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The economic value of volatility timing using a range-based volatility model

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ABSTRACT

There is growing interest in utilizing the range data of asset prices to study the role of volatility in financial markets. In this paper, a new range-based volatility model was used to examine the economic value of volatility timing in a mean–variance framework. We compared its performance with a return-based dynamic volatility model in both in-sample and out-of-sample volatility timing strategies. For a risk-averse investor, it was shown that the predictable ability captured by the dynamic volatility models is economically significant, and that a range-based volatility model performs better than a return-based one.

1. Introduction

In recent years, there has been considerable interest in volatility. The extensive development of volatility modeling has been motivated by related applications in risk management, portfolio allocation, assets pricing and futures hedging. In discussions of econometric methodologies for estimating the volatility of individual assets, ARCH and GARCH have been emphasized most. Various applications in finance and economics are provided as a review in Bollerslev et al. (1992, 1994), and Engle (2004).

Several studies have noted that range data based on the difference of high and low prices in a fixed interval can offer a sharper estimate of volatility than the return data. Range data are available for most financial assets and intuitively have more information than return data for estimating volatility. They utilize the two pieces of information (high and low) comparing with the return data that use only the close to close price. Parkinson (1980) showed that it reduced the variance of the volatility estimator by five times comparing with the traditional return-based volatility estimator. Furthermore, range is an unbiased estimator of the standard deviation. There are quite a few extensions of Parkinson's original results.[1] More recently, Brandt and Jones (2006), Chou (2005, 2006), and Martens and van Dijk (2007). In particular, Chou (2005) proposed a conditional autoregressive range (CARR) model which can easily capture the dynamic volatility structure, and

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[1] See for example, Garman and Klass (1980), Wiggins (1991), Rogers and Satchell (1991), Kunitomo (1992), Yang and Zhang (2000), and Alizadeh et al. (2002).

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provides sharper volatility forecasts comparing with the return-based GARCH model. The CARR model is a conditional mean model and it is easily to incorporate other explanatory variables.

However, the literature above just focuses on the volatility forecast of a univariate asset. It should be noted that there have been some attempts to establish a relationship between multiple assets, such as VECH (see Bollerslev et al., 1988), BEKK (see Engle and Kroner, 1995) and a constant conditional correlation model (CCC) (see Bollerslev, 1990), among others. VECH and BEKK allow time-varying covariance processes which are too flexible to estimate, and CCC with a constant correlation is too restrictive to apply to general applications. Seminal work on solving the puzzle was carried out by Engle (2002a). A dynamic conditional correlation[2] (DCC) model proposed by Engle (2002a) provides another viewpoint to this problem. The estimation of DCC can be divided into two stages. The first step is to estimate univariate GARCH, and the second is to utilize the transformed standardized residuals to estimate time-varying correlations (see Engle and Sheppard, 2001; Cappiello et al., 2006).

A new multivariate volatility, recently proposed by Chou et al. (2009), combines the range data of asset prices with the framework of DCC, namely range-based DCC.[3] The range-based DCC model is flexible and easy to be estimated through the two-step estimation. It also has the relative efficiency of the range data over the return data in estimating volatility. Through the statistical measures RMSE and MAE, based on four benchmarks of implied and realized covariance,[4] they concluded that the range-based DCC model performs better than other return-based models (MA100, EWMA, CCC, return-based DCC, and diagonal BEKK).

Asset allocation efficiency is closely linked to the predictions of asset returns and volatilities. West et al. (1993) was the first to focus on this insight and devise a way to use the utility function to derive the economic value of dynamic volatility models. The economic intuition is simple. A more accurate volatility prediction will render the investors a way to adaptively adjust their portfolio positions to achieve a higher utility level. Hence investors will be willing to pay a fee to switch from a fund manager with poor volatility prediction skill (or model) to another manager offering better volatility predictions. The maximum of such a switching fee is a measure of the difference of economic values of the two competing volatility models. The above described strategy of adjusting portfolio weights according to the prediction of volatility changes is called "volatility timing". This is different from the other type of "market timing" technique in which the portfolios are adjusted following the prediction of changes in expected returns. Market timing is usually not an effective tool given that an efficiency market implied the returns are unpredictable.

Following West et al. (1993), some studies have concentrated on whether some newly devised volatility models have sufficiently high economic values (see Busse, 1999; Fleming et al., 2001, 2003; Marquering and Verbeek, 2004; Thorp and Milunovich, 2007; Corte et al., 2009). The questions upon which we focused were two: first, whether the range-based DCC model contains economic value comparing with a benchmark model using a static or buy-and-hold strategy; and second, whether economic value of range-based volatility model still exists comparing with a return-based DCC model.

In comparing the economic value of return-based and range-based models, it is helpful to use a suitable measure to capture the trade-off between risk and return. Most literature evaluates volatility models through error statistics and related applications but neglects the influence of asset expected returns. A more precise measurement should consider both of them, but only a few such studies have been made at this point. However, a utility function can easily connect them and build a comparable standard. Before entering into a detailed discussion for the economic value of volatility timing, it was necessary to clarify its definition in this paper. In short, the economic value of volatility timing is the gain compared with a static strategy. Our concern was to estimate the willingness of the investor with a mean variance utility to pay for a new volatility model rather than a static one.

In light of the success of the range-based volatility model, the purpose of this paper was to examine its economic value in volatility timing by using the conditional mean–variance framework developed by Fleming et al. (2001). We considered an investor with different risk-averse levels using conditional volatility analysis to allocate three assets: stocks, bonds and cash. Fleming et al. (2001) extended the utility criterion derived from West et al. (1993) to test the economic value of volatility timing for short-horizon investors with different risk tolerance levels.[5] In addition to the short-horizon forecast of selected models, we also examined the economic value of longer horizon forecasts and an asymmetric range-based volatility model in our empirical study. This study may lead to a better understanding of range volatility.

The reminder is laid out as follows. Section 2 introduces the asset allocation methodology, economic value measurement, and the return-based and the range-based DCC. Section 3 describes the properties of data used and evaluates the performance of the different strategies. Finally, the conclusion is showed in Section 4.

[2] See Tsay (2002) and Tse and Tsui (2002) for other related methods for estimating the time-varying correlations.

[3] Fernandes et al. (2005) propose another kind of multivariate CARR model using the formula $\mathrm{Cov}(X,Y)=[\mathrm{Var}(x+Y)-\mathrm{Var}(X)-\mathrm{Var}(Y)]/2$. However, this method can only apply to a bivariate case.

[4] Daily data are used to build four proxies for weekly covariances, i.e. implied return-based DCC, implied range-based DCC, implied DBEKK, and realized covariances.

[5] They found that volatility-timing strategy based on one-step ahead estimates of the conditional covariance matrix (see Foster and Nelson, 1996) significantly outperformed the unconditional efficient static portfolios.

2. Methodologies

To carry out this study we used the framework of a minimum variance strategy, which was conductive to determine the accuracy of the time-varying covariances. We wanted to find the optimal dynamic weights of the selected assets and the implied economic value of a static strategy for a risk-adverse investor. Before applying the volatility timing strategies, we needed to build a time-varying covariance matrix. The details of the methodology are as follows.

2.1. Optimal portfolio weights in a minimum variance framework

Initially, we considered a minimization problem for the portfolio variance subjected to a target return constraint. To derive our strategy, we let $\mathbf{R}_t$ be the $k\times 1$ vector of spot returns at time t.[6] Its conditional expected return $\boldsymbol{\mu}_t$ and conditional covariance matrix $\boldsymbol{\Sigma}_t$ were calculated by $E[\mathbf{R}_t|\Omega_{t-1}]$ and $E[(\mathbf{R}_t-\boldsymbol{\mu}_t)(\mathbf{R}_t-\boldsymbol{\mu}_t)'|\Omega_{t-1}]$, respectively. Here, Ω_t was assumed as the information set at time t. To minimize the portfolio volatility subject to a required target return μ_{target}, it can be formulated as

$$\min_{\mathbf{w}_t} \quad \mathbf{w}_t'\Sigma_t\mathbf{w}_t$$

$$\text{s.t.} \quad \mathbf{w}_t'\boldsymbol{\mu}_t+(1-\mathbf{w}_t'\mathbf{1})R_f=\mu_{\text{target}} \tag{1}$$

where $\mathbf{w}_t$ is a $k\times 1$ vector of portfolio weights for time t. R_f is the return for the risk-free asset. The optimal solution to the quadratic form (1) is

$$\mathbf{w}_t=\frac{(\mu_{\text{target}}-R_f)\boldsymbol{\Sigma}_t^{-1}(\boldsymbol{\mu}_t-R_f\mathbf{1})}{(\boldsymbol{\mu}_t-R_f\mathbf{1})\boldsymbol{\Sigma}_t^{-1}(\boldsymbol{\mu}_t-R_f\mathbf{1})} \tag{2}$$

Under the cost of carry model, we regarded the excess returns as the futures returns by applying regular no-arbitrage arguments.[7] It is clear that the covariance matrix $\boldsymbol{\Sigma}_t$ of the spot returns is the same as that of the excess returns. Eq. (2) can be simply expressed as

$$\mathbf{w}_t=\frac{\mu_{\text{target}}\boldsymbol{\Sigma}_t^{-1}\boldsymbol{\mu}_t}{\boldsymbol{\mu}_t'\boldsymbol{\Sigma}_t^{-1}\boldsymbol{\mu}_t} \tag{3}$$

where the vector $\boldsymbol{\mu}_t$ and the matrix $\boldsymbol{\Sigma}_t$ are redefined in terms of futures. A bivariate case (k=3) of Eq. (3) can be written as

$$w_{1,t}=\frac{\mu_{\text{target}}(\mu_{1,t}\sigma_{2,t}^2-\mu_{2,t}\sigma_{12,t})}{\mu_{1,t}^2\sigma_{2,t}^2+\mu_{2,t}^2\sigma_{1,t}^2-2\mu_{1,t}\mu_{2,t}\sigma_{12,t}}$$

$$w_{2,t}=\frac{\mu_{\text{target}}(\mu_{2,t}\sigma_{1,t}^2-\mu_{1,t}\sigma_{12,t})}{\mu_{1,t}^2\sigma_{2,t}^2+\mu_{2,t}^2\sigma_{1,t}^2-2\mu_{1,t}\mu_{2,t}\sigma_{12,t}} \tag{4}$$

where $\mu_{1,t}$ and $\mu_{2,t}$ are the futures returns of S&P 500 index (S&P 500) and 10-year Treasury bond (T-bond) in our empirical study. In addition, futures contracts are easy to be traded and have lower transaction cost compared to spot contracts. The above analysis pointed out that the optimal portfolio weights were time-varying. Here we assumed that the conditional mean $\boldsymbol{\mu}_t$ was constant.[8] Therefore, the dynamics of weights only depend on the conditional covariance $\boldsymbol{\Sigma}_t$. In this study, the optimal strategy was obtained based on a minimum variance framework subject to a given return. The mean–variance framework above is used to derive the optimal portfolio weights under different target returns. In the following section, we want to build criterion[9] to compare means and variances of the portfolios from the static and dynamic strategies. However, it is not easy to decide the best strategy, especially for the investors with different risk aversions. In this study, we want to apply the quadratic utility function to calculate economic value under some settings.

2.2. Economic value of volatility timing

Fleming et al. (2001) uses a generalization of the West et al. (1993) criterion which builds the relationship between a mean–variance framework and a quadratic utility to capture the trade-off between risk and return for ranking the

[6] Through out this paper, we have used blackened letters to denote vectors or matrices.

[7] There are no costs for futures investment. This means the futures return equals the spot return minus the risk-free rate.

[8] The changes in expected returns are not easy to detect. Merton (1980) points out that the volatility process is more predictable than the return series.

[9] The Sharpe ration is one of the candidates for comparison. However, it may underestimate the performance of dynamic strategies, see Marquering and Verbeek (2004).

performance of forecasting models. According to their work, the investor's utility can be defined as

$$U(W_t) = W_t R_{p,t} - \frac{\alpha \cdot W_t^2}{2} R_{p,t}^2 \tag{5}$$

where W_t is the investor's wealth at time t, α is his absolute risk aversion, and the portfolio return at period t is $\mathbf{w}_t'\mathbf{R}_t$.

For comparisons across portfolios, we assumed that the investor had a constant relative risk aversion[10] (CRRA), $\gamma_t = \alpha \cdot W_t/(1-\alpha \cdot W_t) = \gamma$. This implies $\alpha \cdot W_t$ is a constant. The CRRA setting means an investor's loss tolerance increases in proportion to the investor's wealth. It implies that the expected utility is linearly related to wealth. With this assumption, the average realized utility $\overline{U}(\cdot)$ can be used in estimating the expected utility with a given initial wealth W_0.

$$\overline{U}(\cdot) = W_0 \sum_{t=1}^{T} \left[R_{p,t} - \frac{\gamma}{2(1+\gamma) R_{p,t}^2} \right] \tag{6}$$

where W_0 is the initial wealth.

Therefore, the value of volatility timing calculated by equating the average utilities for two alternative portfolios is expressed as

$$\sum_{t=1}^{T} \left[(R_{b,t} - \Delta) - \frac{\gamma}{2(1+\gamma)} (R_{b,t} - \Delta)^2 \right] = \sum_{t=1}^{T} \left[R_{a,t} - \frac{\gamma}{2(1+\gamma) R_{a,t}^2} \right] \tag{7}$$

where Δ is the maximum expense that an investor would be willing to pay to switch from the strategy a to the strategy b. $R_{a,t}$ and $R_{b,t}$ are the returns of the portfolios from the strategy a and b.[11] If the expense Δ is a positive value, it means the strategy b is more valuable than the strategy a. In our empirical study, we reported Δ as an annualized expense with three risk aversion levels of $\gamma = 1,5$, and 10.

2.3. Return-based and range-based DCC

We used the DCC model of Engle (2002a) to estimate the covariance matrix of multiple asset returns. It is a direct extension of the CCC model of Bollerslev (1990). The covariance matrix $\mathbf{H}_t$ for a vector of k asset returns in DCC can be written as

$$\mathbf{H}_t = \mathbf{D}_t \cdot \mathbf{\Gamma}_t \cdot \mathbf{D}_t \tag{8}$$

$$\mathbf{\Gamma}_t = \operatorname{diag} \mathbf{Q}_t^{-1/2} \cdot \mathbf{Q}_t \cdot \operatorname{diag} \mathbf{Q}_t^{-1/2} \tag{9}$$

where $\mathbf{D}_t$ is the $k \times k$ diagonal matrix of time-varying standard deviations from univariate GARCH models with $\sqrt{h_{i,t}}$ for the i-th return series on the i-th diagonal. Γ_t is a time-varying correlation matrix. The covariance matrix $\mathbf{Q}_t$=[$q_{ij,t}$] of the standardized residual vector $\mathbf{Z}_t = (z_{1,t}, z_{2,t})'$ is denoted as

$$\mathbf{Q}_t = (1-a-b)\overline{\mathbf{Q}} + a \cdot \mathbf{Z}_{t-1}\mathbf{Z}_{t-1}' + b \cdot \mathbf{Q}_{t-1} \tag{10}$$

where $\overline{\mathbf{Q}} = \{\overline{q}_{ij}\}$ denotes the unconditional covariance matrix of $\mathbf{Z}_t$. The coefficients, a and b, are the estimated parameters depicting the conditional correlation process. The dynamic correlation can be expressed as

$$\rho_{12,t} = \frac{(1-a-b)\overline{q}_{12} + a \cdot z_{1,t-1} z_{2,t-1} + b \cdot q_{12,t-1}}{\sqrt{[(1-a-b)\overline{q}_{11} + a \cdot z_{1,t-1}^2 + b \cdot q_{11,t-1}][(1-a-b)\overline{q}_{22} + a \cdot z_{2,t-1}^2 + b \cdot q_{22,t-1}]}} \tag{11}$$

We estimated the DCC model with a two-stage estimation through quasi-maximum likelihood estimation (QMLE) to get consistent parameter estimates. The log-likelihood function can expressed as L=L_{Vol}+L_{Corr}, where L_{Vol}, the volatility component, is $-\frac{1}{2}\sum_t (k \log(2\pi) + \log|\mathbf{D}_t|^2 + \mathbf{r}_t'\mathbf{D}_t^{-2}\mathbf{r}_t)$, and L_{Corr}, the correlation component, is $-\frac{1}{2}\sum_t (k \log|\mathbf{R}_t| + \mathbf{Z}_t'\mathbf{R}_t^{-1}\mathbf{Z}_t - \mathbf{Z}_t'\mathbf{Z}_t)$. The explanation is more fully developed in Engle and Sheppard (2001) and Engle (2002a).

In addition to using GARCH to construct standardized residuals, we can also build them by other univariate volatility models. In this paper, CARR was used as an alternative to verify whether the specification selected adequately suits DCC or not.

The CARR model is a special case of the multiplicative error model (MEM) of Engle (2002b). It can be expressed as

$$\mathcal{R}_{i,t} = \lambda_{i,t} \cdot u_{i,t}, \quad u_{i,t}|I_{t-1} \sim \exp(1,\cdot), \; i = 1,2$$

$$\lambda_{i,t} = \omega_i + \alpha_i \cdot \mathcal{R}_{i,t-1} + \beta_i \cdot \lambda_{i,t-1}$$

$$z_{i,t}^c = \frac{r_{i,t}}{\lambda_{i,t}^*} \quad \text{where } \lambda_{i,t}^* = \text{adj}_i \times \lambda_{i,t}, \text{adj}_i = \frac{\overline{\sigma}_i}{\overline{\hat{\lambda}}_i} \tag{12}$$

[10] West et al. (1993), Fleming et al. (2001), and Corte et al. (2009) also applied CRRA to their studies.

[11] In our setting, we let the strategy pair (a,b) be (OLS, return-based DCC), (OLS, range-based DCC) and (return-based DCC, range-based DCC), respectively. Because the rolling sample method was adopted in the out-of-sample comparison, this type of OLS was named by rollover OLS.

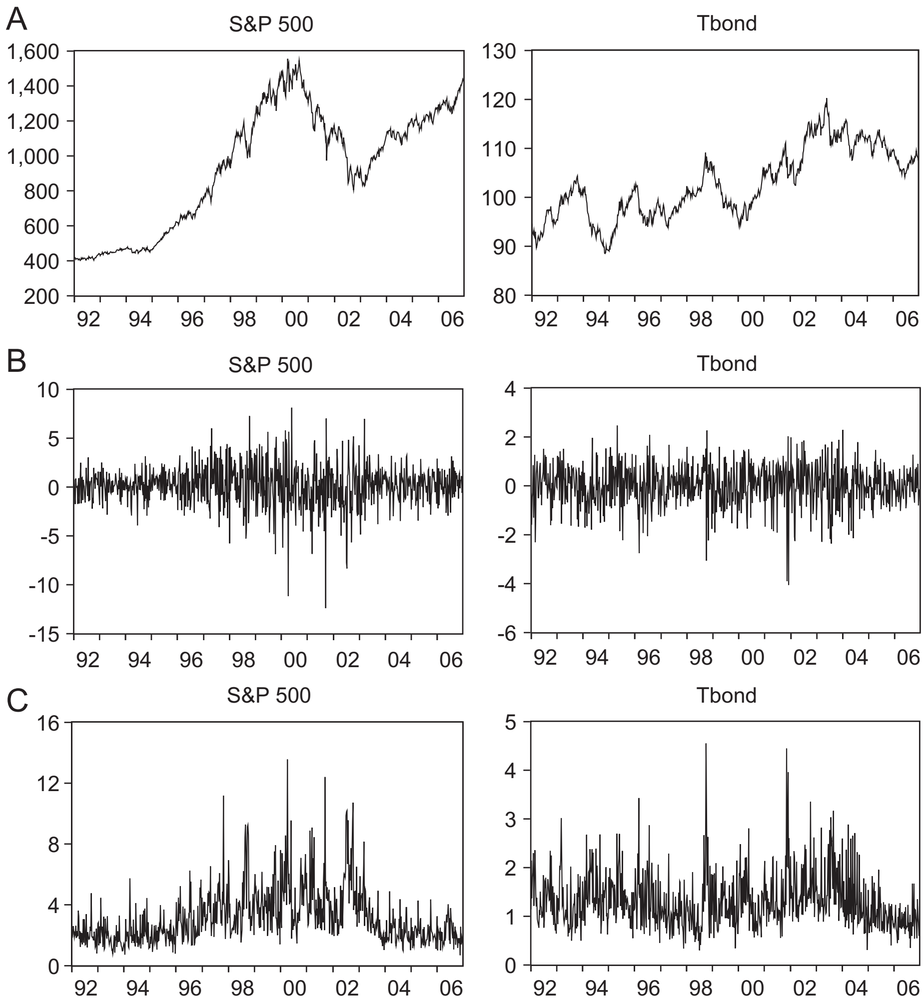

Fig. 1. S&P 500 index futures and T-bond futures weekly closing prices, returns and ranges, 1992–2006. These Panels A, B and C shows the weekly close prices, returns, and ranges of S&P 500 index futures and 10-year treasury bond (T-bond) futures over the sample period.

Table 1
Summary statistics for weekly S&P 500 and T-bond futures return and range data, 1992–2006.

	S&P 500 futures		T-bond futures	
	Return	Range	Return	Range
Mean	0.158	3.134	0.016	1.306
Median	0.224	2.607	0.033	1.194
Maximum	8.124	13.556	2.462	4.552
Minimum	−12.395	0.690	−4.050	0.301
Std. dev.	2.112	1.809	0.855	0.560
Skewness	−0.503	1.756	−0.498	1.390
Kurtosis	6.455	7.232	4.217	6.462
Jarque–Bera	421.317	985.454	80.441	642.367
	(0.000)	(0.000)	(0.000)	(0.000)

The table provides summary statistics for the weekly return and range data on S&P 500 stock index futures and T-bond futures. The returns and ranges were computed by $100\times\log(p_t^{close}/p_t^{open})$ and $100\times\log(p_t^{high}/p_t^{low})$, respectively. The Jarque–Bera statistic is used to test the null of whether the return and range data are normally distributed. The values presented in parentheses are p-values. The annualized values of means (standard deviation) for S&P 500 and T-bond futures were 8.210 (15.232) and 0.853 (6.168), respectively. The simple correlation between stock and bond returns was −0.023. The sample period ranges from January 6, 1992 to December 29, 2006 (15 years, 782 observations) and all futures data were collected from datastream.

Table 2
Estimation results of return-based and range-based DCC model using weekly S&P500 and T-bond futures, 1992–2006.

	S&P 500 futures		T-bond futures	
	GARCH	CARR	GARCH	CARR
Panel A: Volatilities estimation of GARCH and CARR models				
c	0.188		0.008	
	(3.256)		(0.242)	
$\hat{\omega}$	0.019	0.103	0.028	0.075
	(1.149)	(2.923)	(1.533)	(2.810)
$\hat{\alpha}$	0.051	0.248	0.060	0.157
	(3.698)	(9.090)	(2.031)	(5.208)
$\hat{\beta}$	0.946	0.719	0.902	0.785
	(71.236)	(23.167)	(18.645)	(18.041)
Q(12)	26.322	5.647	15.872	23.121
	(0.010)	(0.933)	(0.197)	(0.027)
	S&P 500 and T-bond			
	Return-based DCC		Range-based DCC	
Panel B: Correlation estimation of return- and range-based DCC models				
$\hat{a}$	0.037		0.043	
	(4.444)		(4.679)	
$\hat{b}$	0.955		0.951	
	(85.621)		(80.411)	

$r_{i,t}=c+\varepsilon_{i,t},h_{k,t}=\omega_k+\alpha_k\cdot\varepsilon_{k,t-1}^2+\beta_k\cdot h_{k,t-1},\varepsilon_{k,t}|I_{t-1}\sim N(0,h_{k,t})$, $\mathcal{R}_{i,t}=u_{i,t},\lambda_{k,t}=\omega_k+\alpha_k\cdot\mathcal{R}_{k,t-1}+\beta_k\cdot\lambda_{k,t-1},\mathcal{R}_{k,t}|I_{t-1}\sim\exp(1,\cdot),k=1,2$, $\mathbf{Q}_t=(1-a-b)\overline{\mathbf{Q}}+a\cdot\mathbf{Z}_{t-1}'\mathbf{Z}_{t-1}+b\cdot\mathbf{Q}_{t-1}$, and then $\rho_{12,t}=(1-a-b)\overline{q}_{12}+a\cdot z_{1,t-1}z_{2,t-1}+b\cdot q_{12,t-1}/\sqrt{[(1-a-b)\overline{q}_{11}+a\cdot z_{1,t-1}^2+b\cdot q_{11,t-1}][(1-a-b)\overline{q}_{22}+a\cdot z_{2,t-1}^2+b\cdot q_{22,t-1}]}$ where $\mathcal{R}_t$ is the range variable, $\mathbf{Z}_t$ is the standard residual vector which is standardized by GARCH or CARR volatilities. $\mathbf{Q}_t=\{q_{ij,t}\}$ and $\overline{\mathbf{Q}}_t=\{\overline{q}_{ij,t}\}$ are the conditional and unconditional covariance matrix of $\mathbf{Z}_t$. The three formulas above are GARCH, CARR and the conditional correlation equations, respectively, of the standard DCC model with mean reversion. The table presents estimations of the three models using the MLE method. Panel A is the first step of the DCC model estimation. The estimation results of GARCH and CARR models for two futures were presented here. Q(12) is the Ljung-Box statistic for the autocorrelation test with 12 lags. Panel B is the second step of the DCC model estimation. The values presented in parentheses are t-ratios for the model coefficients and p-values for Q(12).

where the range $\mathcal{R}_{i,t}$ is calculated by the difference between logarithm high and low prices of the i-th asset during a fixed time interval t, and it is also a proxy of standard deviation. $\lambda_{i,t}$ and $\bar{\hat{\lambda}}_i$ are the conditional and unconditional means of the range, respectively. $u_{i,t}$ is the residual which is assumed to follow the exponential distribution. $\overline{\sigma}_i$ is the unconditional standard deviation for the return series. In considering different scales in quantity, the ratio adj_t was used to adjust the range to produce the standardized residuals.[12]

3. Empirical results

The empirical data employed in this paper consists of the stock index futures, bond futures and the risk-free rate. As to the above-mentioned method, we applied the futures data to examine the economic value of volatility timing for return-based and range-based DCC. Under the cost of the carry model, the results in this case can be extended to underlying spot assets (see Fleming et al., 2001). In addition to avoiding the short sale constraints, this procedure reduces the complexity of model setting. To address this issue, we used the S&P 500 futures (traded at CME) and the T-bond futures (traded at CBOT) as the empirical samples. According to Chou et al. (2009), the futures data were taken from datastream, sampling from January 6, 1992 to December 29, 2006 (15 years, 782 weekly observations). Datastream provided the nearest contract and rolls over to the second nearby contract when the nearby contract approaches maturity. We also used the 3-month Treasury bill rate to substitute for the risk-free rate. The Treasury bill rate is available from the Federal Reserve Board.

Fig. 1 shows the graphs for close prices (Panel A) returns (Panel B) and ranges (Panel C) of the S&P 500 and T-bond futures over the sample period. Table 1 presents summary statistics for the return and range data on the S&P 500 and T-bond futures. The return was computed as the difference of logarithm close prices on two continuous weeks. The range was defined by the difference of the high and low prices in a logarithm type. The annualized mean and standard deviation in percentage (8.210, 15.232) of the stock futures returns were both larger than those (0.853, 6.168) of the bond futures

[12] Parkinson (1980) derived the adjustment ratio as a constant, 0.361, but an asset price was required to follow a geometric Brownian motion with zero drift, which is not truly empirical.

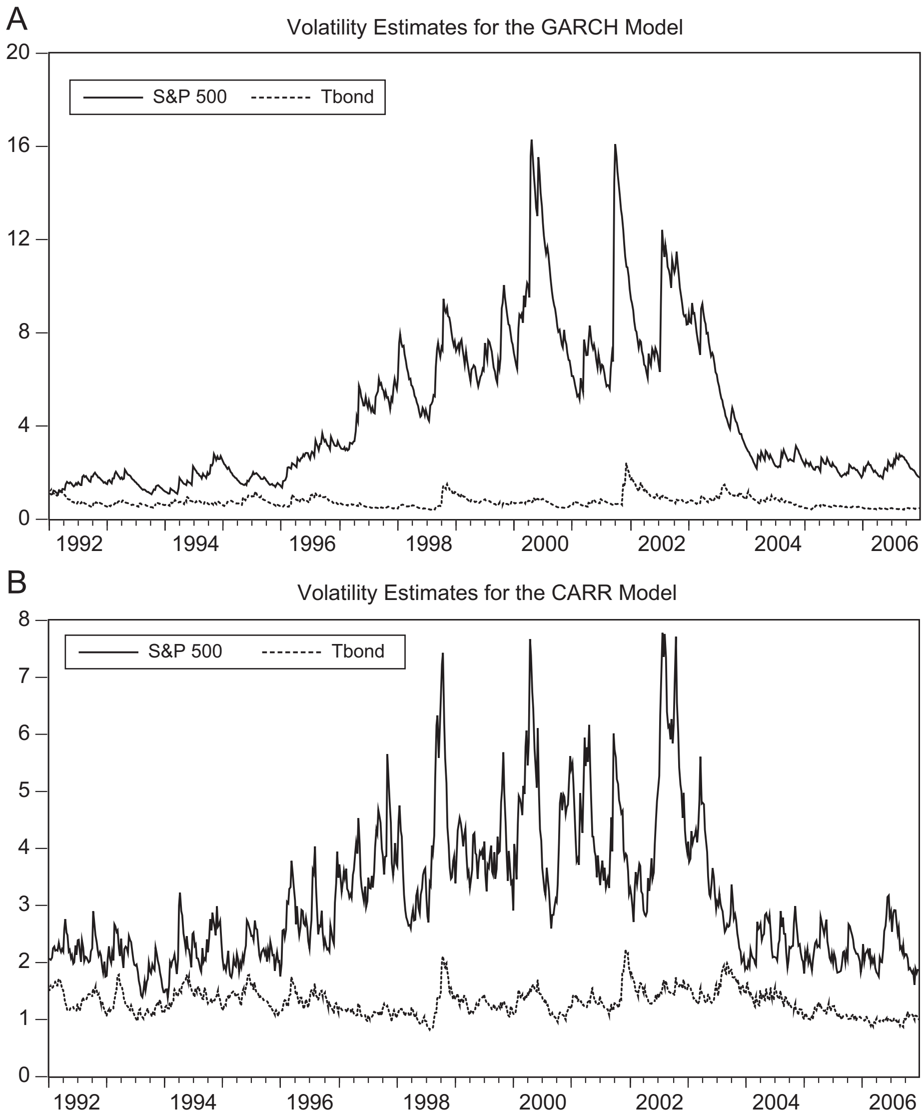

Fig. 2. In-sample volatility estimates for the GARCH and CARR model. Panel A: volatility estimates for the GARCH model and Panel B: volatility estimates for the CARR model.

returns. This fact indicated that the more volatile market may have a higher risk premium. Both futures returns have negative skewness and excess kurtosis, indicating a violation of the normal distribution. The range mean (3.134) of the stock futures prices was larger than that (1.306) of the bond futures prices. This is reasonable because the range is a proxy of volatility. The Jarque–Bera statistic was used to test the null of whether the return and range data were normally distributed. Both return and range data rejected the null hypothesis. The simple correlation between stock and bond returns was small[13] (-0.023), but this does not imply that their relationship was very weak. In our latter analysis, we showed that the dynamic relationship of stocks and bonds will be more realistically revealed by the conditional correlations analysis.

3.1. *The in-sample comparison*

To obtain an optimal portfolio, we used the dynamic volatility models to estimate the covariance matrices. The parameters fitted for return-based and range-based DCC, were both estimated and arranged in Table 2. We divided the

[13] The results are different from the positive correlation value (sample period 1983–1997) in Fleming et al. (2001). After 1997, the relationship between S&P 500 and T-bond presented a reverse condition.

table into two parts corresponding to the two steps in the DCC estimation. In Panel A of Table 2, one can use GARCH (fitted by return) or CARR (fitted by range) with individual assets to obtain the standardized residuals. Fig. 2 provides the volatility estimate of the S&P 500 futures and the T-bond futures based on GARCH and CARR. Then, these standardized residuals series were brought into the second stage for dynamic conditional correlation estimating. Panel B of Table 2 presents the estimated parameters of DCC under the quasi-maximum likelihood estimation (QMLE).

The correlation and covariance estimates for return-based and range-based DCC are shown in Fig. 3. It seems that the correlation became more negative at the end of 1997. This means that it is more desirable to diversify in this period because the bond holding will help offset the volatility caused by the equity component in the portfolio. This conjecture is confirmed in our latter analysis of the estimated portfolio weights. A deeper investigation is also given in Connolly et al. (2005).

Following the model estimation, we constructed the static portfolio (built by OLS) using the unconditional mean and covariance matrices to get the economic values of dynamic models. Under the minimum variance framework, the weights of the portfolio were computed by the given expected return and the conditional covariance matrices estimated by return-based and range-based DCC. Then, we compared the performance of the volatility models on 11 different target annualized returns (5–15%, 1% in an interval).

Table 3 presents how the performance comparisons varied with the target returns and the risk aversions. Panel A of Table 3 presents the annualized means (μ) and volatilities (σ) of the portfolios estimated from three methods, return-based

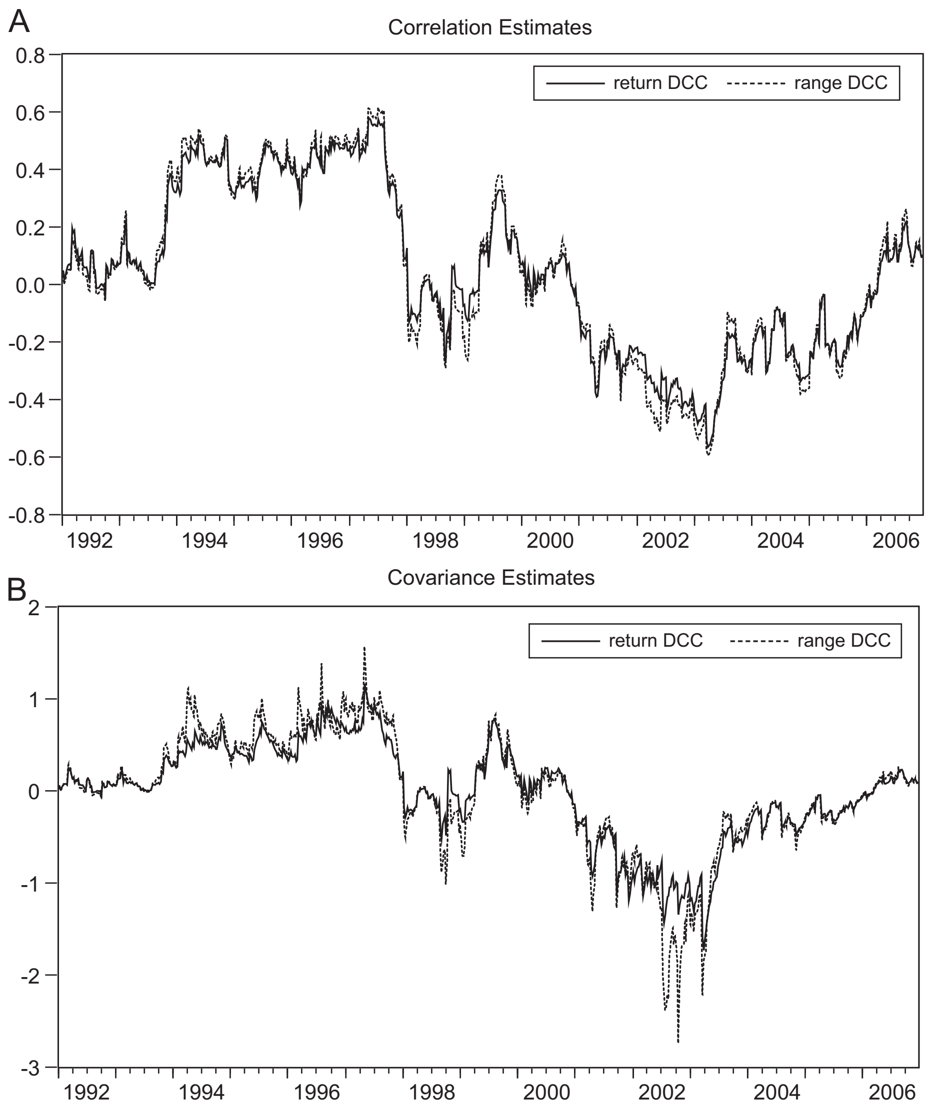

Fig. 3. In-sample correlation and covariance estimates for the return-based and range-based DCC model. Panel A: correlation estimates and Panel B: covariance estimates.

Table 3
In-sample comparison of the volatility timing values in the minimum volatility strategy using different target returns, 1992–2006.

Target return (%)	Return-based DCC		Range-based DCC		Rollover OLS	
	μ	σ	μ	σ	μ	σ
Panel A: Means and volatilities of optimal portfolios						
5	5.201	2.100	5.241	2.100	5.000	2.190
6	6.366	3.814	6.438	3.813	6.000	3.977
7	7.530	5.527	7.635	5.526	7.000	5.764
8	8.694	7.241	8.832	7.239	8.000	7.551
9	9.859	8.954	10.028	8.952	9.000	9.338
10	11.023	10.668	11.225	10.665	10.000	11.125
11	12.187	12.381	12.422	12.378	11.000	12.912
12	13.352	14.095	13.619	14.091	12.000	14.699
13	14.516	15.808	14.815	15.804	13.000	16.486
14	15.680	17.521	16.012	17.517	14.000	18.273
15	16.845	19.235	17.209	19.230	15.000	20.060

Target return (%)	OLS to return DCC			OLS to range DCC			Return to range DCC		
	Δ_1	Δ_5	Δ_{10}	Δ_1	Δ_5	Δ_{10}	Δ_1	Δ_5	Δ_{10}
Panel B: Switching fees with different relative risk aversions									
5	0.303	0.376	0.393	0.343	0.417	0.434	0.040	0.041	0.041
6	0.703	0.950	1.008	0.777	1.025	1.084	0.074	0.076	0.076
7	1.244	1.771	1.897	1.353	1.883	2.009	0.109	0.112	0.112
8	1.929	2.845	3.063	2.073	2.994	3.213	0.144	0.149	0.151
9	2.761	4.173	4.507	2.940	4.360	4.696	0.180	0.189	0.191
10	3.739	5.753	6.224	3.956	5.979	6.453	0.217	0.230	0.233
11	4.866	7.578	8.206	5.121	7.846	8.477	0.255	0.273	0.277
12	6.142	9.641	10.441	6.434	9.951	10.754	0.294	0.318	0.324
13	7.565	11.932	12.914	7.897	12.283	13.270	0.334	0.365	0.373
14	9.135	14.436	15.609	9.507	14.831	16.009	0.375	0.414	0.424
15	10.851	17.142	18.509	11.262	17.580	18.952	0.418	0.466	0.479

The table reports the in-sample performance of the volatility timing strategies with different target returns. The target returns were from 5% to 15% (annualized). The weights for the volatility timing strategies were obtained from the weekly estimates of the conditional covariance matrix and the different target return setting. Panel A presents the annualized means (μ) and volatilities (σ) for each strategy. The estimated Sharpe ratios for the return-based DCC model, the range-based DCC model, and the OLS strategy were 0.680, 0.699, and 0.560, respectively. Panel B presents the average switching annualized fees (Δ_r) from one strategy to another. The values of the constant relative risk aversion γ were 1, 5, and 10.

DCC, range-based DCC and OLS. At a quick look, the annualized Sharpe ratios[14] calculated from return-based DCC (0.680) and range-based DCC (0.699) were higher than the static model (0.560). Panel B of Table 3 presents the average switching fees (Δ_r) from one strategy to another. The value settings of CRRA γ were 1, 5, and 10. As for the performance fees with different relative risk aversions, in general, an investor with a higher risk aversion should be willing to pay more to switch from the static portfolio to the dynamic ones. With higher target returns, the performance fees increased steadily. In addition, Panel B of Table 3 also reports the performance fees for switching from return-based DCC to range-based DCC. Positive values for all cases show that the range-based volatility model can give more significant economic value in forecasting covariance matrices than return-based ones.

In the real practice, the transaction costs should be considered when the dynamic strategies are compared to the static one. For S&P 500 futures, the bid/ask spread and round-trip commission totally cost about \$0.10 index unit. The annualized cost of a one-way transaction in our study can be calculated by $0.05/941.55 \times 52 = 0.28\%$, where 941.55 is an average index level from 1992 to 2006. It means the advantage of the dynamic strategies will not be offset by the transaction costs. For example, with a fixed target return 10%, the economic advantage is about 6% for an investor with relative risk aversion of 5.

Fig. 4 plots the weights of an in-sample minimum volatility portfolio derived from two dynamic models. OLS has constant weights for cash, stocks, and bonds, i.e. -0.1934, 0.7079, and 0.4855.

It is interesting to observe the dynamic patterns of the portfolio weights implied by the two dynamic models. In contrary to the OLS (buy-and-hold strategy), they have substantial fluctuations across the sample periods. The two strategies (panel A for return-based DCC and panel B for range-based DCC) have roughly similar patterns in movements but with noticeably quantitative differences. The stock portfolio weight is most stably fluctuating around <0.8 before 1997 after which it drops to a lower level of about 0.65 with larger variations. It is interesting to observe that the bond weights have been negative or zero before 1997 and become positive after late 1997. The zero or negative weights are the result of the booming equity market in the mid 90s hence it is desirable to invest mostly in the equity market. The mid-crash in the

[14] The Sharpe ratio is constant with different target multipliers. For the further details, see Engle and Colacito (2006).

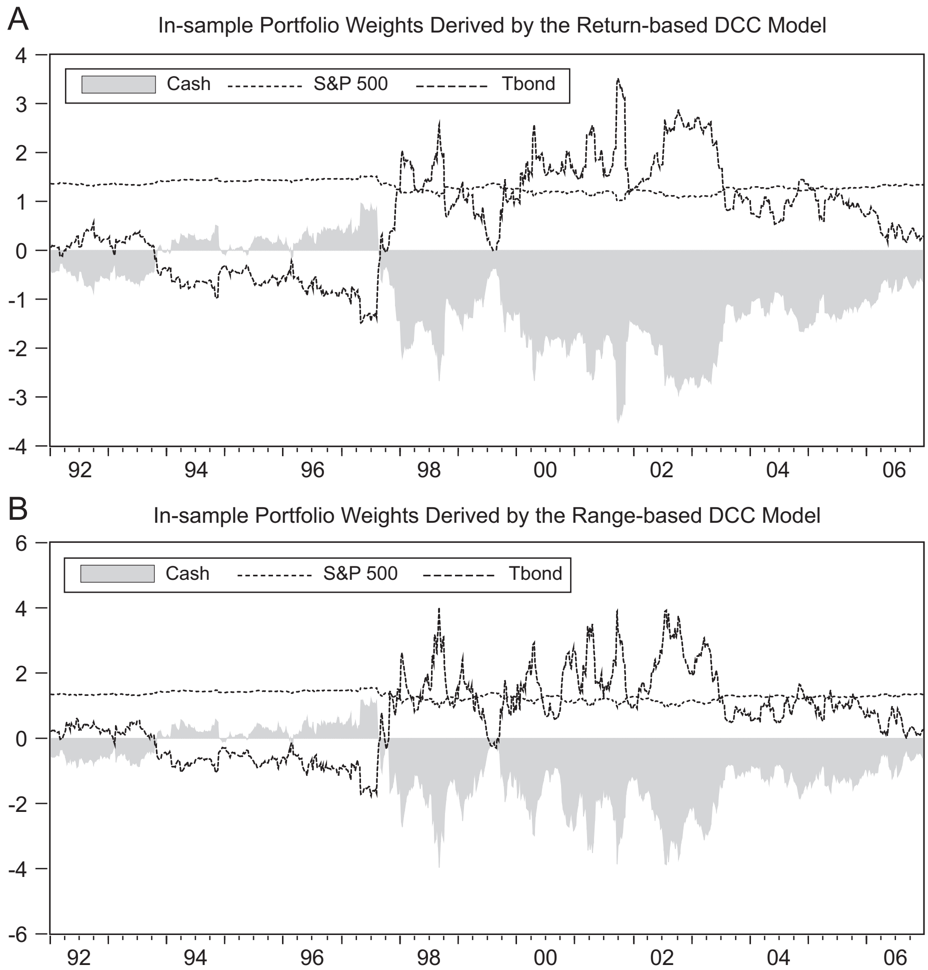

Fig. 4. In-sample minimum volatility portfolio weight derived by the dynamic volatility model. Panels A and B show the weights that minimize conditional volatility while setting the expected annualized return equal to 10%. The OLS model had constant weights for cash, stock, and bond, i.e. −0.1934, 0.7079, and 0.4855. Panel A: In-sample portfolio weights derived by the return-based DCC model and Panel B: in-sample portfolio weights derived by the range-based DCC model.

late 1997 has caused an increase of volatility which would cause a drop in investor's utility and hence should be hedged away. As is seen in Fig. 3, this is a period when the correlation between stock and bond returns became negative. The negative correlation in bond/equity return suggests an increase in the bond position would help to reduce the total portfolio volatilities. The lower level and higher variations of stock weights since then is also a reflection of the fact that the stock/bond correlations in the later periods are mostly negative but with wide swings. Finally, the cash position serves as a residual in the portfolio since the three asset weights add up to one. The movements will be related to the term spread or the term structure of interest rates and the bond volatility.

It is also useful to contrast the time-varying pattern of the bond position to the fixed weight suggested by OLS. The latter suggest that roughly 48% should be invested in the bond market regardless of the movements in the volatility and correlation structures. This is obviously too naive given our discussion above that volatilities and correlations of stock and bond returns do vary over time. A buy-and-hold strategy will therefore yield a poor performance.

3.2. The out-of-sample comparisons

For robust inference, a similar approach was utilized to estimate the value of volatility timing in the out-of-sample analysis. Here the rolling sample approach was adopted for all out-of-sample estimations. This meant that the rollover OLS method replaced the conventional OLS method used in the in-sample analysis. Each forecasting value was estimated by 521 observations over about 10 years. Then, the rolling sample method provided 261 forecasting values for the one period ahead comparison. The first forecasted value occurred the week of January 4, 2002.

Table 4
Out-of-sample comparison for the one period ahead volatility timing values in the minimum volatility strategy with different target returns, 1992–2006.

Target return (%)	Return-based DCC		Range-based DCC		Rollover OLS	
	μ	σ	μ	σ	μ	σ
Panel A: Means and volatilities of optimal portfolios						
5	4.691	1.698	4.747	1.661	4.344	1.749
6	5.438	3.083	5.540	3.016	4.808	3.176
7	6.186	4.468	6.333	4.370	5.273	4.603
8	6.933	5.853	7.127	5.725	5.737	6.030
9	7.681	7.239	7.920	7.080	6.202	7.456
10	8.428	8.624	8.714	8.435	6.667	8.883
11	9.176	10.009	9.507	9.790	7.131	10.310
12	9.923	11.394	10.300	11.145	7.596	11.737
13	10.671	12.779	11.094	12.500	8.060	13.164
14	11.418	14.165	11.887	13.854	8.525	14.591
15	12.166	15.550	12.680	15.209	8.990	16.018

Target return (%)	OLS to return DCC			OLS to range DCC			Return to range DCC		
	Δ_1	Δ_5	Δ_{10}	Δ_1	Δ_5	Δ_{10}	Δ_1	Δ_5	Δ_{10}
Panel B: Switching fees with different relative risk aversions									
5	0.393	0.425	0.433	0.481	0.537	0.550	0.089	0.112	0.118
6	0.781	0.890	0.916	0.991	1.176	1.220	0.210	0.289	0.308
7	1.232	1.463	1.518	1.606	1.998	2.090	0.377	0.545	0.585
8	1.746	2.144	2.239	2.328	3.001	3.159	0.589	0.882	0.953
9	2.323	2.935	3.079	3.156	4.185	4.425	0.848	1.303	1.413
10	2.963	3.834	4.039	4.092	5.545	5.881	1.154	1.810	1.967
11	3.667	4.842	5.116	5.133	7.077	7.522	1.509	2.402	2.617
12	4.435	5.956	6.309	6.280	8.774	9.338	1.913	3.083	3.363
13	5.267	7.174	7.614	7.531	10.629	11.321	2.366	3.851	4.206
14	6.162	8.495	9.029	8.885	12.634	13.460	2.869	4.707	5.146
15	7.121	9.914	10.548	10.340	14.781	15.746	3.422	5.651	6.181

The table reports the one period ahead out-of-sample performance of the volatility timing strategies with different target returns. There were 521 observations in each of the estimated models and the rolling sample approach provided 261 forecasting values for each out-of-sample comparison. The first forecasted value occurred the week of January 4, 2002. The target returns were from 5% to 15% (annualized). The weights for the volatility timing strategies were obtained from the weekly estimates of the one period ahead conditional covariance matrix and the different target return setting. Panel A presents the annualized means (μ) and volatilities (σ) for each strategy. The estimated Sharpe ratios for the return-based DCC model, the range-based DCC model, and the rollover OLS strategy were 0.540, 0.586, and 0.326, respectively. Panel B presents the average switching annualized fees (Δ_r) from one strategy to another. The values of the constant relative risk aversion were 1, 5, and 10.

Table 4 reports how the performance comparisons varied with the target returns and the risk aversions for one period ahead out-of-sample forecast. We obtained a consistent conclusion with Table 3. The estimated Sharpe ratios calculated from return-based DCC, range-based DCC and rollover OLS were 0.540, 0.586 and 0.326, respectively. The performance fees switching from rollover OLS to DCC were all positive. In total, the out-of-sample comparison supported the former inference. Fig. 5 plots the weights that minimize conditional volatility while setting the expected annualized return equal to 10%.

In addition to examining the performance of short-horizon investors, we further reported the results of the long-horizon asset allocations. Table 5 reports one to 13 periods ahead of out-of-sample performance for three methods. Here the rolling sample approach provided 249 forecasting values for each out-of-sample comparison. The portfolio weights for all strategies were obtained from the weekly estimates of the out-of-sample conditional covariance matrices with a fixed target return (10%). In general, the Sharpe ratios taken from range-based DCC were the largest, and return-based DCC were the next. For each strategy, however, we could not find an obvious trend in the Sharpe ratios forecasting periods ahead. As for the result of the performance fees, it seems reasonable to conclude that an investor would still be willing to pay to switch from rollover OLS to DCC. Moreover, the economic value seems to indicate a decreasing trend for forecasting periods ahead. For a longer forecasting horizon (12–13 weeks), however, the results of estimated switching fees were mixed. Switching from return-based DCC to range-based DCC always remains positive.

Thorp and Milunovich (2007) show that a risk-averse investor holding selected international equity indices, with $\gamma = 2,5$, and 10, would pay little for symmetric to asymmetric forecasts. In some cases, the switching fees would even be negative. In order to further understand this argument, we examined it based on the range-based volatility model. Chou (2005) provides an asymmetric range model namely CARRX: $\lambda_t = \omega + \alpha \cdot R_{t-1} + \beta \cdot \lambda_{t-1} + \phi \cdot \mathrm{ret}_{t-1}$. The lagged return in the conditional range equation was used to capture the leverage effect. For building an asymmetric range-based volatility model, CARR in the first step of range-based DCC can be replaced by CARRX. Cappiello et al. (2006) introduced asymmetric DCC: $\mathbf{Q}_t = (1-a-b)\overline{\mathbf{Q}} - c \cdot \overline{\mathbf{N}} + a \cdot \mathbf{Z}_{t-1}\mathbf{Z}'_{t-1} + b \cdot \mathbf{Q}_{t-1} + c \cdot \mathbf{n}_{t-1}\mathbf{n}'_{t-1}$. $\mathbf{n}_t$ is the $k \times 1$ vector calculated by $I(\mathbf{Z}_t < 0) \odot \mathbf{Z}_t$ to allow

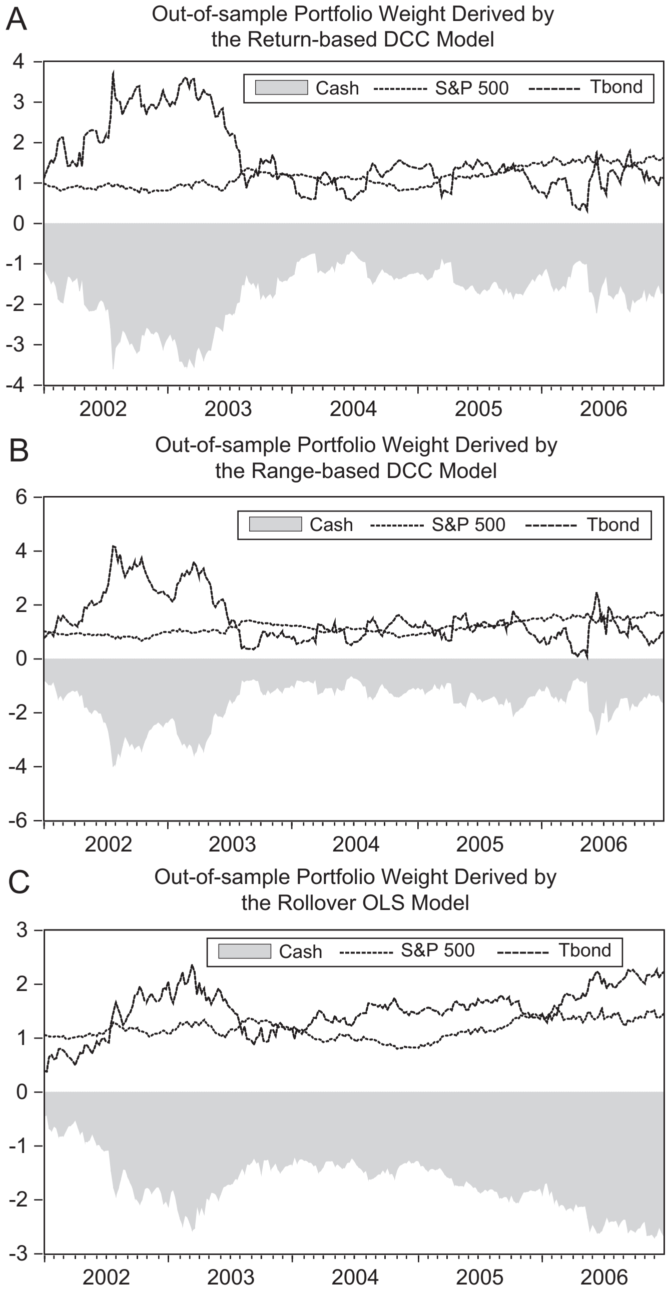

Fig. 5. Out-of-sample minimum volatility portfolio weight derived by the dynamic volatility model for one period ahead estimates. Panels A, B, and C show the one period ahead weights that minimize conditional volatility while the expected annualized return is set at 10%. Different from the in-sample case, the rolling sample method was used in estimating the portfolio weights. The portfolio weights in the rollover OLS model (Panel C) also vary with time. The first forecasted weights occurred the week of January 4, 2002. Panel A: out-of-sample portfolio weight derived by the return-based DCC model, Panel B: out-of-sample portfolio weight derived by the range-based DCC model and Panel C: out-of-sample portfolio weight derived by the rollover OLS model.

correlation to increase more in both falling returns than in both rising returns, and $\overline{\mathbf{N}} = \mathrm{E}(\mathbf{n}_t\mathbf{n}_t')$, where $\odot$ denotes the Hadamard matrix product operator, i.e. element-wise multiplication. Table 6 presents the one period ahead performance of the volatility timing values for asymmetric range-based DCC compared with rollover OLS. The switching fees from rollover OLS to asymmetric range DCC seem to be smaller than the fees from rollover OLS to symmetric range DCC in Table 4. One of the reasons for this may be the poor performance of the bond data. In this case, it is not valuable to switch the symmetric strategy to the asymmetric one.

Table 5

Out-of-sample comparison for one to 13 periods ahead volatility timing values in the minimum volatility strategy, 1992–2006.

Periods ahead	Return-based DCC			Range-based DC			Rollover OLS		
	μ	σ	SR	μ	σ	SR	μ	σ	SR
Panel A: Means and volatilities of optimal portfolios									
1	7.717	8.724	0.452	8.060	8.540	0.502	6.022	8.956	0.251
2	7.868	8.830	0.464	8.562	8.556	0.560	6.068	8.933	0.257
3	7.371	8.807	0.408	8.312	8.572	0.529	6.660	8.931	0.323
4	8.117	8.838	0.491	8.750	8.604	0.578	7.103	8.928	0.373
5	8.464	8.860	0.529	9.200	8.653	0.627	6.869	8.989	0.344
6	9.088	8.903	0.597	9.600	8.637	0.674	7.232	8.973	0.385
7	9.361	8.840	0.632	10.033	8.629	0.725	7.872	8.945	0.458
8	8.853	8.897	0.571	9.429	8.683	0.651	7.644	8.975	0.431
9	9.806	8.878	0.679	10.093	8.664	0.729	8.476	9.023	0.521
10	9.746	8.887	0.672	9.576	8.695	0.667	8.189	8.983	0.491
11	9.436	8.908	0.636	8.986	8.712	0.598	8.031	8.910	0.478
12	8.737	9.003	0.551	8.076	8.791	0.489	7.424	8.853	0.412
13	8.713	9.111	0.542	8.272	8.914	0.505	7.794	8.867	0.453
Panel B: Switching fees with different relative risk aversions									
1	2.772	3.546	3.727	3.944	5.289	5.599	1.196	1.831	1.983
2	2.282	2.633	2.716	4.223	5.448	5.731	1.970	2.914	3.137
3	1.293	1.721	1.823	3.308	4.495	4.772	2.029	2.830	3.019
4	1.440	1.758	1.834	3.152	4.244	4.499	1.728	2.544	2.738
5	2.210	2.665	2.773	3.900	5.032	5.297	1.712	2.446	2.622
6	2.191	2.442	2.503	3.938	5.078	5.345	1.775	2.730	2.958
7	1.993	2.373	2.464	3.647	4.740	4.997	1.674	2.440	2.625
8	1.581	1.861	1.928	3.161	4.172	4.410	1.597	2.369	2.555
9	2.028	2.556	2.683	3.319	4.578	4.875	1.313	2.103	2.295
10	2.019	2.370	2.455	2.753	3.767	4.007	0.753	1.465	1.638
11	1.416	1.424	1.426	1.891	2.591	2.758	0.489	1.209	1.383
12	0.593	0.037	−0.100	0.945	1.164	1.217	0.358	1.128	1.313
13	−0.269	−1.202	−1.436	0.251	0.078	0.035	0.518	1.243	1.417

The table reports the one to 13 periods ahead out-of-sample performance of the volatility timing strategies with the fixed 10% (annualized) target return. The weights for the volatility timing strategies were obtained from the weekly estimates of the one to 13 periods ahead conditional covariance matrix. There were 521 observations in each of the estimated models and the rolling sample approach provided 249 forecasting values for each out-of-sample comparison. The first forecasted mean value occurred the week of January 4, 2002. Panel A presents the annualized means (μ), volatilities (σ), and Sharpe ratios (SR) for each strategy. Panel B presents the average switching annualized fees (Δ_r) from one strategy to another. The values of the constant relative risk aversion were 1, 5, and 10.

Table 6

The one period ahead performance of the volatility timing values for the asymmetric range-based volatility model, 1992–2006.

Target return (%)	Means and volatilities of optimal portfolios for asymmetric range-based DCC		Switching fees from rollover OLS to asymmetric range-based DCC		
	μ	σ	Δ_1	Δ_5	Δ_{10}
5	4.643	1.666	0.373	0.425	0.438
6	5.352	3.025	0.787	0.962	1.003
7	6.060	4.384	1.301	1.670	1.757
8	6.769	5.744	1.915	2.550	2.699
9	7.478	7.103	2.630	3.601	3.827
10	8.187	8.462	3.445	4.818	5.136
11	8.895	9.821	4.361	6.199	6.621
12	9.604	11.180	5.377	7.738	8.274
13	10.313	12.540	6.491	9.428	10.087
14	11.022	13.899	7.703	11.262	12.050
15	11.730	15.258	9.011	13.232	14.155

The table reports the one period ahead out-of-sample performance of the volatility timing strategies for the asymmetric range-based volatility model with different target returns. There were 521 observations in each of the estimated models and the rolling sample approach provided 261 forecasting values for each out-of-sample comparison. The first forecasted value occurred the week of January 4, 2002. The target returns were from 5% to 15% (annualized). The weights for the volatility timing strategies were obtained from the weekly estimates of the one period ahead conditional covariance matrix and the different target return setting. The annualized means (μ) and volatilities (σ) of the optimal portfolio are presented here. Δ_r is the average switching annualized fee from the rollover OLS model to the asymmetric range-based volatility model. The estimated Sharpe ratio for the asymmetric range-based DCC model was 0.521. The values of the constant relative risk aversion were set as 1, 5, and 10.

4. Conclusion

In this paper, we examined the economic value of volatility timing for the range-based volatility model in utilizing range data which combines CARR with a DCC structure. Our analysis is carried out by utilizing S&P 500 and T-bond futures in a mean–variance framework with a no-arbitrage setting. By means of the utility of a portfolio, the economic value of dynamic models can be obtained by comparing it to OLS (a buy-and-hold strategy). Both the in-sample and out-of-sample results show that a risk-averse investor should be willing to switch from OLS to DCC with substantial high switching fees. Moreover, the switching fees from return-based DCC to range-based DCC were always positive. We concluded that the range-based volatility model has more significant economic value than the return-based one. The results gave robust inferences for supporting the range-based volatility model in forecasting volatility. Future studies can consider more general type of utility functions and also include other asset classes such as commodity futures, REIT's and VIX's.

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