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Outlier Detection in the Lognormal Logarithmic Conditional Autoregressive Range Model

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Abstract

An outlier detection procedure in the lognormal logarithmic conditional autoregressive range (lognormal Log-CARR) model is proposed. The proposed test statistic is demonstrated to be well-sized and to have good power using Monte Carlo simulations. Furthermore, the outlier detection procedure suffers less from the masking effect caused by multiple outliers. The results of an empirical investigation show that the proposed method can effectively detect volatility outliers and improve forecasting accuracy.

I. Introduction

Forecasting return volatility is a major concern in financial analysis with regard to asset pricing, asset allocation and risk management. In addition to suitable volatility model and volatility measure, quality data are also essential to obtain good volatility forecasts. In practice, the observed data could present abnormal observations due to impacts of unusual events, such as financial crises or critical announcements. Consequently, the accuracy of parameter estimates and volatility forecasting cannot be guaranteed in the presence of these extreme observations, i.e. volatility outliers.[1]

The aim of this paper is thus to develop a volatility outlier detection procedure and to investigate whether the accuracy of volatility forecasts is influenced by the presence of outliers. Specifically, influences of additive outliers (AO) and innovative outliers (IO) will be examined in this paper.

The volatility proxy used in this paper is the price range, which is defined as the differences between the highest and lowest logarithmic security prices during a predefined

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[1]The outliers are an important issue in time series models, such as ARMA models (Fox, 1972; Peña, 1990; Chen and Liu, 1993) and the GARCH-family models (Franses and Ghijsels, 1999; van Dijk, Franses and Lucas, 1999; Park, 2002; Charles and Darne, 2005, 2006; Charles, 2008; Muler and Yohai, 2002, 2008).

time interval (e.g. an hourly, daily, weekly or monthly frequency). It has been shown in the literature to be a more effective volatility estimator than return-based ones (e.g. Parkinson, 1980; Andersen and Bollerslev, 1998; Alizadeh, Brandt and Diebold, 2002; Brandt and Jones, 2006).

We then adopt a specific price range model, the lognormal logarithmic conditional autoregressive range model[2] (hereafter, lognormal Log-CARR), for examining possible volatility outliers. The conditional autoregressive range (CARR) model introduced by Chou (2005) aims to incorporate the volatility clustering property and to provide more effective volatility estimation compared with the return-based volatility models, such as the GARCH-type ones. On the other hand, the ARMA-form of the lognormal Log-CARR model enables us to achieve volatility outlier detection by modifying the detection procedures used in the ARMA model of Chen and Liu (1993) to avoid the possible masking effect that can arise in volatility outlier detection. The proposed test statistics are demonstrated to perform very well both in size and power simulations. Furthermore, we apply the proposed outlier detection procedure to three stock price range series and demonstrate its effectiveness.

The remainder of this paper is organized as follows: section II introduces the lognormal Log-CARR model. Section III presents the proposed outlier detection method in the lognormal Log-CARR model, while section IV examines the sampling properties of the test statistic and the performance of the outlier detection method. An empirical application of the proposed method to three stock price range series is given in section V, and finally, section VI presents the conclusions of this paper.

II. The lognormal Log-CARR model and statistical properties

The CARR model was first proposed by Chou (2005) to examine the dynamic structure of range series. Similar to the Logarithmic Autoregressive Conditional Duration (Log-ACD) model of Bauwens and Giot (2000) which is an extension of the ACD model of Engle and Russell (1998), the Log-CARR model proposed also relaxes the positivity restrictions on parameters of the conditional mean function.

The Log-ACD model

The Log-ACD model of Bauwens and Giot (2000) is defined as follows:[3]

$$x_i = \exp(\phi_i) v_i, \tag{1}$$

$$\phi_i = \omega + \sum_{j=1}^{p} \alpha_j \ln(x_{i-j}) + \sum_{l=1}^{q} \beta_l \phi_{i-l}, \tag{2}$$

where x_i denotes the duration, which can be the time elapsed between two consecutive trades, two consecutive bid-ask quotes, etc. v_i is usually assumed to follow the distributions with positive supports, such as exponential and Weibull distributions. ϕ_i is the func-

[2]The lognormal Log-CARR model was adopted in Chiang and Wang (2011) as a basic framework to examine the volatility contagion across stock markets.

[3]This model is named the Log-ACD$_1$ model in Bauwens and Giot (2000). Readers can also see the survey paper of Pacurar (2008) for details.

tional form of conditional logarithmic duration, which relaxes the positivity restrictions on parameters of the original ACD model.

The lognormal Log-CARR model

The lognormal Log-CARR model for analyzing the dynamics of price ranges is motivated by the Log-ACD model with a lognormal error distribution setting.

Let P_t be the logarithmic price of an asset at time t. The observed range is defined as $R_t = \max\{P_\tau\} - \min\{P_\tau\}$, where $\tau \in [t-1, t]$. Therefore, the range R_t can be modelled as the Log-CARR(p,q) model:

$$R_t = \exp(\lambda_t)\varepsilon_t, \tag{3}$$

$$\lambda_t = \omega + \sum_{i=1}^{p} \alpha_i \ln(R_{t-i}) + \sum_{j=1}^{q} \beta_j \lambda_{t-j}, \tag{4}$$

where

$$\varepsilon_t | I_{t-1} \overset{iid}{\sim} lognormal(-\sigma^2/2, \sigma^2), \quad \forall t = 1, 2, \ldots, T,$$
$$E(\varepsilon_t | I_{t-1}) = 1, \quad \mathrm{var}(\varepsilon_t | I_{t-1}) = \exp(\sigma^2) - 1, \text{and}$$
$$I_t = \{R_t, R_{t-1}, \ldots, R_1\} \text{ is the information set available up to time } t.$$

The conditional expectation of range at time t based on the information set I_{t-1} is written as $E(R_t|I_{t-1}) = \exp(\lambda_t)$. The exponential transformation of the conditional mean function ensures the positivity of the range regardless of the sign of λ_t. Taking the logarithm of both sides of equation (1), and letting $y_t = \ln(R_t)$, $\psi_t = \lambda_t - \sigma^2/2$, and $\eta_t = \ln(\varepsilon_t) + \sigma^2/2$, the logarithmic range can be expressed as follows:

$$y_t = \lambda_t + \ln(\varepsilon_t) = \psi_t + \eta_t. \tag{5}$$

then equation (4) can be rearranged as

$$\psi_t = \varpi + \sum_{i=1}^{p} \alpha_i y_{t-i} + \sum_{j=1}^{q} \beta_j \psi_{t-j}, \text{ or} \tag{6}$$

$$y_t = \varpi + \sum_{i=1}^{r} (\alpha_i + \beta_i) y_{t-i} - \sum_{j=1}^{q} \beta_j \eta_{t-j} + \eta_t, \tag{7}$$

where $r = \max(p, q)$, $\varpi = \omega + \frac{\sigma^2}{2} \sum_{j=1}^{q} \beta_j - \frac{\sigma^2}{2}$.

Equation (7) is the ARMA(r,q) type of logarithmic range y_t with a normal error term $\eta_t | I_{t-1} \overset{\text{iid}}{\sim} N(0, \sigma^2)$. The log-likelihood function of the lognormal Log-CARR model can be written as follows:

$$l(\boldsymbol{\theta}; R_t) = \sum_{t=1}^{T} l_t(\boldsymbol{\theta}; R_t), \tag{8}$$

$$l_t(\boldsymbol{\theta}; R_t) = -\frac{1}{2} \ln(\sigma^2) - \frac{1}{2\sigma^2} \left(\ln\ R_t - \psi_t\right)^2, \tag{9}$$

where $\boldsymbol{\theta} = (\boldsymbol{\xi}', \sigma^2)'$, $\boldsymbol{\xi} = (\varpi, \boldsymbol{\alpha}', \boldsymbol{\beta}')'$, $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \ldots, \alpha_p)'$, $\boldsymbol{\beta} = (\beta_1, \beta_2, \ldots, \beta_q)'$.

As the Log-CARR model can be rearranged into the ARMA model, as shown in equation (7), for the covariance stationarity of y_t, the roots of $z_1,\ldots,z_r$ of the equation $1-\sum_{i=1}^{r}(\alpha_i+\beta_i)z^{-i}=0$ all lie inside the unit circle.[4] Additionally, the analytical expressions of unconditional moments and the consistency and asymptotic normality of the quasi maximum likelihood estimator of the lognormal Log-CARR model defined in equations (3) and (4) can be also derived using the similar proof procedures in Bauwens, Galli and Giot (2008) and Allen *et al.* (2008) respectively.

III. Outlier detection and estimation method

This section first constructs the outlier detection method to detect the individual AO and IO in the Log-CARR model. The method for dealing with multiple outliers where the number of outliers is more than one is then presented.

Individual outlier detection

As the Log-CARR model can be expressed as the ARMA form as in equation (7), the similar outlier detection approach used in Chen and Liu (1993) can also be adapted and then applied to the Log-CARR model. Consider equation (7) and let

$$\phi(L)=1-\sum_{i=1}^{r}(\alpha_i+\beta_i)L^i,$$
$$\theta(L)=1-\sum_{j=1}^{q}\beta_jL^j, \text{ and}$$
$$\pi(L)=\phi(L)/\theta(L)=1-\pi_1L-\pi_2L^2-\cdots.$$

Assuming that $\theta(L)$ is invertible, then $\pi(L)$ can be defined in an infinite-order polynomial in the lag operator, L.

For simplicity, by ignoring the intercept term ϖ,[5] equation (7) can be formulated as

$$\phi(L)y_t=\theta(L)\eta_t, \quad \text{or} \quad \pi(L)y_t=\eta_t. \tag{10}$$

Let $w_t, t=1,2,\ldots,T$, be a price range series of R_t with an AO or IO at time t_0, and $z_t, t=1,2,\ldots,T$, be the logarithmic range series of w_t, $\ln(w_t)$. As the logarithmic transformation is monotonic, the outlier in z_t is still invariant after a scale change. Assuming that the impact value of an outlier in z_t is k, then the AO and IO are defined as follows:

$$\text{AO: } z_t=y_t+kI_t(t_0), \tag{11}$$

$$\text{IO: } z_t=y_t+\frac{k}{\pi(L)}I_t(t_0), \tag{12}$$

[4] Francq, Wintenberger and Zakoian (2012) show that the roots requirement stated here is equivalent to $|\alpha+\beta|<1$ in log-GARCH model when $p=q=1$, which corresponds to the log-CARR model in this paper and log-ACD$_1$ model in Bauwens and Giot (2000) and Pacurar (2008).

[5] The relative positions of each observation are unchanged when the intercept term is ignored.

where

$$I_t(t_0)=\begin{cases}1 & t=t_0\\ 0 & t\neq t_0\end{cases}$$

Multiply both sides of equations (11) and (12) by $\pi(L)$ and let $e_t, t=1,2,\ldots,T,$ be the error term of z_t in which $e_t=\pi(L)z_t$. Then, z_t with an AO or IO can be modelled as

$$\text{AO: } e_t=\eta_t+k\pi(L)I_t(t_0), \tag{13}$$

$$\text{IO: } e_t=\eta_t+kI_t(t_0), \tag{14}$$

and can also be written as

$$e_t=ku_t+\eta_t, \tag{15}$$

where

$$\text{AO: } u_t=\begin{cases}0, & \text{if} \quad t<t_0\\ 1, & \text{if} \quad t=t_0,\\ -\pi_{t-t_0} & \text{if} \quad t>t_0\end{cases} \quad \text{and}$$

$$\text{IO: } u_t=\begin{cases}0, & \text{if} \quad t<t_0\\ 1, & \text{if} \quad t=t_0\\ 0, & \text{if} \quad t>t_0.\end{cases}$$

The least squares estimator of the impact value k for an outlier at time $t=t_0$ can be obtained as

$$\text{AO: } \tilde{k}_{\text{AO}}(t_0)=\sum_{t=t_0}^{T}\hat{e}_t\hat{u}_t \Big/ \sum_{t=t_0}^{T}\hat{u}_t^2, \tag{16}$$

$$\text{IO: } \tilde{k}_{\text{IO}}(t_0)=\hat{e}_{t_0}, \tag{17}$$

where $\hat{e}_t$ and $\hat{u}_t$ denote the estimates of error terms e_t and u_t, respectively, $\forall t=1,\ldots,T$. The standardized $\tilde{k}(t_0)$ denoted as $\tilde{\tau}$ becomes

$$\text{AO: } \tilde{\tau}_{\text{AO}}(t_0)=\left(\tilde{k}(t_0)\Big/ s(\hat{\eta}_t)\right)\times\sqrt{\sum_{t=t_0}^{T}\hat{u}_t^2}, \tag{18}$$

$$\text{IO: } \tilde{\tau}_{\text{IO}}(t_0)=\tilde{k}(t_0)\Big/ s(\hat{\eta}_t), \tag{19}$$

where $s(\hat{\eta}_t)$ is the standard deviation of the residuals $\hat{\eta}_t$, $t=1,\ldots,T$, calculated by the omit-one method.[6] The statistical distribution property of $\tilde{\tau}_{\text{AO}}(t_0)$ ($\tilde{\tau}_{\text{IO}}(t_0)$) is stated in the following proposition:

Proposition 1. Under the assumption that ε_t in equation (3) are independent and identically distributed as a lognormal distribution and logarithmic range y_t in equation (7)

[6]The omit-one method refers to the calculation of residual standard deviation by excluding the residual at time t when computing test statistics of equations (18) and (19) at time t. Chen and Liu (1993) showed that the omit-one method is able to alleviate the influence of potential outliers.

is covariance stationary, $\widetilde{\tau}_{\mathrm{AO}}(t_0)$ $(\widetilde{\tau}_{\mathrm{IO}}(t_0))$, $\forall t_0 = 1, 2, \ldots, T$ follow an asymptotic standard normal distribution.

Proof. See Appendix A.

Proposition 1 states that $\widetilde{\tau}_{\mathrm{AO}}(t_0)$ $(\widetilde{\tau}_{\mathrm{IO}}(t_0))$ has a limiting distribution which is a standard normal distribution at each time t_0. Fundamentally, the maximum of $\widetilde{\tau}_{\mathrm{AO}}(t_0)$ $(\widetilde{\tau}_{\mathrm{IO}}(t_0))$ is the major candidate of the test statistics for the price range outliers. However, the maximum of $\widetilde{\tau}_{\mathrm{AO}}(t_0)$ $(\widetilde{\tau}_{\mathrm{IO}}(t_0))$ is not a pivotal quantity, which is not suitable for the hypothesis testing in practice. In order to construct a suitable maximum order statistic for the price range outliers, the test statistic considered in this paper is defined as follows:

$$Z_i(t_0) = \frac{\max(\widetilde{\tau}_i(1), \widetilde{\tau}_i(2), \ldots, \widetilde{\tau}_i(T)) - b_T}{a_T}, \tag{20}$$

where $\left\{t_0 : \widetilde{\tau}_i(t_0) = \max(\widetilde{\tau}_i(1), \widetilde{\tau}_i(2), \ldots, \widetilde{\tau}_i(T))\right\}$, $i = \mathrm{AO}$ or IO. $Z_i(t_0)$ is the maximum of $\widetilde{\tau}_{\mathrm{AO}}(t)$ $(\widetilde{\tau}_{\mathrm{IO}}(t))$ statistic with centralization b_T and scale a_T modifications. According to Theorem 1.1.3 in de Haan and Ferreira (2006), the limiting distribution of $Z_i(t_0)$ can be established in the following proposition:

Proposition 2. The limiting distributions of $Z_i(t_0)$ can be approximated by $G(z) = \Pr(Z_i \leqslant z) \to \exp(-\exp(-z))$ with $b_T = (2\log(T) - \log(\log(T)) - \log(4\pi))^{0.5}$ and $a_T = \frac{1}{b_T}$ under the null of no outliers, where G denotes the cumulative density function of $Z_i(t_0)$.

Proof. See Appendix A.

Remark 1. Let C_α be the critical value at the significance level α, then $\alpha = 1 - (G(C_\alpha))$, which means that $C_\alpha = -\log(-\log(1-\alpha))$. Once $Z_i(t_0) > C_\alpha$, the observation at time t_0 is regarded as an AO if i = AO, or regarded as an IO if i = IO.

Joint detection in the presence of multiple outliers

Suppose z_t is subject to m interventions at time periods of $t_1, t_2, \ldots, t_m$. Therefore, we can express z_t as

$$z_t = \sum_{j=1}^{m} k_j L_j(L) I_t(t_j) + y_t, \tag{21}$$

where

$$I_t(t_j) = \begin{cases} 1 & t = t_j \\ 0 & t \neq t_j \end{cases} \quad \text{and} \quad \pi(L) y_t = \eta_t.$$

Multiply both sides of equation (21) by $\pi(L)$ and let $e_t, t = 1, 2, \ldots, T$, be the error term of z_t in which $e_t = \pi(L) z_t$. Therefore, the error terms e_t of z_t can be expressed as

$$e_t = \sum_{j=1}^{m} k_j u_j + \eta_t, \tag{22}$$

where

$$\text{AO: } u_j = \begin{cases} 0, & \text{if } t < t_j \\ 1, & \text{if } t = t_j \text{ and} \\ -\pi_{t-t_j}, & \text{if } t > t_j \end{cases}$$

$$\text{IO: } u_j = \begin{cases} 0, & \text{if } t < t_j \\ 1, & \text{if } t = t_j . \\ 0, & \text{if } t > t_j \end{cases}$$

Equation (22) is a multiple regression with m independent variables which correspond to m price range outliers. Therefore, the impact of multiple price range outliers in the Log-CARR model can then be captured using equation (22).[7]

Detection and estimation procedure

An iterative procedure that is a modification of the outlier detection method developed by Chen and Liu (1993) is presented in this section. This modified detection procedure has four stages.[8] Stage 1 identifies each outlier and removes their effects on the observed ranges. Stage 2 jointly estimates outlier effects and model parameters in the Log-CARR model using the Log-CARR model in equations (3) and (4) and the multiple regression model in equation (22). Some outliers identified in stage 1 can be regarded as normal observations during this stage, and new parameter estimates are calculated using newly identified outliers. In stage 3, the steps in the first and second stages are iterated with new residual series until all parameter estimates are invariant. Stage 4 serves as a safeguard against potential outliers not being detected in the first three stages due to a severe masking effect.

Stage 1: Initial parameter estimation and individual outlier detection

Step 1.1: Estimate the parameters in equation (4) as initial values. Then, compute the logarithmic residuals ($\hat{e}_t, t = 1, 2, \ldots, T$) of the Log-CARR model and calculate $\hat{\pi}(L)$ with the following formula:

$$\hat{\pi}(L) = \left(1 - \sum_{i=1}^{r} (\hat{\alpha}_i + \hat{\beta}_i) L^i\right) \Bigg/ \left(1 - \sum_{j=1}^{q} \hat{\beta}_j L^j\right). \tag{23}$$

Step 1.2: Calculate $\tilde{\tau}_{\mathrm{AO}}(t)$ and $\tilde{\tau}_{\mathrm{IO}}(t)$ for $t = 1, 2, \ldots, T$, using equations (18) and (19). Let $\xi_t = \max(\tilde{\tau}_{\mathrm{AO}}(t), \tilde{\tau}_{\mathrm{IO}}(t))$. If $\max(\xi_t) = \tilde{\tau}_i(t_0) > C_\alpha$, $i =$ AO or IO, there is an outlier of type i at time t_0. The logarithmic observation at t_0 is then modified as follows:

$$\text{AO: } y_{t_0} = z_{t_0} - \tilde{k}(t_0), \quad \text{or} \quad \text{IO: } y_{t_0} = z_{t_0} - \tilde{k}(t_0)/\hat{\pi}(L).$$

The new residuals are calculated using equation (16) and a new omit-one estimate of $s(\hat{\eta}_t)$ is then computed from the modified residuals.

Step 1.3: Re-compute $\tilde{\tau}_{\mathrm{AO}}(t)$ and $\tilde{\tau}_{\mathrm{IO}}(t)$ based on the modified residuals and $s(\hat{\eta}_t)$, and then repeat step 1.2 until all outliers are identified. The initial estimates for $\hat{\pi}(L)$ remain unchanged. When no outliers are found during the first iteration, the procedure stops.

Step 1.4: Suppose step 1.3 is terminated and m outliers have been tentatively identified at time $t_1, t_2, \ldots, t_m$. Then go back to step 1.1 to revise the parameter estimates. Repeat steps 1.2 and 1.3 until no additional outliers are detected; then go to stage 2.

[7] Chen and Liu (1993) indicated that the joint detection of outliers is much more appropriate than the sequential detection due to biased estimates of adjacent outliers under the sequential detection method.

[8] When applying the iterative procedure described here to real data, the sample size should not be too small in order to make sure related statistics approach their respective asymptotic distributions.

Stage 2: Joint estimation of outlier effects and parameters

Step 2.1: The outlier effect $k_j's$ at times $t_1, t_2, \ldots, t_m$ can be estimated jointly using the multiple regression model described in equation (22).

Step 2.2: Compute $\widetilde{\tau}_j$ statistics, $(j = 1, 2, \ldots, m)$. If $\min_j(\widetilde{\tau}_j) = \widetilde{\tau}_v \leqslant C$, then remove the vth observation from the candidate pool and go to step 2.1 with the remaining $m - 1$ outliers; otherwise, go to step 2.3.

Step 2.3: Obtain the adjusted series by adjusting the outlier effects using the most recent estimates of $k_j's$ in step 2.1.

Step 2.4: Estimate the model parameters based on the adjusted series obtained during step 2.3. When the relative change of the standard error of residuals from the previous estimate is larger than a tolerance value of ε, say, 0.001, go to step 2.1 for further iterations; otherwise, go to stage 3.

Stage 3: Detection of outliers based on the new parameter estimates

Compute the residuals by filtering the original series based on the parameter estimates obtained during step 2.4. Iterate the steps in stages 1 and 2 until the parameter estimates in the Log-CARR model are invariant under the preset tolerance value.

Stage 4: Detection of outliers based on the outlier-adjusted series

Perform stages 1–3 iteratively for the outlier-adjusted range series obtained from the previous iteration of stages 1–3 until no more outliers are detected. By doing this, the method is able to detect the outliers masked by those detected previously, and thus alleviate the masking effect.

IV. Monte Carlo simulations

In this section, we investigate the size properties of the test statistic in section ‘Size simulations’, and the power performance for the outlier detection procedure in section ‘Power performance’.

Size simulations

The Log-CARR(1,1) models (equations (3) and (4)) with the parameter settings in Table 1 are used as the data generating processes in the size simulations.[9] Ten thousand samples are generated for each model. Each replication has a sample size of 1,000.[10] The results in Figure 1 show that the sizes are close to the corresponding nominal sizes, which indicates that the test statistic is well-sized for a wide variety of parameter settings of the lognormal Log-CARR model.

[9] For each replication, parameters are estimated and the test statistic is calculated using the estimated residuals. The same procedure is applied to power performance evaluation in next two subsections.

[10] We also perform size and power simulations for the sample sizes of 500 and 2,000. The conclusions are quite similar.

TABLE 1

The parameter settings for size simulations

Model	α	β	σ^2	*Model*	α	β	σ^2	*Model*	α	β	σ^2
1	0.05	0.70	0.10	12	0.05	0.70	0.20	23	0.05	0.70	0.50
2	0.05	0.75	0.10	13	0.05	0.75	0.20	24	0.05	0.75	0.50
3	0.05	0.80	0.10	14	0.05	0.80	0.20	25	0.05	0.80	0.50
4	0.05	0.85	0.10	15	0.05	0.85	0.20	26	0.05	0.85	0.50
5	0.05	0.90	0.10	16	0.05	0.90	0.20	27	0.05	0.90	0.50
6	0.10	0.70	0.10	17	0.10	0.70	0.20	28	0.10	0.70	0.50
7	0.10	0.75	0.10	18	0.10	0.75	0.20	29	0.10	0.75	0.50
8	0.10	0.80	0.10	19	0.10	0.80	0.20	30	0.10	0.80	0.50
9	0.10	0.85	0.10	20	0.10	0.85	0.20	31	0.10	0.85	0.50
10	0.20	0.70	0.10	21	0.20	0.70	0.20	32	0.20	0.70	0.50
11	0.20	0.75	0.10	22	0.20	0.75	0.20	33	0.20	0.75	0.50

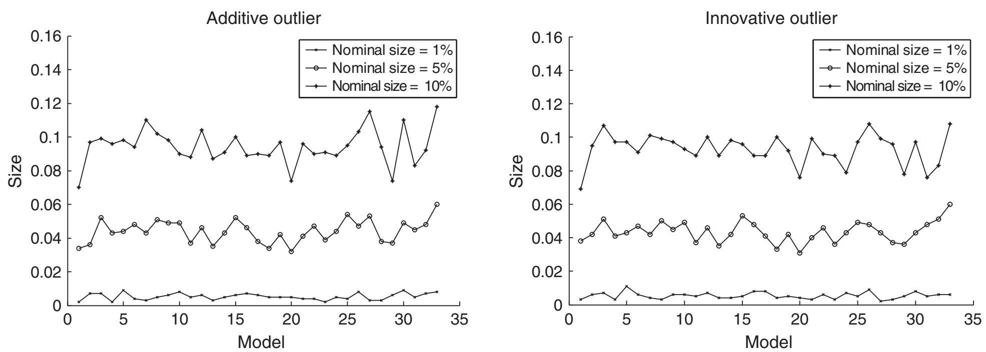

Figure 1. The results of size simulations

Power performance

Six Log-CARR models (Models 5, 8, 10, 27, 30, and 32 in Table 1) are used to investigate the power performance. The outliers are set at three time locations, $0.25T$, $0.5T$, and $0.75T$, for each model, where T is the sample size. Three outlier sizes, $4.5s_i$, $5.0s_i$, and $5.5s_i$, are used to illustrate the outlier effect, where s_i represents the standard deviation of the ith logarithmic sample. Power is calculated as the proportion of the number of samples of correct outlier identification over 10,000 samples, with a sample size of 1,000.

Figure 2 presents the power performance results. The power performance is indifferent with regard to the locations of the outliers. Furthermore, the power increases when the outlier size increases, and overall, the power of the proposed test statistic performs very well when the outlier size is large.

To evaluate whether the proposed test statistic suffers from the masking effect, 10,000 samples with a sample size of 1,000 are generated from the Log-CARR model with the following parameter settings: $(\alpha, \beta, \sigma^2)' = (0.10, 0.80, 0.10)'$, outlier size $k = 5.5s_i$, and the percentages of outliers within the whole sample are 1%, 3% and 5% respectively. For additive outlier detection, the power performance for the 1%, 3%, and 5% outlier settings

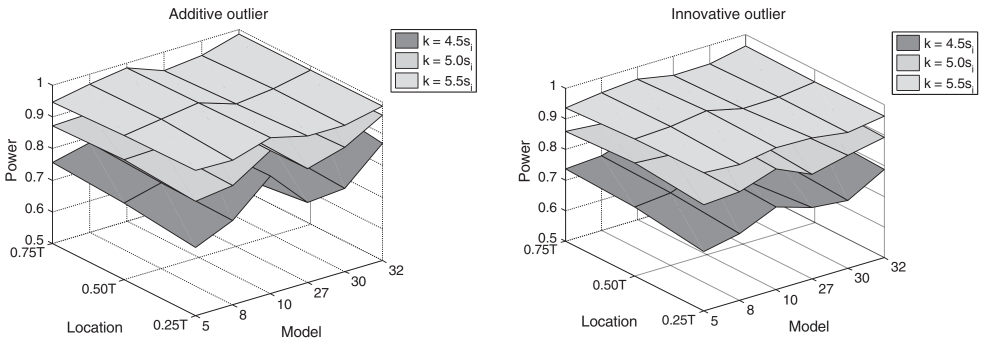

Figure 2. The results of power performance

is 96.3%, 96.3% and 96.1% respectively. This shows that the proposed outlier detection procedure suffers less from the masking effect. The results for IO detection are similar to those for additive outlier detection.

V. Empirical results

The weekly price ranges of three stocks included in the Standard & Poor 500, namely Apple Computer Inc. (AAPL), Merck & Co. Inc. (MRK), and Procter & Gamble Co. (PG), are examined. Data were obtained from the Center for Research in Security Prices (CRSP) database. The data period runs from 07 January 1985 to 06 October 2006, including 5,489 daily maximum and minimum prices.[11] The weekly price ranges are calculated as the difference between the logarithmic maximum and the logarithmic minimum prices in a week. Therefore, the dataset is composed of 1,135 observations.

Table 2 presents the results of parameter estimates and model diagnostics of the ECARR (M1), WCARR (M2), and lognormal Log-CARR (M3) models before and after the outlier detection for AAPL, MRK and PG weekly price ranges. The ECARR and WCARR models proposed in Chou (2005) serve as benchmarks to demonstrate the performance of the Log-CARR model. The parameters of the three models are first estimated using the original (ORI) dataset. The parameters for the outlier-adjusted dataset (ADJ) are then estimated by the proposed outlier detection and estimation procedure of the Log-CARR model. For the Apple (AAPL) stock, one observation (observation No. 821) is detected as an IO, and this was caused by a heavy sell-off on Sep 28, 2000, due to an earnings warning given by Apple that there may be a huge shortfall in revenue and earnings per share in the fourth quarter, 2000, because of poor consumer acceptance of new products. As for the Merck (MRK) stock, two observations (No. 49 and No. 1,030) are detected as outliers. No. 49 is detected as an AO, and was caused by the abnormally low price occurring on Dec 9, 1985.[12] The second outlier (No. 1,030) is identified as an IO. This was due to a stock price decline on

[11] The daily prices are split adjusted, which is able to avoid the problem of extremely high price ranges caused by non-market related activities.

[12] The price high is 131.87 while the price low is 101. The abnormal price low is probably an incorrect record, because the figure is 130.75 as reported in the yahoo website: `http://finance.yahoo.com`.

TABLE 2

Parameter estimates and model diagnostics before and after outlier detection for AAPL, MRK, and PG weekly price ranges

	APPL				*MRK*				*PG*			
	ORI			*ADJ*	*ORI*			*ADJ*	*ORI*			*ADJ*
Parameters	*M1*	*M2*	*M3*	*M3*	*M1*	*M2*	*M3*	*M3*	*M1*	*M2*	*M3*	*M3*
ω	0.441	1.084	0.042	0.058	0.455	**1.015**	**0.104**	**0.089**	0.162	0.046	**0.070**	0.061
	(0.471)	(0.705)	(0.033)	(0.038)	(0.337)	(0.317)	(0.039)	(0.034)	(0.146)	(0.049)	(0.031)	(0.033)
α_1	**0.116**	**0.248**	**0.208**	**0.211**	**0.131**	**0.120**	**0.139**	**0.139**	**0.180**	**0.208**	**0.174**	**0.154**
	(0.060)	(0.037)	(0.031)	(0.039)	(0.057)	(0.026)	(0.027)	(0.028)	(0.061)	(0.024)	(0.036)	(0.036)
α_2		0.001	−0.134	−0.114								
		(0.091)	(0.049)	(0.062)								
β_1	**0.838**	0.130	**0.800**	**0.543**	**0.785**	**0.689**	**0.805**	**0.814**	**0.787**	**0.785**	**0.789**	**0.813**
	(0.100)	(0.218)	(0.367)	(0.204)	(0.107)	(0.078)	(0.044)	(0.043)	(0.078)	(0.025)	(0.051)	(0.052)
β_2		**0.511**	0.110	0.339								
		(0.087	(0.324)	(0.175)								
γ		**2.141**				**2.046**				**2.174**		
		(0.044)				(0.039)				(0.043)		
σ^2			**0.179**	**0.176**			**0.174**	**0.165**			**0.158**	**0.151**
			(0.007)	(0.008)			(0.009)	(0.008)			(0.008)	(0.006)
Skew_res			0.309	0.249			0.370	0.150			0.380	0.195
Kurt_res			3.408	3.193			4.264	3.203			3.954	3.129
KS	<0.01	<0.01	0.261	0.463	<0.01	<0.01	0.149	0.206	<0.01	<0.01	0.091	0.211
indep(1)	**0.038**	0.069	0.280	0.247	0.511	**0.002**	0.680	0.655	0.056	**0.005**	0.059	0.061
indep(2)	0.680	0.292	0.751	0.762	0.589	0.840	0.936	0.772	0.064	0.461	0.684	0.816
indep(3)	0.183	0.063	0.837	0.611	0.247	0.136	0.530	0.553	**0.027**	0.054	0.087	0.086
indep(4)	0.447	0.136	0.814	0.810	**0.002**	0.648	0.929	0.804	**0.006**	0.351	0.272	0.504

(*continued*)

TABLE 2

(*Continued*)

	APPL				*MRK*				*PG*			
	ORI			*ADJ*	*ORI*			*ADJ*	*ORI*			*ADJ*
Parameters	*M1*	*M2*	*M3*	*M3*	*M1*	*M2*	*M3*	*M3*	*M1*	*M2*	*M3*	*M3*
Outlier			NO. 821				NO. 49 NO. 1,030				NO. 146 NO. 792	
Lags	(1,1)	(2,2)	(2,2)	(2,2)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)
T			1043				1043				1043	

Notes: ORI means the original price range data; ADJ means outlier adjusted price range data. M1 and M2 represent the ECARR and WCARR model in Chou (2005), respectively. The error term of the WCARR follows Weibull distribution, i.e. $f(\varepsilon_t|I_{t-1}) = \xi^{\gamma}\gamma\varepsilon_t^{\gamma-1}\exp\left(-(\xi\varepsilon_t)^{\gamma}\right)$, where $\gamma > 0$, $\xi = \Gamma(1+(1/\gamma))$, $\Gamma(\cdot)$ is the gamma function, and γ is the shape parameter. M3 represents the log-CARR model in this study. *Skew_res* and *Kurt_res* denote the skewness and kurtosis of the logarithmic residuals, respectively. *KS* denotes the *P*-value of Kolmogorov–Smirnov statistic for testing the adequate distribution assumption of residuals (The null hypothesis: the residuals follow lognormal distribution). Statistics *indep(k)*, $k = 1, 2, 3, 4$ denote the *P*-value of independence test (The null hypothesis: the kth moment of residuals is serial uncorrelated). *Outlier* represents the detected outliers (AO or IO). The values in parentheses below the parameter estimates are standard errors. The lags (*Lags*) selected for each model are based on the SBC criterion. *T* denotes the sample size. The estimated parameters with *P*-values less than 0.05 are shown in boldface.

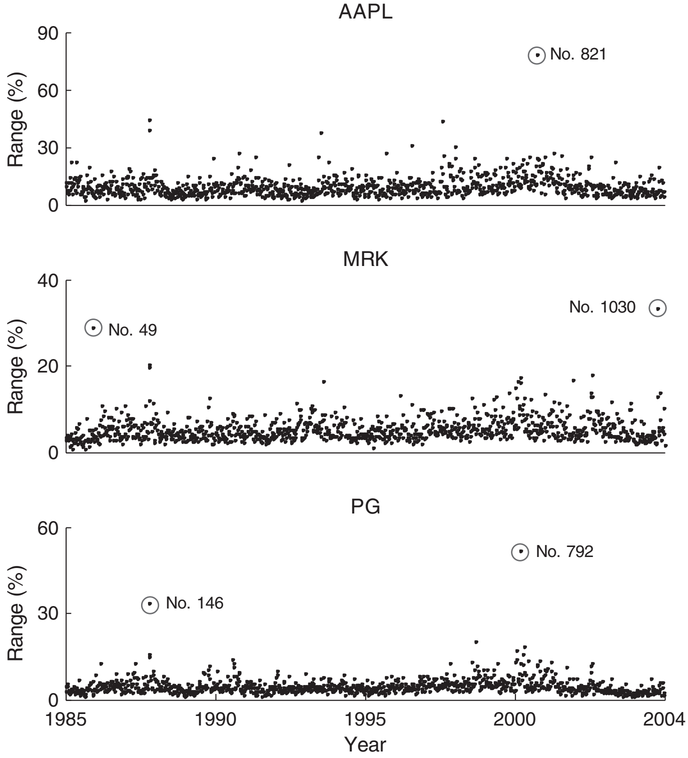

Figure 3. Three weekly price ranges of AAPL, MRK and PG stocks

Sep 30, 2004, because Merck issued a drug recall[13] and withdrew a product (VIOXX®: generic name called Rofecoxib) from the market. For the Procter & Gamble (PG) stock, two IOs (No. 146 and No. 792) are identified. The first IO (No. 146) was the result of Black Monday on Oct 19, 1987, while the second (No. 792) was caused by the decline in stock price on Mar 7, 2000 that occurred because the company issued a warning that earnings for the fiscal year of 2000 would be below expectations. The empirical evidence thus shows that the proposed outlier detection procedure can detect outliers caused by economic, financial, and abnormal trading activities. The critical value of outlier detection at 5% significance level is 2.97. Figure 3 depicts the three weekly price range series with the five outliers marked with circles.

The fitness of the Log-CARR model is much better when using the outlier-adjusted observations than the original data in the three price range series. In general, the estimated volatility in the Log-CARR model using the outlier-adjusted price ranges is lower than that found when using the original price ranges. The decrease in the estimates of parameter σ^2 after the outliers are adjusted corresponds to the reduction in skewness and kurtosis of the logarithmic residual, as shown in Table 2. Additionally, the results in Table 2 indicate that all of the first four moments are serially uncorrelated and lognormally distributed

[13]VIOXX® is a drug used to reduce pain, inflammation and stiffness caused by osteoarthritis and rheumatoid arthritis. This withdrawal was due to safety concerns with regard to the cardiovascular side effects caused by the medication.

TABLE 3

The RMSEs and MAEs of the three price range series

Horizon		*AAPL*		*MRK*		*PG*	
h	*Data*	*RMSE*	*MAE*	*RMSE*	*MAE*	*RMSE*	*MAE*
1	Original data	3.582	2.634	2.648	1.837	1.400	0.967
	Outlier adjusted	3.579	2.632	2.672	1.781	1.390	0.948
5	Original data	3.502	2.686	2.372	1.750	1.263	1.056
	Outlier adjusted	3.499	2.679	2.372	1.695	1.224	1.003
10	Original data	3.546	2.701	2.185	1.788	1.473	1.295
	Outlier adjusted	3.541	2.692	2.131	1.726	1.387	1.199
20	Original data	3.153	2.610	2.288	1.941	1.906	1.765
	Outlier adjusted	3.129	2.577	2.246	1.887	1.744	1.613
40	Original data	3.576	3.130	2.370	2.092	2.542	2.357
	Outlier adjusted	3.521	3.069	2.369	2.087	2.300	2.108

Note: For the definitions of RMSE and MAE, see equations (24) and (25).

for the Log-CARR model before and after outlier detection.[14] The lognormal assumption becomes more solid for the outlier-adjusted series. For the ECARR and WCARR models,[15] the results of the two tests also reveal that neither assumptions of exponential and Weibull error distributions are suitable, nor can serial correlation be completely captured.

To examine whether the forecasting accuracy will be increased, two measures, root-mean-squared-errors (RMSE) and mean-absolute-errors (MAE) are calculated, i.e.

$$\text{RMSE}(h) = \left[T^{-1} \sum_{t=1}^{T} \left(e_{t+h}^{(j)} \right)^2 \right]^{0.5}, \tag{24}$$

$$\text{MAE}(h) = T^{-1} \sum_{t=1}^{T} \left| e_{t+h}^{(j)} \right|, \tag{25}$$

where $T = 52$, $FV_{t+h}^{(j)}$ and $e_{t+h}^{(j)} = R_{t+h} - FV_{t+h}^{(j)}$ are $h-step$ ahead forecast ranges and the forecast errors respectively, $h = 1, 2, \ldots, 40$. The superscript $j = \text{O, A}$ represents the values estimated from the original dataset (O) or outlier-adjusted dataset (A). The forecasting procedure is as follows: First, 1,043 weeks of data prior to the forecast interval are used to estimate the parameters of the Log-CARR(1,1) model. For each forecasting horizon, h, 52 rolling window sample estimations are made. The first end date of the estimated sample is thus 27 December 2004, and the last end date is 19 December 2005. Second, two parameter estimate sets of the Log-CARR(1,1) model estimated from the original data and outlier-adjusted data are available after conducting the outlier detection and estimation procedure for each rolling sample. $h-step$ ahead forecasts, RMSEs and MAEs for the three price ranges of AAPL, MRK and PG, are then calculated.

[14] Two tests are used for diagnosing the independent and identical distribution assumption of the error terms. The Lagrange Multiplier (LM) test provided by Diebold, Gunther and Tay (1998) is used to test the serial independence of the residuals, while the Kolmogorov–Smirnov (KS) test is used to test the correctness of the density specification.

[15] The ECARR and WCARR models with the exponential and Weibull error distributions respectively are proposed in Chou (2005).

TABLE 4

Three tests for comparing forecast accuracy with two loss-differential functions

Series	$h=1$	$h=5$	$h=10$	$h=20$	$h=40$
Panel 1: The loss-differential function is LD1					
AAPL	**0.0410**	0.1450	0.1820	<**0.0001**	<**0.0001**
MRK	0.7765	0.5450	0.0504	**0.0316**	0.4312
PG	**0.0290**	**0.0001**	<**0.0001**	<**0.0001**	<**0.0001**
Panel 2: The loss-differential function is LD2					
AAPL	**0.0425**	**0.0158**	**0.0458**	<**0.0001**	<**0.0001**
MRK	**0.0397**	**0.0250**	**0.0204**	**0.0084**	**0.1810**
PG	**0.0037**	<**0.0001**	<**0.0001**	<**0.0001**	<**0.0001**

Note: $LD1: d1_{t+h}^{(O,A)} = (e_{t+h}^{(O)})^2 - (e_{t+h}^{(A)})^2$ and $LD2: d2_{t+h}^{(O,A)} = |e_{t+h}^{(O)}| - |e_{t+h}^{(A)}|$ are two loss-differential series. The superscript (O or A) represents the values estimated from the original dataset (O) or outlier-adjusted dataset (A). h represents the forecasting horizon. The P-values shown in boldface are those less than 5%, which means that the forecasts calculated from outlier-adjusted dataset are better than those calculated from original dataset.

Table 3 displays the RMSEs and MAEs of the three price range series considering several different horizons. Almost all the RMSEs and MAEs calculated from the outlier-adjusted data are lower than those calculated from the original data, which indicates that the forecasting capability has improved after considering the outliers. In order to evaluate whether the improvements are significant, we implement the predictive ability test of Hansen (2005).[16] Two loss-differential series $LD1: d1_{t+h}^{(O,A)} = (e_{t+h}^{(O)})^2 - (e_{t+h}^{(A)})^2$ and $LD2: d2_{t+h}^{(O,A)} = |e_{t+h}^{(O)}| - |e_{t+h}^{(A)}|$ are employed.

Table 4 displays the test results. They show that the forecasts calculated from the outlier-adjusted data are significantly better than those calculated from the original dataset according to $LD2$, except for the forecast horizon is long term ($h = 40$). The similar comparisons by $LD1$ for PG also prefer the adjusted data; the results for AAPL and MRK are not so clear. Consequently, we can conclude that the Log-CARR(1,1) model estimated by the proposed outlier detection and estimation method has better forecasting performance. Although the proposed outlier detection procedure in this paper has been proven effective empirically, the outliers detected using this method should be treated cautiously in practice if their causes cannot be identified.

VI. Conclusion

The recent development of using price ranges to construct volatility models has led to a number of successes in volatility forecasting (Chou, 2005; Brandt and Jones, 2006). As it is possible that security prices embed some outliers due to the impact of unusual financial and non-financial events, in this work, we examine how to detect additive and IO when modelling price ranges.

[16] The test is conducted by using Oxmetrics.

The detection procedure proposed in this work has been demonstrated to be well-sized and has good power by using Monte Carlo simulations. The results of an empirical application of the proposed detection procedure to three S&P 500 component stocks show that the detection method can effectively detect the outliers of price ranges and improve forecasting accuracy.

Appendix A

Proof of Proposition 1 According to equation (16), $\eta_t = e_t - ku_t$ for $t = 1,\ldots,T$ and $t \neq t_0$. The error sum of squares, $\sum_{t=1,t\neq t_0}^{T} \eta_t^2$, then has $T-2$ degrees of freedom. For the AO, the OLS of the regression equation, $\hat{k}_{\mathrm{AO}}(t_0)$, can be rewritten as $\hat{k}_{\mathrm{AO}}(t_0) = \sum_{t=1}^{T} e_t u_t \big/ \sum_{t=1}^{T} u_t^2$, as $u_t = 0$, for $t < t_0$. $\hat{k}_{\mathrm{AO}}(t_0)$ is normally distributed as the error term, e_t, follows a normal distribution under the null hypothesis ($k=0$), i.e. $e_t \overset{\mathrm{iid}}{\sim} N(0, \sigma^2)$.

The standardized $\hat{k}_{\mathrm{AO}}(t_0)$ is formulated as

$$\hat{\tau}_{\mathrm{AO}}(t_0) = \frac{\hat{k}_{\mathrm{AO}}(t_0)}{\sqrt{s^2(\hat{k}_{\mathrm{AO}}(t_0))}}$$

under the null hypothesis, where

$$s^2(\hat{k}_{\mathrm{AO}}(t_0)) = \frac{\sum_{t=1,t\neq t_0}^{T} \eta_t^2 \big/ (T-2)}{\sum_{t=1}^{T} u_t^2}$$

is an estimator of

$$\sigma^2(\hat{k}_{\mathrm{AO}}(t_0)) = \frac{\sigma^2}{\sum_{t=1}^{T} u_t^2}.$$

Rewrite $\hat{\tau}_{\mathrm{AO}}(t_0)$ as

$$\hat{\tau}_{\mathrm{AO}}(t_0) = \frac{\hat{k}_{\mathrm{AO}}(t_0)}{\sqrt{\sigma^2(\hat{k}_{\mathrm{AO}}(t_0))}} \Bigg/ \frac{\sqrt{s^2(\hat{k}_{\mathrm{AO}}(t_0))}}{\sqrt{\sigma^2(\hat{k}_{\mathrm{AO}}(t_0))}}.$$

Note that

$$\frac{\hat{k}_{\mathrm{AO}}(t_0)}{\sqrt{\sigma^2(\hat{k}_{\mathrm{AO}}(t_0))}} \overset{\mathrm{iid}}{\sim} N(0,1),\ \frac{s^2(\hat{k}_{\mathrm{AO}}(t_0))}{\sigma^2(\hat{k}_{\mathrm{AO}}(t_0))} = \frac{\sum_{t=1,t\neq t_0}^{T} \eta_t^2 \big/ (T-2)}{\sigma^2} = \frac{\sum_{t=1,t\neq t_0}^{T} \eta_t^2}{\sigma^2 (T-2)},$$

$$\text{and } \frac{\sum_{t=1,t\neq t_0}^{T} \eta_t^2}{\sigma^2} \sim \chi_{T-2}^2.$$

The statistic $\hat{\tau}_{\mathrm{AO}}(t_0)$ then follows Student's t-distribution with $T-2$ degrees of freedom under the null hypothesis.

Under certain regularity conditions, the $\hat{e}_t$ and $\hat{u}_t$ are consistent estimators of e_t and u_t, respectively, $\forall t = 1,\ldots,T$. Then $\widetilde{k}_{\mathrm{AO}}(t_0) = \sum_{t=1}^{T} \hat{e}_t \hat{u}_t / \sum_{t=1}^{T} \hat{u}_t^2$ is also a consistent estimator of $\hat{k}_{\mathrm{AO}}(t_0)$. Thus, $\hat{\eta}_t = \hat{e}_t - \widetilde{k}\hat{u}_t$ and

$$s^2(\widetilde{k}_{\mathrm{AO}}(t_0)) = \frac{\sum_{t=1,t\neq t_0}^{T} \hat{\eta}_t^2 / (T-2)}{\sum_{t=1}^{T} \hat{u}_t^2}$$

are consistent estimators of η_t and $s^2(\hat{k}_{\mathrm{AO}}(t_0))$, respectively. The statistic $\widetilde{\tau}_{\mathrm{AO}}(t_0) = \frac{\widetilde{k}_{\mathrm{AO}}(t_0)}{\sqrt{s^2(\widetilde{k}_{\mathrm{AO}}(t_0))}}$ then follows an asymptotic $N(0,1)$under the null hypothesis.

Following the same procedure, for the IO, the standardized $\hat{k}_{\mathrm{IO}}(t_0)$ under the null hypothesis can be formulated as

$$\hat{\tau}_{\mathrm{IO}}(t_0) = \frac{\hat{k}_{\mathrm{IO}}(t_0)}{\sqrt{s^2(\hat{k}_{\mathrm{IO}}(t_0))}} = \frac{e_{t_0}}{\sqrt{s^2(e_{t_0})}} = \frac{e_{t_0}/\sigma}{\sqrt{\sum_{t=1,t\neq t_0}^{T} (\hat{e}_t - \bar{\hat{e}})^2 / (T-2)\sigma^2}},$$

where $\bar{e} = \sum_{t=1,t\neq t_0}^{T} e_t/(T-1)$. The statistic $\hat{\tau}_{\mathrm{IO}}(t_0)$ is distributed as Student's t-distribution with $T-2$ degrees of freedom under the null hypothesis. Consequently,

$$\widetilde{\tau}_{\mathrm{IO}}(t_0) = \frac{\widetilde{k}_{\mathrm{IO}}(t_0)}{\sqrt{s^2(\widetilde{k}_{\mathrm{IO}}(t_0))}} = \frac{\hat{e}_{t_0}/\sigma}{\sqrt{\sum_{t=1,t\neq t_0}^{T} (\hat{e}_t - \bar{\hat{e}})^2 / (T-2)\sigma^2}}$$

follows an asymptotic $N(0,1)$under the null hypothesis, where $\hat{e}_t$ denotes the estimator of e_t, $\forall t = 1,\ldots,T$, and $\bar{\hat{e}} = \sum_{t=1,t\neq t_0}^{T} \hat{e}_t/(T-1)$.

Proof of Proposition 2 In line with Proposition 1, the statistics $\widetilde{\tau}_{\mathrm{AO}}(t_0)$ ($\widetilde{\tau}_{\mathrm{IO}}(t_0)$) for $t_0 = 1,2,\ldots,T$ are independent and identically distributed and asymptotically follow the standard normal distribution under the assumption that ε_t in equation (3) are i.i.d. According to Theorem 1.1.3 in de Haan and Ferreira (2006),[17] the following lemma can be obtained:

Lemma A1. Let $X_1, X_2, \ldots, X_T$ be independent and identically standard normal distributed random variables with distribution $F(\cdot)$ and $Y = \frac{\max(X_1, X_2, \ldots, X_T) - b_T}{a_T}$. $\Pr(Y \leqslant y) \to \exp(-\exp(-y))$ as $T \to \infty$, where $b_T = (2\log(T) - \log(\log(T)) - \log(4\pi))^{0.5}$ and $a_T = 1/b_T$.

The centralization, b_T, and scale constants, a_T, can be chosen as $b_T = F^{-1}\left(1 - \frac{1}{T}\right)$ and $a_T = F^{-1}\left(1 - \frac{1}{T\exp(1)}\right) - b_T$ respectively, to adjust the undersized problem in finite sample simulations in section IV. See theorem 3.3 in Castillo (1988).[18] Additionally, the sample size should not be too small in forming $Z_i(t_0)$ when applying its asymptotic distribution to real data.

Following Lemma A1, the limiting distribution of $Z_i(t_0)$ defined in equation (20) can be approximated as $G(z) = \Pr(Z_i \leqslant z) \to \exp(-\exp(-z))$.

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[18] The approximated distributions are demonstrated to be quite well in section IV.

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