xi \equiv \frac{wL}{Y} = \frac{b_c[1-\rho(1-\delta)] + \Delta\delta\rho\eta(1-\varphi)}{[1-\rho(1-\delta)] + (a_c - a_I)\delta\rho\eta(1-\varphi)} \in (0, 1). \tag{19}$$

When income taxes are progressive, $\eta < 1$ and $\varphi > 0$. Then, although a larger degree of income tax progressivity increases the real interest rate in the steady state, it may decrease or increase the wage share in income in the steady state depending on $\Delta > 0$ or $\Delta < 0$.

In the case of zero income taxes, $\eta = 1$ and $\varphi = 0$. Then, the real interest rate in (18a) and the labor wage share in income in (19) degenerate to:

$$\frac{\tilde{r}}{p} = \frac{[1-\rho(1-\delta)]}{\rho}, \quad \tilde{\Xi} \equiv \frac{wL}{Y} = \frac{b_c[1-\rho(1-\delta)] + \delta\Delta\rho}{(a_c + b_c)[1-\rho(1-\delta)] + \delta\Delta\rho} \in (0, 1).$$

3 Stability analysis

We have obtained the trace and the determinant of the Jacobian matrix associated with the system of difference equations that characterizes the stability property in (17a) and (17b). Let λ_1 and λ_2 be the two eigenvalues. The trace and the determinant are, respectively:

$$\lambda_1 + \lambda_2 = Trace(J) = J_{11} + \frac{-1}{\Gamma} + \frac{\Lambda}{\Gamma} J_{12}, \tag{19a}$$

$$\lambda_1\lambda_2 = Det(J) = (J_{11})\left(\frac{-1}{\Gamma}\right), \tag{19b}$$

[6] To obtain (18a), we use the steady-state version of (17b), along with the form of the income tax $\tau_t^m = \tau_t + \eta\varphi(\bar{Y}/Y_t)^{\varphi}$ and (6). Moreover, we use (18a) and the steady-state version of (17a) to obtain (18b).

where[7]

$$J_{11} = \tilde{J}_{11} - \frac{dY_I}{dK}\left[\frac{a_c}{b_c}\Psi L_K - [1-\eta(1-\varphi)]\right], \quad \text{with}$$

$$J_{11} = \tilde{J}_{11} = 1-\delta + \frac{dY_I}{dK} \quad \text{if } \eta = 1 \text{ and } \varphi = 0;$$

$$J_{12} = \frac{1}{\Delta}\frac{r}{p}[a_c(W_P + L_P)\Psi - b_c R_P] - \delta < 0 \quad \text{if } \theta_c < \theta_I \text{ and } \Delta > 0;$$

$$\Lambda \equiv -\varphi[1-\rho(1-\delta)]\left[\frac{\Xi}{\Psi} + \Xi L_K\right] \leq 0, \quad \text{with } \Lambda = \tilde{\Lambda} = 0 \text{ if } \varphi = 0;$$

$$\Gamma \equiv \tilde{\Gamma} + \varphi\Omega, \quad \text{with } \Gamma = \tilde{\Gamma} = -1 + \frac{[1-\rho(1-\delta)](1-\theta_I)}{\theta_c - \theta_I} \text{ if } \varphi = 0;$$

$$\tilde{\Gamma} < -1 \quad \text{if } \theta_c < \theta_I;$$

$$\Omega \equiv [1-\rho(1-\delta)]\left[\frac{\Xi}{\Psi}R_P + \Xi(W_P + L_P)\right] < 0 \quad \text{if } \theta_c < \theta_I.$$

If both eigenvalues lie inside the unit circle, there is a continuum of the equilibrium path toward the steady state, and thus the equilibrium path is indeterminate. However, if one of the two eigenvalues lies inside the unit circle and the other eigenvalue lies outside the unit circle, the equilibrium path toward a steady state is unique, and thus determinate. To determine the eigenvalues, we start with the model without income taxes, followed by the model with income taxes.

3.1 The model without income taxes

In the absence of income taxes, $\eta = 1$ and $\varphi = 0$. Then, the model degenerates to the models of Benhabib and Nishimura (1998) and Benhabib et al. (2002). We have shown that the two roots are:

$$\tilde{\lambda}_1 = \frac{1}{-\tilde{\Gamma}} \in (0, 1) \quad \text{if } \theta_c < \theta_I, \tag{20a}$$

$$\tilde{\lambda}_2 = \tilde{J}_{11} = \left[1-\delta - \frac{b_c}{\Delta}\frac{\tilde{r}}{p}\right] = (1-\delta) - \frac{b_c}{\Delta}\frac{1-\rho(1-\delta)}{\rho} \in (-1, 1-\delta). \tag{20b}$$

Both roots lie inside the unit circle. To see this, as $\tilde{\Gamma} < -1$ if $(\theta_I - \theta_C) > 0$, the root $\tilde{\lambda}_1$ lies inside (0, 1) when the consumption sector is more labor intensive from the social perspective. The other root $\tilde{\lambda}_2$ lies inside (0, 1), if the consumption good is capital intensive from the private perspective with $\frac{a_c}{b_c} - \frac{a_I}{b_I} > \frac{1}{\rho b_I}$, a condition imposed by Benhabib et al. (2002, Proposition 2). Thus, in the absence of income taxes, the economy is destabilized.

The intuition behind the result of equilibrium indeterminacy is based on self-fulfilling beliefs. The expectations lead to increases in all of p_t, p_{t+1} and $r_{t+1}(p_{t+1})$

[7] A notation with a tilde is denoted as its counterpart in the model without income taxes.

so as to meet (17b). The mechanism goes as follows. With beliefs that the return to capital is higher tomorrow, agents will raise the demand for investment goods today, which increases the investment price today. A higher investment today accumulates the capital stock tomorrow. Given that the consumption good is capital intensive at the private level, via the Rybczynski effect, the increase in the capital stock decreases the output of investment goods and increases the output of consumption goods tomorrow at constant prices. With a reduction in investment goods tomorrow, the investment price increases tomorrow. Now, due to the sector-specific externalities, the investment good is capital intensive from the social perspective. A higher investment price, via the Stolper–Samuelson effect, causes an increase in the return to capital tomorrow. As a result, initial beliefs in a higher return to capital tomorrow are self-fulfilling. Note that higher capital increases consumption goods and decreases investment goods, but the return to capital is increased. Thus, the duality between the Rybczynski and the Stolper–Samuelson effects is destroyed.

Our model considers a utility that is linear in consumption, as in Benhabib and Nishimura (1998). Hence, our aforementioned condition of local indeterminacy only requires relative capital intensity and is independent of the labor supply elasticity. We must note that in general the utility is not linear in consumption. If we allow for a utility with sufficiently finite elasticity of intertemporal substitution in consumption, as in Nishimura and Venditti (2007), in addition to the condition on relative capital intensity, local indeterminacy occurs for sufficiently inelastic labor supply (cf. Nishimura and Venditti 2007, Theorem 5).

3.2 The model with income taxes

In the presence of progressive income taxes, $\eta < 1$ and $\varphi > 0$. A saddle-point property requires that one of the two eigenvalues lie outside the unit circle and the other eigenvalue lie inside the unit circle.

If we solve (19a) and (19b), we obtain the two roots as follows.

$$\lambda_1 = \frac{1}{2}\left[\left(J_{11} - \frac{1}{\Gamma}\right) + \frac{\Lambda J_{12}}{\Gamma} + \sqrt{\left[\left(J_{11} - \frac{1}{\Gamma}\right) + \frac{\Lambda J_{12}}{\Gamma}\right]^2 + 4J_{11}\frac{1}{\Gamma}}\right], \tag{21a}$$

$$\lambda_2 = \frac{1}{2}\left[\left(J_{11} - \frac{1}{\Gamma}\right) + \frac{\Lambda J_{12}}{\Gamma} - \sqrt{\left[\left(J_{11} - \frac{1}{\Gamma}\right) + \frac{\Lambda J_{12}}{\Gamma}\right]^2 + 4J_{11}\frac{1}{\Gamma}}\right]. \tag{21b}$$

We now show that $0 < \lambda_1 < 1$ and $\lambda_2 < -1$, and thus λ_1 lies inside and λ_2 lies outside the unit circle.

First, we show $0 < \lambda_1 < 1$. To start from $\lambda_1 > 0$, the sign of $J_{11} - \frac{1}{\Gamma} + \frac{\Lambda J_{12}}{\Gamma}$ is ambiguous because J_{11} and Γ are negative. If $J_{11} - \frac{1}{\Gamma} + \frac{\Lambda J_{12}}{\Gamma} > 0$, then $\lambda_1 > 0$. Similarly, if $J_{11} - \frac{1}{\Gamma} + \frac{\Lambda J_{12}}{\Gamma} < 0$, then $\lambda_1 = \left(J_{11} - \frac{1}{\Gamma}\right) +$

$\frac{\Lambda J_{12}}{\Gamma} + \sqrt{\left[\left(J_{11} - \frac{1}{\Gamma}\right) + \frac{\Lambda J_{12}}{\Gamma}\right]^2 + 4J_{11}\frac{1}{\Gamma}} > 0$ since $J_{11} < 0$, $\Gamma < 0$ and thus, $4J_{11}\frac{1}{\Gamma} > 0$. Next, to show $\lambda_1 < 1$, we note that $\lambda_1 < \frac{1}{2}\left\{J_{11} + \frac{1}{\Gamma} + \frac{\Lambda J_{12}}{\Gamma} + \sqrt{[(J_{11} - \frac{1}{\Gamma}) + \frac{\Lambda J_{12}}{\Gamma}]^2 + 4J_{11}\frac{1}{\Gamma}}\right\} = -\frac{1}{\Gamma} < -\frac{1}{\bar{\Gamma}} < 1$. Therefore, $0 < \lambda_1 < 1$.

Second, we show $\lambda_2 < -1$. To this end, we restrict the right-hand side of (21b) to be smaller than -1 and obtain the following cubic equation in terms of φ (see "Appendix C"):

$$m_3\varphi^3 + m_2\varphi^2 + m_1\varphi + m_0 > 0, \tag{22}$$

where m_0, m_1, m_2 and m_3 are constant. If we equate the left-hand side of (22) to zero, we obtain three critical values of φ, denoted by $\{\varphi_1, \varphi_2, \varphi_3\}$. Denote by $\varphi_1 = max\{\varphi_1, \varphi_2, \varphi_3\}$, the largest among these three values. In "Appendix", we have shown that there are either (i) three positive critical values or (ii) one positive and two negative critical values. No matter whether it is case (i) or (ii), if $\varphi > \varphi_1$, then condition (22) is met and $\lambda_2 < -1$.

Therefore, if the income tax is sufficiently progressive such that $\varphi > \varphi_1$, one of the eigenvalues lies outside the unit circle and the other eigenvalue lies inside the unit circle. Then, the steady state is a saddle point. We state our main result as follows.

Proposition 1 *The steady state is a saddle point if the income tax schedule is sufficiently progressive.*

Proposition 1 indicates that a sufficiently progressive income tax policy can stabilize an otherwise destabilized economy.

The reasons behind the result are that the duality between the Rybczynski and the Stolper–Samuelson effects otherwise destroyed by sector-specific externalities is now restored by a sufficiently progressive income tax rate. The underlying reason is that, if agents expect a higher return to capital tomorrow, the expectations increase p_t and p_{t+1}. Yet, when the marginal tax rate is sufficiently high, the negative effect via a higher marginal tax rate would dominate the positive effect on the gross return, so the post-tax return to capital decreases. Then, the initial beliefs of a higher return to capital tomorrow is not self-fulfilling.

The effect of a higher capital stock tomorrow on the post-tax return to capital can be derived as follows.

$$\frac{\partial\left[\left(1-\tau_{t+1}^m\right)r_{t+1}\right]}{\partial K_{t+1}} = \underbrace{\frac{\partial\left[\left(1-\tau_{t+1}^m\right)r_{t+1}\right]}{\partial p_{t+1}}}_{(+)}\underbrace{\frac{\partial p_{t+1}}{\partial K_{t+1}}}_{(+)} + r_{t+1}\underbrace{\frac{\partial\left(1-\tau_{t+1}^m\right)}{\partial K_{t+1}}}_{(-)} < 0$$

$$\text{if } \varphi \text{ is large.} \tag{23}$$

As explained in Sect. 3.1, the higher capital stock tomorrow increases the investment price tomorrow. With the investment goods being capital intensive from the social perspective, the gross return to capital is increased tomorrow: $\frac{\partial r_{t+1}}{\partial p_{t+1}} > 0$. Yet, a higher investment price also affects the marginal tax rate. With the use of (13a), (13b) and

(14b), a higher investment price increases the net return to capital in the following way.

$$\begin{aligned}\frac{\partial\left[\left(1-\tau_{t+1}^{m}\right)r_{t+1}\right]}{\partial p_{t+1}} &= \left(1-\tau_{t+1}^{m}\right)\frac{\partial r_{t+1}}{\partial p_{t+1}}+r_{t+1}\frac{\partial\left(1-\tau_{t+1}^{m}\right)}{\partial p_{t+1}}\\ &= \Upsilon\left\{(1-\theta_C)(1-\varphi)\frac{r_{t+1}K_{t+1}}{Y_{t+1}}+[1-\theta_C(1-\varphi)]\frac{w_{t+1}L_{t+1}}{Y_{t+1}}\right\}\\ &> 0,\end{aligned} \tag{24}$$

where $\Upsilon \equiv \frac{(1-\tau_{t+1}^{m})r_{t+1}}{(\theta_I-\theta_C)p_{t+1}} > 0$ if $\theta_C < \theta_I$.

Moreover, in addition to indirectly affecting the marginal tax rate via the investment price, because of a rise in income, the higher capital stock tomorrow also has a direct effect to increase the marginal tax rate. This is the effect of the second term of (23). According to (14b), the effect is negative. In particular, (14b) indicates that a larger degree of income tax progressivity gives a stronger negative effect of capital on the marginal tax rate. If the tax progressivity is sufficiently large, the effect in the second term of (23) dominates the first term. As a result, the post-tax return to capital decreases and the initial beliefs of a higher return to capital tomorrow is not self-fulfilling.

We remark that, from the traditional Keynesian perspective, references to automatic stabilizers of progressive income taxes have always referred to the stabilization of aggregate demand. This is consistent with the assumption that the level of employment is demand-determined. However, in our framework, employment levels are also determined by labor supply conditions. When capital and thus income increases, the higher marginal tax rates also work through incentive effects and discourage labor supply. This is seen from L_K in (16a) wherein, with other things being equal, if φ is larger, the capital stock has a larger effect through which the labor supply is discouraged. Such stabilization on the supply side is consistent with the views expressed in Auerbach and Feenberg (2000, p. 48).

We must note that if the degree of tax progressivity is below the threshold, indeterminacy still arises, because progressive income tax rates cannot restore the duality between the Rybczynski and the Stolper–Samuelson effects destroyed by sector-specific externalities. In this situation, we find that a weaker, rather than a stronger, labor supply elasticity (i.e., a larger χ) makes indeterminacy easier to emerge. This result is understood by observing that in (16a), a larger χ has the same effect as a smaller φ in dampening the negative effect of capital on labor supply. This suggests that a weaker labor supply elasticity also makes it easier for indeterminacy to arise.

Intuitively, the labor supply is determined by the equalization of the marginal disutility of the labor supply and the marginal benefit of the labor supply [cf. (15)]. With other things being equal, a larger χ and thus, a smaller labor supply elasticity, increases the marginal disutility of the labor supply which decreases the labor supply.[8] Moreover, a smaller φ decreases the marginal benefit of the labor supply which decreases the labor supply. Thus, a larger χ and a smaller φ both exert a qualitatively similar

[8] Equation (15) gives $\chi\ln(L_t) = \ln[\eta(1-\varphi)\bar{Y}^{\varphi}] + \varphi[-\ln(1-\tau_t^m)] + \ln w_t$, with $-\ln(1-\tau_t^m) > 0$ as $(1-\tau_t^m) < 1$.

effect on the labor supply. As a result, a larger χ has a role similar to a smaller φ that facilitates indeterminacy.

This result is different from the existing one-sector and two-sector models with social increasing-return technologies put forth by Benhabib and Farmer (1994) as well as Benhabib and Farmer (1996) and Harrison (2001). These strands of research uncovered that it is easier for indeterminacy to emerge if the labor supply elasticity is larger (i.e., a smaller χ). Moreover, our result is also different from the two-sector model set forth by Benhabib and Nishimura (1998). Due to the lack of progressive income taxes, these authors discovered that the indeterminacy result is independent of the labor supply elasticity.

We must note that, in models with the Greenwood et al. (1988) specification for individual preferences and thus the lack of income effects on labor choices, Guo and Harrison (2010) and Dufourt et al. (2015) also found that indeterminacy is more likely if the labor supply elasticity is smaller. They obtain this result, because in their model, due to technologies of social increasing returns, the effect of a smaller labor supply elasticity on employment is like the effect of a larger investment externality on employment. Thus, as the labor supply elasticity decreases, the required minimum degree of the investment externality is decreased, in order for the employment to increase by the same level, so that real return to capital tomorrow equals the marginal rate of substitution between today's and tomorrow's consumption. Our model is different from their model in that, with socially constant-return technology, the mechanism is based on a progressive income tax policy, which restores the duality between the Rybczynski and the Stolper–Samuelson effects.

4 Conclusion

Using the two-sector Benhabib–Farmer–Guo model with technologies of social increasing returns, Guo and Harrison (2001) showed that progressive income taxes functioned like a destabilizer and their subsequent work found the tax implication to be very robust. In this paper, we explore whether such an income tax implication is robust in the two-sector Benhabib–Nishimura model with technologies of social constant returns and a preference which is linear in consumption. We find that sufficiently progressive income taxes function like a stabilizer, and thus the tax implication obtained by Guo and Harrison (2001) and their following work is not robust.

We also find another new result. When the degree of the tax progressivity is below the threshold, the indeterminacy still arises. In this case, a smaller, rather than a larger, labor supply elasticity makes more likely the possibility of indeterminacy.

Finally, we remark that if aggregate business cycle fluctuations are due to exogenous shocks to fundamentals and equilibrium is unique, then alternation of the cycle pattern that occurs in equilibrium will require significant alternation of the incentives faced by private agents and hence significant government intervention in the marketplace at all times. If aggregate fluctuations are due to self-fulfilling revisions to expectations, the fluctuations surely are not efficient. This would necessarily have important consequences for the way we consider the aims of stabilization policy. Yet, it is important not to assume from such a consideration alone that a stabilization scheme necessarily improves welfare. Even a scheme that succeeds in eliminating all sunspot equilibria while not interfering with the deterministic steady state of the economy, like the one discussed in our paper, does not necessarily increase welfare.

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Appendix

A. The trace and determinant of the Jacobian matrix

Using a caret to denote a variable in logarithmic deviations from the steady-state value, the log-linear approximations give[9,10]:

$$\begin{bmatrix} \hat{K}_{t+1} \\ \hat{P}_{t+1} \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \begin{bmatrix} \hat{K}_t \\ \hat{P}_t \end{bmatrix}, \tag{A1}$$

$$\begin{aligned}
J_{11} &= \tilde{J}_{11} - \frac{dY_I}{dK}\left[\frac{a_c}{b_c}\Psi L_K - [1-\eta(1-\varphi)]\right];\\
J_{12} &= \frac{1}{\Delta}\frac{r}{p}\left[a_c(W_P + L_P)\Psi - b_c R_P\right] - \delta < 0 \quad \text{if } \theta_c < \theta_I \text{ and } \Delta > 0;\\
J_{21} &= \frac{\Lambda}{\Gamma} J_{11};\\
J_{22} &= \frac{-1}{\Gamma} + \frac{\Lambda}{\Gamma} J_{12},\\
\Lambda &\equiv -\varphi[1-\rho(1-\delta)]\left[\frac{\Xi}{\Psi} + \Xi L_K\right] \le 0, \text{ and } \Lambda = 0 \quad \text{if } \varphi = 0;\\
\Gamma &\equiv -1 + \frac{[1-\rho(1-\delta)](1-\theta_I)}{\theta_c - \theta_I} + \varphi\Omega;\\
\Omega &\equiv [1-\rho(1-\delta)]\left[\frac{\Xi}{\Psi}R_P + \Xi(W_P + L_P)\right] < 0 \quad \text{if } \theta_c < \theta_I.
\end{aligned}$$

B. Determinacy in the model with income taxes

In the presence of progressive income taxes, $\eta < 1$ and $\varphi > 0$. The two eigenvalues are determined by:

$$\lambda_1 + \lambda_2 = Trace(J) = J_{11} + J_{22} = J_{11} + \frac{-1}{\Gamma} + \frac{\Lambda}{\Gamma} J_{12}, \tag{B1a}$$

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9 $[a_c(W_P + L_P)\Psi - b_c R_P] = -\left[a_c \frac{\chi+1+\varphi\Xi}{\chi+\varphi\Xi}\Psi\frac{\theta_c}{\theta_I-\theta_c} + b_c\frac{1-\theta_c}{\theta_I-\theta_c}\right] < 0$ if $\theta_c < \theta_I$.

10 It is easy to show $[\frac{\Xi}{\Psi}R_P + \Xi(W_P + L_P)] = -\frac{\chi}{\chi+\varphi\Xi}\left[\frac{\Xi-(1-\theta_c)(1-\Xi-\frac{\Xi}{\Psi})}{\theta_I-\theta_c}\right] < 0$ if $\theta_c < \theta_I$, as the labor wage share in income Ξ is close to $1-\theta_c$.

$$\lambda_1\lambda_2 = Det(J) = J_{11}J_{22} - J_{21}J_{12} = (J_{11})\left(\frac{-1}{\Gamma}\right). \tag{B1b}$$

Solving conditions (B1a) and (B1b) gives the following two eigenvalues:

$$\lambda_1 = \frac{1}{2}\left[\left(J_{11} - \frac{1}{\Gamma}\right) + \frac{\Lambda J_{12}}{\Gamma} + \sqrt{\left[\left(J_{11} - \frac{1}{\Gamma}\right) + \frac{\Lambda J_{12}}{\Gamma}\right]^2 + 4J_{11}\frac{1}{\Gamma}}\right], \tag{B2a}$$

$$\lambda_2 = \frac{1}{2}\left[\left(J_{11} - \frac{1}{\Gamma}\right) + \frac{\Lambda J_{12}}{\Gamma} - \sqrt{\left[\left(J_{11} - \frac{1}{\Gamma}\right) + \frac{\Lambda J_{12}}{\Gamma}\right]^2 + 4J_{11}\frac{1}{\Gamma}}\right]. \tag{B2b}$$

If $0 < \lambda_1 < 1$ and $\lambda_2 < -1$, the steady state is a saddle. The part $0 < \lambda_1 < 1$ is proved in the text. Here we prove $\lambda_2 < -1$.

To show $\lambda_2 < -1$, we will find a threshold value so that if φ is larger than the threshold value, then $\lambda_2 < -1$. To derive the threshold value, it suffices to impose $\lambda_2 < -1$ in (B2b). This gives the following cubic equation in terms of φ:

$$m_3\varphi^3 + m_2\varphi^2 + m_1\varphi + m_0 > 0, \tag{B3}$$

where

$$\begin{aligned}
m_3 &= -(2-\delta)\delta(\Delta\rho\eta)^2[(1+\chi)\theta_c H + M] < 0,\\
m_2 &= [M + (1+\chi)\theta_c H]\Delta\rho\eta[(b_c H + 2\delta\Delta\rho\eta)(2-\delta) - (a_c + b_c)\delta H]\\
&\quad - M(2-\delta)\delta\Delta(\rho\eta)^2\chi(a_c - a_I)\\
&\quad - [2(1-\theta_c) - \delta(1-\theta_I)]H^2\Delta\rho\eta\chi a_c,\\
m_1 &= [M + (1+\chi)\theta_c H][(a_c + b_c)H - (2-\delta)\Delta\rho\eta][b_c H + \delta\Delta\rho\eta]\\
&\quad + M(2-\delta)\Delta\rho\eta\chi[H + (a_c - a_I)\delta\rho\eta]\\
&\quad + [2(1-\theta_c) - \delta(1-\theta_I)]H^2\Delta\rho\eta\chi a_c\\
&\quad - \chi M(a_c - a_I)\delta\rho\eta[b_c H + (2-\delta)\Delta\rho\eta],\\
m_0 &= \chi M[H + (a_c - a_I)\delta\rho\eta][b_c H + (2-\delta)\Delta\rho\eta] > 0,\\
M &= 2(\theta_c - \theta_I) + (1-\theta_I)H,\\
H &\equiv [1 - \rho(1-\delta)].
\end{aligned}$$

Note that the condition $\theta_I - \theta_c > 0$ is used in signing m_0 and m_3.

If we set (B3) equal to 0, we obtain $(\varphi - \varphi_1)(\varphi - \varphi_2)(\varphi - \varphi_3) = 0$, and there are three critical values: φ_1, φ_2 and φ_3. Let $max\{\varphi_1, \varphi_2, \varphi_3\}$ be φ_1, the largest value of the three critical values.

To solve the three critical values, we first notice that the product of these three critical values is $\varphi_1\varphi_2\varphi_3 = -m_0/m_3 > 0$. This indicates that there are either case (i) with three positive critical values or case (ii) with one positive and two negative critical values.

Next, we order these three critical values in a way such that $\varphi_1 > \varphi_2 > \varphi_3$. Thus, the largest value is φ_1. Hence, no matter whether it is case (i) or case (ii), the largest value is φ_1. As a result, for the steady state to be a saddle point, the required condition is $\varphi > \varphi_1$.

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