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Abstract

We study the otherwise standard growth model with money except endogenous time preferences determined by resources spent on imagining future pleasures along the line of Becker and Mulligan (1997). Money plays a role in transactions via the cash-in-advance constraint. The resulting steady-state condition can be simplified to the standard textbook diagram in terms of two loci. We analyze the relationship between monetary growth and capital accumulation. If spending on imagining future pleasures is not constrained by cash, the existing relationship no longer holds. The optimum quantity of money is studied.

Keywords: endogenous time preferences; growth; money

JEL classification: E22; E31

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1 Introduction

Patience has long been recognized as an important virtue and time preferences have played a fundamental role in theories of savings, investment, economic growth, and among many other issues. Yet, since Ramsey (1928), time preference rates are almost always taken as exogenous with little discussion concerning what determines their level. Although there have been efforts favoring endogenous time preferences, these time preferences are either a by-product of consumption choices (Uzawa, 1968) or a by-product of fertility choices in the form of a intergenerational discount rate (Becker and Barro, 1988).

Different from these above exogenous and endogenous time preferences, Becker and Mulligan (1997) proposed that a time preference is not a by-product of other choices. Instead, they formally modeled consumers' efforts and activities in order to determine and reduce the discount on future utilities. They postulated that the rate of a time preference is determined by the resources spent on imagining future pleasures and termed it as such, "future oriented capital." Thus, Becker and Mulligan determined the time preference rate by relating it to resources spent on imagining future pleasures and the larger the resources spent the more patient an individual is.

In their concluding remarks, Becker and Mulligan (1997, p. 754) pointed out several directions for future work in order to envisage the implications of endogenous time preferences as the result of resources spent on imagining future pleasures. Since then, many papers have studied the consequences and implications of this type of time preference on the issues regarding addictions and health concerns (Bretteville-Jensen, 1999), cultural transmissions and social status (Bisin and Verdier, 2001), the formation of markets and institutions (Palacios-Huerta and Santos, 2004), religious interactions (Bisin, et al, 2004), and occupational choices and the spirit of capitalism (Doepke and Zilibotti, 2008). However, although time preferences are important determinants of savings and capital accumulation, to the best of our knowledge, no paper has studied the resulting effects on savings and capital accumulation except for Gong (2006) and Stern (2006). Gong (2006) studied a standard one-sector growth model with a money-in-the-utility function and endogenous time preferences, but he found a conventional negative relationship between money and capital in the long run as expected inflation reduces resources spent on imagining future pleasures, thereby a higher time preference rate and lower savings. In a similar standard growth model without money, Stern (2006) characterized the uniqueness and multiplicities of steady state and the stability property in a series of examples with parametric functions for utilities, time preferences and productions.

In this paper, we study the consequence on capital accumulation of endogenous time preferences as determined by resources spent on imagining future pleasures. Our model is the otherwise standard optimal growth model as studied by Gong and Stern except for the role of money. In our model, money plays a role in transactions through the cash-in-advance (henceforth, CIA) constraint and the CIA constraint always binds consumption.[1] At an instantaneous point in time, the representative agent produces and allocates goods to three activities: (i) consumption that increases current utility; (ii) savings that accumulates capital; and, (iii) resources spent on imagining future pleasures that increase one's own appreciation of the future, which lowers the discount and raises the discounted utility of future consumption. The resulting model is easily tractable in steady state. We simplify the analysis to the standard textbook diagram represented by the good-market clearance and Keynes-Ramsey conditions in two loci in the plane of capital and consumption (e.g., Blanchard and Fisher, 1989, Ch. 2; Acemoglu, 2009, Ch. 8). Our simple, analytical method may be useful in the study of other issues in growth models with endogenous time preferences determined by resources spent on imagining future pleasures.

We investigate the consequence of monetary growth on capital in the long run. If the CIA constraint binds resources spent on imagining future pleasures, we find that the conventional neutral or negative long-run relationship between money and capital holds depending on whether the CIA constraint does not or does bind investment (Lucas, 1980; Stockman, 1981). By contrast, if the CIA constraint does not bind resources spent on imagining future pleasures, the long-run relationship between money and capital may be positive or ambiguous depending on whether the CIA constraint does not or does bind investment.

The reason goes as follows. If the CIA constraint binds spending on imagining future pleasures, consumption and spending on imagining future pleasures have the same relative price. When the CIA constraint binds (or, does not bind) investment, monetary growth raises (or, does not change) the price of investment relative to both consumption and spending on imagining future pleasures. As a result, capital decreases (or, does not change) in the long run. By contrast, if the CIA constraint does not bind spending on imagining future pleasures, monetary growth increases the price of consumption relative to spending on imagining future pleasures. In this case, when the CIA constraint does not bind investment, monetary growth leads the agent to substitute away from consumption toward spending on imagining future pleasures which in turn decreases the time preference rate, thus higher savings and capital in the long run. However, when the CIA

[1] Many growth models adopted the CIA constraint when introducing money into models; e.g., Englund and Svensson (1988), Carmichael (1989), Bianconi (1992) and Dotsey and Sarte (2000).

constraint binds investment, monetary growth has an additional direct negative effect on investment, thereby an ambiguous effect on capital.

Mixed empirical evidence about the relationship between money and capital has relevance for our work. Employing 5-year average data across countries, Bruno and Easterly (1998) found a negative relationship between inflation and growth for high inflation countries. Using annual post-war data for 32 countries, however, Karras (1993) showed that monetary growth had a probably neutral effect on output in the long run, but the effect is positive in the short run. Indeed, in a large sample of postwar economies, Bullard and Keating (1995) found a long-run positive relationship between inflation and output in low inflation countries. Utilizing annual time-series data for G-7 countries, Ericsson et al (2001) found that inflation and output were co-integrated and typically output and inflation were positively related in these co-integrating relationships for most countries. The result in our model suggests that a binding or non-binding cash constraint on resources spent on imagining future pleasures may be one of the reasons underlying the ambiguous relationship between money/inflation and capital.

In addition to papers by Lucas (1980), Stockman (1981), Gong (2006) and Stern (2006), our paper is related to Wang and Yip (1992) and Palivos et al. (1993) which found a negative long-run effect of monetary growth when only a fraction of investment is constrained by cash. Recently, based on a utility that is increasing in wealth, Gong and Zhou (2001) and Chang and Tsai (2003) obtained a long-run positive effect of money on capital when a sufficiently small fraction of investment is constrained by cash. If resources spent on imagining future pleasures are not bound by cash, our results are in a sharp contrast to these above papers. The relationship between money and capital is positive if the CIA constraint only binds consumption. If the CIA constraint also binds investment, we find a positive relationship between money and capital when the effect through higher spending on imagining future pleasures is sufficiently large that dominates the adverse effect through the CIA constraint on investment.

As the agent in our model chooses consumption and spending on imagining future pleasures, our paper is also related to two-sector models. In the two-sector, pure-exchange model with one cash good and one credit good by Lucas and Stokey (1987), higher monetary growth increased the consumption of credit goods and decreased the consumption of cash goods. In our model, when spending on imagining future pleasures is not a cash good, higher monetary growth increases not only spending on imagining future pleasures (a credit good) but may also consumption (a cash good) due to higher output. In the growth models by Huo (1997) and Mino (1997), the relationship between money and capital was either neutral or negative in the long run when only consumption is constrained by cash and was unambiguously negative when investment was also constrained by cash. In our paper, the effect of monetary growth on capital is unambiguously positive when only consumption is constrained by cash and may be positive when investment is also constrained by cash. Finally, in the two-sector growth model by Chuang (2004), when only the two types of consumption goods were constrained by cash and were at different degrees, the relationship between money and capital may be positive if the capital intensity in the two sectors are different and opposite to the order of the degree of cash constraints. In our paper there is only one sector, thus only one capital intensity, and if spending on imagining future pleasures is not constrained, the relationship between money and capital is unambiguously positive or ambiguous.

The paper proceeds as follows. Section 2 sets up a model and investigate the equilibrium conditions. Section 3 analyzes long-run effects of permanent monetary growth and the optimum quantity of money. Finally, in Section 4, we make concluding remarks.

2 The model

Our model extends Stern (2006) to a monetary economy in continuous time.[2] The lifetime utility of the representative agent is

$$U = \int_0^\infty u(c_t) X_t dt \,, \tag{1}$$

where c_t is consumption and u is an instantaneous utility function that satisfies the standard concavity properties of $u'>0>u''$. The discount factor is $X_t \equiv \exp[-\int_0^t \rho_\tau d\tau]$, where ρ_t is the instantaneous discount rate. The agent may engage in some activities or sacrifices in order to imagine future pleasures and increase the appreciation of the future. Resources spent on imagining future pleasures decrease the discount rate. Denote by s_t the resource cost spent on imagining future pleasures. Then, $\rho'(s_t)<0$. Following Stern (2006) which was taken from Becker and Mulligan (1997), we assume a diminishing marginal benefit of spending on imagining future pleasures, so $\rho''(s_t)>0$. The discount factor may be rewritten as follows.

$$\dot{X}_t = -\rho(s_t) X_t, \text{ with } X_0 \text{ given.} \tag{2}$$

The representative agent faces the following budget constraint

$$\dot{m}_t = f(k_t) - \pi_t m_t + v_t - c_t - s_t - I_t, \tag{3a}$$

[2] Stern (2006) is in discrete time.

where I is gross investment, k is capital, m is real money balances, π is the inflation rate, and v is real lump-sum transfers. The production function satisfies the neoclassical property, i.e., $f'>0>f''$, and the Inada condition. The budget constraint indicates that available resources including income and transfers may be consumed, spent on imagining future pleasures, or saved. Savings are held in terms of either investment or real balances. Nominal money is initially given and grows at a constant rate μ. We assume that transfers are made by government financed by monetary growth; thus, in aggregates $v_t=\mu m_t$. For simplicity, we assume there is no depreciation of capital. Capital is accumulated as follows.

$$\dot{k}_t = I_t \text{, with } k_0 \text{ given.} \tag{3b}$$

The representative agent also faces the following CIA constraint.

$$c_t + \varphi_I I_t + \varphi_s s_t \leq m_t. \tag{3c}$$

In (3c), if $\varphi_I=\varphi_s=1$, consumption, investment and spending on imagining future pleasures are all bound by cash holdings. If $\varphi_I=1$ and $\varphi_s=0$, consumption and investment both are while spending on imagining future pleasures is not bound by cash. Finally, if $\varphi_I=\varphi_s=0$, only consumption is but investment and spending on imagining future pleasures are not bound by cash.

Finally, let us remark on the utility representation (1) and (2). According to Stern (2006), the representation has two main ways of interpretation, depending on whether one views the optimal program as a dynastic family or as a single individual with an infinite lifetime.

In a dynastic family with each generation living for two periods, childhood and adulthood, the discount factor is the degree to which generation t cares for generation t+1. The connection of the linkage between the parent and the child is endogenously modeled. The resources spent in the appreciation of the future s_t then stand for actions that the parent takes in order to reinforce the connection with the child. The degree to which the parent engages and commits himself to the nurturing and rearing of the child would certainly be a determinant in the intensity of the relationship. Spending more time to read and to play with the child could also be a determinant in the strength of the relationship. As a result, spending in the parent-child relationship would cost the parent either foregone production or current resources.

An alternative explanation is an individual with an infinite lifetime. The agent maximizes the sum of current utility and discounted sum of utilities of the remainder of his life. The discount factor is applied to the individual's own utility in the future, as opposed to that of a dynastic family model that is applied to descendants. Resources spent on imagining future pleasures are used to increase the individual's own appreciation of the future. As put forth by Becker and Mulligan

(1997, pp 2 and 10), activities such as schooling/learning, mortality/health, religion and time spent in trying to appreciate the future all increase the imagination of future pleasures. Like Becker and Mulligan, our specification is in line with the second method of interpretation. Different from Becker and Mulligan wherein the model is in a discrete time and the agent has a finite horizon, our model is in a continuous time and the agent has an infinite horizon.

2.1 Optimization conditions

The representative agent's problem is to maximize (1) subject to (2) and (3a)-(3c), taking as given the monetary growth rate, transfers, initial capital and initial nominal money holdings. Denote by $\lambda_{kt}>0$, $\lambda_{mt}>0$ and $\theta_t>0$ the (current-valued) co-state variable associated with capital, real balances and the discount factor, respectively, and by $\xi_t>0$ the (current-valued) multiplier of the CIA constraint. The necessary conditions are

$$u'(c_t) = \lambda_{mt} + \xi_t \,, \tag{4a}$$

$$-\theta_t \rho'(s_t) = \lambda_{mt} + \varphi_s \xi_t , \tag{4b}$$

$$\lambda_{kt} = \lambda_{mt} + \varphi_I \xi_t , \tag{4c}$$

$$\dot{\lambda}_{kt} = -f'(k_t)\lambda_{mt} + \rho(s_t)\lambda_{kt} , \tag{4d}$$

$$\dot{\lambda}_{mt} = [\pi_t + \rho(s_t)]\lambda_{mt} - \xi_t , \tag{4e}$$

$$\dot{\theta}_t = -u(c_t) + \theta_t \rho(s_t), \tag{4f}$$

along with the transversality constraints: $\underset{t\to\infty}{Lim}\, \lambda_{kt} X_t k_t = 0,$ $\underset{t\to\infty}{Lim}\, \lambda_{mt} X_t m_t = 0$ and $\underset{t\to\infty}{Lim}\, \theta_t X_t = 0.$

Optimal conditions (4a)-(4c) are for consumption, spending on imagining future pleasures and investment. For example, (4a) equates the marginal utility of consumption to the marginal cost of consumption in order to determine optimal consumption. The remaining conditions in (4d)-(4f) are Euler equations that govern how the shadow prices of capital, real balances and the discount factor change over time, respectively.

2.2 Equilibrium

A dynamic equilibrium is a time path $\{c_t, s_t, k_t, m_t, \lambda_{kt}, \lambda_{mt}, \xi_t, \theta_t, \pi_t\}$ that satisfies optimal conditions (4a)-(4f), the binding CIA constraint (3c),[3] the money market clearance condition,

[3] Following Lucas (1980) and Stockman (1981), we assume the CIA constraint is binding in equilibrium. Roughly speaking, it requires that the monetary growth rate be greater than or equal to the discounted marginal rate of substitution between consumption in two consecutive points in time.

$$\dot{m}_t = (\mu - \pi_t) m_t, \tag{5a}$$

and the goods market clearance condition, which, using (3a), (3b) and (5a), is

$$\dot{k}_t = f(k_t) - c_t - s_t. \tag{5b}$$

Below, we explain how the dynamic equilibrium is determined. To save space, in what follows we omit the time subscript t. First, if we substitute ξ in (4a) into (4c), we obtain

$$\lambda_k = \varphi_I u'(c) + (1-\varphi_I)\lambda_m \equiv \lambda_k(c, \lambda_m). \tag{6a}$$

Next, differentiating (4a) with respect to time, with the use of (4c), (4d) and (6a), leads to the following modified Keynes-Ramsey rule

$$\dot{c} = \tfrac{1}{u''(c)}\{(1-\tfrac{1}{\varphi_I})\dot{\lambda}_m + \tfrac{1}{\varphi_I}\rho(s)\lambda_k(c,\lambda_m) - \tfrac{1}{\varphi_I} f'(k)\lambda_m\}. \tag{6b}$$

Moreover, differentiating (4b) with respect to time, with the use of (4d) and (4f) yields

$$\dot{s} = \tfrac{-1}{\theta\rho''(s)}\{(1-\tfrac{\varphi_s}{\varphi_I})\dot{\lambda}_m - [\tfrac{\varphi_s}{\varphi_I} f'(k) + (1-\tfrac{\varphi_s}{\varphi_I})\rho(s)]\lambda_m - u(c)\rho'(s)\}, \tag{6c}$$

which is a variant of the Keynes-Ramsey rule for spending on imagining future pleasures.

Furthermore, as (3b) and (5b) indicate $f(k)=c+I+s$, the binding CIA constraint suggests $m=(1-\varphi_I)c+\varphi_I f(k)+(\varphi_s-\varphi_I)s$. If we differentiate this relationship and use (5a), and then substitute in $\dot{c}$ from (6b), $\dot{k}$ from (5b), and $\dot{s}$ from (6c), along with $\dot{\lambda}_m$ from (4e), we obtain

$$\pi = \mu - \tfrac{(1-\varphi_I)\dot{c}+\varphi_I f'(k)\dot{k}+(\varphi_s-\varphi_I)\dot{s}}{(1-\varphi_I)c+\varphi_I f(k)+(\varphi_s-\varphi_I)s} = \pi(c,k,s,\lambda_m). \tag{6d}$$

Finally, substituting in ξ from (4c) to (4e), together with (6a) and (6d), yields

$$\dot{\lambda}_m = \lambda_m[\rho(s) + \tfrac{1}{\varphi_I} + \pi(c,k,s,\lambda_m)] - \tfrac{1}{\varphi_I}\lambda_k(c,\lambda_m). \tag{6e}$$

Thus, the dynamic equilibrium system is simplified to five equations, including (5b), (6b), (6c), (6e) and (5a) and solves for five equilibrium paths: c, k, s, λ_m, and m. The equilibrium system is block-recursive: when the other four equations simultaneously determine the paths of c, k, s and λ_m, the path of m is determined by (3c). The paths of the remaining variables λ_k, ξ, π and θ are in turn determined by (6a), (4c), (6d) and (4f).

In steady state, $\dot{c} = \dot{k} = \dot{s} = \dot{\lambda}_m = \dot{m} = 0$. To determine steady state, first, the inflation rate is $\pi^*=\mu$, according to (5a).[4] Then, using (6a), (6e) and (4f), we rewrite (6b), (5b) and (6c) as follows.

$$f'(k^*) = \rho(s^*)\{1 + \varphi_I[\rho(s^*) + \mu]\}, \tag{7a}$$

$$f(k^*) = c^* + s^*, \tag{7b}$$

[4] An asterisk will be used to denote a steady-state value.

$$u'(c^*)\rho(s^*)\{1+\varphi_s[\rho(s^*)+\mu]\} = -\rho'(s^*)u(c^*)\{1+\rho(s^*)+\mu\}, \qquad (7c)$$

which determine the values of k^*, c^* and s^*. Finally, (3c) determines m^* and (6e) determines λ^*_m.[5]

3 Long-run effects of monetary growth

Now, we analyze the long-run effect of monetary growth. In our analysis, we can simplify the steady-state condition in (7a)-(7c) to the standard textbook diagrams with two loci summarizing the goods market clearance condition and the Keynes-Ramsey condition in terms of the axis of capital and consumption (e.g., Blanchard and Fisher, 1989, Ch. 2; Acemoglu, 2009, Ch. 8). As the long-run effect depends on how different types of transactions are bounded by the CIA constraint, we start by the case in which the CIA constraint binds spending on imagining future pleasures, followed by the case where the CIA constraint does not bind spending on imagining future pleasures.

3.1 The CIA constraint binds spending on imagining future pleasures

When spending in patience is constrained by cash, φ_s=1. In this case, (7c) implies $u'\rho+\rho'u=0$ and the growth rate of money will not affect the tradeoff between spending on imagining future pleasures and consumption in (7c). In this situation, (7c) indicates a proportional change in spending on imagining future pleasures and consumption.

$$ds = -\tfrac{A}{B}dc, \qquad (8)$$

where $A \equiv u''\rho + \rho'u' < 0$ and $B \equiv u'\rho' + u\rho''$.

Case 1, the CIA constraint does not bind investment; thus φ_I=0.

Under φ_I=0, (7a) and (7b) are independent of the growth rate of money. As (8) is also independent of the growth rate of money, money is superneutral in the long run.

Case 2, the CIA constraint binds investment; thus φ_I=1.

Under φ_I=1, the growth rate of money affects the Keynes-Ramsey condition in (7a). Now, a higher growth rate of money affects the tradeoff between consumption and savings and thus has a direct effect on the tradeoff between current and future consumption. Differentiating (7a) yields

$$f''dk - \rho'(1+2\rho+\mu)ds = \rho\ d\mu. \qquad (9)$$

If we use (8), the modified Keynes-Ramsey rule in (9) is rewritten as

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[5] In the system of (5b), (6b), (6c) and (6e) in c, s, k and λ_m, we have shown that the steady state is a saddle.

$$f''dk + \tfrac{A}{B}\rho'(1+2\rho+\mu)dc = \rho \ d\mu. \qquad (10a)$$

while the goods market equilibrium condition (7b) is

$$f'dk + (-1+\tfrac{A}{B})dc = 0. \qquad (10b)$$

In the (k, c) plane, the goods market clearance condition in (10b) is usually positive sloping, because consumption and capital/output need to be positively correlated in the goods market equilibrium. See locus $\dot{k}=0$ in Figure 1. Moreover, the correspondence principle requires the locus of the modified Keynes-Ramsey condition in (10a) to be positively sloping (locus $\dot{c}=0$) and steeper than locus $\dot{k}=0$.[6] This implies A/B<0 and as A<0, then B>0. Under these conditions, it is obvious that there exists a unique steady state.

[Insert Figure 1 here]

Now, a higher growth rate of money does not influence the goods market equilibrium condition; thus locus $\dot{k}=0$ unaffected. However, a higher growth rate of money has a direct effect on the tradeoff between consumption and savings. As investment is constrained by cash, a higher growth rate of money increases the shadow price of capital relative to the shadow price of real balances in the long run.[7] This lowers the marginal product of capital and decreases household's savings. Thus, locus $\dot{c}=0$ shifts leftwards. As a result, capital and consumption both decrease in the long run.

Our above results indicate that even though resources spent on imagining future pleasures determines the time preference rate and may change the tradeoff between consumption and savings, if resources spent on imagining future pleasures are constrained by cash, the conventional neutral or negative relationship between money and capital in the long run remains hold.

3.2 The CIA constraint does not bind spending on imagining future pleasures

In this situation, the growth rate of money affects the tradeoff between spending on imagining future pleasures and consumption in (7c) in the following way.

$$ds = -\tfrac{\tilde{A}}{\tilde{B}}dc - \tfrac{\rho' u}{\tilde{B}}d\mu, \qquad (11)$$

where $\tilde{A} \equiv u''\rho + \rho' u'(1+\rho+\mu) < 0$ and $\tilde{B} \equiv u'\rho' + u[\rho''(1+\rho+\mu) + (\rho')^2]$.

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[6] Should locus $\dot{c}=0$ be negatively sloping or positively sloping but flatter than locus $\dot{k}=0$, then a higher productivity in production would have led to smaller, not higher, levels of capital and output in the long run.

[7] It is worth noting from (6e) in steady state that the relative price of capital in terms of the price of real balances is $\lambda_k / \lambda_m = 1 + \varphi_l(\rho+\mu)$.

Case 1, the CIA constraint does not bind investment; thus φ_I=0.

If we use (11), the modified Keynes-Ramsey rule in (7a) is

$$f''dk + \tfrac{\tilde{A}}{\tilde{B}}\rho' dc = -\tfrac{(\rho')^2 u}{\tilde{B}} d\mu, \tag{12a}$$

and the goods market clearance condition is

$$f'dk + (-1 + \tfrac{\tilde{A}}{\tilde{B}})dc = -\tfrac{\rho' u}{\tilde{B}} d\mu. \tag{12b}$$

The correspondence principle requires locus $\dot{c} = 0$ in (12a) and locus $\dot{k} = 0$ in (12b) to be positive sloping with $\dot{c} = 0$ steeper than $\dot{k} = 0.$ This implies $\tilde{A}/\tilde{B} < 0$ and as $\tilde{A} < 0,$ then $\tilde{B} > 0.$ Under these conditions, there exists a unique steady state. The relative slopes indicate

$$\Delta \equiv f''(-1 + \tfrac{\tilde{A}}{\tilde{B}}) - f' \tfrac{\tilde{A}}{\tilde{B}}\rho' > 0.$$

Now, higher monetary growth shifts both loci $\dot{k} = 0$ and $\dot{c} = 0$ rightwards. The reason is that, given the goods market equilibrium, higher monetary growth raises the price of consumption relative to spending on imagining future pleasures and increases spending on imagining future pleasures, thus the demand for goods. To maintain market equilibrium, capital needs to increase in order to increase the supply of goods. Thus, $\dot{k} = 0$ shifts rightwards. Moreover, in the Keynes-Ramsey rule, higher monetary growth increases spending on imagining future pleasures and thus decreases the rate of time preference. To satisfy the Keynes-Ramsey rule, capital needs to increase in order to decrease the marginal product of capital. Thus, locus $\dot{c} = 0$ also shifts rightward. As a result, capital is unambiguously increasing in the long run as follows.

$$\tfrac{dk}{d\mu} = \tfrac{1}{\Delta}\tfrac{(\rho')^2 u}{\tilde{B}} > 0. \tag{13a}$$

The effect on consumption is ambiguous, depending on relative rightward shifts in these two loci. We obtain the following result.

$$\tfrac{dc}{d\mu} = \tfrac{1}{\Delta}\tfrac{\rho' u}{\tilde{B}}(-f'' + f'\rho') \geq (resp. \leq)\ 0 \ \text{ if } \ \tfrac{\rho'}{f''} \geq (resp. \leq)\ \tfrac{1}{f'}. \tag{13b}$$

Under $\rho'/f''<1/f'$, the effect on the Keynes-Ramsey rule through increasing spending on imagining future pleasures is small. In this situation, locus $\dot{c} = 0$ shifts rightwards less than locus $\dot{k} = 0$ (see $\dot{c}' = 0$ and E_1 in Figure 1). As a consequence, consumption decreases in the long run. In contrast, under $\rho'/f''>1/f'$, the effect on the Keynes-Ramsey rule through increasing spending on imagining future pleasures is sufficiently large and as a result, locus $\dot{c} = 0$ shifts rightwards more than locus $\dot{k} = 0$ (see $\dot{c}'' = 0$ and E_2 in Figure 1). Therefore, consumption increases in the long run.

The results are understood as follows. When the CIA constraint binds consumption but does not bind resources spent on imagining future pleasures, higher monetary growth increases the opportunity cost of consumption relative to spending on imagining future pleasures. The agent substitutes away from consumption toward spending on imagining future pleasures. As the agent is more patient now, it is optimal to increase savings, thus resulting in higher capital in the long run. If the effect on the tradeoff between consumption and savings through higher spending on imagining future pleasures is sufficiently large, incentives for savings are so large that capital is increasing sufficiently higher. The level of consumption will also increase in the long run.

To summarize the above results,

Proposition 1 *In an optimal growth model with a binding CIA constraint on consumption and a non-binding CIA constraint on spending on imagining future pleasures, if the CIA constraint does not bind investment, higher monetary growth unambiguously increases capital in the long run. Consumption is also increasing if the effect through increasing spending on imagining future pleasures is sufficiently large.*

Case 2, the CIA constraint binds investment: φ_I=1.

In this case, when monetary growth is higher, the change in the goods market equilibrium condition is expressed in (12b). If we use (11), the change in the Keynes-Ramsey rule is

$$f''dk + \tfrac{\tilde{A}}{\tilde{B}}\rho'(1+2\rho+\mu)dc = [\rho-(1+2\rho+\mu)(\rho')^2 \tfrac{u}{\tilde{B}}]d\mu. \tag{14}$$

The correspondence principle requires locus $\dot{c}=0$ in (14) and locus $\dot{k}=0$ in (12b) to be positive sloping with $\dot{c}=0$ steeper than $\dot{k}=0$. Under these conditions, there exists a unique steady state. This also indicates

$$\Lambda \equiv f''(-1+\tfrac{\tilde{A}}{\tilde{B}}) - f'\tfrac{\tilde{A}}{\tilde{B}}\rho'(1+2\rho+\mu) > 0.$$

Now, higher monetary growth increases spending on imagining future pleasures and, thus, increases the demand for goods. In the market equilibrium, capital needs to increase in order to increase the supply of goods. Locus $\dot{k}=0$ thus shifts rightwards to $\dot{k}'=0$. See Figure 2.

[Insert Figure 2]

However, higher monetary growth exerts two effects on the Keynes-Ramsey condition. First, as investment is constrained by cash, higher monetary growth has a direct negative effect on investment through increasing the shadow price of capital relative to the shadow price of real balances and thus discouraging savings. This effect is represented by the first term in the large brackets in the right-hand side of (14). Second, as spending on imagining future pleasures is not bound by cash, the price of spending on imagining future pleasures relative to consumption is lower. The household increases spending on imagining future pleasures which lowers the time preference rate and enhances savings. Such an effect is represented by the second term in the large brackets in the right-hand side of (14). If the former effect through the shadow price of capital relative to the shadow price of real balances dominates the latter effect through higher spending on imagining future pleasures, locus $\dot{c}=0$ shifts leftwards. Then, both capital and consumption decrease in the long run. In contrast, if the latter positive effect through higher spending on imagining future pleasures dominates the former direct negative effect, locus $\dot{c}=0$ shifts rightwards. Then, capital increases when locus $\dot{c}=0$ shifts downward more than locus $\dot{k}=0$ and consumption increases when locus $\dot{c}=0$ shifts rightward more than locus $\dot{k}=0$. It is obvious that when locus $\dot{c}=0$ shifts rightward more than locus $\dot{k}=0$ does, it must be the situation that locus $\dot{c}=0$ shifts downwards more than locus $\dot{k}=0$. Therefore, we have

$$\tfrac{dk}{d\mu} \geq (resp. \leq)\ 0 \quad \text{and} \quad \tfrac{dc}{d\mu} \geq (resp. \leq)\ 0 \quad \text{if} \quad \tfrac{\rho-(\rho')^2(1+2\rho+\mu)u/\tilde{B}}{f''} \geq (resp. \leq)\ \tfrac{-\rho' u/\tilde{B}}{f'} > 0. \tag{15}$$

Intuitively, as investment is constrained by cash, higher monetary growth has a direct negative effect on capital. This is the conventional effect initiated since Stockman (1981). However, the agent in our model also chooses resources spent on imagining future pleasures. If spending on imagining future pleasures is not bound by cash, higher monetary growth induces the agent substitute away from consumption toward spending on imagining future pleasures. As a result, the agent is more patient and it is optimal to save more. This effect increases capital in the long run. If this latter effect is sufficiently large and dominates the former effect, capital and output are increased sufficiently large. Then, consumption is increasing in the long run.

To summarize the results,

Proposition 2 *In an optimal growth model with a binding CIA constraint on consumption and a non-binding CIA constraint on spending on imagining future pleasures, when the CIA constraint binds investment, higher monetary growth unambiguously increases capital and consumption in the long run if the effect through spending on imagining future pleasures is sufficiently large.*

Another class of endogenous time preferences was the one proposed by Uzawa (1968). The Uzawa time preference is in general a function of an individual's consumption. In a one-sector growth model with CIA constraints and the Uzawa time preference, in Appendix we show that steady state is characterized by $f'(k^*) = \rho(c^*)[1 + \varphi_I(\rho(c^*) + \mu)]$ and $c^* = f(k^*)$. Then, no matter whether the time preference rate is increasing or decreasing in consumption, the conventional neutral or negative relationship between money and capital holds.

3.3 The optimum quantity of money

It is interesting to analyze the optimum quantity of money. As the Friedman rule of a zero money growth rate does not hold in the case when spending on imagining future pleasures is not constrained by cash, we focus on this case.

The optimum growth rate of money is determined by differentiating the discounted life-time utility in the long run with respect to the growth rate of money. If we use (11), the effect on welfare is

$$\frac{dU}{d\mu} = \frac{u(c^*)}{\rho(s^*)}\Big[\frac{u'(c^*)}{u(c^*)}\frac{dc^*}{d\mu} + \frac{[-\rho'(s^*)]}{\rho(s^*)}\frac{ds^*}{d\mu}\Big] = \frac{u}{\rho}\Big[\Big(\frac{u'}{u} + \frac{\rho'}{\rho}\frac{\tilde{A}}{\tilde{B}}\Big)\frac{dc^*}{d\mu} + \frac{(\rho')^2}{\rho}\frac{u}{\tilde{B}}\Big]. \tag{16}$$

As the price of spending in patience relative to consumption is reduced, higher monetary growth has a direct positive effect on welfare through higher spending in patience and lower discount rates (second term in the right-hand side of (16)).

Case 1, The CIA constraint does not bind investment

When higher monetary growth shifts locus $\dot{c} = 0$ rightwards more than locus $\dot{k} = 0$, consumption is higher. In this situation, higher monetary growth unambiguously increases the representative agent's welfare. On the other hand, when higher monetary growth shifts locus $\dot{c} = 0$ rightwards less than locus $\dot{k} = 0$, consumption is lower, thus leading to a lower welfare. Then, there is an optimum growth rate of money that balances these two opposing effects. Using (16) and (13), the optimum growth rate of money, μ^*, is characterized by

$$\Big\{\frac{u'(c^*(\mu^*))}{u(c^*(\mu^*))}\Big/\Big[\frac{-\rho'(s^*(\mu^*))}{\rho(s^*(\mu^*))}\Big] - \frac{\tilde{A}}{\tilde{B}}\Big\}\frac{1}{\Delta}[-f''(k^*(\mu^*)) + f'(k^*(\mu^*))\rho'(s^*(\mu^*))] = 1.$$

Case 2, The CIA constraint binds investment

When locus $\dot{c} = 0$ shifts rightward more than locus $\dot{k} = 0$, consumption will increase. In this situation, higher money growth unambiguously increases welfare. On the other hand, when locus $\dot{c} = 0$ is shifted rightward less than locus $\dot{k} = 0$, or when locus $\dot{c} = 0$ is shifted leftwards, consumption is lower. However, as an increase in monetary growth shifts locus $\dot{k} = 0$ rightwards via reducing the price of spending on imagining future pleasures relative to consumption, there is a direct positive effect upon welfare. Then, there is an optimum growth rate of money that balances the two opposing effects. Using (16) and (15), the optimum growth rate of money, μ^*, is determined by

$$[\tfrac{u'(c^*(\mu^*))}{u(s^*(\mu^*))}/(\tfrac{-\rho'(s^*(\mu^*))}{\rho(s^*(\mu^*))})-(\tfrac{\tilde{A}}{\tilde{B}})]\tfrac{1}{\Lambda}\{-f''(k^*(\mu^*))+f'(k^*(\mu^*))[\rho-(\rho')^2(1+2\rho+\mu)]/(\tfrac{-\rho'u}{\tilde{B}})\}=1.$$

To summarize the results,

Proposition 3 *In an optimal growth model with a binding CIA constraint on consumption and a non-binding CIA constraint on spending on imagining future pleasures, either (i) higher monetary growth makes households better off, or (ii) there is an optimum quantity of money.*

4. Concluding remarks

In the standard optimal growth model with money, this paper analyzed the consequence upon savings and thus capital accumulation of endogenous time preferences affected by resources spent on imagining future pleasures along the lines of Becker and Mulligan (1997). Money plays the role of transactions in the way of the cash-in-advanced constraint and consumption is always bound by the constraint. This resulting model is analytically simple, so we simplify the steady-state condition to the standard textbook version with two loci that correspond to the good-market clearance condition and the Keynes-Ramsey condition. Under the correspondence principle, we find that there exists a unique steady state.

We investigate the effects of monetary growth on capital accumulation. If the CIA constraint binds resources spent on imagining future pleasures, we find that the neutral or negative conventional long-run relationship between money and capital holds. By contrast, if the CIA constraint does not bind resources spent on imagining future pleasures, there is a positive or ambiguous long-run relationship between money and capital depending on whether the CIA constraint on investment is not or is bound. Moreover, if the CIA constraint does not bind resources spent on imagining future pleasures, the Friedman rule does not hold.

5 Appendix

In the model with time preferences along the lines of Uzawa (1968), the discount factor is $X_t \equiv \exp[-\int_0^t \rho(c_\tau)d\tau]$. In this model, the steady state is characterized by

$$\frac{f'(k^*)}{1+\varphi_I[\rho(c^*)+\mu]}=\rho(c^*), \tag{A1}$$

$$c^* = f(k^*), \tag{A2}$$

where φ_I =0 if only consumption is constrained by cash and φ_I =1 if consumption and investment are equally constrained by cash. Case 1, the CIA constraint does not bind investment: φ_c=1 and φ_I=0. In this case, the long-run relationship between money and capital is neutral. Case 2, the CIA constraint binds investment: φ_c=φ_I=1. In this case, monetary growth, μ, appears in the left-hand side of (A1). It is clear that a higher monetary growth rate reduces the discounted marginal product of capital in (A1). As a result, the long-run relationship between money and capital is always negative.

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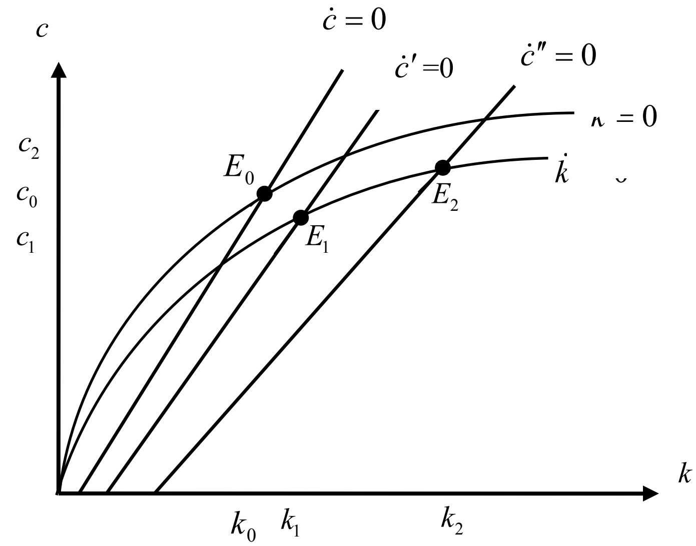

Figure 1. Steady state and comparative-static effects under $\varphi_l=\varphi_s=0$.

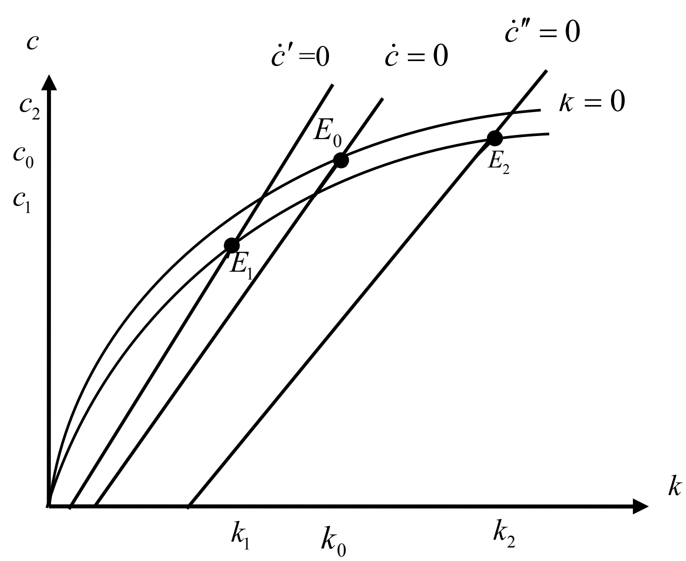

Figure 2. Comparative-static effects under $\varphi_l=1$ and $\varphi_s=0$.

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