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Range-based multivariate volatility model with double smooth transition in conditional correlation

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This paper proposes a multivariate model named Double Smooth Transition Conditional Correlation Conditional Autoregressive Range (DSTCC-CARR for short). Determined by two transition variables, the correlations smoothly transit from one state to another. Together with the DSTCC-GARCH model, the model is employed to investigate the interdependence between Hong Kong's and international stock markets. It is proved by the empirical analysis that the DSTCC-CARR model is more credible and efficient than the DSTCC-GARCH model. Linkages among Hong Kong's and other world's markets captured by these two models are testified to be consistent with history, and have meaningful interpretations.

1. Introduction

Since the 1970s, research on volatility model has become a prime and active theme in financial economics and econometrics. One of the most famous theory frames is the ARCH/GARCH family, which was first introduced by Engle (1982) and generalized by Bollerslev (1986). The univariate ARCH/GARCH model, which was extended by many economists on top of the original paper, became one of the most efficient tools in estimating and forecasting in financial markets. For a detailed survey, see Bollerslev, Chou, and Kroner (1992), Engle (2002a) and Engle (2004).

The multivariate ARCH/GARCH model has been developed very promptly during the same period. Researchers focused on conditional variance and covariance and developed the models. CCC-GARCH (Constant

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Conditional Correlation GARCH) model of Bollerslev (1990) made the hypothesis of constant conditional correlation. An extension to CCC-GARCH was introduced by Jeantheau (1998), which relaxes the auto-correlation structure in the original CCC-GARCH model and allow dynamic interactions between the conditional variance equations. Its moment structure was later considered in the paper of He and Teräsvirta (2004). Oppositely, there are some other representative models in early researches, including VECH model of Bollerslev, Engle, and Wooldridge (1988), the BEKK model of Engle and Kroner (1995), which are flexible in allowing time-varying covariance. But these two models have their own disadvantages. Test developed by Tse (2000) and Bera and Kim (2002) often rejects the constancy of conditional correlations as the latter models have difficulty in estimating the parameters.

Both Tse and Tsui (2002) and Engle (2002b) created dynamic conditional correlation GARCH models entitled VC-GARCH and DCC-GARCH respectively. The models set the conditional correlations as well as the conditional variance with the GARCH-type dynamic structure. Because of the conditional correlations generated by first-order GARCH processes with identical parameters, the number of parameters in DCC-GARCH model remains relatively small.

To avoid the difficulty of estimating the parameters in the VC-GARCH and DCC-GARCH models, three types of simplified models are proposed. The first one is the threshold approach, which separates the possible correlations into several states and has been used by Kwan, Li, and Ng (2005) to extend the VC-GARCH model. The second one is the regime-switching approach. Pelletier (2006) constructed a regime switching correlation by an unobserved state variable following a first-order Markov chain. The last one is the smooth transition approach used in models proposed by this paper. The latest researches are Silvennoinen and Teräsvirta (2005, 2007). They built the STCC-GARCH (Smooth Transition Conditional Correlation GARCH) model and the DSTCC-GARCH (Double Smooth Transition Conditional Correlation GARCH) model respectively, in which the correlations vary smoothly among the extreme constant states. The dynamic characteristics are caught by some observed transition variables such as time trend, average of returns, business cycle indices and so on.

Besides the return-based volatility models we introduced above, there is another branch of models named range-based volatility models in estimating the volatility of asset prices. It is proved that using the high/low range data of asset prices to do estimation can acquire more efficient results than the return data based on close prices. The information sets are enlarged and more efficient results regarding the volatility are obtained. The main studies of this branch include Parkinson (1980), Garman and Klass (1980), Wiggins (1991), Rogers and Satchell (1991), Andersen and Bollerslev (1997), Galant, Hsu, and Tauchen (1999), Yang and Zhang (2000), Alizadeh, Brandt, and Diebold (2002), Brandt and Jones (2006), Chou (2005, 2006). Chou proposed the Conditional Autoregressive Range (CARR) model, which combines range analysis and the GARCH model. The CARR model is proved to perform well on dynamical volatility process. Chou, Wu, and Liu (2004) applied this model to the Taiwan stock market and captured some insightful results while Ding and Xia (2005) applied CARR model to the Shanghai stock market and found better-forecast behavior. According to recent studies, we find the range-based univariate volatility model as a good alternative to return-based volatility models.

An extension of CARR model to multivariate framework named the Dynamic Conditional Correlations CARR (DCC-CARR) was proposed by Chou, Wu, and Liu (2009) in which they used a univariate CARR model to replace the standard univariate GARCH structure. The empirical results showed that the DCC model can fit the CARR model as well as the GARCH model, and the range-based DCC model outperformed the return-based models in estimating and forecasting covariance matrices. Zhang Xiaoqing (2007) extended the STCC model using the range-based framework called the STCC-CARR model. It showed that the STCC-CARR model had a better performance in describing the transiting process of the conditional correlations than the STCC-GARCH model.

As we introduce above, there are kinds of methods on estimating the correlations. DCC-GARCH is a popular model for estimating the correlations in recent researches. However, there is little published work on testing the constancy against this model as we know. Moreover, the dynamic structure of the time-varying correlations in the VC-GARCH or DCC-GARCH models is connected with past returns, but it is difficult to introduce the exogenous variables into these models because of the technical limitation. In order to introduce the exogenous variables[1] and make the constant correlation tests more easily, we propose a range-based smooth transition model called the Double Smooth Transition Conditional Correlation CARR

[1] As mentioned in Silvennoinen and Teräsvirta (2008), correlations between markets are affected by some exogenous variables, such as business cycle, market volatilities and so on.

(DSTCC-CARR) in our paper. Compared with STCC-CARR model, this model allows two exogenous variables in it, so it could contain other economic variables related with correlations besides the calendar time.

The organization of this paper is as follows: In Section 2, the structure of the DSTCC-CARR model is introduced. Section 3 gives the methodology of estimation and model specification tests. Section 4 is the empirical study, where seven different stock indices are employed to find the correlation between Hong Kong's and other international markets. Comparison of the DSTCC-CARR and DSTCC-GARCH models are also involved. Section 5 concludes.

2. The DSTCC-CARR model

The structure of the DSTCC-CARR model is similar to that of DSTCC-GARCH[2] model. The basic framework consists of two parts: the multivariate CARR and the double smooth transition structure of the conditional correlation. The multivariate CARR model is defined as Eq. (1):

$$\begin{aligned} &R_{i,t} = \lambda_{i,t}\varepsilon_{i,t} \quad \varepsilon_{i,t}|I_{t-1} \sim f(1;.) \quad t = 1,2,...,T; i = 1,2 \\ &\lambda_{i,t} = \omega_i + \alpha_i R_{i,t-1} + \beta_i \lambda_{i,t-1} \end{aligned} \tag{1}$$

where $R_{i,t}$ denotes the high/low range in logarithm type, of the ith asset during the time interval t. $\lambda_{i,t}$ is the conditional mean of the range on the basis of all information up to time t. The distribution of the disturbance term $\varepsilon_{i,t}$ is assumed to be distributed with a density function $f(\cdot)$ with a unit mean. $z^*_{i,t}$ is defined in order to convert the errors of return variables, and is given in Eq. (2):

$$z^*_{i,t} = r_{i,t} / \lambda^*_{i,t}, \quad \text{where} \quad \lambda^*_{i,t} = \text{adj}_i \times \lambda_{i,t}, \quad \text{adj}_i = \overline{\sigma}_i / \overline{\hat{\lambda}}_i. \tag{2}$$

The variable $r_{i,t}$ symbols the return of asset i. $\overline{\sigma}_i$ and $\overline{\hat{\lambda}}_i$ are the unconditional standard deviation of the return series i and the sampling mean of the estimated conditional range of the series i respectively. $\lambda^*_{i,t}$ denotes the conditional standard deviation computed from a scaled expected range in the CARR model. The specification of the exponential distribution of the disturbance term provides a consistent but inefficient, estimator for the parameters. Detailed discussion can be found in Chou (2005).

The first and second moments of vector z^*_{it} are given by Eq. (3):

$$E\left[\mathbf{z}^*_\mathbf{t} | \Omega_{\mathbf{t}-1}\right] = 0, \; E\left[\mathbf{z}^*_\mathbf{t}\mathbf{z}^{*'}_\mathbf{t} | \Omega_{\mathbf{t}-1}\right] = \mathbf{P}_\mathbf{t} \tag{3}$$

$\mathbf{z}^*_\mathbf{t}$ is derived from the univariate CARR and $E[\mathbf{r_t r'_t}] = E[\boldsymbol{\varepsilon}_\mathbf{t}\boldsymbol{\varepsilon}'_\mathbf{t}] = \mathbf{H_t} = \mathbf{S_t P_t S_t}$, where $\mathbf{S_t} = \text{diag}(\lambda^*_{1t},..., \lambda^*_{Nt})$.

The smooth transition correlation structure is represented as Eq. (4):

$$\mathbf{P_t} = (1 - G_{1t})\mathbf{P}_{(1)\mathbf{t}} + G_{1t}\mathbf{P}_{(2)\mathbf{t}}, \quad \textit{where} \quad \mathbf{P}_{(j)\mathbf{t}} = (1 - G_{2t})\mathbf{P}_{(j1)} + G_{2t}\mathbf{P}_{(j2)}, \quad j = 1,2 \tag{4}$$

where the transition functions are logistic in Eq. (5):

$$G_{jt} = \left(1 + e^{-\gamma_j(s_{jt} - c_j)}\right)^{-1}, \quad \gamma_j > 0 \quad j = 1,2. \tag{5}$$

There are two parameters which are location parameter c_j and speed parameter γ_j in a transition function. c_j determines the location of the transition, that is, when does a transition from one state to the other happen. γ_j controls the slope of the function, which could be considered as speed of the transition. The higher γ_j is, the higher the speed of transition. For the extreme condition, the transition function become a jump function when $\gamma_j \rightarrow \infty$.

In this paper, we use a special type of DSTCC models called the Time-varying STCC (TVSTCC) model in which we fix one of the transition variables as time trend[3], i.e. $s_{2t} = t/T$. We can transform Eq. (4) into Eq. (6).

$$\begin{aligned} &\mathbf{P_t} = (1 - G_{2t})\left((1 - G_{1t})\mathbf{P}_{(11)} + G_{1t}\mathbf{P}_{(21)}\right) + G_{2t}\left((1 - G_{1t})\mathbf{P}_{(12)} + G_{1t}\mathbf{P}_{(22)}\right) \quad \textit{or} \\ &\mathbf{P_t} = (1 - G_{1t})\left((1 - G_{2t})\mathbf{P}_{(11)} + G_{2t}\mathbf{P}_{(12)}\right) + G_{1t}\left((1 - G_{2t})\mathbf{P}_{(21)} + G_{2t}\mathbf{P}_{(22)}\right) \end{aligned} \tag{6}$$

[2] Details of constructing the DSTCC-GARCH model are introduced in Silvennoinen and Teräsvirta (2007).
[3] Silvennoinen and Teräsvirta (2008) summarize that calendar time is a nice transition variable.

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Eq. (6) shows the intuitive meanings of the parameters and their relationships. First, as the transition variables vary from the location parameters, the four constant states have different effects on the correlation. When $s_{1t}<c_1$ and $s_{2t}<c_2$, the correlation is closer to the state "$\mathbf{P}_{(11)}$" than any other three states; when $s_{1t}>c_1$ and $s_{2t}<c_2$, the correlations is closer to the state "$\mathbf{P}_{(21)}$"; when $s_{1t}<c_1$ and $s_{2t}>c_2$, the correlation is closest to the state "$\mathbf{P}_{(12)}$" among the four states; when $s_{1t}>c_1$ and $s_{2t}>c_2$, the correlation is closest to the state "$\mathbf{P}_{(22)}$". Secondly, γ_{1t} and γ_{2t} control the smoothness of the transition. Particularly, the closer γ_{1t} and γ_{2t} are to zero, the more slowly they change from one state to another state, and while γ_{1t} and γ_{2t} approach infinity, there is a jump from one state to another.

3. Estimation and model specification tests

3.1. Estimation of DSTCC-CARR model

We assume joint conditional normality for errors for maximum likelihood estimation, that is $\mathbf{z}_t^* | \mathbf{\Omega}_{t-1} \sim N(\mathbf{0}, \mathbf{P_t})$. The log-likelihood is shown in Eq. (7):

$$l_t(\boldsymbol{\theta}) = -\frac{N}{2}log(2\pi) - \sum_{i=1}^{N} log\lambda_{it}^* - \frac{1}{2}log|\mathrm{P}_t| - \frac{1}{2}\mathbf{z}_t^{*\prime}\mathbf{P}_t^{-1}\mathbf{z}_t^*, \quad t = 1,\ldots T \tag{7}$$

where $\boldsymbol{\theta}$ denotes the vector of all the parameters in the model, and maximizing $\sum_{t=1}^{T} l_t(\boldsymbol{\theta})$ with respect to $\boldsymbol{\theta}$ yields the maximum likelihood estimator $\hat{\boldsymbol{\theta}}_{\mathbf{T}}$.

Because of the addition of non-linear smooth transition function into the model, maximizing the log likelihood can be difficult due to numerical problems. To increase the efficiency of the estimation we use the iterative method defined in Silvennoinen and Teräsvirta's (2007). The procedure works as follows: maximization of log likelihood is carried out iteratively by concentrating on the likelihood. In each round the estimated parameters are split into three sets: CARR, correlation, and transition parameters. The log likelihood is maximized with respect to one set at the time keeping another two sets of parameters fixed as the previous estimating values. Once all the values of the parameters are identical with the values obtained in the previous iterations, the convergence is reached.[4]

However, some supplements must be pointed out. First, we assume the asymptotic distribution of ML-estimator is normal: $\sqrt{T}\left(\hat{\boldsymbol{\theta}}_{\mathrm{T}} - \boldsymbol{\theta}_0\right) \xrightarrow{d} N\left(0, \mathrm{I}^{-1}(\boldsymbol{\theta}_0)\right)$, where $\boldsymbol{\theta}_0$ is the true parameter vector and $\mathbf{I}^{-1}(\boldsymbol{\theta}_0)$ denotes the population information matrix evaluated at $\boldsymbol{\theta} = \boldsymbol{\theta}_0$. But the asymptotic normality has not yet been proved even for univariate non-linear GARCH models.[5] As the same consideration of Silvennoinen and Teräsvirta (2007), we persist in the normality assumption and jump over the formal proofs. Secondly, as mentioned in their paper, when the speed parameter γ become closer to infinity, the likelihood function becomes flat with respect to that parameter, the convergence becomes difficult, and therefore, we have to set an upper limit. Once it reaches 500, it will make the rest of the parameters be conditional upon this speed parameter. Finally, the non-linear models usually have local maxima solutions. There are all kinds of numerical recipes trying to solve this problem, but there is no authoritative recipe until now. We use an algorithm of selecting different sets of starting values to choose the final solution.

3.2. Model specification test

Before doing the estimation using the DSTCC-CARR model, model specification tests must be executed. Making the time-varying correlation hypothesis arbitrarily by ignoring the possibility of constant conditional correlations ("CCC" hereafter) in the true model or adding an unnecessary transition variable

[4] The estimation tool we used is OX, version 4.10. The latest version can be found in the website http://www.doornik.com. The GnuDraw package is used to draw the figures.

[5] See Meitz and Saikkonen (2006) for details of conditions about the stability and ergodicity of these models.

into the model is not appropriate. Estimating with a wrong model will cause problems such as inconsistent parameter estimators and so on.[6]

Following Luukkonen, Saikkonen, and Teräsvirta (1988), transition function could make a linearization by first-order Taylor expansion around $\gamma_i = 0$, $i = 1, 2$, the detailed specification shows as Eq. (8).

$$G_{it} \cong 1/2 + 1/4(\gamma_i(s_{it} - c_i)) + \Re. \tag{8}$$

$\Re$ is the error term above the second-order. For simplicity, each of the individual series follows a CARR (1, 1) process. Let $\boldsymbol{\varphi}_\mathbf{i} = (\alpha_{i0}, \alpha_{i1}, \beta_{i1})'$ be the vector of parameters for the conditional mean $\lambda_{i,t}$.

3.2.1. Tests for CCC hypothesis against a STCC-CARR model

The model under the null hypothesis is the CCC-CARR model while it is the STCC-CARR model under the alternative hypothesis. The null hypothesis is constructed as: H_0: $\gamma = 0$, and the dynamic correlations can be given as Eq. (9)

$$\mathbf{P}_\mathbf{t}^* = \mathbf{P}_1^* + s_t\mathbf{P}_2^*;\ \text{where}\ \mathbf{P}_1^* = \frac{1}{2}(\mathbf{P}_1 + \mathbf{P}_2) + \frac{1}{4}c(\mathbf{P}_1 - \mathbf{P}_2)\gamma,\ \mathbf{P}_2^* = \frac{1}{4}(\mathbf{P}_1 - \mathbf{P}_2)\gamma. \tag{9}$$

Thus we construct an auxiliary null hypothesis: H_0^{aux}: $\boldsymbol{\rho}_\mathbf{2}^* = \mathbf{0}$, this null hypothesis can be tested by an LM test, and the LM statistic is listed as Eq. (10):

$$\text{LMccc} = T^{-1}\left(\sum_{t=1}^{T}\frac{\partial l_t\left(\hat{\boldsymbol{\theta}}\right)}{\partial \boldsymbol{\rho}_2^{*'}}\right)\left[\hat{\mathbf{I}}_\mathbf{T}\left(\hat{\boldsymbol{\theta}}\right)\right]^{-1}_{\left(\rho_2^*, \rho_2^*\right)}\left(\sum_{t=1}^{T}\frac{\partial l_t\left(\hat{\boldsymbol{\theta}}\right)}{\partial \boldsymbol{\rho}_2^{*'}}\right) \tag{10}$$

where $\boldsymbol{\theta} = (\boldsymbol{\varphi}_1',\ldots, \boldsymbol{\varphi}_N', \boldsymbol{\rho}_\mathbf{1}^{*\prime}, \boldsymbol{\rho}_\mathbf{2}^{*\prime})$. $\boldsymbol{\rho}_i^* = vecl\mathbf{P}_i^*$[7], $\hat{\mathbf{I}}_\mathbf{T}(\hat{\boldsymbol{\theta}})$ is a consistent estimator of the asymptotic information matrix, and $[\hat{\mathbf{I}}_T(\hat{\boldsymbol{\theta}})]^{-1}_{(\rho_2^*, \rho_2^*)}$ is the south-east $\frac{N(N-1)}{2} \times \frac{N(N-1)}{2}$ block of the inverse of $\hat{\mathbf{I}}_\mathbf{T}$. The LM statistic has an asymptotic χ^2 distribution with $\frac{N(N-1)}{2}$ degrees of freedom in this case.

3.2.2. Tests for CCC hypothesis against a DSTCC-CARR model

The model under the null hypothesis is the CCC-CARR model while it is the DSTCC-CARR model under the alternative hypothesis. The null hypothesis is defined as: H_0: $\gamma_1 = \gamma_2 = 0$, and the dynamic correlations can be given as Eq. (11):

$$\mathbf{P}_\mathbf{t}^* = \mathbf{P}_{(1)}^* + s_{1t}\mathbf{P}_{(2)}^* + s_{2t}\mathbf{P}_{(3)}^* + s_{1t}s_{2t}\mathbf{P}_{(4)}^*;$$

where the four correlation states can be illustrated as follows:

$$\begin{aligned}
\mathbf{P}_{(1)}^* &= 1/4\left(\mathbf{P}_{(11)} + \mathbf{P}_{(12)} + \mathbf{P}_{(21)} + \mathbf{P}_{(22)}\right) + 1/8c_1\gamma_1\left(\mathbf{P}_{(11)} + \mathbf{P}_{(12)} - \mathbf{P}_{(21)} - \mathbf{P}_{(22)}\right)\\
&\quad + 1/8c_2\gamma_2\left(\mathbf{P}_{(11)} - \mathbf{P}_{(12)} + \mathbf{P}_{(21)} - \mathbf{P}_{(22)}\right) + 1/16c_1\gamma_1c_2\gamma_2\left(\mathbf{P}_{(11)} - \mathbf{P}_{(12)} - \mathbf{P}_{(21)} + \mathbf{P}_{(22)}\right)\\
\mathbf{P}_{(2)}^* &= -1/8\gamma_1\left(\mathbf{P}_{(11)} + \mathbf{P}_{(12)} - \mathbf{P}_{(21)} - \mathbf{P}_{(22)}\right) - 1/16c_2\gamma_1\gamma_2\left(\mathbf{P}_{(11)} - \mathbf{P}_{(12)} - \mathbf{P}_{(21)} + \mathbf{P}_{(22)}\right)\\
\mathbf{P}_{(3)}^* &= -1/8\gamma_2\left(\mathbf{P}_{(11)} - \mathbf{P}_{(12)} + \mathbf{P}_{(21)} - \mathbf{P}_{(22)}\right) - 1/16c_1\gamma_1\gamma_2\left(\mathbf{P}_{(11)} - \mathbf{P}_{(12)} - \mathbf{P}_{(21)} + \mathbf{P}_{(22)}\right)\\
\mathbf{P}_{(4)}^* &= -1/16\gamma_1\gamma_2\left(\mathbf{P}_{(11)} - \mathbf{P}_{(12)} - \mathbf{P}_{(21)} + \mathbf{P}_{(22)}\right)
\end{aligned} \tag{11}$$

[6] Silvennoinen and Teräsvirta (2007) summarized general testing methods in different conditions. In this paper, we verify three of their tests. For more discussion, you can refer to Tse (2000), Bera and Kim (2002), Silvennoinen and Teräsvirta (2005, 2007) and Zhang Xiaoqing (2007).

[7] The "vecl" operator stacks the columns of the strict lower diagonal (obtained by excluding the diagonal elements) of the square argument matrix.

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under the null hypothesis there are: $\mathbf{P}^*_{(1)} = 1/4(\mathbf{P}_{(11)} + \mathbf{P}_{(12)} + \mathbf{P}_{(21)} + \mathbf{P}_{(22)}$, $\mathbf{P}^*_{(2)} = \mathbf{0}_{N\times N}$, $\mathbf{P}^*_{(3)} = \mathbf{0}_{N\times N}$ and $\mathbf{P}^*_{(4)} = \mathbf{0}_{N\times N}$. Thus we construct the auxiliary null hypothesis: H_0^{aux}: $\boldsymbol{\rho}^*_{(2)} = \boldsymbol{\rho}^*_{(3)} = \boldsymbol{\rho}^*_{(4)} = \mathbf{0}$. The null hypothesis can be tested by an LM test, and the LM statistic shows as Eq. (12)

$$\mathrm{LMccc} = T^{-1}\left(\sum_{t=1}^{T}\frac{\partial l_t(\hat{\theta})}{\partial\left(\boldsymbol{\rho}^{*\prime}_{(2)},\boldsymbol{\rho}^{*\prime}_{(3)},\boldsymbol{\rho}^{*\prime}_{(4)}\right)}\right)\left[\hat{\mathrm{I}}_{\mathrm{T}}(\hat{\theta})\right]^{-1}_{\left(\boldsymbol{\rho}^*_{(2-4)},\boldsymbol{\rho}^*_{(2-4)}\right)}\left(\sum_{t=1}^{T}\frac{\partial l_t(\hat{\theta})}{\partial\left(\boldsymbol{\rho}^{*\prime}_{(2)},\boldsymbol{\rho}^{*\prime}_{(3)},\boldsymbol{\rho}^{*\prime}_{(4)}\right)}\right) \tag{12}$$

where $\boldsymbol{\theta} = (\boldsymbol{\varphi}_1',\ldots,\boldsymbol{\varphi}_N', \boldsymbol{\rho}^{*\prime}_{(1)},\ldots,\boldsymbol{\rho}^{*\prime}_{(4)}$. $\boldsymbol{\rho}^*_{(i)} = vecl\mathbf{P}^*_{(i)}$, and $\hat{\mathbf{I}}_{\mathbf{T}}(\hat{\boldsymbol{\theta}})$ is a consistent estimator of the asymptotic information matrix, a $\left[\hat{\mathrm{I}}_T(\hat{\theta})\right]^{-1}_{\left(\rho^*_{(2-4)},\rho^*_{(2-4)}\right)}$ is the south-east $\frac{3N(N-1)}{2}\times\frac{3N(N-1)}{2}$ block of the inverse of $\hat{\mathbf{I}}_{\mathbf{T}}$. The LM statistic has an asymptotic χ^2 distribution with $\frac{3N(N-1)}{2}$ degrees of freedom in this case.

3.2.3. Tests for the additional transition effect in a DSTCC-CARR model

The model under the null hypothesis is the STCC-CARR model while it is the DSTCC-CARR model under the alternative hypothesis. The null hypothesis is defined as: H_0: $\gamma_2 = 0$, and the dynamic correlations can be given as Eq. (13)

$$\mathbf{P}^*_t = (1 - G_{2t})\mathbf{P}^*_{(1)} + G_{1t}\mathbf{P}^*_{(2)} + s_{2t}\mathbf{P}^*_{(3)};$$

where the four correlation states can be illustrated as follows:

$$\begin{aligned}
\mathbf{P}^*_{(1)} &= 1/2\gamma_1\left(\mathbf{P}_{(11)} + \mathbf{P}_{(12)}\right) + 1/4c_2\gamma_2\left(\mathbf{P}_{(11)} - \mathbf{P}_{(12)}\right)\\
\mathbf{P}^*_{(2)} &= 1/2\left(\mathbf{P}_{(21)} + \mathbf{P}_{(22)}\right) + 1/4c_2\gamma_2\left(\mathbf{P}_{(21)} - \mathbf{P}_{(22)}\right)\\
\mathbf{P}^*_{(3)} &= -1/4(1 - G_{1t})\gamma_2\left(\mathbf{P}_{(11)} - \mathbf{P}_{(12)}\right) - 1/4G_{1t}\gamma_2\left(\mathbf{P}_{(21)} - \mathbf{P}_{(22)}\right)
\end{aligned} \tag{13}$$

under the null hypothesis there are: $\mathbf{P}^*_{(1)} = 1/2(\mathbf{P}_{(11)} + \mathbf{P}_{(12)})$, $\mathbf{P}^*_{(2)} = 1/2(\mathbf{P}_{(21)} + \mathbf{P}_{(22)})$, and $\mathbf{P}^*_{(3)} = \mathbf{0}_{N\times N}$. Thus we construct an auxiliary null hypothesis: H_0^{aux}: $\boldsymbol{\rho}^*_{(3)} = \mathbf{0}$. This null hypothesis can be tested by an LM test, and the LM statistic shows as Eq. (14):

$$\mathrm{LMccc} = T^{-1}\left(\sum_{t=1}^{T}\frac{\partial l_t(\hat{\theta})}{\partial\boldsymbol{\rho}^{*\prime}_{(3)}}\right)\left[\hat{\mathrm{I}}_{\mathrm{T}}(\hat{\theta})\right]^{-1}_{\left(\boldsymbol{\rho}^*_{(3)},\boldsymbol{\rho}^*_{(3)}\right)}\left(\sum_{t=1}^{T}\frac{\partial l_t(\hat{\theta})}{\partial\boldsymbol{\rho}^{*\prime}_{(3)}}\right) \tag{14}$$

where $\boldsymbol{\theta} = (\boldsymbol{\varphi}_1',\ldots,\boldsymbol{\varphi}_N', \boldsymbol{\rho}^{*\prime}_{(1)}, \boldsymbol{\rho}^{*\prime}_{(2)}, \boldsymbol{\rho}^{*\prime}_{(3)})$. $\boldsymbol{\rho}^*_{(i)} = vecl\mathbf{P}^*_{(i)}$, and $\hat{\mathbf{I}}_{\mathbf{T}}(\hat{\boldsymbol{\theta}})$ is a consistent estimator of the asymptotic information matrix, and $\left[\hat{\mathrm{I}}_T(\hat{\theta})\right]^{-1}_{\left(\rho^*_{(3)},\rho^*_{(3)}\right)}$ is the southeast $\frac{N(N-1)}{2}\times\frac{N(N-1)}{2}$ block of the inverse of $\hat{\mathbf{I}}_{\mathbf{T}}$. The LM statistic has an asymptotic χ^2 distribution with $\frac{N(N-1)}{2}$ degrees of freedom in this case.

4. Empirical analysis

4.1. Data description

Seven stock indices are selected including daily high, low and close prices. There are HSI (Hong Kong), GSPC (USA), N225 (Japan), STI (Singapore), TWII (Taiwan), FTSE (UK) and SSEC (China). The original data is extracted from the website Finance, Yahoo of China.[8] Generally, stock returns and ranges are computed by $100\times\log(p_t^{close}/p_{t-1}^{close})$ and $100\times\log(p_t^{high}/p_t^{low})$ respectively. To compare the correlations among different indices, we revise dataset afterwards. The rules for filtrating data are as follows: 1) Delete data of

[8] Unfortunately the data of TWII is not integrated. As a result, we use the data from TEJ Instead.

Table 1
The summary statistic of seven stock returns (1992.1.6–2007.5.30).

	FTSE	GSPC	HSI	N225	SSEC	STI	TWII
Mean	0.003	0.018	0.029	−0.012	0.081	−0.001	0.077
Median	0.027	0.043	0.041	−0.012	0.035	0.009	0.002
Maximum	5.904	5.574	17.247	7.655	30.000	8.159	23.267
Minimum	−5.589	−7.113	−10.000	−7.234	−10.000	−9.153	−10.000
Std. Dev.	1.012	0.987	1.578	1.436	2.518	1.231	2.198
Skewness	−0.111	−0.120	0.095	0.137	2.276	−0.376	0.947
Kurtosis	6.387	6.811	11.129	5.209	28.529	9.577	14.648
Jarque-Bera	1509.519	1910.213	8660.802	649.038	88,089.110	5740.191	18,242.650

Table 2
The summary statistic of seven stock ranges (1992.1.6–2007.5.30).

	FTSE	GSPC	HSI	N225	SSEC	STI	TWII
Mean	1.226	1.213	1.592	1.616	2.337	1.253	1.641
Median	1.005	1.015	1.338	1.414	1.675	1.030	1.405
Maximum	9.937	8.479	13.724	8.929	79.308	12.951	9.501
Minimum	0.166	0.177	0.000	0.291	0.000	0.000	0.146
Std. Dev.	0.812	0.773	0.994	0.904	2.780	0.931	0.956
Skewness	2.568	2.266	2.666	2.002	10.577	3.086	1.844
Kurtosis	15.279	12.616	18.008	9.952	223.811	23.090	8.676
Jarque-Bera	23,205.770	14,803.310	33,230.930	8433.108	6,445,838.000	57,859.360	6000.983

the day when some markets have missing values to keep the consistency. 2) Cut off the outliers to avoid probable estimation problem. 3) Set range of the day to the mean value when there is no change during the day. The valid dataset range after filtrating is from 1992.1.6 to 2007.5.30, and the sample size is 3144. Details are given in Tables 1 and 2 and Fig. 1. All returns and ranges exhibit excess kurtosis and Jarque-Bera tests clearly reject the null of a Gaussian distribution in all cases, so it is appropriate to use the CARR model proposed by Chou (2005).

4.2. Choosing the transition variables

It was referred to the facility of choosing different transition variables in the DSTCC structure for researchers. According to capture different characteristics of international interdependence, three variables are employed as the transition variables. They are time[9], st_1 (the seven-day average of lagged absolute HSI return) and st_2 (the lagged HSI return over three days). The statistics of the transition variables are described in Table 3 and Fig. 2.

There are several reasons for these choices. Firstly, we want to find whether time could influence the correlations among different stock markets. Secondly, in order to focus on the correlations between HSI and other six indices, HSI indicators are chosen as the main transition variables. Finally, the market turbulence and shock asymmetry, which could be described by absolute return and the lag return respectively, have an effect on the conditional correlations.[10]

4.3. Tests for the combinations of the transition variables

Corresponding to what introduced in Section 3.2, three tests are brought in.

[9] The time stamp, each value represents the position of the date in the whole period, compared to the normal equidistant time values. We do not simply split time with proportional spacing but consider the different values of the time-interval. This makes the data more convincible.

[10] See the details in Silvennoinen and Teräsvirta (2005).

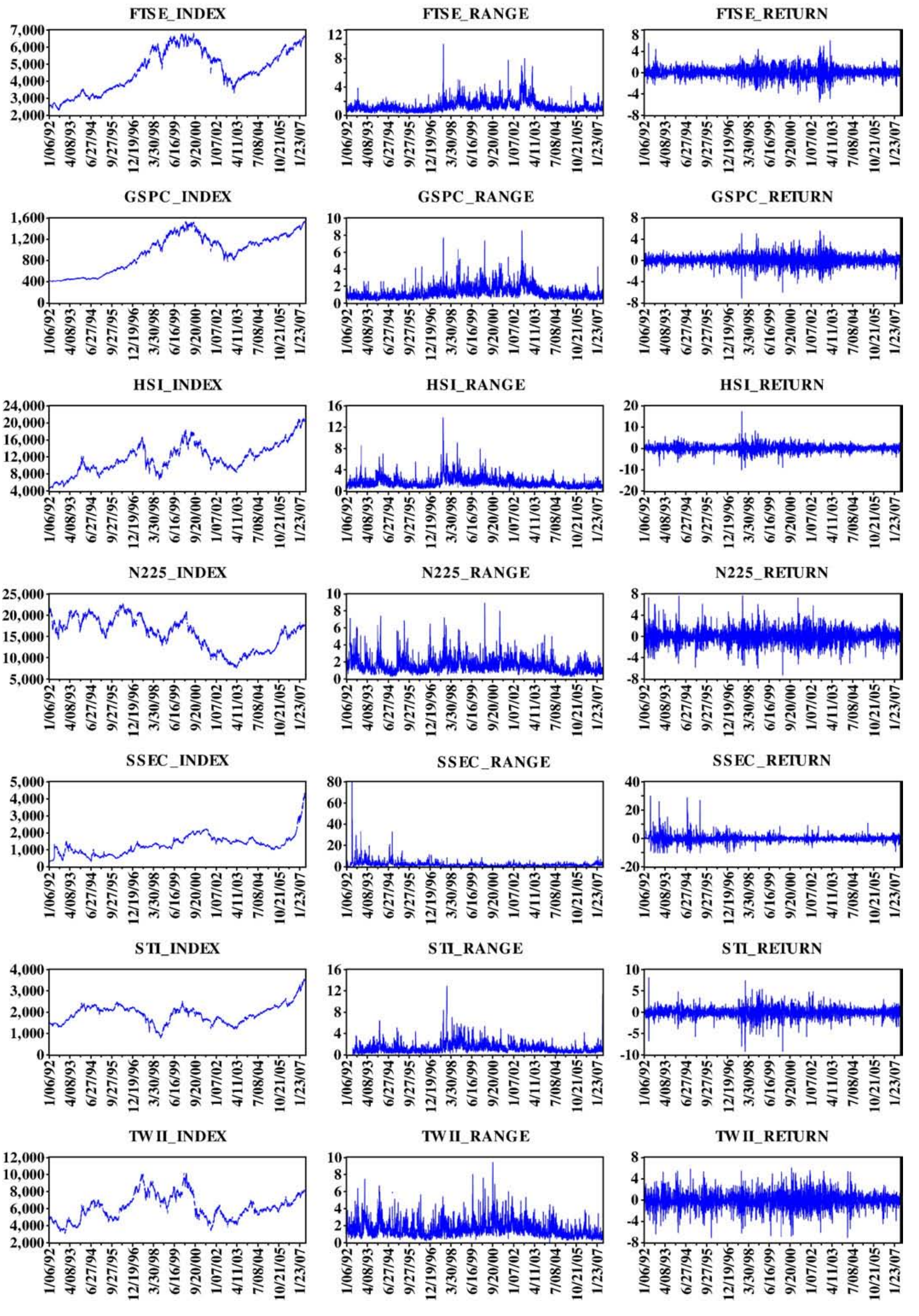

Fig. 1. Seven indices, returns and ranges (1992.1.6–2007.5.30).

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Table 3
The summary statistic of transition variables.

	Mean	Median	Maximum	Minimum	Std. Dev.	Skewness	Kurtosis	Jarq-Bera
st_1	1.099	0.925	9.493	0.190	0.666	3.115	23.987	62784.640
st_2	0.095	0.161	14.913	−10.000	2.716	−0.169	5.041	560.403
Time	0.498	0.493	1.000	0.000	0.289	0.013	1.801	188.401

We construct Test 1 for all bivariate instances against the STCC-CARR and STCC-GARCH models with every single transition variable respectively. Table 4 gives the LM statistics and the corresponding *p*-values. The instances using time as the transition variable reject the null hypothesis at a significant level of 1% for both the STCC-CARR and STCC-GARCH models except the case of "HIS&GSPC". Four out of six instances in both models reject the null hypothesis at a significant level of 10% when st_1 selected as transition variable. However, st_2 does not perform well in both models; only half of the cases in these models rejected the null hypothesis at a significant level of 10%.

Test 2 examines CCC hypothesis against DSTCC-CARR and DSTCC-GARCH models with the components "time& st_1" and "time& st_2". The LM statistics and corresponding p-values are listed in Table 5. Except for the case "HSI&GSPC", all the cases reject the hypothesis at a significant level of 1%. It indicates that we may do the estimations for bivariate models with these two transition variable components.

Test 3 checks up additional effects produced by st_1 and st_2 based on STCC model with time as transition variable. It is proved in Test 1 that the correlations of most models are time-varying. We questioned whether adding a new transition variable could provide more interpretation in correlations. Test 3 may offer an answer. The LM statistics and the p-values are listed in Table 6. Three (four) and Five (three) cases reject the null hypothesis at a significant level of 10% when we set st_1 and st_2 as an additional transition variable using DSTCC-CARR (DSTCC-GARCH) respectively. It implies that st_1 and st_2 can explain some additional change in the correlations.

4.4. The estimation and discussion

Based on the model specification tests introduced in Section 4.3, we construct DSTCC-CARR and DSTCC-GARCH model with "time& st_1" and "time& st_2"as transition variables. The estimation results are reported in Tables 7 and 8.[11] Figs. 3 and 4 outline the correlations. Most coefficients are statistically significant, which is consistent with the tests results above. The results indicate that the DSTCC-CARR model outperforms the DSTCC-GARCH model not only in terms of the significance of the coefficients, but also the sensitivity to transition variables. Some interesting stories may be underneath the estimated results, as discussed in following paragraphs.

4.4.1. The time effect

Correlations of all the instances are increasing as time passes by, which suggests that Hong Kong are more related with the world financial markets. Time effect on correlations could be classified as "mild change" and "sharp shock". By literal translation, "mild change" implies a slow correlation adjustment during a long term, while "sharp shock" describes an abrupt change in the correlation structure in the short run. Different characteristics are represented because of the distinct speed.

On one side, for those cases with "mild change", the speed parameters (γ_2) are small, and the location parameters (c_2) indicate the medium date of the time change. Some of the location coefficients are insignificant. It's attributed to the slow speeds which make it hard for the models to find the exact turning point. "HSI&N225" and "HSI&STI" stand on this side.[12] It indicates that the stock markets of Japan and

[11] For the sake of parsimony, both the univariate CARR and GARCH coefficients are not reported here, but are available from the authors upon request.

[12] When we use "time& st_1" and "time& st_2" as transition variable components, "HSI&FTSE" and "HSI&SSEC" are also on this side respectively.

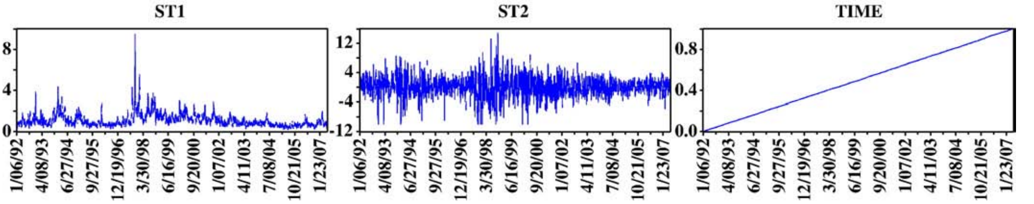

Fig. 2. Three transition variables.

Singapore are stably related to Hong Kong's stock market as time passes by. Since Japan, Singapore and Hong Kong are important financial hubs in Asia, it is most likely that they act in tandem and try to keep the state steady between each other.

The counterpart is quite different. The instances with the "sharp shock" always have large speed parameters (γ_2), and the location coefficients indicate that there are some incidents or structure changes around. Table 9 shows the transformations from c_2 to exact time stamp (date). "HSI&FTSE", "HSI&GSPC", "HSI&SSEC" and "HSI&TWII" belong to this set. Interpretations are as follows. (a). When selected the component "time& st_2" as transition variables, the time location of "HSI&FTSE" is nearby last September 1997. Although there is no special incident in the stock market of London, it should be mentioned that Hong Kong met the bubbles of stock and realty markets and both collapsed in October 1997 after Hong Kong reunified. Since Hong Kong has an unusual relationship with the UK, i.e. many investors and companies in Hong Kong were from UK, the interdependence between them became closer once the economic bubble came out. (b). "HSI&SSEC" focus on the second quarter of the year 1997 when we select the component "time& st_1" as transition variables. China increased the stamp tax by 0.3% on May 13th, 1997 and with that, the SSEC dropped 120 dots on May 22nd, 1997 and began a two-year adjustment. The return of Hong Kong to China on July 1st, 1997 also provided some shock to the interdependence. (c). "HSI&TWII" locates c_2 at the third quarter of the year 2001 in both transition components. It is coincident with the history: the stock market of Taiwan was affected by the depreciation of Taiwan dollar, and then went through the biggest winter after the shock of 9/11. The index "TWII" drops to the lowest level since 1993. (d). The case "HSI&GSPC" is particular which capture different events with different transition variable components. It is well known that the USA is the biggest financial center of the world so that the change of its stock market will certainly affect other stock markets. The interest rate of Hong Kong follows on the heels of Fed funds rate of the USA. As a result the correlation between them is sensitive as time changes. The corresponding date for c_2 of "HSI&GSPC" is Sep 13th, 2001 in DSTCC-CARR (st_1, time)/Sep 11th, 2001 in DSTCC-GARCH (st_1,

Table 4
Tests of CCC hypothesis against STCC-CARR and STCC-GARCH models.

	st_1		st_2		time	
	LM	*p*-value	LM	*p*-value	LM	*p*-value
Against STCC-CARR						
HSI–FTSE	4.873	0.027	*2.452*	*0.117*	23.350	0.000
HSI–GSPC	*1.490*	*0.222*	4.440	0.035	*1.720*	*0.190*
HSI–N225	*2.665*	*0.103*	*2.219*	*0.136*	123.450	0.000
HSI–SSEC	10.736	0.001	3.792	0.052	15.458	0.000
HSI–STI	8.890	0.003	23.510	0.000	162.950	0.000
HSI–TWII	6.811	0.009	*0.630*	*0.428*	94.642	0.000
Against STCC-GARCH						
HSI–FTSE	7.517	0.006	*0.989*	*0.320*	16.189	0.000
HSI–GSPC	*1.381*	*0.240*	6.367	0.012	*1.155*	*0.282*
HSI–N225	3.043	0.081	*2.165*	*0.141*	139.040	0.000
HSI–SSEC	9.955	0.002	*2.213*	*0.137*	17.422	0.000
HSI–STI	*0.000*	*0.987*	15.983	0.000	98.641	0.000
HSI–TWII	7.540	0.006	3.015	0.082	98.618	0.000

Note: italicized data indicates insignificance at the 10% level.

Table 5
Tests of CCC hypothesis against DSTCC-CARR and DSTCC-GARCH models.

	Against DSTCC-CARR				Against DSTCC-GARCH			
	st_1 & time		st_2 & time		st_1 & time		st_2 & time	
	LM	*p*-value	LM	*p*-value	LM	*p*-value	LM	*p*-value
HSI–FTSE	45.189	0.000	33.186	0.000	40.559	0.000	25.340	0.000
HSI–GSPC	*2.990*	*0.393*	9.121	0.028	*2.167*	*0.539*	10.924	0.012
HSI–N225	160.030	0.000	147.180	0.000	157.970	0.000	139.930	0.000
HSI–SSEC	27.918	0.000	22.159	0.000	29.847	0.000	21.118	0.000
HSI–STI	209.670	0.000	190.500	0.000	117.400	0.000	112.760	0.000
HSI–TWII	97.466	0.000	105.200	0.000	100.700	0.000	112.440	0.000

Note: italicized data indicates insignificance at the 10% level.

time), on which 9/11 occurred. In the DSTCC models with st_2 and time, another significant structure change is pointed out. The time is Jun 2nd 2000 when the American economic bubble struck the financial market heavily, and many high-tech companies went bankrupt at the same time.

4.4.2. The "volcano effect" and "lake effect" under st_1

The indicator "st_1" describes the volatility of Hong Kong stock market and could be used to explain the correlations during Hong Kong's turbulent periods as well as its calm periods. The "volcano effect" occurs in the turbulent periods while the "lake effect" occurs in the calm periods. How does the mechanism work?

As a whole, the speeds of the transition (γ_1) are mostly large[13] so that correlations will jump from one state to the other quickly according to the change of volatility, which are determined by st_1. The locations are useful for the analysis. From Table 3, we get the mean value of st_1, which is 1.099. Many of the c_1 coefficients are around the mean values, except two extreme cases "HSI&N225" and "HSI&SSEC". The former case means that Japan is easily affected by the volatility of Hong Kong stock market, while the latter one implies China has a very stable relationship with Hong Kong, unless there is some extraordinary volatility in the stock market of Hong Kong. The remaining cases behave normally.

We concentrate on those significant correlation coefficients and explain how the "volcano effect" and "lake effect" perform on different cases. Firstly, "HSI&FTSE" shows larger correlations in the turbulence than in the calmness at all times, it may be attributed to the same fluctuations that Hong Kong and UK suffer from. Secondly, the opposite situation happens in the case "HSI&SSEC", which indicates that correlation is higher when Hong Kong is in the calmness all the while. It implies that large correlation between them is not related to the volatility of Hong Kong's market. It is good news for diversification in these two markets. Thirdly, correlations of Hong Kong and Taiwan are higher in the turbulence at first but make a reverse after later. It shows that the stock market of Taiwan was active before the crash in 2001. "This coincides with reality." Oppositely, in recent period, correlation was fixed at some high level and changed rarely. It is possibly caused by the breakdown of Taiwan's market. Finally, the remaining cases show the lower correlations in the turbulence in the early period and make a reverse in the latest period. It turns out that Hong Kong is integrated with world's financial markets and fluctuates with the world.

Table 6
Tests for the additional transition based on "time" in the DSTCC-CARR and DSTCC-GARCH models.

	Against DSTCC-CARR				Against DSTCC-GARCH			
	st_1		st_2		st_1		st_2	
	LM	*p*-value	LM	*p*-value	LM	*p*-value	LM	*p*-value
HSI–FTSE	3.085	0.079	2.768	0.096	6.812	0.009	*1.189*	*0.275*
HSI–GSPC	*1.628*	*0.202*	5.589	0.018	*2.020*	*0.155*	6.503	0.011
HSI–N225	*0.287*	*0.592*	2.926	0.087	*1.275*	*0.259*	*1.415*	*0.234*
HSI–SSEC	4.895	0.027	4.722	0.030	2.824	0.092	2.883	0.090
HSI–STI	*0.967*	*0.325*	10.896	0.001	11.401	0.001	10.580	0.001
HSI–TWII	6.898	0.009	*0.085*	*0.770*	7.335	0.007	*0.861*	*0.354*

Note: italicized data indicates insignificance at the 10% level.

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Table 7
Estimation results of all bivariate DSTCC-CARR and DSTCC-GARCH models based on st_1 and time transition variables.

Series	ρ_1	ρ_2	ρ_3	ρ_4	c_1	c_2	γ_1	γ_2
(a) Estimation results for DSTCC-CARR								
HSI	−0.344	0.332	−0.213	0.424	0.952	0.010	500.000	5.516
FTSE	(0.634)	(0.035)	(0.565)	(0.034)	(0.011)	(0.284)	(–)	(1.613)
HSI	0.117	0.106	0.098	0.250	1.092	0.629	500.000	500.000
GSPC	(0.038)	(0.035)	(0.027)	(0.058)	(0.010)	(0.006)	(–)	(–)
HSI	−0.068	0.642	−0.257	0.792	1.273	0.273	500.000	3.523
N225	(0.183)	(0.042)	(0.227)	(0.108)	(0.008)	(0.177)	(–)	(2.432)
HSI	0.069	0.179	−0.729	−0.101	2.925	0.348	1.813	500.000
SSEC	(0.108)	(0.061)	(0.344)	(0.122)	(0.483)	(0.006)	(1.916)	(–)
HSI	0.270	0.806	−0.025	0.990	1.025	0.431	500.000	2.923
STI	(0.084)	(0.047)	(0.117)	(0.081)	(0.009)	(0.134)	(–)	(0.815)
HSI	−0.012	0.545	0.180	0.535	0.711	0.624	500.000	39.770
TWII	(0.080)	(0.034)	(0.023)	(0.028)	(0.005)	(0.018)	(–)	(3.318)
(b) Estimation results for DSTCC-GARCH								
HSI	0.009	0.317	0.080	0.433	0.951	0.219	500.000	5.515
FTSE	(0.174)	(0.037)	(0.140)	(0.045)	(0.011)	(0.196)	(–)	(1.249)
HSI	0.123	0.101	0.087	0.293	1.192	0.629	60.456	500.000
GSPC	(0.036)	(0.036)	(0.027)	(0.073)	(0.070)	(0.004)	(22.752)	(–)
HSI	0.009	0.769	−0.174	0.990	1.363	0.462	500.000	3.192
N225	(0.100)	(0.065)	(0.146)	(0.140)	(0.007)	(0.144)	(–)	(1.006)
HSI	0.033	0.150	−0.990	−0.077	3.171	0.348	2.759	500.000
SSEC	(0.035)	(0.027)	(0.293)	(0.159)	(0.325)	(0.007)	(1.973)	(–)
HSI	0.301	0.785	0.297	0.990	0.999	0.652	38.977	4.061
STI	(0.054)	(0.058)	(0.050)	(0.074)	(0.045)	(0.087)	(5.562)	(1.062)
HSI	−0.008	0.579	0.171	0.564	0.711	0.637	500.000	37.413
TWII	(0.084)	(0.034)	(0.023)	(0.028)	(0.006)	(0.016)	(–)	(3.425)

Table 8
Estimation results of all bivariate DSTCC-CARR and DSTCC-GARCH models based on st_2 and time transition variables.

Series	ρ_1	ρ_2	ρ_3	ρ_4	c_1	c_2	γ_1	γ_2
(a) Estimation results for DSTCC-CARR								
HSI	0.185	0.395	0.160	0.285	2.394	0.366	500.000	475.724
FTSE	(0.031)	(0.021)	(0.060)	(0.046)	(0.018)	(0.006)	(–)	(7.229)
HSI	0.060	0.151	0.160	0.117	0.526	0.546	500.000	500.000
GSPC	(0.030)	(0.033)	(0.036)	(0.046)	(0.041)	(0.013)	(–)	(–)
HSI	−0.257	0.618	−0.941	0.705	0.667	0.000	500.000	3.129
N225	(0.831)	(0.070)	(1.534)	(0.130)	(0.012)	(0.533)	(–)	(1.905)
HSI	−0.007	0.259	0.036	0.736	−2.419	1.000	500.000	5.295
SSEC	(0.045)	(0.662)	(0.045)	(1.404)	(0.019)	(0.612)	(–)	(1.640)
HSI	−0.070	0.830	−0.657	0.988	−0.044	0.040	500.000	1.964
STI	(1.374)	(0.278)	(2.436)	(0.473)	(0.011)	(1.235)	(–)	(2.770)
HSI	0.187	0.514	0.141	0.576	−0.039	0.629	500.000	38.489
TWII	(0.030)	(0.030)	(0.032)	(0.031)	(0.023)	(0.017)	(–)	(3.317)
(b) Estimation results for DSTCC-GARCH								
HSI	0.193	0.378	0.160	0.285	2.393	0.367	500.000	500.000
FTSE	(0.033)	(0.021)	(0.062)	(0.046)	(0.017)	(0.006)	(–)	(–)
HSI	0.056	0.148	0.166	0.110	0.526	0.546	500.000	500.000
GSPC	(0.029)	(0.033)	(0.037)	(0.049)	(0.033)	(0.012)	(–)	(–)
HSI	−0.167	0.769	−0.763	0.990	0.667	0.196	500.000	2.183
N225	(1.068)	(0.232)	(2.012)	(0.439)	(0.011)	(0.873)	(–)	(2.505)
HSI	0.009	0.248	0.060	0.816	−2.377	1.000	500.000	10.708
SSEC	(0.035)	(0.340)	(0.024)	(0.657)	(0.010)	(0.129)	(–)	(1.417)
HSI	0.134	0.878	−0.139	0.990	0.260	0.360	500.000	1.914
STI	(0.507)	(0.166)	(0.750)	(0.236)	(0.013)	(0.695)	(–)	(1.315)
HSI	0.175	0.537	0.141	0.609	−0.037	0.640	500.000	37.978
TWII	(0.029)	(0.031)	(0.032)	(0.029)	(0.017)	(0.015)	(–)	(3.317)

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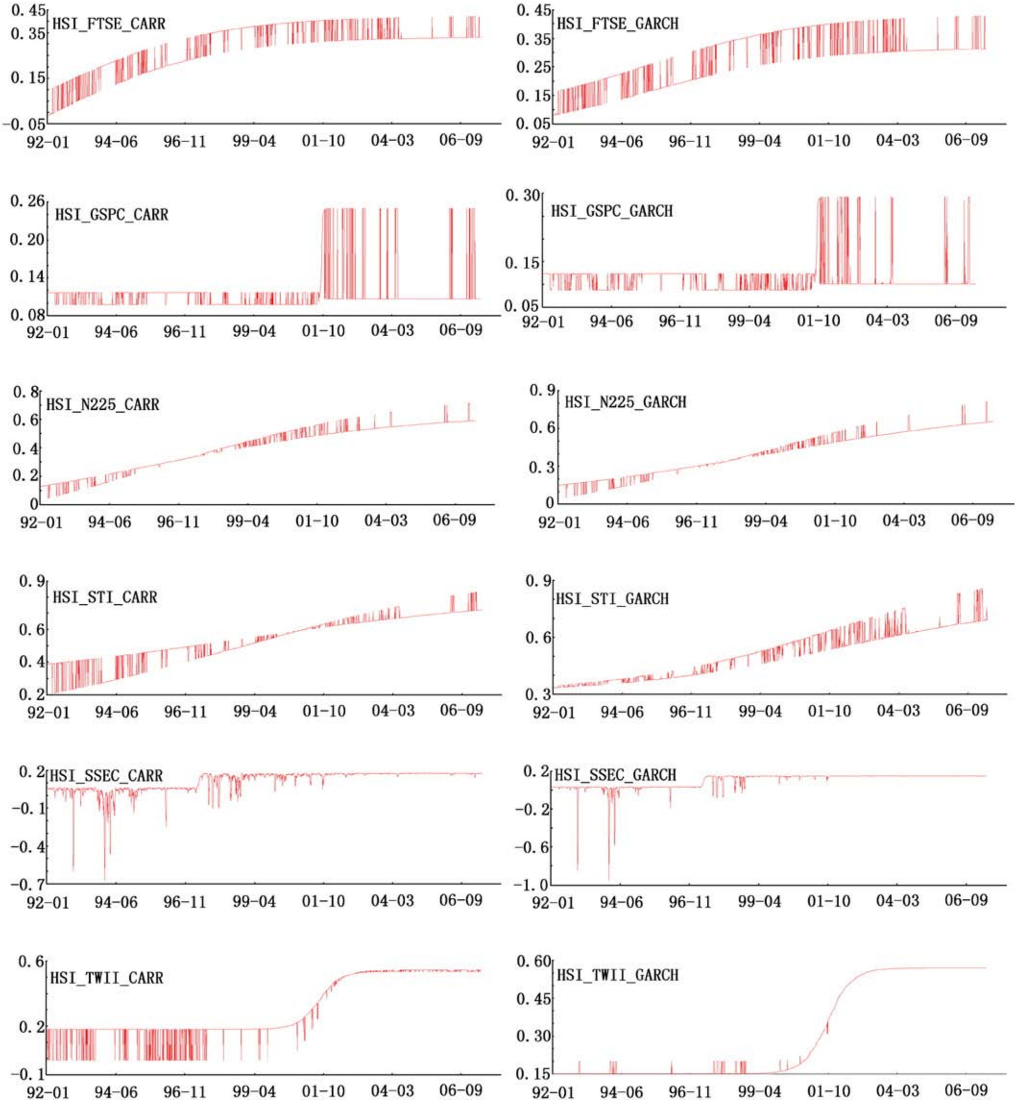

Fig. 3. Time-varying conditional correlation estimated by bivariate DSTCC-CARR (left) and DSTCC-GARCH (right) models when the transition variables are st_1 and time.

4.4.3. The boom effect and the depression effect under st_2

Similarly, "st_2" indicates the boom effect and depression effect on every case. The speed parameters (γ_1) are all large which imply jumps between two states instead of gradual transition. The location parameters (c_1) control the turning points. The mean of st_2 is 0.095, which is different from location coefficients. In the cases "HSI&TWII" and "HSI&STI", c_1s are around the mean value, which represent neutral responses to st_2. For the cases "HSI&GSPC", "HSI&N225" and "HSI&FTSE", c_1s are sensitive to positive lag returns. On the contrary, in the case "HSI&SSEC", c_1 is sensitive to negative lag returns. From the distribution, we may obtain the preference of different markets. America, Japan and UK are concerned with the potential benefits they may acquire, especially for the UK, the location is 2.394 (2.393) in DSTCC-CARR (DSTCC-GARCH)

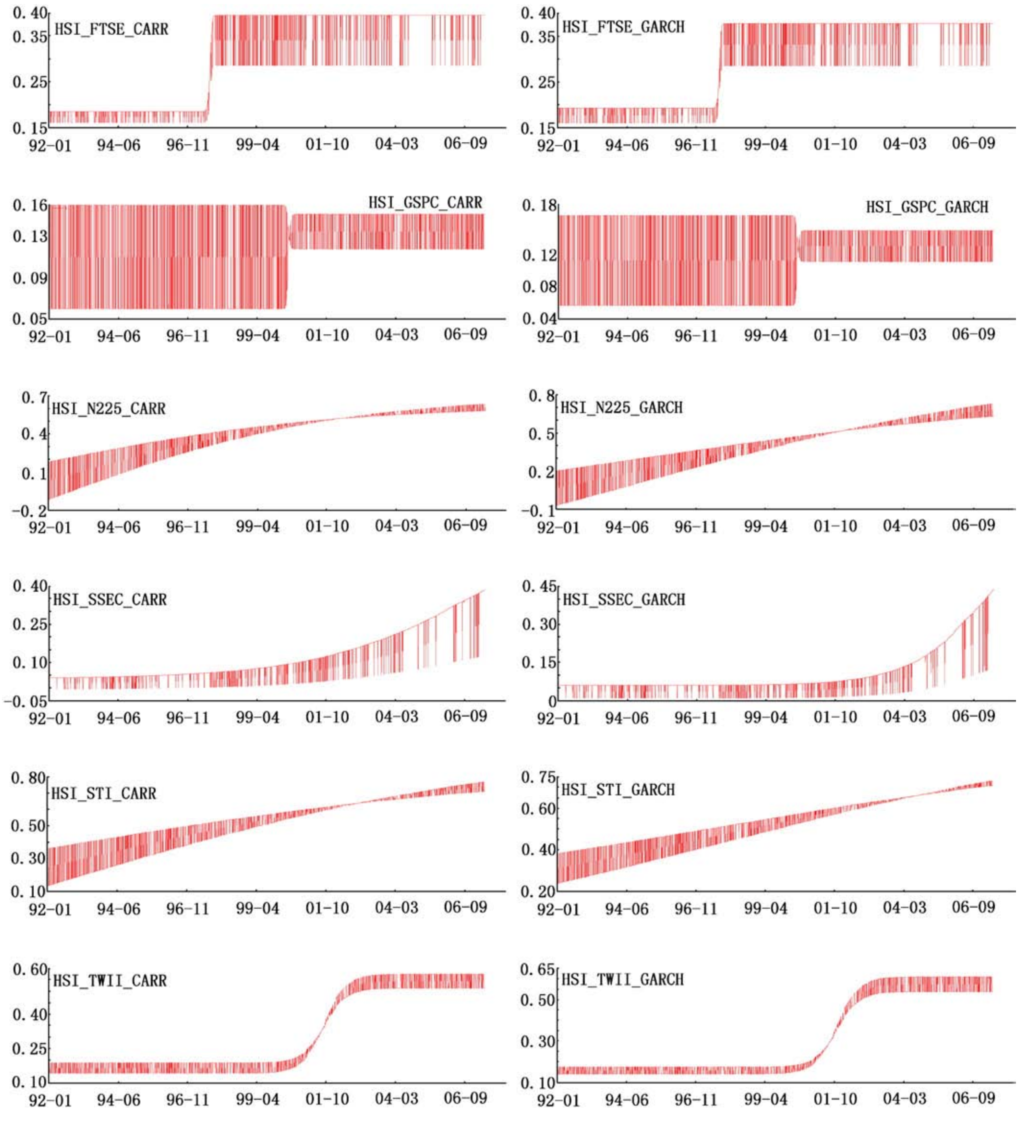

Fig. 4. Time-varying conditional correlation estimated by bivariate DSTCC-CARR (left) and DSTCC-GARCH (right) models when the transition variables are st_2 and time.

model. China keeps an eye on the possible loss of Hong Kong, and it is proved that Chinese government is always giving a hand to Hong Kong's development. Taiwan and Singapore tend to regard Hong Kong as an agency when they engage in international investments. For example, Hong Kong is a springboard to the investors who want to invest in China from Taiwan.

Beside the "HSI&GSPC" case, the rest have increasing correlations along time. Five branches are classified. (1). The correlation of "HIS&GSPC" was low during Hong Kong's recession while it was high during its boom in the early years. However, after the bubble in the year 2000, the American economy was not as active as before and correlation is lower when Hong Kong was in boom. It could be seen from the figure that the movements were slacker after the bubble. (2). Japan and Singapore had highly negative correlations with Hong Kong, but changed into highly positive correlations in the latest time. Correlations

Table 9
The transformation from location parameter c_2 to the date in DSTCC-CARR and DSTCC-GARCH models.

	st_1 and time				st_2 and time			
	DSTCC-CARR		DSTCC-GARCH		DSTCC-CARR		DSTCC-GARCH	
	c_2	Date	c_2	Date	c_2	Date	c_2	date
HSI–FTSE	0.010	1992-3-4	0.219	1995-5-24	0.366	1997-8-22	0.367	1997-8-29
HSI–GSPC	0.629	2001-9-12	0.629	2001-9-11	0.546	2000-6-2	0.546	2000-6-2
HSI–N225	0.273	1996-3-17	0.462	1999-2-17	0.000	1992-1-6	0.196	1995-1-13
HSI–SSEC	0.348	1997-5-15	0.348	1997-5-14	1.000	2007-5-30	1.000	2007-5-30
HSI–STI	0.431	1998-8-26	0.652	2002-1-16	0.040	1992-8-18	0.360	1997-7-24
HSI–TWII	0.624	2001-8-14	0.637	2001-10-24	0.629	2001-9-13	0.640	2001-11-14

were highest when Hong Kong was in boom these years. This means that they were the competitors in the early periods for the purpose of being the financial center of Asia. The cooperation among them increased these years and they were closely bounded. (3). The "HSI&TWII" case was stable until an abrupt change in the year 2001, when Taiwan was searching for a new economic development by investing outside when marginal product inside was at a low level. (4). Correlation between markets of Hong Kong and UK was high in the recession, low in the boom, but it was not obvious. (5). Correlation between Chinese and Hong Kong's markets was unchanged in early period, and the correlation was changed not as much in latest period when Hong Kong went through a winter. However, if Hong Kong went through a boom, the correlations are extremely high.

5. Conclusions

Compared to DSTCC-GARCH model presented by Silvennoinen and Teräsvirta (2007), we propose a range-based multivariate model named DSTCC-CARR in this paper. The estimation method is discussed, as well as three model specification tests, which are tests for CCC hypothesis against a STCC-CARR model, test for CCC hypothesis against a DSTCC-CARR model, and test for the additional effect in a DSTCC-CARR model based on the STCC-CARR model.

The empirical study is employed by concentrating on correlations between Hong Kong and other six international markets. The seven-day average of lagged HSI return and the lagged HSI return over three days together with the weighted time are chosen as transition variables. It is proved that the DSTCC-CARR model outperforms the DSTCC-GARCH model in our paper.

As a financial center of Asia, Hong Kong always catches the eyes of other markets. Firstly, the financial market of Hong Kong has increasing correlations with other international markets as time passes by. It separates into two types of transition: "mild change" and "sharp shock". The correlations with US, Taiwan, UK and China jumped when there were some structure changes, while the correlations with Japan and Singapore made smooth adjustments from a low level to a high level. Secondly, The "volcano effect" and the "lake effect" attributed to Hong Kong's turbulence and calmness are found out. On one side, Japan is easily affected by the volatility of Hong Kong's stock market, and on the contrary, China has a stable correlation with Hong Kong whether or not it is going through a turbulence or calmness. Finally, the boom and depression effects divide into three groups. America, Japan and UK are active in booming time as they focus on the benefits. Taiwan and Singapore act indifferently with two effects, since they regard Hong Kong as a springboard for international investments. China pays more attention to the loss of Hong Kong's market.

Although the result looks nice, it is not the end of the story. The model can be extended in many ways. The researchers can do some deeper study to tell a new story of the linkages between different stock markets or financial markets.

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