The turnover costs and the Solow condition in an efficiency wage model with intertemporal optimization

Chung-cheng Lin, Ching-chong Lai*
Sun Yat-Sen Institute for Social Sciences and Philosophy, Academia Sinica, Nankang, Taipei 11529, Taiwan
Received 13 September 1993; accepted 7 December 1993

Abstract

This paper sets out an intertemporal optimizing model embodying the efficiency wage hypothesis, and examines whether the Solow condition is valid. It is shown that, unless the turnover costs are absent and the voluntary quitting rate is independent from wage offer, the effort–wage elasticity is less than unity. Our result thus can be regarded as a theoretical solution of the Akerlof and Yellen criticism on the efficiency wage theory.

JEL classification: J41

1. Introduction

In recent years the efficiency wage theory has been regarded as a powerful vehicle to explain why involuntary unemployment is persistent in the labor market. Among the literature, Blinder (1988, p. 290) claims that, ‘the simplest, and to me the most appealing, of [the theories addressing involuntary unemployment question] is the efficiency wage model. It also seems to accord best with common sense.’ Blanchard and Fischer (1989, p. 463) also say that, ‘... efficiency wage theory is surely one of the most promising directions of research at this stage.’

A well-known result in the standard efficiency wage model is that the optimizing firm will set its wage offer at the level which the elasticity of work effort with respect to the real wage is unity. This result is dubbed the Solow condition by Akerlof and Yellen (1986). However, in their review of efficiency wage models, Akerlof and Yellen (1986, p. 14) question that an effort–wage elasticity of unity is too high, and propose a simple model with external costs to
illustrate an effort–wage elasticity lower than unity.\textsuperscript{1} The Akerlof and Yellen argument attracts some studies that give a theoretical explanation for the lower elasticity. Schmidt-Sørensen (1990) introduces fixed employment costs and Pisauro (1991) takes into account specific taxes on labor. A common feature of these studies is that their models are static in nature.

In a well-cited survey of labor demand theory, Nickell (1986) states that, ‘workers who walk out of the factory gate on a Friday afternoon will typically return to the same gate on a Monday morning, if not before... The firm’s demand for labour cannot, therefore, be described by a static model.’ In line with this viewpoint, this paper makes a new attempt to examine the effort–wage elasticity in the context of an intertemporal optimizing model.

The rest of the paper is arranged as follows. The intertemporal optimizing model embodying an efficiency wage hypothesis is outlined in section 2. Section 3 investigates the validity of the Solow condition. Finally, section 4 summarizes the main findings of our analysis.

2. The model

As claimed by Malcomson (1981) and Lindbeck and Snower (1989, pp. 62–63), the efficiency wage theory rests on two fundamental assumptions: (i) the firm exerts market power in the wage-setting process; and (ii) the productivity of employees increases as the real wage received by them is increased. Define \( e \) to be the effort per employee, \( n \) the number of employees, and \( \lambda = en \) the firm’s workforce in efficiency units. The firm then has an effort–argumented production function \( f \):

\[
y = f(\lambda) = f(e(w)n) ,
\]

where \( w \) is the wage offer. In Eq. (1), according to the efficiency wage hypothesis, the productivity of employees is specified as a positive function of the wage they receive.

Let \( c \) denote the turnover costs, \( l \) the hired (or fired) employees, \( \delta \) the rate of voluntary quitting, \( r \) the interest rate, and \( n \) total employees. A representative firm operating in the competitive goods market thus faces the following intertemporal optimization problem:

\[
\begin{align*}
\max_{l,w} \int_0^\infty [y - wn - c(l)]e^{-rt}dt , \\
\text{s.t. } n = l - \delta(w)n .
\end{align*}
\]

Equation (2) states that the firm will choose \( l \) and \( w \) to maximize the discounted profit over an infinite horizon. Moreover, in accordance with the Nickell (1986) specification, the turnover cost function has the properties \( c_l \geq 0 \) as \( l \geq 0 \) and \( c_{ll} > 0 \). Equation (3) defines total employees as equal to the difference between the number of hirings (or firings) and the number of hirings.

\textsuperscript{1} Akerlof and Yellen (1986, p. 14) state that ‘the downside risk from shirking labor is usually not limited by the smaller amount of its own labor input: It may include the wastage of scarce opportunities inherent in the job itself, poor use of other current inputs, or, in some cases, the destruction, theft, or wastage of capital assets.’
voluntary quittings. Following the viewpoint of Fallon and Verry (1988, p. 93) and Moutos (1991), we specify that the rate of voluntary quitting will decrease as the firm raises its wage offer (i.e., \( \delta_w < 0 \)). It is worth noting that, without loss of generality, we set the price of goods equal to unity. As a consequence, the real wage is equivalent to the nominal wage.

Substituting Eq. (1) into (2), the current-value Hamiltonian function, \( H \), can be expressed as

\[
H = [ f(e(w)n - wn - c(l)) + \psi[l - \delta(w)n] ,
\]

where \( \psi \) is the costate variable associated with state variable \( n \).

The first-order conditions necessary for this optimization problem are

\[
c_l(l) = \psi ,
\]

\[
e'_w(w)f_c(e(w)n)n = n + \delta_w(w)n\psi ,
\]

\[
\dot{\psi} = [r + \delta(w)]\psi + w - e(w)f_c(e(w)n) ,
\]

\[
\dot{n} = l - \delta(w)n .
\]

Equation (5) indicates that the firm will hire (or fire) labor until the marginal cost of labor is equal to the marginal contribution. Equation (6) is the optimal condition for wage setting. It states that the marginal revenue of adjusting wages equals the marginal cost. Equations (7) and (8) state how the costate variable and state variable will change over time.

3. The effort-wage elasticity

It follows from Eqs. (5), (6), and (7) that

\[
e'_w(w) = \frac{w}{e(w)} - \frac{1 + \delta_w(w)c_l(l)}{1 + [(r + \delta)c_l(l) - \psi]/w} .
\]

We are now in a position to discuss whether the Solow condition (i.e., the elasticity of work effort with respect to the wage is unity) is valid. To save space, we only focus on the long-run situation.

In the steady state, \( \dot{\psi} = \dot{n} = 0 \) holds, and it follows from Eq. (8) that \( \dot{I} = \delta(w)n > 0 \). According to the definition of turnover costs, \( c_l(l) > 0 \) must hold as \( \dot{I} > 0 \). Under such a circumstance, the numerator is less than unity and the denominator is greater than unity in Eq. (9). We thus have

\[
e'_w(w) = \frac{w}{e(w)} < 1 .
\]

Using Eqs. (5) and (6) we have

\[
e'_w(w)f_c(e(w)n) = 1 + \delta_w(w)c_l(l) .
\]

Recalling that \( e'_w(w) > 0 \) and \( f_c(e(w)n) > 0 \), \( 1 + \delta_w(w)c_l(l) > 0 \) should hold unambiguously.
Equation (10) indicates that the wage elasticity of effort is less than unity; the Solow condition thus is no longer valid. This result can be treated as theoretical support for the Akerlof and Yellen (1986) argument.

It is interesting to discuss the following special situations. First, if the turnover costs are absent (i.e. \( c = c_j = 0 \)), Eq. (9) then reduces to

\[
e_w(w) \frac{w}{e(w)} = 1 + \delta_w(w).
\]

(11)

The equilibrium effort-wage is definitely less than unity, and the Solow condition does not hold.

Next, in accordance with Nickell (1986), if the voluntary quitting rate has nothing to do with the wage offer (i.e. \( \delta_w = 0 \)), Eq. (9) reduces to

\[
e_w(w) \frac{w}{e(w)} = \frac{1}{1 + [(r + \delta)c_j(l)].}
\]

(12)

The firm will set its wage offer at the level where the elasticity of work effort with respect to the real wage is less than unity. As a consequence, the Solow condition is invalid.

4. Concluding remarks

A typical result of the standard efficiency wage model is that the elasticity of work effort with respect to wage is unity. This result is called the Solow condition in the efficiency wage literature. This paper sets out an intertemporal optimizing model embodying the efficiency wage hypothesis, and examines whether the Solow condition is valid. It is shown that, unless the turnover costs are absent and the voluntary quitting rate is independent of the wage offer, the effort-wage elasticity is less than unity. Our result thus can be regarded as a plausible way to explain the argument of Akerlof and Yellen (1986).

References