# Capital, Credit Constraints and the Comovement between Consumer Durables and Nondurables*


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**Abstract**

Evidence indicates that consumer durables are more flexibly priced than nondurable goods and services. In otherwise standard two-sector neoclassical sticky-price models with flexible durable prices, following monetary tightening, nondurables decrease but consumer durables increase. Friction in lending between households can resolve the comovement problem if durable prices are sticky. However, if durable prices are flexible, friction in lending fails to generate joint decline. This paper resolves the co-movement problem by adding capital into a model with flexible durable prices and friction in lending. When capital is needed in production, monetary tightening reduces the relative price of durables which induces investment and decreases firms' real profits in the short run. Due to fewer profits remitted from firms, savers have a lower disposable income and cannot increase expenditures on consumer durables as much as otherwise. As a consequence, aggregate consumer durables decrease and there is a joint decline of nondurables and consumer durables.



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## 1. Introduction

Empirical evidence documented by Barsky et al. (2003) and Erceg and Levin (2006) indicates that expenditures on nondurables and consumer durables comove closely. However, in otherwise standard two-sector, neoclassical New Keynesian models calibrated to U.S. data, where nondurable goods have sticky prices and durable goods have flexible prices, as documented by Bils and Klenow (2004) and others, Barsky et al. (2003, 2007) found that in response to monetary tightening, it appears that nondurables decrease but consumer durables increase, thus causing the inconsistent comovement problem between nondurables and durables. This comovement puzzle is robust, whether monetary policy is assumed to follow a money supply rule, as in Barsky et al. (2003, 2007), or an interest rate rule, as in Carlstrom and Fuerst (2010).

To reconcile the inconsistency between the empirical findings and the model-implied responses, Barsky et al. (2003) suggested introducing friction in lending.[1] With frictional financial markets, monetary tightening raises nominal interest rate and causes the severity of borrowing constraints that is negatively related to cash flows or collateral, and agents who face binding credit constraints may spend less income on consumer durables even if durable goods become relatively cheaper. Monacelli (2009) formalized the idea in a model with sticky nondurable prices wherein impatient households borrow from patient households and are subject to collateral constraints. Monacelli's model can resolve the comovement problem when durable prices have some degrees of stickiness. However, the comovement puzzle is not resolved when durable prices are flexible, because relatively cheaper durables make savers increase expenditures on consumer durables so much so that aggregate consumer durables remain increase. Moreover, in an otherwise identical model, Sterk (2010) found that the presence of collateral constraints increases the purchase of consumer durables more than that in the absence of collateral constraints, indicating the Monacelli model with collateral constraints being more difficult to generate the joint decline of durables and nondurables.

The upshot is that with sticky durable prices, the Monacelli model with collateral constraints produces the joint decline, but with flexible durable prices, the Monacelli model with collateral constraints cannot yield the joint decline. The purpose of this paper is to show that if capital is added into the Monacelli model with collateral constraints and flexible durable prices, the comovement problem can be resolved. The result emerges because, with flexible durable prices, monetary tightening reduces the relative price of durables. By way of increases in investment, a lower relative price of durables reduces firms' real profits in the short run. With fewer profits remitted from firms, savers, thus owners of firms, have a lower disposable income and cut their expenditures on

[1] Barsky et al. (2003) also proposed that the introduction of friction in the price setting behavior such as sticky wages and complementarities such as consumption habits may resolve such an inconsistency.

consumer durables largely so that aggregate expenditure on consumer durables falls. There is thus a joint decline of nondurables and consumer durables.

It should be noted that in studying the comovement between durables and nondurables, Tsai (2010) introduced a setting where goods producers borrow to finance their payment to production inputs at the beginning of a period. Since a monetary contraction shock is nothing but a positive innovation in the interest rate that firms pay to banks, the nominal marginal cost of durable goods is affected by the sluggish time path of a policy rate which leads to weaker substitution effects. Other than consumers being borrowers in our model and producers being borrowers in the Tsai model, there are two essential differences. First, the mechanisms of borrowing constraints work differently. In Tsai, the borrowing constraint acts to *increase* the cost and thus the durable price which causes less durable production. By contrast, in our model with borrowing constraints, durable prices *decrease* which is consistent with the data.[2] A lower durable price induces investment and reduces firms' profits remitted to savers, so savers decrease expenditures on consumer durables. Next, with our borrowing constraints on consumers, durable goods expenditures are more sensitive to the interest rate than spending on nondurables which is consistent with empirical observation but, if there is no habit formation, Tsai's borrowing constraint on firms cannot generate durable goods expenditures that are more interest-sensitive than nondurables.

Our study complements recent studies which resolved the comovement puzzle using alternative mechanisms. These mechanisms include the input-output linkage between the durable and nondurable sectors (e.g., Sudo, 2012) and its interaction with labor immobility across the two sectors (e.g., Bouakez et al., 2011). In this line of research, the key channel to resolve the comovement puzzle is that nondurable goods are inputs into the durable sector. As a result, the price stickiness of the nondurable sector moderates the decline of the marginal cost in the durable sector and hence, its price decrease which weakens the substitution effect and resolves the comovement problem. Although durable investment goods are inputs into the nondurable sector in our paper, nondurable goods are not input in the durable sector and thus the substitution effect is not weakened. Instead, the key mechanism acts through a firm's increases in investment which lower profits remitted to households and strengthen the income effect.

The second mechanism involves sticky nominal wages as the main channel to resolve the comovement problem. Carlstrom and Fuerst (2010) showed that the co-movement between housing and nondurable consumption can arise under sticky nominal wages, adjustment costs in housing construction, and a large degree of complementarity between the consumption of housing services

[2] Using Romer dates as indicators of distinct monetary tightening, Barsky et al. (2003) found that the price of new houses relative to the consumer price index for nondurables fell by 12% and the relative price of cars fell by more than 6% after a Romer date. The price of durables relative to nondurables fell by 4.8% following a Romer date.

and nondurable goods, whereas their model indicated that habit formation in consumption helps to move the volatility of nondurable production relative to residential investment closer to that in the data. Finally, the third mechanism includes preference-related contributions to resolve the co-movement problem. Kim and Katayama (2010) examined the implications for sectoral co-movement of the non-separability between consumption and leisure, imperfect capital mobility, and variable capacity utilization. Huang et al. (2013) resolved the co-movement problem by exploring the dynamic interaction between habit formation and non-homothetic preferences on durable and non-durable consumption goods, of which the former weakens the role of the substitution effect and the latter enhances the role of the income effect in shaping the composition of preferred consumption bundles. Through decreases in profits which strengthen the income effect, the primary mechanism in our paper is different from those of sticky nominal wages and preference-related contributions.

This paper is organized thus. In Section 2, we set up basic two-sector sticky-price models. In Section 3, we calibrate the model and envisage the impulse responses of a monetary tightening. Finally, concluding remarks are offered in Section 4.

## 2. Three Sticky-price Models with and without Collateral Constraints and Capital

Three neoclassical sticky-price models with consumer durables and nondurables are analyzed. We start with our model; the Monacelli (2009) model; and, the Sterk (2010) model.

Our model is similar to the Monacelli (2009) model except for the inclusion of capital. The economy includes two sectors, the durable and nondurable sectors. Each sector has a continuum of firms which produce and sell final goods at competitive prices and has a continuum of producers which produce and sell intermediates at monopolistic prices. Nondurable final goods are used as consumption, but unlike Monacelli (2009), durable final goods can be used either for consumption or for investment. The economy consists of a continuum of households with a unit mass. There are two types of households, named borrowers and savers, of the measures $\varpi$ and 1-$\varpi$, respectively. Savers have a lower time preference rate than borrowers. Households with different time preference rates trade nominal private debts, with borrowers being subject to collateral constraints. Both savers and borrowers supply labor elastically and consume both nondurables and consumer durables.

### 2.1 Households

A typical household consumes an index of consumption $X_t$ defined as

$$X_t \equiv [(1-\mu)^{\frac{1}{\eta}}(C_t)^{\frac{\eta-1}{\eta}} + \mu^{\frac{1}{\eta}}(D_t)^{\frac{\eta-1}{\eta}}]^{\frac{\eta}{\eta-1}},$$

where $C_t$ is nondurables and $D_t$ is services from the stock of consumer durables. The parameter $\mu$>0 is the share of durable services in consumption, and $\eta \geq 0$ is the elasticity of substitution between durable services and nondurables.

The *borrower* maximizes the following expected lifetime utility

$$E_0 \left\{ \sum_{t=0}^{\infty} (\beta)^t (\log X_t - \nu \tfrac{(L_t)^{1+\phi}}{1+\phi}) \right\}, \tag{1}$$

where $E_t$ is conditional expectations at any given period $t$. The borrower's discount factor $\beta \in (0, 1)$ is smaller than a saver's discount factor $\gamma \in (0, 1)$. $L_t$ is the hours of work. Parameter $\nu > 0$ is the coefficient associated with the disutility of work, and $\phi > 0$ is the inverse of the Frisch labor supply elasticity.

In each period $t$, the borrower faces the following budget and collateral constraints.

$$C_t + p_t[D_t - (1-\delta)D_{t-1}] + R_{t-1} \tfrac{b_{t-1}}{\pi_{c,t}} = b_t + w_t L_t + \tfrac{T_t}{P_{c,t}}, \tag{2a}$$

$$R_t b_t \leq m(1-\delta) E_t (p_{t+1} D_t \pi_{c,t+1}) = m(1-\delta) p_t E_t (D_t \pi_{d,t+1}), \tag{2b}$$

where $P_{c,t}$ is the price of nondurables, $P_{d,t}$ is the price of durables and $p_t \equiv P_{d,t}/P_{c,t}$ is the relative price of durables, with $\pi_{c,t} \equiv P_{c,t}/P_{c,t\text{-}1}$ and $\pi_{d,t} \equiv P_{d,t}/P_{d,t\text{-}1}$ being the gross inflation of nondurable goods and durable goods, respectively. $B_t$ is end-of-period $t$ nominal debts and $W_t$ is nominal wage, with $b_t \equiv B_t/P_{c,t}$ and $w_t \equiv W_t/P_{c,t}$ being real debt and wage, respectively. $T_t$ is lump-sum transfers, $R_{t\text{-}1}$ is the gross nominal interest rate on a loan between $t$-1 and $t$, and $\delta$ is the depreciation rate of consumer durables. The collateral constraint in (2b) states that debt servicing at the end of period $t$ cannot exceed a fraction $m$ of the expected value of the depreciated consumer durables one period ahead. Hence, (1-$m$) can be interpreted as a down payment requirement, and $m$ is thus a borrower's "loan-to-value" ratio. Like Monacelli (2009), we assume that the collateral constraint always binds.

Let $\xi_t$ and $\xi_t \psi_t$ be the current-valued Lagrange multipliers of constraints (2a) and (2b), respectively. We denote $U_{C,t}$, $U_{D,t}$ and $U_{L,t}$, respectively, as the marginal utility of nondurables, consumer durables, and labor in $t$. The first-order conditions for $C_t$, $L_t$, $D_t$ and $b_t$ are

$$U_{C,t} = \xi_t, \tag{3a}$$

$$\tfrac{-U_{L,t}}{U_{C,t}} = w_t, \tag{3b}$$

$$p_t U_{C,t} = U_{D,t} + \beta(1-\delta) E_t (p_{t+1} U_{C,t+1}) + m(1-\delta) \psi_t p_t U_{C,t} E_t (\pi_{d,t+1}), \tag{3c}$$

$$R_t \psi_t = 1 - \beta E_t \left( \tfrac{U_{C,t+1}}{U_{C,t}} \tfrac{R_t}{\pi_{c,t+1}} \right), \tag{3d}$$

along with $\lim_{t\to\infty}(\beta)^t \xi_t D_t = 0$ and $\lim_{t\to\infty}(\beta)^t \xi_t \psi_t b_t = 0$ which ensure that there is no "Ponzi scheme."

These conditions are the same as those obtained in Monacelli (2009). In particular, (3d) is a modified Euler equation. If the shadow price of borrowing constraints $\psi_t$ equals zero, (3d) is reduced to the standard Euler condition. However, if $\psi_t>0$, (3d) suggests that $\beta E_t(U_{C,t+1}R_t/\pi_{c,t+1})<U_{C,t}$, and thus the discounted expected marginal utility of shifting a unit of nondurables to the next period is less than the marginal utility of nondurables today. Condition (3c) indicates that the demand for consumer durables is determined by the equalization of the real marginal cost and the real marginal benefit from a marginal unit of consumer durables. The real marginal cost is the relative price of durables in terms of the marginal utility of nondurables. The real marginal gain is the shadow value of consumer durables to the borrower which is the immediate marginal utility and two marginal gains. One of the marginal gains is the discounted expected value of the undepreciated part of consumer durables in the next period (the second term), and the other marginal gain is the marginal utility of relaxing collateral constraints (the third term).

The utility of the *saver* is the same as (1) with the discount factor being replaced by $\gamma$. Savers have otherwise the same budget constraint (2a) as borrowers except for nominal profits/dividends $F_t$ remitted from firms as they are endowed with the ownership of firms.[3]

$$C_t^* + p_t[D_t^* - (1-\delta)D_{t-1}^*] + R_{t-1}\tfrac{b_{t-1}^*}{\pi_{c,t}} = b_t^* + w_t L_t^* + \tfrac{T_t^*}{P_{c,t}} + \tfrac{F_t/(1-\varpi)}{P_{c,t}}. \tag{4}$$

The saver maximizes the expected lifetime utility subject to (4). The first-order conditions for $C_t^*$, $L_t^*$, $D_t^*$ and $b_t^*$ are the same as (3a)-(3d), respectively, except for $\psi_t^*=0$ and $\beta$ being replaced by $\gamma$.

## 2.2 Firms

In each sector, a perfectively competitive *final goods* producer operates the production function $Y_{j,t} = [\int_0^1 (Y_{j,t}(z))^{(\varepsilon_j-1)/\varepsilon_j} dz]^{\varepsilon_j/(\varepsilon_j-1)}$, $j$=$c$, $d$, where $Y_{j,t}(z)$ is the intermediate good $z$ used in sector $j$ and $\varepsilon_j>1$ is the elasticity of substitution between intermediates. Maximization of profits in sector $j$ gives the demand for intermediate $z$.

$$Y_{j,t}(z) = (\tfrac{P_{j,t}(z)}{P_{j,t}})^{-\varepsilon_j} Y_{j,t},\ z\in(0,1),\ j=c,\ d, \tag{5a}$$

where $P_{j,t}(z)$ is the price of the intermediate $z$ in sector $j$. A zero profit of final goods in sector $j$ implies $P_{j,t} = [\int_0^1 (P_{j,t}(z))^{1-\varepsilon_j} dz]^{1/(1-\varepsilon_j)}$.

In each sector $j$, a continuum of *intermediate producers* of a unit mass, indexed by $z\in(0,1)$, uses capital and labor to produce intermediate goods. The production technology is

$$Y_{j,t}(z) = A_j K_{j,t}(z)^{\alpha_j} L_{j,t}(z)^{1-\alpha_j},\ j=c,\ d, \tag{5b}$$

where $\alpha_j\in(0,1)$ is the capital share and $A_j>0$ is a coefficient.

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[3] As savers have a lower discount rate than borrowers, eventually they own all wealth in the economy. The simplest method is to assume that savers are endowed with the ownership of firms. A superscripted asterisk is used to differentiate variables for savers from those for borrowers.

An intermediate producer $z$ in sector $j$ sells intermediates to final goods producers in sector $j$. Following Monacelli (2009), in setting its monopolistic price $P_{j,t}(z)$, an intermediate producer faces the adjustment cost $\Theta(P_{j,t}(z)) = \frac{\vartheta_j}{2}(\frac{P_{j,t}(z)}{P_{j,t-1}(z)} - 1)^2 P_{j,t} Y_{j,t}$, where $\vartheta_j$ measures the degree of nominal rigidities in sector $j$.[4] In each period, the representative intermediate firm has to decide how much capital to invest, how much labor to hire and what prices to set. Following Jermann (1998), managers of the firm maximize the value to its owners (savers) which is the present discounted value of all current and future expected cash flows.

$$E_0 \sum_{t=0}^{\infty} (\frac{\gamma^t U_{C,t}^*}{U_{C,0}^*})[P_{j,t}(z) Y_{j,t}(z) - W_t L_{j,t}(z) - P_{d,t} I_{j,t}(z) - \Theta(P_{j,t}(z))],\ j{=}c,\ d, \tag{6a}$$

with the discount factor $\frac{\gamma^t U_{C,t}^*}{U_{C,0}^*}$ being the marginal rate of substitution of the owners.

The firm's capital stock obeys the following intertemporal accumulation equation.

$$K_{j,t+1}(z) = I_{j,t}(z) + (1-\delta_K) K_{j,t}(z), \tag{6b}$$

where $\delta_K$ is the depreciation rate of capital.

Give the technology (5b), managers choose $\{K_{j,t+1}(z),\ L_{j,t}(z),\ P_{j,t}(z)\}_{t=0}^{\infty}$ to maximize the cash flows in (6a) subject to the demand for intermediates in (5a) and the accumulation of capital in (6b).

Let $\lambda_{j,t}(z)$ denote the current-valued Lagrange multiplier of the demand for intermediate $z$ in (5a), $j$=$c$, $d$. Denote $MP_{j,t}^{\ L}$ and $MP_{j,t}^{\ K}$ as the marginal product of labor and capital in period $t$, respectively. The first-order conditions for $L_{j,t}(z)$, $P_{j,t}(z)$ and $K_{j,t+1}(z)$, $j$=$c$, $d$, are

$$w_t = \lambda_{j,t}(z) \frac{P_{j,t}(z)}{P_{c,t}} MP_{j,t}^L, \tag{7a}$$

$$\left\{[1-(1-\lambda_{j,t}(z))\varepsilon_j](\frac{P_{j,t}(z)}{P_{j,t}})^{-\varepsilon_j} - \vartheta_j(\pi_{j,t}(z)-1)\frac{P_{j,t}}{P_{j,t-1}(z)} + \lambda_{j,t}(z)[\frac{Y_{j,t}(z)}{Y_{j,t}} - (\frac{P_{j,t}(z)}{P_{j,t}})^{-\varepsilon_j}]\right\} Y_{j,t} + E_t[(\frac{\gamma U_{C,t+1}^*}{U_{C,t}^*})\vartheta_j(\pi_{j,t+1}(z)-1)\frac{\pi_{j,t+1}(z)}{\pi_{c,t+1}}\frac{P_{j,t+1}}{P_{j,t}(z)} Y_{j,t+1}] = 0, \tag{7b}$$

$$p_t U_{C,t}^* = \gamma E_t \left\{[\lambda_{j,t+1}(z) \frac{P_{j,t+1}(z)}{P_{c,t+1}} MP_{j,t+1}^K + p_{t+1}(1-\delta_K)] U_{C,t+1}^*\right\}, \tag{7c}$$

where $\pi_{j,t}(z) \equiv P_{j,t}(z)/P_{j,t-1}(z)$ is the gross inflation of intermediate $Y_{j,t}(z)$ in sector $j$.

While the conditions in (7a) and (7b) are the same as those in Monacelli (2009), (7c) is new. The last condition equates the real marginal cost of a marginal unit of capital to the real marginal revenue. The real marginal revenue is the discounted sum of the expected real marginal product of capital and the relative price of durables of undepreciated capital in terms of the marginal utility of nondurables in the next period. The real marginal cost is the relative price of durables in terms of the marginal utility of nondurables in this period. Notice that the real cost of capital is the same as the real cost of

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[4] The quadratic cost of price adjustment is based on Rotemberg (1982). An alternative method of price adjustment is random price durations based on Calvo (1983). According to Galí (2008, p.41), these two methods generate the same inflation dynamics.

a marginal unit of consumer durables (cf. (3c) for savers) since durables goods may be used as either capital or consumer durables. Due to such a linkage, the demand for capital is expected to affect savers' expenditure on consumer durables as analyzed in the next section.

By imposing the symmetry conditions of $K_{j,t}(z)=K_{j,t}$, $L_{j,t}(z)=L_{j,t}$, $Y_{j,t}(z)=Y_{j,t}$, $P_{j,t}(z)=P_{j,t}$ and $\lambda_{j,t}(z)=\lambda_{j,t}$ for all $z$, $j=c$, $d$, (7a)-(7c) give, respectively,

$$w_t = \lambda_{j,t} \tfrac{P_{j,t}}{P_{c,t}} MP^L_{j,t}, \tag{8a}$$

$$\tfrac{1}{\lambda_{j,t}} = \tfrac{\varepsilon_j}{\varepsilon_j - \Omega_{j,t}}, \tag{8b}$$

$$p_t U^*_{C,t} = \gamma E_t \left\{ [\lambda_{j,t+1} \tfrac{P_{j,t+1}}{P_{c,t+1}} U^*_{C,t+1} MP^K_{j,t+1} + p_{t+1} U^*_{C,t+1}(1-\delta_K)] \right\}, \tag{8c}$$

where $\Omega_{j,t} \equiv 1 - \vartheta_j(\pi_{j,t}-1)\pi_{j,t} + E_t[(\tfrac{\gamma U^*_{C,t+1}}{U^*_{C,t}})\vartheta_j(\pi_{j,t+1}-1)\tfrac{(\pi_{j,t+1})^2}{\pi_{c,t+1}}\tfrac{Y_{j,t+1}}{Y_{j,t}}]$, $j=c$, $d$.

The multiplier $\lambda_{j,t}$ stands for the real marginal cost of intermediates, and its inverse is the markup over the marginal cost. Since the durable prices are flexible, $\vartheta_d=0$ and thus $\Omega_{d,t}=1$ for all $t$. In this case, the markup $1/\lambda_{d,t}=\varepsilon_d/(\varepsilon_d-1)$ is constant for all $t$. Moreover, in a steady state, $\pi_{j,t}=\pi_{j,t+1}=1$ and $\Omega_{j,t}=\Omega_{j,t+1}=1$ for all intermediate prices. Then, the markup $1/\lambda_j=\varepsilon_j/(\varepsilon_j-1)$ is constant in a steady state.

### 2.3 Equilibrium

In equilibrium, the nondurable and the durable final goods markets are clear.

$$Y_{c,t} - \tfrac{\vartheta_c}{2}(\pi_{c,t}-1)^2 Y_{c,t} = \bar{C}_t, \tag{9a}$$

$$Y_{d,t} - \tfrac{\vartheta_d}{2}(\pi_{d,t}-1)^2 Y_{d,t} = [\bar{D}_t - (1-\delta)\bar{D}_{t-1}] + [K_{t+1} - (1-\delta_K)K_t], \tag{9b}$$

where $\bar{C}_t = \varpi C_t + (1-\varpi)C^*_t$ is aggregate nondurables and $\bar{D}_t = \varpi D_t + (1-\varpi)D^*_t$ is the stock of aggregate consumer durables. We abstract from redistribution via the fiscal policy and hence $T^*_t=T_t=0$. Moreover, the capital, the labor and the debt markets are all clear.

$$K_t = K_{c,t} + K_{d,t}, \tag{10a}$$

$$L_{c,t} + L_{d,t} = \varpi L_t + (1-\varpi)L^*_t, \tag{10b}$$

$$\varpi b_t + (1-\varpi)b^*_t = 0. \tag{10c}$$

The model is closed by the following monetary policy rule.

$$\tfrac{R_t}{R} = (\tfrac{\pi_t}{\pi})^{\chi} \exp(\varsigma_t), \quad \chi>1, \tag{11a}$$

where $\pi_t \equiv (\pi_{c,t})^{1-\mu}(\pi_{d,t})^{\mu}$ is a composite inflation index with the weight for durable goods inflation being the share of consumer durables in consumption $\mu$, and $R$ and $\pi$ are steady-state values. We assume that $\chi$ is sufficiently large in order to ensure equilibrium determinacy. Policy parameter $\varsigma_t$ is a shock which evolves according to

$$\exp(\varsigma_t) = \exp(\varsigma_{t-1})^{\rho} u_t, \quad 0<\rho<1, \tag{11b}$$

where the innovation $u_t$ is assumed to be independent and identically distributed.

### 2.4 The model with credit constraints without capital

The Monacelli (2009) model is otherwise the same as our model except for no capital. As a result, firms cannot accumulate capital. In the Monacelli model, the production function (5b) is linear in labor and is modified as $Y_{j,t}(z)=A_jL_{j,t}(z)$, $j=c, d$. Moreover, (6b), (8c) and (10a) are not equilibrium conditions. Equilibrium conditions (8a), (8b) and (9b) involve capital and are also modified.

### 2.5 The model without credit constraints

The Sterk (2010) model underlines the absence of credit constraints. The model is otherwise the same as our model except for setting the fraction of borrowers equal to zero, $\varpi=0$. Thus, debts equal zero in equilibrium ($b_t^*=b_t=0$) and collateral constraints are irrelevant ($\psi_t=0$). Moreover, (2a)-(2b), (3d) and (10c) are not equilibrium conditions. Condition (3c) is modified by replacing $\beta$ by $\gamma$ and setting $\psi_t=0$ and (9a)-(9b) and (10b) are modified by setting $\varpi=0$.

## 3. Comparisons of the Three Models

In this section, we study the effects of monetary tightening on the impulse responses of nondurables, consumer durables, capital and other variables.

### 3.1 Calibration

The time frequency is quarters. The subsequent parameter values follow from Monacelli (2009). The fraction of borrowers is $\varpi=1/2$, the TFPs in the nondurable and the durable sectors are $A_c=A_d=1$, the elasticity of substitution between nondurables and consumer durables is $\eta=1$, the elasticity of substitution in the intermediate production is $\varepsilon_c=\varepsilon_d=6$, the share of consumer durables in consumption is $\mu=0.2$, the inverse of the Frisch labor supply elasticity is $\phi^*=\phi=1$,[5] the discount factor is $\gamma=0.99$ for savers and $\beta=0.98$ for borrowers, the borrower's loan-to-value ratio is $m=0.75$, the depreciation rate of consumer durables is $\delta=0.01$, hours of work are $L^*=L=1/3$, the autocorrelation of error to the interest rate is $\rho=0.5$, and the coefficient of the inflation in the policy rule is $\chi=1.5$.

For production, we follow Acemoglu and Guerrieri (2008) and set the capital shares of the durable and nondurable sectors equal to $\alpha_d=0.27$ and$\alpha_c=0.47$, respectively, to match with the average

[5] The results are robust under $\phi$, $\phi^*\in[0.01, 100]$.

capital share in the respective sector in 1987-2005. We follow Hansen (1985) and set the rate of depreciation of capital equal to $\delta_K$=0.025 which implies a 10% annual depreciation rate.[6] For the preference, with $L^*$=$L$=1/3, we use the consumption-leisure tradeoff condition for borrowers in (3b) to calibrate the parameter of leisure in preference at $\nu$=8.602. In a similar fashion, we use the corresponding condition for savers to calibrate $\nu^*$=3.797. Finally, for the degree of price-stickiness, we set $\vartheta_d$=0 so durable prices are flexible. We target the stickiness of nondurable prices at four quarters and pin down $\vartheta_c$=58.25.[7]

Parameter values in the benchmark model are summarized in Table 1. In the Table, we also list parameter values used in the Monacelli model where $\alpha_c$=$\alpha_d$=$\delta_K$=0 and the Sterk model where $\varpi$=$\phi$=$m$=$\beta$=$\nu$=0.

[Insert Table 1 here]

### 3.2 Effects of monetary tightening

We carry out a monetary shock in the same way as Monacelli (2009). Specifically, the innovation of the monetary shock $u_t$ is increased by 25 basis points in the policy rule with the initial value of the shock parameter $\varsigma_t$ normalized at zero. The results of impulse responses are illustrated in Figure 1. An increase in $u_t$ raises the nominal interest rate and is monetary tightening.

[Insert Figure 1 here]

To begin with, we duplicate the results in both the Monacelli model and the Sterk model. The Monacelli model has binding credit constraints and no capital. The results are illustrated by dashed blue lines in Figure 1. As a result of monetary tightening and since durable prices are flexible, the relative price of durables decreases (cf. Panel B). A tighter credit constraint increases the shadow price of borrowing and real debt decreases (cf. Panels C-D). Because of fewer loans, borrowers reduce expenditure on nondurables and durable services (cf. Panels E-F). As durable prices fall relative to nondurable prices, savers substitute away from nondurables to durables, thereby decreasing expenditure on nondurables and increasing expenditure on durables (cf. Panels H-I). Thus, aggregate nondurables are reduced (cf. Panel K). As the reduction in borrowers' expenditure on consumer

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[6] This value of the rate of depreciation of capital gives a quarterly capital-to-output ratio of 9.625 which is in the range of the data. Alternatively, if we use a lower value of $\delta_K$=0.02 or a higher value of $\delta_K$=0.03, the quarterly capital-to-output ratio would be 11.296 and 8.385, respectively. Our results are robust under these different values.

[7] To obtain the value of $\vartheta_c$, we use the log-linearization of the optimal pricing condition in (8b) to yield the slope of the Phillips curve equal to $(\varepsilon_c-1)/\vartheta_c$. Then, we equate the slope to the slope of the New Keynesian Phillips curve in the standard Calvo-Yun model, which is $(1-\theta_c)(1-\gamma\theta_c)/\theta_c$, where $\theta_c$ is the probability of not resetting prices for nondurables. Thus, we obtain $\vartheta_c=(\varepsilon_c-1)\theta_c/[(1-\theta_c)(1-\gamma\theta_c)]$. See supplementary data in Monacelli (2009). By setting the stickiness of nondurable prices at four quarters so $1-\theta_c$=0.25, with $\varepsilon_c$=6 and $\gamma$=0.99, we obtain $\vartheta_c$=58.25.

durables is less than the increase in savers' expenditure on consumer durables, aggregate durable services are increased (cf. Panel L). As a result, the model fails to generate the joint decline.

In the Sterk model, there are no credit constraints. Like the foregoing model, the relative price of durables falls (cf. red line in Panel B). This makes the households substitute away from nondurables toward consumer durables. Firms decrease capital, and thus profits are increased (cf. Panels G and J). As there is only one type of households (savers), all households increase expenditure on consumer durables and decrease expenditure on nondurables (cf. Panels K-L). As a result, the model cannot generate a joint decline.

In our model, there are credit constraints and capital. First, like the Monacelli model, a fall in the relative price of durables raises the shadow price of borrowing and decreases private loans and, because of fewer loans, borrowers reduce expenditure on nondurables and consumer durables (cf. green lines in Panels B-F). Unlike the Monacelli model, a lower relative price of durables induces firms to increase investment, and thus more capital is accumulated (cf. Panel G). As a result of more investment, firms' real profits decrease (cf. Panel J). With fewer profits remitted from firms, savers cut consumption expenditures. Thus, savers increase less expenditure on consumer durables than that in the Monacelli model (cf. Panel I). As a result, aggregate durable services fall in our model (cf. Panel L) and therefore, aggregate durable services co-move with aggregate nondurables. In particular, aggregate durable services and aggregate nondurables respond negatively to the monetary contraction policy shock at least for four quarters (cf. Panels K-L) which is consistent with the VAR results in Erceg and Levin (2006).

We should mention that by including capital in our model, due to an increase in investment, monetary tightening leads to about a 9% decline in consumer durables along with a 0.35% decline in nondurables on impact (cf. Figure 1) indicating that consumer durables are more sensitive to interest rates than nondurables. The result is consistent with the empirical evidence in the postwar US data put forth by Barsky et al. (2003) and Erceg and Levin (2006).

Finally, it should be noted that with credit constraints and capital, our model uncovers decreases in aggregate output, employment and consumption (nondurables and durables) in response to monetary tightening which are consistent with data (see Figure 2). By contrast, without credit constraints and with capital in the Sterk model, monetary tightening decreases aggregate output and employment, but aggregate consumption is increased and thus does not co-move with aggregate output. With credit constraints and without capital in the Monacelli model, monetary tightening increases all aggregate output, employment and consumption and inconsistent with data.

[Insert Figure 2 here]

### 3.3 Sensitivity analysis

In the earlier calibration exercises, all the preset parameters are well justified. In setting up nominal price stickiness, we follow Monacelli (2009) and use the price adjustment cost function with the parameter $\vartheta_j$, $j$=$c$, $d$, controlling the degree of stickiness of the nominal price adjustment. Monacelli (2009, Figure 4, p.253) conducted a sensitivity analysis of increasing the value of $\vartheta_d$ from $\vartheta_d$=0 so durable prices are sticky. Here, we perform a sensitivity analysis to see the contribution of the parameter $\vartheta_c$ into the co-movement. To this end, we decrease the value of $\vartheta_c$ from the baseline of $\vartheta_c$=58.25 wherein the probability of adjusting nondurable prices is 1-$\theta_c$=1/4. A decreasing value of $\vartheta_c$ lowers the cost of the nondurable price adjustment and thus nondurable prices become more flexible than the baseline case. With more flexible nondurable prices, in response to a monetary tightening, more of the firms lower nondurable prices and thus the durable price relative to the nondurable price is not decreased as much as it is in the baseline. Then, nondurable consumption is decreased to a less degree. Aggregate consumer durables would change little as savers' increases offset borrowers' decreases. Even if the value of $\vartheta_c$ is decreased to zero so nondurable prices are flexible, our results indicate that aggregate consumer durables still decrease. Figure 3 illustrates the impulse responses by decreasing the parameter value of $\vartheta_c$ to 29.5, 9.9 and then 0, with these values of $\vartheta_c$ being obtained by setting the probability of adjusting prices 1-$\theta_c$ equal to 1/3, 1/2 and 1, respectively. It is clear to see that when the nondurable price adjusts more frequently, the relative price of durables is decreased to a less degree and may be even increased when nondurable prices are flexible (cf. Panel B). Nevertheless, with lower profits, aggregate durable services decrease and co-move with nondurables (cf. Panels K and L).

[Insert Figure 3 here]

Moreover, in setting up the fraction of borrowers, we follow Monacelli (2009) and use $\varpi$=50%. It is interesting to see the importance of the share of borrowers in generating comovement in durable and nondurable consumption. When the share of borrowers in the economy decreases and thus the fraction of savers increases, we would expect that aggregate consumer durables are reduced to a less degree and it is more difficult to generate comovement between durable and nondurable consumption. Figure 4 demonstrates the impulse responses when the share of borrowers is decreased to $\varpi$=40% and then, $\varpi$=30%. The figure indicates that aggregate consumer durables still decrease and thus co-move with nondurables. Yet, aggregate consumer durables are decreased to a less degree by less when $\varpi$ is decreased. Indeed, our results suggest that if $\varpi$ is less than 27%, aggregate consumer durables and nondurables do not co-move. Although the estimated share of liquidity or credit constrained households is diverse in the literature, in general the share is more than 40%. For example, using data on nondurable consumption up to 1983, Jappelli and Pagano (1989) estimated 40%-51% of U.K. consumption to be associated with liquidity constraints. Moreover, using the U.S. quarterly time series data obtained from the Data Resources, Campbell and Mankiw (1989) estimated 52.3% of consumers are credit-constrained. Finally, using the data in 10 UK regions collected for the period 1971-1987, Bayoumi (1993) estimated 60% of consumers are credit-constrained. Thus, the share of borrowers that generates the comovement is within the range of the estimated values in the UK and the US.

[Insert Figure 4 here]

### 3.4 Why does the model with credit constraints and capital resolve the co-movement problem?

This subsection explains the underlying reasons why capital can resolve the comovement problem in an otherwise standard neoclassical sticky-price model with credit constraints and flexible durable prices. At the center of the analysis is the fact that in a model with credit constraints, the capital demand changes the shadow value of consumer durables for savers who are owners of firms.

In the Monacelli model, a saver's shadow value of consumer durables in period $t$ is (3c) with $\psi_t^*=0$ and $\beta$ being replaced by $\gamma$. Denote $V_t^* \equiv p_t U_{C,t}^*$. Then, a saver's shadow value of consumer durables can be rewritten as

$$V_t^* = U_{D,t}^* + \gamma(1-\delta)E_t(V_{t+1}^*) = E_t\sum_{\tau=0}^{\infty}\gamma^{\tau}(1-\delta)^{\tau}U_{D,t+\tau}^*, \tag{12}$$

in which the second equality follows from the law of iterated expectations. The condition indicates that a saver's shadow value of consumer durables in this period is equal to the expected discounted sum of the marginal utility of undepreciated consumer durables from period $t$ onward. A saver's shadow value of consumer durables is quasi-constant, since variations in durable flows in this period have little effects on the stock of consumer durables upon which the marginal utility of consumer durables depends. With a quasi-constant shadow value of consumer durables for savers, the relative price of durables and the marginal utility of nondurables move in opposite directions. In response to monetary tightening, a fall in the relative price of durables $p_t$ is accompanied by an increase in the marginal utility of nondurables $U_{C,t}^*$ which is associated with decreases in expenditures on nondurables. Moreover, a lower relative durable price increases savers' expenditure on consumer durables so much so that nondurables and consumer durables move in opposite directions.

Thus, using (12), a saver's consumer durables are determined by

$$U_{D,t}^* = V_t^* - \gamma(1-\delta)E_t(V_{t+1}^*), \tag{13}$$

which indicates that a saver's marginal utility of consumer durables in this period is the saver's shadow value of consumer durables in this period minus the saver's expected discounted shadow value of undepreciated consumer durables in the next period. If there is no capital, (13) determines the saver's consumer durables which are the level in the Monacelli model.

Notice that in our model, capital and a saver's consumer durables are not perfectly substitutable, because capital goods are inputs in production and consumer durable goods are final goods which provide service flows to households. As capital goods are inputs in production, capital affects a saver's consumer durables via the effects of the marginal product of capital on the saver's shadow value of consumer durables. To see this, the demand for capital in (8c) is rewritten as

$$V_t^* = \gamma E_t\{[(\lambda_{d,t+1} MP_{d,t+1}^K) + (1-\delta_K)]V_{t+1}^*\}. \tag{14}$$

Thus, the demand for capital is such that a saver's shadow value of consumer durables in this period is equal to the discounted expected marginal revenue of a marginal unit of capital in the next period which is the discounted saver's shadow value of consumer durables in the next period multiplied by the sum of the marginal product of capital and the undepreciated unit of capital. Hence, the demand for capital affects a saver's shadow value of consumer durables in this period. Note that the discount factor $\gamma$, the undepreciated fraction of capital (1-$\delta_K$) and the real marginal cost of durable intermediates $\lambda_{d,t}$ are all constant.[8]

With capital, (14) modifies (13) as

$$U_{D,t}^{K*} = \gamma E_t\{[(\lambda_{d,t+1} MP_{d,t+1}^K) + (1-\delta_K)]V_{t+1}^*\} - \gamma(1-\delta)E_t(V_{t+1}^*), \tag{15}$$

which indicates that a saver's marginal utility of consumer durables in this period is the saver's discounted expected marginal revenue of capital in the next period minus the saver's discounted expected shadow value of undepreciated consumer durables in the next period.

By comparing (15) with (13), the difference of these two marginal utilities lies in the first terms in the right-hand side of these two expressions. In (13), the first term is the shadow value of consumer durables in *this period* $V_t^*$. By contrast, in (15), the first term is the discounted expected shadow value of consumer durables in the *next period* $V_{t+1}^*$ multiplied by the sum of the marginal product of capital and the undepreciated fraction of capital in the *next period.* Following monetary tightening, a saver's shadow value of consumer durables falls on impact and then increases toward its steady state. Thus, a saver's shadow value of consumer durables in this period $V_t^*$ is smaller than the shadow value of consumer durables in the next period $V_{t+1}^*$. Moreover, due to the stock-flow relationship, the marginal product of capital falls by a very small margin. (See Panel B, Figure 5.) Thus, the first term in the right-hand side of (15) is likely to be larger than the first term in the right-hand side of (13). Indeed, our results indicate that it is the case, as seen in Panel A, Figure 5 wherein a saver's shadow value of consumer durables when there is capital (i.e., the first term in (15)) is decreased less than a saver's shadow value of consumer durables when there is no capital (i.e., the

[8] Since durable prices are flexible, durable prices have a constant markup in equilibrium, and thus the real marginal cost of durable intermediates $\lambda_{d,t}$ is constant.

first term in (13)). Therefore, a saver's marginal utility of consumer durables in this period in (15) is larger than that in (13). (See Panel C, Figure 5).

[Insert Figure 5 here]

The foregoing results indicate that, in the economy with capital, the saver's expenditure on consumer durables is smaller than that in the economy without capital. In particular, as the demand for capital is sizable, firms' profits fall in the short run and thus profits remitted to savers decrease, thereby reducing savers' disposable income. Hence, savers cut their expenditures on consumer durables. As a result, aggregate durable services fall in the economy with capital as seen in Figure 1.

Finally, in the Sterk model, there is no borrower. In response to monetary tightening, the relative durable price falls, and thus the household increases expenditure on consumer durables and decreases expenditure on nondurables. The model fails to generate the joint decline. Moreover, as there are only savers, increases in consumer durables are so strong that capital decreases.

## 4. Concluding Remarks

In otherwise standard two-sector neoclassical sticky-price models, if nondurable prices are sticky and durable prices are flexible, following monetary tightening nondurables decrease, but consumer durables increase. Frictions in lending between households can resolve the comovement problem if durable prices have some degrees of stickiness. However, if durable prices are flexible, frictions in lending fail to generate the joint decline. This paper resolves the comovement problem by adding capital in otherwise standard neoclassical sticky-price models with frictions in lending and flexible durable prices. We find that with capital as input in production, monetary tightening lowers relative prices of durable goods which induces investment and decreases firms' real profits in the short run. Due to reduced profits remitted from firms, savers have a lower disposable income and hence cannot increase expenditures on consumer durables as much as otherwise. As a consequence, aggregate consumer durables fall and there is comovement between nondurables and consumer durables.

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**Table 1: Parameter Setting** (frequency: quarterly)

| Description | Parameter | This model | Monacelli | Sterk |
|---|---|---|---|---|
| Fraction of borrowers | $\varpi$ | 0.5 | 0.5 | 0 |
| TFP in nondurable and durable sector | $A_c$, $A_d$ | 1 | 1 | 1 |
| Elasticity of sub. between nondurables and durables | $\eta$ | 1 | 1 | 1 |
| Elasticity of sub. between intermediates | $\varepsilon_c$, $\varepsilon_d$ | 6 | 6 | 6 |
| Share of durables in consumption | $\mu$ | 0.2 | 0.2 | 0.2 |
| Inverse elasticity of labor supply for savers | $\phi^*$ | 1 | 1 | 1 |
| Inverse elasticity of labor supply for borrowers | $\phi$ | 1 | 1 | — |
| Discount factor of savers | $\gamma$ | 0.99 | 0.99 | 0.99 |
| Discount factor of borrowers | $\beta$ | 0.98 | 0.98 | — |
| Borrowers' loan-to-value ratio | $m$ | 0.75 | 0.75 | — |
| Depreciation rate of durables | $\delta$ | 0.01 | 0.01 | 0.01 |
| Hours of work | $L^*$, $L$ | 1/3 | 1/3 | 1/3 |
| Autocorrelation of error to the interest rate | $\rho$ | 0.5 | 0.5 | 0.5 |
| Coefficient of inflation rates in Taylor rule | $\chi$ | 1.5 | 1.5 | 1.5 |
| Capital share in durables sector | $\alpha_d$ | 0.27 | 0 | 0.27 |
| Capital share in nondurables sector | $\alpha_c$ | 0.47 | 0 | 0.47 |
| Depreciation rate of capital | $\delta_K$ | 0.025 | — | 0.025 |
| Parameter of labor in utility for savers | $v^*$ | 3.797 | 5.515 | 5.290 |
| Parameter of labor in utility for borrowers | $v$ | 8.602 | 8.602 | — |
| Coefficient of price adjustment in durables | $\vartheta_d$ | 0 | 0 | 0 |
| Coefficient of price adjustment in nondurables | $\vartheta_c$ | 58.25 | 58.25 | 58.25 |

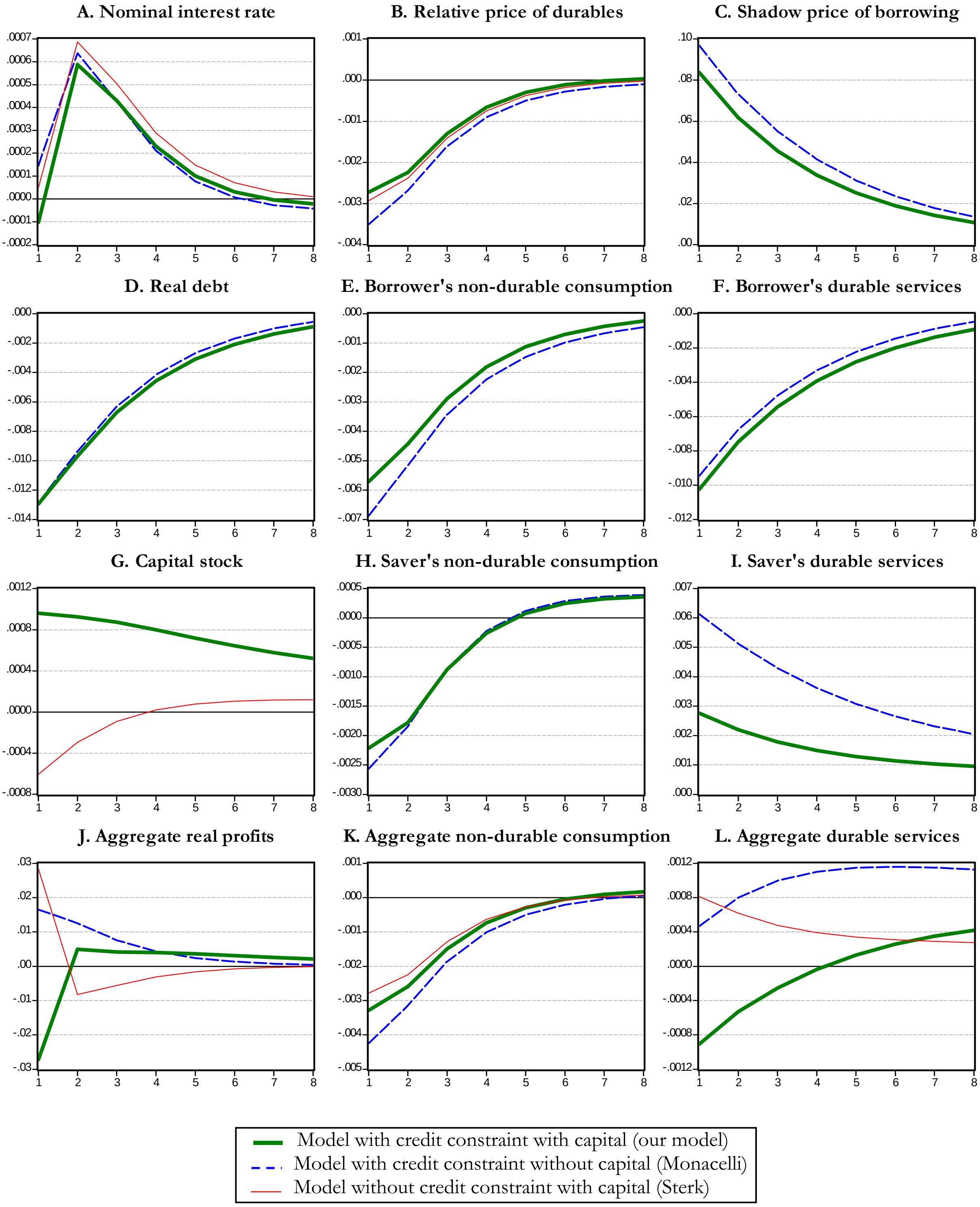


**Figure 1. Impulse responses to monetary tightening in different models**

Note: The horizontal axis is divided in quarters; the vertical axis represents percentage deviations from a steady state.

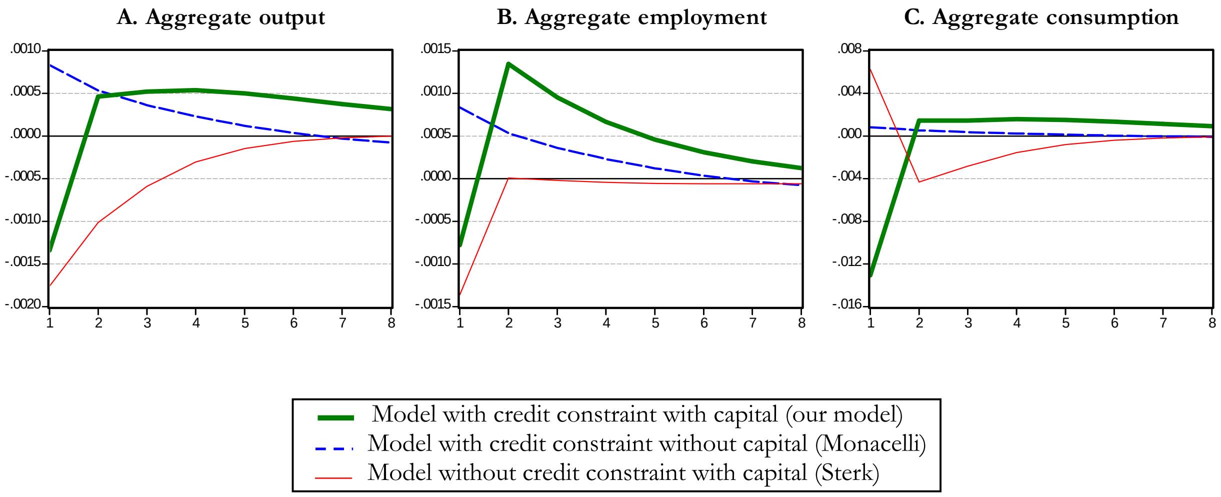


**Figure 2. Impulse responses of aggregate output, employment and consumption to monetary tightening**

Note: The horizontal axis is divided in quarters; the vertical axis represents percentage deviations from a steady state.

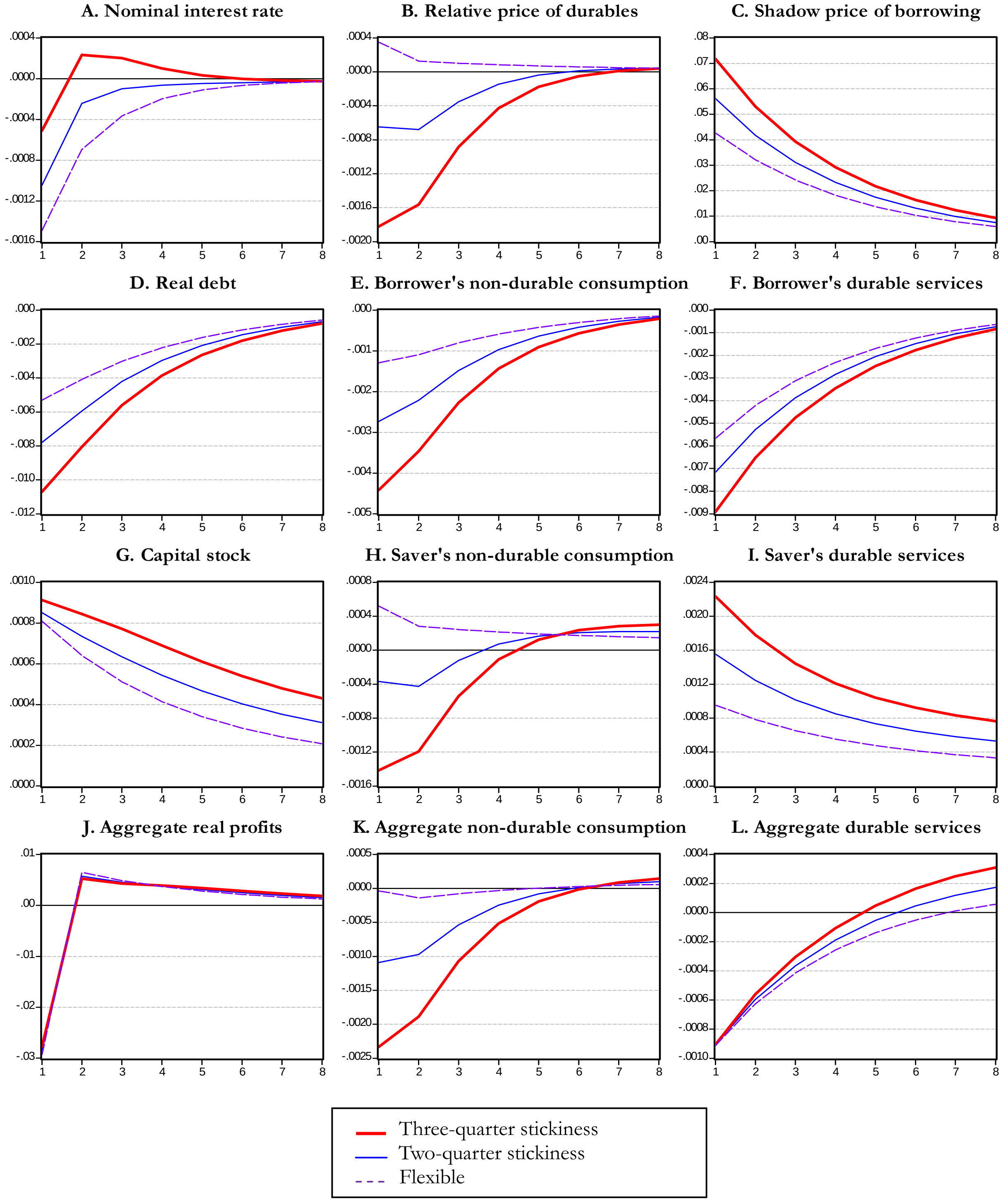


**Figure 3. Sensitivity analysis: impulse responses with decreasing degrees of nondurable price stickiness**

Note: The horizontal axis is divided in quarters; the vertical axis represents percentage deviations from a steady state.

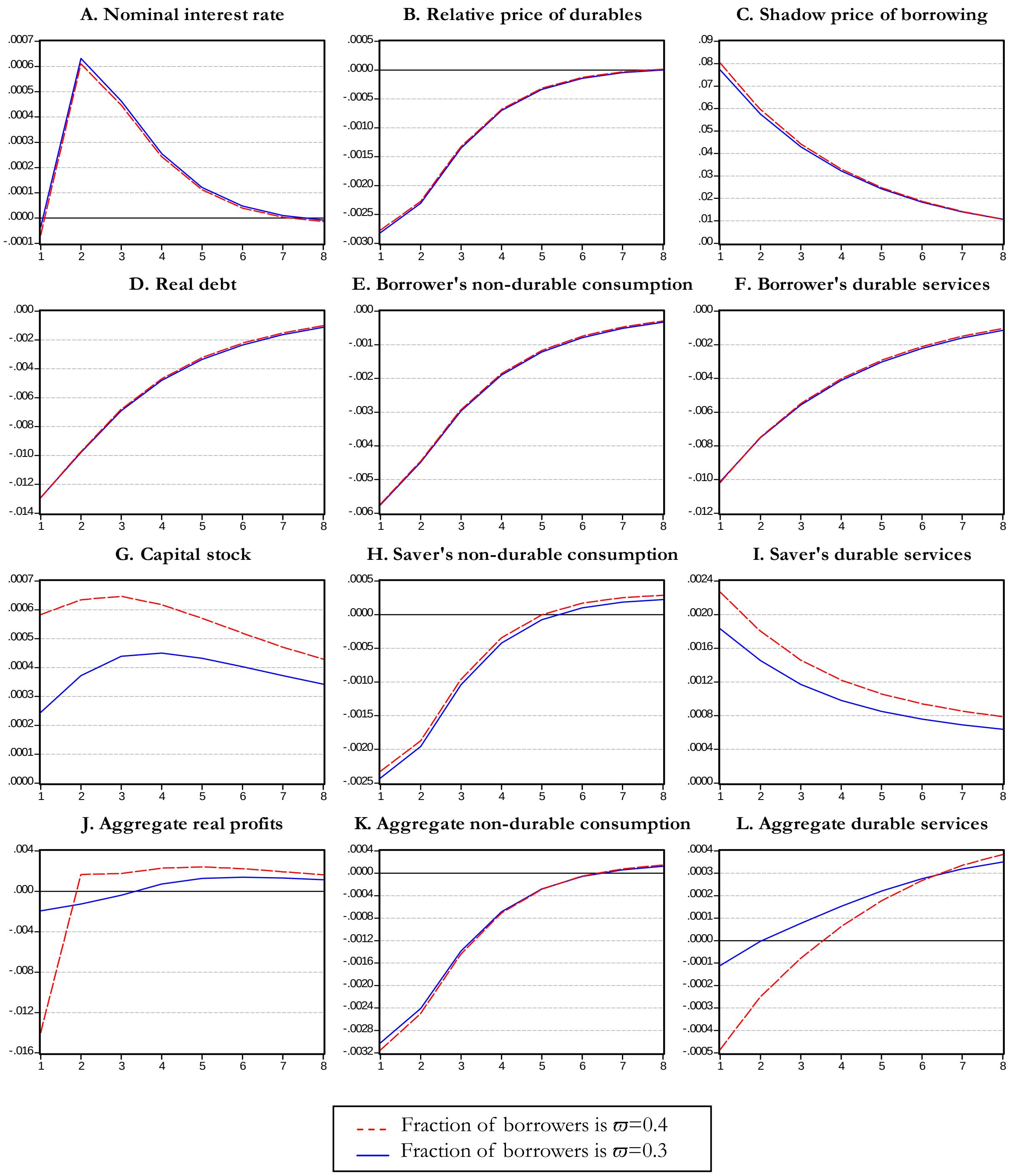


**Figure 4. Sensitivity analysis: impulse responses with decreasing fractions of borrowers**

Note: The horizontal axis is divided in quarters; the vertical axis represents percentage deviations from a steady state.

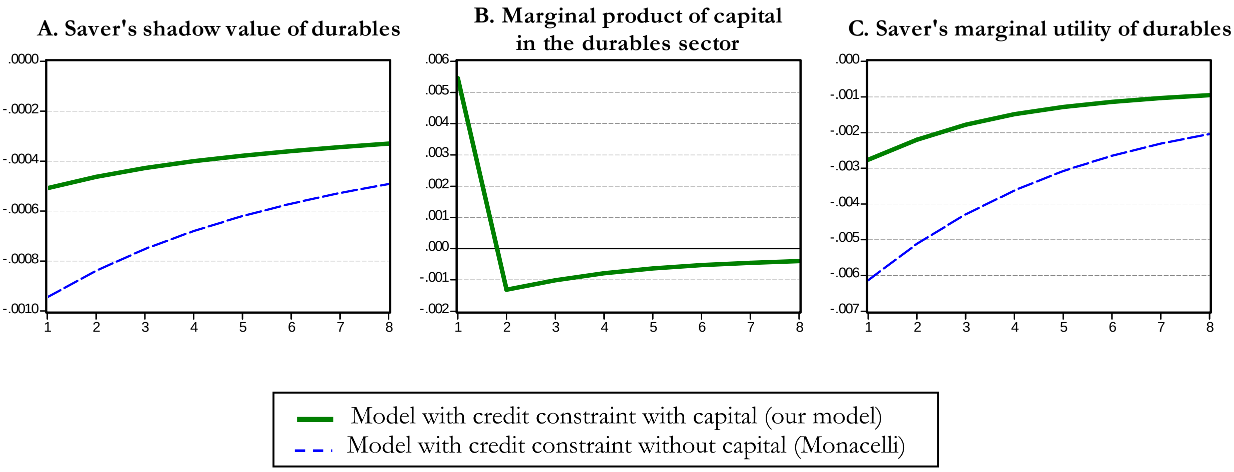


**Figure 5. Impulse responses of saver's shadow values of durables and marginal product of capital to monetary tightening**

Note: The horizontal axis is divided in quarters; the vertical axis represents percentage deviations from a steady state.