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Optimal factor tax incidence in two-sector human capital-based models[☆]

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ABSTRACT

This paper studies the optimal factor tax incidence in a standard two-sector, human capital-based endogenous growth model elucidated by Lucas (1988). Capital income taxes generate dynamic inefficiency for capital accumulation and labor income taxes create dynamic inefficiency for human capital accumulation. A factor tax incidence is a tradeoff between these two inefficiencies. A switch from capital income taxes to labor income taxes reduces the long-run welfare coming from lower leisure and increases the long-run welfare originated from higher economic growth and higher consumption. Because the representative agent's learning time and human capital are inseparable and thus affect learning activities at the same degree, we find that based on the current US income tax code, it is optimal to first tax capital income, and to resort to taxing labor income only when tax revenue is insufficient to cover government expenditure.

1. Introduction

If the government's only option is to tax factor income, should the government tax capital income or labor income? The pivotal work by Judd (1985) and Chamley (1986) argued that the government should tax only labor income and not capital income in the long run. Their claim was based on dynastic growth models in which physical capital is accumulated over time while labor does not embody human capital. Several studies have revisited the issue by relaxing key assumptions and found the result to be robust; for example, see Chari et al. (1994), Chari and Kehoe (1999) and Atkeson et al. (1999). The reason for the result is that capital accumulates over time and the capital income taxation creates a dynamic inefficiency for capital accumulation. Therefore, it is not optimal to tax capital income.

If labor embodies human capital, one might wonder whether capital income should be taxed instead of labor income. Lucas (1990) was the first to study the question of the optimal factor tax incidence in a two-sector human capital-based endogenous growth model. In a two-sector, human capital-based growth model, both physical capital and human capital accumulate over time; thus the capital income taxation creates a dynamic inefficiency for capital accumulation and labor income taxation generates a dynamic inefficiency for human capital accumulation. Factor income taxation then is a tradeoff between these two kinds of dynamic inefficiencies. In Lucas (1990), the learning activity that forms human capital is linear with regard to the representative agent's current level of human capital and *concave* in the representative agent's learning time. Because the contribution of the representative agent's learning time to the formation of human capital in the future is smaller than the contribution of the agent's human capital, in the long run it is optimal to tax labor/human capital income and not to tax capital income. Later, Jones et al. (1993, Model II) studied an otherwise identical model. In their model, although material goods are an input, their learning function is intrinsically like that revealed by Lucas (1990) wherein the contribution of the agent's learning time is smaller than the contribution of the agent's human capital in the formation of future human capital. As a consequence, the paper reached the same conclusion as Lucas (1990). The upshot was preserved in a similar but more complicated model by Jones et al. (1997).[1]

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[1] In Jones et al. (1997), the learning technology activity is the same as that demonstrated by Jones et al. (1993, Model II). Moreover, the labor input into the goods sector is generated not only by human capital-embodied labor hours into the working activity but also by the investment flow allocated to the working activity. Readers are also referred to Milesi-Ferretti and Roubini (1998) and Judd (1999, Section 7) who obtained optimal factor taxation similar to that of Jones et al. (1997) in endogenous growth models wherein the learning function uses material goods as input. In a similar model, Reis (2007) showed that if the government cannot distinguish between consumption and human capital investment, then the optimal capital tax is zero when the level of capital does not influence the relative productivity of human capital.

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In this paper, we revisit the question of the optimal factor tax incidence in an otherwise standard two-sector, human capital-based growth model by Lucas (1988) as well as Bond et al. (1996) and Mino (1996), wherein human capital is embodied in and inseparable from learning hours and they are not separable. The representative agent's learning time and the embodied human capital affect the learning function at the same degree and thus, the contribution of the representative agent's learning time is the same as the contribution of the agent's human capital in the formation of human capital in the future.[2] Ever since the form was proposed by Lucas (1988), it has been used by many authors; to name just a few, Lucas (1993), Benhabib and Perli (1994), Bond et al. (1996), Ladrón-de-Guevara et al. (1997, 1999). The fact that many established authors have used the form indicates that this is an important alternative form. It is thus valuable and essential to understand the implication on the capital tax in a model when the form is used. As it turns out, the implication on the capital tax is exactly opposite to that in Lucas (1990). While the optimal capital tax is zero in the Lucas (1990) model, capital should be taxed to a maximum in our model. As a result, it is optimal to tax capital income first. Labor income is to be taxed only if the revenue gained from capital income taxes is insufficient to cover government expenditure.

Our benchmark framework is based on the model put forth by Lucas (1988) with an extension to consider leisure and factor income taxes in the goods sector in order to finance a given stream of the government expenditure as a lump-sum transfer. We analyze the long-run welfare effect of a switch from a pre-existing tax code to a higher capital tax rate and a lower labor tax rate in order to finance given government expenditure. There are three effects at work. First, leisure is lower as a lower labor tax rate increases the price of leisure. Second, the economic growth rate is higher because a higher capital tax rate discourages physical capital accumulation. Because capital and human capital are complements in the production of goods, less labor is allocated to the goods production and more is allocated to the learning activities. Learning activity is the engine of economic growth, so the economic growth rate increases. Finally, consumption is increased relative to physical capital because a higher capital income tax rate reduces the price of consumption relative to investment. While lower leisure leads to a negative welfare effect, higher economic growth and higher consumption create positive welfare effects. If the first effect dominates, it is optimal to tax labor income completely. If the latter two effects dominate, it is optimal to tax capital income completely. Finally, if these three effects completely offset each other, there is an interior mix of optimal income tax rates.

We calibrate our benchmark model in the long run in order to reproduce key features representative of the US economy with the pre-existing average capital income and average labor income tax rates at 30% and 20%, respectively, in 1960–2007, thus with tax revenue accounting for 23% of total output. In a revenue-neutral tax reform experiment, as we switch from the current US income tax code by increasing the capital income tax rate and decreasing the labor income tax rate in order to finance a given fraction of the government expenditure in output as a lump-sum transfer, we find that leisure is decreasing while the ratio of consumption to capital and economic growth are increasing in the long run. As the welfare is increasing in leisure, the ratio of consumption to capital and economic growth, lower leisure reduces welfare while higher economic growth and a higher consumption to capital ratio increase welfare in the long run. It turns out that the negative welfare effect coming from lower leisure is quantitatively dominated by the positive welfare effect originated from higher consumption and economic growth. As a consequence, it is optimal to tax capital income.

In particular, we find that when capital income is taxed above a rate, the labor tax rate is negative, thus providing a subsidy to working labor. Then, the incentive to accumulate human capital is so large and the incentive to accumulate physical capital is so small such that physical capital accumulates slowly and the human capital to capital ratio is increased by a large margin. Because of large increases in human capital, the economic growth rate is almost double that from the baseline rate. With such a high economic growth rate accompanied by a small incentive to accumulate physical capital, consumption is increased substantially, increasing the consumption to capital ratio by more than several times from its baseline level. Conversely, the decrease in the leisure time is flat which indicates a small welfare loss. As a result of large welfare gains due to a higher consumption to capital ratio and higher economic growth and a small welfare loss due to lower leisure time, it is always welfare improving if the capital tax is increased. We find that it is optimal to raise the capital tax rate to the highest possible rate that gives feasible allocation. We also find that a feasible allocation is obtained and the agent still saves when the capital tax rate is less than 100%. Our quantitative exercises indicate that the optimal tax mix is to tax 99.99% of the capital income and −10.00% of the labor income.

In order to obtain an optimal capital tax rate below 99%, we restrict to a corner solution by imposing a non-negativity constraint on the labor tax rate. This assumption is reasonable because in reality the labor tax rate cannot be negative on average, although there are situations wherein some laborers' income is subsidized. After imposing the constraint, we find that the optimal capital tax rate is $\tau_k=$ 76.67%, a drop by more than 23 percentage points from an interior solution. Yet, the restriction to a corner solution also reduces the welfare gain of a tax reform.

Finally, we study a model with a general learning function that uses both human capital and physical capital proposed by Bond et al. (1996) and Mino (1996). In this economy, although capital is used in the learning activity, the representative agent's learning time contributes to the formation of human capital in the future at the same degree as the embodied human capital. As a result, this economy features a similar taxation effect as the benchmark economy. Our calibration results stipulate that as the capital income tax rate is increasing and the labor income tax rate is decreasing from a pre-existing tax code, the effects are quantitatively similar to those in the benchmark model. Thus, it is optimal to tax capital income and not to tax labor income. We find that our results are robustness with regard to progressive or regressive tax policies and different choices in spending the factor tax revenue. Even if the human capital formation is via learning-by-doing, it is optimal to tax capital income.

Other related literature found a positive optimal capital tax rate, including Guo and Lansing (1999), Cassou and Lansing (2006) and Chen (2007) which incorporated positive externalities, productive public capital or market imperfections into dynastic models. In life-cycle models, the driving mechanism for positive taxes on capital is to mimic age-dependent taxes on labor income. Garriga (2001) and Erosa and Gervais (2002) demonstrated that mimicking an age-dependent tax on labor income was a substantial motive for a positive tax on capital. Conesa et al. (2009) discovered that this motive was large in a model with exogenous human capital accumulation. Peterman (2012) showed that including endogenous human capital

[2] We cannot find empirical evidence which will directly support either our form or the form used in Lucas (1990). Nevertheless, our setup is reasonable within a private perspective because one's human capital is embodied in the length of time one commits to learning something. One cannot tell which of one's learning time and the human capital embodied in learning time contributes more to one's learning. It is not easy to tell one's learning time and the human capital embodied in one's learning time in the data. Moreover, in the data an individual's learning may be affected by others, an externality which is assumed to be zero in our model. When one learns things in a group with more knowledgeable or more creative people, one learns better and is more creative; for example, casual evidence might show that economists do not just read papers but also try very hard to participate in a conference/group in order to learn things along with well-established economists. Given these constraints, it is thus difficult to use data to distinguish the contribution of private learning time from the contribution of the human capital embodied in learning time.

accumulation further enhanced the motive.[3] Our paper is different from those papers in that human capital is embodied in the labor hour in our model. In particular, the optimal factor tax mix in our model is to tax only capital income and the tax reform based on the current US tax code results in a large welfare gain.

We organize this paper as follows. We study the Lucas (1988) model in Section 2 and analyze and quantify the resulting optimal factor tax incidence in Section 3. In Section 4 we envisage the robustness of optimal positive capital taxes in a series of departures from the basic model. The basic model is expanded to consider capital in the learning technology in Section 5. Finally, concluding remarks are offered in Section 6.

2. The basic model

Our basic model is the Lucas (1988) framework which was extended by Benhabib and Perli (1994) and Ladrón-de-Guevara et al. (1997, 1999) to include leisure. We extend these models to consider factor taxes. The economy is populated with a continuum of representative households of mass one and with a continuum of representative firms of mass one.

2.1. Households

The representative household is endowed with L units of time. At an instant in time, n units are used for labor activities and the remaining $l=L$-n units are for leisure activities. In labor activities, a fraction of time u is devoted to working and the remaining fraction $(1-u)$ is devoted to learning. The representative household obtains utility from consumption and leisure. The agent's lifetime utility is represented as follows.

$$U=\int_0^\infty u(c,l)e^{-\rho t}\mathrm{d}t,$$

where

$$u(c,l)=\ \ln c+\psi\frac{(l)^{1-\sigma}-1}{1-\sigma},\sigma>0, \tag{1}$$

and c is consumption and $\rho>0$ is the time preference rate.

In Eq. (1), we follow Benhabib and Perli (1994) and Ladrón-de-Guevara et al. (1997, Section 4.2) and use a utility function separable in consumption and leisure with a unit intertemporal elasticity of substitution (hereafter, IES) for consumption that is different from the IES of labor which is $(L-n)/(\sigma n)$.[4] Parameter ψ is the degree of leisure in utility relative to consumption.

At any point of time, the representative agent's flow budget constraint is

$$\dot{k}=(1-\tau_{\mathrm{k}})rk+(1-\tau_{\mathrm{h}})wunh-c+G,k(0)given, \tag{2}$$

in which k is physical capital and h is human capital with given initial values $k(0)$ and $h(0)$. The variable r is the rental rate of capital, w is the wage rate per effective unit of capital, τ_k is the capital income tax rate, τ_{h} is the labor income tax rate, and G is the government lump-sum transfer. For simplicity, we assume that capital does not depreciate. The flow budget constraint indicates that unspent income accumulates physical capital. If $\tau_{\mathrm{k}}=\tau_{\mathrm{h}}$, then taxing both types of factor income is equivalent to taxing the output produced in the goods sector.[5]

The household's human capital is accumulated via the learning activity as follows.

$$\dot{h}=B(1-u)nh,h(0)\text{given}, \tag{3}$$

where $B>0$ is the efficiency coefficient that measures the maximum rate of human capital accumulation. This linear human capital formation function is the one employed in Lucas (1988) with a simplified assumption of zero depreciation. In this linear learning function, the representative agent's learning time $(1-u)n$ and his or her level of human capital h contribute to the human capital formation at the same degree. This same feature appears in the goods production technology which will be specified below.

The representative household's problem is to maximize utility by choosing between consumption, leisure, and investment in the goods and the education sectors, subject to the constraints (2), (3), and $l=L-n$, taking as given the tax rates, transfers, factor prices, and the given initial physical capital and human capital, $k(0)$ and $h(0)$. Let $\lambda>0$ and $\lambda_{\mathrm{h}}>0$ be the co-state variables associated with physical capital and human capital, respectively. The necessary conditions are

$$\frac{1}{c}=\lambda, \tag{4a}$$

$$\psi l^{-\sigma}=\lambda(1-\tau_{\mathrm{h}})wuh+\lambda_{\mathrm{h}}B(1-u)h, \tag{4b}$$

$$\lambda(1-\tau_{\mathrm{h}})w=\lambda_{\mathrm{h}}B, \tag{4c}$$

$$\dot{\lambda}=[\rho-(1-\tau_{\mathrm{k}})r]\lambda, \tag{4d}$$

$$\dot{\lambda}_{\mathrm{h}}=\left[\rho-\frac{\dot{h}}{h}\right]\lambda_{\mathrm{h}}-\lambda(1-\tau_{\mathrm{h}})wnu, \tag{4e}$$

along with the transversality conditions,

$$\lim_{t\to\infty}e^{-\rho t}\lambda_{\mathrm{t}}k_{\mathrm{t}}=0 \tag{4f}$$

$$\lim_{t\to\infty}e^{-\rho t}\lambda_{\mathrm{ht}}h_{\mathrm{t}}=0. \tag{4g}$$

The conditions above are standard: Eq. (4a) determines optimal consumption, while Eq. (4b) indicates tradeoffs between labor supply and leisure. Eq. (4c) allocates labor supply optimally between working and learning. Finally, Eqs. (4d) and (4e) are two Euler equations, and Eqs. (4f) and (4 g) are the two usual transversality or "no Ponzi game" conditions on physical and human capital.

2.2. Firm

The representative firm produces a single output by renting capital and hiring labor from the household. Following Lucas (1988), the Cobb–Douglas production function is used.

$$y=A(k)^{\alpha}(unh)^{1-\alpha}, \tag{5}$$

where $0<\alpha<1$ is the share of physical capital and $A>0$ is the technology coefficient.[6] It is worth noting that as in the learning activity the representative agent's labor hours and its embodied human capital affect the production activity in Eq. (5) in the same way. Thus, the

[3] See also Kapicka (2006) who studied models with and without endogenous human capital and found that if human capital is unobservable, endogenous human capital formation lowers marginal tax rates.

[4] As noted in Ladrón-de-Guevara et al. (1999, p. 613) and Jones et al. (2005, p. 809), only two forms of felicity are consistent with a balanced growth path (BGP) in our two-sector endogenous growth model: a separable form and a non-separable form. A general non-separable form is $\{[c^{\theta}g(l)]^{(1-\sigma)}-1\}/(1-\sigma)$, with the special case of $\{[c^{\theta}l^{\psi}]^{(1-\sigma)}-1\}/(1-\sigma)$ used in Lucas (1990) which imposes the same IES for consumption and leisure. A general separable felicity is $u(c,l)=\log c+g(l)$ and since a constant IES of consumption is necessary in order to be consistent with a BGP, we employ the parametric form for the felicity of leisure as used in Benhabib and Perli (1994).

[5] We assume that learning is a non-market good and thus, is not subject to taxation, an assumption in line with Lucas (1990) and Jones et al. (1993, 1997).

[6] Lucas (1990) employed the following CES form: $y=A\left[\alpha(k)^{1-\frac{1}{\varepsilon}}+(1-\alpha)(unh)^{1-\frac{1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}$. Our (5) is a special case that arises when $\varepsilon=1$.

contribution of the labor hour to production is the same as the contribution of human capital.

Taking factor prices as given, the representative firm chooses capital and labor in order to maximize the profit. The optimal conditions are

$$r = \alpha\frac{y}{k}, \tag{6a}$$

$$w = (1-\alpha)\frac{y}{unh}, \tag{6b}$$

which equate factor prices to marginal products.

2.3. The government

The government's objective is to maximize social welfare. At an instant point in time, the government receives capital income taxes and labor income taxes. The government uses the tax revenue to finance a direct lump-sum transfer G under a balanced budget as follows.

$$\tau_{\mathrm{k}} rk + \tau_{\mathrm{h}} whun = G, \tag{7}$$

It is worth noting that the transfer is included to ensure the government budget is balanced in the presence of pre-existing factor taxes that fit the data observation. As the government transfers tax revenues to the representative agent, the goods market clearance condition is

$$\dot{k} = y - c. \tag{8}$$

3. Equilibrium and optimal tax

Perfect-foresight competitive equilibrium defines the time paths of quantities $\{c, k, h, l, u, G\}$ and prices $\{r, w, \lambda, \lambda_h\}$ that satisfy Eqs. (3), (4a)–(4e), (6a)–(6b), (7) and (8). Denote $z \equiv c/k$, $q \equiv h/k$ and $p \equiv \lambda_h/\lambda$. We simplify the equilibrium conditions by transforming them into a three-dimensional dynamical system with state vector (l, q, z).

First, if we use Eqs. (4a) and (4d), we obtain

$$\frac{\dot{c}}{c} = (1-\tau_{\mathrm{k}})r - \rho. \tag{9a}$$

If we utilize Eqs. (3) and (6b), then Eq. (4c) becomes

$$\dot{h}\frac{1}{y} = (1-\tau_{\mathrm{h}})(1-\alpha)\frac{\lambda}{\lambda_{\mathrm{h}}}\frac{1-u}{u}, \tag{9b}$$

where the left-hand side is the marginal product of time devoted to learning and the right-hand side is the marginal product of time allocated to working.

Next, if we use Eqs. (3), (6b) and (9b), we rewrite Eq. (4b) as

$$\psi l^{-\sigma} = Bh\lambda_{\mathrm{h}}. \tag{9c}$$

Moreover, manipulating Eq. (9c) with the use of Eqs. (3), (4a) and (4b) yields

$$p = \frac{\psi z}{Bql^{\sigma}} \equiv p(l, q, z). \tag{10a}$$

Furthermore, using Eqs. (6b) and (10a), (4c) gives

$$u = p^{\frac{-1}{\alpha}}\frac{1}{q}\frac{1}{L-l}\left[(1-\tau_{\mathrm{h}})(1-\alpha)\frac{A}{B}\right]^{\frac{1}{\alpha}} \equiv u(p(l, q, z), l, q, \tau_{\mathrm{h}}). \tag{10b}$$

Finally, with p and u expressed in Eqs. (10a) and (10b), respectively, we can write down our equilibrium system in terms of l, q and z. Differentiating Eq. (9c) and using Eqs. (4e) and (10b) yields,

$$\frac{\dot{l}}{l} = \frac{-1}{\sigma}\left\{\rho - B\left[(1-\tau_{\mathrm{h}})(1-\alpha)\frac{A}{B}\right]^{\frac{1}{\alpha}} p^{\frac{-1}{\alpha}}\frac{1}{q}\right\}, \tag{11a}$$

while with the use of Eqs. (10b), (3) and (8) give

$$\frac{\dot{q}}{q} = B\frac{1-u}{u}\left[(1-\tau_{\mathrm{h}})(1-\alpha)\frac{A}{B}\right]^{\frac{1}{\alpha}} p^{\frac{-1}{\alpha}}\frac{1}{q} - A\left[(1-\tau_{\mathrm{h}})(1-\alpha)\frac{A}{B}\right]^{\frac{1-\alpha}{\alpha}} p^{\frac{-(1-\alpha)}{\alpha}} + z, \tag{11b}$$

and Eqs. (9a), (6a), (8) and (10b) lead to

$$\frac{\dot{z}}{z} = z - [1-\alpha(1-\tau_{\mathrm{k}})]A\left[(1-\tau_{\mathrm{h}})(1-\alpha)\frac{A}{B}\right]^{\frac{1-\alpha}{\alpha}} p^{\frac{-(1-\alpha)}{\alpha}} - \rho, \tag{11c}$$

Given τ_{k} and τ_{h}, we can use Eqs. (11a)–(11c) to determine l, q and z.

If we differentiate Eq. (10a) and use Eqs. (11a)–(11c), we can derive

$$\frac{\dot{p}}{p} = B\left[(1-\tau_{\mathrm{h}})(1-\alpha)\frac{A}{B}\right]^{\frac{1}{\alpha}} p^{\frac{-1}{\alpha}}\left\{\alpha(1-\tau_{\mathrm{k}})[(1-\tau_{\mathrm{h}})(1-\alpha)]^{-1}p - \frac{1}{q}\frac{1}{u}\right\}. \tag{12}$$

It is easier to determine the balanced growth path (BGP) if we use Eqs. (11a), (11b) and (12).

3.1. The balanced growth path

On any BGP, we have $\dot{l} = \dot{q} = \dot{z} = \dot{p} = 0$ and thus l, q, z and p are constant. Along that path, the fraction u is constant, while c, k and h grow at the same rate as do λ and λ_h.

To determine the BGP, it is useful if we start by rewriting p as a function of u in a BGP, and sequentially each of q, z and l as a function of u in a BGP. Then, we can determine the existence and uniqueness of a BGP in terms of u. First, with the help of Eq. (12), we use Eq. (11a) to obtain

$$p = (1-\alpha)\frac{A}{B}(1-\tau_{\mathrm{h}})\left[\frac{1}{\rho}\alpha A(1-\tau_{\mathrm{k}})\right]^{\frac{\alpha}{1-\alpha}} u^{\frac{\alpha}{1-\alpha}} \equiv p(u; \tau_{\mathrm{h}}, \tau_{\mathrm{k}}), \tag{13a}$$

where $\frac{\partial p}{\partial u} = \frac{\alpha}{1-\alpha}\frac{p}{u} > 0$, $\frac{\partial p}{\partial \tau_{\mathrm{h}}} = \frac{-p}{1-\tau_{\mathrm{h}}} < 0$, and $\frac{\partial p}{\partial \tau_{\mathrm{k}}} = \frac{\alpha}{1-\alpha}\frac{-p}{1-\tau_{\mathrm{k}}} < 0$.

To explain the signs in Eq. (13a), first, the relationship between u and p is positive in the long run, because when a larger fraction of labor hours is allocated to working, given τ_{h} and τ_{k}, there will less labor hours to learning which reduces human capital formation. This increases the shadow price of human capital relative to physical capital in the long run. With u being held constant, the two factor tax rates directly lower the shadow price of human capital relative to physical capital in the long run for the following reasons.[7] The reason is that a higher labor income tax rate generates two offsetting effects; it reduces the marginal cost of leisure (cf. right-hand side of Eq. (4b)) and the net marginal product of labor (cf. left-hand side of Eq. (4c)) in the goods sector. The former effect increases leisure (cf. left-hand side of Eq. (4b)) and reduces the learning activity so the shadow price of human capital relative to physical capital is increasing. However, the latter effect increases the marginal product of labor in the learning activity relative to the goods production (cf. right-hand side of Eq. (4c)) so the shadow price of human capital relative to physical capital is decreasing. As the latter effect dominates, a higher labor tax rate reduces the shadow price of human capital relative to physical capital. Finally, a higher capital tax rate reduces the net marginal product of capital (cf. right-hand side of Eq. (4d)) which decreases the shadow price of human capital relative to physical capital.

[7] These are only the direct effects of τ_{h} and τ_{k} on p. There are indirect effects of τ_h and τ_k via changes in u. The indirect effects can be analyzed when u is determined in (14) below. Similar remarks apply to Eqs. (13b)–(13e).

Next, if we substitute Eq. (13a) to Eq. (12), we attain

$$q = B\rho^{\frac{-\alpha}{1-\alpha}}[\alpha A(1-\tau_{\mathrm{k}})]^{\frac{-1}{1-\alpha}} u^{\frac{-1}{1-\alpha}} \equiv q(u;\tau_{\mathrm{k}}), \tag{13b}$$

where $\frac{\partial q}{\partial u} = \frac{-1}{1-\alpha}\frac{q}{u} < 0$ and $\frac{\partial q}{\partial \tau_{\mathrm{k}}} = \frac{1}{1-\alpha}\frac{q}{(1-\tau_{\mathrm{k}})} > 0$. The signs in Eq. (13b) are explained as follows. First, a larger fraction of labor allocated to working (u) is negatively related to the ratio of human capital to physical capital in the long run because it increases goods production and decreases the fraction of labor allocated to learning $(1-u)$. Next, given u, a larger capital tax rate τ_{k} discourages capital accumulation and thus increases the ratio of human capital to physical capital in the long run.

Moreover, with the use of Eq. (13a) and Eqs. (13b), (11b) leads to

$$z = \frac{\rho}{\alpha}\frac{1-\alpha(1-\tau_{\mathrm{k}})(1-u)}{(1-\tau_{\mathrm{k}})u} \equiv z(u;\tau_{\mathrm{k}}), \tag{13c}$$

where $\frac{\partial z}{\partial u} = \frac{\rho}{\alpha}\frac{-1+\alpha(1-\tau_{\mathrm{k}})}{(1-\tau_{\mathrm{k}})u^2} < 0$ and $\frac{\partial z}{\partial \tau_{\mathrm{k}}} = \frac{\rho}{\alpha}\frac{1}{(1-\tau_{\mathrm{k}})^2 u} > 0$. Intuitively, for a given ratio of consumption to physical capital, a larger fraction of labor allocated to working exerts two counteracting effects. First, it decreases goods production relative to capital and thus the growth rate of physical capital (cf. Eqs. (5) and (8)). Next, it decreases the marginal product of capital and thus the growth rate of consumption (cf. Eq. (9a)). As the decrease in the growth rate of consumption is smaller than that of the growth rate of capital,[8] in a BGP the ratio of consumption to capital needs to decrease in order to increase the growth rate of capital so as to equate the two growth rates. Moreover, a larger capital tax rate reduces the net marginal product of capital and thus the growth rate of consumption. In the long run, the ratio of consumption to physical capital needs to increase in order to equate the growth rates of physical capital and consumption.

Finally, substituting Eqs. (13a)–(13c), (10a) gives

$$l = \left\{\psi\frac{\rho}{B(1-\alpha)}\frac{[1-\alpha(1-\tau_{\mathrm{k}})(1-u)]}{(1-\tau_{\mathrm{h}})}\right\}^{\frac{1}{\sigma}} \equiv l(u;\tau_{\mathrm{h}},\tau_{\mathrm{k}}), \tag{13d}$$

where $\frac{\partial l}{\partial u} = \Upsilon\frac{\alpha(1-\tau_{\mathrm{k}})}{(1-\tau_{\mathrm{h}})} \equiv l_{\mathrm{u}} > 0$, $\frac{\partial l}{\partial \tau_{\mathrm{h}}} = \Upsilon\frac{1-\alpha(1-u)(1-\tau_{\mathrm{k}})}{(1-\tau_{\mathrm{h}})^2} \equiv l_{\tau\mathrm{h}} > 0$, $\frac{\partial l}{\partial \tau_{\mathrm{k}}} = \Upsilon\left[\frac{\alpha(1-u)}{(1-\tau_{\mathrm{h}})}\right] \equiv l_{\tau\mathrm{k}} > 0$, and $\Upsilon \equiv \psi\frac{1}{\sigma}\frac{\rho}{B(1-\alpha)}l^{1-\sigma} > 0$.

To understand these effects, first, a larger fraction of labor allocated to working reduces the net marginal product of labor in production, while the marginal product of learning is constant in the education time. As a lower marginal product of labor in production decreases the marginal cost of leisure, leisure is thus increasing in the fraction of labor allocated to working.[9] Next, a larger labor tax rate reduces the net marginal product of labor in the goods sector and thus the marginal cost of leisure. Thus, leisure is increasing in the labor income tax rate. Finally, a larger capital tax rate lowers the net marginal product of capital and thus lowers the shadow price of human capital relative to physical capital. A lower shadow price of human capital relative to physical capital reduces the marginal product of labor in the educational sector which lowers the marginal cost of leisure. As a result, leisure increases in order to reduce the marginal utility of leisure.

We are now ready to determine the BGP. Using Eq. (13d), the labor supply is $n^s = L - l(u;\tau_h,\tau_k)$. Moreover, using Eq. (6b), with the help of Eqs. (4c), (5), (13a) and (13b), the labor demand is $n^d = \rho/Bu$. The labor marker clearance condition is

$$n^{\mathrm{d}}(u) \equiv \frac{\rho}{B}\frac{1}{u} = L - l(u;\tau_{\mathrm{h}},\tau_{\mathrm{k}}) \equiv n^{\mathrm{s}}(u). \tag{14}$$

Thus, for given capital and labor tax rates, Eq. (14) determines u^* in a BGP. The value of p^*, l^*, q^*, z^* and G^* can then be obtained by substituting u^* into Eqs. (13a)–(13d) and (7). The left-hand side of Eq. (14), for simplicity, is referred to as $n^d(u)$ and the right-hand side as $n^s(u)$.

The shape of $n^{\mathrm{d}}(u)$ and $n^{\mathrm{s}}(u)$ is illustrated in Fig. 1 in which both loci are decreasing in u. We obtain the following result.

Proposition 1. *Assume that* $L > \left\{\frac{\psi}{B}\frac{\rho}{1-\alpha-\beta+\tau_k\alpha}\right\}^{1/\sigma} + \frac{\rho}{B}$. *Then, there exists a BGP.*

The condition in Proposition 1 serves to ensure $n^s(1) > n^{\mathrm{d}}(1) = \rho/B$ so that the loci $n^{\mathrm{d}}(u)$ and $n^{\mathrm{s}}(u)$ intersect once for all u in (0, 1). It is worth noting that $n^s(u)$ is decreasing in u and we cannot rule out the possibility of multiple BGPs. This is a feature common to Benhabib and Perli (1994) and Ladrón-de-Guevara et al. (1997, 1999) who studied an otherwise similar model except for factor taxes. If $\psi = 0$ and thus no leisure exists, the labor supply is a horizontal line L and is independent of u. In this case, if $L > \frac{\rho}{B}$, there exists a unique BGP.

To see the effects of factor tax rates on u, it is clear that both tax rates shift the labor supply curve downward in Fig. 1. Thus, u increases in a BGP. Intuitively, a higher factor tax rate does not affect the $n^{\mathrm{d}}(u)$ locus, but leisure is unambiguously increasing just as labor is unambiguously decreasing. As the labor supply is reduced, the labor demand needs to be reduced in equilibrium. Because of a diminishing marginal product of labor, the labor demand is inversely related to the fraction of labor allocated to working; the fraction of labor allocated to working increases resulting in a decrease in the fraction of the time devoted to education. Hence, the two factor tax rates affect p, q, z and l indirectly via their effects on u. By substituting these indirect effects, we can obtain the net effect of factor tax rates on the shadow price of human capital relative to physical capital p in Eq. (13a), the ratio of human capital to physical capital q in Eq. (13b), the ratio of consumption to capital z in Eq. (13c) and leisure l in Eq. (13d). In particular, using Eq. (13d), it is easy to see that both factor tax rates increase leisure l due to both a direct positive effect and an indirect positive effect via u. However, using Eq. (13c), a higher labor tax rate unambiguously decreases the ratio of consumption to capital z, but a higher capital tax rate has an ambiguous effect

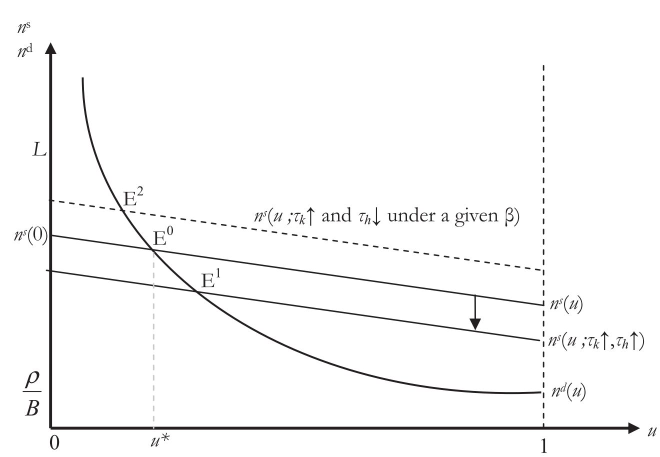


Fig. 1. The existence of BGP and the effect of the tax incidence in an economy with a linear learning function.

[8] Let $\Omega \equiv A\left[\frac{(1-\tau_h)(1-\alpha)A}{B}\right]^{\left(\frac{1-\alpha}{\alpha}\right)}(p)^{-\left(\frac{1-\alpha}{\alpha}\right)} > 0$. Eqs. (5a) and (8) give $\frac{\dot{k}}{k} \equiv \Omega - z$ and Eq. (9a) yields $\frac{\dot{c}}{c} \equiv (1-\tau_k)\alpha\Omega - \rho$. Given that p is increasing in u in Eq. (13a), Ω is decreasing in u. Since $(1-\tau_{\mathrm{k}})\alpha < 1$, under a given z, a higher u deceases $\frac{\dot{c}}{c}$ less than the decrease in $\frac{\dot{k}}{k}$. To attain $\frac{\dot{c}}{c} = \frac{\dot{k}}{k}$ in a BGP, z needs to decrease in u.

[9] If, as in Lucas (1990) and Jones et al. (1993, 1997), the learning technology is linear in human capital and concave in the education time, the resulting smaller fraction of the labor time allocated to learning will increase the marginal product of learning. This will increase the marginal cost of leisure and when this effect dominates the effect on the marginal product in the goods sector, leisure is decreasing, rather than increasing, in the fraction of labor time allocated to working.

on the ratio of consumption to capital z as the direct positive effect offsets the indirect negative effect via a higher u.

3.2. Effect of factor taxation and tax incidence

We are now ready to analyze the effect of the factor taxation. Using Eq. (1), the agent's welfare in the long run is

$$U^* = \frac{1}{\rho}\left[\ln k_0 + \ln z^* + \frac{\theta^*}{\rho} + \psi\frac{l^{*1-\sigma}-1}{1-\sigma}\right], \tag{15a}$$

where

$$\theta^* = \frac{\rho}{u^*} - \rho \tag{15b}$$

is the economic growth rate in the BGP.

Dividing both side of Eq. (7) by the total output in the economy gives

$$\tau_k r\frac{k}{y} + \tau_h w\frac{hun}{y} = \frac{G}{y}.$$

As in Lucas (1990) and Jones et al. (1993, 1997), we assume that the government transfer is a (exogenous) lump sum and accounts a given fraction of total output in the economy, $\beta<1$. As a result, the balanced government budget leads to

$$\tau_h = \frac{1}{1-\alpha}(\beta - \alpha\tau_k) \equiv \Gamma(\tau_k), \tag{16}$$

where $\frac{\partial\tau_h}{\partial\tau_k} = -\frac{\alpha}{1-\alpha} \equiv \Gamma_{\tau_k} < 0$.

Before investigating the tax incidence exercise, recall that both factor income tax rates have direct effects in increasing both leisure l and the fraction of labor allocated to working u (Fig. 1). Thus, the fraction of the education time $(1-u)$ is decreased which unambiguously decreases economic growth. However, a higher capital tax rate has an ambiguous effect on the ratio of consumption to capital z in Eq. (13c) while a higher labor tax rate unambiguously decreases the ratio of consumption to capital q in Eq. (13b). As both tax rates increase leisure but reduce economic growth, they both have ambiguous effects on welfare. Yet, a higher labor tax rate has an unambiguously negative effect on the ratio of consumption to capital as opposed to an ambiguous effect in the case of a higher capital tax rate. It seems that labor income taxation is more detrimental on welfare and it is worthwhile to replace the labor tax rate with the capital tax rate in a revenue-neutral tax reform.

Now, we conduct a tax incidence exercise. In a revenue-neutral tax reform, the government chooses two factor tax rates to finance a fixed fraction of the government expenditure in output that maximizes the representative agent's welfare in the long run.[10] Specifically, the government determines the capital income tax rate and the labor income tax rate to maximize the long-run social welfare in Eq. (15a) subject to the government budget in Eq. (16) and the equilibrium conditions summarized in Eqs. (13a)–(13d), (14) and (15b).

Suppose that the government increases τ_k with the corresponding decrease in τ_h so as to satisfy Eq. (16). First, from Eq. (14), the tax incidence exercises exert effects only on the $n^s(u)$ locus through the effect on leisure. From Eq. (13d), such a tax incidence has two opposing effects on leisure. First, a higher capital tax rate encourages leisure (through the term $l_{\tau k}>0$), but to meet the government balanced-budget constraint there is a corresponding decrease in the labor tax rate which discourages leisure (via the term $l_{\tau h}\Gamma_{\tau k}<0$). It is easy to show that the latter effect dominates the former effect: $l_{\tau k} + l_{\tau h}\Gamma_{\tau k} = \frac{1}{\sigma}l^*\frac{-\alpha[1-(1-u)(1-\beta)]}{(1-\alpha-\beta+\alpha\tau_k)[1-\alpha(1-u)(1-\tau_k)]} < 0$. Thus, leisure is decreasing and the labor supply is increasing. As a result, the labor supply locus $n^s(u)$ shifts upward. See Fig. 1.

As the tax incidence does not directly affect the labor demand locus $n^d(u)$, the fraction of labor allocated to working is smaller. Specifically, Eq. (14) gives the effects on u^*

$$\frac{du^*}{d\tau_k} = \frac{-1}{\Delta^*}(l_{\tau k} + l_{\tau h}\Gamma_{\tau k}) < 0, \tag{17}$$

where $\Delta^* \equiv \left(-\frac{\rho}{Bu^2} + l_u\right) < 0$ since the negatively sloping $n^d(u)$ locus is steeper than the negatively sloping $n^s(u)$ locus. See E^2 in Fig. 1.

Intuitively, when the government increases the capital tax rate and decreases the labor tax rate to finance a given fraction of government spending in output, the higher capital tax rate decreases the labor supply while the lower labor income tax rate increases the labor supply. Because the effect through the lower labor tax rate dominates, the labor supply is increasing. In order to clear the labor market, the labor demand must increase and a higher marginal product of labor in production is necessary. As a result, the fraction of the labor supply allocated to working must be decreasing.

The total effect on leisure is a combination of the direct negative effect of the tax incidence and an indirect negative effect as the result of a smaller fraction of labor allocated to working. Thus, leisure is unambiguously decreasing.

$$\frac{dl^*}{d\tau_k} = l_u\frac{du}{d\tau_k} + (l_{\tau k} + l_{\tau h}\Gamma_{\tau k}) < 0. \tag{18a}$$

Moreover, a smaller fraction of labor allocated to working and thus, a larger education time generates a positive effect on economic growth.

$$\frac{d\theta^*}{d\tau_k} = -\frac{\rho}{(u^*)^2}\frac{du^*}{d\tau_k} > 0. \tag{18b}$$

Furthermore, using Eqs. (17) and (13c), a larger capital tax rate in combination with the resulting smaller labor income tax rate increases the ratio of consumption to capital.

$$\frac{dz^*}{d\tau_k} = \underset{(+)}{\frac{\partial z^*}{\partial\tau_k}} + \underset{(-)}{\frac{\partial z^*}{\partial u}}\underset{(-)}{\frac{du^*}{d\tau_k}} > 0. \tag{18c}$$

Two positive effects are at work in Eq. (18c). There is a direct positive effect on consumption because of a lower price of consumption relative to investment. There is an indirect effect because of a smaller fraction of labor allocated to working which results in a larger ratio of consumption to capital. Both effects increase the ratio of consumption to capital.

Finally, the effects in Eqs. (18a)–(18c) stipulate that the effect on the social welfare in the long run is

$$\frac{dU^*}{d\tau_k} = \frac{1}{\rho}\left[\underset{(+)}{\left(\frac{1}{z^*}\frac{dz^*}{d\tau_k}\right)} + \left(\underbrace{-\frac{1}{(u^*)^2}\frac{du^*}{d\tau_k}}_{(+)}\right) + \psi(l^*)^{-\sigma}\underset{(-)}{\frac{dl^*}{d\tau_k}}\right] > (<) 0. \tag{19a}$$

It is clear that because of a smaller level of leisure, the revenue-neutral tax reform of increasing the capital income tax rate and reducing the labor income tax rate reduces social welfare. Yet, due to a higher economic growth rate and a larger ratio of consumption to physical capital, the tax reform increases social welfare.

If the effect coming from lower leisure dominates, it is best just to tax labor income and not to tax capital income. If the effects originated from higher economic growth rates and higher ratios of consumption to capital dominate, it is best just to tax just capital income alone and not labor income. Finally, if these two opposite effects completely offset each other, there exists an interior optimal capital tax rate that

[10] That is, we use the dual approach in which the tax rates are viewed as governmental decision variables, as opposed to the primal approach used in Lucas (1990) in which the government chooses a feasible allocation that is implementable by the set of taxes.

maximizes the social welfare in the long run. In this case, the optimal capital tax rate τ_k is determined by

$$\left(\frac{1}{z^*}\frac{dz^*}{d\tau_k}\right)+\frac{1}{(u^*)^2}\left(-\frac{du^*}{d\tau_k}\right)=\psi l^{*-\sigma}\left(-\frac{dl^*}{d\tau_k}\right). \tag{19b}$$

Our analysis thus indicates that the optimal capital tax rate may be an interior solution or a corner solution. It is worth noting that when $\psi=0$, there is no leisure effect. Then, Eq. (19a) is always positive, indicating that it is always optimal to tax the capital income and not to tax the labor income. The reason is that leisure has no utility and the labor endowment is used either in working or in learning. Increasing the capital income tax rate and decreasing the labor income tax rate discourage physical capital accumulation. As capital and human capital are complements in the goods production, a smaller fraction of labor is allocated to working and a larger fraction is allocated to learning. As a result of enhanced learning processes, human capital formation is accumulated faster and thus the economic growth rate is higher. Moreover, consumption is also increasing because of a positive direct effect and a positive indirect effect. Thus, if a given fraction of government expenditure in output is needed as a transfer, the tax burden should be entirely on capital if the tax rate is less than 100%.[11]

Notice that as ψ increases, the effect through lower leisure is stronger and then there is a gain in welfare of a switch from the capital income tax to the labor income tax rate. It is also worth noting that the first-best policy is to have a zero fraction of government expenditure since the transfer is a lump sum while factor taxes have distortions. In the first-best policy, neither the labor income tax nor the capital income tax is needed in optimum.

3.3. Quantitative optimal factor income taxes

This subsection calibrates our model to quantitatively determine the optimal factor tax rates. We calibrate the model in the BGP in order to reproduce key features representative of the U.S. economy in annual frequencies. Using the tax rates in McDaniel (2007), the average tax rates of the capital income and the labor income during 1960–2007 are around 0.3 and 0.2, respectively.[12] Thus we set initial tax rates at $\tau_k=0.3$ and $\tau_h=0.2$.

The time endowment is assumed to be $L=100$ units. As pointed out by Prescott (2006), the fraction of time allocated to market is around 25 percent. We choose $n^*=25$, and thus, $l^*=75$. Initial physical capital stock is assumed to be $k(0)=100$. There is no data for human capita, but human capital is as large as physical capital, as argued by Kendrick (1976). Based on this and following Chen et al. (2011), we normalize the ratio of physical capital to human capital at $q^*(0)=h(0)/k(0)=1$.

The IES for labor ranges from close to 0 (MaCurdy, 1981) to 3.8 (Imai and Keane, 2004). Following Hansen and Imrohoroglu (2009), we choose a middle value of the Frisch labor elasticity at $(L\text{-}n)/(\sigma n)=2$ as our benchmark case, which implies $\sigma=1.5$. Later, we will study the sensitivity of results by changing the value of the IES for labor to 1 and 3, in which case $\sigma=3$ and 1, respectively.

We choose the share of physical capital in the goods sector at $\alpha=0.3$. The rate of time preference is set at $\rho=4\%$ as used by Kydland and Prescott (1991).

Using the parameter values given above, we calibrate the fraction of labor allocated to the goods sector (u) to match the $\theta=2\%$ per capita economic growth rate in the long run and we obtain $u^*=0.6667$. Given this, we use Eq. (13c) to compute the ratio of consumption to capital and obtain $z^*=0.2657$. Equations (13a) and (13b) together, with the value of u^* and q^*, yield the ratio of the shadow price of human capital to the shadow price of capital $p^*=4.0000$. Then, we use Eqs. (13a) and (10b) to calibrate $A=0.0399$ and $B=0.0024$. Finally, using Eq. (13d), we obtain the degree of leisure in utility relative to consumption at $\psi=23.4665$.

We summarize the benchmark parameter values and the calibrated values in Table 1. Under these benchmark parameter values, there exists a unique BGP in our model. The equilibrium values for time allocation, welfare level and economic growth rate in the BGP are $n^*=25$, $l^*=75$, $U^*=1132.3368$, and $\theta=2\%$. The pre-existing tax rates imply that in the BGP, the share of government spending in output is $\beta=0.2300$.

According to Eq. (15a), the representative household's welfare in the long run is determined by the ratio of consumption to physical capital, the level of leisure and the economic growth rate in the long run. Before we quantify the effects of the tax incidence, we calculate the effect of raising one of the two tax rates by one percentage point while maintaining the other tax rate as fixed at benchmark level. The results are in Table 2. When the government increases the capital income tax rate (τ_k) by 1.0 percentage point, the government transfer (β) increases by 0.3 percentage points. Under these changes, the fraction of labor allocated to the learning $(1-u)$ is decreasing thus reducing the economic growth rate (θ). Yet, leisure (l) and the ratio of consumption to capital (z) are both increasing. Although a lower economic growth rate reduces welfare, our quantitative result indicates that the positive welfare effects through more leisure and a larger ratio of consumption to capital dominate the negative welfare effect resulted from the lower economic growth rate. As a result, the agent's welfare is increasing in the long run. Alternatively, when the government increases the labor income tax rate (τ_h) by 1 percentage point, the government transfer increases by 0.7 percentage points. Under these changes, the fraction of labor allocated to the learning is decreasing and thus the economic growth rate decreases. Moreover, the ratio of consumption to capital decreases. Yet, leisure increases. Our quantitative result suggests that the negative welfare

Table 1
Benchmark parameter values and calibration under the Lucas (1988) learning technology.

Benchmark parameters and observables		
Per capita real economic growth rate	θ	0.02
Time preference rate	ρ	0.04
Tax rate on capital	τ_k	0.30
Tax rate on labor	τ_h	0.20
Physical capital's share	α	0.3
Initial capital stock	k_0	100
Human capital-physical capital ratio	$q=(h/k)$	1.0
Total labor hours	L	100
Total employment hours	n	25
The IES for leisure	IES	2
Coefficient about the IES for leisure	σ	1.5
Calibration		
Coefficient of goods technology	A	0.0399
Coefficient of learning technology	B	0.0024
Fraction of employment time in working	u	0.6667
Consumption-physical capital ratio	$z=(c/k)$	0.2657
Ratio of shadow price of human to physical capital	p	4.0000
Leisure time	l	75.000
Degree of leisure in utility relative to consumption	ψ	23.4665
The fraction of government spending to output	β	0.2300

[11] A capital tax rate equal to (or larger than) 100% is not consistent with positive shadow prices of human capital and capital, positive stock of human capital and capital and a positive and bounded consumption to capital ratio. Should the capital tax rate be equal to (or larger than) 100%, the allocation is not feasible. Obviously, Eq. (13a) would have then implied a zero (or a negative) shadow price of human capital in terms of the shadow price of capital, Eq. (13b) an infinite (or a negative) ratio of human capital to capital and Eq. (13c) an infinite (or a negative) consumption to capital ratio. In either case, the allocation is not feasible. Moreover, as a referee pointed out, a 100% capital tax rate is not consistent with a positive economic growth rate in the BGP as then the transversality condition is violated. Thus, the capital tax rate must be less than 100%.

[12] McDaniel (2007) calculated a series of average tax rates on consumption, investment, labor and capital using national account statistics in 15 OECD countries. The data has been used by Rogerson (2008) and Ohanian et al. (2008).

Table 2
The effects of changing one of the two tax rates under the Lucas (1988) learning technology.

τ_k (%)	τ_h (%)	β (%)	u^* (%)	θ^* (%)	l^*	z^*	U^*
Benchmark case							
30	20	23.00	66.67	2.00	75.000	0.2657	1132.3368
Effect of an increase in one of the tax rates by one percentage point							
31	20	23.30	66.87	1.98	75.0765	0.2692	1132.6124
30	21	23.70	69.22	1.78	75.9219	0.2574	1130.9833

effects resulted from both a lower economic growth rate and a smaller ratio of consumption to capital are larger than the positive welfare effect resulted from higher leisure. Thus, the agent's welfare is decreasing in the long run.

We are now ready to quantify the effect of the tax incidence between τ_k and τ_h in the long run. We conduct changes in τ_k from 15% to 99.99%,[13] and determine the resulting τ_h that finances the same fraction of the government spending in output as in the benchmark case. The results are reported in Fig. 2. It is clear that as τ_k increases from 15.00% and thus τ_h decreases from 26.43% (in the top left diagram), as expected from Eq. (18a) leisure decreases in the top right diagram of Fig. 2. Moreover, as expected from Eqs. (18b) and (18c), we see the ratio of consumption to capital in the center diagram and the economic growth rate in the middle right diagram both increase. While the leisure effect decreases welfare, the last two effects increase welfare. As the last two effects quantitatively dominate the leisure effect, the welfare in the bottom left diagram is increasing. Our quantitative results indicate that if the welfare-maximization is to be achieved, the optimal tax mix is at $(\tau_k, \tau_h)=(99.99\%, -10.00\%)$. This is also the optimal tax mix under the growth maximization standard. Moving from the benchmark tax mix to the optimal tax mix leads to a negative welfare cost and thus, a welfare gain. See the bottom middle and right diagrams of Fig. 2. Our results indicate that the welfare gain is 99.99% both in terms of changes in consumption equivalence and changes in output equivalence. The results are in Table 3.

One may wonder whether taxing capital at a rate near 100% really improves the welfare because the agent loses incentives to save. First, the agent still saves as long as the capital tax rate is less than 100%. This is because according to Eq. (4d), the agents demand for capital is positive if the after-tax return to capital is $(1-\tau_k)r$ is positive (and is equal to the time preference rate ρ minus the growth rate of capital's shadow prices $\frac{\dot{\lambda}}{\lambda}$, which is a capital gain). As $\tau_k<1$, then $(1-\tau_k)r>0$ and Eq. (4d) holds with the equality with the positive stock of capital. Thus, as long as $\tau_k<100\%$, the agent still saves and accumulates capital.

Next, a capital tax rate near 100% is an interior solution and is optimal. The reason is as follows. According to Eq. (15a), the long-run welfare depends positively on (i) the consumption to capital ratio ($z\equiv c/k$), (ii) the economic growth rate (θ) and (iii) the leisure time (l). In the tax incidence exercise, we have shown in Eqs. (18a)–(18c) that, alongside a corresponding decrease in the labor tax rate, an increase in the capital tax rate always leads to a higher z, a higher θ and a lower l. Positive effects on z and θ increase welfare but a negative effect on l decreases welfare and the net effect on welfare thus depends on which effects dominate. To see the net effect, our quantitative results in Fig. 2 indicate that starting from the baseline tax rates of $(\tau_k, \tau_h)=(0.3, 0.2)$, an increase in the capital tax rate accompanied by a corresponding decrease in the labor tax rate raises the consumption to capital ratio z and the economic growth rate θ by large margins and lowers the leisure time by a small margin.

In particular, according to Fig. 2, when the capital tax is increased to a rate above 76.67%, the labor tax rate is negative and is, thereby, a subsidy to working labor. Then, the incentive to accumulate human capital is so large and the incentive to accumulate physical capital is so small so that the accumulation of physical capital is slowed and the human capital to capital ratio h/k is increased by a large margin (cf. the middle left diagram of Fig. 2). Because of large increases in human capital, the economic growth rate is almost double from the baseline rate of $\theta=2\%$ (cf. the middle right diagram). With such a high economic growth rate accompanied by a small incentive to accumulate physical capital, consumption is increased substantially; further, the consumption to capital ratio z is increased by more than 5 times from its baseline level at $z=0.2657$ (cf. the middle center diagram). Conversely, the decrease in leisure time is flat indicating a small welfare loss (cf. the top right diagram). As a result of large welfare gains due to higher consumption to capital ratio and higher economic growth and a small welfare loss caused by lower leisure time, it is always welfare- improving if the capital tax is increased. Then, the optimal capital tax rate is the highest possible rate that provides for a feasible allocation. Thus, in Table 3, the optimal capital tax rate is 99.99%.[14]

In order to obtain an optimal capital tax rate below 99%, we restrict to a corner solution by imposing a non-negativity constraint on the labor tax rate. The restriction is reasonable because in reality the labor tax rate is not negative on average, although there are situations wherein some laborers' income is subsidized. By imposing the constraint of a non-negative labor tax rate, then the optimal tax rate is lower. The optimal factor tax mix is thus $(\tau_k, \tau_h)=(76.67\%, 0\%)$. See Table 4. In this case, there is no subsidy to labor, so the optimal tax rate on capital income is smaller. The optimal capital tax rate is then a drop by more than 23 percentage points from an interior solution. However, the welfare gain of the tax reform is 80.10% in terms of changes in consumption equivalence and 77.26% in terms of changes in output equivalence, which is clearly smaller than the welfare gain in the absence of the non-negativity constraint.

While the positive optimal capital tax rate above is obtained based on the value of the IES for labor equal 2, our result still holds true if the IES of labor is changed. When we decrease the value of the IES for labor to 1 (so the value of σ is 3) or increase the value of the IES for labor to 3 (so the value of σ is 1), we find the same results. Our quantitative exercises indicate that, under both values of the IES for labor, the optimal tax mix is still at $(\tau_k, \tau_h)=(99.99\%, -10.00\%)$. The results are also in Table 3.[15]

Since a larger value of ψ increases the negative effect of leisure, one may wonder whether one can increase the value of ψ and change the above result. In this model, we can change the value of ψ to a value less than 34 above which the equilibrium allocation is infeasible as $u>1$ and n becomes negative. For values of ψ that produce feasible allocation, we obtain the same results.

In our previous calibration, the benchmark factor tax rates of $(\tau_k, \tau_h)=(30\%, 20\%)$ are smaller than those used in Lucas (1990) which are $(\tau_k, \tau_h)=(36\%, 40\%)$. We also calibrate our model under $(\tau_k, \tau_h)=(36\%, 40\%)$ while maintaining other parameter values the same as those in Table 1.[16] We then change the value of τ_k from the smallest feasible value of 24% to the largest value of 99.99% and determine the resulting τ_h that finances the same fraction of the government spending in output as in the benchmark case. The results (not reported) are similar to those in Fig. 2. We find that as τ_k is increasing from 24% and thus τ_h is decreasing from 45.14%, leisure decreases while the ratio of consumption to capital and the economic growth rate both increase. Since the latter two effects dominate the leisure effect, welfare is increasing with regard to the capital tax rate. As the feasible

[13] This range of τ_k is chosen in order to assure a reasonable BGP in that (i) $0<u<1$, (ii) $\theta\geq 0$, and (iii) q and z are non-negative, real numbers. As noted earlier, the allocation is feasible only when $\tau_k<100\%$.

[14] We have shown that by allowing for the depreciation of capital at 10%, quantitative results are like those in Fig. 2. The optimal capital tax rate is still 99.99%.

[15] For robustness analysis that follows, we also calculate the optimal factor tax mix under the constraint of a non-negative labor tax rate.

[16] The resulting calibrated parameter values are $A=0.0436$, $B=0.0024$, $\psi=17.4871$ and $\beta=0.3880$. The equilibrium values in BGP are: $u^*=0.6667$, $z^*=0.2925$, $p^*=3.2812$, $n^*=25$, $l^*=75$, $\theta=2\%$ and $U^*=870.2880$.

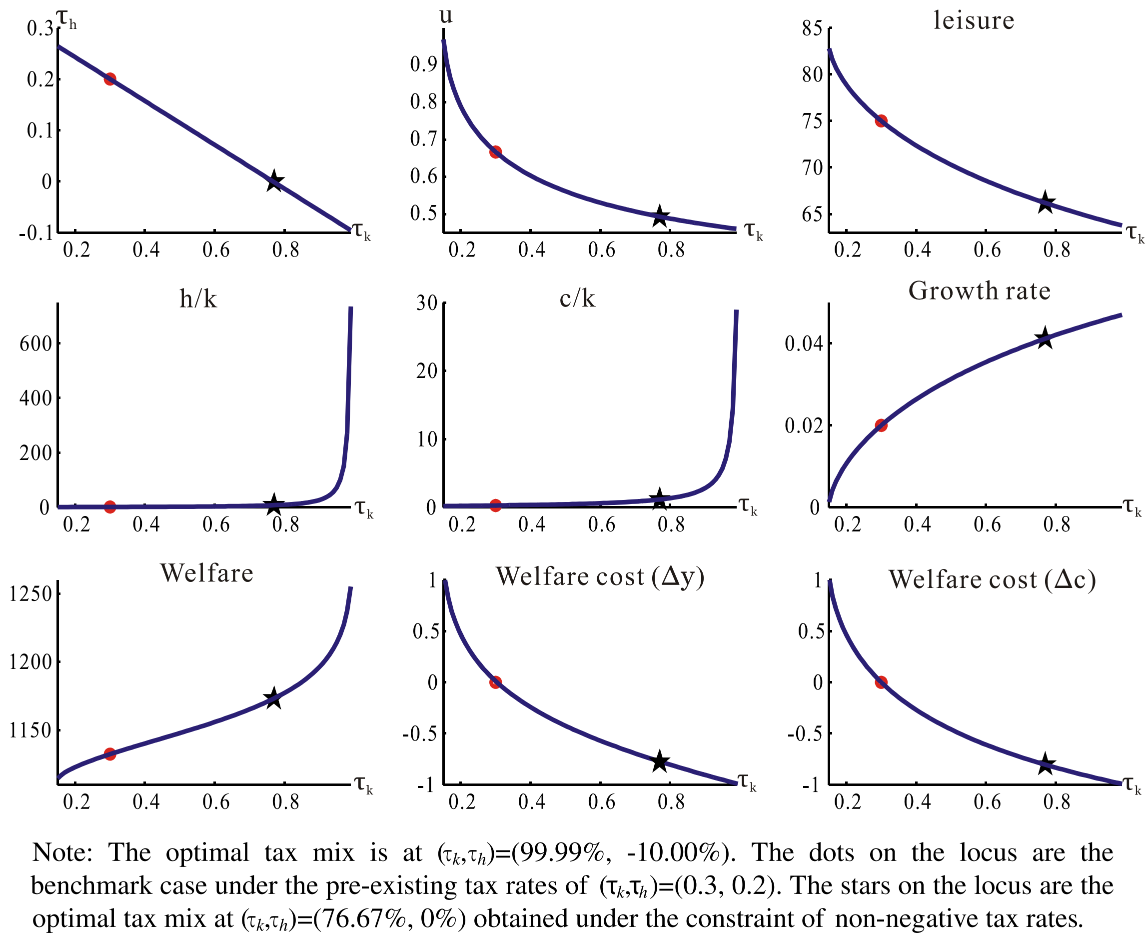


Fig. 2. The results of dynamic tax incidence under a linear learning function.

capital tax rate is no more than 100%, the optimal tax mix is at $(\tau_k, \tau_h) = (99.99\%, 12.58\%)$. This is also the tax mix under the growth-maximizing standard. It is worth noting that the required share of government expenditure in output ($\beta = 0.388$) is larger than the share of capital income in output ($\alpha = 0.3$). When capital income is taxed at 99.99%, the revenue cannot finance the government expenditure. It is required to tax labor income. As compared with those in the benchmark case, the welfare gain at the optimal tax rates is 99.99% both in terms of changes in consumption equivalence and changes in output equivalence.

Our benchmark case reproduces key features representative of the U.S. economy. One may wonder whether our results hold true in rapid growing economies like China. Unfortunately, the factor tax rates are not available for China. Alternatively, Taiwan had a similar experience with very high economic growth before 1990 similar to what China has achieved over the past 30 years. Taiwan had an annual growth rate in real gross domestic product (GDP) of over 10% from 1952–1990. Data for the factor income taxes in Taiwan are available from 1970. Thus, we quantify the model for Taiwan for the period 1970 to 1990, wherein the average annual growth rate of real GDP is $\theta = 9\%$. The average tax rates of capital income and labor income are around $\tau_k = 0.19$ and $\tau_h = 0.11$, respectively.[17] We re-calibrate our model under $(\tau_k, \tau_h) = (19\%, 11\%)$ and $\theta = 9\%$ while maintaining other parameters with the same values as those in Table 1.[18] We then change the value of τ_k from the smallest feasible value of -17% to the largest value of 99.99% and determine the resulting τ_h that finances the same fraction of the government spending in output as in the benchmark case. The results (not reported) are similar to those in Fig. 2. We find that as τ_k increases from -17% and thus τ_h decreases from 26.43%, leisure decreases while the ratio of consumption to capital and the economic growth rate both increase. Since the last two effects dominate the leisure effect, welfare is increasing with regard to the capital tax rate. The optimal tax mix is at $(\tau_k, \tau_h) = (99.99\%, -23.71\%)$. Note that this optimal subsidy rate to the labor income is larger than a 10.00% in an otherwise identical but slower growing economy. This is because high economic growth rates are driven by human capital formation. Thus, a switch to taxing capital along with providing a larger subsidy to labor gives a larger welfare gain in terms of higher economic growth.

Finally, we should note that if the learning curve is not found the form of Eq. (3), but is the one used in Lucas (1990), the quantitative result is completely reversed. To see this result, the learning function elucidated by Lucas (1990) is

$$\dot{h} = B[(1-u)n]^{\eta}h,\ h(0)\ \text{given}, \eta < 1. \tag{20}$$

Equations for preferences, individuals' budgets, the goods technology, and the government budget are the same as those in Eqs. (1), (2), (5) and (7).[19]

In quantitative analysis, starting from pre-existing tax rates of $(\tau_k, \tau_h) = (30\%, 20\%)$, if the capital tax rate is increased and the labor tax rate is decreased, we find that the negative welfare effect from

[17] The data with regard to the real economic growth rate for Taiwan is taken from the website, National Statistics, http://ebas1.ebas.gov.tw/pxweb/Dialog/statfile9L.asp, collected by the Directorate-General of Budget, Accounting and Statistics. The data with regard to the factor tax rates is taken from the *Yearbook of Tax Statistics* 1970–1990, collected by the Ministry of Finance.

[18] The resulting calibrated parameter values are $A = 0.1283$, $B = 0.0052$, $\psi = 63.2442$ and $\beta = 0.1340$. The equilibrium values in BGP are: $u^* = 0.3077$, $z^* = 0.4450$, $p^* = 8.3323$, $n^* = 25$, $l^* = 75$, $\theta = 9\%$ and $U^* = 2948.2038$.

[19] See the Appendix for an analysis of the model and a quantitative analysis.

Table 3
Optimal taxation.

Model	IES	σ	τ_k (%)	τ_h (%)	Δ_y (%)	Δ_c (%)
(1) Lucas (1988) model						
Basic model						
Benchmark case	2	1.5	99.99	−10.00	−99.99	−99.99
	1	3	99.99	−10.00	−99.99	−99.99
	3	1	99.99	−10.00	−99.99	−99.99
Initial $(\tau_k, \tau_h)=$ (36%, 40%)	2	1.5	99.99	12.58	−99.99	−99.99
Taiwan case	2	1.5	99.99	−23.71	−99.99	−99.99
Progressive tax in labor income						
	2	1.5	99.82	−9.92	−99.81	−99.83
	1	3	99.91	−9.96	−99.88	−99.89
	3	1	99.66	−9.85	−99.69	−99.75
G is thrown away						
	2	1.5	99.99	−10.00	−76.99	−99.99
	1	3	99.99	−10.00	−76.99	−99.99
	3	1	99.99	−10.00	−76.99	−99.99
G enters the household's utility in a separate form $u(c, l, G)$						
$\psi_2=1$	2	1.5	99.99	−10.00	−77.00	−100.00
$\psi_2=1$	1	3	99.99	−10.00	−77.00	−100.00
$\psi_2=1$	3	1	99.99	−10.00	−77.00	−100.00
$\psi_2=0.5$	2	1.5	99.99	−10.00	−77.00	−100.00
$\psi_2=2$	2	1.5	99.99	−10.00	−77.00	−100.00
Different utility form $u(c, n)$						
	2	0.5	99.99	−10.00	−99.99	−99.99
	1	1	99.99	−10.00	−99.99	−99.99
	3	0.3333	99.99	−10.00	−99.99	−100.00
General two-sector model: both sectors use k and h						
	2	1.5	99.85	−9.94	−99.30	−99.30
	1	3	99.87	−9.94	−99.13	−99.14
	3	1	99.84	−9.93	−99.51	−99.51
Initial $(\tau_k, \tau_h)=$ (36%, 40%)	2	1.5	99.93	12.60	−99.33	−99.33
(2) Lucas (1990) model ($\eta=0.9$)						
	2	1.5	−20.41	41.60	−86.72	−86.83
	1	3	−7.89	36.24	−64.44	−65.73
	3	1	−2.53	33.94	−86.19	−86.52
Initial $(\tau_k, \tau_h)=$ (36%, 40%)	2	1.5	−3.32	56.85	−54.48	−54.53
(3) Learning time is more important than the human capital ($\eta=1.1$)						
	2	1.5	99.99	−10.00	−99.99	−99.99
	1	3	99.99	−10.00	−99.99	−99.99
	3	1	99.99	−10.00	−99.99	−99.99
(4) Learning-by-doing model						
$\eta_1=1$	2	1.5	99.99	−10.00	−99.99	−99.99
$\eta_1=1$	1	3	99.99	−10.00	−99.99	−99.99
$\eta_1=1$	3	1	99.99	−10.00	−99.99	−99.99
$\eta_1=0.9$	2	1.5	99.99	−10.00	−99.99	−99.99
$\eta_1=0.9$	1	3	99.99	−10.00	−99.99	−99.99
$\eta_1=0.9$	3	1	99.99	−10.00	−99.98	−99.99
$\eta_1=0.5$	2	1.5	99.99	−10.00	−99.98	−99.99
$\eta_1=0.1$	2	1.5	99.99	−10.00	−99.98	−99.99

Note. Δ_c and Δ_y are the welfare cost at the optimal tax rates in terms of changes in consumption equivalence and in terms of changes in output equivalence, respectively. Thus, $\Delta_c<0$ and $\Delta_y<0$ are welfare gains.

lower leisure dominates the positive welfare effect from higher consumption and economic growth. See Fig. 3 for the results under the pre-existing tax rates of $(\tau_k, \tau_h)=(30\%, 20\%)$. As seen from the figure, a decrease in the capital tax rate accompanied with an increase in the labor tax rate from their benchmark tax mix increases leisure in the top right diagram, but the consumption to capital ratio is reduced in the center diagram and the economic growth rate is also reduced in the middle right diagram. As the last two effects are dominated by the leisure effect, the welfare is decreasing in the capital tax rate in the bottom left diagram. The result of a negative capital tax rate is robust if the value of the IES for labor is decreased to 1 or increased to 3. The result holds if the pre-existing tax rates is $(\tau_k, \tau_h)=(36\%, 40\%)$ as in Lucas (1990). See Table 3. This is why in Lucas (1990) it was optimal to tax labor income taxes but not to tax capital income taxes.

4. Robustness of positive capital tax rates

In our basic model, we assume flat factor tax rates with the revenue transferred to the household in a lump-sum fashion, but there may be progressive or regressive tax policies and different ways of spending the tax revenue. Moreover, we use a utility function with leisure so the IES for labor depends on the labor supply. Furthermore, our

Table 4
Optimal taxation under non-negative tax rates.

Model	IES	σ	τ_k (%)	τ_h (%)	Δ_y (%)	Δ_c (%)
(1) Lucas (1988) model						
Basic model						
Benchmark case	2	1.5	76.67	0.00	−77.26	−80.10
	1	3	76.67	0.00	−72.87	−75.15
	3	1	76.67	0.00	−80.42	−83.71
Initial (τ_k, τ_h) = (36%, 40%)	2	1.5	99.99	12.58	−99.99	−99.99
Taiwan case	2	1.5	44.67	0.00	−41.45	−47.12
Progressive tax in labor income						
	2	1.5	76.67	0.00	−75.55	−78.23
	1	3	76.67	0.00	−70.81	−72.91
	3	1	76.67	0.00	−78.98	−82.12
G is thrown away						
	2	1.5	76.67	0.00	−57.02	−77.57
	1	3	76.67	0.00	−54.57	−73.73
	3	1	76.67	0.00	−58.89	−80.53
G enters the household's utility in a separate form $u(c, l, G)$						
$\psi_2=1$	2	1.5	76.67	0.00	−73.28	−99.68
$\psi_2=1$	1	3	76.67	0.00	−73.61	−99.46
$\psi_2=1$	3	1	76.67	0.00	−72.99	−99.81
$\psi_2=0.5$	2	1.5	76.67	0.00	−71.55	−97.33
$\psi_2=2$	2	1.5	76.67	0.00	−73.51	−100.00
Different utility form $u(c, n)$						
	2	0.5	76.67	0.00	−79.33	−82.47
	1	1	76.67	0.00	−73.54	−75.90
	3	0.3333	76.67	0.00	−83.96	−87.83
General two-sector model: both sectors use k and h						
	2	1.5	76.67	0.00	−45.55	−48.37
	1	3	76.67	0.00	−46.06	−48.53
	3	1	76.67	0.00	−45.28	−48.35
Initial (τ_k, τ_h) = (36%, 40%)	2	1.5	99.93	12.60	−99.33	−99.33
(2) Lucas (1990) model ($\eta=0.9$)						
	2	1.5	0.00	32.86	−79.87	−80.59
	1	3	0.00	32.86	−58.38	−60.30
	3	1	0.00	32.86	−85.33	−85.75
Initial (τ_k, τ_h) = (36%, 40%)	2	1.5	0.00	55.43	−52.84	−52.91
(3) Learning time is more important than the human capital ($\eta=1.1$)						
	2	1.5	76.67	0.00	−75.30	−77.62
	1	3	76.67	0.00	−71.51	−73.49
	3	1	76.67	0.00	−78.07	−80.68
(4) Learning-by-doing model						
$\eta_1=1$	2	1.5	76.67	0.00	−65.86	−67.60
$\eta_1=1$	1	3	76.67	0.00	−66.08	−67.77
$\eta_1=1$	3	1	76.67	0.00	−65.74	−67.50
$\eta_1=0.9$	2	1.5	76.67	0.00	−65.67	−67.38
$\eta_1=0.9$	1	3	76.67	0.00	−65.95	−67.62
$\eta_1=0.9$	3	1	76.67	0.00	−65.52	−67.24
$\eta_1=0.5$	2	1.5	76.67	0.00	−65.03	−66.65
$\eta_1=0.1$	2	1.5	76.67	0.00	−64.44	−65.99

Note: Δc and Δy are the welfare cost at the optimal tax rates in terms of changes in consumption equivalence and in terms of changes in output equivalence, respectively. Thus, $\Delta_c<0$ and $\Delta_y<0$ are welfare gains.

learning technology assumes an equal contribution of the training time and the stock of human capital and does not take into account learning by doing. In this Section, we will investigate the robustness of positive capital tax rates by departing the setup from our basic model.

4.1. Progressive or Regressive Tax Policies

In this subsection, we depart from the basic model by considering progressive or regressive tax policies. The revised model is the same as the basic model in Section 2 except for the tax function. Following Conesa et al. (2009), we assume that the set of tax revenue function is given by

$$T=\{T_{\mathrm{h}}(y_{\mathrm{h}}),T_{\mathrm{k}}(y_{\mathrm{k}}):T_{\mathrm{h}}(y_{\mathrm{h}})=T_{\mathrm{h}}(y_{\mathrm{h}};\kappa_0,\kappa_1,\kappa_2)\text{ and }T_{\mathrm{k}}(y_{\mathrm{k}})=\tau_{\mathrm{k}}y_{\mathrm{k}}\},$$

where κ_0, κ_1 and κ_2 are parameters and $y_{\mathrm{h}}=wunh$ and $y_{\mathrm{k}}=rk$ are labor income and capital income, respectively.[20]

The tax function $T_i(y_i;\kappa_0,\kappa_1,\kappa_2)=\kappa_0\left(y_i-\left(y_i^{-\kappa_1}+\kappa_2\right)^{-1/\kappa_1}\right)$ was proposed by Gouveia and Strauss (1994), where y_i is taxable income. However, as κ_2 is a constant, this form is not consistent with sustainable growth. To be compatible with perpetual growth, we allow for the term associated with κ_2 to vary with the stock of human capital. Thus, we adopt the following form.

$$T_h(y_{\mathrm{h}})=\kappa_0\left(y_{\mathrm{h}}-\left(y_{\mathrm{h}}^{-\kappa_1}+\kappa_2h^{-\kappa_1}\right)^{-1/\kappa_1}\right). \tag{21}$$

The government's budget constraint is $\tau_{\mathrm{k}}rk+T_{\mathrm{h}}(y_{\mathrm{h}})=G$ and the representative agent's budget constraint is $\dot{k}=(1-\tau_{\mathrm{k}})rk+wunh-T_{\mathrm{h}}(y_{\mathrm{h}})-c+G$. The average rate of the labor income tax is $\tau_{\mathrm{h}}=T_{\mathrm{h}}(y_{\mathrm{h}})/y_{\mathrm{h}}=T_{\mathrm{h}}(y_{\mathrm{h}})/(wunh)$.

We then solve the model and carry out a quantitative analysis.[21] In quantitative exercises, we also start from pre-existing tax rates of $(\tau_{\mathrm{k}},\tau_{\mathrm{h}})=(30\%,20\%)$ and maintain other parameter values the same as those in Table 1. Gouveia and Strauss (1994) estimated the parameters $(\kappa_0,\kappa_1,\kappa_2)$ that best approximate taxes paid under the actual US income tax system and found $\kappa_0=0.258$ and $\kappa_1=0.768$. Following Conesa et al. (2009) and Peterman (2012), we set $\kappa_0=0.258$ and $\kappa_1=0.768$ and calibrate parameter κ_2 to ensure government budget balance. We obtain $\kappa_2=7.3878$.

We then change the value of τ_{k} from the smallest feasible value of 17% to the largest value of 99.99% and use Eq. (21) to determine parameter κ_2 and the resulting total tax revenue of labor income $T_{\mathrm{h}}(y_{\mathrm{h}})$ that finances the same fraction of the government spending in output as in the benchmark case. We obtain similar results as those in the benchmark model in subsection 3.3. The optimal tax mix is at $(\tau_{\mathrm{k}},\tau_{\mathrm{h}})=(99.82\%,-9.92\%)$. See Table 3.

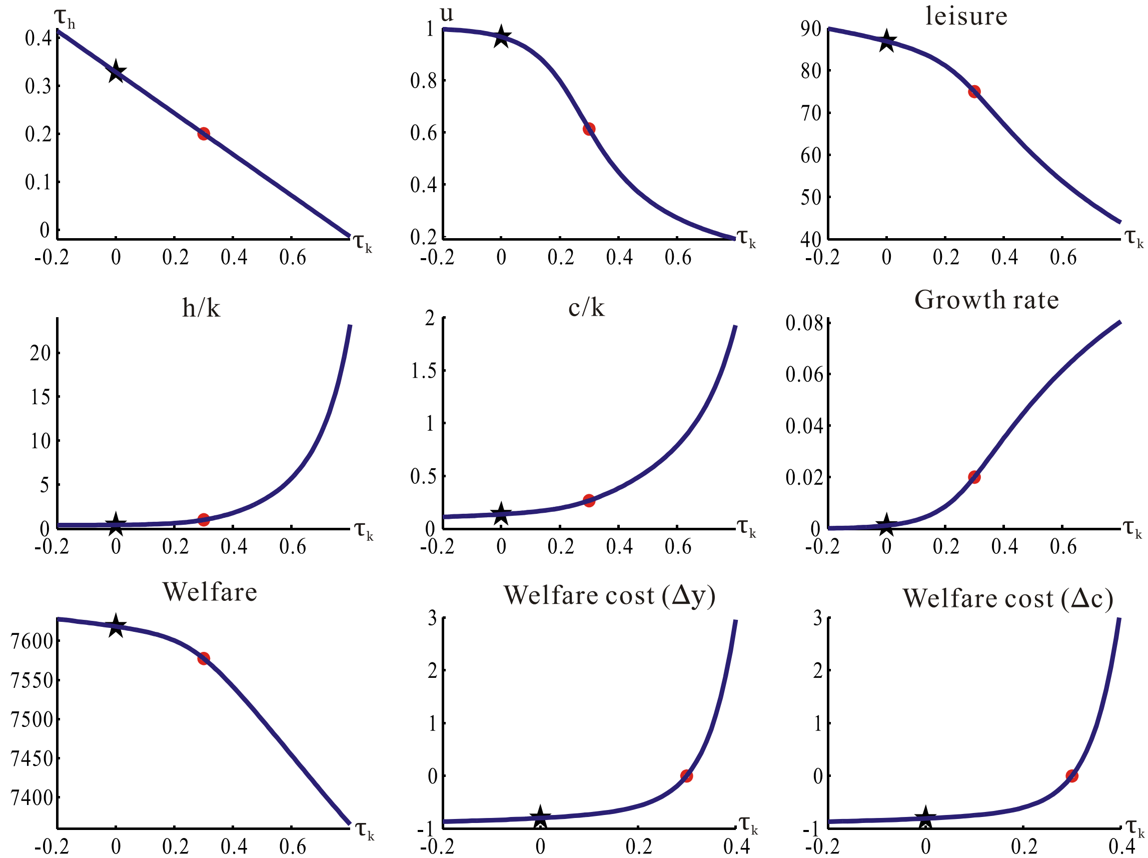
Like Peterman (2012), we find that the optimal tax policies are flat tax rates. The result of optimal flat tax policies is not a surprise and is reasoned as follows. The government's balanced budget along the BGP is $\tau_{\mathrm{k}}rk+T_{\mathrm{h}}(y_{\mathrm{h}})=\beta y$. This form gives $\tau_{\mathrm{k}}\alpha+T_{\mathrm{h}}(y_{\mathrm{h}})/y=\beta$ which, given constant α, β and τ_{k}, implies that $T_{\mathrm{h}}(y_{\mathrm{h}})/y$ is constant. Thus, the average labor tax rate in equilibrium is $\tau_h=\frac{1}{1-\alpha}(\beta-\tau_{\mathrm{k}}\alpha)$,[22] which is the same as that in Eq. (16) and is a constant.

4.2. Different ways of government spending

In our benchmark model, the tax revenue is transferred to the household in a lump-sum fashion. One might wonder whether different ways of government expenditure may alter the optimal factor tax mix. In this subsection, we consider two alternative ways of government spending. In the first instance, the government spends the tax revenue in an unproductive sector or, equivalently, the government throws away the tax revenue. In contrast, the government spending directly enters the household utility function in a separable fashion.

In the case that the tax revenue is spent in an unproductive sector, the representative agent's budget constraint in Eq. (2) is modified as $\dot{k}=(1-\tau_k)rk+(1-\tau_h)wunh-c$. In this model, the equilibrium

[20] If we use a flat labor income tax rate with a progressive capital income tax rate, the results are the same.
[21] See the Appendix for analysis of the model and a quantitative analysis.
[22] The average labor tax rate is $\tau_{\mathrm{h}}=T_{\mathrm{h}}(y_{\mathrm{h}})/y_{\mathrm{h}}=(T_h(y_{\mathrm{h}})/y)(y/y_{\mathrm{h}})=(\beta-\tau_{\mathrm{k}}\alpha)/(1-\alpha)$.



Note: The optimal tax mix is at (τ_k,τ_h)=(-20.41%, 41.60%). The dots on the locus are the benchmark case under the pre-existing tax rates of (τ_k,τ_h)=(0.3, 0.2). The stars on the locus are the optimal tax mix at (τ_k,τ_h)=(0%, 32.86%) obtained under the constraint of non-negative tax rate.

Fig. 3. The results of dynamic tax incidence under the learning function that is concave in the education time.

conditions are the same as Eqs. (3)–(12) in basic model except for Eqs. (8), (11b) and (11c) which now become, respectively

$$\dot{k} = (1-\beta)y - c, \tag{8'}$$

$$\frac{\dot{q}}{q} = B\frac{1-u}{u}\left[(1-\tau_h)(1-\alpha)\frac{A}{B}\right]^{\frac{1}{\alpha}} p^{\frac{-1}{\alpha}}\frac{1}{q} - (1-\beta)A\left[(1-\tau_h)(1-\alpha)\frac{A}{B}\right]^{\frac{1-\alpha}{\alpha}} p^{\frac{-(1-\alpha)}{\alpha}} + z, \tag{11b'}$$

$$\frac{\dot{z}}{z} = z - [1-\beta-\alpha(1-\tau_k)]A\left[(1-\tau_h)(1-\alpha)\frac{A}{B}\right]^{\frac{1-\alpha}{\alpha}} p^{\frac{-(1-\alpha)}{\alpha}} - \rho. \tag{11c'}$$

Thus, the conditions determining the BGP are Eqs. (3a), (13b) and

$$z = \frac{\rho}{\alpha}\frac{1-\beta-\alpha(1-\tau_k)(1-u)}{(1-\tau_k)u}, \tag{13c'}$$

$$l = \left\{\psi\frac{\rho}{B(1-\alpha)}\frac{[1-\beta-\alpha(1-\tau_k)(1-u)]}{(1-\tau_h)}\right\}^{\frac{1}{\sigma}}. \tag{13d'}$$

To quantify the optimal factor tax mix in this model,[23] we change the value of τ_k from the smallest feasible value of 19% to the largest value of 99.99% and determine the resulting τ_h that finances the same fraction of the government spending in output as in the benchmark case. The results (not reported) are similar to those in Fig. 2. We find that as τ_k is increasing from 19% and thus τ_h is decreasing from 24.71%, leisure decreases while the ratio of consumption to capital and the economic growth rate both increase. Since the last two effects dominate the leisure effect, welfare is increasing with regard to the capital tax rate. Thus the optimal tax mix is still at (τ_k, τ_h) = (99.99%, −10.00%). See the results in Table 3.

Next, we analyze the case when the government spending directly enters the household utility function in a separable fashion. In this case, the instantaneous utility in Eq. (1) is modified as

$$u(c,l,G) = \ln c + \psi_1\frac{(l)^{1-\sigma}-1}{1-\sigma} + \psi_2\ln(G). \tag{1'}$$

In this model, all equilibrium conditions and the conditions for the BGP are the same as those in the model when the government throws away the tax revenue. However, as the public spending affects the utility, the representative household's welfare in the long run is

$$U^* = \frac{1}{\rho}\left[(1+\psi_2)\ln k_0 + \ln z^* + (1+\psi_2)\frac{\theta^*}{\rho} + \psi_1\frac{l^{*1-\sigma}-1}{1-\sigma} + \psi_2\ln\beta + \psi_2\ln\left(\frac{y}{k}\right)^*\right]. \tag{15a'}$$

Note that Eq. (15a′) is Eq. (15a) plus the term $\ln(y/k)^*$ which is increasing in the ratio of human capital to physical capital and is

[23] We also start from pre-existing tax rates of (τ_k, τ_h) = (30%, 20%) and maintain other parameter values the same as those in Table 1.The resulting calibrated parameter values are A = 0.0399, B = 0.0024, ψ = 31.1769, and β = 0.2300. The equilibrium values in BGP are: u^* = 0.6667, z^* = 0.2000, p^* = 4.0000, n^* = 25, l^* = 75, θ = 2% and U^* = 1466.239.

unfavorable of a labor tax. In a quantitative analysis,[24] when we change the value of τ_k and determine the resulting τ_h that finances the same fraction of the government spending in output as in the benchmark case, we find that the results are similar to those when the tax revenue is spent in an unproductive sector. The optimal tax mix is still at $(\tau_k, \tau_h)=(99.99\%, -10.00\%)$. When we change the value of ψ_2, the results still hold. See Table 3.

4.3. Different utility form

The utility function in the basic model is a function of leisure and thus the IES of labor depends on the labor supply. Now, we consider a utility form so the IES of labor is independent of the labor supply. In this model, the instantaneous utility in Eq. (1) is modified as

$$u(c,n) = \ln c - \psi\frac{(n)^{1+\sigma}}{1+\sigma}, \sigma > 0, \tag{1''}$$

where without abuse of notations we use the same parameters as those in Eq. (1). Now, the IES of labor is $1/\sigma$ and is thus independent of the labor supply.

Now, the necessary conditions are Eqs. (4a)–(6b) in the basic model except for Eq. (4b) which becomes

$$\psi n^{\sigma} = \lambda(1-\tau_h)wuh + \lambda_h B(1-u)h. \tag{4b''}$$

The equilibrium conditions and the conditions in the BGP are the same as those in the basic model except for Eqs. (10a), (11a) and (13d) which are now[25]

$$p = \frac{\psi z(n)^{\sigma}}{Bq} \equiv p(n,q,z), \tag{10a''}$$

$$\frac{\dot{n}}{n} = \frac{1}{\sigma}\left\{\rho - B\left[(1-\tau_h)(1-\alpha)\frac{A}{B}\right]^{\frac{1}{\alpha}} p^{\frac{-1}{\alpha}} \frac{1}{q}\right\}, \tag{11a''}$$

$$L-l = n = \left\{\frac{B(1-\alpha)(1-\tau_h)}{\psi\rho[1-\alpha(1-\tau_k)(1-u)]}\right\}^{\frac{1}{\sigma}} \equiv n(u;\tau_h,\tau_k). \tag{13d''}$$

The agent's welfare in the long run in Eq. (15) is modified as

$$U^* = \frac{1}{\rho}\left[\ln k_0 + \ln z^* + \frac{\theta^*}{\rho} - \psi\frac{(n^*)^{1+\sigma}}{1+\sigma}\right]. \tag{15a''}$$

To quantify the optimal factor tax mix,[26] we change the value of τ_k and determine the resulting τ_h that finances the same fraction of the government spending in output as in the benchmark case. The results (not reported) are similar to those in Fig. 2. The optimal tax mix is still at $(\tau_k, \tau_h)=(99.99\%, -10.00\%)$. See other results in Table 3.

4.4. Learning time is more important than the stock of human capital

Now, we envisage the optimal factor tax mix when the learning time is more important than the stock of human capital in learning technology. In this case, the learning function is similar to that in Lucas (1990) except for $\eta>1$ and is modified as follows.[27]

$$\dot{h} = B[(1-u)n]^{\eta}h,\ h(0)\text{ given}, \eta > 1. \tag{20'}$$

When the learning function is Eq. (20′), the analysis is the same as the Lucas (1990) model. See the Appendix.

In the quantitative analysis, we set $\eta=1.1$.[28] When we change the value of τ_k and determine the resulting τ_h that finances the same fraction of the government spending in output as in the benchmark case, the results (not reported) are similar to those in Fig. 2. We find that the negative welfare effect from lower leisure is dominated by the positive welfare effect from higher ratios of consumption to capital and economic growth and thus the welfare is increasing in the capital tax rate. The optimal factor tax mix is $(\tau_k, \tau_h)=(99.99\%, -10.00\%)$. See other results in Table 3.

4.5. Learning by doing

Human capital is accumulated through the training time in the basic model. We now analyze the optimal taxation when the human capital is accumulated via learning by doing. Now, as there is no training time, the labor endowment is allocated to either work n or leisure $l=L-n$. Specifically, the model is otherwise the same as the basic model except that u and $1-u$ both are equal to 1 in Eqs. (2), (5) and (7) and Eq. (3) is now replaced by

$$\dot{h} = B(n)^{\eta_1}h,\ \eta_1 \le 1. \tag{22}$$

Our above model allows for $\eta_1 \le 1$ which includes the Lucas (1988, Section 5) model as a special case that emerges if $\eta_1=1$. The analysis of the model is shown in the Appendix.

In the quantitative analysis, we set $\eta_1=1$ with parameter values calibrated.[29] We then change the value of τ_k and determine the resulting τ_h to finance the same fraction of the government spending in output as in the benchmark case. The results (not reported) are similar to those in Fig. 2. The optimal tax mix is still at $(\tau_k, \tau_h)=(99.99\%, -10.00\%)$. When we lower the value of η_1 to 0.9 with parameter values recalibrated, the optimal tax mix is still at $(\tau_k, \tau_h)=(99.99\%, -10.00\%)$. Even when we lower the value of η_1 to 0.5 and even further to 0.1, the optimal tax mix is still at $(\tau_k, \tau_h)=(99.99\%, -10.00\%)$. See other results in Table 3.

Thus, it is optimal to tax capital income even if the learning-by-doing effect is much less important than the human capital in the formation of human capital. The reason is as follows. As the human capital is formed via learning by doing, the tax on labor income directly hurts not only the labor supply in the goods sector n but also the human capital formation via learning by doing. Thus, the labor income tax is more harmful in the learning-by-doing model than in the training model. Hence, even if the learning technology is concave in learning-by-doing,

[24] We also start from pre-existing tax rates of $(\tau_k, \tau_h)=(30\%, 20\%)$, and maintain other parameter values the same as those in Table 1.The resulting calibrated parameter values are $A=0.0399$, $B=0.0024$, $\psi_1=31.1769$, and $\beta=0.2300$. We set $\psi_2=1$ and obtain the following equilibrium values in BGP: $u^*=0.6667$, $z^*=0.2000$, $p^*=4.0000$, $n^*=25$, $l^*=75$, $\theta=2\%$ and $U^*=1525.8073$.

[25] The condition to assume the existence of a BGP in Proposition 1 becomes $\left\{\frac{B}{\psi}\frac{1-\alpha-\beta+\tau_k\alpha}{\rho}\right\}^{1/\sigma} > \frac{\rho}{B}$.

[26] We also start from pre-existing tax rates of $(\tau_k, \tau_h)=(30\%, 20\%)$ and maintain other parameter values the same as those in Table 1 except for $\sigma=0.5$ which gives the IES of labor equal 2. The resulting calibrated parameter values are $A=0.0399$, $B=0.0024$, $\psi=0.0072$, and $\beta=0.2300$. The equilibrium values in BGP are: $u^*=0.6667$, $z^*=0.2657$, $p^*=4.0000$, $n^*=25$, $l^*=75$, $\theta=2\%$ and $U^*=79.4421$.

[27] When the learning time is more important than the stock of human capital, the learning technology may be $\dot{h} = B[(1-u)n]h^{v}-\chi h,\ 0<v<1,\ \chi>0$. We do not consider this learning technology as human capital is fixed at $h^*=\{B[1-u]n]/\chi\}^{1/(1-v)}$ in a BGP and there is thus a zero growth rate of human capital.

[28] We start from pre-existing tax rates of $(\tau_k, \tau_h)=(30\%, 20\%)$ and calibrate the model. We obtain $A=0.0383$, $B=0.0022$, $\psi=8.2343$ and $\beta=0.23$. The equilibrium values in the BGP are: $l^*=75$, $n^*=25$, $u^*=0.7051$, $\theta^*=0.02$, $z^*=0.2657$ and $U^*=458.6699$.

[29] We also start from pre-existing tax rates of $(\tau_k, \tau_h)=(30\%, 20\%)$. The resulting calibrated parameter values are $A=0.0300$, $B=0.0008$, $\psi=23.4665$, and $\beta=0.2300$. The equilibrium values in BGP are: $z^*=0.2657$, $p^*=4.0000$, $n^*=25$, $l^*=75$, $\theta=2\%$ and $U^*=1132.3368$.

it is optimal to tax capital income but not to tax labor income in the learning-by-doing model.

5. A general learning function model

In previous models, the learning activity uses only labor as input and not capital. In this subsection, we consider a model with a general learning function wherein both labor and capital are inputs as used in Bond et al. (1996) and Mino (1996). Thus, the capital income tax now exerts a direct distortion on learning activities.

Denote the fraction of capital used in goods production as s and thus the fraction in the learning activity as $1-s$. The learning function in Eq. (3) is modified as

$$\dot{h} = B((1-s)k)^{\gamma}((1-u)nh)^{1-\gamma},\ h(0) \text{ given}, 0<\gamma<1, \tag{23a}$$

and the goods production function in Eq. (5) is now modified as

$$y = A(sk)^{\alpha}(unh)^{1-\alpha}. \tag{23b}$$

We assume that the goods sector is relatively more capital intensive than the human capital sector; thus, $\alpha > \gamma$.

At any point of time, the representative agent's flow budget constraint in Eq. (2) is modified as

$$\dot{k} = (1-\tau_k)rsk + (1-\tau_h)wunh - c + G,\ k_0 \text{ given}, \tag{23c}$$

and the balanced government budget in Eq. (7) is now

$$\tau_k rsk + \tau_h whun = G. \tag{23d}$$

As in the previous sections, the representative household chooses consumption, leisure and labor allocation between working and learning, and capital and human capital accumulation over time. The representative firm chooses capital and labor so that the factor price is equal to the marginal product. The optimal conditions are the same as those in Section 2, except for the appearance of s due to the modification of the model in Eqs. (23a)–(23d). There is an additional condition the household chooses capital allocation between working and learning so as to equate the marginal product between the two sectors. As the result turns out, the fraction of capital allocated to the goods sector is positively correlated with the fraction of labor allocated to the goods sector, given by

$$s = \left[1 + \frac{1-\alpha}{\alpha}\frac{\gamma}{1-\gamma}\frac{1-\tau_h}{1-\tau_k}\left(\frac{1}{u}-1\right)\right]^{-1} = s(u;\tau_h,\tau_k), \tag{24}$$

where $\frac{ds}{du} = \frac{(1-s)s}{(1-u)u} > 0$.

The relationship in Eq. (24) arises because capital and labor are complements in production activities as well as in learning activities. Substituting this relationship into the learning function in Eq. (23a) gives $\dot{h} = B(k)^{\gamma}(nh)^{1-\gamma}\left[\frac{\chi}{\chi+u(1-\chi)}\right]^{\gamma}(1-u)$, where $\chi \equiv \frac{1-\alpha}{\alpha}\frac{\gamma}{1-\gamma}\frac{1-\tau_h}{1-\tau_k}$.

It is obvious to see that the marginal product of learning is not decreasing in the education time, $1-u$.

For a tax incidence analysis, by increasing the capital tax rate and decreasing the labor income tax rate to balance the government budget, the welfare effect is similar to the benchmark model.[30]

In calibration, we use the pre-existing tax rates of $(\tau_k, \tau_h) = (30\%, 20\%)$ in the data and the parameter values as listed in Table 1. Further, since we assume that the physical capital share in the educational sector is less than that in the goods sector, we choose $\gamma = 0.2$. Under this set of parameter values, we calibrate and obtain $A = 0.0320$, $B = 0.0061$, $\psi = 21.7297$ and $\beta = 0.2300$. The equilibrium values in BGP are: $u^* = 0.7333$, $s^* = 0.8049$, $z^* = 0.2100$, $p^* = 2.9268$, $n^* = 25$, $l^* = 75$, $\theta = 2\%$ and $U^* = 1049.6365$.

We then conduct tax incidence exercises. We change the values of τ_k and τ_h which finance the government expenditure at 23% of the output in the economy. The outcome is reported in Fig. 4. It is clear to see that the results in Fig. 4 are the same as those in Fig. 2. As τ_k increases from -17% and thus τ_h decreases from 40.14%, in the top right diagram we see that leisure first decreases and then increases when the tax rate of capital income is very large, while the ratio of consumption to capital increases in the center diagram and the economic growth rate first increases and decreases in the middle right diagram when the tax rate of capital income is very large. Since the positive welfare effects dominate the negative welfare effect when the capital tax rate is less than 100%, in the bottom left diagram the level of welfare increases as the tax rate of capital income increases. The result indicates that it is best to use the capital income tax to replace the labor income tax. The optimal tax mix is $(\tau_k, \tau_h) = (99.85\%, -9.94\%)$. However, the optimal tax mix is different from the growth-maximizing tax mix which is $(\tau_k, \tau_h) = (70.56\%, 2.62\%)$. Moving from the benchmark tax mix to the optimal tax mix, the welfare gain is 99.30% both in terms of changes in consumption equivalence and changes in output equivalence.

Finally, we also calibrate our model under $(\tau_k, \tau_h) = (36\%, 40\%)$ while maintaining other parameters at the same values. We calibrate and obtain $A = 0.0359$, $B = 0.0063$, $\psi = 16.1165$ and $\beta = 0.3880$. The equilibrium allocations are $u^* = 0.7333$, $s^* = 0.8341$, $z^* = 0.2407$, $p^* = 2.4882$, $l^* = 75$, $n^* = 25$, $\theta = 2\%$ and $U^* = 804.7951$. We then change the values of τ_k and τ_h that finance the same fraction of the government spending in output. As τ_k increases from 0.01% and τ_h decreases from 55%, we find similar results (not reported) as those in Fig. 4. Since the positive welfare effect dominates the negative effect, welfare is increasing in the capital income tax rate. The optimal tax mix is $(\tau_k, \tau_h) = (99.93\%, 12.60\%)$. As the government expenditure share in output (β, which is 38.8%) is larger than the share of capital in the goods sector (α, which is 30%), the amount of capital tax revenue is not enough to cover the government expenditure. Thus, a positive labor income tax is needed.[31] Moving from the benchmark tax mix to the optimal tax mix, the welfare gain is 99.33% both in terms of changes in consumption equivalence and changes in output equivalence.

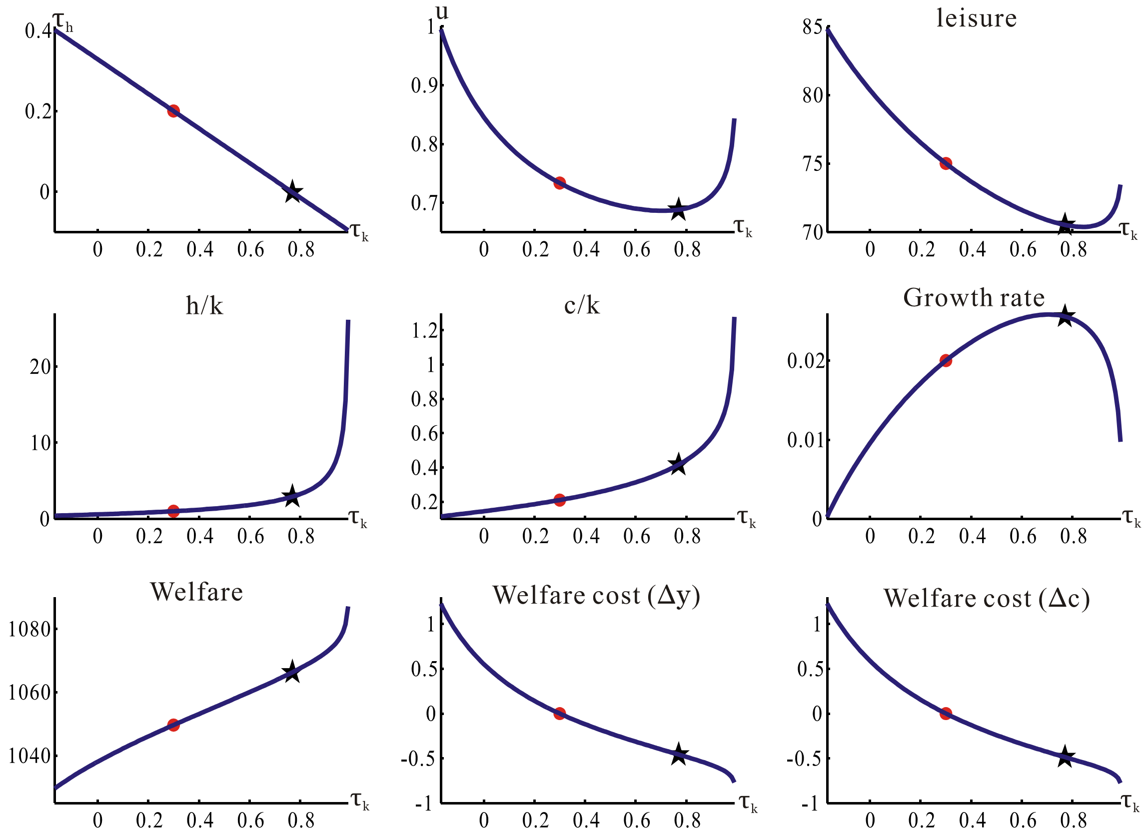
6. Concluding remarks

In this paper, we study optimal factor tax incidence in a standard two-sector, human capital-based endogenous growth model. In the existing research with regard to an optimal factor tax incidence in human capital-based endogenous growth models, learning time and human capital affect learning activity at different degrees such that the representative agent's learning time contributes to the formation of human capital at a rate smaller than the representative agent's human capital. As a result, existing research revealed the conclusion that, in order to finance a positive stream of the government transfer as a given fraction of total output, it was optimal to tax labor income and not to tax capital income.

Our model is different from these existing studies in that as human capital is embodied in labor hours but they are inseparable, the representative agent's learning time and his or her embodied human capital affect learning activities at the same degree. Under this learning function, the representative agent's learning time and his or her embodied human capital contributes to the formation of human capital at the same degree. In our model, a switch from the labor income tax to the capital income tax generates a positive welfare effect coming through higher growth and higher ratios of

[30] See the Appendix for a complete analysis of this section.

[31] The optimal tax mix is different from the growth-maximizing tax mix which is $(\tau_k, \tau_h) = (76.95\%, 22.45\%)$.



Note: The optimal tax mix is at (τ_k,τ_h)=(99.85%, -9.94%). The dotson the locus are the benchmark case under the pre-existing tax rates of (τ_k,τ_h)=(0.3, 0.2). The stars on the locus are the optimal tax mix at (76.67%, 0%) obtained under the constraint of non-negative tax rate.

Fig. 4. The results of dynamic tax incidence under a general learning function.

consumption to capital and a negative welfare effect originated from lower leisure. If the effects arising from higher growth and ratios of consumption to capital dominate, it is optimal to switch from labor income taxes completely to capital income taxes.

Our calibration exercises indicate that based on the current US income tax code, a switch from labor income taxes to capital income taxes generates positive welfare effects coming through higher economic growth and higher ratios of consumption to capital which quantitatively dominate the negative welfare effect of lower leisure. As a consequence, it is always best to tax capital income taxes only. Labor income is to be taxed if the revenue gained from capital income taxes is insufficient to cover government expenditure. Moreover, we find the welfare gain of such a tax reform is quantitatively large.

Appendix 1. Proof of Proposition 1.

Examining the left-hand side of Eq. (14), the $n^d(u)$ locus is decreasing in u. Moreover, it is clear to see that as u increases from 0 to 1, the $n^d(u)$ locus is monotonically decreasing from $n^d(0)=\infty$ to $n^d(1)=\rho/B>0$.

Next, the right-hand side of Eq. (14) is $n^s(u)$. According to Eq. (13d), we have $\partial l/\partial u>0$, which indicates that the $n^s(u)$ locus is monotonically decreasing in u from $n^s(0)<\infty$ to $n^s(1)$, where

$$n^s(0) = L-\left\{\frac{\psi}{B}\frac{\rho}{1-\alpha-\beta+\tau_k\alpha}[1-(1-\tau_k)\alpha]\right\}^{1/\sigma} < \infty$$

$$n^s(1) = L-\left\{\frac{\psi}{B}\frac{\rho}{1-\alpha-\beta+\tau_k\alpha}\right\}^{1/\sigma} < n^s(0).$$

Therefore, there exists a BGP if $n^s(1)>\rho/B$ or, equivalently, if $L>\left\{\frac{\psi}{B}\frac{\rho}{1-\alpha-\beta+\tau_k\alpha}\right\}^{1/\sigma}+\frac{\rho}{B}$.

Appendix 2. Derivation of the Model with the Lucas (1990) Learning Technology

Suppose that the learning technology is not the form in Eq. (3), but is the one used in Lucas (1990). Specifically, let the technology be

$$\dot{h} = B[(1-u)n]^{\eta}h, h(0) \text{ given}, \eta<1. \tag{20}$$

Equations for preferences, an individual's budget, the goods technology, and the government budget are the same as those in Eqs. (1), (2), (5) and (7). While we restrict to $\eta<1$ in Eq. (20), the following analysis applies to the case in Subsection 4.4 when $\eta>1$ and thus the learning time is more important than the stock of human capital.

The first order conditions of the household's problem are

$$\frac{1}{c}=\lambda, \tag{A2.1a}$$

$$\psi l^{-\sigma} = \lambda(1-\tau_h)wuh+\lambda_h B\eta(1-u)^{\eta}n^{\eta-1}h, \tag{A2.1b}$$

$$\lambda(1-\tau_h)w = \lambda_h B\eta[(1-u)n]^{\eta-1}, \tag{A2.1c}$$

$$\dot{\lambda} = [\rho-(1-\tau_k)r]\lambda, \tag{A2.1d}$$

$$\dot{\lambda}_h = \left[\rho - \frac{\dot{h}}{h}\right]\lambda_h - \lambda(1-\tau_h)wnu, \quad \text{(A2.1e)}$$

along with the transversality conditions: $\lim_{t\to\infty} e^{-\rho t}\lambda_t k_t = 0$ and $\lim_{t\to\infty} e^{-\rho t}\lambda_{ht}h_t = 0$.The firm's first order conditions are the same as Eqs. (6a) and (6b) in Section 2.The equilibrium conditions are simplified to a three-dimensional equilibrium system in the state vector (l, q, z) as follows.

$$\begin{aligned}\frac{\dot{l}}{l} &= \frac{-1}{\sigma}\left\{\rho + B\frac{p^{\frac{\eta}{1-\alpha-\eta}}q^{\frac{\alpha\eta}{1-\alpha-\eta}}}{[(1-\tau_{\mathrm{h}})(1-\alpha)\frac{A\eta}{B}]^{\frac{\eta}{1-\alpha-\eta}}}\left[\left(\frac{u}{1-u}\right)^{\frac{\alpha\eta}{1-\alpha-\eta}} - \frac{1-u(1-\eta)}{\eta^2 u}\left(\frac{u}{1-u}\right)^{\frac{(1-\alpha)(1-\eta)}{1-\alpha-\eta}}\right]\right.\\ &\quad \left. + \frac{\eta-1}{1-\alpha-\eta}\left(\frac{\dot{p}}{p} + \alpha\frac{\dot{q}}{q} + \alpha\frac{\dot{u}}{u(1-u)}\right)\right\}, \end{aligned} \quad \text{(A2.2a)}$$

$$\frac{\dot{q}}{q} = B\frac{p^{\frac{\eta}{1-\alpha-\eta}}q^{\frac{\alpha\eta}{1-\alpha-\eta}}\left(\frac{u}{1-u}\right)^{\frac{\alpha\eta}{1-\alpha-\eta}}}{[(1-\tau_{\mathrm{h}})(1-\alpha)\frac{A\eta}{B}]^{\frac{\eta}{1-\alpha-\eta}}} - A\frac{p^{\frac{1-\alpha}{1-\alpha-\eta}}q^{\frac{(1-\alpha)(1-\eta)}{1-\alpha-\eta}}\left(\frac{u}{1-u}\right)^{\frac{(1-\alpha)(1-\eta)}{1-\alpha-\eta}}}{[(1-\tau_h)(1-\alpha)\frac{A\eta}{B}]^{\frac{1-\alpha}{1-\alpha-\eta}}} + z, \quad \text{(A2.2b)}$$

$$\frac{\dot{z}}{z} = z - A\frac{p^{\frac{1-\alpha}{1-\alpha-\eta}}q^{\frac{(1-\alpha)(1-\eta)}{1-\alpha-\eta}}\left(\frac{u}{1-u}\right)^{\frac{(1-\alpha)(1-\eta)}{1-\alpha-\eta}}}{[(1-\tau_h)(1-\alpha)\frac{A\eta}{B}]^{\frac{1-\alpha}{1-\alpha-\eta}}}[1-(1-\tau_k)\alpha] - \rho. \quad \text{(A2.2c)}$$

Moreover, we also derive

$$\frac{\dot{p}}{p} = \frac{\alpha A(1-\tau_k)p^{\frac{1-\alpha}{1-\alpha-\eta}}q^{\frac{(1-\alpha)(1-\eta)}{1-\alpha-\eta}}\left(\frac{u}{1-u}\right)^{\frac{(1-\alpha)(1-\eta)}{1-\alpha-\eta}}}{[(1-\tau_h)(1-\alpha)\frac{A\eta}{B}]^{\frac{1-\alpha}{1-\alpha-\eta}}} - B\frac{1-u(1-\eta)}{\eta^2 u}\frac{p^{\frac{\eta}{1-\alpha-\eta}}q^{\frac{\alpha\eta}{1-\alpha-\eta}}\left(\frac{u}{1-u}\right)^{\frac{(1-\alpha)(1-\eta)}{1-\alpha-\eta}}}{[(1-\tau_h)(1-\alpha)\frac{A\eta}{B}]^{\frac{\eta}{1-\alpha-\eta}}}, \quad \text{(A2.2d)}$$

$$p = \left(\frac{l^{\sigma}B\eta}{\psi z}\right)^{\frac{1-\alpha-\eta}{\alpha}}\left[(1-\tau_h)(1-\alpha)\frac{A\eta}{B}\right]^{\frac{\eta-1}{\alpha}}q^{\frac{(1-\alpha)(1-\eta)-\alpha}{\alpha}}\left(\frac{u}{1-u}\right)^{\eta-1} \equiv p(l,q,z), \quad \text{(A2.2e)}$$

$$\frac{u}{(1-u)^{\frac{1-\eta}{\alpha}}} = p^{\frac{-1}{\alpha}}\frac{1}{q}(L-l)^{\frac{1-\alpha-\eta}{\alpha}}\left[(1-\tau_{\mathrm{h}})(1-\alpha)\frac{A\eta}{B}\right]^{\frac{1}{\alpha}} \equiv u(p(l,q,z),l,q,\tau_{\mathrm{h}}). \quad \text{(A2.2f)}$$

Given τ_{k} and β and with the help of (A2.2d)–(A2.2f), we can use (A2.2a)–(A2.2c) to determine l, q and z. Along the BGP, $\dot{l} = \dot{q} = \dot{z} = \dot{p} = 0$. Variables l, q and z are constant in the BGP and so are u and p. To determine the BGP, as in Section 3 it is useful if we rewrite p, z, q and l as functions of u and then determine u.

$$\begin{aligned} p &= \frac{(1-\tau_h)(1-\alpha)A\eta}{B^{\frac{1}{\eta}}}\left[\frac{\alpha A(1-\tau_k)}{\rho}\right]^{\frac{\alpha}{1-\alpha}}\left[\frac{\left(1-\eta^2\right)(1-u)+u\eta}{\rho\eta^2(1-u)}\right]^{\frac{\eta-1}{\eta}}\left[\frac{\left(1-\eta^2\right)(1-u)+u\eta}{1-u(1-\eta)}\right]^{\frac{\alpha}{1-\alpha}}\\ &\equiv p(u;\tau_h,\tau_k), \end{aligned} \quad \text{(A2.3a)}$$

$$\begin{aligned} q &= \left\{Bp^{\frac{\eta}{1-\alpha-\eta}}\left[\frac{(1-\tau_{\mathrm{h}})(1-\alpha)A\eta}{B}\right]^{\frac{\eta}{\alpha+\eta-1}}\left(\frac{u}{1-u}\right)^{\frac{\alpha\eta}{1-\alpha-\eta}}\frac{\left(1-\eta^2\right)(1-u)+u\eta}{\rho\eta^2(1-u)}\right\}^{\frac{\alpha+\eta-1}{\alpha\eta}}\\ &\equiv q(p(u;\tau_{\mathrm{h}},\tau_{\mathrm{k}}),u;\tau_{\mathrm{k}},\tau_{\mathrm{k}}), \end{aligned} \quad \text{(A2.3b)}$$

$$\begin{aligned} z &= Aq^{\frac{(1-\alpha)(1-\eta)}{1-\alpha-\eta}}\left(\frac{u}{1-u}\right)^{\frac{(1-\alpha)(1-\eta)}{1-\alpha-\eta}}p^{\frac{1-\alpha}{1-\alpha-\eta}}\left[\frac{(1-\tau_{\mathrm{h}})(1-\alpha)A\eta}{B}\right]^{\frac{1-\alpha}{\alpha+\eta-1}} - \frac{\rho\eta^2(1-u)}{(1-\eta^2)(1-u)+u\eta}\\ &\equiv z(u;\tau_{\mathrm{h}},\tau_{\mathrm{k}}), \end{aligned} \quad \text{(A2.3c)}$$

$$l = \left\{\frac{\psi}{B\eta}\frac{z}{q}p^{\frac{\alpha}{1-\alpha-\eta}}q^{\frac{\alpha(1-\eta)}{1-\alpha-\eta}}\left[\frac{(1-\tau_{\mathrm{h}})(1-\alpha)A\eta}{B}\right]^{\frac{1-\eta}{1-\alpha-\eta}}\left(\frac{u}{1-u}\right)^{\frac{\alpha(1-\eta)}{1-\alpha-\eta}}\right\}^{\frac{1}{\sigma}} \equiv l(u;\tau_{\mathrm{h}},\tau_{\mathrm{k}}). \quad \text{(A2.3d)}$$

Using the above relationships, the labor marker clearance condition is given by

$$n^{\mathrm{d}}(u) \equiv \left[\frac{\rho\eta^2(1-u)}{(1-\eta^2)(1-u)+u\eta}\frac{1}{B}\right]^{\frac{1}{\eta}}\frac{1}{1-u} = L - l(u;\tau_{\mathrm{h}},\tau_{\mathrm{k}}) \equiv n^{\mathrm{s}}(u), \quad \text{(A2.4)}$$

which determines the value of u^* in the BGP.

While the agent's lifetime welfare in the long run is in Eq. (15a), the long-run economic growth rate now is modified as

$$\theta^* = \frac{\rho\eta^2(1-u^*)}{(1-\eta^2)(1-u^*)+u^*\eta}, \frac{\partial\theta^*}{\partial u^*} = \frac{-\rho\eta^3}{\left[(1-\eta^2)(1-u^*)+u^*\eta\right]^2} < 0. \quad \text{(A2.5)}$$

The tax incidence analysis is the same as Section 3. By increasing the capital tax rate and decreasing the corresponding labor income tax rate to balance the government budget, the welfare effect is similar to the benchmark model with positive effects from higher ratios of consumption to capital and growth effects and a negative effect from lower leisure.

In calibration, we use the pre-existing tax rates of $(\tau_{\mathrm{k}},\tau_{\mathrm{h}}) = (30\%, 20\%)$ in the data with other parameter values the same as those in Table 1. It turns out that there is no BGP if $\eta < 0.8$. If we set $\eta = 0.8$, then our calibration obtains $\psi > 50{,}000$, which is implausibly large, meaning that leisure is 50,000 times more important than consumption. As such, we set $\eta = 0.9$. We calibrate and obtain $A = 0.0422$, $B = 0.0026$, $\psi = 169.1976$ and $\beta = 0.23$. The equilibrium values in the BGP are: $l^* = 75$, $n^* = 25$, $u^* = 0.6137$, $\theta^* = 0.02$, $z^* = 0.2657$ and $U^* = 7577.5134$. We then increase the value of τ_{k} from the smallest feasible value of -0.2041 and determine the resulting τ_{h} that finances the same fraction of the government spending in output as in the benchmark case. See the results in Fig. 3. We find that as τ_{k} increases and thus τ_{h} decreases, leisure decreases while the ratio of consumption to capital and the economic growth rate both increase. Since the negative welfare effect from lower leisure dominates the positive welfare effect from higher consumption and economic growth, welfare is decreasing in the capital tax rate. Thus, it is optimal to tax labor income first. Therefore, it is optimal to switch from capital income taxes to labor income taxes. As there is no feasible BGP for $\tau_{\mathrm{k}} < -20.41\%$, the optimal tax mix is $(\tau_{\mathrm{k}},\tau_{\mathrm{h}}) = (-20.41\%, 41.60\%)$.

We also recalibrate the model under the pre-existing tax rates of $(\tau_{\mathrm{k}}, \tau_{\mathrm{h}}) = (36\%, 40\%)$ while maintaining other parameters at the same values. We calibrate and obtain $A = 0.0462$, $B = 0.0026$, $\psi = 103.4289$ and $\beta = 0.3880$. The equilibrium allocations are $u^* = 0.6137$, $z^* = 0.2925$, $l^* = 75$, $n^* = 25$, $\theta = 2\%$ and $U^* = 4671.195$. We then calculate the effects of changes in the values of τ_{k} and τ_{h} that finance the same fraction of the government spending in output. We find the same effects (not reported) as in Fig. 3 as the capital tax rate increases and the labor tax rate decreases, the negative welfare effect due to lower leisure always dominates the positive welfare effect from higher ratios of consumption to capital and economic growth. Thus, it is optimal to tax labor income first. If $\tau_{\mathrm{k}} < -3.32\%$, there is no feasible BGP. Thus, the optimal tax mix is $(\tau_{\mathrm{k}},\tau_{\mathrm{h}}) = (-3.32\%, 56.85\%)$.

Appendix 3. Progressive or Regressive Tax Policies

When the tax rate of labor income is not flat, the representative agent's budget constraint is changed to $\dot{k} = (1-\tau_{\mathrm{k}})rk + wunh - T_{\mathrm{h}}(y_{\mathrm{h}}) - c + G$. The first order conditions of the household's problem are

$$\frac{1}{c} = \lambda, \quad \text{(A3.1a)}$$

$$\psi l^{-\sigma} = \lambda\left\{wuh(1-\kappa_0) + \frac{\kappa_0\left[(wunh)^{-\kappa_1} + \kappa_2 h^{-\kappa_1}\right]^{\frac{-1}{\kappa_1}}}{n[1+\kappa_2(\frac{1}{wun})^{-\kappa_1}]}\right\} + \lambda_h B(1-u)h, \quad \text{(A3.1b)}$$

$$\lambda\left\{wnh(1-\kappa_0) + \frac{\kappa_0\left[(wunh)^{-\kappa_1} + \kappa_2 h^{-\kappa_1}\right]^{\frac{-1}{\kappa_1}}}{u[1+\kappa_2(\frac{1}{wun})^{-\kappa_1}]}\right\} = \lambda_h Bnh, \quad \text{(A3.1c)}$$

$$\dot{\lambda} = [\rho-(1-\tau_{\mathrm{k}})r]\lambda, \quad \text{(A3.1d)}$$

$$\dot{\lambda}_h = \left[\rho - \frac{\dot{h}}{h}\right]\lambda_h - \lambda\left\{wun(1-\kappa_0) + \frac{\kappa_0}{h}\left[(wunh)^{-\kappa_1} + \kappa_2 h^{-\kappa_1}\right]^{\frac{-1}{\kappa_1}}\right\}, \quad \text{(A3.1e)}$$

and the transversality conditions $\lim_{t\to\infty} e^{-\rho t}\lambda_t k_t = 0$ and $\lim_{t\to\infty} e^{-\rho t}\lambda_{ht}h_t = 0$.

The firm's first order conditions are the same as Eqs. (6a) and (6b) as in Section 2.

The equilibrium conditions are simplified to a three-dimensional equilibrium system in the state vector (l, q, z) as follows.

$$\frac{\dot{l}}{l} = \frac{-1}{\sigma}\left[\rho - \frac{(1-\tau_h)(1-\alpha)}{p}A(un)^{1-\alpha}q^{-\alpha}\right], \tag{A3.2a}$$

$$\frac{\dot{q}}{q} = B(1-u)n - A(un)^{1-\alpha}q^{1-\alpha} + z, \tag{A3.2b}$$

$$\frac{\dot{z}}{z} = z - [1-(1-\tau_k)\alpha]A(un)^{1-\alpha}q^{1-\alpha} - \rho. \tag{A3.2c}$$

Moreover, we also derive

$$\frac{\dot{p}}{p} = \left[(1-\tau_k)\alpha - \frac{(1-\tau_h)(1-\alpha)}{pq}\right]A(un)^{1-\alpha}q^{1-\alpha} - B(1-u)n, \tag{A3.2d}$$

$$p = \psi l^{-\sigma}\frac{z}{Bq} \equiv p(l,q,z), \tag{A3.2e}$$

$$(1-\alpha)(1-\kappa_0) + [\kappa_0(1-\alpha) - (\beta - \alpha\tau_k)]\left\{1 + \kappa_2\left[(1-\alpha)A(un)^{1-\alpha}q^{-\alpha}\right]^{\kappa_1}\right\}^{-1}$$
$$= \frac{pB}{A}(unq)^{\alpha}. \tag{A3.2f}$$

Given τ_{k} and β and with the help of Eqs. (A3.2d)-(A3.2f) and Eqs. (21), we can use Eqs. (A3.2a)–(A3.2c) to determine l, q and z. Along the BGP, $\dot{l} = \dot{q} = \dot{z} = \dot{p} = 0$. Variables l, q and z are constant in the BGP and so is p. To determine the BGP, as in Section 3 it is useful if we rewrite n (or l), p, z and q as functions of u and then determine u.

$$n = \frac{\rho}{B(1-\tau_{\mathrm{h}})(1-\alpha)u}\left\{(1-\alpha)(1-\kappa_0) + [\kappa_0(1-\alpha) - (\beta - \alpha\tau_{\mathrm{k}})]\left[1 - \frac{\beta - \alpha\tau_{\mathrm{k}}}{\kappa_0(1-\alpha)}\right]^{\kappa_1}\right\}$$
$$\equiv n(u;\tau_{\mathrm{h}},\tau_{\mathrm{k}}), \tag{A3.3a}$$

$$p = \frac{(1-\tau_{\mathrm{h}})(1-\alpha)}{\rho}A\left[\frac{\rho + B(1-u)n}{(1-\tau_{\mathrm{k}})\alpha A}\right]^{\frac{-\alpha}{1-\alpha}}un \equiv p(u,n;\tau_{\mathrm{h}},\tau_{\mathrm{k}}), \tag{A3.3b}$$

$$q = \left[\frac{\rho + B(1-u)n}{(1-\tau_{\mathrm{k}})\alpha A}\right]^{\frac{-\alpha}{1-\alpha}}\frac{1}{un} \equiv q(u,n;\tau_{\mathrm{k}}), \tag{A3.3c}$$

$$z = [1-(1-\tau_k)\alpha]\frac{\rho + B(1-u)n}{(1-\tau_k)\alpha} + \rho \equiv z(u,n;\tau_k). \tag{A3.3d}$$

Using the above relationships, the labor market clearance condition is given by

$$\psi(L-n)^{-\sigma}\left\{[1-(1-\tau_k)\alpha]\frac{\rho + B(1-u)n}{(1-\tau_k)\alpha} + \rho\right\} = \frac{B(1-\tau_h)(1-\alpha)}{\rho}\frac{\rho + B(1-u)n}{(1-\tau_k)\alpha}, \tag{A3.4}$$

which determines the value of u^* in the BGP.

While the agent's lifetime utility in the long run is in Eq. (15a), the long-run economic growth rate now is modified as

$$\theta^* = B(1-u^*)n^*. \tag{A3.5}$$

In a further calibration, we maintain parameters with the same values as those in Table 1 and set $\kappa_0 = 0.258$ and $\kappa_1 = 0.768$. Combining Eq. (21), the government's budget constraint, Eqs. (A3.3c) and (A3.5), we calibrate parameter values and obtain $\kappa_2 = 7.3878$, $A = 0.0404$, $B = 0.0023$ and $u^* = 0.6553$. Thus $z^* = 0.2657$ and $p^* = 4.0000$. Using (A3.4), we calibrate $\psi = 22.6928$. Other equilibrium values in BGP are: $\beta = 0.2300$, $n^* = 25$, $l^* = 75$, $\theta = 2\%$ and $U^* = 1098.1173$.

Appendix 4. Derivation of the Learning-by-doing Model

In the learning-by-doing model, the learning technology is Eq. (22), and the representative agent's flow budget constraint is

$$\dot{k} = (1-\tau_k)rk + (1-\tau_h)wnh - c + G, k_0 \text{ given} \tag{A4.1}$$

The necessary conditions of the representative household are

$$\frac{1}{c} = \lambda, \tag{A4.2a}$$

$$\psi l^{-\sigma} = \lambda(1-\tau_h)wh + \lambda_h Bh\eta_1 n^{\eta_1 - 1}, \tag{A4.2b}$$

$$\dot{\lambda} = [\rho - (1-\tau_k)r]\lambda, \tag{A4.2c}$$

$$\dot{\lambda}_h = \left[\rho - \frac{\dot{h}}{h}\right]\lambda_h - \lambda(1-\tau_h)wn, \tag{A4.2d}$$

and the transversality conditions $\lim_{t\to\infty} e^{-\rho t}\lambda_t k_t = 0$ and $\lim_{t\to\infty} e^{-\rho t}\lambda_{ht} h_t = 0$.

The production function is $y = A(k)^{\alpha}(nh)^{1-\alpha}$, and the firm's first order conditions are

$$r = \alpha\frac{y}{k}, \tag{A4.2e}$$

$$w = (1-\alpha)\frac{y}{nh}. \tag{A4.2f}$$

The government's balanced budget is $\tau_k rk + \tau_h wnh = G$.

The equilibrium conditions are simplified to a three-dimensional equilibrium system in the state vector (n, q, z) as follows.

$$\frac{\dot{n}}{n} = \frac{1}{\frac{\sigma\psi(L-n)^{-\sigma-1}n + (1-\eta_1)\eta_1 pBn^{\eta_1-1}q/z}{\psi(L-n)^{-\sigma} - \eta_1 pBn^{\eta_1-1}q/z} + \alpha}\left\{\frac{\psi(L-n)^{-\sigma}}{\psi(L-n)^{-\sigma} - \eta_1 pBn^{\eta_1-1}q/z}\left[\rho - \frac{(1-\tau_h)(1-\alpha)An^{1-\alpha}q^{-\alpha}}{p}\right] - \frac{\dot{p}}{p} - \alpha\frac{\dot{q}}{q}\right\}, \tag{A4.3a}$$

$$\frac{\dot{q}}{q} = Bn^{\eta_1} - An^{1-\alpha}q^{1-\alpha} + z, \tag{A4.3b}$$

$$\frac{\dot{z}}{z} = z - [1-(1-\tau_k)\alpha]An^{1-\alpha}q^{1-\alpha} - \rho. \tag{A4.3c}$$

Moreover, we also derive

$$\frac{\dot{p}}{p} = \left[(1-\tau_k)\alpha - \frac{(1-\tau_h)(1-\alpha)}{pq}\right]An^{1-\alpha}q^{1-\alpha} - Bn^{\eta_1}, \tag{A4.3d}$$

$$p = \left[\psi(L-n)^{-\sigma} - (1-\tau_h)(1-\alpha)An^{-\alpha}q^{1-\alpha}\frac{1}{z}\right]\frac{z}{\eta_1 Bn^{\eta_1-1}q} \equiv p(n,q,z). \tag{A4.3e}$$

Given τ_k and β and with the help of (A4.3d) and (A4.3e), we can use (A4.3a)-(A4.3c) to determine n, q and z. Along the BGP, $\dot{n} = \dot{q} = \dot{z} = \dot{p} = 0$. Variables n, q and z are constant in the BGP and so is p. To determine the BGP, as in Section 3 it is useful if we rewrite p, z and q as functions of n and then determine n.

$$p = \frac{(1-\tau_h)(1-\alpha)}{\rho}An\left[\frac{\rho + Bn^{\eta_1}}{(1-\tau_k)\alpha A}\right]^{\frac{-\alpha}{1-\alpha}} \equiv p(n;\tau_h,\tau_k), \tag{A4.4a}$$

$$q = \left[\frac{\rho + Bn^{\eta_1}}{(1-\tau_k)\alpha A}\right]^{\frac{-\alpha}{1-\alpha}}\frac{1}{n} \equiv q(n;\tau_k), \tag{A4.4b}$$

$$z = [1-(1-\tau_k)\alpha]\frac{\rho + Bn^{\eta_1}}{(1-\tau_k)\alpha} + \rho \equiv z(n;\tau_k). \tag{A4.4c}$$

Using the above relationships, the labor marker clearance condition is given by

$$\psi(L-n)^{-\sigma}\left\{[1-(1-\tau_k)\alpha]\frac{\rho+Bn^{\eta_1}}{(1-\tau_k)\alpha}+\rho\right\}n=\frac{(\rho+\eta Bn^{\eta_1})(1-\tau_h)(1-\alpha)}{\rho}\frac{\rho+Bn^{\eta_1}}{(1-\tau_k)\alpha}. \tag{A4.5}$$

The left-hand side of (A4.5) is positively sloping in n whose locus starts from 0 as $n=0$ and goes to infinity when $n=L$. The right-hand side of (A4.5) is also positively sloping in n whose locus starts from a constant as $n=0$ and goes to a larger constant when $n=L$. Thus, (A4.5) determines a unique value of n^* in the BGP.

While the agent's welfare in the long run is in Eq. (15a), the long-run economic growth rate now is modified as

$$\theta^*=B(n^*)^{\eta_1}. \tag{A4.6}$$

In calibration, we maintain other parameters at the same values in Table 1 except for $\eta_1=1$ or 0.9, and use (A4.4a) and (A4.4b) to calibrate the values of A and B, and use (A4.5) to calibrate the value of ψ.

Appendix 5. Derivation of the General Learning Function Model

The first order conditions of the household's problem are

$$\frac{1}{c}=\lambda, \tag{A5.1a}$$

$$\psi l^{-\sigma}=\lambda(1-\tau_h)wuh+\lambda_h B(1-u)h\left[(1-\gamma)((1-s)k)^{\gamma}((1-u)nh)^{-\gamma}\right], \tag{A5.1b}$$

$$\lambda(1-\tau_h)w=\lambda_h B\left[(1-\gamma)((1-s)k)^{\gamma}((1-u)nh)^{-\gamma}\right], \tag{A5.1c}$$

$$\lambda(1-\tau_k)r=\lambda_h B\left[\gamma((1-s)k)^{\gamma-1}((1-u)nh)^{1-\gamma}\right], \tag{A5.1d}$$

$$\dot{\lambda}=[\rho-(1-\tau_k)rs]\lambda-\frac{\dot{h}}{k}\gamma\lambda_h, \tag{A5.1e}$$

$$\dot{\lambda}_h=\left[\rho-(1-\gamma)\frac{\dot{h}}{h}\right]\lambda_h-\lambda(1-\tau_h)wnu, \tag{A5.1f}$$

and the transversality conditions $\lim_{t\to\infty}e^{-\rho t}\lambda_t k_t=0$ and $\lim_{t\to\infty}e^{-\rho t}\lambda_{ht}h_t=0$.

The first order conditions of the firm's problem

$$r=\alpha\frac{y}{sk}, \tag{A5.2a}$$

$$w=(1-\alpha)\frac{y}{unh}, \tag{A5.2b}$$

The equilibrium conditions are simplified to a three-dimensional equilibrium system in the state vector (l, q, z) as follows.

$$\frac{\dot{l}}{l}=\frac{-1}{\sigma}\left\{\frac{\alpha}{\alpha-\gamma}\frac{\dot{p}}{p}+\rho+A\alpha\left[\frac{\gamma}{\alpha}\frac{B}{A}\left(\frac{1-\tau_k}{1-\tau_h}\frac{\alpha}{1-\alpha}\frac{1-\gamma}{\gamma}\right)^{1-\gamma}\right]^{\frac{\alpha-1}{\alpha-\gamma}}(1-\tau_k)^{\frac{1-\gamma}{\alpha-\gamma}}p^{\frac{\alpha-1}{\alpha-\gamma}}\left[-1+\frac{1-s}{\gamma}\frac{1}{pq}\right]\right\}, \tag{A5.3a}$$

$$\frac{\dot{q}}{q}=A\left[\frac{\gamma}{\alpha}\frac{B}{A}\left(\frac{1-\tau_k}{1-\tau_h}\frac{\alpha}{1-\alpha}\frac{1-\gamma}{\gamma}\right)^{1-\gamma}\right]^{\frac{\alpha-1}{\alpha-\gamma}}\left\{(1-\tau_k)^{\frac{1-\gamma}{\alpha-\gamma}}\frac{\alpha}{\gamma}\frac{1}{q}(1-s)p^{\frac{\gamma-1}{\alpha-\gamma}}-(1-\tau_k)^{\frac{1-\alpha}{\alpha-\gamma}}sp^{\frac{\alpha-1}{\alpha-\gamma}}\right\}+z, \tag{A5.3b}$$

$$\frac{\dot{z}}{z}=z+\left[1-\frac{s}{\alpha(1-\tau_k)}\right]A\alpha\left[\frac{\gamma}{\alpha}\frac{B}{A}\left(\frac{1-\tau_k}{1-\tau_h}\frac{\alpha}{1-\alpha}\frac{1-\gamma}{\gamma}\right)^{1-\gamma}\right]^{\frac{\alpha-1}{\alpha-\gamma}}p^{\frac{\alpha-1}{\alpha-\gamma}}(1-\tau_k)^{\frac{1-\gamma}{\alpha-\gamma}}-\rho. \tag{A5.3c}$$

Moreover, we also derive

$$\frac{\dot{p}}{p}=A\left[\frac{\gamma}{\alpha}\frac{B}{A}\left(\frac{1-\tau_k}{1-\tau_h}\frac{\alpha}{1-\alpha}\frac{1-\gamma}{\gamma}\right)^{1-\gamma}\right]^{\frac{\alpha-1}{\alpha-\gamma}}(1-\tau_k)^{\frac{1-\gamma}{\alpha-\gamma}}p^{\frac{\gamma-1}{\alpha-\gamma}}\left[\alpha p-(1-\alpha)\frac{1-\tau_h}{1-\tau_k}\frac{1}{q}\frac{s}{u}\right], \tag{A5.4a}$$

$$p=\left[\frac{\psi z}{l^{\sigma}(1-\tau_h)(1-\alpha)Aq}\right]^{\frac{\alpha-\gamma}{\alpha}}\frac{(1-\tau_k)}{\left[\frac{\gamma}{\alpha}\frac{B}{A}\left(\frac{1-\tau_k}{1-\tau_h}\frac{\alpha}{1-\alpha}\frac{1-\gamma}{\gamma}\right)^{1-\gamma}\right]}=p(l,q,z), \tag{A5.4b}$$

$$u=p^{\frac{-1}{\alpha-\gamma}}\frac{1}{q}\frac{1}{L-l}(1-\tau_k)^{\frac{1}{\alpha-\gamma}}\left[\frac{\gamma}{\alpha}\frac{B}{A}\left(\frac{1-\tau_k}{1-\tau_h}\frac{\alpha}{1-\alpha}\frac{1-\gamma}{\gamma}\right)^{1-\gamma}\right]^{\frac{-1}{\alpha-\gamma}}s(u)=u(l,q,z). \tag{A5.4c}$$

Given τ_k and β and with the help of (A5.3a)-(A5.3c), we can use (A5.4a)-(A5.4c) to determine l, q and z. Along a BGP, $\dot{l}=\dot{q}=\dot{z}=\dot{p}=0$. Variables l, q and z are constant in the BGP and so are u, s and p. To determine the BGP, as in Section 3 it is useful if we rewrite p, z, q and l as functions of u only and then determine u.

$$p=\left(\frac{\alpha A}{(1-\gamma)\rho}\right)^{\frac{\alpha-\gamma}{1-\alpha}}(1-\tau_k)^{\frac{1-\gamma}{1-\alpha}}\left[\frac{\gamma}{\alpha}\frac{B}{A}\left(\frac{1-\tau_k}{1-\tau_h}\frac{\alpha}{1-\alpha}\frac{1-\gamma}{\gamma}\right)^{1-\gamma}\right]^{-1}(u-\gamma)^{\frac{\alpha-\gamma}{1-\alpha}}\equiv p(u;\tau_h,\tau_k), \tag{A5.5a}$$

$$q=(1-\gamma)^{\frac{1-\gamma}{1-\alpha}}\left(\frac{\rho}{\alpha A}\right)^{\frac{\alpha-\gamma}{1-\alpha}}\left[\frac{\gamma}{\alpha}\frac{B}{A}\left(\frac{1-\tau_k}{1-\tau_h}\frac{\alpha}{1-\alpha}\frac{1-\gamma}{\gamma}\right)^{1-\gamma}\right](1-\tau_k)^{-\frac{1-\gamma}{1-\alpha}} \tag{A5.5b}$$

$$\frac{(u-\gamma)(1-\tau_h)(1-\alpha)}{(1-\tau_k)(1-\gamma)\alpha u+\gamma(1-u)(1-\tau_h)(1-\alpha)}\left(\frac{1}{u-\gamma}\right)^{\frac{1-\gamma}{1-\alpha}}\equiv q(u;\tau_h,\tau_k),$$

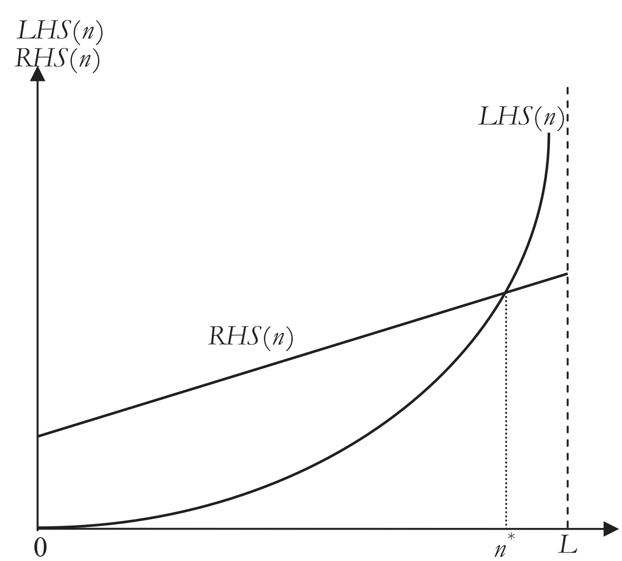
$$z=\rho\frac{u-1}{u-\gamma}+\rho\frac{1-\gamma}{u-\gamma}\frac{(1-\gamma)u}{(1-\tau_h)(1-\alpha)\gamma(1-u)+(1-\tau_k)\alpha(1-\gamma)u}\equiv z(u;\tau_h,\tau_k), \tag{A5.5c}$$

$$l=\left\{\frac{\psi}{(1-\tau_h)(1-\alpha)A}p(u)^{\frac{\alpha}{\gamma-\alpha}}(1-\tau_k)^{\frac{\alpha}{\alpha-\gamma}}\frac{z(u)}{q(u)}\left[\frac{\gamma}{\alpha}\frac{B}{A}\left(\frac{1-\tau_k}{1-\tau_h}\frac{\alpha}{1-\alpha}\frac{1-\gamma}{\gamma}\right)^{1-\gamma}\right]^{\frac{\alpha}{\gamma-\alpha}}\right\}^{\frac{1}{\sigma}}$$

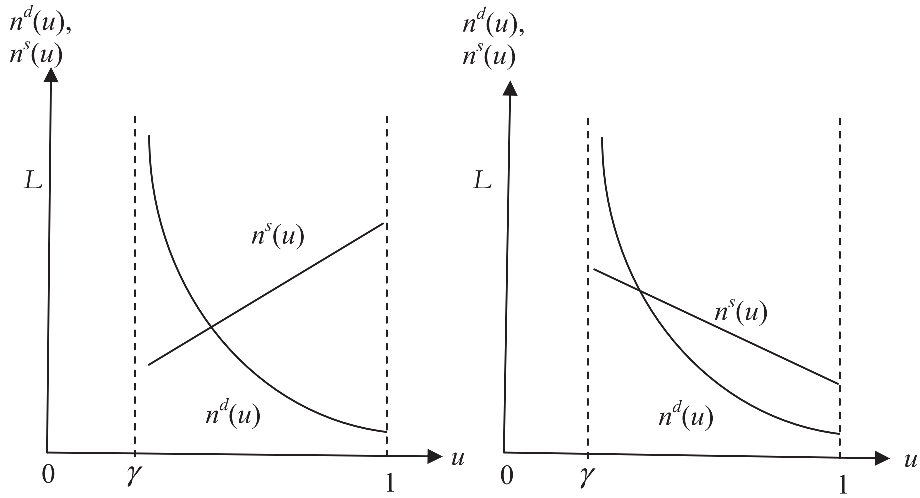
$$\equiv l(u;\tau_h,\tau_k). \tag{A5.5d}$$

Finally, the labor supply is $n^s=L-l(u;\tau_h,\tau_k)$. Moreover, if we use the firm's optimal condition for labor, along with the optimal condition of the labor allocation between the two sectors, the goods production technology, (A5.5a)-(A5.5d) and (A5.4c), the labor demand is

$$n^d=\frac{\alpha^2A}{(1-\alpha)\gamma B}\left(\frac{1-\alpha}{\alpha}\frac{\gamma}{1-\gamma}\right)^{1-\gamma}\left[\frac{\rho(1-\gamma)}{\alpha A}\right]^{\left(1+\frac{\gamma}{1-\alpha}\right)}\frac{(1-\tau_k)^{\gamma-\frac{\gamma}{1-\alpha}}}{(1-\tau_h)^{\gamma}}\left(\frac{1}{u-\gamma}\right)^{\left(1+\frac{\gamma}{1-\alpha}\right)}.$$



Appendix Fig. 1. The existence of BGP in a learning-by-doing model.



Appendix Fig. 2. The existence of BGP in an economy with a general learning function.

The labor market clearance condition is thus

$$n^d(u) \equiv \frac{\alpha^2 A}{(1-\alpha)\gamma B}\left(\frac{1-\alpha}{\alpha}\frac{\gamma}{1-\gamma}\right)^{1-\gamma}\left[\frac{\rho(1-\gamma)}{\alpha A}\right]^{\left(1+\frac{\gamma}{1-\alpha}\right)}\frac{(1-\tau_k)^{\gamma-\frac{\gamma}{1-\alpha}}}{(1-\tau_h)^{\gamma}}\left(\frac{1}{u-\gamma}\right)^{\left(1+\frac{\gamma}{1-\alpha}\right)}$$

$$= L - l(u) \equiv n^s(u). \tag{A5.6}$$

Thus, for a given capital tax rate, (A5.6) determines u in a BGP. As in Section 3, the left-hand side of (A5.6) is referred to as the $n^d(u)$ locus and the right-hand side as the $n^s(u)$ locus. In (A5.6), it is required that $u > \gamma$ in order to assure a feasible allocation so that $n^d(u) \geq 0$ and $n^s(u) \geq 0$. This condition is also necessary in order for $z(u) \geq 0$, $q(u) \geq 0$ and $p(u) \geq 0$. The shape of $n^d(u)$ and $n^s(u)$ are illustrated in the Appendix Figs. 1 and 2 wherein $n^d(u)$ is decreasing in u and locus $n^s(u)$ may be increasing or decreasing in u. We find that under (i) L is sufficiently large and (ii) $u > \gamma$, there exists a BGP.

While the agent's lifetime welfare in the long run is in Eq. (15a), the long-run economic growth rate is modified as

$$\theta^* = \rho\frac{1-u^*}{u^*-\gamma}, \frac{\partial\theta^*}{\partial u^*} = \rho\frac{\gamma-1}{(u^*-\gamma)^2} < 0.$$

In analyzing the tax incidence, the balanced government budget is Eq. (23d). Dividing both sides of Eq. (23d) by the total output in the economy gives

$$\tau_k r\frac{sk}{y} + \tau_h w\frac{hun}{y} = \frac{G}{y},$$

which, under the condition $G/y = \beta$, leads to a relationship identical to Eq. (16).

Now, we experiment with an increase in τ_k and a corresponding decrease in τ_h so as to balance the government budget (Eq. (16)). For a given u, the relationship $l(u;\ \tau_h,\ \tau_k)$ in (A5.5d) indicates that the effect through a higher τ_k and a corresponding lower τ_h is ambiguous. So the $n^s(u)$ locus may shift upward or downward.

Next, for a given u, we envisage the effect on the $n^d(u)$ locus of an increase in τ_k with a corresponding lower τ_h to meet Eq. (16). The $n^d(u)$ locus may shift upward or downward depending on the value of τ_k and τ_h.[32] As a result, u may decrease or increase in BGP and thus, the economic growth rate may increase or decrease.

Thus, a higher capital tax rate in combination with a corresponding lower labor tax rate has ambiguous effects on leisure and the ratio of consumption to capital,

$$\frac{dl^*}{d\tau_k} = \underset{(?)}{\frac{\partial l^*}{\partial \tau_k}} + \underset{(?)}{\frac{\partial l^*}{\partial u}}\underset{(?)}{\frac{du^*}{d\tau_k}}, \tag{A5.7a}$$

$$\frac{dz^*}{d\tau_k} = \underset{(?)}{\frac{\partial z^*}{\partial \tau_k}} + \underset{(?)}{\frac{\partial z^*}{\partial u}}\underset{(?)}{\frac{du^*}{d\tau_k}}. \tag{A5.7b}$$

As a result, the effect on the representative agent's long-run welfare is ambiguous

$$\frac{dU^*}{d\tau_k} = \frac{1}{\rho}\left[\left(\underset{(?)}{\frac{1}{z^*}\frac{dz^*}{d\tau_k}}\right) + \left(\underbrace{-\frac{1-\gamma}{(u^*-\gamma)^2}\frac{du^*}{d\tau_k}}_{(?)} + \psi(l^*)^{-\sigma}\underset{(?)}{\frac{dl^*}{d\tau_k}}\right)\right] > (<) 0. \tag{A5.8a}$$

In comparing Eq. (A5.8a) with Eq. (19a), the use of physical capital in the learning activity brings ambiguous effects on the ratio of consumption to capital, the economic growth rate and the level of leisure in Eq. (A5.8a). Thus, the effect on welfare is ambiguous in Eq. (A5.8a) and is different from unambiguous positive effects through higher ratios of consumption to capital and higher economic growth and an unambiguous negative effect through lower leisure in Eq. (19a) when physical capital is not used in the learning activity. Nevertheless, when the effect arising from the use of physical capital in the learning activity is not too strong, there will be a positive welfare effect through a larger ratio of consumption to capital and a larger economic growth rate and a negative welfare effect via a larger level of leisure. If the negative effect dominates the positive effects, it is optimal to tax only labor income and not to tax capital income. If the positive effects on the economic growth rate and on the ratio of consumption to capital dominate, it is optimal just to tax capital income and not to tax labor income. If these two opposite effects are completely cancelled, there exists an interior optimal capital tax rate that maximizes the social welfare in the long run. In this situation, the optimal capital tax rate τ_k is determined by

$$\left(\frac{1}{z^*}\frac{dz^*}{d\tau_k}\right)_{(?)} + \left(\underbrace{-\frac{1-\gamma}{(u^*-\gamma)^2}\frac{du^*}{d\tau_k}}_{(?)}\right) = \psi(l^*)^{-\sigma}\left(-\underset{(?)}{\frac{dl^*}{d\tau_k}}\right). \tag{A5.8b}$$

32 $\frac{dn^d(u)}{d\tau_k} = \frac{\alpha\gamma}{1-\alpha}n^d(u)\frac{\tau_k-\tau_h}{(1-\tau_k)(1-\tau_h)} > (<)0$ if $\tau_k > (<)\tau_h$.

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