Welfare Implications and Equilibrium Indeterminacy in a Two-sector Growth Model with Consumption Externalities*

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Abstract

In one-sector neoclassical growth models, consumption externalities lead to an inefficient allocation in a steady state and indeterminate equilibrium toward a steady state only if there is a labor-leisure tradeoff. This paper shows that in a two-sector neoclassical growth model, even without a labor-leisure tradeoff, consumption spillovers easily lead to an inefficient allocation in a steady state and indeterminate equilibrium toward a steady state. Negative consumption spillovers that yield over-accumulation of capital in a one-sector model may lead to under-accumulation or an over-accumulation of capital in two-sector models depending on the relative capital intensity between sectors. Moreover, a two-sector model economy with consumption externalities is less stabilized than an otherwise identical one-sector model economy.

Keywords: two-sector model; consumption externalities; efficiency; indeterminacy

JEL Classification: E21; E32; O41

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1. Introduction

Economists have paid a great deal of attention to consumption externalities since the inception of modern economic thought.[1] Some empirical and experimental studies provide convincing support for the significance of consumption externalities associated with social comparison: see, for example, Clark and Oswald (1996), Frank (1997) and Luttmer (2005).[2] Many macro models specify preferences wherein an average consumption level of the economy affects individuals' felicity. In particular, this departure from standard preference specifications has been presented in growth models in order to account for some empirical phenomena that cannot be explained under more traditional specifications.

Generally speaking, we can classify consumption externalities as the "keeping up with the Joneses" (hereinafter, *KUJ*) effect and the "catching up with the Joneses" (henceforth, *CUJ*) effect. The *KUJ* effect emerges when *contemporary* average consumption in the economy makes more valuable a marginal increase of personal current consumption, while the *CUJ* effect arises when *past* average consumption in the economy makes more valuable a marginal increase of personal current consumption. Many studies found the *KUJ* effect to be of central importance in accounting for asset prices (Gali, 1994), capital accumulation (Arrow and Dasguputa, 2009; Fisher and Hof, 2000; Dupor and Liu, 2003; Liu and Turnovsky, 2005), consumption across generations (Abel, 2005), business cycles (Chen and Hsu, 2007; Alonso-Carrera et al., 2008a), and dynamic altruism (Alonso-Carrera et al., 2008b). Moreover, a number of analyses uncovered the *CUJ* effect to be crucial upon asset prices (Abel, 1990; Campbell and Cochrane, 1999), business cycles (Ljungqvist and Uhlig, 2000), capital accumulation (Turnovsky and Monteiro, 2007), and long-term balanced

[1] The concept of the consumption externalities and habits may be traced to Hume (1748) who argued that preferences were influenced not simply by what a person did in the past, what his/her parents did, and what contemporary peers were doing but also by the behavior of past generations of peers. Contemporary ideas dated to Marshall (1898) and Veblen (1912) and were first formalized by Duesenberry (1949) as a determinant of aggregate consumption in his development of the relative income hypothesis. More recently, the Easterlin paradox, first mentioned in Easterlin (1974) and since that time further expanded upon along with a lively debate, posits that income growth does not necessarily enhance individual welfare, and instead, emphasizes the role of consumption externalities and social comparison; see for example Easterlin (2001).

[2] Clark and Oswald (1996) presented some direct empirical evidence for British workers, showing that their reported satisfaction levels are inversely related to their comparison wage rates. Based on both the psychological evidence and the more fragmentary evidence in behavioral economics, Frank (1997) concluded that both these sources support the claim that satisfaction depends upon the agent's relative position, again emphasizing the role of the externalities it generates. Luttmer (2005) presented a detailed empirical study showing that the level of earning relative to average earning may significantly affect the individual welfare. He indicated further that the relative consumption is the most relevant proxy of unobservable earnings of other households. See also Clark et al. (2008), Frank (2005), and Maurer and Meier (2008) for further discussion.

growth (Alvarez-Cuadrado et al., 2004; Doi and Mino, 2008).[3]

In one-sector neoclassical growth models with constant time preference rates, only if there is a labor-leisure tradeoff, contemporary consumption externalities can lead to an inefficient allocation in a steady state and generate indeterminate dynamic equilibrium paths toward a steady state. The reason why the labor-leisure tradeoff plays such a role may be explained by using as an illustrative example a negative consumption externality. First, Liu and Turnovsky (2005) and Turnovsky and Monteiro (2007) show that negative spillover leads to over-consumption that is associated with inefficiently less leisure time and more labor supply and, with fixed capital intensity in the long run, capital is thus over-accumulated.[4] Moreover, Alonso-Carrera et al. (2008) shows that, if negative consumption spillovers make the Frisch labor supply to have a certain shape, the dynamic equilibrium toward a steady state is indeterminate.[5]

This paper shows that in a two-sector neoclassical growth model, even without a labor-leisure tradeoff, contemporary consumption spillovers can yield not only an inefficient allocation in a steady state but also indeterminate dynamic equilibrium paths toward a steady state. We show these results in a two-sector model wherein the general goods sector produces goods that are used as consumption and investment and the consumption goods sector produces goods that are used only as consumption. The reasons to have these results lie in relative prices of these two goods and the factor allocation between the two sectors. In a producer's optimization, the price of general goods relative to consumption goods equals the marginal rate of transformation (hereinafter, *MRT*), the ratio of the marginal product in the consumption sector to the marginal product in the general goods sector. In a consumer's optimization, the relative price of general goods equals the marginal rate of substitution (hereinafter, *MRS*), the ratio of the marginal utility of general goods to that of

[3] External consumption habits are accumulated consumption externalities. As external habits are formed by both current and past average consumption, the habit effect is a mix of the *KUJ* effect and the *CUJ* effect (e.g., Carroll et al., 1997; Abel, 1999; Turnovsky and Monteiro; 2007; Chen et al., 2013). Conversely, if the habit is formed only by past average consumption, the habit effect involves only the *CUJ* effect (e.g., Campbell and Cochrane, 1997; Grishchenko, 2010).

[4] Liu and Turnovsky (2005) and Turnovsky and Monteiro (2007) found that consumption externalities cause long-run distortions only if there is a labor-leisure tradeoff. Other than depending on personal consumption and leisure, Liu and Turnovsky (2005) assumed that an individual's preference depends on contemporary average consumption, while Turnovsky and Monteiro (2007) assumed that an individual's preference depends on external habits. As the habits are a weighted average of current and past average consumption, Liu and Turnovsky (2005) is a special case of Turnovsky and Monteiro (2007) when the weights in the past are zero.

[5] Alonso-Carrera et al. (2008) extended the model used by Liu and Turnovsky (2005) with elastic leisure and studied the dynamic properties of equilibrium. These authors showed that indeterminacy cannot arise if the utility is homothetic in personal consumption and average consumption. They found that only if the utility is not homothetic can indeterminacy arise.

consumption goods. The presence of consumption externalities distorts the relationship between *MRS* and *MRT,* which may generate inefficiency and indeterminacy.

Our two-sector setting has two advantages over the foregoing studies with regard to the role of consumption externalities in dynamic macroeconomics. First, our model is more general than the models found in extant literature. In effect, when the consumption goods sector does not use capital, the goods are the consumption of leisure and our model is reduced to a one-sector model with elastic leisure which was also studied by Liu and Turnovsky (2005) and Alonso-Carrera et al. (2008). The second advantage of our modeling is that, unlike homogeneous-good models, we can treat commodity-specific externalities. We show that asymmetric external effects may play a key role for equilibrium determinacy. It should be noted that Ravn et al. (2006), Doi and Mino (2008) and Hori (2011) studied commodity-specific consumption externalities. In analyzing models with monopolistic competition and a variety of consumption goods, those authors assume that consumers put the same importance on all types of consumption of other households. In such a situation, a consumer would be concerned equally about the consumption of food and autos of other consumers. However, recent studies in behavioral economics have offered evidence that consumers placed different importance on different types of consumption of other consumers. See, for example, Solnick and Hemenway (1998), Alpizar et al. (2005), and Carlsson et al. (2007). They investigated the positionality degree for different goods based on empirical or experimental approaches. Our formulation follows such a research agenda.

In this paper we first explore the relation between consumption externalities and efficiency of the steady-state equilibrium. To see how inefficiency emerges, we consider as an example a negative externality of general goods consumption. Since there is no production externality, the *MRT* in the market is equal to the *MRT* in a centrally planned economy, thereby indicating the same capital intensity in both a market and centrally planned economy. The negative externality leads to a lower individual marginal utility of general goods consumption, thus, the *MRS* in a market economy is lower than the *MRS* in a centrally planned economy. With a concave utility, a lower market *MRS* indicates an equilibrium general goods consumption being too high and equilibrium consumption goods being too low in the long run compared to their respective social optimum. As a result, the allocation in the market equilibrium is different from the efficient level.

Next, we explain the indeterminacy in terms of the *KUJ* effect of general goods consumption. Indeterminate equilibrium emerges if self-fulfilling expectations can be supported as an equilibrium. Suppose that the representative agent expects an increase in the relative price of general goods. The agent will reallocate input factors from the consumption goods sector to the general goods sector which increases the *MRT*. Yet, more production of general goods increases general goods consumption. If general goods consumption has no *KUJ* effect, more general goods consumption lowers the marginal utility of general goods consumption, so the *MRS* is smaller than the higher *MRT*. Then, self-fulfilling expectations of a higher relative price of general goods cannot be supported as an equilibrium. However, when general goods consumption has a *KUJ* effect, then more general goods consumption can increase the *MRS* so as to keep pace with the higher *MRT* and the higher relative price of general goods. As a result, self-fulfilling expectations about higher relative prices of general goods can be supported as an equilibrium.

We should note that in one-sector growth models, when production externalities establish indeterminacy, it requires that labor supply be elastic and in particular the labor supply and demand curves cross with the "wrong slopes" (e.g., Benhabib and Farmer, 1994; Farmer and Guo, 1994). Moreover, when consumption externalities create indeterminacy in one-sector growth models, it still requires a labor-leisure tradeoff so the externality can cause the Frisch labor supply to have a certain shape, even though the labor supply need not cross the labor demand with the "wrong slopes" (Alonso-Carrera et al., 2008). In our two-sector model, general goods consumption externalities produce indeterminacy even when there is no labor-leisure tradeoff.[6]

Our primary findings are as follows. First, even with a negative spillover of general goods consumption, capital is over (*resp.* under) accumulated if the general goods sector is more (*resp.* less) capital-intensive than the consumption sector. This result is in sharp contrast to the one-sector model studied by Liu and Turnovsky (2005) and Turnovsky and Monteiro (2007) wherein a negative externality of general goods consumption must lead to an over-accumulation of capital. Although consumption externalities distort the allocation of capital in a two-sector model, we find that no active capital taxes are necessary once the sources of consumption distortions are identified and corrected by consumption taxes.

Next, when the general goods sector is more capital intensive, each of general goods consumption externalities and consumption externalities can easily establish indeterminacy, and it is easier for the consumption externality in the consumption goods sector to produce indeterminacy. The consumption good is reduced to leisure if capital is not an input in this sector. This foregoing result is contrary to existing one-sector models studied by Benhabib and Farmer (2000) and Weder

[6] In one-sector and two-sector neoclassical growth models with productive externalities, Kim (2005) showed that the utility curvature does not matter for indeterminacy. Unlike Kim, a specific utility curvature is not our focus.

(2004) who found that the leisure externality itself cannot create indeterminacy. Further, if there are symmetric degrees of consumption externalities from both goods, the utility is homothetic and the competitive equilibrium is efficient in a steady state. However, we still find indeterminate equilibrium because symmetric externalities produce different shadow prices of capital between a market and a socially planned economy which cause market failures in transitions. The result is different from the one-sector growth model studied by Alonso-Carrera et al. (2008) wherein consumption externalities in a homothetic utility do not lead to indeterminacy. Finally, it does not matter whether consumption externalities are from general goods, consumption goods or both goods, we find that it is much easier for a two-sector growth model to trigger indeterminacy than an otherwise identical one-sector growth model with elastic leisure.

We organize this paper as follows. We set a two-sector model with consumption externalities in Section 2. In Section 3, we study welfare properties. In Section 4, we investigate the dynamic properties of equilibrium. Finally, concluding remarks are found in Section 5.

2. The Basic Model

The economy is populated by a representative firm and a representative household. There are two sectors: the general goods (y_1) and consumption goods sectors (y_2). The general goods sector produces goods that are used as consumption and investment and the consumption goods sector produces pure consumption goods only. We will also refer to general goods as goods 1 and to consumption goods as goods 2.[7] The representative firm hires labor and rents capital in order to produce goods in the two sectors. The representative household has a fixed supply of labor which is normalized to unity and chooses savings and consumption of both goods.

2.1 Technology

The production function is

$$y_i = f^i(k_i, l_i),\ \ i = 1, 2, \tag{1}$$

where k_i and l_i are capital and labor allocated to sector i.

We assume that the function f^i is twice continuously differentiable and is homogenous of degree one with respect to both inputs. Moreover, the function is strictly increasing and strictly

[7] Our model is a variant of those posited by in Whelan (1993), Rogerson (2008) and Duarte and Restuccia (2010). Goods 1 may be thought of as manufactured goods and goods 2 as service goods. Alternatively, goods 2 may be interpreted as home goods as in Benhabib et al. (1991) and Greenwood and Hercowitz (1991).

concave in inputs and satisfies the Inada condition.[8] Our basic assumption is that sector 1 is more capital intensive than sector 2, but we also consider the opposite case.

2.2 Preference

The representative household supplies all its labor to work and there is thus no leisure activity. The household's utility is affected not only by personal consumption but also by average consumption in the society. Let $\rho>0$ denote the time preference rate, c_i denote personal consumption of goods i=1, 2 and $\overline{c}_i$ denote average consumption of goods i in the society. The agent's lifetime utility is represented by

$$U = \int_0^\infty e^{-\rho t} u(c_1, c_2, \overline{c}_1, \overline{c}_2) dt. \tag{2}$$

We assume that the instantaneous utility function is twice continuously differentiable and is strictly increasing and strictly concave in c_1 and c_2. The effect of $\overline{c}_i$ may be positive or negative. It is said that the household is "*admiring*" in good c_i if $\partial u / \partial \overline{c}_i > 0$ and "*jealous*" in good c_i if $\partial u / \partial \overline{c}_i < 0$ (e.g., Dupor and Liu, 2003; Liu and Turnovsky, 2005). Moreover, the consumption activity is described as "*KUJ*" if $\partial^2 u / (\partial c_i \partial \overline{c}_i) > 0$ (e.g., Gali, 1994; Ljungqvist and Uhlig, 2000).

2.3 Resource constraints and markets

Denote k as total capital (per capita) in the economy at a point in time and k_i as capital allocated to sector i=1, 2. The resource constraints in the economy at a point in time are given by

$$k = k_1 + k_2 = sk + (1-s)k, \tag{3a}$$

$$1 = l_1 + l_2 = l + (1-l). \tag{3b}$$

With a fixed labor supply normalized at unity, y_i in (1) is output per capita produced in sector i. Let the endogenous fraction of total capital in the economy allocated to sector 1 be denoted by $s \in (0,1)$; thus $k_1 = sk$. Then, the fraction of total capital going to sector 2 is $1-s$; thus $k_2 = (1-s)k$. We denote the endogenous fraction of labor allocated to sector 1 as $l \in (0,1)$ with the remaining fraction going to sector 2 being $(1- l)$.

Finally, the goods market clearance conditions in the economy are

$$\dot{k} = f^1(sk, l) - c_1 - \delta k, \tag{4a}$$

[8] The Inada condition is $\lim_{k_i \to 0} f^i(k_i, l_i) = \infty$ and $\lim_{k_i \to \infty} f^i(k_i, l_i) = 0$.

$$c_2 = f^2((1-s)k, 1-l), \tag{4b}$$

where δ is the rate of depreciation of capital.

Note that when y_2 does not use capital, then c_2 provides only leisure services. In this case, our model is reduced to a one-sector neoclassical growth model with leisure and leisure externalities. If leisure exhibits no externalities, this is the model studied by Liu and Turnovsky (2005) and Alonso-Carrera et al. (2008).

3 Inefficiency of Allocation in a Steady State

3.1 Allocation in a Decentralized Economy and in a Socially Planned Economy

In a competitive market economy, the representative agent takes $\overline{c_i}$ as given by the society. By substituting (4b) into (2), the representative agent's problem is to choose c_1, s, l and k in order to maximize (2) subject to (4a). The optimal conditions for c_1, s, l and k are

$$u_1(c_1, c_2, \overline{c_1}, \overline{c_2}) = \lambda, \tag{5a}$$

$$\frac{\lambda}{u_2(c_1, c_2, \overline{c_1}, \overline{c_2})} = \frac{f_2^2((1-s)k, 1-l)}{f_2^1(sk, l)} = p, \tag{5b}$$

$$\frac{f_2^2((1-s)k, 1-l)}{f_2^1(sk, l)} = \frac{f_1^2((1-s)k, 1-l)}{f_1^1(sk, l)}, \tag{5c}$$

$$\frac{\dot{\lambda}}{\lambda} = (\rho + \delta) - f_1^1(sk, l), \tag{5d}$$

where λ is the shadow price of capital and p is the relative price of general goods in terms of consumption goods.

In these optimal conditions, (5a) equates the marginal utility of general goods consumption to the shadow price of capital, which determines general goods consumption. In (5b), the *MRS* between two types of consumption is equal to the *MRT* between two sectors and is also equal to the relative price of general goods. In (5c), the *MRT* between the two sectors must equal for both capital and labor. Finally, (5d) is the Euler equation that governs how the shadow price of capital changes over time.

In a socially planned economy, the planner internalizes the consumption externality $\overline{c_i}$. Then, the optimal conditions for c_1, s, l and k give

$$u_1(c_1, c_2, \overline{c_1}, \overline{c_2}) + u_3(c_1, c_2, \overline{c_1}, \overline{c_2}) = \lambda^p, \tag{6a}$$

$$\frac{\lambda^p}{u_2(c_1,c_2,\overline{c}_1,\overline{c}_2)+u_4(c_1,c_2,\overline{c}_1,\overline{c}_2)}=\frac{f_2^{\,2}((1-s)k,1-l)}{f_2^{\,1}(sk,l)}, \tag{6b}$$

$$\frac{f_2^{\,2}((1-s)k,1-l)}{f_2^{\,1}(sk,l)}=\frac{f_1^{\,2}((1-s)k,1-l)}{f_1^{\,1}(sk,l)}, \tag{6c}$$

$$\frac{\dot{\lambda}^p}{\lambda^p}=(\varrho+\delta)-f_1^{\,1}(sk,l), \tag{6d}$$

where λ^p is the shadow price of capital.

3.2 Efficiency of the Allocation in a Decentralized Economy

The equilibrium conditions in a competitive market include (4a)-(4b) and (5a)-(5d) along with $\overline{c}_i=c_i$. The efficient allocation conditions in a socially planned economy are (4a)-(4b) and (6a)-(6d) with $\overline{c}_i=c_i$. In a steady state when $\dot{k}=\dot{\lambda}=\dot{\lambda}^p=0$, these two sets of conditions are the same except for (5a) and (5b) in a decentralized economy and (6a) and (6b) in a socially planned economy. Combining (5a) and (5b) gives

$$MRS\equiv\frac{u_1(c_1,c_2,\overline{c}_1,\overline{c}_2)}{u_2(c_1,c_2,\overline{c}_1,\overline{c}_2)}=\frac{f_2^{\,2}((1-s)k,1-l)}{f_2^{\,1}(sk,l)}\equiv MRT, \tag{7a}$$

and (6a) and (6b) yields

$$MRS^p\equiv\frac{u_1(c_1,c_2,\overline{c}_1,\overline{c}_2)+u_3(c_1,c_2,\overline{c}_1,\overline{c}_2)}{u_2(c_1,c_2,\overline{c}_1,\overline{c}_2)+u_4(c_1,c_2,\overline{c}_1,\overline{c}_2)}=\frac{f_2^{\,2}((1-s)k,1-l)}{f_2^{\,1}(sk,l)}\equiv MRT^p. \tag{7b}$$

Since the *MRT* in (7a) is the same as the MRT^p in (7b), the allocation in a decentralized economy is the same as the allocation in a socially planned economy if the *MRS* in (7a) is the same as the MRS^p in (7b). We obtain the following result.

Proposition 1. *In a neoclassical growth model with general goods and consumption goods, the competitive equilibrium allocation is efficient in a steady state if and only if*

$$\frac{u_1(c_1,c_2,\overline{c}_1,\overline{c}_2)}{u_2(c_1,c_2,\overline{c}_1,\overline{c}_2)}=\frac{u_3(c_1,c_2,\overline{c}_1,\overline{c}_2)}{u_4(c_1,c_2,\overline{c}_1,\overline{c}_2)}. \tag{8}$$

for all feasible c_1 and c_2, where $\overline{c}_1=c_1$ *and* $\overline{c}_2=c_2$.

Condition (8) stipulates that the *MRS* between personal consumption of goods 1 and 2 must be equal to the MRS^p between the *social* consumption of goods 1 and 2. Only when this condition is met, the equilibrium allocation of consumption, labor and capital in a competitive market is efficient in the long run; otherwise, the equilibrium allocation is inefficient.

A typical specification of the utility function that satisfies condition (8) is a function that is multiplicatively separable between (c_1, c_2) and $(\overline{c}_1, \overline{c}_2)$ such that

$$u(c_1, c_2, \overline{c}_1, \overline{c}_2) = v(c_1, c_2)h(v(\overline{c}_1, \overline{c}_2)), \tag{9a}$$

where the h(.) is a monotonically increasing or decreasing function.

Another functional form satisfying (8) is that u(.) is weakly separable between $(c_1, \overline{c}_1)$ and $(c_2, \overline{c}_2)$ such that

$$u(c_1, c_2, \overline{c}_1, \overline{c}_2) = V(h(c_1, \overline{c}_1)h(c_2, \overline{c}_2)), \tag{9b}$$

where h(.) is a homothetic function. In this case, (8) is written as $\frac{h_1(c_1,\overline{c}_1)}{h_1(c_2,\overline{c}_2)} = \frac{h_2(c_1,\overline{c}_1)}{h_2(c_2,\overline{c}_2)}$. Since h(.) is homothetic, we obtain $h_1(\frac{c_1}{\overline{c}_1},1)h_2(\frac{c_2}{\overline{c}_2},1) = h_1(\frac{c_2}{\overline{c}_2},1)h_2(\frac{c_1}{\overline{c}_1},1)$, which always holds when $\overline{c}_i = c_i$.

When condition (8) fails to hold, a competitive equilibrium allocation is inefficient in a steady state. Then, the consumption externality can lead to an inefficient allocation even though there is no leisure choice. In the one-sector neoclassical growth model studied by Liu and Turnovsky (2005), the consumption externality produces an inefficient allocation in a steady state only when there is a labor-leisure tradeoff. In particular, in a one-sector growth model, a jealous (i.e., negative) consumption externality leads to an over-accumulation of capital and an admiring (i.e., positive) consumption externality results in under-accumulation of capital. In contrast, in our two-sector growth model, a jealous (or admiring) general goods consumption externality may cause an over-accumulation or under-accumulation of capital depending on relative capital intensities. To see this, we use a parametric version of our model with the following utility function with a constant elasticity of substitution and the Cobb-Douglas production function

$$u(c_1, c_2, \overline{c}_1, \overline{c}_2) = [\gamma(\frac{c_1}{\overline{c}_1^{\theta_1}})^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)(\frac{c_2}{\overline{c}_2^{\theta_2}})^{\frac{\varepsilon-1}{\varepsilon}}]^{\frac{\varepsilon}{\varepsilon-1}}, \quad 0<\gamma\leq 1, \tag{10a}$$

$$f^i(k_i, l_i) = A_i k_i^{a_i} l_i^{1-a_i}, A_i > 0, \ 0 \leq a_i < 1, \ i = 1, 2, \tag{10b}$$

where a_i is the capital intensity in sector i. Parameter γ>0 is the share of goods 1 relative to goods 2 in utility, and θ_i measures the degree of the external consumption effect arising from $\overline{c}_i$. A negative (i.e., jealous) consumption externality emerges if θ_i>0 while a positive (i.e., admiring) consumption externality occurs if θ_i<0. We assume that $\theta_i < 1$ to keep a positive social level of the marginal utility of each good. Parameter ε>0 is the elasticity of substitution (hereinafter, *ES*) between two goods. Consumption i displays a *KUJ* effect if $\theta_i(\varepsilon$-1)<0.

Several special cases are worthy of mention.

1. If ε=1 and γ=1, goods 2 is not demanded and (10a) becomes $u = \log[c_1(\overline{c}_1)^{-\theta_1}]$. Our model is reduced to a one-sector growth model with consumption externalities and an inelastic labor supply. Then, the competitive equilibrium allocation is efficient in the long run.

2. If ε=1 and $\theta_1=\theta_2=\theta$, (10a) becomes $u = \log[(c_1^{\gamma}c_2^{1-\gamma})(\overline{c}_1^{\gamma}\overline{c}_2^{1-\gamma})^{-\theta}]$, which satisfies (9a). Thus, (8) is satisfied and a competitive equilibrium allocation is efficient in the long run.

3. If $\varepsilon\neq1$ and $\theta_1=\theta_2=\theta$, then (10a) is homothetic and satisfies (9b). Thus, (8) is met and a competitive equilibrium allocation is efficient in the long run.

4. If $\varepsilon\neq1$ and $\theta_1\neq\theta_2$, the degree of externalities is asymmetric. Then, (8) fails to hold and a competitive equilibrium allocation is inefficient in a steady state. Condition (8) fails to satisfy since

$$\frac{u_3(.)}{u_4(.)} = \frac{\theta_1}{\theta_2}\frac{u_1(.)}{u_2(.)} \neq \frac{u_1(.)}{u_2(.)} = \frac{\gamma}{1-\gamma}\left[\frac{(c_1)^{-1-\theta_1(\varepsilon-1)}}{(c_2)^{-1-\theta_2(\varepsilon-1)}}\right]^{\frac{1}{\varepsilon}} \quad \textit{if } \varepsilon \neq 1,\ \theta_1 \neq \theta_2.$$

In what follows we focus on the case $\varepsilon\neq1$ and $\theta_1\neq\theta_2$. The following proposition characterizes a steady-state competitive equilibrium allocation for the parametric version of our model.

Proposition 2. *Suppose $\varepsilon\neq1$. Then, in a steady state, capital is over-accumulated if (i) $a_1>a_2$ and $\theta_1>\theta_2$ or (ii) $a_1<a_2$ and $\theta_1<\theta_2$ and under-accumulated if (i) $a_1>a_2$ and $\theta_1<\theta_2$ or (ii) $a_1<a_2$ and $\theta_1>\theta_2$.*

To see the properties in proposition 2, we rewrite (3a) as

$$lx_1 + (1-l)x_2 = k, \tag{11a}$$

where $x_1 = \frac{k_1}{l_1} = \frac{sk}{l}$ and $x_2 = \frac{k_2}{l_2} = \frac{(1-s)k}{1-l}$ are the capital intensity in sector 1 and 2, respectively. Then, the production function in (1) becomes

$$y_1 = lg_1(x_1),\ \ y_2 = (1-l)g_2(x_2),$$

where $g_i(x_i) \equiv f^i(x_i,1) = \frac{f^i(k_i,l_i)}{l_i}$. Then, the relative price satisfies $p = \frac{f_1^2(.)}{f_1^1(.)} = \frac{g_2'(x_2)}{g_1'(x_1)}$ and (5c) becomes

$$\frac{g_2(x_2)-x_2g_2'(x_2)}{g_2'(x_2)} = \frac{g_1(x_1)-x_1g_1'(x_1)}{g_1'(x_1)}. \tag{11b}$$

In a steady state, $\rho+\delta = f_1^1(x_1,1) = g_1'(x_1)$, and, in view of (11b), the steady-state levels of capital intensity, x_1 and x_2, are uniquely determined and are independent of the presence of consumption externalities. The relative price p is thus uniquely determined. The steady-state expression of (7a) in terms of the utility function (10a) is

$$MRS \equiv \frac{u_1(.)}{u_2(.)} = \frac{\gamma}{1\text{-}\gamma}\left[\frac{\left(c_2\right)^{1+\theta_2(\varepsilon\text{-}1)}}{\left(c_1\right)^{1+\theta_1(\varepsilon\text{-}1)}}\right]^{\frac{1}{\varepsilon}} = \frac{A_2(1-a_2)(\frac{(1-s)k}{1-l})^{a_2}}{A_1(1-a_1)(\frac{sk}{l})^{a_1}} = \frac{g_2'(x_2)}{g_1'(x_1)} \equiv MRT = p, \tag{12a}$$

When the external effects are internalized, (7b) is expressed as

$$MRS^p \equiv \frac{u_1(.)+u_3(.)}{u_2(.)+u_4(.)} = \frac{\gamma}{1\text{-}\gamma}\left(\frac{1-\theta_1}{1-\theta_2}\right)\left[\frac{\left(c_2{}^p\right)^{1+\theta_2(\varepsilon\text{-}1)}}{\left(c_1{}^p\right)^{1+\theta_1(\varepsilon\text{-}1)}}\right]^{\frac{1}{\varepsilon}} = \frac{g_2'(x_2^p)}{g_1'(x_1^p)} \equiv MRT^p = p, \tag{12b}$$

With no production externalities, (5d) and (6d) determine the same long-run capital-labor ratio $x_1^p = x_1 = (\frac{A_1 a_1}{\varrho+\delta})^{1/(1-a_1)}$ for a social optimum and an equilibrium and thus the *MRT* in (12a) and the MRT^p in (12b) are equal. As a result, the *MRS* in (12a) and the MRS^p in (12b) are equal. Under $\theta_1=\theta_2=\theta$, there are symmetric consumption externalities and the utility is homothetic. Due to the homothetic utility, the same allocation in a competitive equilibrium and in a social optimum leads to an equalization of the planner's MRS^p in (12b) and the market's *MRS* in (12a). As a result, an allocation in a market economy is the same as an allocation in a centrally planned economy and is thus efficient in the long run. Conversely, under $\theta_1 \neq \theta_2$ and thus $\frac{1-\theta_1}{1-\theta_2} \neq 1$, the equalization of a planner's MRS^p in (12b) and a market's *MRS* in (12a) indicates that a competitive equilibrium allocation is different from a social optimum allocation and thus is inefficient.

To see how the relative consumption externalities interact with relative capital intensities between the two sectors and determine whether capital is over or under accumulated, note that the equilibrium shares and the social optimum shares of capital and labor between the two sectors satisfy $\frac{1-l^p}{l^p}\frac{s^p}{1-s^p} = \frac{1-l}{l}\frac{s}{1-s} = \frac{a_1(1-a_2)}{a_2(1-a_1)}$. When using this relationship, the equilibrium consumption good is

$$y_2 = c_2 = A_2(\frac{sk}{l})^{a_2}(\frac{l}{1-l}\frac{1-s}{s})^{a_2}(1-l) = A_2(\frac{A_1 a_1}{\varrho+\delta})^{\frac{a_2}{1-a_1}}[\frac{a_2(1-a_1)}{a_1(1-a_2)}]^{a_2}(1-l), \tag{13a}$$

which is linear in the fraction of equilibrium labor allocated to the consumption goods sector.

When $\theta_1>\theta_2>0$ and thus $(1\text{-}\theta_1)/(1\text{-}\theta_2)<1$, the concave utility indicates that long-run equilibrium consumption c_1 is too high and equilibrium consumption c_2 is too low compared to the social optimum $c_1{}^p$ and $c_2{}^p$, respectively. From (13a), c_2 is proportional to $1\text{-}l$ and thus $1\text{-}l$ is too low compared to the social optimum $1\text{-}l^p$. A fixed labor supply in the economy indicates that equilibrium labor l is over-valued, thus $dl>0$. The equilibrium capital stock k is higher than the social optimum k^p if the increase in l is larger than the increase in s. Specifically, a fixed $\frac{sk}{l}$ gives[9]

$$\frac{dk}{k}=\frac{dl}{l}-\frac{ds}{s}=\frac{dl}{l}-\frac{dl}{l}\frac{1-s}{1-l}\gtrless 0 \ \ if\ s \gtrless l. \tag{13b}$$

If the capital intensity is the same in both sectors (i.e., $a_1=a_2$), the fraction of capital and the fraction of labor in the economy allocated to sector 1 are the same ($s=l$). Then, (13b) suggests that, despite consumption externalities, the equilibrium capital stock k is the same as the social optimum k^p. Conversely, if sector 1 is more capital-intensive than sector 2 (i.e., $a_1>a_2$), then the fraction of capital allocated to sector 1 is larger than the fraction of labor allocated to sector 1 (i.e., $s>l$); if sector 1 is less capital-intensive than sector 2 (i.e., $a_1<a_2$), then the fraction of capital allocated to sector 1 is smaller than the fraction of labor allocated to sector 1 (i.e., $s<l$). Then, (13b) indicates that the steady-state equilibrium capital is over-accumulated if sector 1 is more capital-intensive ($a_1>a_2$) and under-accumulated if sector 1 is less capital-intensive ($a_1<a_2$).

Alternatively, when $\theta_2>\theta_1>0$ and thus $(1-\theta_1)/(1-\theta_2)>1$, steady-state equilibrium consumption c_1 is too low and equilibrium consumption c_2 is too high compared to the social optimum c_1^p and c_2^p, respectively. Over-valued c_2 indicates that equilibrium $1-l$ is too high compared to the social optimum $1-l^p$. Then, l is under-valued, thus $dl<0$. As a result, (13b) indicates that the steady-state capital is under-accumulated if sector 1 is more capital-intensive than sector 2 ($a_1>a_2$) and over-accumulated if sector 1 is less capital-intensive ($a_1<a_2$)

This completes the proof of Proposition 2.

Finally, it should be noted that that under $a_2=0$, $c_2=(1-l)$ is leisure. If $\theta_2=0$, then our model is reduced to the one-sector growth model with elastic leisure studied by Liu and Turnovsky (2005). In this case, if the consumption of general goods is jealousy (i.e., $\theta_1>0$), capital is over-accumulated in a steady state in a one-sector growth model. In addition to jealousy consumption, now we introduce jealousy leisure ($\theta_2\neq0$) into a one-sector growth model which was not analyzed by Liu and Turnovsky (2005). In particular, if $\theta_2>\theta_1>0$, since $a_1>a_2=0$, equilibrium consumption c_1 is too low and equilibrium leisure $c_2=1-l$ is too high compared to the social optimum c_1^p and $c_2^p=1-l^p$, respectively. Thus, equilibrium labor supply l is under-valued, hence $l<l^p$. With the same capital-labor ratio for both the market and the social optimum in a steady state, the equilibrium of capital stock k is lower than the social optimum k^p.

As consumption externalities distort resource allocation in the long run, it follows that the

[9] Condition $\frac{1-l^*}{l^*}\frac{s^*}{1-s^*}=\frac{a_1(1-a_2)}{a_2(1-a_1)}$ is used in the second equality.

allocation in the short run must be inefficient.[10] There is an opportunity for government tax policy to improve efficiency. In the next subsection, we analyze the optimal tax policy

3.3 Optimal Tax Policy

Consider again a decentralized economy. Let τ_k, τ_{c1}, τ_{c2} be denoted as the tax rates on capital and the consumption of goods 1 and goods 2, respectively, and let T denote lump-sum transfers (or taxes).[11] The representative household maximizes the lifetime utility (2), subject to the following budget constraint,

$$\dot{k} = (1-\tau_k)rk - (1+\tau_{c1})c_1 - (1+\tau_{c2})\frac{c_2}{p} - \delta k + T, \tag{14a}$$

where $r = f_1^1(sk, l) = \frac{1}{p} f_1^2((1-s)k, 1-l)$ is the return to capital. The government maintains a balanced budget, rebating all tax revenues to the households in a lump-sum fashion

$$\tau_k rk + \tau_{c1}c_1 + \tau_{c2}\frac{c_2}{p} = T. \tag{14b}$$

The objective is to characterize an optimal tax structure such that the decentralized economy mimics the dynamic equilibrium path of the centrally planned economy in (6a)–(6d). To achieve this, we allow the tax rates τ_k, τ_{c1}, τ_{c2} to be time-varying. First, we derive the optimal conditions of a decentralized economy with taxes. Then, we replicate the allocation in a decentralized economy with that in a centralized economy. Replication involves setting time-varying tax rates such that the allocation of capital and consumption is equal in a decentralized economy and in a centralized economy, which requires $\frac{\dot{\lambda}}{\lambda} = \frac{\dot{\lambda}^p}{\lambda^p}$, or equivalently $\varkappa\lambda = \lambda^p$, where $\varkappa > 0$ is an arbitrary constant. The main results are as follows (see the Appendix).

Proposition 3. . *In a neoclassical economy with general goods and consumption goods, the entire time path of the optimal resource allocation can be obtained by setting tax rates at each instant of time in accordance with*

$$\tau_k = \overline{\tau}_k; \quad 1+\tau_{c2}(t) = \xi \frac{u_2(c_1,c_2,c_1,c_2)}{u_2(c_1,c_2,c_1,c_2)+u_4(c_1,c_2,c_1,c_2)}; \quad \frac{1+\tau_{c1}(t)}{1+\tau_{c2}(t)} = \frac{\frac{u_1(c_1,c_2,c_1,c_2)}{u_1(c_1,c_2,c_1,c_2)+u_3(c_1,c_2,c_1,c_2)}}{\frac{u_2(c_1,c_2,c_1,c_2)}{u_2(c_1,c_2,c_1,c_2)+u_4(c_1,c_2,c_1,c_2)}},$$

where $\overline{\tau}_k$ *and* ξ *are arbitrary constants.*

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[10] Liu and Turnovsky (2005) have shown that even with inelastic leisure, a one-sector growth model with consumption externalities can distort the transitional path, albeit it cannot distort the steady-state allocation.

[11] As labor is supplied in-elastically, it is clear that the labor taxation will not improve the efficiency and thus we do not consider the labor tax.

Intuitively, as there are consumption externalities and no production externalities, optimal consumption taxes are needed. If there are negative (or positive) consumption externalities of goods i=1, 2 and thus u_i<0 (or u_i>0), the consumption of goods i is more (or less) than an efficient level. Then, the government should tax (or subsidize) the consumption of goods i and therefore, τ_{ci}>0 (or τ_{ci}<0). Although both consumption externalities distort the allocation of capital, no active capital taxes are necessary once the sources of consumption distortions are identified and corrected by consumption taxes. The capital income tax can thus be set to zero.

In a special case when goods 2 is leisure, then the tax on the consumption of goods 2 is a tax on leisure; thus, it is a negative labor income tax. This is the case analyzed in Liu and Turnovsky (2005) wherein leisure has no externalities and thus u_4=0. In this case of u_4=0, the labor income tax is $\tau_w(t) = -\tau_{c2}(t)$ and is an arbitrary constant. Setting an arbitrary constant tax on labor income at $\overline{\tau}_w$ and identifying $\xi = (1-\overline{\tau}_w)$ would give the following optimal tax rate on the consumption of goods 1

$$\frac{1+\tau_{c1}(t)}{1-\overline{\tau}_w} = \frac{u_1(c_1,c_2,c_1,c_2)}{u_1(c_1,c_2,c_1,c_2)+u_3(c_1,c_2,c_1,c_2)}, \tag{14c}$$

which is identical to that in Liu and Turnovsky (2005, p.1117).

In a more general case when goods 2 is leisure and leisure has externalities ($u_4 \neq 0$), then the optimal tax rate on labor income varies over time given by $\tau_w \equiv -\tau_{c2} = \xi \frac{u_4}{u_2+u_4}$. In particular, in the most general case wherein the production of goods 2 requires both capital and labor, the tax on the consumption of goods 2 is different from the tax on labor income. In these general cases, the tax rate on the consumption of goods 1 is the expression in proposition 3 which is different from the expression in (14c) above.

4. Indeterminacy in Transitional Dynamics

Consumption externalities cause not only an inefficient allocation in a steady state, but they also generate indeterminate equilibrium paths toward a steady state. This section analyzes the conditions of indeterminacy.

4.1 Conditions of Indeterminacy

Dynamic equilibrium conditions in a competitive market are summarized by (4a)-(4b) and (5a)-(5d) with six variables: c_1, c_2, s, l, λ and k. Different from a one-sector model, our model involves two goods and it is difficult to simplify these equations to a dynamic system with a state vector $\{k, c_1\}$. We will simplify them to a system with a state vector $\{k, \lambda\}$ as follows.[12]

First, with the help of (4b) and (5a)-(5c), we use (5d) to obtain the Keynes-Ramsey condition.

$$\dot{\lambda} = \lambda[(\rho + \delta) - f_1^1(s(\lambda, k)k, l(\lambda, k))] \equiv J_1(\lambda, k). \tag{15a}$$

Next, with (4b) and (5a)-(5c), we rewrite the general goods market clearance condition (4a).

$$\dot{k} = f^1(s(\lambda, k)k, l(\lambda, k)) - c_1(\lambda,, k) - \delta k \equiv J_2(\lambda, k). \tag{15b}$$

Equations (15a) and (15b) constitute a simplified dynamic system. The steady state is determined by $\dot{k} = 0$ and $\dot{\lambda} = 0$. The dynamic property of equilibrium is analyzed if we take Taylor's linear expansion of (15a)-(15b) around the steady state (k, λ). The expansion gives

$$\begin{bmatrix} \dot{\lambda} \\ \dot{k} \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \begin{bmatrix} \lambda(t) - \lambda \\ k(t) - k \end{bmatrix}. \tag{15c}$$

The dynamic system (15c) includes a state variable whose initial value is given at $k(0)$. There are two roots associated with the Jacobian matrix in (15c), denoted by J. The steady state is a saddle if there is only one root with a negative real part and a sink if there are two roots with negative real parts. If the steady state is a sink, the equilibrium path toward the steady state is indeterminate.

Before analyzing dynamic properties, we investigate the existence and uniqueness of a steady state in the case without consumption externalities ($\theta_1=\theta_2=0$). First, for the slope of the $\dot{k}^* = 0$ locus in the (k, λ) plane,[13] we find $J^*_{21}>0$ and $J^*_{22}>0$ when k is small and $J^*_{22}<0$ when k is large. The sign of J^*_{21} is positive, since a higher λ (a higher shadow price of capital) attracts more capital and labor to the general goods sector and thus increases general goods production. It also reduces general goods consumption, due to an increased cost of general goods consumption relative to consumption goods. Moreover, for the sign of J^*_{22}, when k is small, because of a high marginal product of capital, larger capital increases more production of general goods than consumption of general goods. When k is very large, because of a diminishing marginal product of capital, the production of general goods is increased less than the increase in the consumption of general goods. Thus, $J^*_{22}>0$ when k is small and $J^*_{22}<0$ when k is large. As a result, the $\dot{k}^* = 0$ locus is downward sloping when k is smaller than a threshold and upward sloping when k is larger than the

[12] This method follows from that found in Benhabib and Farmer (1996) which is a two-sector growth model with one consumption goods and one investment goods. In their model, sector-specific externalities in production are the mechanism leading to indeterminacy.

[13] In what follows, an asterisk superscript is used to denote the case of a two-sector growth model without consumption externalities.

threshold, just like the $\dot{k}^* = 0$ locus in the standard one-sector growth model. See Figure 1. Moreover, it is easy to show that $\dot{k}^* = -c_1 < 0$ when k=0 and $\dot{k}^* = -\infty < 0$ when k=∞, implying that the $\dot{k}^* = 0$ locus approaches λ=∞ on both ends of k=0 and k=∞.

[Insert Figure 1 about here]

Next, for the slope of the $\dot{\lambda}^* = 0$ locus, we find $J^*_{11}<0$ and $J^*_{12}>0$. For a given k, a higher λ decreases the consumption of general goods which increases the marginal utility of general goods consumption and leads to a higher *MRS* between c_1 and c_2 in (5b). In optimum, the marginal product of labor and capital in general goods relative to that in consumption goods in (5b) and (5c) needs to increase. Thus, $J^*_{11}<0$. Further, a larger capital has two effects. It decreases the marginal product of capital which directly increases the shadow price of capital. As sector 1 is more capital intensive than sector 2 under construction, the Rybczynski theorem stipulates that larger capital and labor shares are allocated to sector 1. A larger labor share increases the marginal product of capital which indirectly decreases the shadow price of capital. In general, the direct effect dominates the indirect effect and thus $J^*_{12}>0$. As a result, the $\dot{\lambda}^* = 0$ locus is upward sloping in the (λ, k) plane. See A1 and A2 in Figure 1. Moreover, it is clear to see $\dot{\lambda}^* = -\infty < 0$ at (k, λ)=(0, 0) as the Inada condition implies an infinite marginal product of capital. Thus, the $\dot{\lambda}^* = 0$ locus will start from a finite value of k so that at λ=0, the marginal product of capital can equal the sum of the discount rate and the depreciation rate.

The shape of the two loci indicates that the $\dot{\lambda}^* = 0$ locus intersects the $\dot{k}^* = 0$ locus only once and there is a unique steady state (k, λ). The $\dot{\lambda}^* = 0$ locus may intersect the $\dot{k}^* = 0$ locus at the downward (A1) or upward (A2) sloping segment as illustrated in Figure 1.

In the two-sector growth model without consumption externalities, the steady state is a saddle. This indicates that there is only one root with a negative real part and the condition is $Det(J^*)=J^*_{11}J^*_{22}-J^*_{21}J^*_{12}<0$. Under $J^*_{21}>0$ and $J^*_{11}<0$, $Det(J^*)<0$ indicates $(-J^*_{12}/J^*_{11})>(-J^*_{22}/J^*_{21})$ >(or<)0. Thus, the positive slope of the $\dot{\lambda}^* = 0$ locus is larger than the slope of the $\dot{k}^* = 0$. See the steady states E_1 and in A1 and A2, Figure 1, wherein the intersection is, respectively, at the negative and positive slope section of the $\dot{k}^* = 0$

However, if consumption exhibits externalities, the steady state may be a sink. Consider the *KUJ* effect as follows.

Condition *KUJ*: $\partial^2 u / (\partial c_i \partial \overline{c}_i) > 0$.

The *KUJ* effect may lead to J_{11}>0 so the locus $\dot{\lambda} = 0$ is negatively sloping. To illustrate this point, suppose that general goods consumption exhibits a *KUJ* effect. Then, when the *KUJ* effect is sufficiently large, a higher λ increases rather than decreases general goods consumption which reduces the *MRS* between c_1 and c_2. In optimum, the marginal product of labor and capital in the general goods sector relative to that in the consumption goods sector in (5b) and (5c) needs to decrease. Thus, J_{11}>0, so the $\dot{\lambda} = 0$ locus is negatively sloping.

When the $\dot{\lambda} = 0$ locus is negatively-sloping, the dynamic property of equilibrium may change. In particular, when the negatively-sloping $\dot{\lambda} = 0$ is steeper than locus $\dot{k} = 0$ as illustrated in B1 and B2 of Figure 1, the steady state is a sink. This requires two roots with negative real parts and the conditions are $Det(J)=J_{11}J_{22}-J_{21}J_{12}>0$ and $Tr(J)=J_{11}+J_{22}<0$, which are equivalent to $(-J_{12}/J_{11})<(-J_{22}/J_{21})>$(or<)0. Thus, the negatively-sloping $\dot{\lambda} = 0$ is steeper than locus $\dot{k} = 0$.

4.2 A Parametric Version

For ease of exposition, in this subsection we use the parametric version of the utility function (10a) and the production function (10b) to illustrate the dynamic properties of equilibrium. The utility function stipulates that if $[-\theta_i(\varepsilon-1)]>0$, then goods i consumption exhibits the *KUJ* effect. The 2x2 dynamical equations in (15a) and (15b) are derived as follows.

First, with the production function (10b), factor allocation between sectors in (5c) leads to

$$l = l(s) \equiv [1 + \frac{a_1(1-a_2)}{a_2(1-a_1)}(\frac{1}{s} - 1)]^{-1}, \tag{16a}$$

where $l'(s) = \frac{a_1(1-a_2)}{a_2(1-a_1)} \frac{l^2}{s^2} > 0$. The positive sign is due to the complement of capital and labor. Feasibility of l restricts $s < \frac{a_1(1-a_2)}{a_2(1-a_1)}$. Note that a_1>a_2 implies $\frac{a_1(1-a_2)}{a_2(1-a_1)} \frac{l}{s} > 1$ and thus $l'(s) > \frac{l}{s}$.

Next, the production function (9b) and the consumption goods equilibrium (4b) indicate

$$c_2 = c_2(s,k) \equiv c_2(s,k) A_2[(1-s)k]^{a_2}[1-l(s)]^{1-a_2}, \tag{16b}$$

where $\frac{\partial c_2}{\partial s} = -c_2[\frac{a_2}{1-s} + \frac{1-a_2}{1-l} l'(s)] < 0$,

$\frac{\partial c_2}{\partial k} = \frac{a_2 c_2}{k} > 0$.

Intuitively, a smaller share of capital allocated to sector 2 (i.e., a larger s) reduces the production and consumption of goods 2. Moreover, larger capital in the economy indicates more capital allocated to sector 2 which increases the production and consumption of goods 2.

Further, with the utility (10a), (5a) is $\gamma c_1^{\frac{(\varepsilon-1)(1-\theta_1)}{\varepsilon}} + (1-\gamma) c_2^{\frac{(\varepsilon-1)(1-\theta_2)}{\varepsilon}} = [\lambda c_1^{\frac{[1+\theta_1(\varepsilon-1)]}{\varepsilon}} \gamma^{-1}]^{\varepsilon-1}$ which gives

$$c_1 = c_1(\lambda, c_2), \tag{16c}$$

where $\frac{\partial c_1}{\partial \lambda} = \frac{\varepsilon c_1}{\lambda} \frac{1+p(c_1/c_2)}{B} < 0$ if θ_1=0 ; ambiguous if otherwise;

$\frac{\partial c_1}{\partial c_2} = \text{-}(1-\theta_2) \frac{c_1}{c_2} \frac{1}{B} > 0$ if θ_1=θ_2=0; ambiguous if otherwise;

$B = -1 + [-\theta_1(\varepsilon-1)][1 + p(c_1 / c_2)\varepsilon / (\varepsilon-1)] < 0$ if θ_1=0; ambiguous if otherwise.[14]

Thus, if θ_1=θ_2=0, $(\frac{\partial c_1}{\partial \lambda})^* = \text{-}\varepsilon \lambda^{-1} c_1 (1 + p c_1 / c_2) < 0$. Intuitively, it follows that without consumption externalities, for a given c_2, a higher shadow price of capital must decrease c_1 in order to increase the marginal utility of c_1. Moreover, $(\frac{\partial c_1}{\partial c_2})^* = \frac{c_1}{c_2} > 0$ if θ_1=θ_2=0. This result comes because given λ, c_1 and c_2 are complementary in utility. However, with externalities from $\overline{c}_1$, a higher shadow price of capital may increase c_1 if the *KUJ* effect ([-θ_1(ε-1)]>0) is sufficiently large. In this case, c_1 and c_2 are negatively related.

Furthermore, using (4b), (5a) and (16a)-(16c), the utility function (10a) and the production function (10b), the equalization of the *MRS* to the *MRT* in (5b) is rewritten as

$$\frac{\gamma\ c_2(s,k)^{\frac{1+\theta_2(\varepsilon\text{-}1)}{\varepsilon}}}{(1\text{-}\gamma)\ c_1(\lambda, c_2)^{\frac{1+\theta_1(\varepsilon\text{-}1)}{\varepsilon}}} = \frac{A_2(1-a_2)}{A_2(1-a_1)} \frac{(1\text{-}s)^{a_2}(1\text{-}l(s))^{a_2}}{(s)^{a_1}(l(s))^{a_1}} \frac{1}{k^{(a_1-a_2)}},$$

where the *KUJ* effect affects the *MRS* between goods 2 and 1 via $c_1(\lambda, c_2)$. This condition gives

$$s = s(\lambda, k), \tag{16d}$$

where $\frac{\partial s}{\partial \lambda} = [1 + \theta_1(\varepsilon - 1)] \frac{1}{\varepsilon} \frac{1}{c_1} \frac{\partial c_1}{\partial \lambda} \frac{1}{\Xi} > 0$ if θ_1=θ_2=0; ambiguous if otherwise;

$\frac{\partial s}{\partial k} = [\frac{1+\theta_1(\varepsilon-1)}{\varepsilon} \frac{c_2}{c_1} \frac{\partial c_1}{\partial c_2} - \frac{1+\theta_2(\varepsilon-1)}{\varepsilon}] \frac{a_2}{k} \frac{1}{\Xi} - \frac{a_1-a_2}{k} \frac{1}{\Xi} > 0$ if θ_1=θ_2=0; ambiguous if otherwise;

$\Xi = [\frac{a_2}{1-s} - \frac{a_2}{1-l} l'(s)] + [\frac{a_1}{s} - \frac{a_1}{l} l'(s)] + [\frac{1+\theta_1(\varepsilon-1)}{\varepsilon} \frac{c_2}{c_1} \frac{\partial c_1}{\partial c_2} - \frac{1+\theta_2(\varepsilon-1)}{\varepsilon}][\frac{a_2}{1-s} + \frac{1-a_2}{1-l} l'(s)] < 0$ if θ_1=θ_2=0; ambiguous if otherwise.

Note that $\frac{c_2}{c_1}(\frac{\partial c_1}{\partial c_2})^* = 1$ under θ_1=θ_2=0, which gives $\Xi^* = [\frac{a_2}{1-s} - \frac{a_2}{1-l} l'(s)] + [\frac{a_1}{s} - \frac{a_1}{l} l'(s)] < 0$. Thus, under θ_1=θ_2=0, $(\frac{\partial s}{\partial \lambda})^* = -\frac{1+p(c_1/c_2)}{\lambda^*} \frac{1}{\Xi^*} > 0$ and $(\frac{\partial s}{\partial k})^* = -(\frac{a_1}{a_2} - 1) \frac{a_2}{k} \frac{1}{\Xi^*} > 0$. Intuitively, it follows that without consumption externalities, for a given k, a higher shadow price of capital must

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[14] In the parametric version, $p = f_2^2 / f_2^1 = (1-a_2) A_2 [(1-s)k / (1-l)]^{a_2} [(1-a_1) A_1 (sk / l)^{a_1}]^{-1}$.

decrease c_1 in order to increase the *MRS* between c_1 and c_2. Optimally, a larger share of capital needs to be allocated to the general goods sector (i.e., increases s) in order to decrease the marginal product of capital in general goods relative to consumption goods so as to increase the *MRT*. For a given λ, if capital increases, since sector 1 is more capital intensive than sector 2, the Rybczynski theorem stipulates that more capital is allocated to sector 1 and thus s increases. However, with consumption externalities, these relationships are ambiguous. In particular, with the *KUJ* effect, a higher shadow price of capital decreases the share of capital allocated to the general goods sector.

Finally, with the use of (16a)-(16d), the dynamic equations in (15a) and (15b) are

$$\dot{\lambda} = J_1(\lambda, k) \equiv \lambda\{(\rho+\delta) - A_1 a_1 [\frac{l(s(\lambda,k))}{s(\lambda,k)}]^{1-a_1} \frac{1}{k^{1-a_1}}\}, \tag{(17a)}$$

$$\dot{k} = J_2(\lambda, k) \equiv A_1 s(\lambda,k)^{a_1} k^{a_1} l(s(\lambda,k))^{1-a_1} - c_1(\lambda, c_2(s(\lambda,k),k)) - \delta k. \tag{17b}$$

To envisage the dynamic property, by differentiating (17a) and (17b) around the steady state, with the use of (16a), we obtain the elements in the Jacobian matrix in (15c) as follows.

$$J_{11} = -\lambda(\rho+\delta)(1-a_1)\frac{(a_1-a_2)l}{a_2(1-a_1)s}\frac{\partial s}{\partial \lambda}, \tag{18a}$$

$$J_{12} = \lambda(\rho+\delta)(1-a_1)[\frac{1}{k} - \frac{(a_1-a_2)l}{a_2(1-a_1)s}\frac{\partial s}{\partial k}], \tag{18b}$$

$$J_{21} = \{(c_1+\delta k)[\frac{a_1}{s} + \frac{1-a_1}{l}l'(s)] + c_2[\frac{a_2}{1-s} + \frac{1-a_2}{1-l}l'(s)]\frac{\partial c_1}{\partial c_2}\}\frac{\partial s}{\partial \lambda} - \frac{\partial c_1}{\partial \lambda}, \tag{18c}$$

$$J_{22} = \{(c_1+\delta k)[\frac{a_1}{s} + \frac{1-a_1}{l}l'(s)] + c_2[\frac{a_2}{1-s} + \frac{1-a_2}{1-l}l'(s)]\frac{\partial c_1}{\partial c_2}\}\frac{\partial s}{\partial k} + \frac{c_1}{k}[a_1 - a_2\frac{c_2}{c_1}\frac{\partial c_1}{\partial c_2}] - (1-a_1)\delta. \tag{18d}$$

In the case of $\theta_1=\theta_2=0$ and thus, the two-sector growth model without consumption externalities, (16d) indicates $\frac{\partial s}{\partial \lambda} = (\frac{\partial s}{\partial \lambda})^* > 0$ and $(\frac{\partial s}{\partial k})^* > 0$. Thus, $J^*_{11}<0$. As sector 1 is more capital intensive than sector 2 under construction, an expansion of capital increases capital (sk) and labor (l) allocated to sector 1; moreover, capital sk is increased proportionally more than the proportional increase of labor l, thus $J^*_{12}>0$. As a result, the $\dot{\lambda}^* = 0$ locus is positive sloping: $\frac{d\lambda}{dk}\big|_{\dot{\lambda}=0} = -J^*_{12} / J^*_{11} > 0$.

Further, the slope of the $\dot{k}^* = 0$ locus is $\frac{d\lambda}{dk}\big|_{\dot{k}=0} = -J^*_{22} / J^*_{21}$. Under $\theta_1=\theta_2=0$, (16c) indicates $\frac{\partial c_1}{\partial \lambda} = (\frac{\partial c_1}{\partial \lambda})^* < 0$ and thus $J^*_{21}>0$. The sign of J^*_{22} depends on the threshold of k, denoted by $\tilde{k}$, wherein the marginal product of capital minus the effect of capital on consumption is equal to the deprecation rate, and $J^*_{22}>(resp.<)0$ when $k<(resp.>)\tilde{k}$.[15] Thus, $\dot{k}^*=0$ is a U-shaped locus as illustrated in Figure 1.

In a two-sector growth model without consumption externalities, the steady state is a saddle. The condition is $Det(J^*)<0$ and is equivalent to $-J^*_{12}/J^*_{11}>-J^*_{22}/J^*_{21}$. The condition requires the positive slope of the $\dot{\lambda}^*=0$ locus to be larger than the slope of the $\dot{k}^*=0$ locus, as seen in A1 and A2, Figure 1. The $\dot{\lambda}^*=0$ locus needs to start at a positive and finite value of $k=\hat{k}>0$ in order for the marginal product of capital to equal the sum of the discount rate and the depreciation rate at $\lambda=0$.[16] With consumption externalities, the dynamic property of equilibrium determinacy may change.

Based on the analytical relations derived above, we investigate the indeterminacy conditions for special cases. When discussing calibrated models in these special cases below, we use the model without consumption externalities as the benchmark case. Although authors such as Maurer and Meier (2008) and Ravin (2007) have estimated the degree of external consumption effects in models with one type of consumption goods,[17] their estimates correspond to the value of $\theta_1(\varepsilon$-1) if the two sectors are interpreted as general goods and home goods, or to an average over the two values of $\theta_1(\varepsilon$-1) and $\theta_2(\varepsilon$-1) if the two sectors are interpreted as manufacture and services. These estimates are not θ_1 and θ_2. In fact, it is difficult to specify the magnitudes of θ_1 and θ_2 by using the standard estimation strategy based on the Euler equation of the representative household. Therefore, in what follows, we use the model with empirically plausible parameter values and $\theta_1=\theta_2=0$ as the benchmark setting. Then, with all other values in the baseline parameterization remaining unchanged, we examine the levels of θ_1 and θ_2 with variations of ε and γ in preference specifications under which indeterminacy may emerge. As to the factor-intensity ranking between the two sectors, we focus mainly on the case where the general good sector is more capital intensive than the pure consumption good sector. We also briefly consider the opposite case.

[15] $\tilde{k}$ is determined by $[D\frac{\partial s}{\partial k}+a_1A_1\frac{s^{a_1}l^{1-a_1}}{k^{1-a_1}}]-\frac{\partial c_1}{\partial c_2}\frac{\partial c_2}{\partial k}=\delta$, where $D\equiv\{A_1(\frac{s}{l})^{a_1}k^{a_1}[a_1\frac{l}{s}+(1-a_1)l'(s)]-\frac{\partial c_1}{\partial c_2}\frac{\partial c_2}{\partial s}\}>0.$

[16] The value $k=\hat{k}$ is determined by $(\rho+\delta)=A_1a_1[l(s(0,\hat{k}))/s(0,\hat{k})]^{1-a_1}(1/\hat{k})^{1-a_1}$.

[17] Maurer and Meier (2008) and Ravina (2007) estimated the *KUJ* effect based on one-sector models. Maurer and Meier (2008) employed US micro data of the Panel Study of Income Dynamics wherein the *KUJ* effect is based on peer groups. They obtained the *KUJ* effect in the range of [0.3, 0.45]. Ravina (2007) used California micro data of the credit card holders wherein the *KUJ* effect is based on a geographic notion of neighborhood effects. She found the *KUJ* effect in the range of [0.26, 0.29].

4.2.1 Only goods 1 has consumption externalities

First, we consider the externality arising only from general goods consumption; that is, $\theta_1 \neq 0$ and $\theta_2=0$. This is the type of externalities analyzed in existing one-sector models studied by Gali (1994), Dupor and Liu (2003), Liu and Turnovsky (2005) and Alonso-Carrera et al. (2008).

With general goods consumption externalities, when there is the *KUJ* effect, self-fulfilling expectations can be supported as an equilibrium. To explain the reasons, we use (5a)-(5c) to obtain

$$MRS = \frac{\lambda}{u_2} = \frac{\gamma}{1-\gamma}\left[\frac{c_2}{(c_1)^{1-[-\theta_1(\varepsilon-1)]}}\right]^{\frac{1}{\varepsilon}} = MRT = \frac{f_2^2((1-s)k,(1-l))}{f_2^1(sk,l)} = p. \tag{19}$$

Suppose that there are sunspot expectations that the relative price of general goods in terms of consumption goods will increase (higher p). This raises the *MRT* between general goods and consumption goods. Thus, the agent allocates more capital and labor to the general goods sector (thus sk and l are increased) which will lower the marginal product in the general goods sector and raise the marginal product in the consumption goods sector. Yet, more capital and labor in the general goods sector increases the production of general goods which increases general goods consumption. If there is no consumption externality ($\theta_1=0$), (19) indicates a lower *MRS* between c_1 and c_2 which will not equal the *MRT*. As a result, anticipations of higher prices of general goods relative to consumption goods cannot be supported as an equilibrium. Suppose instead that $\theta_1 \neq 0$ and there is the *KUJ* effect, $[-\theta_1(\varepsilon-1)]>0$. If the *KUJ* effect is sufficiently large and is within a proper range, then the increase in general goods consumption can raise the *MRS* so as to equal the *MRT*. In this situation, self-fulfilling expectations can be supported as equilibrium.

To see how a sufficiently large *KUJ* effect leads to indeterminacy, with goods 1 consumption externalities, the elements of the Jacobian matrix in (18a)-(18d) are: $J^{\theta 1}{}_{11}$, $J^{\theta 1}{}_{12}$, $J^{\theta 1}{}_{21}$ and $J^{\theta 1}{}_{22}$.[18] The change in the dynamic property of equilibrium comes mainly from the change in the sign from $J^*{}_{11}<0$ to $J^{\theta 1}{}_{11}>0$ as a result of the *KUJ* effect. When $\theta_1 \neq 0$, $(\frac{\partial s}{\partial \lambda})$ in (18a) is modified as $(\frac{\partial s}{\partial \lambda})^{\theta_1}$ and the sign of $J^{\theta 1}{}_{11}>0$ is opposite to the sign of $(\frac{\partial s}{\partial \lambda})^{\theta_1}$ given by

$$\left(\frac{\partial s}{\partial \lambda}\right)^{\theta_1} = \frac{1-(-\theta_1(\varepsilon-1))}{\varepsilon}\frac{1}{c_1}\left(\frac{\partial c_1}{\partial \lambda}\right)^{\theta_1}\frac{1}{\Xi^{\theta_1}}, \tag{20}$$

where $(\frac{\partial c_1}{\partial \lambda})^{\theta_1} = -\frac{\varepsilon c_1}{\lambda}\frac{1+p(c_1/c_2)}{1-[-\theta_1(\varepsilon-1)][1+p(c_1/c_2)\varepsilon/(\varepsilon-1)]}$,

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[18] A superscript θ_i is used to represent the source of externalities from consumption c_i.

$$\Xi^{\theta_1}=[\tfrac{a_2}{1-s}-\tfrac{a_2}{1-l}l'(s)]+[\tfrac{a_1}{s}-\tfrac{a_1}{l}l'(s)]+\{\tfrac{[1-(-\theta_1(\varepsilon-1))]}{\varepsilon}\tfrac{c_2}{c_1}(\tfrac{\partial c_1}{\partial c_2})^{\theta_1}-\tfrac{1}{\varepsilon}\}(\tfrac{a_2}{1-s}+\tfrac{1-a_2}{1-l}l'(s))\equiv\Xi^*+\Lambda^{\theta_1},$$

$$\Lambda^{\theta_1}=\{\tfrac{[1-(-\theta_1(\varepsilon-1))]}{\varepsilon}\tfrac{c_2}{c_1}(\tfrac{\partial c_1}{\partial c_2})^{\theta_1}-\tfrac{1}{\varepsilon}\}(\tfrac{a_2}{1-s}+\tfrac{1-a_2}{1-l}l'(s)),$$

$$(\tfrac{\partial c_1}{\partial c_2})^{\theta_1}=\tfrac{c_1}{c_2}\tfrac{1}{1-[-\theta_1(\varepsilon-1)][1+p(c_1/c_2)\varepsilon/(\varepsilon-1)]}.$$

Thus, if $(\frac{\partial s}{\partial \lambda})^{\theta_1}<0$, then $J^{\theta 1}{}_{11}>0$. It is clear that the *KUJ* effect leads to $\frac{c_2}{c_1}(\frac{\partial c_1}{\partial c_2})^{\theta_1}>1$ and $\frac{[1-(-\theta_1(\varepsilon-1))]}{\varepsilon}<\frac{1}{\varepsilon}$. Moreover, the *KUJ* effect can make $\{\frac{[1-(-\theta_1(\varepsilon-1))]}{\varepsilon}\frac{c_2}{c_1}(\frac{\partial c_1}{\partial c_2})^{\theta_1}-\frac{1}{\varepsilon}\}>0$, which gives $\Lambda^{\theta_1}>0$. Further, the *KUJ* effect causes $\Lambda^{\theta_1}>-\Xi^*>0$, which gives $\Xi^{\theta_1}>0$. Finally, the *KUJ* effect maintains $(\frac{\partial c_1}{\partial \lambda})^{\theta_1}<0$. With $\Xi^{\theta_1}>0$ and $(\frac{\partial c_1}{\partial \lambda})^{\theta_1}<0$, then $(\frac{\partial s}{\partial \lambda})^{\theta_1}<0$ and thus, $J^{\theta 1}{}_{11}>0$.

The same reasoning also indicates $(\frac{\partial s}{\partial k})^{\theta_1}<0$ which gives $J^{\theta 1}{}_{12}>0$ and $J^{\theta 1}{}_{21}>0$. As a result, the slope of $\dot{\lambda}=0$ locus is negative: $-J_{12}^{\theta_1}/J_{11}^{\theta_1}<0$. When the negative slope of the $\dot{\lambda}=0$ locus is smaller than the slope of the $\dot{k}=0$ locus, as illustrated in B1 and B2, Figure 1, $Det(J^{\theta 1})>0$ and $Tr(J^{\theta 1})<0$ and the steady state is a sink.

To obtain the required *KUJ* effect under which the steady state is a sink, in the Appendix we have shown that the relative slope condition gives

$$\varphi_1[-\theta_1(\varepsilon-1)]^2+b_1[-\theta_1(\varepsilon-1)]+Det(J^*)>0, \tag{21}$$

where φ_1 and b_1 are coefficients that are functions of consumption and the shadow price of capital evaluated at the steady state. This inequality gives the *KUJ* effect wherein the steady state is a sink.

Recall that when a_2=0, $c_2=A_2(1-l)$ is leisure.[19] With θ_2=0, our model in this subsection is reduced to the one-sector growth model with elastic labor supply studied by Alonso-Carrera et al. (2008). In the model of Alonso-Carrera et al. (2008), indeterminacy arises only if consumption externalities make the Frisch labor supply to have certain shape. Differing from Alonso-Carrera et al. (2008), in our two-sector model, general goods consumption externalities lead to indeterminacy even though the labor supply is inelastic.

It is interesting to explore whether the conditions of indeterminacy are quantitatively easy to meet under proper preference specifications summarized by parameters θ_1, ε and γ. Without estimated values of θ_1 and θ_2, our quantitative strategy is as follows. We calibrate our model to the

[19] In this case, s=1 and there is no relationship (16a). If we normalize A_2=1, then (16b) is changed to c_2=(1-l), which is leisure, (16c) is $c_1=c_1(\lambda,\ 1\text{-}l)$, and (16d) is $l^{a_1}[(1-l)(c_1)^{-1}]^{-[1+(\varepsilon-1)\theta_2]/\varepsilon}=\gamma(1-a_1)A_1(1-\gamma)^{-1}k^{a_1}$ which implies $l=l(\lambda, k)$,where $\frac{\partial l}{\partial \lambda}>0$ and $\frac{\partial l}{\partial k}>0$ when $\theta_1=\theta_2=0$ but $\frac{\partial l}{\partial \lambda}$ and $\frac{\partial l}{\partial k}$ are ambiguous if $\theta_1\neq 0$ or $\theta_2\neq 0$. Using $l=l(\lambda, k)$, (16c) becomes $c_1=c_1(\lambda,\ 1\text{-}l(\lambda, k))$.

baseline economy without consumption externalities (i.e., $\theta_1=\theta_2=0$). Then, with all other values in the baseline parameterization remain unchanged we vary the values of preference parameters θ_1, ε and γ that satisfy the equilibrium conditions and calculate the required smallest degree of the *KUJ* effect (i.e., $\theta_1(\varepsilon\text{-}1)$) which causes indeterminacy. If the required smallest value of the *KUJ* effect is smaller than the foregoing estimates found by Maurer and Meier (2008) and Ravin (2007), then local indeterminacy is plausible.

To calibrate the model, the capital share in the general goods sector is set at $a_1=0.32$ following Herrendorf and Valentinyi (2008). The consumption goods sector is thought of as the service sector which includes restaurants, transportation services, and financial and retail services. Lee and Wolpin (2006) provided estimates of the share of labor earnings in the service sector in selected years by using the data coming from the Bureau of Economic Analysis in the U.S. Following their estimation, the labor share in the consumption sector is set equal to 72% which is the average share of labor earnings in the service sector from 1985 to 2000. Hence, the implied value of a_2 is 0.28. Moreover, these two authors pointed out that as a fraction of total employment, service-sector employment grew from 57 percent to 75 percent between 1950 and 2000. We take this number and set $1\text{-}l=0.75$. For the value of the *ES* between two consumption goods, we choose $\varepsilon=1.25$ as our baseline value, which is in the range estimated by Ogaki and Reinhart (1998). Under these values, we use (5b) to calculate $\gamma=0.1654$. Then, according to (5c), we calculate $s= 0.2874$. If we set the depreciation rate to equal $\delta=0.05$ and the discount rate to equal $\rho=0.04$, as conventionally suggested, we can use the steady-state condition in (14a) to compute $k^*=5.6179$. Finally, we use (4a) and (7a) to obtain $c_1{}^*=0.1732$ and $c_2{}^*=1.1987$. We found that the steady state is a saddle. Then, with all other parameter values in the baseline remaining unchanged, we change preference parameters θ_1, ε and γ that cause local indeterminacy.

To start, we consider a special model of $a_2=0$, the one-sector growth model with elastic leisure studied by Alonso-Carrera et al. (2008). We hold $a_2=0$ and all other parameter values unchanged and adjust the values of θ_1, ε and γ that meet equilibrium conditions. In the right diagram of Figure 2, the shaded area is the region of (ε, $-\theta_1$) under which the steady state is a sink. In this case, indeterminacy arises when $\varepsilon<1$ and $-\theta_1<0$. Thus, when the labor supply is elastic, the general goods consumption externality generates indeterminacy only when the consumption externality is negative. The estimates in Zabalza et al. (1980) indicates that the *ES* between general goods and leisure is around ε=0.5.[20] At ε=0.5, indeterminacy is established at the smallest value of θ_1=2.2798.[21] The required smallest degree of the *KUJ* effect is [(–θ₁)(ε-1)]=1.1399, which is larger than the estimates obtained by Maurer and Meier (2008) and Ravina (2007). The result suggests that it is difficult for consumption externalities to create indeterminacy in a one-sector model.

[Insert Figure 2 here]

Now, we quantify our general two-sector growth model and thus y_2 is consumption goods. We hold a_2=0.28 and all other parameter values unchanged and vary the values of θ_1, ε and γ that meet equilibrium conditions. The results are illustrated in the left diagram of Figure 2. In the diagram, the shaded area is the region of (ε, -θ_1) under which the steady state is a sink. Here, indeterminacy arises only when ε>1 and $-\theta_1$>0. Thus, with the consumption goods sector, general goods consumption externalities generate indeterminacy only when the externality is positive. At the empirically plausible value of ε=1.25 estimated by Ogaki and Reinhart (1998), the smallest absolute value of $-\theta_1$ where indeterminacy can be established is $-\theta_1$=4.2%.[22] The required smallest degree of the *KUJ* effect is $(-\theta_1)(\varepsilon\text{-}1)$=0.0105 which is smaller than the estimates obtained by Maurer and Meier (2008) and Ravina (2007). Indeed, the requirement is only 1% as large as the smallest degree required in a one-sector model. The results imply that it is easy for consumption externalities to establish indeterminacy in a two-sector model.

4.2.2 Only goods 2 has consumption externalities. (θ_1=0, $\theta_2\neq$0).

Next, we consider the externality arising only from consumption goods. To see why the *KUJ* effect of consumption goods can generate indeterminacy, we use (5a)-(5c) to obtain

$$MRS = \frac{\lambda}{u_2} = \frac{\gamma}{1\text{-}\gamma}\left[\frac{(c_2)^{1\text{-}[-\theta_2(\varepsilon\text{-}1)]}}{c_1}\right]^{\frac{1}{\varepsilon}} = MRT = \frac{f_2^2((1-s)k,(1-l))}{f_2^1(sk,l)} = p. \tag{22}$$

To see that sunspot expectations equilibrium can emerge, suppose that the representative agent expects that the price of general goods relative to consumption goods is increasing (higher *p*). This raises the *MRT* between general goods and consumption goods. Thus, the agent allocates more input to the general goods sector and less input to the consumption goods sector which

[20] Using data in the UK, Zabalza et al. (1980) found that the ES between income and leisure is 0.25 for men and 1.30 for women. As men are the major labor force, ε=0.5 is about the average of men and women in the labor force estimated by these authors.

[21] In the simulation that gives rise to a sink, at ε=0.5, θ_1 is larger than or equal to 2.2972. When θ_1 is larger, γ is larger. At θ_1=2.2972, γ= 0.6585.

[22] In the simulation that yields a sink, at ε=1.25, -θ_1 is in [0.042, 0.726] and γ is in [0.1675, 0.2036].

reduces the marginal product in the general goods sector, increases the marginal product in the consumption goods sector and reduces the production of consumption goods. When the *KUJ* effect of consumption goods ($[-\theta_2(\varepsilon-1)]>0$) is sufficiently large, then consumption goods are consumed less so as to increase the *MRS* and equal the *MRT*. Thus, self-fulfilling expectations can be supported as an equilibrium.

In the case, the elements of the Jacobian matrix in (18a)-(18d) are: $J^{\theta 2}{}_{11}$, $J^{\theta 2}{}_{12}$, $J^{\theta 2}{}_{21}$ and $J^{\theta 2}{}_{22}$. The *KUJ* effect may affect the sign of $J^{\theta 2}{}_{11}$ and $J^{\theta 2}{}_{12}$. To see how the *KUJ* effect works when $\theta_2\neq 0$, $(\frac{\partial s}{\partial \lambda})$ and $(\frac{\partial s}{\partial k})$ in (18a) and (18b) become, respectively,

$$\left(\frac{\partial s}{\partial \lambda}\right)^{\theta_2} = \frac{1}{\varepsilon}\frac{1}{c_1}\left(\frac{\partial c_1}{\partial \lambda}\right)^{\theta_2}\frac{1}{\Xi^{\theta_2}}, \tag{23a}$$

$$\left(\frac{\partial s}{\partial k}\right)^{\theta_2} = -\theta_2\frac{a_2}{k}\frac{1}{\Xi^{\theta_2}} - \frac{a_1-a_2}{k}\frac{1}{\Xi^{\theta_2}}, \tag{23b}$$

where $(\frac{\partial c_1}{\partial \lambda})^{\theta_2} = -\frac{\varepsilon c_1[1+p(c_1/c_2)]}{\lambda} < 0$,

$$\Xi^{\theta_2} = [\tfrac{a_2}{1-s} - \tfrac{a_2}{1-l}l'(s)] + [\tfrac{a_1}{s} - \tfrac{a_1}{l}l'(s)] - \theta_2[\tfrac{a_2}{1-s} + \tfrac{1-a_2}{1-l}l'(s)] \equiv \Xi^* + \Lambda^{\theta_2},$$

$$\Lambda^{\theta_2} = -\theta_2[\tfrac{a_2}{1-s} + \tfrac{1-a_2}{1-l}l'(s)].$$

Since $(\frac{\partial c_1}{\partial \lambda})^{\theta_2} < 0$, the *KUJ* effect has to give $\Xi^{\theta_2} > 0$ in order to obtain $(\frac{\partial s}{\partial \lambda})^{\theta_2} < 0$. With $\Xi^*<0$, this is possible only if $\Lambda^{\theta_2} > 0$, which requires $\theta_2<0$ and, under the *KUJ* effect, $\varepsilon>1$. Moreover, if $-\theta_2>0$ is sufficiently large such that $\Xi^{\theta_2} > 0$, then $(\frac{\partial s}{\partial \lambda})^{\theta_2} < 0$. and thus $J^{\theta 2}{}_{11}>0$. However, the value of $-\theta_2>0$ cannot be too large because a large $-\theta_2>0$ gives a large $(\frac{\partial s}{\partial k})^{\theta_2} > 0$ such that $J^{\theta 2}{}_{12}<0$ and the slope of the $\dot{\lambda}^{\theta_2} = 0$ locus is positive. Thus, it is required that $[-\theta_2(\varepsilon-1)]$ lay within a range so the slope of locus $\dot{\lambda}^{\theta_2} = 0$ is negative as illustrated in B1 and B2, Figure 1.

Then, a sink arises if $Det(J^{\theta 2})>0$ and $Tr(J^{\theta 2})<0$ which requires that the slope of locus $\dot{\lambda}^{\theta_2} = 0$ be smaller than the slope of locus $\dot{k}^{\theta_2} = 0$. In the Appendix, we have shown that the relative slope condition is met under

$$\varphi_2[-\theta_2(\varepsilon-1)]^2 + b_2[-\theta_2(\varepsilon-1)] + Det(J^*) > 0, \tag{24}$$

where φ_2 and b_2 are coefficients that are functions of consumption and the shadow price of capital evaluated at a steady state. Then, we obtain the range of the *KUJ* effect wherein the steady state is a sink.

It is worth noting that when $a_2=0$, with $\theta_2\neq 0$ our model is reduced to a one-sector growth model with a leisure externality. Benhabib and Farmer (2000) and Weder (2004) have studied the role of positive leisure externalities in establishing indeterminacy.[23] These authors showed that positive leisure externalities help establish indeterminacy as leisure externalities make it easier for the Frisch labor supply curve to slope down as a function of the real wage. However, they both found that it is difficult for the leisure externality alone to generate indeterminacy.

To quantify the conditions of indeterminacy in our model with the externality arising only from consumption goods, we offer the results in the left diagram of Figure 3. The shaded area in the diagram is the region of (ε, -θ_2) under which the steady state is a sink. At ε=1.25, indeterminacy can be established at the smallest absolute value of $-\theta_2$=0.6%.[24] The required smallest degree of the *KUJ* in consumption goods is $(-\theta_2)(\varepsilon-1)$=0.0015. This required degree is only 10% as large as the required smallest *KUJ* effect of general goods consumption externalities at 0.0105 in Figure 2. The results indicate that indeterminacy emerges even more easily under consumption goods externalities than under general goods consumption externalities.

[Insert Figure 3 here]

We also quantify the case under a_2=0. The right diagram of Figure 3 offers the region of (ε, -θ_2) that leads to indeterminacy. The results indicate that indeterminacy can arise under both positive and negative leisure externalities. At the empirically plausible *ES* between general goods and leisure at ε=0.5, indeterminacy can be established at the smallest value of θ_2=8.0002.[25] This indicates that the required smallest degree of the *KUJ* effect in leisure externalities is 4.0001. This required smallest *KUJ* effect is almost 4 times as large as the required smallest *KUJ* effect in general goods in a one-sector model (cf. 1.1399) in the right diagram of Figure 2. The result thus confirms the conclusions found in by Benhabib and Farmer (2000) and Weder (2004) in that it is difficult for the leisure externality alone to generate indeterminacy in one-sector growth models.

4.2.3 Goods 1 and 2 exhibit symmetric consumption externalities. ($\theta_1 \neq 0$, $\theta_2 \neq 0$).

We have derived the conditions under which general goods consumption externalities and consumption goods externalities each can be a source of indeterminacy. If we combine these conditions, externalities in both types of consumption together can establish indeterminacy.

[23] The analysis provided by Benhabib and Farmer (2000) is a one-sector model with real balances but without capital in the production function. Weder (2004) is a one-sector growth model with externalities in production. In these two models, leisure is elastic. They both allow for leisure externalities in utility.

[24] In the simulation a sink emerges, at ε=1.25, -θ_2 is in [0.006, 0.1] and γ is in [0.1655, 0.1659].

[25] In the simulation that gives a sink, at ε=0.5, θ_2 is larger than or equal to 8.0002. When θ_1=8.0002, γ= 0.0197. When θ_1 is larger, γ is larger.

A more appealing case is $\theta_1=\theta_2=\theta\neq 0$ when consumption externalities from both goods are symmetric. In this case, we have shown in Section 3 that if $\varepsilon\neq 1$, the utility is homothetic and the market equilibrium is efficient in a steady state. Yet, symmetric consumption externalities can cause inefficiency in transitions as the shadow price of capital in a market is no longer a fixed proportion of the shadow price of capital in a socially planned economy. As we will see below, indeterminacy arises when the symmetric *KUJ* effect is in a proper range,.

To see how this works, we use (5a)-(5c) to obtain

$$MRS=\frac{\lambda}{u_2}=\frac{\gamma}{1\text{-}\gamma}[\frac{c_2}{c_1}]^{\frac{1\text{-}[\text{-}\theta(\varepsilon\text{-}1)]}{\varepsilon}}=MRT=\frac{f_2^2((1-s)k,(1-l))}{f_2^1(sk,l)}=p. \tag{25}$$

To see that sunspot expectations equilibrium can emerge, suppose that the representative agent expects a higher price of general goods relative to consumption goods (higher p). This raises the *MRT* between general and consumption goods. Thus, the agent allocates more input to general goods which decreases the marginal product in general goods and increases the marginal product in consumption goods. Then, there are more production of general goods and less production of consumption goods. When these have symmetric *KUJ* effects (i.e., $(-\theta(\varepsilon-1))>0$), then general goods may be consumed more and consumption goods may be consumed less so as to increase the *MRS* and equal the *MRT*. In this situation, self-fulfilling expectations can be supported as an equilibrium.

To derive the conditions, the elements of the Jacobian matrix in (18a)-(18d) are J^θ_{11}, J^θ_{12}, J^θ_{21} and J^θ_{22} which are combinations of those in subsections 4.2.1 and 4.2.2. To see how the *KUJ* effect works when $\theta_1=\theta_2=\theta\neq 0$, $(\frac{\partial s}{\partial \lambda})^\theta$ and $(\frac{\partial s}{\partial k})^\theta$ are, respectively,

$$\left(\frac{\partial s}{\partial \lambda}\right)^\theta=[1-(-\theta(\varepsilon-1))]\frac{1}{\varepsilon}\frac{1}{c_1}\left(\frac{\partial c_1}{\partial \lambda}\right)^\theta\frac{1}{\Xi^\theta}, \tag{26a}$$

$$\left(\frac{\partial s}{\partial k}\right)^\theta=\frac{1-(-\theta(\varepsilon-1))}{\varepsilon}[\frac{c_2}{c_1}(\frac{\partial c_1}{\partial c_2})^\theta-1]\frac{a_2}{k}\frac{1}{\Xi^\theta}-\frac{a_1-a_2}{k}\frac{1}{\Xi^\theta}, \tag{26b}$$

where $(\frac{\partial c_1}{\partial \lambda})^\theta=-\frac{\varepsilon c_1}{\lambda}\frac{1+p(c_1/c_2)}{1-[-\theta(\varepsilon-1)][1+p(c_1/c_2)\varepsilon/(\varepsilon-1)]}$,

$(\frac{\partial c_1}{\partial c_2})^\theta=\frac{c_1}{c_2}\frac{1-\theta}{1-[-\theta(\varepsilon-1)][1+p(c_1/c_2)\varepsilon/(\varepsilon-1)]}$,

$\Xi^\theta=[\frac{a_2}{1-s}-\frac{a_2}{1-l}l'(s)]+[\frac{a_1}{s}-\frac{a_1}{l}l'(s)]+\frac{1-[-\theta(\varepsilon-1)]}{\varepsilon}[\frac{c_2}{c_1}(\frac{\partial c_1}{\partial c_2})^\theta-1][\frac{a_2}{1-s}+\frac{1-a_2}{1-l}l'(s)]\equiv\Xi^*+\Lambda^\theta$,

$\Lambda^\theta=\frac{1-[-\theta(\varepsilon-1)]}{\varepsilon}[\frac{c_2}{c_1}(\frac{\partial c_1}{\partial c_2})^\theta-1][\frac{a_2}{1-s}+\frac{1-a_2}{1-l}l'(s)]$.

Similar to the case with only general goods consumption externalities (cf. (20)), here the *KUJ* effect ($-\theta(\varepsilon$-1)) appears in a multiplicative term in $(\frac{\partial s}{\partial \lambda})^{\theta}$ and $(\frac{\partial s}{\partial k})^{\theta}$ If the *KUJ* effect lays within a range,[26] it is possible to obtain $(\frac{\partial s}{\partial \lambda})^{\theta} < 0$ and $(\frac{\partial s}{\partial k})^{\theta} < 0$ Then, $J^{\theta}_{11}>0$, $J^{\theta}_{12}>0$ and $J^{\theta}_{21}<0$. The required condition is $Det(J^{\theta})>0$ and $Tr(J^{\theta})<0$ which gives a negative slope of $\dot{\lambda} = 0$ that is steeper than the slope of $\dot{k} = 0$. In the Appendix, we have shown that the relative slope condition is met if

$$\varphi_3[-\theta(\varepsilon-1)]^3 + b_3[-\theta(\varepsilon-1)]^2 + d_3[-\theta(\varepsilon-1)] + Det(J^*) > 0, \tag{27}$$

where φ_3, b_3 and d_3 are coefficients that are functions of consumption and the shadow price of capital evaluated at a steady state. Then, we obtain the range of the *KUJ* effect wherein the steady state is a sink.

Our analysis indicates that in a two-sector growth model, symmetric consumption externalities lead to indeterminacy even when the utility is homothetic. The result is different from that in the one-sector growth model provided by Alonso-Carrera et al. (2008) wherein consumption externalities do not lead to indeterminacy when the utility is homothetic. The difference arises because there is a relative price of the two goods in our model.

Figure 4 offers quantitative results about the region of (ε, $-\theta$) under which the steady state is a sink (see left diagram). At the empirically plausible value of ε=1.25, indeterminacy is established at the smallest absolute value of $-\theta$=0.5%. The required smallest *KUJ* effect of symmetric consumption externalities is $-\theta(1-\varepsilon)$=0.00125 which is smaller than the required smallest *KUJ* effect of general goods consumption (0.00105). Hence, the consumption externality helps the general goods consumption externality to establish indeterminacy.

[Insert Figure 4 here]

When a_2=0, $\theta_2>0$ is a leisure externality.[27] With $\theta_1=\theta_2=\theta$, quantitative results are shown in the right diagram of Figure 4. Notice that this case may be thought of as the case of the general goods consumption externality with an additional symmetric leisure externality. Thus, similar to the left diagram of Figure 2 with only the general goods consumption externality, indeterminacy here arises only when externalities are positive ($-\theta_1>0$). However, with additional symmetric, positive leisure externalities $-\theta_2=-\theta>0$, the required degree of general goods consumption externalities here is

[26] In the range of the *KUJ* effect, (i) the signs of $(\frac{\partial c_1}{\partial \lambda})^{\theta}$ and $(\frac{\partial c_1}{\partial c_2})^{\theta}$ do not deviate from their counterparts of $(\frac{\partial c_1}{\partial \lambda})^{*} < 0$ and $(\frac{\partial c_1}{\partial c_2})^{*} > 0$ under $\theta_1=\theta_2=0$ and (ii) $\Lambda^{\theta} > -\Xi^{*} > 0$.

[27] In the case under a_2=0, as there is the leisure externality, our model is different from the Alonso-Carrera et al. (2008) model which has no leisure externality.

much larger than that in Figure 2. For example, at ε=1.25, indeterminacy can be established at the smallest absolute value of $-\theta$=50.76% here in Figure 4, as opposed to $-\theta_1$=4.2% in Figure 2. Thus, a positive leisure externality does not help the general goods consumption externality to establish indeterminacy in a one-sector model. This result is different from the findings uncovered by Benhabib and Farmer (2000) and Weder (2004) wherein the leisure externality helps establish indeterminacy in a model with both money and externalities in production.

4.2.4 Goods 2 is more capital intensive than goods 1

We have so far assumed the plausible case that the general goods sector is more capital intensive than the pure consumption goods sector, $a_1>a_2$. Nevertheless, from a theoretical point of view, it might be interesting to consider the opposite case that $a_1<a_2$. As shown in Section 3, with the jealousy effect of general goods consumption, the case of $a_1<a_2$ produces under-accumulation, rather than, over-accumulation of capital in the long run. It is interesting to quantitatively investigate local dynamic properties of equilibrium under a different intensity of capital.

To this end, we set a_1=0.28 and a_2=0.32 so the general goods sector is less capital intensive. We recalibrate the model following the same method used in subsection 4.2.1. The baseline parameter values are not different but the values of the steady state change.[28] We quantify the *KUJ* effect with an adjustment of *ES* between the two goods to see whether the steady state is a sink. The results are as follows.

First, if the *ES* between the two goods is smaller than unity (ε<1), the steady state is always a saddle. Second, if the *ES* between the two goods is larger than one (ε>1), whether the steady state is a sink or not depends on the source of consumption externalities and is as follows.

Case 1. $\theta_1\neq 0$ and $\theta_2=0$

In this case, only general goods consumption has externalities. We find that the steady state is either a saddle or a source, not a sink. The result is thus different from those in the case of $a_1>a_2$ as illustrated in the left diagram in Figure 2.

Case 2. $\theta_1=0$ and $\theta_2\neq 0$

In this case, only the consumption of consumption goods has externalities. We find that the steady state is a sink only if $1<\varepsilon<1.45$. See the left diagram of Figure 5. As we see from the diagram, a sink arises only if the value of $-\theta_2$ lies above a threshold. Note the difference from the corresponding left diagram of Figure 3 wherein the threshold of the degree of consumption

[28] The baseline parameters are a_1=0.28, a_2=0.32, ρ=0.04, δ=0.05, $\theta_1=\theta_2$=0. Then γ and s are recalibrated to 0.1267 and 0.216, respectively. Under the parameter values we obtain k^*=5.5995, c_1^*=0.1087 and c_2^*=1.3202.

externalities is constant, the threshold here is increasing in the value of the *ES* between the two goods ε. Moreover, for a given value of ε, the required smallest *KUJ* effect here is much larger. For example, at ε=1.25, the required smallest *KUJ* effect here is 5.68, as opposed to 0.0015 in Figure 3.

[Insert Figure 6 here].

Case 3. $\theta_1 = \theta_2 = \theta \neq 0$**.**

In this case, two types of consumption have symmetric externalities. We find that the steady state is a sink only if $1<\varepsilon<1.1$ (the right diagram of Figure 5). In this range of the *ES*, a sink arises if the value of $-\theta$ is larger than a large threshold. Note that different from the corresponding left diagram of Figure 4 wherein the required smallest degree of consumption externalities is constant, the threshold here is increasing in the value of the *ES* between the two goods. Moreover, for a given value of ε, the required degree of consumption externalities is large. For example, at ε=1.1, a sink emerges at the smallest value of $-\theta$=148.6 and thus the required smallest *KUJ* effect is $-\theta(\varepsilon-1)$=14.86, which is very large as compared to 0.0005 in the left diagram of Figure 4. Even at ε=1.01, a sink requires a large smallest *KUJ* at 0.39069.

4.3. Economic Intuition for Indeterminacy Conditions

So far, we have presented a detailed analysis concerning the mathematical conditions for indeterminacy. To sum up, we have found: (i) regardless of the capital intensity ranking between the two sectors, indeterminacy tends to emerge under *KUJ* with negative levels θ_1 and θ_2; (ii) if the general goods sector is more capital intensive than the consumption goods sector, then local indeterminacy holds rather easily, and; (iii) if the general goods sector is less capital intensive than the consumption goods sector, then indeterminacy holds only when the consumption goods are associated with extremely high degrees of external effects.

To find insights within those results, it is useful to focus on the relation between the relative price of the general goods p and the shadow price of capital λ. Recall that the capital-labor ratio in sector i is denoted by $x_i = \frac{k_i}{l_i}$ and output per unit of labor in sector i is $g_i(x_i) = \frac{f^i(k_i,l_i)}{l_i}$. It is well-known that the two-sector model with the Cobb-Douglas production function satisfies

$$x_i = x_i\left(p\right), \quad \text{where} \quad \operatorname{sign} x_i'(p) = \operatorname{sign}\,(a_2 - a_1), \tag{27a}$$

$$r = g_1'\left(x_1\left(p\right)\right) = r\left(p\right), \quad \text{where} \quad \operatorname{sign} r'(p) = \operatorname{sign}\,(a_1 - a_2), \tag{27b}$$

$$y_1 = \frac{k - x_2(p)}{x_1(p) - x_2(p)} g_1(x_1(p)) = y^1(k,p), \quad c_2 = y_2 = \frac{x_1(p) - k}{x_1(p) - x_2(p)} g_2(x_2(p)) = y^2(k,p), \tag{27c}$$

where $\operatorname{sign} y_k^1(k,p) = \operatorname{sign}(a_1 - a_2)$, $y_p^1(k,p) > 0$, $\operatorname{sign} y_k^2(k,p) = \operatorname{sign}(a_2 - a_1)$, $y_p^2(k,p) < 0$.

First, consider the model without externalities (θ_1=θ_2=0). In this case, conditions (5a) and (11b) are written respectively as

$$\gamma\left[\gamma + (1-\gamma)(\frac{c_2}{c_1})^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{1}{\varepsilon-1}} = \lambda, \tag{28a}$$

$$(\frac{\gamma}{1-\gamma})^{\varepsilon}(\frac{c_2}{c_1}) = p^{\varepsilon}. \tag{28b}$$

These equations give the relation of p and λ such that $p = p(\lambda)$, $p'(\lambda) > 0$. The dynamic system (15a)-(15b) in the absence of consumption externalities is alternatively summarized as

$$\dot{\lambda} = \lambda\left(\varrho + \delta - r(p(\lambda))\right), \tag{29a}$$

$$\dot{k} = y^1\left(k, p(\lambda)\right) - p(\lambda)^{-\varepsilon} y^2\left(k, p(\lambda)\right) - \delta k. \tag{29b}$$

Note that the price system (29a) is independent of the quantity system (29b). It is clear that from (27a) to (27c), in spite of the factor intensity ranking between the two sectors, it holds that

$$\operatorname{sign}(\frac{\partial\dot{\lambda}}{\partial\lambda}) = \operatorname{sign}\left(a_2 - a_1\right),\ \operatorname{sign}(\frac{\partial\dot{k}}{\partial k}) = \operatorname{sign}\left(a_1 - a_2\right),$$

which implies that the dynamic system exhibits a saddle-point property.

Now consider the effect of consumption externalities. For simplicity, we focus upon the case of symmetric externalities, θ_1=θ_2=θ≠0. Equations (5a) and (12b) are given respectively by

$$\gamma(c_1)^{\theta(\frac{1}{\varepsilon}-1)}\left[\gamma + (1-\gamma)(\frac{c_2}{c_1})^{\frac{(1-\theta)(\varepsilon-1)}{\varepsilon}}\right]^{\frac{1}{\varepsilon-1}} = \lambda, \tag{30a}$$

$$(\frac{\gamma}{1-\gamma})^{\frac{\varepsilon}{1+\theta(\varepsilon-1)}}(\frac{c_2}{c_1}) = p^{\frac{\varepsilon}{1+\theta(\varepsilon-1)}}, \tag{30b}$$

which can be summarized as

$$\gamma(\frac{\gamma}{1-\gamma})^{\theta(\frac{1}{\varepsilon}-1)}\left[p^{-\frac{\varepsilon}{1+\theta(\varepsilon-1)}} y^2(k,p)\right]^{\theta(\frac{1}{\varepsilon}-1)}\left[\gamma + (1-\gamma)(\frac{\gamma}{1-\gamma})^{\frac{(1-\theta)(\varepsilon-1)}{1+\theta(\varepsilon-1)}} p^{\frac{(1-\theta)(\varepsilon-1)}{1+\theta(\varepsilon-1)}}\right]^{\frac{1}{\varepsilon-1}} = \lambda. \tag{30c}$$

Given the specification used in the numerical examples, it is assumed that θ<0, ε<1 and 1+θ(ε-1)>0. Thus, the foregoing equation gives

$$p = \pi\left(k,\lambda\right),\quad \text{where}\ \ \operatorname{sign} \pi_k\left(k,\lambda\right) = \operatorname{sign}\left(a_2 - a_1\right),\ \ \pi_\lambda\left(k,\lambda\right) > 0.$$

As a result, an alternative representation of the dynamic system (15a)-(15b) in the presence of symmetric consumption externalities is given by

$$\dot{\lambda} = \lambda\left[\varrho + \delta - r\left(\pi\left(k,\lambda\right)\right)\right], \tag{31a}$$

$$\dot{k} = y^1\left(k,\pi\left(k,\lambda\right)\right) - c^1\left(k,\pi\left(k,\lambda\right)\right) - \delta k, \tag{31b}$$

where $c^1\left(k,\pi\left(k,\lambda\right)\right) = [\pi\left(k,\lambda\right)]^{-\frac{\varepsilon}{1+\theta(\varepsilon-1)}} y^2\left(k,\pi\left(k,\lambda\right)\right)$.

Notice that the price system (31a) is no longer independent of the quantity system (31b). A key condition is to require a negative sign for the trace of the Jacobian matrix as follows.

$$\frac{\partial\dot{\lambda}}{\partial\lambda} + \frac{\partial\dot{k}}{\partial k} = -\lambda r'\left(p\right)\pi_\lambda + y_k^1\left(k,p\right) + y_p^1\pi_k - c_k^1\left(k,p\right) - c_p^1\left(k,p\right)\pi_k - \delta.$$

We see that indeterminacy tends to emerge easily when the general good sector is more capital-intensive than the consumption good sector ($a_1 > a_2$), because the shadow price of capital shows self-stabilizing behavior, $\partial\dot{\lambda} / \partial\lambda < 0$. Moreover, a negative effect of an increase in capital on the relative price of general goods p gives rise to a possibility of $\partial\dot{k} / \partial k < 0$. Thus, while an increase in capital enhances the production of general goods and reduces the consumption of general goods, a lower relative price of general goods caused by a rise in capital reduces the production and increases the consumption of general goods. This stabilizing effect never exists in the model without consumption externalities.

Conversely, when the general good sector is less capital-intensive than the consumption good sector, the shadow price of capital exhibits self-destabilizing behavior, $\partial\dot{\lambda} / \partial\lambda > 0$. It is thus necessary to hold $\partial\dot{k} / \partial k < 0$. for indeterminacy. In addition, the self-stabilizing effect of capital should dominate the self-destabilizing effect of the shadow price of capital. With $a_2 > a_1$, since the relative price of capital goods increases, a rise in capital stock reduces the production of general goods and increases its consumption, which promotes self-stabilizing behavior of capital. However, a higher k increases p, which raises the production of general goods. Therefore, if the external effects are small, the stabilizing effect of capital is not large enough to dominate the destabilizing effect of the shadow price of capital.

To sum up, indeterminacy in the presence of consumption externalities stems from the interaction between the price system and the quantity system. If there is no consumption externality, the price system is independent of the quantity system. Such an independency no longer holds in models with consumption externalities even if external effects are symmetric so that the preference is homothetic. Numerical examples shown above illustrate magnitudes of preference parameters under which the interaction between price and capital generates indeterminacy of converging paths.

5. Concluding Remarks

In one-sector neoclassical growth models, consumption externalities can produce an inefficient allocation in a steady state and create indeterminate equilibrium paths toward a steady state only if there is a labor-leisure tradeoff. In our paper, we have shown that in a two-sector neoclassical growth model with general goods and consumption goods, even if there is no labor-leisure tradeoff, consumption spillovers can yield inefficient allocation in a steady state and generate indeterminate equilibrium paths toward a steady state.

In our two-sector model, the factor reallocation and the consumption allocation between sectors are the mechanisms that generate these results. Consumption externalities change the *MRS* between sectors and affect the *MRT* which gives rise to an inefficient allocation in a steady state. Moreover, equilibrium paths toward a steady state are indeterminate because these externalities generate the *KUJ* effect that influences the *MRS* and the *MRT* in such a way that self-fulfilling expectations about relative prices of the two goods can be supported as an equilibrium.

We find that even with negative general goods consumption externalities, capital is over (under) accumulated only if the general goods sector is more (less) capital-intensive than the consumption sector. Although consumption externalities distort the allocation of capital, no active capital taxes are necessary once the sources of consumption distortions are identified and corrected by consumption taxes. Next, when the general goods sector is more capital intensive, general goods consumption externalities generate indeterminacy more easily in a two-sector model than in a one-sector model. The consumption goods are leisure if the consumption goods sector does not use capital, but consumption externalities easily cause indeterminacy, while it is difficult for leisure externalities to generate indeterminacy. Finally, when there are symmetric consumption externalities so the utility is homothetic, the allocation is efficient in a steady state but the equilibrium path may be indeterminate. As a result, it does not matter whether consumption externalities are from general goods, consumption goods or both, it is much easier for consumption externalities to exhibit indeterminacy in a two-sector growth model than in a one-sector growth model. A two-sector model economy with consumption externalities is thus less stabilized than a one-sector model economy.

Appendix

A Optimal Tax Policy

The household maximizes the lifetime utility (2) subject to (14a). The first order conditions are

$$\frac{u_1(c_1,c_2,c_1,c_2)}{(1+\tau_{c1})}=\lambda, \tag{A1a}$$

$$\frac{u_2(c_1,c_2,c_1,c_2)}{(1+\tau_{c2})}=\frac{\lambda}{p}, \tag{A1b}$$

$$\frac{\dot{\lambda}}{\lambda}=(\varrho+\delta)-(1-\tau_k)r. \tag{A1c}$$

The representative firm maximizes the profit. Let w denote the wage. The first order conditions of the representative firm in sectors 1 and 2 are, respectively,

$$f_1^1(sk,l)=r, \quad f_2^1(sk,l)=w, \tag{A2a}$$

$$\frac{1}{p}f_1^2((1-s)k,1-l)=r, \quad \frac{1}{p}f_2^2((1-s)k,1-l)=w. \tag{A2b}$$

Equalization of the wage in (A2a) and (A2b) across sectors gives

$$\frac{f_2^2((1-s)k,1-l)}{f_2^1(sk,l)}=p, \tag{A3a}$$

and if we substitute this expression into (A1b), we obtain

$$\frac{\lambda(1+\tau_{c2})}{u_2(c_1,c_2,c_1,c_2)}=\frac{f_2^2((1-s)k,1-l)}{f_2^1(sk,l)}=p. \tag{A3b}$$

Moreover, equalization of the rental to capital in (A2a) and (A2b) across sectors gives

$$\frac{f_2^2((1-s)k,1-l)}{f_2^1(sk,l)}=\frac{f_1^2((1-s)k,1-l)}{f_1^1(sk,l)}. \tag{A4a}$$

Finally, by using the rental to capital in (A2a), we rewrite (A1c) as

$$\frac{\dot{\lambda}}{\lambda}=(\varrho+\delta)-(1-\tau_k)f_1^1(sk,l). \tag{A4b}$$

The optimal conditions of the decentralized economy include (A1a), (A3b), (A4a) and (A4b).

Our objective is to determine a tax structure such that the decentralized economy replicates the dynamic equilibrium time path of the centrally planned economy, as described in (6a)-(6d). Let a variable with a superscript p denote the variable in the centralized economy. Replication involves setting time-varying tax rates such that $k=k^p$, $c_1= c_1^p$ and $c_2= c_2^p$, which requires $\frac{\dot{\lambda}}{\lambda}=\frac{\dot{\lambda}^p}{\lambda^p}$, or equivalently $\varkappa\lambda=\lambda^p$, where $\varkappa>0$ is an arbitrary constant. It is clear that (A4a) is the same as (6c).

First, replicating (A4b) with (6d) gives

$$\tau_k = 0. \tag{A5a}$$

Next, replicating (A3b) with (6b) gives

$$(1+\tau_{c2}) = \xi \frac{u_2(c_1,c_2,c_1,c_2)}{u_2(c_1,c_2,c_1,c_2)+u_4(c_1,c_2,c_1,c_2)}. \tag{A5b}$$

Finally, we substitute λ in (A3b) into (A1a) and λ^p in (6b) into (6a). Then, replicating the resulting (A1a) with the resulting (6a) gives

$$\frac{(1+\tau_{c1})}{(1+\tau_{c2})} = \frac{\dfrac{u_1(c_1,c_2,c_1,c_2)}{u_1(c_1,c_2,c_1,c_2)+u_3(c_1,c_2,c_1,c_2)}}{\dfrac{u_2(c_1,c_2,c_1,c_2)}{u_2(c_1,c_2,c_1,c_2)+u_4(c_1,c_2,c_1,c_2)}}. \tag{A5c}$$

B Stability Conditions

Differentiating the general case in (15a) and (15b), the elements in the Jacobian matrix are:

$$J_{11} = -\tfrac{(1+pc_1/c_2)(\varrho+\delta)(1-a_1)}{s}[-1+\tfrac{a_1(1-a_2)}{a_2(1-a_1)}\tfrac{l}{s}]\tfrac{1+\theta_1(\varepsilon\text{-}1)}{B}\tfrac{1}{\Xi},$$

$$J_{12} = \tfrac{\lambda(\varrho+\delta)(1-a_1)}{s}\{\tfrac{s}{k}-[-1+\tfrac{a_1(1-a_2)}{a_2(1-a_1)}\tfrac{l}{s}][\tfrac{1+\theta_1(\varepsilon\text{-}1)}{\varepsilon}\tfrac{1\text{-}\theta_2}{B}+\tfrac{1+\theta_2(\varepsilon\text{-}1)}{\varepsilon}+\tfrac{a_1-a_2}{a_2}]\tfrac{a_2}{k}\tfrac{1}{\Xi}\},$$

$$J_{21} = \tfrac{(1+pc_1/c_2)}{\lambda B}\{[1+\theta_1(\varepsilon-1)]\tfrac{\Gamma}{\Xi}-\varepsilon c_1\},$$

$$J_{22} = [\tfrac{1+\theta_1(\varepsilon\text{-}1)}{\varepsilon}\tfrac{1\text{-}\theta_2}{B}+\tfrac{1+\theta_2(\varepsilon\text{-}1)}{\varepsilon}+\tfrac{a_1-a_2}{a_2}]\tfrac{a_2}{k}\tfrac{\Gamma}{\Xi}+\tfrac{a_1c_1}{k}+\tfrac{(1-\theta_2)}{B}\tfrac{a_2c_1}{k}-(1-a_1)\delta,$$

where $B = -1+[-\theta_1(\varepsilon-1)][1+p(c_1/c_2)(\varepsilon/\varepsilon-1)]$,

$$\Xi = -[\tfrac{1+\theta_1(\varepsilon-1)}{\varepsilon}\tfrac{1-\theta_2}{B}+\tfrac{1+\theta_2(\varepsilon-1)}{\varepsilon}]\Psi+\Xi^*,$$

$$\Psi = [\tfrac{a_2}{1-s}+\tfrac{1-a_2}{1-l}l'(s)],$$

$$\Xi^* = [\tfrac{a_2}{1-s}-\tfrac{a_2}{1-l}l'(s)]+[\tfrac{a_1}{s}-\tfrac{a_1}{l}l'(s)] < 0,$$

$$\Gamma = (c_1+\delta k)(\tfrac{a_1}{s}+\tfrac{1-a_1}{l}l')-\tfrac{c_1(1-\theta_2)}{B}\Psi .$$

In the baseline model of the two-sector growth model without consumption externalities ($\theta_1=\theta_2=0$), the elements in the Jacobean matrix are as follows.[29]

$$J_{11}^* = \tfrac{(1+pc_1/c_2)(\varrho+\delta)(1-a_1)}{s}[-1+\tfrac{a_1(1-a_2)}{a_2(1-a_1)}\tfrac{l}{s}]\tfrac{1}{\Xi^*} < 0,$$

$$J_{12}^* = \tfrac{\lambda^*(\varrho+\delta)(1-a_1)}{s}\{\tfrac{s}{k}-[-1+\tfrac{a_1(1-a_2)}{a_2(1-a_1)}\tfrac{l}{s}]\tfrac{a_1-a_2}{k}\tfrac{1}{\Xi^*}\} > 0,$$

[29] Notice that $a_1 > a_2$ implies $\tfrac{a_1(1\text{-}a_2)}{a_2(1\text{-}a_1)}\tfrac{l}{s} > 1$.

$$J_{21}^{*}=\frac{(1+pc_1/c_2)}{\lambda^{*}}\{[(c_1+\delta k)(\frac{a_1}{s}+\frac{1-a_1}{l}l')+c_1\Psi]\frac{1}{\Xi^{*}}-\varepsilon c_1\}>0,$$

$$J_{22}^{*}=\frac{a_1-a_2}{k}[(c_1+\delta k)(\frac{a_1}{s}+\frac{1-a_1}{l}l')+c_1\Psi]\frac{1}{\Xi^{*}}+\frac{(a_1-a_2)c_1}{k}-(1-a_1)\delta \underset{<}{\overset{>}{}} 0$$

$$if\ \frac{a_1-a_2}{k}[(c_1+\delta k)(\frac{a_1}{s}+\frac{1-a_1}{l}l')+c_1\Psi]\frac{1}{\Xi^{*}}+\frac{(a_1-a_2)c_1}{k}\underset{<}{\overset{>}{}}(1-a_1)\delta.$$

1: $\theta_1=\theta_2=0$

In a standard two-sector growth model wherein $\theta_1=\theta_2=0$, the steady state is a saddle, which indicates that the dynamic system has one root with a negative real part and one root with a positive real part. The required condition is $Det(J^*)=J_{11}^*J_{22}^*-J_{21}^*J_{12}^*<0$.

2: the Case of $\theta_1\neq 0$ and $\theta_2=0$

The elements in the Jacobian matrix are:

$$J^{\theta_1}{}_{11}=-\frac{1+\theta_1(\varepsilon\text{-}1)}{B}\frac{\Xi^{*}}{\Xi^{*}+\Xi^{\theta_1}}J_{11}^{*},$$

$$J^{\theta_1}{}_{12}=\frac{\lambda}{\lambda^{*}}J_{12}^{*}+\frac{\lambda(\rho+\delta)(1-a_1)}{s}[1-\frac{a_1(1-a_2)}{a_2(1-a_1)}\frac{l}{s}]\frac{a_1-a_2}{k}\frac{1-D^{\theta_1}}{\Xi^{*}},$$

$$J^{\theta_1}{}_{21}=-\frac{[1+\theta_1(\varepsilon-1)]}{\lambda B}\frac{(1+pc_1/c_2)}{\Xi^{*}+\Xi^{\theta_1}}\{\frac{\lambda^{*}\Xi^{*}}{(1+pc_1/c_2)}J_{21}^{*}+\frac{[-\theta_1(\varepsilon-1)][1+p(c_1/c_2)\varepsilon/(\varepsilon-1)]c_1\Psi}{B}-\varepsilon c_1\Xi^{*}\}-\frac{\varepsilon c_1(1+pc_1/c_2)}{\lambda B},$$

$$J^{\theta_1}{}_{22}=D^{\theta_1}J_{22}^{*}+\frac{[-\theta_1(\varepsilon-1)][1+p(c_1/c_2)\varepsilon/(\varepsilon-1)]c_1}{B}[\frac{(a_1-a_2)\Psi D^{\theta_1}}{k\Xi^{*}}+\frac{a_2}{k}]+[\frac{(a_1-a_2)c_1}{k}-(1-a_1)\delta](1-D^{\theta_1}),$$

where $\Xi^{\theta_1}=-[\frac{1+\theta_1(\varepsilon-1)}{\varepsilon}\frac{1}{B}+\frac{1}{\varepsilon}]\Psi$ and $D^{\theta_1}=(1-\frac{a_2}{a_1-a_2}\frac{\Xi^{\theta_1}}{\Psi})\frac{\Xi^{*}}{\Xi^{*}+\Xi^{\theta_1}}$.

The steady state is a sink if the Jacobean matrix J has two roots with negative real parts, whose conditions are $Tr(J^{\theta_1})<0$ and $Det(J^{\theta_1})>0$. These conditions require (i) $-J^*_{11}>J^*_{22}$ and (ii) the slope condition:

$$\left.\frac{dc_1}{dk}\right|_{\dot{c}_1=0}=-\frac{J^{\theta_1}{}_{12}}{J^{\theta_1}{}_{11}}>\left.\frac{dc_1}{dk}\right|_{\dot{k}=0}=-\frac{J^{\theta_1}{}_{22}}{J^{\theta_1}{}_{21}}. \tag{B1}$$

Denote

$$a_1\equiv\frac{[1+p(c_1/c_2)(\varepsilon/\varepsilon-1)]\Xi^2}{(1+pc_1/c_2)^2}[Det(J^{*})+\Omega_1]-\frac{p(c_1/c_2)(1/(\varepsilon-1))\Xi^{*}\Psi}{(1+pc_1/c_2)^2}[Det(J^{*})+\Omega_1+(1+\frac{a_2}{a_1-a_2}\frac{\Xi^{*}}{\Psi})(\Omega_2-\Omega_3-\frac{J_{11}^{*}c_1(a_1-a_2)}{k}\frac{\Psi}{\Xi^{*}})],$$

$$b_1\equiv-\frac{[1+p(c_1/c_2)\varepsilon/(\varepsilon-1)]\Xi^2}{(1+pc_1/c_2)^2}\Omega_1-\frac{p(c_1/c_2)(1/(\varepsilon-1))\Xi^{*}\Psi}{(1+pc_1/c_2)^2}[\Omega_4-(1+\frac{a_2}{a_1-a_2}\frac{\Xi^{*}}{\Psi})\Omega_2]-\frac{[2+p(c_1/c_2)\varepsilon/(\varepsilon-1)]\Xi^2}{(1+pc_1/c_2)^2}\Omega_4,$$

where

$$\Omega_1=\frac{J_{12}^{*}c_1(1+pc_1/c_2)}{\lambda^{*}}(\varepsilon-\frac{\Psi}{\Xi^{*}})+\frac{J_{11}^{*}c_1(a_1-a_2)}{k}\frac{\Psi}{\Xi^{*}}(1+\frac{a_2}{a_1-a_2}\frac{\Xi^{*}}{\Psi}),$$

$$\Omega_2=\frac{J_{11}^{*}[(a_1-a_2)c_1-(1-a_1)\delta k-J_{22}^{*}k]}{k}-\frac{(\rho+\delta)(1-a_1)(a_1-a_2)}{sk\Xi^{*}}[1-\frac{a_1(1-a_2)}{a_2(1-a_1)}\frac{l}{s}][J_{21}^{*}\lambda^{*}-\varepsilon c_1(1+pc_1/c_2)],$$

$\Omega_3 = -(1+pc_1/c_2)(\varrho+\delta)(1-\alpha_1)[1-\frac{a_1(1-a_2)}{a_2(1-a_1)}\frac{l}{s}]\frac{a_1-a_2}{sk}\frac{c_1\Psi}{\Xi^{*2}}$,

$\Omega_4 = Det(J^*)+\frac{1}{\lambda^*}J_{12}^*\varepsilon c_1(1+pc_1/c_2)$.

Then, the relative slope condition that yields a sink is rewritten as

$$\varphi_1[-\theta_1(\varepsilon-1)]^2 + b_1[-\theta_1(\varepsilon-1)] + Det(J^*) > 0. \tag{B2}$$

If we set (B2) to equal zero, we obtain two critical values for $[-\theta_1(\varepsilon\text{-}1)]$, denoted by ζ_1 and ζ_2,

$$\zeta_1, \zeta_2 = \frac{1}{2\varphi_1}\{-b_1 \pm [(b_1)^2 - 4\varphi_1 Det(J^*)]^{1/2}\}.$$

Let $\zeta_1<\zeta_2$ when $\varphi_1>0$. Thus, when $\varphi_1<0$, $\zeta_1>\zeta_2$. Then, we have the following results.

(i) If $\varphi_1>0$, (B2) requires $[-\theta_1(\varepsilon\text{-}1)]<\zeta_1$ or $[-\theta_1(\varepsilon\text{-}1)]>\zeta_2$.

(ii) If $\varphi_1<0$, (B2) require $\zeta_2<[-\theta_1(\varepsilon\text{-}1)]<\zeta_1$.

If we combine Conditions *KUJ* and (B2), the required conditions of a steady state that is a sink are summarized as follows.

(i) $0<[-\theta_1(\varepsilon\text{-}1)]<\zeta_1$ or $[-\theta_1(\varepsilon\text{-}1)]>\max\{\zeta_2, 0\}$ if $\varphi_1>0$;

(ii) $\max\{\zeta_2, 0\}<[-\theta_1(\varepsilon\text{-}1)]<\zeta_1$ if $\varphi_1<0$.

3: the Case of θ_1=0 and $\theta_2\neq$0

In this case, the elements in the Jacobian matrix are:

$J^{\theta_2}{}_{11} = \frac{\Xi^*}{\Xi^*-\theta_2\Psi}J_{11}^*$,

$J^{\theta_2}{}_{12} = \frac{\lambda}{\lambda^*}J_{12}^* + \frac{\lambda(\varrho+\delta)(1-a_1)}{s}[1-\frac{a_1(1-a_2)}{a_2(1-a_1)}\frac{l}{s}]\frac{a_1-a_2}{k}\frac{1-D^{\theta_2}}{\Xi^*}$,

$J^{\theta_2}{}_{21} = \frac{1}{\lambda(\Xi^*-\theta_2\Psi)}[\Xi^*\lambda^*J_{21}^* + \theta_2 c_1(1+pc_1/c_2)(1-\varepsilon\Psi)]$,

$J^{\theta_2}{}_{22} = D^{\theta_2}J_{22}^* + \theta_2 c_1[\frac{(a_1-a_2)\Psi D^{\theta_2}}{k\Xi^*} + \frac{a_2}{k}] + [\frac{(a_1-a_2)c_1}{k} - (1-a_1)\delta](1-D^{\theta_2})$,

where $D^{\theta_2} = (1-\frac{a_2}{a_1-a_2}\frac{\Xi^{\theta_2}}{\Psi})\frac{\Xi^*}{\Xi^*+\Xi^{\theta_2}}$.

To obtain two roots with negative real parts, the conditions are $Tr(J^{\theta 2})=J^{\theta 2}{}_{11}+J^{\theta 2}{}_{22}<0$ and $Det(J^{\theta 2})=J^{\theta 2}{}_{11}J^{\theta 2}{}_{22}\text{-}J^{\theta 2}{}_{12}J^{\theta 2}{}_{21}<0$ which lead to the relative slope condition that gives a sink.

Denote

$\varphi_2 = \frac{\Psi^2}{(\varepsilon-1)(1+pc_1/c_2)^2}[(1+\frac{a_2}{a_1-a_2}\frac{\Xi^*}{\Psi})\frac{\Xi^*}{\Psi}\frac{\Omega_3}{(1+pc_1/c_2)} - \frac{J_{12}^*c_1}{\lambda^*}]$,

$b_2 \equiv \frac{\Xi^*\Psi}{(\varepsilon-1)(1+pc_1/c_2)^2}[Det(J^*) - \frac{J_{11}^*c_1(a_1-a_2)}{k} + \frac{J_{12}^*c_1(1+pc_1/c_2)(1-\varepsilon\Xi^*/\Psi)}{\lambda^*} + (1+\frac{a_2}{a_1-a_2}\frac{\Xi^*}{\Psi})(\Omega_2 + \frac{\varepsilon\Xi^*}{\Psi}\Omega_3) - \frac{a_2c_1J_{11}^*\Xi^*}{k\Psi}]$,

Then, the relative slope condition that gives a sink is rewritten as

$$\varphi_2[-\theta_2(\varepsilon-1)]^2 + b_2[-\theta_2(\varepsilon-1)] + Det(J^*) > 0. \tag{B3}$$

If we set (B3) to equal zero, we obtain the two critical values for $[-\theta_2(\varepsilon\text{-}1)]$, denoted by η_1 and η_2. et$\eta_1<\eta_2$ when $\varphi_2>0$. Thus, when $\varphi_2<0$, $\eta_1>\eta_2$. Then, we have the following results.

(i) If $\varphi_2>0$, (B3) requires $[-\theta_2(\varepsilon\text{-}1)]<\eta_1$ or $[-\theta_2(\varepsilon\text{-}1)]>\eta_2$.

(ii) If $\varphi_2<0$, (B3) require$\eta_2<[-\theta_2(\varepsilon\text{-}1)]<\eta_1$.

If we combine Conditions *KUJ* and (B3), the required conditions of a steady state that is a sink are summarized as

(i) $0<[-\theta_2(\varepsilon\text{-}1)]<\eta_1$ or $[-\theta_2(\varepsilon\text{-}1)]>\max\{\eta_2, 0\}$ if $\varphi_2>0$;

(ii) $\max\{\eta_2, 0\}<[-\theta_2(\varepsilon\text{-}1)]<\eta_1$ if $\varphi_2<0$.

4: the Case of $\theta_1=\theta_2=\theta\neq 0$

In the case, the elements in the Jacobian matrix are:

$$J^{\theta}{}_{11} = -\frac{1+\theta(\varepsilon\text{-}1)}{B}\frac{\Xi^*}{\Xi^*+\Xi^{\theta}}J^*_{11}, \quad J^{\theta}{}_{12} = \frac{\lambda}{\lambda^*}J^*_{12} + \frac{\lambda(\varrho+\delta)(1-\alpha_1)}{s}[1-\frac{a_1(1-a_2)}{a_2(1-a_1)}\frac{l}{s}]\frac{a_1-a_2}{k}\frac{1-D^{\theta}}{\Xi^*},$$

$$J^{\theta}{}_{21} = -\frac{[1+\theta(\varepsilon-1)]}{\lambda B}\frac{(1+pc_1/c_2)}{\Xi^*+\Xi^{\theta}}\{\frac{\lambda^*\Xi^*}{(1+pc_1/c_2)}J^*_{21} + \frac{[-\theta_1(\varepsilon-1)][1+p(c_1/c_2)]\varepsilon/(\varepsilon-1)c_1\Psi}{B} - \varepsilon c_1\Xi^*\} - \frac{\varepsilon c_1(1+pc_1/c_2)}{\lambda B},$$

$$J^{\theta}{}_{22} = D^{\theta}J^*_{22} + \frac{[-\theta_1(\varepsilon-1)][1+p(c_1/c_2)]\varepsilon/(\varepsilon-1)c_1}{B}[\frac{(a_1-a_2)\Psi D^{\theta}}{k\Xi^*} + \frac{a_2}{k}] + [\frac{(a_1-a_2)c_1}{k} - (1-a_1)\delta](1-D^{\theta}),$$

where $\Xi^{\theta} = -\frac{1+\theta(\varepsilon-1)}{\varepsilon}[1+\frac{1-\theta}{B}]\Psi$ and $D^{\theta_1} = (1-\frac{a_2}{a_1-a_2}\frac{\Xi^{\theta}}{\Psi})\frac{\Xi^*}{\Xi^*+\Xi^{\theta}}$.

Denote

$$\varphi_3 \equiv \frac{\Xi^*\Psi}{(\varepsilon-1)(1+pc_1/c_2)}[\Omega_4 + (1+\frac{a_2\Xi^*}{a_1-a_2})\Omega_2],$$

$$b_3 \equiv \frac{\Xi^{*2}}{(1+pc_1/c_2)^2}[(1+p\frac{c_1}{c_2}\frac{\varepsilon}{\varepsilon-1})\Omega_4 - (1+\frac{a_2\Xi^*}{a_1-a_2})\frac{\varepsilon\Omega_3(1+pc_1/c_2)}{\Psi}] + \frac{\Xi^*\Psi\varepsilon c_1}{(\varepsilon-1)(1+pc_1/c_2)}[(1+\frac{a_2\Xi^*}{a_1-a_2})\frac{J^*_{11}(a_1-a_2)}{k} + \frac{J^*_{12}(1+pc_1/c_2)}{\lambda^*}],$$

$$d_3 \equiv \frac{\Xi^{*2}(1+p\frac{c_1}{c_2}\frac{\varepsilon}{\varepsilon-1})}{(1+pc_1/c_2)}\frac{J^*_{12}\varepsilon c_1}{\lambda^*} - \frac{\Xi^{*2}(2+p\frac{c_1}{c_2}\frac{\varepsilon}{\varepsilon-1})}{(1+pc_1/c_2)^2}\Omega_4 + \frac{\Xi^*\Psi}{(\varepsilon-1)(1+pc_1/c_2)}[Det(J^*) + (1+\frac{a_2\Xi^*}{a_1-a_2})(\Omega_2 + \frac{\varepsilon\Xi^*\Omega_3}{\Psi} - \frac{J^*_{11}\varepsilon c_1(a_1-a_2)}{k})],$$

The relative slope condition that yields a sink is rewritten as

$$\varphi_3[-\theta(\varepsilon-1)]^3 + b_3[-\theta(\varepsilon-1)]^2 + d_3[-\theta(\varepsilon-1)] + Det(J^*) > 0. \tag{B4}$$

Define $\varphi=b_3/\varphi_3$, $b=d_3/\varphi_3$, and $d_3=Det(J^*)/\varphi_3$. If we set (B4) equal 0, we obtain three critical values of $[-\theta(\varepsilon\text{-}1)]$: $m+n\text{-}\varphi/3$, $m\omega+n\omega^2\text{-}\varphi/3$and $m\omega^2+n\omega\text{-}\varphi/3$, in which

$$m = \frac{1}{3}\sqrt[3]{-\frac{27d-9\varphi b+2\varphi^3}{2} + \sqrt{(\frac{27d-9\varphi b+2\varphi^3}{2})^2 + (3b-\varphi^2)^3}},$$

$$n = \frac{1}{3}\sqrt[3]{-\frac{27d-9\varphi b+2\varphi^3}{2} - \sqrt{(\frac{27d-9\varphi b+2\varphi^3}{2})^2 + (3b-\varphi^2)^3}},$$

$$\omega = \frac{-1+\sqrt{3}i}{2}.$$

Denote the three critical values as ι_1, ι_2 and ι_3. Let ι_3 be the largest and ι_1 be the smallest value of the three critical values. Then, the required conditions of a steady state that is a sink are summarized as

(i) $\max\{0, \iota_1\}<[-\theta(\varepsilon\text{-}1)]<\max\{0, \iota_2\}$ *or* $[-\theta(\varepsilon\text{-}1)]>\max\{0, \iota_3\}$, *if* $\varphi_3>0$;

(ii) $0<[-\theta(\varepsilon\text{-}1)]<\max\{0, \iota_1\}$ *or* $\max\{0, \iota_2\}<[-\theta(\varepsilon\text{-}1)]<\max\{0, \iota_3\}$, *if* $\varphi_3<0$.

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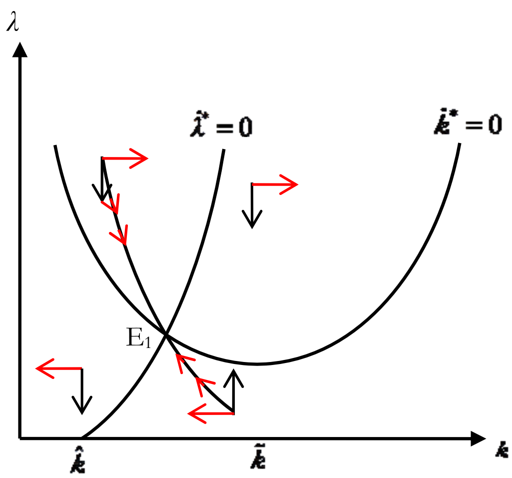

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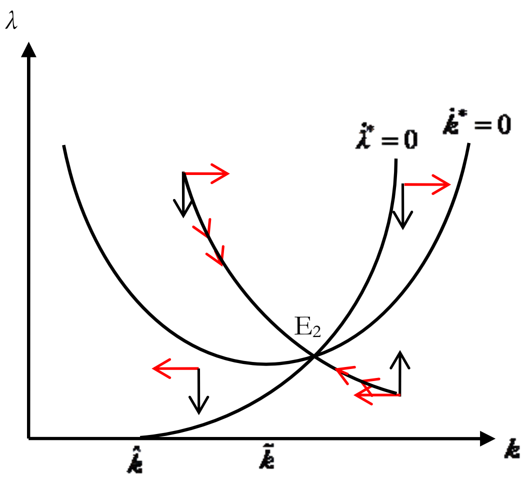

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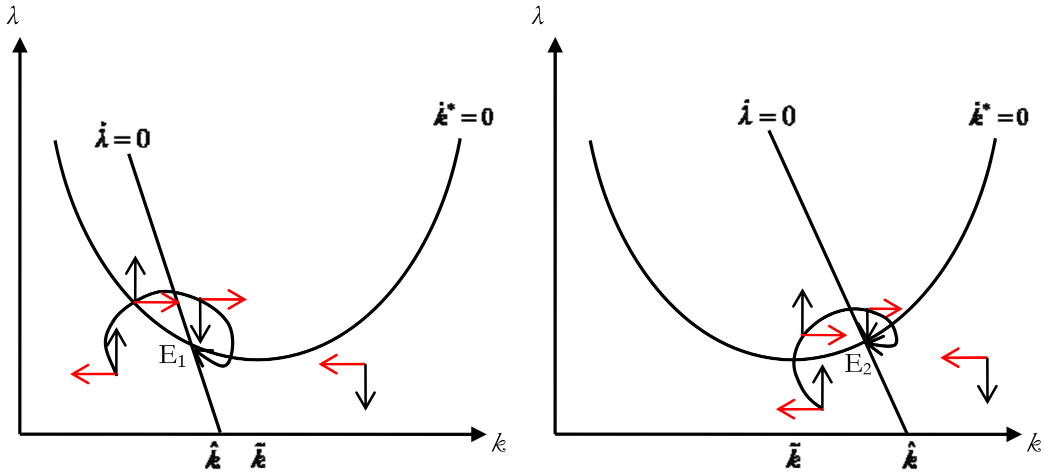

B1 **B2**

Figure 1: Two-sector Growth Model with a Sufficiently Large *KUJ* Effect: a Sink

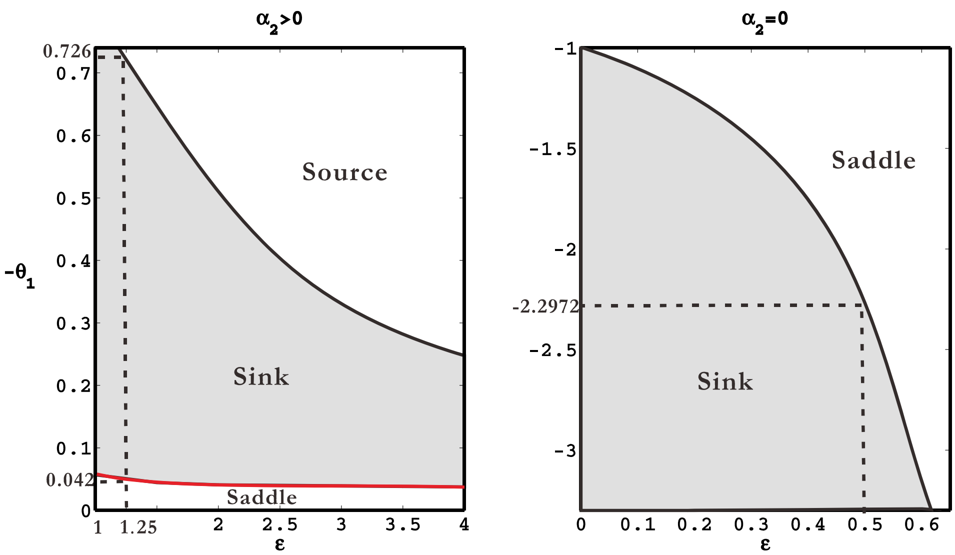

Figure 2: $\theta_1 \neq 0$

(Baseline parameters: a_1=0.32, a_2=0.28, δ=0.05, ρ=0.04, γ=0.1654, θ_1=θ_2=0.)

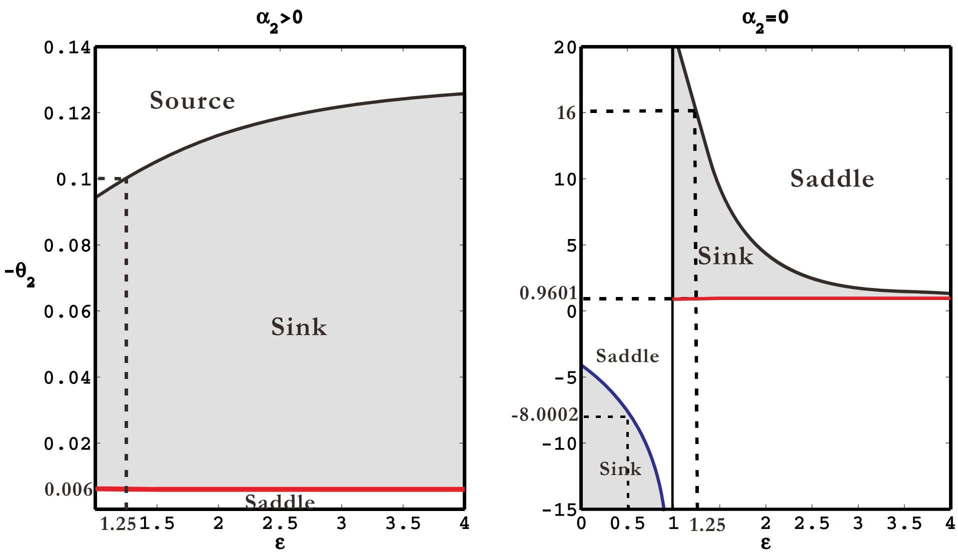

Figure 3: $\theta_2 \neq 0$

(Baseline parameters: same as Figure 2.)

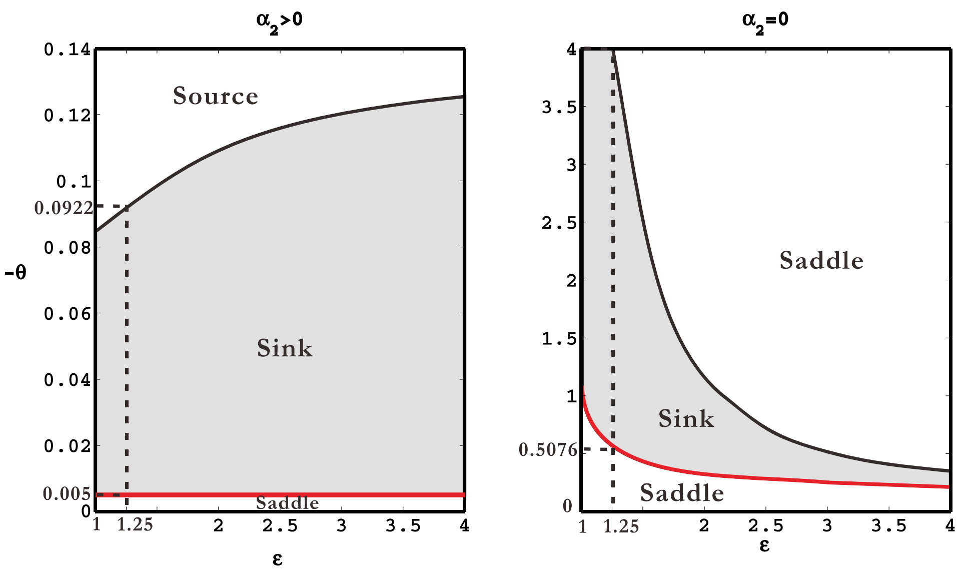

Figure 4: $\theta_1 = \theta_2 = \theta \neq 0$

(Baseline parameters: same as Figure 2.)

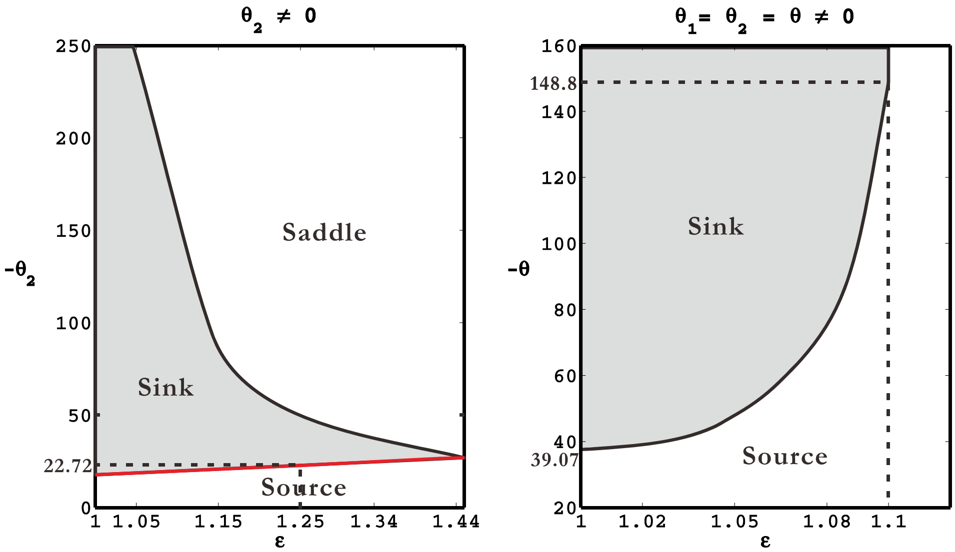

Figure 5: Factor Intensity Reversal

(Baseline parameters: a_1=0.28, a_2=0.32, δ=0.05, ϱ=0.04, γ=0.1654, θ_1=θ_2=0.)