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Time preference and two-country trade

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We present a dynamic two-country model of international trade with endogenous time preference. We show that if the two countries have similar preferences, production technologies and labor endowments, there exists a unique and stable steady state such that both consumption and investment goods are produced in both countries. Unlike the case of constant time preferences, the steady state is independent of the initial international distribution of capital. We prove a dynamic Heckscher–Ohlin theorem such that the labor-abundant country exports the labor-intensive good.

Key words dynamic two-country trade, endogenous time preferences, Heckscher–Ohlin theorem

JEL classification F11, E20, H20

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1 Introduction

The determination of trade patterns is a central topic in international economics. This paper presents a dynamic Heckscher–Ohlin model that explains the long-run pattern of international trade.

The literature on dynamic Heckscher–Ohlin models originated in Oniki and Uzawa (1965). While Oniki and Uzawa assume an exogenous savings rate in each trading country, most subsequent contributions, including Stiglitz (1970), Baxter (1992), Chen (1992), Shimomura (1993), Ventura (1997), Atkeson and Kehoe (2000) and Nishimura and Shimomura (2002) assume that households maximize their discounted sum of utility (i.e. savings rates are endogenously determined).

Another common assumption shared in most dynamic models of international trade is that the discount rate is exogenously given and constant. Few papers analyze trade-related issues under endogenous time preferences. Assuming endogenous time preferences, Obstfeld (1982) considers how an economy responds to permanent and unanticipated

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terms-of-trade shocks and Shi and Epstein (1993) examine international trade in an exogenous growth model of an open economy. However, both contributions assume a small open economy such that international prices and interest rates are given.

In contrast, Devereux and Shi (1991) analyze international distribution of wealth in a two-country and one-sector model that is based on variable discount rates. They assume a competitive world credit market and/or that physical capital is freely mobile between countries.

Those contributions are based on the endogenous time preference introduced through the presence of a variable discount rate originated by Uzawa (1968) and later developed by Epstein (1987). A non-constant time preference rate has been emprically documented through panel data and cross-country data by Hong (1988), Lawrence (1991) and Ogawa (1993).

The purpose of the present paper is to reformulate a two(-country, say Home and Foreign) by two(-factor, capital and labor) by two(-good, a pure consumption good and an investment good) dynamic Heckscher–Ohlin model under the Uzawa–Epstein endogenous time preference and to derive long-run trade-pattern propositions.[1]

Let us discuss why such a reformulation is required. Suppose, for the time being, that the discount rate of each household is given and constant. As is well known, in the steady rate the real interest rate is equal to the given discount rate

$$\rho = R - \delta, \tag{1}$$

where ρ, R and δ denote the discount rate, the rental rate measured by the investment good, and the rate of capital depreciation. Hence, $R - \delta$ can be interpreted as the real interest rate. It is well known in the standard static Heckscher–Ohlin model that, for a given international good price p, the rental rate R in each country is weakly decreasing in the country's capital endowment, say K. There are two crucial levels of K, $\underline{K}(p)$ and $\overline{K}(p)$, such that

1 If $0 < K < \underline{K}(p)$, the country produces only a labor-intensive good (complete specialization). R is equal to the value of marginal productivity of capital of the labor-intensive good, and it is decreasing in K, say $\underline{R}(K)$.
2 If $\underline{K}(p) < K < \overline{K}(p)$, it produces both labor-intensive and capital-intensive goods (incomplete specialization). R is a function of p only, and does not depend on K, say $R(p)$.
3 If $\overline{K}(p) < K$, it produces only a capital-intensive good (complete specialization). R is equal to the value of marginal productivity of capital, and it is decreasing in K, say $\overline{R}(K)$.

(See the slope of the gross domestic product (GDP) function, OCBA, in Figure 4, where we assume that the investment good is more labor-intensive than the consumption good.)

In Foreign we also have

$$\rho^* = R^* - \delta^*, \tag{2}$$

[1] We assume that labor endowment in each country is given and constant over time. Thus, our model is regarded as an exogenous growth model.

where the variables with an asterisk (*) attached are those belonging to Foreign. Comparing (1) and (2), we see that it is purely a matter of chance that both countries are incompletely specialized. That is, except for "measure zero" cases, production of at least one country is completely specialized. For example, suppose that production technologies are common in both countries. Then, if $\rho + \delta >$ (respectively $<$)$\rho^* + \delta^*$, then $R > R^*$. It follows from 1–3 above that Home (respectively, Foreign) is completely specialized to the production of the labor-intensive good and/or Foreign (respectively, Home) is completely specialized to the production of the capital-intensive good.

As Baxter (1992) makes clear, this property of the standard dynamic Heckscher–Ohlin model has an implication such that capital income taxation adopted by the Home and Foreign governments drastically affects the long-run production/trade structure. Denote by τ and τ^* the Home and Foreign rates of tax on capital income. Then, under internationally identical production technologies, capital depreciation ($\delta = \delta^*$) and preferences ($\rho = \rho^*$), (1) and (2) are replaced by

$$\rho = (1 - \tau)R - \delta$$

and

$$\rho = (1 - \tau^*)R^* - \delta,$$

which means that the production/trade structure drastically change according to the fiscal policies employed by Home and Foreign governments. Let us show this. If initially $1 > \tau > \tau^* > 0$ (respectively, $1 > \tau^* > \tau > 0$) so that

$$R = \frac{\rho + \delta}{1 - \tau} > (\text{respectively} <) \frac{\rho + \delta}{1 - \tau^*} = R^*,$$

then the above 1–3 again imply that Home is completely specialized to the production of the labor (respectively capital)-intensive good and/or Foreign is completely specialized to the production of the labor (respectively capital)-intensive good. Those results do not depend on how small the difference $|\tau - \tau^*|$ is. Thus, the long-run production/trade structure of each country drastically responds to a small change in fiscal policies of either country. One can find similar drastic properties under trade policies involving tariffs and subsidies.

Such drastic properties do not seem to be justified by reality. Moreover, although the standard static Heckscher–Ohlin theorem holds even if preferences and technologies are slightly different among countries, the dynamic Heckscher–Ohlin theorem that was proved by Chen (1992) holds *only if* preferences and technologies are strictly identical among countries. The state of arts in dynamic trade theory is apparently unsatisfactory.

The purpose of this paper is to construct a dynamic general equilibrium model of international trade in which a change in parameters and policy variables *continuously* influences the steady-state production/trade structure, at least as long as the change is sufficiently small. Making use of such a model, one can derive fundamental propositions concerning the relationship between trade patterns and international differences in preferences, technologies and factor endowments.

To achieve this purpose, we introduce what Uzawa calls the time preference function,[2] $\rho(u(c))$, into the dynamic Heckscher–Ohlin model, where c is the Home consumption, $u(c)$ is the instantaneous utility (the felicity function) and $\frac{d}{du}\rho(u)$ is assumed to be positive. Replacing it by the above constant discount rate, we have

$$\rho(u(c)) = (1-\tau)R - \delta \tag{3}$$

and

$$\rho^*(u(c^*)) = (1-\tau^*)R^* - \delta^*. \tag{4}$$

As we see later, these two equations are compatible with $R = R^*$, even if the rate of capital income tax, the rate of capital depreciation, discount rate functions, and the rental rate functions are internationally different. Putting it in another way, small international differences in these parameters are possible in the steady state such that the production of each country is incompletely specialized, like in the static Heckscher–Ohlin model. The two equations and the world market-clearing condition together determine the steady-state c, c^* and the market-clearing steady-state price p of the dynamic general equilibrium model in the present paper. Hence, under the conditions for the existence, uniqueness and (saddlepoint-) stability, which will be obtained in this paper, we can study the aforementioned relationship. The steady-state pattern of trade is independent of the initial international distribution of capital and comparative statical analysis will be much simpler than the case of constant rate of time preferences.

The paper is organized as follows. Section 2 states the main assumptions and sets up the model. Section 3 states the basic technical proposition concerning the existence, uniqueness and stability of the steady state in which the production of both Home and Foreign is incompletely specialized. Section 4 derives the trade-pattern propositions. Section 5 concludes. The Appendix includes the proof for the basic technical proposition.

2 The model

Let us set up the two-country dynamic general equilibrium model of international trade.

2.1 Consumers

Each consumer maximizes the discounted sum of utility subject to his or her budget constraint:

$$\max \int_0^\infty u(c) X dt \tag{5}$$

[2] See Uzawa (1968).

subject to

$$\dot{a} = ra + wl - pc \tag{6}$$

$$\dot{X} = -\rho(u(c))X, \tag{7}$$

where a is his or her net (physical and financial) asset, r is the real interest rate, w is the wage rate, l is the consumer's labor supply, and p is the price of the pure consumption good in terms of the investment good. We assume that l is normalized to be unity.

$u(c)$ is the felicity function of his or her consumption c that is assumed to satisfy[3]

$$\begin{aligned} u(c) &> 0, \quad u'(c) > 0 \quad \text{and} \quad u''(c) < 0 \text{ for all } c > 0 \\ u(0) &= 0 \text{ and } \lim_{c\to\infty} u(c) = \infty. \end{aligned} \tag{8}$$

Following Uzawa (1968), we call $\rho(u)$ the time preference function and keep his assumption about the function,

$$\begin{aligned} \rho(u) &> 0, \quad \rho'(u) > 0, \quad \rho''(u) > 0, \quad \rho(u) - u\rho'(u) > 0 \text{ for all } u > 0 \\ \rho^0 &\equiv \rho(0) > 0. \end{aligned} \tag{9}$$

As Obstfeld (1990) mentions, there is a linear function $a + bu$, $a \geq 0$ and $b > 0$ such that

$$\rho(u) > a + bu \text{ for all } u > 0, \quad \text{and } \lim_{c\to\infty} \frac{\rho(u)}{u} = b.$$

See Figure 1.

Associated with the above problem is the Hamiltonian

$$H = u(c)X + \lambda[ra + w - pc] - \theta\rho(u(c))X, \tag{10}$$

where λ and θ are shadow prices of a and X. The necessary conditions for optimality are

$$0 = u'(c)X[1 - \theta\rho'(u(c))] - \lambda p \tag{11}$$

$$\dot{\lambda} = -\lambda r \tag{12}$$

$$\dot{\theta} = -u(c) + \theta\rho(u(c)). \tag{13}$$

Letting $y \equiv \lambda/X$ and combining (7) and (12), we can rewrite (11) and (12) as

$$yp = u'(c)[1 - \theta\rho'(u(c))] \tag{14}$$

$$\dot{y} = y[\rho(u(c)) - r]. \tag{15}$$

[3] We add the last two restrictions on the felicity to Uzawa's assumptions. As we show in the Appendix, the restrictions make the proof of the existence of the steady state simpler.

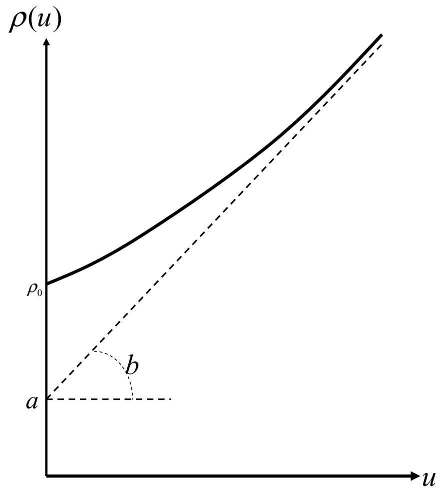

Figure 1 The time preference function.

2.2 Firms

There are two sectors in each country that produce a pure consumption good and an investment good by using physical capital and labor, respectively. Following the Oniki and Uzawa tradition, we assume that while the two goods are tradable, capital stock is not internationally mobile and depreciates at a constant rate δ. We assume away an international credit market, while each country has a competitive domestic financial market. Thus, through arbitration, the real interest rate is equal to the rental rate, say R, minus the rate of depreciation and the net asset is equal to the capital stock at each point in time. That is,

$$r = R - \delta \quad \text{and } A \equiv \sum a = \sum k \equiv K,$$

where A and K are the aggregate national financial asset and the aggregate national physical capital stock, respectively.

Both sectors are competitive, and production technology in each sector is described by a neoclassical constant-returns-to-scale (CRS) production function. If both goods are produced under perfect competition, the price of each good has to be equal to its unit cost.

$$p = \Lambda^C(w,\ R) \tag{16}$$

$$1 = \Lambda^I(w,\ R), \tag{17}$$

where $\Lambda^C(w,\ R)$ is the unit cost of the pure consumption good and $\Lambda^I(w,\ R)$ is the unit cost of the investment good.

The unit cost functions have all the standard properties. Moreover, we impose a couple of further conditions on them. First, for any positive p, w and R satisfying (16) and (17) are uniquely determined, $R(p)$ and $w(p)$.[4] Second, factor intensity reversal is assumed away, and the following holds.

The consumption good is more capital-intensive (respectively, labor-intensive) than the investment good in the sense that for any $p > 0$ the capital–labor ratio of the consumption good, say K^C/L^C, is greater (respectively, smaller) than that of the investment good, say K^I/L^I,where K^i and L^i are capital and labor allocated to Sector i, $i = C$ and I. That is, for any $p > 0$

$$\frac{K^C}{L^C} = \frac{\frac{\partial}{\partial R}\Lambda^C(w(p),\, R(p))}{\frac{\partial}{\partial w}\Lambda^C(w(p),\, R(p))} > (\text{respectively } <) \frac{\frac{\partial}{\partial R}\Lambda^I(w(p),\, R(p))}{\frac{\partial}{\partial w}\Lambda^I(w(p),\, R(p))} = \frac{K^I}{L^I}. \tag{18}$$

Third, the partial derivative of the national income

$$w(p)L + R(p)K,$$

where L is the aggregate labor supply, is equal to the aggregate national output of the consumption good:

$$Y_C(p) \equiv w'(p)L + R'(p)K. \tag{19}$$

It is also well known in trade theory[5] that the second derivative of the national income function is positive, $Y'_C(p) > 0$, when both goods are produced.

2.3 The dynamic system of a two-country world

We assume that there are two countries, Home and Foreign, which may have different production technologies, preferences and initial factor endowments. The population of Home may be different from the Foreign one.[6] Based on the foregoing argument and if both countries are incompletely specialized, we can describe the dynamic general equilibrium two-coutry model as follows:

$$\dot{K} = (R(p) - \delta)K + w(p)L - pcL \tag{20}$$

$$\dot{K}^* = (R^*(p) - \delta^*)K^* + w^*(p)L^* - pc^*L^* \tag{21}$$

$$\dot{y} = y[\rho(u(c)) + \delta - R(p)] \tag{22}$$

[4] If production technologies are of Cobb-Douglas, the unit-cost functions are also Cobb-Douglas,

$$\Lambda^i(w,\, R) = w^{\alpha_i} R^{1-\alpha_i}, \quad 0 < \alpha_i < 1.$$

If $\alpha_1 \neq \alpha_2$, the system of equations, (16) and (17), has a unique solution for any given $p > 0$. Denote it by a pair $(w(p),\, R(p))$.

[5] See the textbooks of international trade, for example, Dixit and Norman (1980), Woodland (1982) and Wong (1995).

[6] Because we assume that each household is endowed with one unit of labor, the populatioon of Home and Foreign is equal to L and L^*, respectively.

$$\dot{y}^* = y^*[\rho^*(u^*(c^*)) + \delta^* - R^*(p)] \tag{23}$$

$$\dot{\theta} = -u(c) + \theta\rho(u(c)) \tag{24}$$

$$\dot{\theta}^* = -u^*(c^*) + \theta^*\rho^*(u^*(c^*)) \tag{25}$$

$$yp = u'(c)[1 - \theta\rho'(u(c))] \tag{26}$$

$$y^* p = u^{*\prime}(c^*) - \theta^*\rho^{*\prime}(u^*(c^*)) \tag{27}$$

$$0 = [R'(p)K + w'(p)L] + [R^{*\prime}(p)K^* + w^{*\prime}(p)L^*] - cL - c^*L^*, \tag{28}$$

where an asterisk (*) is attached to foreign variables and functions. Recall that (19) means that the first two terms of (28) are the Home and Foreign supplies of the consumption good. Therefore, (28) is the world market-clearing condition for the good. The system determines the equilibrium paths of two state variables, K and K^*, and seven jump variables, c, c^*, p, y, y^*, θ and θ^*.

3 The steady state

Let us define the steady state as $(K^e, K^{*e}, c^e, c^{*e}, p^e, y^e, y^{*e}, \theta^e, \theta^{*e})$, a time-invariant solution to the above dynamic system. Thus, the following equalities are established:

$$y^e p^e = u'(c^e) - \theta^e\rho'(u(c^e)) \tag{29}$$

$$y^{e*} p^e = u^{*\prime}(c^{e*}) - \theta^{e*}\rho^{*\prime}(u^*(c^{e*})) \tag{30}$$

$$0 = (R(p^e) - \delta)K^e + w(p^e)L - p^e c^e L \tag{31}$$

$$0 = (R^*(p^e) - \delta^*)K^{e*} + w^*(p^e)L^* - p^e c^{e*} L^* \tag{32}$$

$$0 = y^e[\rho(u(c^e)) + \delta - R(p^e)] \tag{33}$$

$$0 = y^{e*}[\rho^*(u^*(c^{e*})) + \delta^* - R^*(p^e)] \tag{34}$$

$$0 = -u(c^e) + \theta^e\rho(u(c^e)) \tag{35}$$

$$0 = -u^*(c^{e*}) + \theta^{e*}\rho^*(u^*(c^{e*})) \tag{36}$$

$$0 = w'(p^e)L + w^{*\prime}(p^e)L^* + R'(p^e)K^e + R^{*\prime}(p^e)K^{e*} - c^e L - c^{e*} L^*. \tag{37}$$

Concerning the steady state, we can prove the following proposition.

The basic technical proposition on the existence, uniqueness and stability of the steady state *Suppose that the differences in preferences, technologies and initial factor endowments between Home and Foreign are not very large. Then, there exists a unique steady state* ($K^e > 0$, $K^{*e} > 0$, $c^e > 0$, $c^{*e} > 0$, $p^e > 0$, $y^e > 0$, $y^{*e} > 0$, $\theta^e > 0$, $\theta^{*e} > 0$), *which is saddlepoint-stable. There does not exist a steady state such that complete specialization holds in at least one country.*

PROOF: See the Appendix.

Remark 1 One may naturally wonder if there is a steady state such that at least one country is completely specialized to the production of either the pure consumption good or an investment good. The Appendix shows, however, that such complete specialization is impossible as long as the above differences are not very large.

Before deriving the main results of this paper, let us state the Stolper–Samuelson theorem, one of the most fundamental theorems in trade theory by using the notation defined in this paper.

The Stolper–Samelson theorem

1. *If the consumption good is more capital-intensive than the investment good, then*

$$pR'(p)/R(p) > 1 > 0 > w'(p) \tag{38}$$

2. *If the consumption good is more labor-intensive than the investment good, then*

$$pw'(p)/w(p) > 1 > 0 > R'(p) \tag{39}$$

The first inequalities of (38) and (39) are often called “magnification effects” in trade theory.

Remark 2 Because $R(p^e) > R(p^e) - \delta = \rho(u(c^e)) > 0$, (38) implies that

$$\frac{p^e R'(p^e)}{R(p^e) - \delta} > \frac{p^e R'(p^e)}{R(p^e)} > 1. \tag{40}$$

4 Trade-pattern propositions

Let us focus on the Home excess demand for the pure consumption good in the steady state,

$$ED(p) \equiv c(R(p) - \delta)L - [w'(p)L + R'(p)K(p)], \tag{41}$$

where $c(.)$ is the inverse function of the composite function $\rho(u(.))$ and

$$K(p) \equiv \frac{[pc(R(p)-\delta) - w(p)]L}{R(p)-\delta}. \tag{42}$$

We see from (31) that $K(p^e) = K^e$. Denote by p_a and p_a^* the autarkic steady-state prices of Home and Foreign, which satisfy $ED(p_a) = 0 = ED^*(p_a^*)$. If the two countries are completely identical, the steady-state price p^e has to be equal to $p_a(= p_a^*)$.

As is shown in Lemma 4 in the Appendix, the excess demand curve $ED(p)$ is negatively sloped (at least) in a neighborhood of p_a. We are now ready to obtain the following trade-pattern propositions.

4.1 Labor endowments

Substituting (42), let us rewrite (41) as

$$ED(p) = \left[\left(1 - \frac{pR'(p)}{R(p)-\delta}\right)c(R(p)-\delta) + \left(\frac{pR'(p)}{R(p)-\delta} - \frac{pw'(p)}{w(p)}\right)\frac{w(p)}{p}\right]L. \tag{43}$$

Suppose that Home and Foreign differ only in the labor endowments. Then, we see from (43) that $sign[ED(p)] = sign[ED^*(p)]$ for any p and that $ED(p^e) = ED^*(p^e) = 0$. That is, the two excess demand curves can be depicted as in Figure 2, which shows that neither country has the comparative advantage to any good and that international trade does not take place. Summarizing, we have the first main result.

Proposition 1 *Suppose that Home and Foreign differ only in the labor endowments, L and L^*. Then international trade does not take place in the steady state.*

Remark 1 *We have assumed that each household is endowed with one unit of labor and the population may be different between countries. Alternatively, we could assume that each Home household is endowed with a different labor endowment from the Foreign household and the poplulation of each country is the same. In the latter case, we can prove that in the*

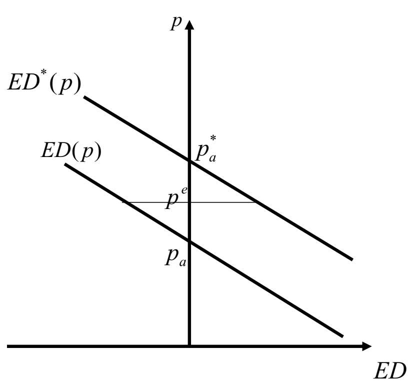

Figure 2 $R'(p) > 0$ and $L^* > L$.

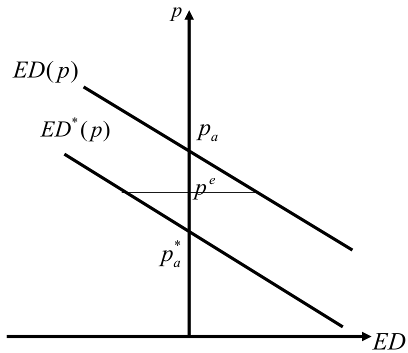

Figure 3 $R'(p) < 0$ and $L^* > L$.

steady state the labor abundant country exports (respectively imports) the labor (respectively capital)-intensive good.[7]

4.2 Preferences

Next, let us assume that Foreign is more patient than Foreign in the sense that

$$\rho^*(u(c^*)) \equiv \rho(u(c^*))/\eta,$$

where $\eta > 1$ is a parameter. Then

$$c^* = c(\eta(R(p) - \delta)).$$

Thus, (43) can be written as

$$ED^*(p) = \left[\left(1 - \frac{pR'(p)}{R(p) - \delta}\right) c(\eta(R(p) - \delta)) + \left(\frac{pR'(p)}{R(p) - \delta} - \frac{pw'(p)}{w(p)}\right) \frac{w(p)}{p}\right] L. \tag{44}$$

Differentiating (44) with respect to η, we derive

$$\frac{\partial}{\partial \eta} ED^*(p) = \left[1 - \frac{pR'(p)}{R(p) - \delta}\right] (R(p) - \delta) c'(\eta(R(p) - \delta)) L,$$

which is positive (respectively, negative) due to the Stolper–Samelson theorem and (40) if the pure consumption good is more labor(respectively, capital)-intensive than the investment good. Then, we can depict the excess demand curves of the impatient and patient countries in a way similar to Figure 3. Inspection of the figure ensures us the second trade-pattern result:

Proposition 2 *Other things being equal, the patient country imports the labor-intensive good in the steady state.*

[7] The proof is available from the authors on request.

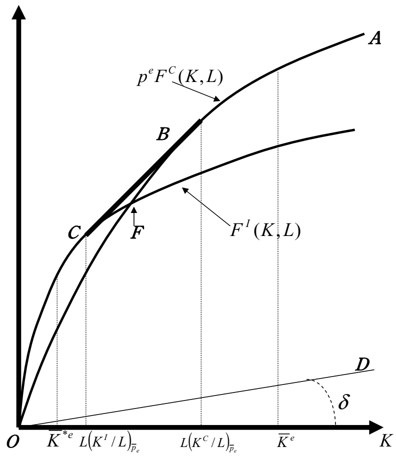

Figure 4 Gross domestic product function (ABCO). The difference between ABCD and OD is increasing in K.

4.3 Fiscal policy

Suppose that the Home government imposes income tax and transfers the tax revenue to the Home households in a lump-sum manner. How may the income taxation affect the pattern of international trade?

First, let us consider the flow-budget constraint of the representative Home household. Denoting by τ_R and τ_w the rates of capital and labor income taxes, it is

$$\dot{k} = (1-\tau_R)(R(p)-\delta)k + (1-\tau_w)w(p) + f - pc,$$

where f is the lump-sum transfer from the government to the Home household, which is equal to $\tau_R(R(p)-\delta)k + \tau_w w(p)$. Solving the utility-maximization problem for the Home household, we obtain, in the steady state

$$0 = \rho(u(c)) - (1-\tau_R)(R(p)-\delta),$$

from which we obtain

$$c = c((1-\tau_R)(R(p)-\delta)). \tag{45}$$

Using this "consumption function", and considering $T = Lf = \tau_R(R(p)-\delta)K + \tau_w w(p)L$, we can rewrite the above flow-budget constraint in the steady state as

$$K = \frac{[pc((1-\tau_R)(R(p)-\delta)) - w(p)]L}{R(p)-\delta}. \tag{46}$$

Substituting (45) and (46) into the steady-state excess demand (41), we obtain

$$ED(p) = \left[c((1-\tau_R)(R(p)-\delta)) - \left\{w'(p) + R'(p)\frac{(pc((1-\tau_R)(R(p)-\delta)) - w(p))}{R(p)-\delta}\right\}\right]L$$
$$= \left[\left(1 - \frac{pR'(p)}{R(p)-\delta}\right)c((1-\tau_R)(R(p)-\delta)) - \left(w'(p) - \frac{R'(p)w(p)}{R(p)-\delta}\right)\right]L. \quad (47)$$

For a given p, the differentiation of (47) with respect to the income tax rate τ yields

$$\frac{\partial}{\partial \tau_R} ED(p) = -(R(p)-\delta)c'((1-\tau_R)(R(p)-\delta))\left[1 - \frac{pR'(p)}{R(p)-\delta}\right]L,$$

which implies that the imposition of the capital income tax shifts the excess demand curve to the right (respectively, left) direction if the pure consumption good is more labor(respectively, capital)-intensive, whereas the labor income tax has no effect on trade pattern. Therefore, we can derive the following proposition that relates trade pattern to fiscal policy.

Proposition 3 *Suppose that two countries are initially identical so that there is no trade even if there is no barrier to free trade, and that the Home government starts imposing a small income tax permanently at time zero on. The imposition of a small tax on capital income at Home makes the country import the capital-intensive good in the tax-ridden steady state, whereas the imposition of tax on labor income does not affect the long-run pattern of international trade.*

4.4 Trade policy

Let us examine the effect of import tariffs on trade patterns.

First, suppose that Home imports the pure consumption good under free trade and that the Home government imposes a small import tariff. Following the standard textbook of international trade, we assume that the Home government transfers the tariff revenue to the Home households in a lump-sum fashion. The flow-budget constraint of the Home household is

$$\dot{k} = (R(p+s)-\delta)k + w(p+s) - pc + \gamma,$$

where γ is the lump-sum transfer of the tariff revenue and s is the import tariff rate. Therefore, $p+s$ is the Home domestic price of the pure consumption good, and

$$\Gamma = L\gamma = s[cL - \{(R'(p+s)K + w(p+s)L\}].$$

In the steady state we have

$$c = c(R(p+s)-\delta). \quad (48)$$

Substituting it to the steady-state flow-budget constraint, we derive

$$K = K(p+s) \equiv \frac{[pc(R(p+s)-\delta) - (w(p+s) - sw'(p+s))]L}{R(p+s) - \delta - sR'(p+s)}. \tag{49}$$

From (48) and (49), we can express the Home excess demand for the pure consumption good as

$$\begin{aligned} ED(p) &= c(R(p+s)-\delta)L - \{R'(p+s)K(p+s) + w'(p+s)L\} \\ &\quad - \left\{ \frac{R'(p+s)[pc(R(p+s)-\delta) - \{w(p+s) - sw'(p+s)\}]}{R(p+s)-\delta - sR'(p+s)} \right. \\ &\quad \left. + w'(p+s) \right\} L. \end{aligned} \tag{50}$$

Differentiating (50) with respect to s and evaluating the derivative at $s = 0$, we have

$$\begin{aligned} \frac{\partial}{\partial s} ED(p)|_{s=0} = c' \cdot R'(p) \left[1 - \frac{pR'(p)}{R(p)} \frac{R(p)}{(R(p)-\delta)} \right] L \\ - [R''(p)K(p) + w''(p)L], \end{aligned}$$

which is negative, irrespective of the factor-intensity ranking.

Next, let us examine the case such that Home imports the investment good. Let $\bar{S}$, P_1, P_2, $\bar{W}$ and $\bar{R}$ be the tariff rate, the price of the pure consumption good, the price of the investment good, the wage rate and the rental rate in the nominal term. The flow-budget constraint is expressed as

$$\begin{aligned} (\bar{S} + P_2)\dot{K} &= \bar{R}K - \delta(\bar{S} + P_2)K + \bar{W}L - P_1 cL \\ &\quad + \bar{S}\left[\delta K - \frac{1}{\bar{S} + P_2}\{\bar{R}K + \bar{W}L - P_1 Y_1\} \right], \end{aligned}$$

where Y_1 is the Home output of the pure consumption good. Letting $\bar{s} \equiv \frac{\bar{S}}{\bar{S}+P_2}$, $R \equiv \frac{\bar{R}}{\bar{S}+P_2}$, $w \equiv \frac{\bar{W}}{\bar{S}+P_2}$, $\bar{s} \equiv \frac{\bar{S}}{\bar{S}+P_2}$ and $p \equiv \frac{P_1}{P_2}$, the above constraint can be rewritten as

$$\dot{K} = (R-\delta)K + wL - pcL + \bar{s}[\delta K - \{RK + wL - (1-\bar{s})pY_1].$$

Note that $(1-\bar{s})p = \frac{P_2}{\bar{S}+P_2}\frac{P_1}{P_2} = \frac{P_1}{\bar{S}+P_2}$ is the domestic price of the pure consumption good. Hence, R and w are the functions of $(1-\bar{s})p$, and $Y_1 = R'((1-\bar{s})p)K + w'(1-\bar{s})p)L$. In the steady state we have

$$c = c(R((1-\bar{s})p) - \delta)$$

and

$$K = \frac{[pc(R((1-\bar{s})p)-\delta) - \{w((1-\bar{s})p) + \bar{s}w'((1-\bar{s})p)\}]L}{R((1-\bar{s})p) - \delta + \bar{s}R'((1-\bar{s})p)}.$$

Thus, in the steady state the excess demand for the pure consumption good is

$$ED(p) = \Bigg[c((1-\bar{s})p)-\delta) - w'((1-\bar{s})p) \\ - \frac{R'((1-\bar{s})p)-\delta)[pc(R((1-\bar{s})p)-\delta) - \{w((1-\bar{s})p) + \bar{s}w'((1-\bar{s})p)\}}{R((1-\bar{s})p)-\delta+\bar{s}R'((1-\bar{s})p)} \Bigg] L.$$

Differentiating the excess demand with respect to $\bar{s}$ and evaluating the derivative at $\bar{s}=0$, we see that

$$\frac{\partial}{\partial \bar{s}} ED(p) = c' LpR' \left[\frac{pR'}{R-\delta} - 1 \right] + p(R''K + w''L) > 0.$$

Summarizing the foregoing exercises, we obtain the following proposition:

Proposition 4 *The imposition of a small import tariff always makes the volume of imports smaller.*

5 Concluding remarks

We have presented a basic dynamic two-coutry model of international trade, which is regarded as an integration of Uzawa's two seminal contributions to economic theory. We show that there exists a unique and stable steady state with both countries being incompletely specialized. The steady state is independent of the initial international distribution of capital, which is different from the dynamic trade model with constant rate of time preference.

Because the model is a basic one, there seem to be many ways to discuss trade issues from the point of view of trade dynamics. One direction is to apply the model to the issues in the normative side of international trade like dynamic trade gains and international transfers.

Appendix

Here we shall prove the basic technical proposition.

A The existence of the steady state with incomplete specialization

Assume that Home and Foreign are completely identical in preferences, technologies and endowments. Moreover, in what follows, we normalize L to be unity. Then, there exists a steady state if the following system of equations have a solution (K, p),

$$0 = (R(p)-\delta)K + w(p) - pc(R(p)-\delta) \tag{51}$$

$$0 = c(R(p)-\delta) - (R'(p)K + w'(p)). \tag{52}$$

Rewriting (51) as

$$K = \frac{pc(R(p)-\delta) - w(p)}{R(p)-\delta}, \tag{53}$$

and substituting (53) into (52), we obtain

$$\begin{aligned} ED(p) &\equiv c(R(p)-\delta) - \left[\frac{R'(p)pc(R(p)-\delta)-w(p)}{R(p)-\delta} + w'(p)\right] \\ &= \left[1 - \frac{pR'(p)}{R(p)}\frac{R(p)}{(R(p)-\delta)}\right]c(R(p)-\delta) + \left[\frac{pR'(p)}{R(p)}\frac{R(p)}{(R(p)-\delta} - \frac{pw'(p)}{w(p)}\right]\frac{w(p)}{pR(p)} \\ &= 0. \end{aligned} \tag{54}$$

Suppose that (54) has a solution p^e. Defining

$$K^e \equiv \frac{p^e c(R(p^e)-\delta) - w(p^e)}{R(p^e)-\delta}, \tag{55}$$

we see that the pair (p^e, K^e) is a solution to the system (51) and (52).

To ensure the existence of p^e such that $ED(p^e) = 0$, we specify production technologies a little more.

Assumption 1

(i) If the pure consumption good is more capital-intensive than the investment good,

$$\begin{aligned} R(0) &= 0, \\ \infty &> \sup_{p>0} pR'(p)/R((p) \geq \inf_{p>0} pR'(p)/R(p) > 1, \\ \infty &> \sup_{p>0} \left|pw'(p)/w(p)\right| \geq \inf_{p>0} \left|pw'(p)/w(p)\right| > 0. \end{aligned}$$

(ii) If the pure consumption good is more labor-intensive than the investment good,

$$\begin{aligned} w(0) &= 0, \\ \infty &> \sup_{p>0} pw'(p)/w(p) \geq \inf_{p>0} pw'(p)/w(p) > 1, \\ \infty &> \sup_{p>0} \left|pR'(p)/R(p)\right| \geq \inf_{p>0} \left|pR'(p)/R(p)\right| > 0. \end{aligned}$$

(iii) There exists $p_0 > 0$ such that $R(p_0) - \delta = \rho^0$.
We define in Section 2 that $\rho^0 = \rho(0)$. Because we assumed $u(0) = 0$, it follows that $c(\rho^0) = c(R(p_0) - \delta) = 0$.
Note that Cobb-Douglas technologies satisfy Assumption 1(i)–(iii).

Lemma 1 (54) *has a unique and positive solution p^e.*

PROOF: Considering the properties of the functions $\rho(u)$ and $u(c)$ stated in Section 2, we see that the inverse function $c(.)$ has to satisfy the following properties:

$$\begin{aligned} c'(\rho) &> 0 \quad \text{and} \quad c''(\rho) < 0 \text{ for any } \rho > \rho^0, \\ c(\rho^0) &= 0 \quad \text{and} \quad \lim_{c\to\infty} c(\rho) = \infty. \end{aligned} \tag{56}$$

First, let us consider the case where the pure consumption good is more capital-intensive than the investment good. That is, the conditions in Assumption 1(i) hold. Considering Assumption 1(iii), we see that

$$ED(p_0) = \left[\frac{p_0 R'(p_0)}{R(p_0)}\frac{R(p_0)}{(R(p_0)-\delta)} - \frac{p_0 w'(p_0)}{w(p_0)}\right]\frac{w(p_0)}{p_0 R(p_0)},$$

which is positive if the pure consumption good is more capital-intensive than the investment good. In contrast, Assumption 1(i) implies that while the term in (54),

$$\left[1-\frac{pR'(p)}{R(p)}\frac{R(p)}{(R(p)-\delta)}\right]c(R(p)-\delta),$$

diverges to negative infinite as $p\to\infty$, the second term

$$\left[\frac{pR'(p)}{R(p)}\frac{R(p)}{(R(p)-\delta}-\frac{pw'(p)}{w(p)}\right]\frac{w(p)}{pR(p)}$$

never diverges to positive infinite as $p\to\infty$. It follows that for a sufficiently large p, $ED(p)<0$. Because $ED(p)$ is continuos in p, it follows from $ED(p_0)>0$ that there exists a positive p^e such that $p_0<p^e<\infty$ and $ED(p^e)=0$.

Second, let us consider the other case that the pure consumption good is more labor-intensive than the investment good. That is, the conditions in Assumption 1(ii) hold. On the one hand, considering Assumption 1(iii), we see that

$$ED(p_0)=\left[\frac{p_0R'(p_0)}{R(p_0)}\frac{R(p_0)}{(R(p_0)-\delta)}-\frac{p_0w'(p_0)}{w(p_0)}\right]\frac{w(p_0)}{p_0R(p_0)},$$

which is negative if the pure consumption good is more labor-intensive than the investment good.

On the other hand, Assumption 1(ii) implies that the first term in (54),

$$\left[1-\frac{pR'(p)}{R(p)}\frac{R(p)}{(R(p)-\delta)}\right]c(R(p)-\delta),$$

is positive for any nonnegative $p<p_0$. It also implies that, when $p=0$, the second term

$$\left[\frac{pR'(p)}{R(p)}\frac{R(p)}{(R(p)-\delta}-\frac{pw'(p)}{w(p)}\right]\frac{w(p)}{pR(p)}$$

is zero.

Let us prove it. First, Assumption 1(ii) implies that, as $p\to 0$, a part of the term

$$\left[\frac{pR'(p)}{R(p)}\frac{R(p)}{(R(p)-\delta}-\frac{pw'(p)}{w(p)}\right]$$

never converges to zero. Second, Assumption 1(ii) also implies that, as $p\to 0$, $w(p)/p$ converges to zero.[8]

Therefore, (54) must be positive when p is close to zero. Because $ED(p)$ is continuos in p, it follows from $ED(p_0)<0$ and $\lim_{p\to 0}ED(p)>0$ that there exists a positive p^e such that $0<p_0<p^e$ and $ED(p^e)=0$. □

Lemma 2 K^e, *which is defined in (55), is positive.*

PROOF: From (51) and (52), we obtain

$$K^e=\frac{pc(R(p^e)-\delta)-w(p^e)}{R(p^e)-\delta}=\frac{c(R(p^e)-\delta)-w'(p^e)}{R'(p^e)}. \tag{57}$$

[8] Let us prove $\lim_{p\to 0}\frac{w(p)}{p}=0$. Suppose $\lim_{p\to 0}\frac{w(p)}{p}>0$. Because $w(0)=0$, we have

$$\lim_{p\to 0}\frac{w(p)}{p}=\lim_{p\to 0}w'(p)>0,$$

which implies $\lim_{p\to 0}\frac{pw'(p)}{w(p)}=1$. However, this result contradicts Assumption 1(ii).

Solving the second equality for $c(R(p^e)-\delta)$, we have

$$c(R(p^e)-\delta)=\frac{w(p)R'(p)-w'(p)(R(p)-\delta)}{pR'(p)-(R(p)-\delta)}. \tag{58}$$

The substitution of (58) into (57) yields

$$K^e=\frac{w(p^e)-p^e w'(p^e)}{p^e R'(p^e)-(R(p^e)-\delta)}, \tag{59}$$

which is positive due to the Stolper–Samuelson theorem. □

It is well known that incomplete specialization (i.e. positive products in both production sectors), is guaranteed if K^e is in between the factor intensities of both sectors at $p=p^e$.

Lemma 3 *Incomplete specialization is established at the steady state* (p^e, K^e).

PROOF: As is shown in the footnote,[9] the factor intensities at $p=p^e$ are

$$K^C=\frac{w(p^e)-p^e w'(p^e)}{p^e R'(p^e)-R(p^e)} \text{ (the pure consumption good)} \tag{60}$$

and

$$K^I=-\frac{w'(p^e)}{R'(p^e)} \text{ (the investment good).} \tag{61}$$

Subtracting (61) from (60),

$$K^C-K^I=\frac{R'(p^e)w(p^e)-w'(p^e)R(p^e)}{R'(p^e)[p^e R'(p^e)-R(p^e)]}. \tag{62}$$

It is clear that (62) is positive (respectively negative) if the pure consumption good is more capital(respectively labor)-intensive than the investment good.

Subtracting each of (60) and (61) from (59),

$$K^e-K^C=\frac{-\delta\{w(p^e)-p^e w'(p^e)\}}{\{p^e R'(p^e)-(R(p^e)-\delta)\}\{p^e R'(p^e)-R(p^e)\}} \tag{63}$$

$$K^e-K^I=\frac{R'(p^e)w(p^e)-w'(p^e)(R(p^e)-\delta)}{\{p^e R'(p^e)-(R(p^e)-\delta)\}R'(p^e)}. \tag{64}$$

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[9] Let us start from (19); that is,

$$Y_C(p^e)\equiv w'(p^e)L+R'(p^e)K^e.$$

K^C/L must be equal to the factor endowment ratio such that only the pure consumption good is produced in an economy, which means that the national income is equal to $p^e Y_C(p^e)$. That is,

$$\begin{aligned} w(p^e)L+R(p^e)K^C &= p^e Y_C(p^e) \\ &= p^e[w'(p^e)L+R'(p^e)K^C]. \end{aligned}$$

Rearranging, we obtain (60).
Next, K^I/L must be equal to the factor endowment ratio such that only the investment good is produced in an economy, which means $Y_C(p^e)=0$. That is,

$$0=w'(p^e)L+R'(p^e)K^I,$$

from which we obtain (61).

The foregoing results, (62), (63) and (64), can be summarized as follows:

$$\min[K^C, K^I] < K^e < \max[K^C, K^I]. \tag{65}$$

That is, incomplete specialization is established at (p^e, K^e). □

Finally, let us prove the uniqueness of (p^e, K^e).

Lemma 4 *The pair (p^e, K^e) is unique.*

PROOF: Due to the definition of K^e stated in (55), it is sufficient to prove that p^e is unique. Differentiating $ED(p)$ with respect to p at $p = p^e$, we see that

$$\begin{aligned}\left.\frac{dED(p)}{dp}\right|_{p=p^e} &= c'(R(p^e)-\delta)R'(p^e) - [R''(p^e)K^e + w''] - \frac{R'(p^e)}{R(p^e)-\delta}[p^e R'(p^e)c'(R(p^e)-\delta) \\ &\quad + \{c(R(p^e)-\delta) - w'(p^e) - R'(p^e)K^e\}] \\ &= c'(R(p^e)-\delta)R'(p^e) - [R''(p^e)K^e + w''] - \frac{p^e(R'(p^e))^2 c'(R(p^e)-\delta)}{R(p^e)-\delta} \\ &= R'(p^e)\left[1 - \frac{p^e R'(p^e)}{R(p^e)-\delta}\right]c'(R(p^e)-\delta) - [R''(p^e)K^e + w''],\end{aligned}$$

which is negative. Due to the Stolper–Samuelson theorem and $c' > 0$, the first term

$$R'(p^e)\left[1 - \frac{p^e R'(p^e)}{R(p^e)-\delta}\right]c'(R(p^e)-\delta)$$

is negative, irrespective of the factor-intensity ranking. Under the standard neoclassical technologies, the static supply function of the pure consumption good is positively sloped,

$$dY_C(p)/dp|_{K^e \text{ kept unchanged}} = w''(p) + R''(p)K^e > 0,$$

irrespective of the factor-intensity ranking. Therefore, $ED(p)$ is negatively sloped at $p = p^e$, which implies that p^e is unique, as is the pair (p^e, K^e). □

Once the unique existence result is established in the case such that Home and Foreign are identical, we can assert that the result holds for the case in which the two countries are slightly different from each other, if the Jacobian is not zero at the symmetric case, which will be established in the subsequent subsection concerning stability.

B The non-existence of the steady state with complete specialization in Home and/or Foreign

The foregoing result does not exclude the possibility that production is completely specialized at least in one of the two countries. This section examines the possibility. We assume that $L = L^* = 1$ and that the production functions $F^1(K,1)$ and $F^2(K,1)$ satisfy $F^i(0,1) = 0$, $F^i_K(K,1) > 0$ and $F^i_{KK}(K,1) < 0$, for any $K > 0$ and $i = 1, 2$.

We have denoted the steady-state price and Home capital by p^e and K^e. We shall denote the steady-state Foreign capital by K^{*e}.

Let us examine the case such that the pure consumption good is more capital-intensive than the investment good (i.e. $R' > 0$). Let us define a function of K as follows

$$\Psi_1(K) \equiv \begin{cases} [F^2(K,1) - \delta K] - p^e c\big(F_K^2(K,1) - \delta\big) & \text{for} \quad 0 < K < K^I \\ [R(p^e)K + w(p^e) - \delta K] - p^e c(R(p^e) - \delta) & \text{for} \quad K^I \le K \le K^C \\ [p^e F^1(K,1) - \delta K] - p^e c\big(p^e F_K^1(K,1) - \delta\big) & \text{for} \quad K^C < K. \end{cases} \tag{66}$$

Considering the definitions of K^C and K^I, we see that $\Psi_1(K)$ is differentiable in K. We can derive

$$\frac{d}{dK}\Psi_1(K) \equiv \begin{cases} \big(F_K^2(K,1) - \delta\big) - p^e Lc'\big(F_K^2(K,1) - \delta\big) F_{KK}^2 & \text{for} \quad 0 < K < K^I \\ (R(p^e) - \delta) & \text{for} \quad K^I \le K \le K^C \\ \big(p^e F_K^1(K,1) - \delta\big) - p^e c'\big(p^e F_K^1(K,1) - \delta\big) p^e F_{KK}^1 & \text{for} \quad K^C < K, \end{cases} \tag{67}$$

which is negative.[10]

Making use of the function $\Psi_1(K)$, we can prove that it is impossible that the production of Home and/or Foreign is completely specialized.

First, suppose that Home is completely specialized to the production of the pure consumption good in the steady state. Then, $K^e \ge K^C > K^{*e}$. Because $\dot{K} = \dot{K}^* = 0$ at the steady state and $(R^e, w^e) = (p^e F_K^1(K^e, 1), p^e F_L^1(K^e, 1))$ due to the complete specialization to the production of the pure consumption good in Home, we must have the Home flow-budget condition as follows.

$$\begin{aligned} 0 &= (R^e - \delta)K^e + w^e - p^e c^e \\ &= \big(p^e F_K^1(K^e,1) - \delta\big)K^e + p^e F_L^1(K^e,1) - p^e c\big(p^e F_K^1(K^e,1) - \delta\big) \\ &= [p^e F^1(K^e,1) - \delta K^e] - p^e c\big(p^e F_K^1(K^e,1) - \delta\big) \\ &= \Psi_1(K^e), \end{aligned} \tag{68}$$

where the second-last equality comes from a property of linearly homogeneous functions.[11]

In contrast, $\dot{K}^* = 0$ means that K^{*e} has to satisfy either

$$\begin{aligned} 0 &= [R(p^e)K^{*e} + w(p^e) - \delta K^{*e}] - p^e c(R(p^e) - \delta) \\ &= \Psi_1(K^{*e}), \end{aligned}$$

if Foreign is incompletely specialized, or

$$\begin{aligned} 0 &= (R^{*e} - \delta)K^{*e} + w^{*e} - p^e c^{*e} \\ &= \big(F_K^2(K^{*e},1) - \delta\big)K^{*e} + F_L^2(K^{*e},1) - p^e c\big(F_K^2(K^{*e},1) - \delta\big) \\ &= (F^2(K^{*e},1) - \delta K^{*e}) - p^e c\big(F_K^2(K^{*e},1) - \delta\big) \\ &= \Psi_1(K^{*e}), \end{aligned}$$

[10] Let us denote by $\bar{K}$ the solution to $\rho^0 = p^e F_K^1(K, L) - \delta$. Because the steady-state consumption must be positive, we do not need to consider the case such that K is greater than $\bar{K}$. Therefore, we can assume, without loss, that

$$p^e F_K^1(K, L) - \delta > 0 \text{ for } K^I < K < \bar{K},$$

which implies that

$$0 < R(p^e) - \delta < F_K^2(K, L) - \delta$$

for $0 < K < K^I$.

[11] $F^1(K, L) = KF_K^1(K, L) + LF_L^1(K, L)$.

if Foreign completely specializes to the production of the investment good.[12] In any case, we have

$$0 = \Psi_1(K^e) = \Psi_1(K^{*e}) \text{ and } K^e > K^{*e},$$

which contradicts the fact that $\Psi_1(K)$ is decreasing in K.

Second, suppose that Home specializes in the production of the investment good. Then, $K^e \leq K^I < K^{*e}$. Because $\dot{K} = \dot{K}^* = 0$ at the steady state and $(R^e, w^e) = (F_K^2(K^e, 1), F_L^2(K^e, 1))$, due to the complete specialization to the production of the investment good in Home, we must have the Home flow-budget condition as follows.

$$\begin{aligned} 0 &= (R^e - \delta)K^e + w^e - p^e c^e \\ &= \big(F_K^2(K^e, 1) - \delta\big)K^e + F_L^2(K^e, 1) - p^e c\big(F_K^2(K^e, 1) - \delta\big) \\ &= (F^2(K^e, 1) - \delta K^e) - p^e c\big(F_K^2(K^{*e}, 1) - \delta\big) \\ &= \Psi_1(K^e). \end{aligned}$$

However, $\dot{K}^* = 0$ means that K^{*e} has to satisfy either

$$\begin{aligned} 0 &= [R(p^e)K^{*e} + w(p^e) - \delta K^{*e}] - p^e c(R(p^e) - \delta) \\ &= \Psi_1(K^{*e}), \end{aligned}$$

if Foreign is incompletely specialized, or

$$\begin{aligned} 0 &= (R^{*e} - \delta)K^{*e} + w^{*e} - p^e c^{*e} \\ &= \big(p^e F_K^1(K^{*e}, 1) - \delta\big)K^{*e} + F_L^1(K^{*e}, 1) - p^e c\big(p^e F_K^1(K^{*e}, 1) - \delta\big) \\ &= (p^e F^1(K^{*e}, 1) - \delta K^{*e}) - p^e c\big(p^e F_K^1(K^{*e}, 1) - \delta\big) \\ &= \Psi_1(K^{*e}), \end{aligned}$$

if Foreign is completely specialized to the production of the pure consumption good.[13] In any case, we have

$$0 = \Psi_1(K^e) = \Psi_1(K^{*e}) \text{ and } K^e < K^{*e},$$

which again contradicts to the fact that $\Psi_1(K)$ is decreasing in K.

We can make a parallel argument for the case such that the pure consumption good is more labor-intensive than the investment good, if, instead of $\Psi_1(K)$, we use the function,

$$\Psi_2(K) \equiv \begin{cases} [p^e F^1(K, 1) - \delta K] - p^e c(p^e F_K^1(K, 1) - \delta) & \text{for} \quad 0 < K < K^C \\ [R(p^e)K + w(p^e) - \delta K] - p^e c(R(p^e) - \delta) & \text{for} \quad K^C \leq K \leq K^I \\ [F^2(K, 1) - \delta K] - p^e c\big(F_K^2(K, 1) - \delta\big) & \text{for} \quad K^I < K. \end{cases} \tag{69}$$

We arrive at the following lemma.

Lemma 5 *When the two countries are sufficiently identical, it is impossible that there is a country whose production is completely specialized to one of the two goods in the steady state.*

[12] Note that

$$(R^{*e}, w^{*e}) = \big(F_K^2(K^{*e}, L),\, F_L^2(K^{*e}, L)\big).$$

[13] Note that at the present case

$$(R^{*e}, w^{*e}) = \big(p^e F_K^1(K^{*e}, L),\, pF_L^1(K^{*e}, L)\big).$$

C Local saddlepoint-stability

Let us assume that the two countries are identical. Our dynamic general equilibrium model is described as

$$\begin{aligned}
\dot{K} &= (R(p)-\delta)K + w(p) - pc \\
\dot{K}^* &= (R(p)-\delta)K^* + w(p) - pc^* \\
\dot{y} &= y[\rho(u(c)) + \delta - R(p)] \\
\dot{y}^* &= y^*[\rho(u(c^*)) + \delta - R(p)] \\
\dot{\theta} &= -u(c) + \theta\rho(u(c)) \\
\dot{\theta}^* &= -u^*(c^*) + \theta^*\rho(u(c^*)) \\
0 &= -yp + u'(c)[1-\theta\rho'(u(c))] \\
0 &= -y^*p + u'(c^*)[1-\theta^*\rho'(u(c^*))] \\
0 &= w'(p) + w'(p) + R'(p)K + R'(p)K^* - c - c^*.
\end{aligned}$$

Linearizing around the steady state, we obtain

$$\begin{bmatrix} \dot{x}_K \\ \dot{x}_K^* \\ \dot{x}_y \\ \dot{x}_y^* \\ \dot{x}_\theta \\ \dot{x}_\theta^* \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \rho^e & 0 & 0 & 0 & 0 & 0 & -p^e & 0 & 0 \\ 0 & \rho^e & 0 & 0 & 0 & 0 & 0 & -p^e & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & u'\rho' y^e & 0 & -R'y^e \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & u'\rho' y^e & -R'y^e \\ 0 & 0 & 0 & 0 & \rho^e & 0 & -y^e p^e & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho^e & 0 & -y^e p^e & 0 \\ 0 & 0 & -p^e & 0 & -u'\rho' & 0 & \begin{matrix}(1-\varepsilon_\rho)u'' \\ -(u')^2\theta\rho''\end{matrix} & 0 & -y \\ 0 & 0 & 0 & -p^e & 0 & -u'\rho' & 0 & \begin{matrix}(1-\varepsilon_\rho)u'' \\ -(u')^2\theta^e\rho''\end{matrix} & -y \\ R' & R' & 0 & 0 & 0 & 0 & -1 & -1 & 2\Xi \end{bmatrix} \begin{bmatrix} x_K \\ x_K^* \\ x_y \\ x_y^* \\ x_\theta \\ x_\theta^* \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

where $x_K \equiv K - K^e$, $x_K^* \equiv K^* - K^e$, $x_y \equiv y - y^e$, $x_y^* \equiv y^* - y^e$, $x_\theta \equiv \theta - \theta^e$, $x_\theta^* \equiv \theta^* - \theta^e$, $\varepsilon_\mu \equiv u^e\rho'/\rho^e$, and $\Xi \equiv (R''K^e + w''L) > 0$. Note that (9) implies that $0 < \varepsilon_\mu < 1$.

Denote the above matrix by J, and the corresponding eigenvalue as z. Then z is determined by the characteristic equation

$$\Omega(z) \equiv |J - zI| = 0, \ \text{where } I \equiv \begin{bmatrix} I_6 & 0 \\ 0 & O_3 \end{bmatrix}.$$

If we subtract from the 9th row of $\Omega(z)$ the first row multiplied by $R'/(\rho^e - z)$, and the second row multiplied by $R'/(\rho^e - z)$, we obtain

$$\Omega(z) = (\rho^e - z)\begin{vmatrix} -z & 0 & 0 & 0 & \rho' u' y^e & 0 & R'y^e \\ 0 & -z & 0 & 0 & 0 & \rho' u' y^e & R'y^e \\ 0 & 0 & \rho^e - z & 0 & -y^e p^e & 0 & 0 \\ 0 & 0 & 0 & \rho^e - z & 0 & -y^e p^e & 0 \\ -p^e & 0 & -\rho' u' & 0 & \begin{matrix}(1-\varepsilon_\rho)u'' \\ -(u')^2\theta^e\rho''\end{matrix} & 0 & -y^e \\ 0 & -p^e & 0 & -\rho' u' & 0 & \begin{matrix}(1-\varepsilon_\rho)u'' \\ -(u')^2\theta^e\rho''\end{matrix} & -y^e \\ 0 & 0 & 0 & 0 & p^e R' - (\rho^e - z) & p^e R' - (\rho^e - z) & 2(\rho^e - z)\Xi \end{vmatrix}.$$

Next, the fifth row minus the first row multiplied by p^e/z, and the sixth row minus the second row multiplied by p^e/z, obtain:

$$\Omega(z) = (\rho^e - z)\begin{vmatrix} \rho^e - z & 0 & -y^e p^e & 0 & 0 \\ 0 & \rho^e - z & 0 & -y^e p^e & 0 \\ -\rho' u' z & 0 & \begin{matrix}[(1-\varepsilon_\rho)u'' \\ , -(u')^2\theta^e\rho'']z - p^e\rho' u' y^e\end{matrix} & 0 & -zy^e + p^e R' y^e \\ 0 & -\rho' u' z & 0 & \begin{matrix}[(1-\varepsilon_\rho)u'' \\ -(u')^2\theta^e\rho'']z - p^e\rho' u' y\end{matrix} & -zy^e + pR' y^e \\ 0 & 0 & p^e R' - (\rho^e - z) & p^e R' - (\rho^e - z) & 2(\rho^e - z)\Xi \end{vmatrix}.$$

Finally, we add the first row multiplied by $\rho' u' z/(\rho^e - z)$ to the third row and the second row multiplied by $\rho' u' z/(\rho^e - z)$ to the fourth row, to get

$$\begin{aligned} \Omega(z) &= (\rho^e - z)^2 \begin{vmatrix} A(z) & 0 & F(z) \\ 0 & A(z) & F(z) \\ B(z) & B(z) & C \end{vmatrix} \\ &= (\rho^e - z)^2 A(AC - 2BF), \end{aligned}$$

where

$$\begin{aligned} A(z) &\equiv [\{(1-\varepsilon_\rho)u'' - (u')^2\theta^e\rho''\}z - p^e\rho' u' y^e](\rho^e - z) - z\rho' u' p^e y^e \\ B(z) &\equiv p^e R' - (\rho^e - z) \\ C &\equiv 2\Xi > 0 \\ F(z) &\equiv y(p^e R' - z). \end{aligned}$$

For $\Omega(z) = 0$, we have $z_1 = z_2 = \rho^e > 0$, $A(z) = 0$, and $A(z)C - 2B(z)F(z) = 0$. We rewrite

$$\begin{aligned} A(z) = &-\{(1-\varepsilon_\rho)u'' - (u')^2\theta^e\rho''\}z^2 \\ &+ \rho^e\{(1-\varepsilon_\rho)u'' - (u')^2\theta^e\rho''\}z - p^e y^e \rho' u' \rho^e. \end{aligned}$$

As $-\{(1-\varepsilon_\rho)u'' - (u')^2\theta^e\rho''\} > 0$ (the second-order condition) and $-p^e\rho' u' y^e \rho^e < 0$, there exist two roots $z_3 > 0 > z_4$ such that $A(z_3) = A(z_4) = 0$.

Finally,

$$\begin{aligned} &H(z) \\ \equiv\ & A(z)C - 2B(z)F(z) \\ =\ & 2\Xi[[\{(1-\varepsilon_\rho)u'' - (u')^2\theta^e\rho''\}z - p^e\rho' u' y^e](\rho^e - z) - z\rho' u' p^e y^e] \\ & -2y^e(p^e R' - z)(p^e R' - \rho^e + z) \\ =\ & 2[y - \Xi\{(1-\varepsilon_\rho)u'' - (u')^2\theta^e\rho''\}]z^2 \\ & -2\rho^e\{y - \Xi\{(1-\varepsilon_\rho)u'' - (u')^2\theta^e\rho''\}\}z \\ & -2p^e y^e\{\Xi\rho' u' \rho^e + (p^e R' - \rho^e)R'\} \\ =\ & 0. \end{aligned}$$

Because $2\{y^e - \Xi(u'' - \theta\rho'')\} > 0$ and $2p^e y^e\ \{\Xi\rho' u' \rho^e + (p^e R' - \rho^e)R'\} > 0$, there exist $z_5 > 0 > z_6$ such that $H(z_5) = H(z_6) = 0$.

Therefore, there are four positive characteristic roots (i.e. z_1, z_2, z_3 and z_6) and two negative roots (i.e. z_4 and z_6). Because there are two state variables, K and K^*, it follows that the steady state is a saddle point.

Lemma 6 *When the two countries are sufficiently identical, the steady state with both countries being incompletely specialized is saddlepoint-stable.*

Lemmas 1–6 together imply the basic technical proposition.

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