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ORIGINAL RESEARCH

Forecasting time-varying covariance with a range-based dynamic conditional correlation model

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Abstract This paper proposes a range-based dynamic conditional correlation (DCC) model combined by the return-based DCC model and the conditional autoregressive range (CARR) model. The substantial gain in efficiency of volatility estimation can boost the accuracy for estimating time-varying covariances. As to the empirical study, we use the S&P 500 stock index and the 10-year treasury bond futures to examine both in-sample and out-of-sample results for six models, including MA100, EWMA, CCC, BEKK, return-based DCC, and range-based DCC. Of all the models considered, the range-based DCC model is largely supported in estimating and forecasting the covariance matrices.

Keywords CARR · DCC · Dynamic covariance · Range · Volatility

JEL Classification C1 · C5 · G11

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1 Introduction

It is of primary importance in the practice of portfolio management, asset allocation and risk management to have an accurate estimate of the covariance matrices for asset returns. Meanwhile, a useful approach for estimating volatilities and covariances in valuing derivatives is necessary. Surveying from a bundle of past related literature, the univariate ARCH/GARCH family of models have provided effective tools in estimating the volatility of individual asset. Tailored to the needs of different asset classes, these various models have achieved remarkable success [see Bollerslev et al. (1992), and Engle (2004), for a comprehensive review]. However, estimating the covariance and correlation matrices of multiple variables, especially large sets of asset prices, is still an active research issue. Early attempts include the VECH model[1] of Bollerslev et al. (1988), the BEKK (Baba-Engle-Kraft-Kroner) model[2] of Engle and Kroner (1995), and the constant conditional correlation (CCC) model of Bollerslev (1990), among others. To our knowledge, the constant correlation model is too restrictive in that it imposes stringent constraints whereby the dynamic structure of the covariance is completely determined by individual volatilities. VECH and BEKK are, however, more flexible in that they allow time-varying correlations. While the BEKK parameterization for a bivariate model involves 11 parameters, for higher-dimensional systems, the additional parameters in BEKK make estimation very difficult.

In a series of related papers, Engle and Sheppard (2001), Engle (2002a), and Cappiello et al. (2006) provide another viewpoint to this problem by using a model referred to the dynamic conditional correlation (DCC) multivariate GARCH.[3] Intuitively, the conditional covariance estimation for two variables is simplified by estimating univariate GARCH models for each asset's variance process. Then, the estimation of the time-varying conditional correlation is performed by using the transformed standardized residuals. A meaningful and excellent performance of this model is demonstrated in these studies.

The objective of this article is to propose an alternative to the return-based DCC approach. In this paper, we consider a refinement of the return-based DCC model by utilizing the high/low range data of asset prices during a fixed time interval. In estimating the volatility of asset prices, there is a growing recognition of the fact that the range data of asset prices can provide sharper estimates and forecasts than the return data based on close-to-close prices. Many insightful studies have provided powerful evidence including Parkinson (1980), Garman and Klass (1980), Wiggins (1991), Rogers and Satchell (1991), Kunitomo (1992) and, more recently, Gallant et al. (1999), Yang and Zhang (2000), Alizadeh et al. (2002), Brandt and Jones (2006), Chou (2005, 2006), and Martens and van Dijk (2007). Above all, Chou (2005) proposes the conditional autoregressive range (CARR) model which can capture the dynamic volatility process and has obtained some insightful evidence in terms of real trading data. In other words, a range-based volatility model can serve as a useful substitution for the return-based volatility model in describing the process of volatility.

[1] The k-dimensional VECH model is written as $\text{vech}(H_t) = A + B\,\text{vech}\left(\xi_{t-1}\xi'_{t-1}\right) + C\,\text{vech}(H_{t-1})$, where H_t is the conditional covariance matrix at time t and $\text{vech}(H_t)$ is the vector that stacks all the elements of the covariance matrix.

[2] It is a general parameterization that involves the minimum number of parameters while imposing no cross equation restrictions and ensuring positive definiteness for any parameter value.

[3] Other econometric methods for estimating the time-varying correlation are proposed by Tsay (2002) and by Tse and Tsui (2002).

Range data intuitively have more information than return data for estimating volatility. Again, they are easy to obtain for many financial markets. The previous studies have proved that range is an efficient volatility estimator.[4] Moreover, Chou (2005) puts the range into the dynamic process, and verifies that the range model can also fit time-varying volatility well. In light of the success of the range-based univariate volatility models, it is natural to inquire whether the efficiency of the range structure can be extended and incorporated into a multivariate framework[5] for constructing covariance process.

The remainder of this paper is laid out in the following manner. Section 2 introduces the range-based volatility model. Section 3 reviews the bivariate models for estimating the covariance process. Section 4 describes the properties of data used and discusses the empirical results. Finally, the conclusion is showed in Sect. 5.

2 The range-based volatility model

The asset high/low range, $\Re_t$, is defined as the difference between the daily high and low prices in a logarithm type over a fixed time period. It is readily available for some assets and can be written as:

$$\Re_t = \ln(H_t) - \ln(L_t), \tag{1}$$

where H_t and L_t are the highest and lowest intraday price over a fixed period such as daily, weekly, or monthly. For weekly data, the highest price of a week is its intraday highest price that we can observe over the trading time in the week. Unlike the intraday realized volatility, the range therefore does not have a time-aggregation problem.

The previous studies indicated that range has relative efficient, but did not empirical support. Chou (2005) argues that its poor performance is due to the poor dynamic fitting, and further, proposes the CARR model to capture its dynamic structure. The CARR can be expressed as:

$$\begin{aligned} \Re_t &= \lambda_t u_t,\ u_t | I_{t-1} \sim \exp(1;\cdot) \\ \lambda_t &= \omega + \alpha \Re_{t-1} + \beta \lambda_{t-1}, \end{aligned} \tag{2}$$

where $\Re_t$ and λ_t is the high/low range and the conditional mean of the range during the time interval t, respectively. u_t is the innovation assumed to follow the exponential distribution with a unit mean.

The CARR model is a special case of the multiplicative error model (MEM) of Engle (2002b).[6] The specification of the exponential distribution for the disturbance term provides a consistent estimator of the parameters. For specific discussions, see Chou (2005) for a review. This paper extends this range model to a multivariate case by the DCC model, which will be introduced in the next section.

4 Shu and Zhang (2006) provide relative performance of different range-based volatility estimators, and find that the range estimators all perform very well when an asset price follows a continuous geometric Brownian motion.

5 Fernandes et al. (2005) utilize the formula $\mathrm{Cov}(X,Y) = [V(X+Y) - V(X) - V(Y)]/2$ to propose a kind of multivariate CARR model. However, this method limits the multivariate CARR model to a bivariate case only.

6 The MEM model is designed to fit a non-negative series, like duration or realized volatility.

3 Covariance estimation review and the DCC model

This section provides an overview of methods for describing the current level of covariance. Conventionally, the conditional covariance estimation between two return series is defined as:

$$\mathrm{COV}_{12,t} = E_{t-1}[(r_{1,t} - \mu_1)(r_{2,t} - \mu_2)], \tag{3}$$

where $\mu_i = E(r_{i,t})$. In most applications, asset returns are assumed to have zero means. This common viewpoint is adopted in our study. Thus, Eq. 3 can be expressed as $\mathrm{COV}_{12,t} = E_{t-1}(r_{1,t}r_{2,t})$.

It is useful to estimate time-varying covariance parameters between asset returns in many financial applications. For example, they can be used to deal with the hedging ratio for futures, the optimal weights for the portfolio allocation, the time-varying beta for the market model, and so on. The information of the conditional covariance is derived from previous trading data. One commonly used method is to compute the historical covariance. For capturing the time-varying property of covariances, however, one approach we use works with a moving average with a 100-week window, namely MA100, which is rich enough to be relevant and yet simple enough to permit a streamlined exposition:

$$\mathrm{COV}^{\mathrm{MA100}}_{12,t} = \frac{1}{100}\sum_{s=t-100}^{t-1} r_{1,s}r_{2,s}. \tag{4}$$

Intuitively, it is reasonable to attach more weight to recent data. Going by this, we introduce an exponentially weighted moving average (EWMA) model where the weights decrease exponentially as we move back through time. Exponential smoothing is used to model the unobservable variables for volatility in J.P. Morgan's RiskMetrics, too. EWMA has an attractive feature in that relatively little data need to be stored. Exponential averages arrange the most weight to the most recent observations, with weights declining exponentially as observations go back in time. It turns out that EWMA for covariance estimation can briefly be illustrated as follows.

$$\mathrm{COV}^{\mathrm{EWMA}}_{12,t} = (1-\lambda)\sum_{s=1}^{\infty} \lambda^{s-1} r_{1,t-s}r_{2,t-s}, \tag{5}$$

where the smoothing parameter λ lies between zero and unity. The value of λ governs how sensitive the estimate of the current variable is to percent changes in the most recent period. The popular RiskMetrics approach adopts exponential moving averages[7] to estimate future volatility because it believes the method responds rapidly to market shocks.

The conditional variance–covariance matrix can build a multivariate ARCH model. This approach has been extracted by Engle and Kroner (1995), who proposed the so-called BEKK model. The parameters, however, easily diverge from the acceptable scope when the type of the full-rank BEKK model is adopted. In the related literature, the diagonal BEKK (DBEKK) model is adopted more frequently due to its property of convergence of parameters used in general empirical research. Considering the bivariate case for DBEKK, its covariance matrix $H^{\mathrm{DBEKK}}_t = [h_{ij,t}]$ is shown as below:

[7] The RiskMetrics database uses the exponentially-weighted moving average model with $\lambda = 0.94$ for updating daily volatility estimates. J.P. Morgan found that, across variant market variables, this value of λ results in forecasts of the volatility that come closest to the realized volatility. Following J.P. Morgan's suggestion, the variable λ equals 0.94 for the time being in the later empirical discussion.

$$H_t^{\mathrm{DBEKK}} = \begin{bmatrix} c_{11} & 0 \\ c_{12} & c_{22} \end{bmatrix} \begin{bmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{bmatrix} + \begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t-1}^2 & \varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ \varepsilon_{1,t-1}\varepsilon_{2,t-1} & \varepsilon_{t-1}^2 \end{bmatrix} \begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix} + \begin{bmatrix} b_{11} & 0 \\ 0 & b_{22} \end{bmatrix} \begin{bmatrix} h_{1,t-1} & h_{12,t-1} \\ h_{21,t-1} & h_{22,t-1} \end{bmatrix} \begin{bmatrix} b_{11} & 0 \\ 0 & b_{22} \end{bmatrix}, \quad (6)$$

where a_{ij}, b_{ij}, c_{ij} are estimated parameters. ε_t represents the innovation term of the mean equation under the assumption $\varepsilon_t | I_{t-1} \sim (0, H_t)$.

Bollerslev (1990) proposed the CCC model with a constant correlation matrix, where univariate GARCH models are estimated for each asset and then the corresponding correlation matrix is constructed. An illustration of CCC is shown below. The covariance matrix H_t^{CCC} for a vector of k asset returns can be decomposed as follows:

$$H_t^{\mathrm{CCC}} = D_t R D_t, \quad (7)$$

where R is the correlation matrix and D_t is the $k \times k$ diagonal matrix of time-varying standard deviations from univariate GARCH models with $\sqrt{h_{i,t}}$ on the ith diagonal. As for the $\sqrt{h_{i,t}}$, it is the square root of the estimated variance for the ith return series. The assumption of a constant correlation makes estimating a large model feasible and ensures that the estimator is positive definite, simply requiring each univariate conditional variance to be non-zero and the correlation matrix to be of full rank. Under such a situation, the estimate of the conditional covariance can be obtained, based on information regarding the fixed correlation and the product of the two conditional standard deviations.

Although CCC is meaningful, the setting of constant conditional correlations could sometimes be too restrictive and the estimators in the constant correlation setting, as proposed, do not offer a rule to construct consistent standard errors, using the multi-stage estimation process. Another shortcoming for the constant correlation model is that the correlation coefficient tends to change over time in real applications. Engle (2002a) extended CCC to the more comprehensive DCC type. DCC retains the parsimony of the univariate GARCH model of individual assets' volatilities with a simple GARCH-like time varying correlation. Meanwhile, DCC differs from CCC mainly in that it allows the correlation matrix to be changed over time. Accordingly, we can write DCC as:

$$H_t^{\mathrm{DCC}} = D_t R_t D_t, \quad (8)$$

$$R_t = \mathrm{diag}\{Q_t\}^{-1/2} Q_t \mathrm{diag}\{Q_t\}^{-1/2}, \quad (9)$$

$$Q_t = S \circ (ii' - A - B) + A \circ Z_{t-1} Z_{t-1}' + B \circ Q_{t-1}, \quad (10)$$

where D_{t} is defined as in Eq. 7 and R_t is the possibly time-varying correlation matrix. $Q_t = [q_{ij,t}]$ denotes the conditional covariance matrix of the standardized residuals.

In Eq. 10, A and B are parameter matrices and o denotes the Hadamard matrix product operator, i.e. element-wise multiplication. The symbol i denotes a vector of ones and S denotes the unconditional covariance matrix of the standardized residuals. Finally, $Z_t = [z_{i,t}]$ is the standardized but correlated residual vector, and its conditional correlation matrix is given by variable R_t. If A and B are zeros, then the DCC model can revert to the structure of CCC. Related literature shows that if A, B, and $(ii' - A - B)$ are positive semi-definite, then Q_t will also be positive semi-definite. If any one of the matrices is positive definite, then Q_t will also be so. For the ijth element of R_t, the conditional correlation matrix is given by $q_{ij,t} / \sqrt{q_{ii,t} q_{jj,t}}$. In our study, we focus on the comparison of forecasting covariances for two assets and Eq. 10 has the following structure in a bivariate case,

$$\begin{bmatrix} q_{11,t} & q_{12,t} \\ q_{12,t} & q_{22,t} \end{bmatrix} = (1-a-b)\begin{bmatrix} 1 & \bar{q}_{12} \\ \bar{q}_{12} & 1 \end{bmatrix} + a\begin{bmatrix} z_{1,t-1}^2 & z_{1,t-1}z_{2,t-1} \\ z_{1,t-1}z_{2,t-1} & z_{2,t-1}^2 \end{bmatrix} + b\begin{bmatrix} q_{11,t-1} & q_{12,t-1} \\ q_{12,t-1} & q_{22,t-1} \end{bmatrix} \tag{11}$$

where a and b are parameters. In most cases, they can substitute for complicated matrices A and B. $\bar{q}_{12}$ is the unconditional covariance of the two standardized residuals.

The DCC model is constructed to permit for two-stage estimation of the conditional covariance matrix H_t. Briefly speaking, during the first step, a univariate volatility model is fitted for each of the assets and the estimates of $h_{i,t}$ are obtained. In the second step, the asset returns transformed by their estimated standard deviations are used to estimate the parameters of the conditional correlation.

The log-likelihood of this estimator is straightforward. One simply maximizes the log-likelihood:

$$\begin{aligned} L &= -\frac{1}{2}\sum_t \left(k\log(2\pi) + \log|H_t| + r_t' H_t^{-1} r_t\right) \\ &= -\frac{1}{2}\sum_t \left(k\log(2\pi) + \log|D_t R_t D_t| + r_t' D_t^{-1} R_t^{-1} D_t^{-1} r_t\right) \\ &= -\frac{1}{2}\sum_t \left(k\log(2\pi) + 2\log|D_t| + \log|R_t| + Z_t' R_t^{-1} Z_t\right). \end{aligned} \tag{12}$$

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Following Engle (2002a)'s argument, one can perform the estimation by means of qusi-maximum likelihood estimation (QMLE) to yield consistent parameter estimates. The advantages of QMLE are its simplicity and consistency. However, its disadvantages are that the estimates are inefficient, even asymptotically, and more importantly, its small-sample properties are suspect. (also see Hafner and Franses (2003) for a review.) Let the parameters in D_t be denoted by θ_1 and the additional parameters in R_t be denoted by θ_2. According to Engle (2002a), one can divide the log-likelihood function into two parts:

$$L(\theta_1, \theta_2) = L_{\mathrm{Vol}}(\theta_1) + L_{\mathrm{Corr}}(\theta_1, \theta_2). \tag{13}$$

The former term in the right hand side of Eq. 13 represents the volatility part:

$$L_{\mathrm{Vol}}(\theta_1) = -\frac{1}{2}\sum_t \left(k\log(2\pi) + \log|D_t|^2 + r_t' D_t^{-2} r_t\right), \tag{14}$$

and the latter term can be viewed as the correlation component:

$$L_{\mathrm{Corr}}(\theta_1, \theta_2) = -\frac{1}{2}\sum_t \left(\log|R_t| + Z_t' R_t^{-1} Z_t - Z_t' Z_t\right). \tag{15}$$

Following the recipe for the first stage, we can pick up a suitable θ_1 easily, which satisfies Eq. 14 and is maximized after the estimate of $\hat{\theta}_1$ is computed. Subsequently, in the second stage, the correlation part in Eq. 15 can be maximized with respect to the optimized θ_1 and θ_2 simultaneously. Consequently, the formidable task of maximizing Eq. 13 is attainable. Estimates for $\hat{\theta}_1$ and $\hat{\theta}_2$ are useful in subsequent analysis.

It is interesting and important to recognize that although the dynamics of the D_t matrix has usually been structured as a standard GARCH model, it can be easily extended to many other types of models. For instance, one could adopt the EGARCH or GJR-GARCH model to replace the simple GARCH model for describing the asymmetric phenomenon in the actual volatility process or use the FIGARCH model to allow for the long memory volatility processes. In this paper, the CARR model of Chou (2005) will be used as an alternative to verify if the specification selected adequately fit the DCC model.

When the specific GARCH model is fitted, the term of volatility in the likelihood function can be demonstrated as below:

$$L_{\text{Vol}}^{\text{GARCH}}(\theta) = -\frac{1}{2}\sum_{t}\sum_{i=1}^{k}\left(\log(2\pi) + \log(h_{i,t}) + \frac{r_{i,t}^2}{h_{i,t}}\right). \tag{16}$$

By the same token, if D_t is determined by a CARR specification, then the likelihood function of the volatility term will be modified as:

$$L_{\text{Vol}}^{\text{CARR}}(\theta) = -\frac{1}{2}\sum_{t}\sum_{i=1}^{k}\left(\log(2\pi) + 2\log(\lambda_{i,t}^{*}) + \frac{r_{i,t}^2}{\lambda_{i,t}^{*2}}\right), \tag{17}$$

where $\lambda_{i,t}^{*}$ denotes the conditional standard deviation as computed from a scaled expected range, using the CARR model.

The second part of the likelihood function will be used to estimate the parameters for correlations. As the squared residuals are not dependent on these parameters, they will not appear in the first-order conditions and can be neglected. A simple transformation of the two-stage framework to maximize the likelihood function is achieved. Apparently, $\hat{\theta}_1 = \arg\max\{L_{\text{Vol}}(\theta_1)\}$ and then we extract this value $\hat{\theta}_1$ as given, into the second step, $\max_{\theta_2}\left\{L_{\text{Corr}}\left(\hat{\theta}_1, \theta_2\right)\right\}$. It is shown in Engle and Sheppard (2001) that under some regularity conditions, the condition for consistency will be satisfied. Maximization of Eq. 15 will be a function of the parameter estimates from Eq. 14. These conditions are similar to those given in White (1994), where the asymptotic normality and the consistency of the two-step QMLE estimator are established.

The following GARCH and CARR structures can be performed in the first step of the DCC estimation. As to the GARCH volatility structure, the function form can be illustrated as below:

$$\begin{aligned} r_{i,t} &= \varepsilon_{i,t}\ \varepsilon_{i,t}|I_{t-1} \sim N(0, h_{i,t}), \quad i = 1, 2 \\ h_{i,t} &= \omega_i + \alpha_i\varepsilon_{i,t-1}^2 + \beta_i h_{i,t-1}, \\ z_{i,t}^{\text{GARCH}} &= r_{i,t}\big/\sqrt{h_{i,t}}. \end{aligned} \tag{18}$$

In addition to the original GARCH model embedded in DCC, one can replace it with the CARR framework. CARR is powerful in capturing the volatility process. It is intuitive to put CARR into the first stage, which is particularly convenient for complex dynamic systems in operation. It means the new standardized residuals can be obtained from the CARR model, that is $z_{i,t}^{\text{CARR}} = r_{i,t}/\lambda_{i,t}^{*}$, where $\lambda_{i,t}^{*} = \text{adj}_i \times \lambda_{i,t}$ and $\text{adj}_i = \bar{\sigma}_i\Big/\bar{\hat{\lambda}}_i$. The rescaled expected range $\lambda_{i,t}^{*}$ is used to replace the conditional standard deviation. It is computed by a product of $\lambda_{i,t}$ and the adjusted coefficient adj_i which is the ratio of unconditional standard deviations $\bar{\sigma}_i$ for the return series to the sample mean $\bar{\hat{\lambda}}_i$ of the estimated conditional range.

In performing a comparison of the in-sample data during subsequent empirical analysis of the covariance matrices, several related and conventional models are included—MA100, EWMA[8] with $\lambda = 0.94$, CCC, and DBEKK models.

For robustness of inference, we also perform out-of-sample forecast comparisons. The out-of-sample forecast of the DCC model for correlations can be obtained using the standard forward iterative approach; given T as the sample size, the $T + 1$th observation will be obtained.

At time T, the out-of-sample forecast for conditional correlation in the period $(T + 1)$ is presented by:

$$\begin{bmatrix} q_{11,T+1} & q_{12,T+1} \\ q_{12,T+1} & q_{22,T+1} \end{bmatrix} = (1-a-b)\begin{bmatrix} 1 & \bar{q}_{12} \\ \bar{q}_{12} & 1 \end{bmatrix} + a\begin{bmatrix} z_{1,T}^2 & z_{1,T}z_{2,T} \\ z_{1,T}z_{2,T} & z_{2,T}^2 \end{bmatrix} + b\begin{bmatrix} q_{11,T} & q_{12,T} \\ q_{12,T} & q_{22,T} \end{bmatrix}, \tag{19}$$

The estimated correlation at time $T + 1$ can be calculated as $\rho_{T+1} = q_{12,T+1} \Big/ \sqrt{q_{11,T+1} q_{22,T+1}}$. The out-of-sample prediction for correlation for the period $(T + p)$, where $p \geq 2$, can be expressed as shown below:

$$\begin{bmatrix} q_{11,T+p} & q_{12,T+p} \\ q_{12,T+p} & q_{22,T+p} \end{bmatrix} = (1-a-b)\begin{bmatrix} 1 & \bar{q}_{12} \\ \bar{q}_{12} & 1 \end{bmatrix} + (a+b)\begin{bmatrix} q_{11,T+p-1} & q_{12,T+p-1} \\ q_{12,T+p-1} & q_{22,T+p-1} \end{bmatrix} \tag{20}$$

In addition to range-based and return-based DCC, MA100, EWMA, CCC and DBEKK are introduced for an out-of-sample predictive comparison.[9] For distinguishing the forecasting abilities of these models, as in Taylor (2004), we still use root mean square error (RMSE) and mean absolute error (MAE) as two criterions for comparison.

4 Comparison of various methods for conditional covariance forecasts

The data employed for our empirical study comprise 782 weekly observations on the S&P 500 stock index (S&P 500) futures, and the 10-year treasury bond (Tbond) futures spanning the period from January 6, 1992 to December 29, 2006 (15 years). We retrieve range and return data for the entire period from Datastream.

Figure 1 shows the graphs for close prices (Panel A), returns (Panel B) and ranges (Panel C) of S&P 500 and Tbond futures over the sample period. The descriptive statistics for the returns and ranges of the series are given in Table 1. For the weekly returns and ranges of the S&P 500 and Tbond futures, they are computed by $100 \times \log\left(p_t^{\text{high}} \Big/ p_t^{\text{low}}\right)$ and $100 \times \log\left(p_t^{\text{close}} / p_{t-1}^{\text{close}}\right)$, respectively. Table 1 shows that the means of two futures returns are positive. Both the standard deviations and the means of the ranges indicate that S&P 500 is more volatile than Tbond. For higher moments of the return data, each of them has negative skewness and excess kurtosis. As to the range data, they also have excess kurtosis values, but positive skewness coefficients. These largely contribute to the rejection for the null hypothesis of a normal distribution with the Jarque–Bera statistic.

[8] The estimate of λ is 0.94 approximately for the returns that we adopted in this study.

[9] It is also intuitively clear that the out-of-sample forecasts for the covariance are all constant in the EWMA model.

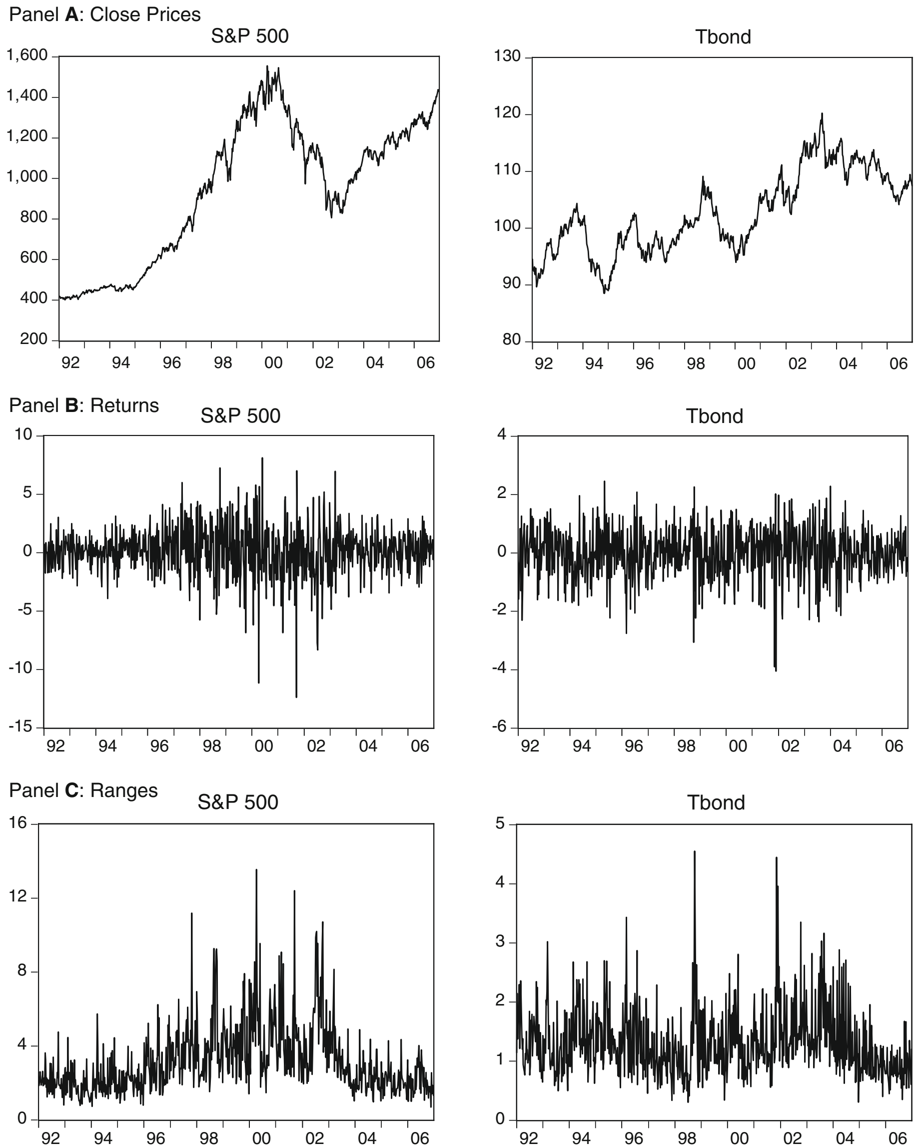

Fig. 1 S&P 500 and Tbond futures weekly closing prices, returns and ranges, 1992–2006. Figure 1 shows the weekly close prices, returns, and ranges of S&P 500 and Tbond Futures over the sample period

4.1 Measured covariances

Like the specific property of volatilities, the covariance matrices are also unobservable. In this work, we use daily data to construct the proxies for the weekly covariances. The purpose behind doing this is to extract the values of the measured covariances (MCOVs), as one kind of benchmark for determining the relative performance of return-based DCC and range-based DCC, for the time being.

Table 1 Summary statistics for the weekly returns and ranges, 1992–2006

	S&P 500		Tbond	
	Return	Range	Return	Range
Mean	0.158	3.134	0.016	1.306
Median	0.224	2.607	0.033	1.194
Maximum	8.124	13.556	2.462	4.552
Minimum	−12.395	0.690	−4.050	0.301
Standard deviation	2.112	1.809	0.855	0.560
Skewness	−0.503	1.756	−0.498	1.390
Kurtosis	6.455	7.232	4.217	6.462
Jarque–Bera	421.317	985.454	80.441	642.367

Daily data are used to build four proxies for covariances, including implied return-based DCC, implied range-based DCC, implied DBEKK, and realized covariances. Initially, the sample period for daily data from 1/6/1992 to 12/29/2006 is extracted. In total, we collect 3,779 daily data for model fitting with return-based DCC, range-based DCC and DBEKK, respectively. Meanwhile, the implied daily covariances are calculated in this stage. Sequentially, it is easy to get the implied weekly estimates for covariance series, followed by the computation below:

$$\mathrm{MCOV}_t^{\mathrm{implied}} = \sum_j \mathrm{cov}_t^j, \tag{21}$$

where cov_t^j denotes implied daily covariance on the jth trading day during the corresponding week t. Ferland and Lalancette (2006) also use this idea to build the weekly covariance and correlation.

As to the realized volatility, its concept has been used productively by French et al. (1987) and Andersen et al. (2001). The realized covariance can be expressed as:

$$\mathrm{MCOV}_t^{\mathrm{realized}} = \sum_j \left(r_{1t}^j \times r_{2t}^j\right), \tag{22}$$

where r_{it}^j denotes return for the asset i on the jth trading day during the corresponding week t.

Checking Fig. 2, we depict the different covariance patterns between S&P 500 and Tbond series for return-based DCC, range-based DCC, DBEKK and the realized pattern, respectively. Some useful insights can be obtained from these figures. It seems to reflect strong interactions around these MCOVs. Furthermore, the realized covariances are more volatile than other implied ones. This shows that the realized pattern is not easy to be fitted. The empirical result also demonstrates this conjecture.

4.2 In-sample forecast comparison

In this section, we present the empirical results for the in-sample forecast comparison of covariances. Mainly, we exhibit the in-sample forecasting ability of return-based DCC, ranged-based DCC and some related models for the purpose of performance comparison. As for the parameters fitted for DCC, we estimate and arrange them in Table 2. Due to the procedure for parameters estimated under the DCC setting, we have to cope with two

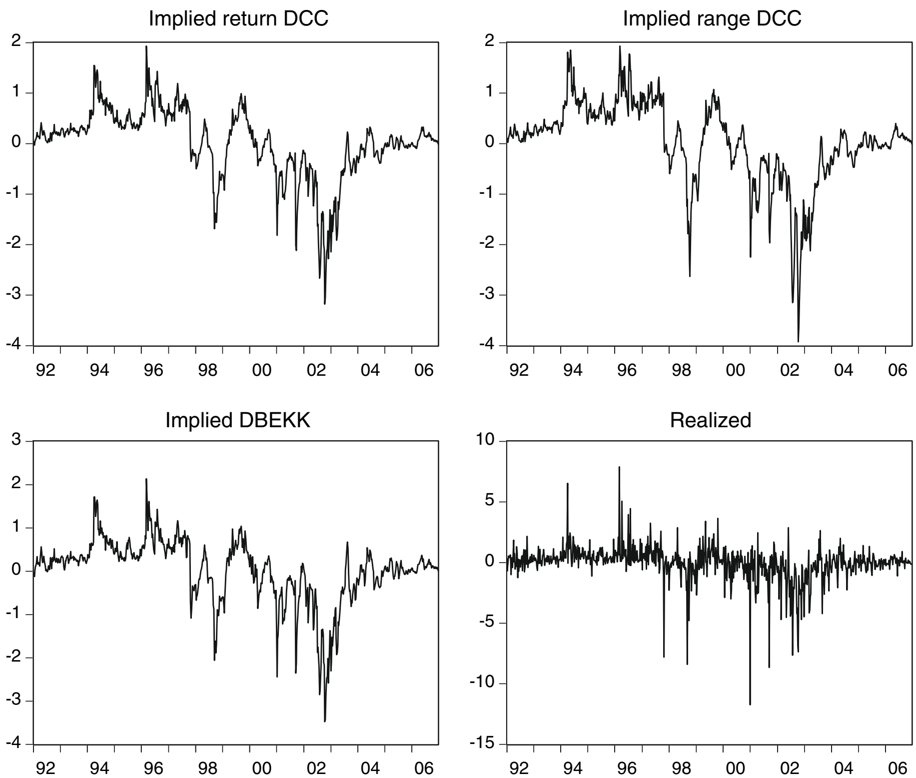

Fig. 2 Four Measured Covariances between S&P 500 and Tbond Futures, 1992–2006. Figure 2 plots the four measured weekly covariances between S&P 500 and Tbond futures. The measured weekly covariances are built from the daily data and are used to be the weekly covariance proxies in our empirical comparison. For getting the implied and realized weekly covariance series, we sum their daily covariances on the trading days of the corresponding week

inherent stages. In the first stage, one can utilize GARCH fitted by returns, or CARR fitted by ranges, with individual assets, for obtaining standardized residuals. Afterwards, we bring these standardized residuals series into the second stage for dynamic conditional correlation estimating.

Table 3 illustrates some brief results of covariances estimated for in-sample prediction, based on different econometrical models that we have mentioned previously. We draw clear inference from Table 3 to the effect that they all appeared to be more accurate in range-based DCC than in the other five models, regardless of what criterion is adopted. This appears to be consistent not only in RMSE but also in MAE. The worst performance in predicting the covariance under the in-sample analysis is the MA100.

Generally speaking, there are no significant differences in covariance forecasting performance between return-based DCC and DBEKK under the in-sample context. In addition, predicting results of CCC perform even worse than EWMA. One reasonable conjecture is that the simple correlation between S&P 500 and Tbond is just an average and rough value. In contrast to the dynamic correlation process generated by other models, the correlations are very volatile in this sample period. For example, see Fig. 3 for an illustration. Looking at the FCOVs generated by return-based DCC and CCC, the only

Table 2 Estimation of bivariate return-based and range-based DCC model using weekly S&P 500 and Tbond futures, 1992–2006

Panel A: Step 1 of DCC estimation

	S&P 500		Tbond	
	GARCH	CARR	GARCH	CARR
$\hat{\omega}$	0.018	0.103	0.027	0.075
	(1.170)	(2.923)	(1.533)	(2.809)
$\hat{\alpha}$	0.048	0.248	0.059	0.157
	(3.744)	(9.090)	(2.046)	(5.208)
$\hat{\beta}$	0.949	0.719	0.903	0.785
	(76.443)	(23.167)	(18.994)	(18.041)

Panel B: Step 2 of DCC estimation

S&P 500 versus Tbond

	Return-based DCC	Range-based DCC
$\hat{a}$	0.034	0.041
	(4.323)	(4.624)
$\hat{b}$	0.960	0.954
	(96.873)	(86.943)

Numbers in parentheses are *t*-values

Table 3 In-sample forecast errors for covariances between the S&P 500 and Tbond futures, 1992–2006

Forecast errors	Base model	Forecasting model					
		MA100	EWMA	Return DCC	Range DCC	CCC	DBEKK
RMSE	Return DCC	0.741	0.420	0.392	0.296	0.693	0.395
	Range DCC	0.861	0.467	0.469	0.344	0.807	0.475
	DBEKK	0.780	0.490	0.469	0.377	0.732	0.457
	Realized	1.515	1.302	1.301	1.261	1.426	1.300
MAE	Return DCC	0.543	0.305	0.274	0.219	0.502	0.261
	Range DCC	0.638	0.324	0.316	0.240	0.588	0.302
	DBEKK	0.566	0.350	0.322	0.270	0.529	0.298
	Realized	0.897	0.789	0.764	0.753	0.842	0.765

difference between them is the estimated correlation process. However, we can find that their covariance process have salient difference. Accordingly, it seems inappropriate to assume that the correlation parameter between different assets is constant over time.

4.3 Out-of-sample forecast comparison

For completeness, we assess the out-of-sample forecasting performance for different models by using RMSE and MAE, discussed in the previous in-sample comparison. Given that the data set contains a total of 782 usable observations, it is possible to use a holdback period of observations. This way, there are 521 observations (10 years) in each estimated

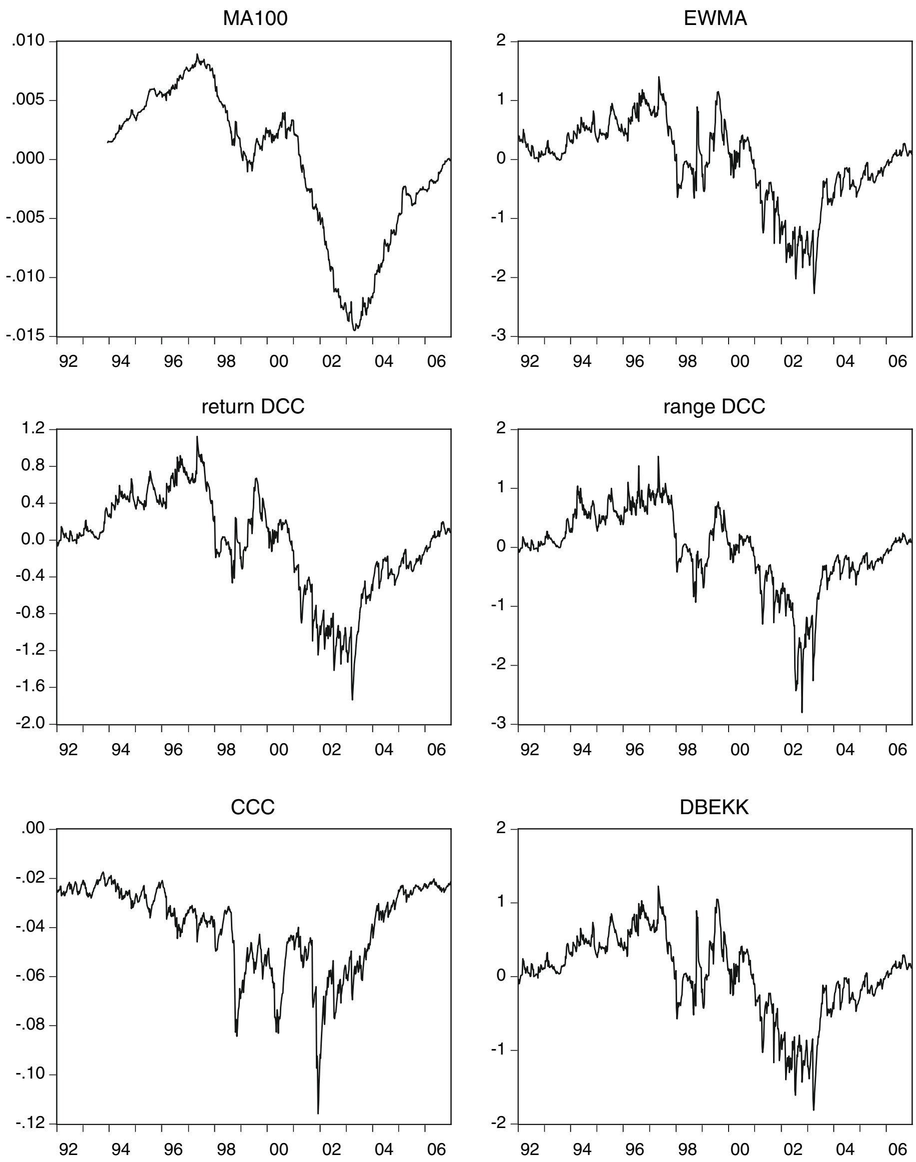

Fig. 3 In-sample forecasting covariances between S&P 500 and Tbond Futures for six models, 1992–2006. Figure 3 provides the fitted covariances between S&P 500 and Tbond futures for six different models. We lost some former values in MA100. This is because the first estimated value must be derived by the former 100 observation. The covariances of CCC are all negative and quite smaller than ones of DCC. The reasonable explanation is its negative and small constant correlation (−0.0229)

model and 258 out-of-sample forecasting values for comparison. Here, the rolling sample approach for out-of-sample measurement is adopted and the first forecasted value for one period ahead forecast respectively occurs on the week of January 4, 2002. Table 4 reports one, two, and four periods ahead of out-of-sample forecasting results for covariance.

Table 4 One, two, and four periods ahead out-of-sample forecast errors for covariances between the S&P 500 and Tbond futures, 1992–2006

	Base model	Forecasting model					
		MA100	EWMA	Return DCC	Range DCC	CCC	DBEKK
Panel A: One period ahead forecast errors							
RMSE	Return DCC	0.823	0.439	0.439	0.283	0.883	0.596
	Range DCC	0.935	0.469	0.495	0.301	0.994	0.684
	DBEKK	0.875	0.519	0.528	0.354	0.935	0.655
	Realized	1.508	1.254	1.285	1.183	1.556	1.366
MAE	Return DCC	0.495	0.331	0.320	0.219	0.557	0.344
	Range DCC	0.562	0.323	0.322	0.223	0.622	0.395
	DBEKK	0.523	0.389	0.384	0.279	0.596	0.385
	Realized	0.877	0.807	0.800	0.756	0.923	0.805
Panel B: Two periods ahead forecast errors							
RMSE	Return DCC	0.823	0.454	0.456	0.312	0.885	0.638
	Range DCC	0.935	0.481	0.511	0.344	0.996	0.729
	DBEKK	0.875	0.537	0.548	0.387	0.938	0.701
	Realized	1.507	1.263	1.294	1.214	1.557	1.403
MAE	Return DCC	0.495	0.342	0.336	0.237	0.558	0.366
	Range DCC	0.561	0.336	0.336	0.242	0.622	0.415
	DBEKK	0.523	0.403	0.403	0.302	0.598	0.408
	Realized	0.875	0.816	0.813	0.771	0.921	0.843
Panel C: Four periods ahead forecast errors							
RMSE	Return DCC	0.823	0.482	0.487	0.380	0.889	0.656
	Range DCC	0.935	0.514	0.546	0.432	0.999	0.748
	DBEKK	0.875	0.567	0.581	0.461	0.942	0.725
	Realized	1.506	1.281	1.312	1.252	1.558	1.411
MAE	Return DCC	0.494	0.359	0.357	0.285	0.560	0.392
	Range DCC	0.559	0.360	0.357	0.291	0.623	0.434
	DBEKK	0.523	0.425	0.428	0.351	0.601	0.441
	Realized	0.872	0.826	0.822	0.792	0.916	0.846

We obtain a consistent inference for covariance prediction's performance based on different competitive models. All of the inferences demonstrate an overwhelming phenomenon, namely, that the range-based DCC approach dominates other methods in accuracy from out-of-sample forecasting. Various forecasting results for covariance with different periods ahead are presented in Table 4. Except for MA100 in the forecasting models, the results in Table 4 appear to show a trend that the forecasting errors are proportionate to the forecasted periods. One period ahead out-of-sample forecasting covariances of all compared models are given in Fig. 4.

Exploring other characteristics of out-of-sample forecasting, CCC, among these competitive models is the worst one, even inferior than MA100. One possible explanation for this is that the relationship between S&P 500 and Tbond in the post-sample has structural change. Unlike previous results in the in-sample comparison, however, return-based DCC performs significantly better than DBEKK. With the exception of range-based DCC, it is

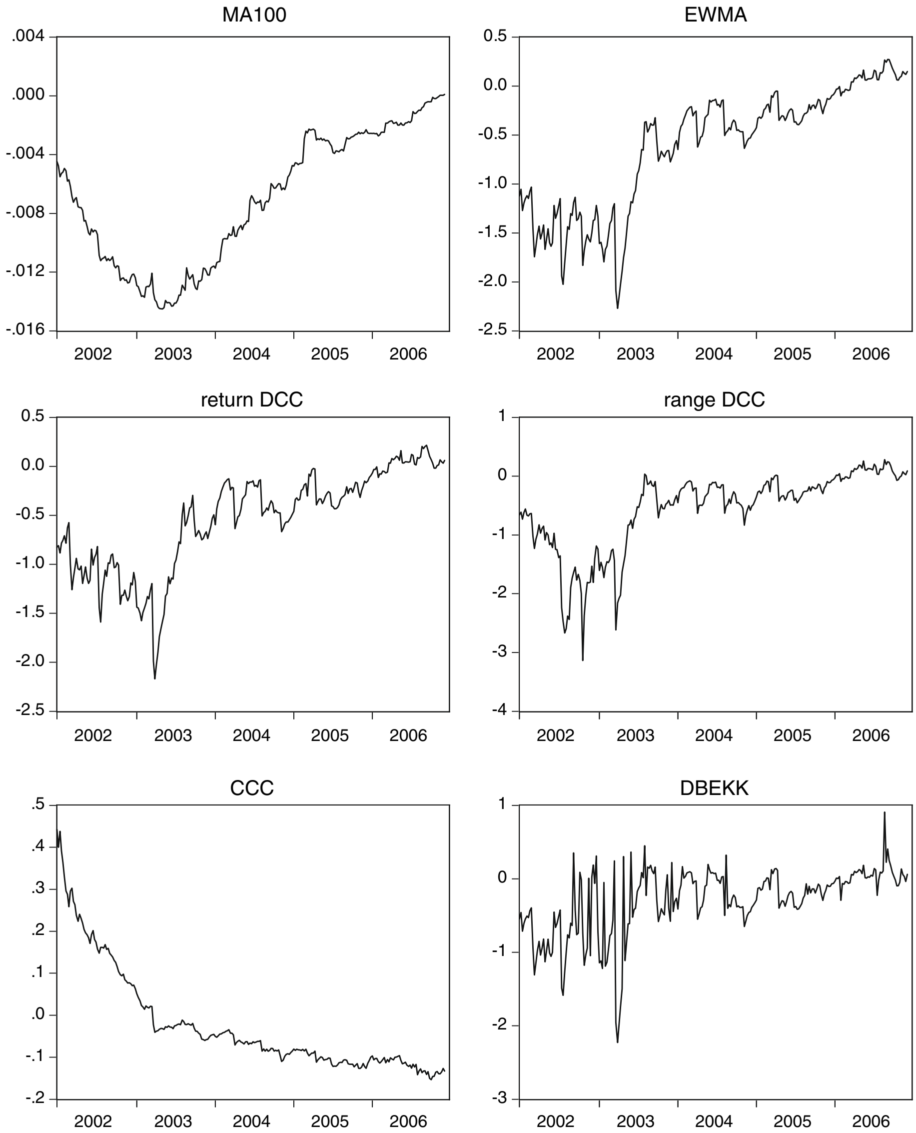

Fig. 4 One period ahead out-of-sample forecasting covariances between S&P 500 and Tbond Futures for six models, 1992–2006. Figure 4 shows one period ahead out-of-sample forecasting result of six different models. The rolling sample approach is adopted for each model with 521 observations (10 years). The first forecasted value for one period ahead forecast respectively, occurs the week of January 4, 2002. In all, we have 258 out-of-sample forecasting covariances

surprising that EWMA, holding constant post-sample covariance, even has outstanding performance compared to those of the other models.

Moreover, we can take another look at the out-of-sample forecast comparison. Table 5 shows the simple correlations between MCOVs and FCOVs for one, two, and four periods ahead covariance forecasts. The results show a clear and strong relationship between the

Table 5 Simple correlations between MCOVs and FCOVs for one, two, and four periods ahead out-of-sample covariance forecasts, 1992–2006

		FCOVs					
		MA100	EWMA	Return DCC	Range DCC	CCC	DBEKK
Panel A: Correlations for one period ahead forecast							
MCOVs	Return DCC	0.646	0.836	0.815	0.941	−0.543	0.579
	Range DCC	0.660	0.833	0.815	0.940	−0.537	0.578
	DBEKK	0.600	0.803	0.778	0.922	−0.514	0.568
	Realized	0.340	0.471	0.428	0.557	−0.408	0.310
Panel B: Correlations for two periods ahead forecast							
MCOVs	Return DCC	0.635	0.821	0.794	0.922	−0.571	0.504
	Range DCC	0.651	0.822	0.799	0.916	−0.563	0.502
	DBEKK	0.588	0.783	0.751	0.898	−0.545	0.485
	Realized	0.334	0.458	0.411	0.520	−0.430	0.236
Panel C: Correlations for four periods ahead forecast							
MCOVs	Return DCC	0.616	0.793	0.754	0.867	−0.624	0.469
	Range DCC	0.631	0.791	0.757	0.855	−0.612	0.474
	DBEKK	0.567	0.749	0.703	0.836	−0.604	0.436
	Realized	0.321	0.433	0.380	0.470	−0.455	0.220

FCOV built by range-based DCC and MCOVs. The correlation coefficients in the CCC case are negative and all are lower than −0.4. It is clear that the assumption of the constant correlation may cause the serious influence. In general, the correlations show a declining trend along with forecasting horizons.

Table 5 reports the simple correlations between MCOVs and FCOVs for one, two and four periods ahead out-of-sample covariance forecasts. MCOV represents the covariance proxy derived from the base model. FCOV is the forecast covariance for the forecasting model. Daily data are used to compute the weekly implied MCOVs (Return DCC. Range DCC, and DBEKK), and the realized MCOV (Realized). MA100, EWMA, Return DCC, Range DCC, CCC and DBEKK, are estimated from the weekly data to build FCOVs. There are 521 observations (10 years) in each of the estimated models. Additionally, the rolling sample method provides 258 forecasting values for every out-of-sample comparison. The first forecasted values for one, two, and four periods ahead forecasts respectively occur the week of January 4, 11, and 25 in 2002.

In view of in-sample and out-of-sample empirical results, we can not clearly put all forecasting models in a proper order. However, it is undoubted that the range-based DCC model possesses the optimal forecasting power in covariance.

4.4 A simple application in futures hedging

At last, we provide a simple application for the range-based DCC model. The range model is used to calculate the optimal hedge ratio for coffee futures,[10] ranging from January 2, 1997 to December 29, 2006 (10 years, 521 observations), and is compared with Naïve,[11]

[10] The coffee data is obtained from Datastream.

[11] Naïve here is the short hedge with selling one unit futures.

Table 6 Comparisons of hedging performance, 1997–2006

Hedging Portfolio variances for different strategies				
Naïve	OLS	CCC	Return-based DCC	Range-based DCC
13.290	9.936	8.278	8.244	8.118

OLS, return-based CCC and return-based DCC models. Here we use the maximum and minimum price respectively, among the daily close prices in a week to replace the real intraday high and low prices.

The hedging portfolio variance can be calculated by $\mathrm{Var}\left(r_{S,t} - h_t^* r_{F,t}\right)$, where $r_{S,t}$ and $r_{F,t}$ are the spot and futures returns of coffee, respectively. The optimal hedge ratios from different strategies are denoted by h_t^*, which is the ratio of the covariance between spot and futures returns to the futures variance.

The hedging performance is shown in Table 6. For coffee futures hedging, the dynamic hedging methods obviously perform better than the static hedging strategies. One of reasons for this result is the commodity we choose, indicating the dynamic relationship between its spot and futures returns. Even though the real range data are not available, the range-based DCC model still performs better than the return-based CCC and DCC models. It points to new possibilities for future research.

5 Conclusions

In this paper, we propose a new estimator of the time-varying covariance matrices, utilizing the range data that combines the CARR model with the framework of the DCC model. The advantage of this range-based DCC model, in terms of its forecasting ability to outperform the standard return-based DCC model, hinges on the relative efficiency of the range data over the return data in estimating volatilities. Using weekly futures data of S&P 500 and Tbond, we find a consistent result that the range-based DCC model outperforms the return-based models in estimating and forecasting covariance matrices for both in-sample and out-of-sample analysis.

In addition to using conventional realized covariance for the purpose of comparison, we introduce the viewpoint of implied covariance, which is derived from return-based DCC, range-based DCC and DBEKK for benchmarking robustness. Nonetheless, no matter what realized covariance or implied covariances are adopted for comparison, we obtain a consistent conclusion that the range-based DCC approach is the best one for predicting covariance process.

Although we only applied this estimator to the bivariate systems, it can also be applied to larger systems in a manner which is similar to the application of the DCC model structures, having already been demonstrated in Engle and Sheppard (2001). It will be surely useful to utilize more diagnostic statistics or to test based on value-at-risk calculations as proposed by Engle and Manganelli (2004) in future research. Other applications such as estimating the optimal portfolio weighting matrices and calculating the dynamic hedge ratio in the futures market will also bear fruit.

An interesting issue is to investigate how data frequency affects the accuracy of the covariance estimate. In a study of the "realized range" as a volatility estimator, Martens and van Dijk (2007) reports that the accuracy of the range-based estimator improves as the measuring frequency becomes higher. This result holds as long as there is no bid-ask bounce. It will be useful to exam whether such a result carries out in the estimation of the covariance using the range-based estimator such as ours. We leave this as a topic for future study.

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